2 LUMPED PARAMETER MODELS

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2.1 INTRODUCTION

A comprehensive description of the lumped-parameter models applicable to the interpretation of environmental tracers in groundwater systems is given. It will be shown that the lumped-parameter models are particularly useful for interpreting the tracer data which were obtained at separate sampling sites, when it is neither possible, nor justified, to use distributed-parameter models, as the latter require more detailed knowledge of the investigated system, which is often unavailable. A more detailed description of the approach and a number of examples can be found in Maloszewski and Zuber (1996) and in other references given further. A user-friendly programme (FLOWPC) for the interpretation of environmental tracer data by several most commonly used models is available from the IAEA.

For a better understanding of the tracer method and the interpretation of the tracer data, several definitions are recalled. Some of these definitions are more or less generally accepted and frequently used (e.g., Gardner and Ely 1967, Levenspiel 1972, Lohman et al. 1972, NEA 1990); whereas remaining are unfortunately used only occasionally. As a consequence of infrequent use of adequate definitions, a lot of misunderstandings occur in literature, especially when radioisotope ages versus water ages are considered, or when mathematical models equivalent to the behaviour of a well-mixed reservoir are used for groundwater systems in which good mixing never occurs. As explained further, some misunderstandings also result from a common identification of tracer ages with water ages in fractured rocks whereas in fact these two physical quantities differ considerably.

The tracer method is a technique for obtaining information about a system or some part of a system by observing the behaviour of a specific substance, the tracer, which has been added (injected) to the system. Environmental tracers are added by natural processes whereas their production is either natural or results from the global activity of man.
An **ideal tracer** is a substance behaving in the system exactly as the traced material, at least as far as the sought parameters are concerned, and which has one property that distinguishes it from the traced material. For an ideal tracer, there should be neither sources nor sinks in the system other than those related to the sought parameters. In practice a substance which has other sources or sinks can also be regarded as suitable tracer, if they can be properly accounted for, or if their influence is negligible within the required accuracy.

A **conservative tracer** is an ideal tracer without sinks (there is no decay, sorption or precipitation).

A **conceptual model** is a qualitative description of a system and its representation (e.g. description of geometry, parameters, initial and boundary conditions) relevant to the intended use of the model.

A **mathematical model** is a mathematical representation of a conceptual model for a physical, chemical, and/or biological system by expressions designed to aid in understanding and/or predicting the behaviour of the system under specified conditions.

In a **lumped-parameter model** (black-box model) spatial variations of parameters are ignored and the system is described by adjustable (fitted) parameters.

**Verification of a mathematical model**, or its computer code, is obtained when it is shown that the model behaves as intended, i.e., that it is a proper mathematical representation of the conceptual model and that the equations are correctly encoded and solved.

**Model calibration** is a process in which the mathematical model assumptions (e.g., type of the model) and parameters are varied to fit the model to observations. Usually, calibration is carried out by a trial-and-error procedure, and it can be quantitatively described by the goodness of fit. Model calibration is a process in which the inverse problem (ill-posed problem) is solved, i.e., from known input-output relations the values of parameters are determined by fitting the model results to experimental data. Sought (fitted, matched) parameters are found in the process of calibration. The direct problem is solved if for known or assumed parameters the output results are calculated (model prediction). In the FLOWPC programme an option is included (when no observations exist) which serves for direct calculations. Testing of hypotheses is performed by comparison of model predictions with experimental data.

**Validation** is a process of obtaining assurance that a model is a correct representation of the process or system for which it is intended. Ideally, validation is obtained if the predictions derived from a calibrated model agree with new observations, preferably for other conditions than those used for calibration (e.g., larger distances and longer times). Contrary to calibration, the validation process is qualitative and based on the modeller’s judgement. In the case of the tracer method the validation is often performed by comparison of the values of found parameters with the values obtainable independently from other methods. In such a case it is perhaps more adequate to state that the model is confirmed, or partially confirmed.
Lumped parameter models

In spite of contradictions expressed by some authors (e.g., Konikow and Bredehoeft 1993), the difference between validation and confirmation is rather verbal, and mainly depends on the definitions used and their understanding (e.g., some authors by the working definition of validation understand the process of calibration).

Partial validation can be defined as validation performed with respect to some properties of a model. For instance, in the modelling of artificial tracer tests or pollutant transport, the dispersion equation usually yields proper solute velocities (i.e., can be validated in that respect), but seldom adequately describes the dispersion process in predictions at much larger distances.

The turnover time \( t_w \); other terms: age of water leaving a system, mean exit age, mean residence time of water, mean transit time, hydraulic age, kinematic age) is usually defined as the ratio of the mobile water volume \( (V_m) \) to the volumetric flow rate \( (Q) \) through the system:

\[
t_w = \frac{V_m}{Q} \tag{2.1}
\]

For vertical flow in the recharge area, especially in the unsaturated zone, \( Q \) in Eq. 2.1 can be expressed by recharge rate \( (I) \):

\[
t_w = \frac{V_m}{I} \tag{2.1a}
\]

If a system can be approximated by unidimensional flow pattern, this definition yields \( t_w = \frac{x}{v_w} \), where \( x \) is the length for which \( t_w \) is determined, and \( v_w \) is the mean water velocity, defined below. Darcy’s velocity \( (v_f) \) is defined as the ratio of \( Q/S \), \( S \) being the cross-section area perpendicular to flow lines. The effective porosity is understood as that in which the water movement takes place (Lohman et al. 1972). Consequently, the mean water velocity \( (v_w) \) is defined as the ratio of Darcy’s velocity to the effective porosity, \( v_w = v_f/n_e \) (other equivalent terms: pore velocity, interstitial velocity, travel velocity, transit velocity). Other definitions of the effective porosity are also common. For instance, it is common to define the effective porosity as that which is effective to a given physical process, e.g., diffusion. Of course, in such cases, the effective porosity differs from that which is directly related to Darcy’s law.

The mean tracer age \( (t_t) \); other terms: mean transit time of tracer, mean travel time of tracer) can be defined as:

\[
t_t = \frac{\int_0^\infty t' C_1(t') \, dt'}{\int_0^\infty C_1(t') \, dt'}
\]

where \( C_1 \) is the tracer concentration observed at the measuring site (the outlet of a system) as the result of an instantaneous injection at the entrance.
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The mean tracer age is equal to the mean water age only if there are no stagnant zones in the system, and the tracer is injected and measured in flux. Flux injection and measurement mean that at both the entrance and outlet the amounts of tracer in particular flow lines are proportional to their volumetric flow rates. That condition is automatically satisfied in natural systems for tracers entering the system with infiltrating water and measured in outflows. However, if sampling is performed at a certain depth of a borehole, that condition may perhaps be satisfied for the sampled flow line, but surely not for the whole system. Radiocarbon most probably does not satisfy in some cases the flux injection because it enters groundwater systems mainly due to the production of CO$_2$ by plant roots. Therefore, its natural injection is not necessarily proportional to the volumetric flow rates. The problem of a proper injection and measurement is more acute in artificial tracing, however, one should be aware that even an ideal environmental tracer may in some cases yield an age which differs from the water age. The problem of stagnant zones, which is of particular importance for fissured rocks, will be discussed further.

Immobile systems are beyond the scope of this work, but for the consistency of age definitions they should mentioned. The water age of an immobile system is usually understood as the time span for which the system has been separated form the atmosphere. In such cases, the radioisotope age of an airborne radioisotope, which has no other sources and sinks than the radioactive decay, can be identified with the age of water. The radioisotope age ($t_a$) is defined by the radioactive decay:

$$C(t_a)/C(0) = \exp(-\lambda t_a)$$  \hspace{1cm} (2.3)

where $C(t_a)$ and $C(0)$ are the actual and initial radioisotope concentrations, respectively, and $\lambda$ is the radioactive decay constant.

Unfortunately, few radioisotope tracers are available for dating both mobile and immobile old groundwater systems. Therefore, for such systems, the accumulation of some decay products is rather used (e.g., $^4$He and $^{41}$Ar). Similarly, the dependence of $\delta^{2}H$ and $\delta^{18}O$ in meteoric waters on the climatic conditions which existed when the recharge took place may supply information on the age of both mobile and immobile systems in terms of geological periods of known climates. Obviously, the ages of immobile systems, or systems which were immobile for some time, should not be interpreted directly in terms of hydraulic parameters.

2.2 BASIC PRINCIPLES OF THE LUMPED-PARAMETER APPROACH FOR CONSTANT FLOW SYSTEMS

In the lumped-parameter approach the groundwater system is treated as a whole and the flow pattern is assumed to be constant. Usually the flow rate through the system is also assumed to be constant because variations in the flow rate through the system and changes in its volume were shown to be negligible when distinctly shorter than the mean age (Zuber et al. 1986).
Lumped parameter models

Detailed description of the lumped-parameter approach can be found in a number of papers (Amin and Campana 1996, Maloszewski and Zuber 1982, 1996, Zuber 1986). For the most commonly applied models, the schematic presentation of underground water systems is given in Fig.2.1, and the relation between the variable input \((C_{in})\) and output \((C)\) concentrations is:

\[
C(t) = \int_{-\infty}^{t} C_{in}(t') g(t - t') \exp[-\lambda(t - t')] dt'
\]  

(2.4)

An equivalent form is:

\[
C(t) = \int_{0}^{\infty} C_{in}(t - t') g(t') \exp(-\lambda t') dt'
\]  

(2.5)

where \(t'\) is time of entry, \(t-t'\) is the transit time, and the \(g(t-t')\) function is called the response function, which describes the output distribution of a conservative substance (tracer) injected instantaneously at the inlet, and the integration from or to infinity means that the whole input curve \((C_{in})\) has to be included to get a correct output concentration \((C_{out} \text{ in Fig.2.1})\). Other common terms for the \(g(t)\) function are: transit time distribution, residence time distribution (RTD) of tracer, tracer age distribution, and weighting function. As discussed further the RTD of tracer is not necessarily equivalent to the RTD of the investigated fluid.

![Schematic presentations of groundwater systems in the lumped-parameter approach.](image)

Sometimes it is convenient to express Eq.2.4 or 2.5, as a sum of two convolution integrals, or two input functions. The most common case is that one component is either free of tracer, or the tracer concentration can be regarded as being constant. As shown further in some cases such approach is justified by independent information, which defines the fraction of tracer
free (or constant) component. In other cases, the fraction of tracer free component is used as an additional fitting parameter. In the FLOWPC programme an option is included for an older fraction of water ($\beta$) which either contains a constant tracer concentration, or is free of tracer.

The response function represents the normalised output concentration, i.e., the concentration divided by the injected mass, which results from an instantaneous injection of a conservative tracer at the inlet. It is impossible to determine the response functions of groundwater systems experimentally. Therefore, functions known from other fields of science are used. The response function, which is either chosen by the modeller, or found by calibration, defines the type of the model whereas the parameters of the model are found by calibration. Calibration means finding a good fit of concentrations calculated by Eq.2.4 or 2.5, to the experimental data, for a known or estimated input function (time record of $C_{in}$). Usually, when referring to a model of good fitting, the type of the model and the values of its parameters are reported.

In chemical engineering, the response function is often identified with the $E(t)$ function which describes the exit time distribution (or the residence time distribution, RTD) of the investigated fluid. By definition, the mean value of the $E$-function is equal the volume of the system divided by the volumetric flow rate, and is equal to the mean exit age of the fluid (i.e., to the mean residence time of the fluid). In the case of groundwater systems, the response function, which describes the exit distribution of the tracer, can be identified with the exit time distribution of water flow only under favourable conditions, which exist when there are no stagnant zones in the investigated system. When stagnant zones are present, even an ideal tracer may be delayed in respect to the water flow due to diffusion exchange between mobile and immobile zones. That problem will be discussed further in more detail.

### 2.3 MODELS

#### 2.3.1 THE PISTON FLOW MODEL

In the piston flow model (PFM) approximation the flow lines are assumed to have the same transit time, and the hydrodynamic dispersion and diffusion are negligible. Therefore, the tracer moves from the recharge area as if it was in a can. The response function is given by the well-known Dirac delta function, $g(t') = \delta(t' - t_t)$, which inserted into Eq.2.4 yields:

$$C(t) = C_{in} (t - t_t) \exp(-\lambda t_t)$$

Eq.2.6 means that for the PFM the output concentration at a given time is equal to the input concentration at the time $t_t$ earlier, and changed only by the radioactive decay during the time span $t_t$. The transit time of the tracer ($t_t$) is the only parameter of the model, and the shape of the input concentration function is followed by the output concentration. It will be shown further that the PFM is applicable only to systems with constant tracer input. The most commonly used are the three models considered in the following sections.
2.3.2 THE EXPONENTIAL MODEL

In the exponential model (EM) approximation, the flow lines are assumed to have the exponential distribution of transit times, i.e., the shortest line has the theoretical transit time equal to zero, and the longest line has the transit time equal to infinity. It is assumed that there is no exchange of tracer between the flow lines, and then the following response function is obtained:

\[ g(t') = t_i^{-1} \exp(-t'/t_i) \]  

(2.7)

This relationship is mathematically equivalent to the response function of a well-mixed reservoir, known in chemical engineering. Some investigators reject the EM because in principle no good mixing may occur in aquifers whereas others claim the applicability of the EM to be indicative of good mixing conditions in a groundwater system. Both opinions are wrong because, as mentioned, the model is based on an assumption of no exchange (mixing) of tracer between particular flow lines (Eriksson 1958, Maloszewski and Zuber 1982, 1996, Zuber 1986). If tracer exchanges between the flow lines with an exponential distribution of travel times, its distribution will tend to be described by the dispersion model discussed further. Expected effects are similar to the effects shown for tracer distributions in a laminar flow in a capillary (Maloszewski and Zuber 1996, Fig.A.1). Understanding of all effects which may lead to differences between the tracer response function and the distribution of flow lines is very useful for a proper interpretation of tracer data.

For the exponential model approximation, mixing occurs only at the sampling site (spring, abstraction well, stream or river). In general, groundwater systems are never well mixed, and they may contain mixed waters only if two, or more, water flows meet, or in transition zones where the hydrodynamic dispersion and diffusion play an important role.

Similarly to the PFM, the mean transit time (age) of tracer is the only parameter of the EM, which unambiguously defines the whole transit time distribution (Fig.2.2). Therefore, when reporting the tracer age, the model used, or the response function should also be given. The response function of the EM shows the model to be inapplicable to systems in which infinitesimally short flow lines do not exist. In other words, the EM is not applicable when samples are taken well below ground surface, e.g., from boreholes screened at large depths, mines, and artesian outflows. Experience shows that very often, due to a too short record of the tracer data, the exponential model yields a good fit though its use is not justified. In such cases, it should be remembered that the obtained result is a rough approximation, and the real situation can be described more adequately by one of the models discussed in the next sections. Evidently in such cases no unique solution is available.

The EM and other models with a broad distribution of ages describe situations in which only the shortest flow lines supply to the sampling site a decaying tracer (e.g., tritium or \(^3\)H), or a non-decaying tracer with the input function starting from zero (e.g., freons). Therefore, in the
case of a large value of the mean tracer age, no information is in fact available on the part of the system with flow lines without tracer. In consequence, the knowledge on the whole system is derived from the information available for its fraction with low ages (short transit times). In other words, the remaining part of the system, which does not supply tracer to the sampling site, may have a quite different distribution of flow lines than that assumed in the model.

2.3.3 THE COMBINED EXPONENTIAL-PISTON FLOW MODEL

In the exponential-piston flow model (EPM) approximation, the aquifer is assumed to consist of two parts in line, one with the exponential distribution of transit times, and another with the distribution approximated by the piston flow. The response function of the EPM is:

\[
g(t') = \left( \frac{\eta}{t_t} \right) \exp\left( -\frac{\eta t'}{t_t} + \eta - 1 \right) \quad \text{for} \quad t' \geq t_t(1 - \eta - 1) \\
= 0 \quad \text{for} \quad t' < t_t(1 - \eta - 1)
\]

(2.8)

where \( \eta \) is the ratio of the total volume to the volume with the exponential distribution of transit times, i.e., \( \eta = 1 \) means the exponential flow model. The response function is independent of the sequence in which EM and EPM are combined. The EPM has two fitting (sought) parameters, i.e., \( t_t \) and \( \eta \). Examples of the response functions are shown in Fig.2.3. For low values of \( \eta \) that model is close to the EM whereas for large values of \( \eta \) it is somewhat similar to the dispersion model with a low value of the apparent dispersion parameter. That model is somewhat more realistic than the exponential model because it allows for the existence of a delay of the shortest flow lines.

Fig.2.2 Examples of response functions of the exponential model (EM).
2.3.4 THE DISPERSION MODEL

In the dispersion model (DM), the following uni-dimensional solution to the dispersion equation for a semi-infinite medium is used as the response function (Kreft and Zuber 1978):

\[
g(t') = (4\pi P_D t' / t_i)^{-1/2} t'^{-1} \exp[-(1 - t'/t_i)^2 / (4P_D t'/t_i)]
\]

where \( P_D \) is the apparent dispersion parameter (reciprocal of the Peclet number), which is unrelated to the common dispersivity of groundwater systems, and mainly depends on the distribution of travel times. The higher the value of the dispersion parameter, the wider and the more asymmetrical the distribution of the travel times. Examples of the response functions are shown in Fig.2.4, for the \( P_D \) values of 0.05 and 0.5, which bracket the most common situations. However, in some published case studies, the interpretation of \(^3\)H records yielded the values of the dispersion parameter as high as 2.5 (Małoszewski and Zuber 1982, Zuber 1986, Zuber et al. 2000) whereas lower values than 0.05 are rather unexpected. Some authors, instead of Eq.2.9, apply the solution to the dispersion equation for an infinite medium, which is inadequate, especially in cases of high values of the dispersion parameter (Kreft and Zuber 1978, Małoszewski and Zuber 1982, Zuber 1986).
2.4 CASES OF CONSTANT TRACER INPUT

The lumped-parameter approach is applicable for any tracer with variable input. It is also applicable for radioisotopes with a constant input concentration. However, in the latter case, a unique interpretation is possible only for models with a single sought parameter, because two unknown values cannot be found from a single equation. The most typical solutions to Eq.2.4 for a constant input \( C_0 \) are:

\[
C = C_0 \exp(-\lambda t_t) \quad \text{for PFM} \tag{2.10}
\]
\[
C = \frac{C_0}{1 + \lambda t_t} \quad \text{for EM} \tag{2.11}
\]
\[
C = C_0 \exp\left\{-(2P_D)^{-1} \times [1 - (1 + 4\lambda P_D t_t)^{1/2}]\right\} \quad \text{for DM} \tag{2.12}
\]

The investigators who apply Eq.2.10 for dating and understand its limitations often use the term *apparent tracer age* for the PFM tracer age (e.g., Solomon and Cook 1996). Eqs.10-12 demonstrate that the radioisotope age found from Eq.2.3 is a correct representation of the mean tracer age \( t_t \) only for the PFM, which, as mentioned, is equal to the mean water age \( t_w \) under favourable conditions. In spite of a number of works, in which differences between the ages resulting from particular models and the radioisotope age were shown, it is a quite common mistake to identify the radioisotope age, given by Eq.2.3 with the mean tracer age. It is especially common in the case of \(^{14}\)C measurements of samples taken from systems with either an unknown flow pattern, or with a flow pattern described by evidently another model than the PFM. For a graphical presentation of \( C/C_0 \) values yielded by different models see Fig.2.2 in Maloszewski and Zuber (1982), Fig.4 in Maloszewski and Zuber (1986), or Fig.27 in Zuber (1986). As it is impossible to get a unique solution if two or more sought parameters
Lumped parameter models

are used, the age cannot be found from \( C/C_0 \). Therefore, if no other information is available, at least bracket age values yielded by the PFM and EM should be given.

In general, when the flow pattern is unknown, the interpretation should be performed for different models, and the ages obtained can be regarded as brackets of the real values. That problem is serious only for large relative ages. As mentioned above, it can easily be shown that if the tracer age is lower than the half-life of the radioisotope \( (t_s < T_{1/2} = 0.693/\lambda) \), all the models yield close values of ages independently of the assumed flow pattern (Małoszewski and Zuber 1982, 1996, Zuber 1986).

In the case of constant tracer input, the age can be found from a single measurement. The only way to validate, or confirm, a model is to compare its results with other independent data, if available. However, the environmental tracers are particularly useful in investigations of little known systems, where no other data are available for comparisons. Therefore, the general validity of particular models is judged on the basis of vast literature of the subject.

2.5 CASES OF VARIABLE TRACER INPUT

2.5.1 THE TRITIUM METHOD

Tritium \((^3\text{H}; T_{1/2} = 12.43 \text{ years})\) concentrations in atmospheric waters were constant and very low \((5-10 \text{ TU})\) before the hydrogen bomb tests, which started in 1952. The highest concentrations, up to about 6000 TU during summer months in the northern hemisphere, were observed in 1962-63. Since then, the atmospheric concentrations exponentially decrease reaching 10-20 TU in late 90-90-ties, with characteristic maximum contents in spring and summer months and minimum contents in autumn and winter months. High \(^3\text{H}\) concentrations in the precipitation of early sixties offer a unique opportunity for dating young groundwater systems in a relatively wide range of ages. In the case of piston flow, or systems with very low dispersivity, the \(^3\text{H}\) method yields ages of waters recharged after 1952 because for older waters the present concentrations are close to zero. However, for systems approximated by the exponential model, even the ages of the order of 1000 years can be determined. For typical dispersive systems, the ages of 100-200 years are often observed. Therefore, the environmental \(^3\text{H}\) is still the most useful tracer for dating young waters, especially in the northern hemisphere. Unfortunately, in the tropics the atmospheric \(^3\text{H}\) peak was much lower, and in the southern hemisphere its was even more damped and delayed (Gat 1980), which makes the dating more difficult or even impossible.

Seasonal variations of the \(^3\text{H}\) concentration in precipitation as well as variations in the precipitation and infiltration rates cause difficulties in the estimation of the input function, i.e., \( C_{\text{in}}(t) \). For each calendar year the value of the input can be expressed as:
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\[ C_{in} = \sum_{i=1}^{12} C_i I_i / \sum_{i=1}^{12} I_i = \sum_{i=1}^{12} C_i \alpha_i P_i / \sum_{i=1}^{12} \alpha_i P_i \]  

(2.13)

where \( C_i, P_i \) and \( I_i \) are the \(^3\)H concentration in precipitation, precipitation rate, and infiltration rate for the \( i \)-th month, respectively. The infiltration coefficient \( (\alpha_i = I_i/P_i) \) represents the fraction of precipitation which enters the groundwater system in the \( i \)-th month. The record of \( C_{in} \) values, calculated for each year prior to the latest sampling date, represents the input function. For the interpretation of old \(^3\)H data, the record of \( C_{in} \) should include constant \( C_{in} \) values observed prior to the beginning of the rise in 1954 caused by hydrogen bomb test in the atmosphere, in other cases the calculations of the input function can be started since 1954.

![Graph](image)

**Fig. 2.5** \(^3\)H concentration (logarithmic scale) in precipitation \((\alpha = 1.0)\) and input functions for \(\alpha = 0.7\) and 0.0 calculated for Cracow (Poland) station. Shown for comparison is the input function for Świeradów station (Sudetes Mts., Poland), and one of the input functions corrected for the radioactive decay to 1998.

Some authors tried to estimate the infiltration coefficients for particular months (Andersen and Sevel 1974, Przewłocki 1975). In general, these coefficients usually remain unknown, and approximations have to be applied. If it is assumed that the infiltration coefficient in the summer months \((\alpha_s)\) of each year is the same fraction of the infiltration coefficient in the winter month \((\alpha_w)\), i.e., \(\alpha = \alpha_s/\alpha_w\), Eq. 2.13 simplifies into Eq. 2.14 (Grabczak et al. 1984).

\[ C_{in} = [(\alpha \sum_{i=4}^{9} C_i P_i)_s + (\sum_{i=10}^{12} C_i P_i)_w] / [(\alpha \sum_{i=4}^{9} P_i)_s + (\sum_{i=10}^{12} P_i)_w] \]  

(2.14)
In the northern hemisphere the summer months are from April to September (from the fourth to the ninth month), and the winter months are from October to March (from the tenths to the third month of the next calendar year). Monthly precipitation amounts should be taken from the nearest meteorological station, and the $^3$H data should be taken from the nearest station of the IAEA network. As complete records are usually unavailable, the record of a given station has to be completed by extrapolating correlation with another station for which a complete record exists, either in original or correlated form (Davis et al. 1967). Experience shows that in a rough approximation, the input functions from distant stations, with climatic conditions similar to those of the investigated area, can be used, especially for ages larger than about 20 years. $^3$H concentrations in precipitation and examples of the input functions are given in Figs.2.5 and 2.6. The logarithmic scale of Fig.2.5 gives a better idea about the concentrations which have been observed since 1954, and the long tail of the $^3$H pulse whereas the linear scale of Fig.2.6 serves for a better understanding the pulse character of the $^3$H input. That pulse character and low values of the tail gave reasons to opinions that the $^3$H method would be of little use in near future. However, it seems that the $^3$H method will remain the best method for dating young waters for two decades at least. It is also evident that for large values of $\alpha$ no drastic changes in the input function are observed.

It is well known that under moderate climatic conditions the recharge takes place mainly in winter months. Therefore, in some early publications the $\alpha$ coefficient was assumed to be equal to zero or 0.05. However, the isotopic composition of shallow groundwaters is usually equal, or close, to the yearly mean weighted isotopic composition of precipitation, even in areas of prevailing potential evapotranspiration over precipitation in summer months. It means that in the summer months the evapotranspiration partly removes water stored in the unsaturated zone both in the summer and winter months. In consequence, the remaining
water, which reaches the groundwater table, represents the winter and summer precipitation. When local precipitation and isotope data exists, or if they are available from a nearby station, the value of the $\alpha$ coefficient can be estimated from Eq.2.15.

$$\alpha = \left[ \frac{\left( \sum_{i=10}^{3} P_i \delta_i \right)_w - \delta \left( \sum_{i=10}^{3} P_i \right)_w }{\left[ \delta \left( \sum_{i=4}^{9} P_i \right)_s - \left( \sum_{i=4}^{9} P_i \delta_i \right)_s \right]} \right]$$

(2.15)

In that equation $\delta_w$, $\delta_w$ are the stable isotope compositions of the precipitation in the summer months and winter months, respectively; and $\delta$ is the mean isotopic composition of local groundwater ($\delta^{18}O$ or $\delta^2H$) (Grabczak et al. 1984). Eq.2.15 is useful if sufficiently long (a few years) records of the isotopic composition and precipitation rates are available. However, for moderate and humid tropical climates, the $\alpha$ coefficient is commonly within the range of 0.4-0.8, and experience shows that within this range the accuracy of modelling only slightly depends on the assumed $\alpha$ value, if the ages are greater than 10-20 years. In general, if the input function is not found independently, the $\alpha$ coefficient is either arbitrarily chosen by the modeller, or tacitly used as a hidden fitting (sought) parameter. As mentioned, the larger the number of sought parameters, the lower the reliability of modelling. Therefore, the number of sought parameters should be kept as low as possible. In any case, the method used for the calculation of the input function should also be reported. It is a common mistake to assume $\alpha = 0$ on the basis of conventional hydrological observations, which indicate the lack of net recharge in some areas during summer months, because it does not mean the lack of the summer $^3$H in recharging water, as mentioned above.

### 2.5.2 THE $^3$H-$^3$He METHOD

$^3$H concentrations in the atmosphere are now much lower than during the bomb test peak and they still decrease, which cause the $^3$H method to be less useful in near future than in the last four decades. In consequence, other tracer methods are considered as potential tools, which may either replace the $^3$H method or prolong its applicability (e.g., Plummer et al. 1993). As $^3$H decays to $^3$He, the measurements of the tritiogenic $^3$He accumulated in groundwater systems potentially prolong the dating range resulting from the $^3$H peak (Maloszewski and Zuber 1983). In the $^3$H-$^3$He method the $^3$He to $^3$H ratio is usually considered, which for the PFM yields a well-known formula in which the tracer age is independent of the input (Torgersen et al. 1979):

$$t_i = \lambda_{T}^{-1} \ln \left[ 1 + \frac{^3\text{He}_T}{^3\text{H}} \right]$$

(2.16)

where $\lambda_T$ is the radioactive decay constant for $^3$H ($1/\lambda_T = T_{1/2}/\ln2 = 17.9$ a), $^3$H is the $^3$H content, and $^3\text{He}_T$ is the tritiogenic $^3$He content expressed in $^3$H units (for $^3$He expressed in ml STP of gas per gram of water, the factor is $4.01 \times 10^{14}$ to obtain the $^3$He content in TU).
Unfortunately, Eq. 2.16 is not applicable to other flow models. If Eq. 2.5 is used for the calculation of the theoretical $^3$H output function, the following equation should be used or the daughter $^3$He theoretical output (Małoszewski and Zuber 1983):

$$C_{He} = \int_{0}^{\infty} C_{Tim}(t-t')g(t')[1-\exp(-\lambda_{t}t')]dt$$

(2.17)

where $C_{Tim}$ is the $^3$H input function, and $C_{He}$ is the helium concentration expressed in the same units as in Eq. 2.16.

Several recent studies showed the applicability of Eq. 2.16 for vertical transport through the unsaturated or saturated zone, where samples are taken at different depths of a chosen profile, and the dispersivity is negligible. Then, the $^3$H-$^3$He method in the PFM approximation is advantageous to the $^3$H method because only several samples taken at different depths close to the surface supply the same information as the $^3$H peak and allow to determine recharge rate (Eq. 2.1a) as shown by Cook and Solomon (1997). That is especially important as in most cases the $^3$H peak, which corresponds to the atmospheric peak in 1963, has disappeared, or is preserved in vertical profiles only under exceptionally favourable conditions, and at large depths. The method can also be used in horizontal flow in the saturated zone, if the particular flow lines are observed with the aid of multi-lever samplers. In the latter case the $^3$H-$^3$He method has been shown to be particularly useful to calibrate flow and transport models in shallow aquifers. As mentioned, another advantage of the method is its potentially longer applicability in near future in comparison with the $^3$H method.

Specific limitations of the $^3$He method result from the need to separate the tritiogenic helium from helium originating from other sources (atmospheric solubility, excess air and radiogenic production) as discussed in detail by Torgersen et al. (1979), Weise and Moser (1987), and Schlosser et al. (1989). For the PFM approximation, age uncertainties caused by these sources, and by fast diffusion of $^3$He in comparison with the diffusion of $^3$H$^1$HO, were shortly reviewed by Solomon et al. (1998).

Other difficulties are common to all gaseous tracers and they are mainly related to possible escapes or gains by enhanced diffusion when water is in contact with air in the unsaturated zone or in karstic channels. For instance, Grabczak et al. (1982) determined the models and $^3$H ages for withdrawal wells exploiting an unconfined aquifer with thick loess and sandy covers, and for several karstic springs. In all the cases the concentrations of $^{85}$Kr, $^3$He and freon-11 (CCl$_3$F) were in disagreement with the values expected on the basis of the $^3$H models. These disagreements were explained as diffusion losses or gains caused by sharp differences in concentration between water and air either in the unsaturated zone of the recharge areas or in channels partly filled with water near the outflows from a karstic aquifer. In the case of $^3$H, the age is counted from the moment of recharge at the surface whereas for gas tracers it starts rather at the water table (Solomon et al. 1993, 1998), which makes...
additional difficulty in other applications than the observations of vertical profiles for recharge studies.

Fig. 2.7 Specific activity of $^{85}$Kr in the air of the northern hemisphere (Cook and Solomon 1997, Sartorius 1998, and CGGC 1999), directly applicable as the input function.

2.5.3 THE KRYPTON-85 METHOD

The presence of radioactive $^{85}$Kr ($T_{1/2} = 10.76$ years) in the atmosphere results from emissions from nuclear power stations and plutonium production for military purposes. In spite of large spatial and temporal variations, the input function based on yearly averages is quite smooth as shown in Fig. 2.7 for the northern hemisphere. For the southern hemisphere, the specific activity is about 0.2 Bq/m$^3$ lower (Sartorius 1993). The $^{85}$Kr concentration is expressed in units of the specific activity, and, therefore, it is independent of the krypton solubility in water, and of the possible excess of air in water, which is related to a common effect of incorporation of air bubbles in the recharge area. The $^{85}$Kr method was initially hoped to replace the $^3$H method in near future. However, serious limitations result from large samples required due to low solubility of Kr and low concentrations of $^{85}$Kr, and possible excess or deficit of $^{85}$Kr caused by exchange with the atmosphere, especially in karstic channels and thick unsaturated zones, similarly to the discussed earlier $^3$He tracer. In spite of these limitations the krypton-85 method is probably the most promising replacement of the $^3$H method in future. Other potential gaseous tracers are discussed further.

Depth profiles for vertical flow or multi-level samplers make the method useful for studies of recharge rates (Cook and Solomon 1997). However, for the typical applications of the lumped parameter models (interpretation of data obtained in abstraction well and springs), the solutions of the direct problem, i.e., the calculations of the output concentrations show a need
Lumped parameter models

of prolonged records of sampling (Małoszewski and Zuber 1983). For short tracer ages, say, up to about 5 to 10 years, the differences between particular models are slight, similarly to the constant tracer inputs. For larger ages, the differences are not negligible.

2.5.4 THE CARBON-14 METHOD

Usually the $^{14}$C content is not measured in young waters in which $^{3}$H is present unless mixing of components having distinctly different ages is investigated. However, in principle, due to a distinct bomb peak of $^{14}$C concentration, the lumped-parameter approach for variable input can be applied. A high cost of $^{14}$C analyses and a low accuracy related to the problem of the so-called initial carbon content make that approach impractical. However, it is suggested that when the $^{14}$C data are available, the lumped-parameter approach can be used to check if they are consistent with the results obtained from the $^{3}$H modelling.

2.5.5 THE OXYGEN-18 AND DEUTERIUM METHOD

Seasonal variations of $\delta^{18}$O and $\delta^{2}$H in precipitation are under favourable conditions observed in outlets of small catchments with the mean ages up to about 4 years (a common definition of a small catchment is that with the surface area up to 100 km$^{2}$ (Buttle, 1998). Due to a strong damping of the seasonal input variations in outflows, a frequent sampling over several years is usually required both at the input and the outlet. The input data should be taken from a local precipitation collector and the outlet data form a chosen drainage site, i.e., a spring or stream draining the investigated retention basin (Bergman et al. 1986, Małoszewski et al. 1992).

$$\delta_i(t) = \bar{\delta} + \alpha \frac{\sum P \delta_i}{P / n}$$

where $\bar{\delta}$ is the mean input which must be equal to the mean output of $\delta^{18}$O or $\delta^{2}$H values and $n$ is the number of months (or weeks, or two-weeks periods, because in that method a shorter time unit is preferable) for which the observations are available. When the information on $\alpha_i$ is not available, it can be replaced by $\alpha$, similarly to the $^{3}$H method. Then $\alpha$ is either calculated from Eq.2.15, or assumed, and appears as a coefficient for the precipitation of the summer months whereas $\alpha = 1$ is put for the winter months.

The stable isotope method is also useful for determining the fraction of river, or lake, water flowing to pumping wells near rivers (lakes), and the travel time of that water from the river (lake) to the well, if the isotopic composition of the river (lake) water sufficiently varies seasonally. The isotopic composition in the pumping well is the mixture of the river and groundwater (Stichler et al. 1986, Hötzl et al. 1989, Małoszewski et al. 1990):

$$\delta_w(t) = p\delta_r(t) + (1-p)\delta_g(t)$$

21
where $p$ is the fraction of the river water and subscripts $w$, $r$ and $g$ stay for the pumped, river and local groundwater, respectively. The value of $p$ can be found by rearranging Eq.2.19 and using the mean isotopic compositions of particular components:

$$p = \frac{(\delta_w - \delta_g)}{(\delta_r - \delta_g)} \quad (2.20)$$

The isotopic composition of local groundwater ($\delta_g$) is either constant or only slightly varies in comparison with the isotopic composition of river water ($\delta_r$). In consequence, the travel time from river to the withdrawal well is found by fitting Eq.2.21 whereas the fraction of the river water is obtained from Eq.2.20 (Stichler et al. 1986, Hötzl et al. 1989, Małoszewski et al. 1990).

$$\delta_w(t) = p \int_0^\infty \delta_r(t - t') g(t') \, dt' + (1 - p) \delta_g \quad (2.21)$$

The stable isotope method used for small retention basins or bank filtration usually requires a frequent sampling, which makes it costly. Therefore, in the case of small retention basins its applicability is limited to research purposes. In the case of bank filtration, the method is undoubtedly cheaper than a number of drilled wells needed to obtain data for construction of a numerical flow and transport model.

2.5.6 OTHER POTENTIAL METHODS

Among other environmental tracers with variable input the most promising for age determinations of young waters are freons (chlorofluorocarbons), particularly freon-12 ($\text{CCl}_2\text{F}_2$), and sulphur hexafluoride ($\text{SF}_6$) which has been shown to be a good atmospheric tracer. Their input functions monotonically increase due to the global contamination of the atmosphere by industry (Fig.2.8). In the southern-hemisphere their concentrations are somewhat lower. Freons enter groundwater systems similarly to other gases with infiltrating water in which they are dissolved in low concentrations. As mentioned, exchange with the air in the unsaturated zone makes the input function less accurately defined than for the $^3\text{H}$. Under extremely favourable conditions (low filtration rate and high diffusion coefficient in the unsaturated zone), the response function should probably start at the water table. The use of freons is also limited due to sorption effects, which are still little known. Another difficulty results from the dependence of the input function on their solubility, i.e., on the pressure and temperature at the recharge area, which is especially serious when the altitude of the recharge area remains unknown. However, the most serious difficulties are related to possible local contamination of shallow groundwaters by industry, and legal and illegal disposal sites (e.g., disposal of refrigerators into sinkholes in karstic areas). Therefore, chlorofluorocarbons are more commonly used to observe the contaminant transport in groundwater systems, and to calibrate numerical transport models, than to determine the age of water. Due to stripping effects, all gaseous tracers are not applicable in investigations of waters rich in CO$_2$ and CH$_4$. 

22
Increased $^3$H concentrations in groundwaters is a temporary phenomenon due to a short half-life of that radioisotope and a short duration of the atmospheric peak. Theoretically, the atmospheric peak of bomb produced $^{36}$Cl, with half-life of about $3.01 \times 10^5$ years, should be an ideal tracer for relatively young groundwaters. However, the spatial differences in peak concentrations make this tracer difficult to apply in a similar way to $^3$H. An exception was for early recharge rate studies where the position of the peak in vertical profiles was measured and interpreted by the PFM approximation (see Bentley et al. 1986 for a review).

2.6 EXAMPLES OF $^3$H AGE DETERMINATIONS

Examples of $^3$H age determinations for relatively long records of $^3$H data can be found in references given earlier whereas in Figs.2.9 and 2.10 two other examples are given after Zuber and Ciężkowski (in press) who gave a number of examples with short records of data. In the first case a large number of models can be fitted whereas in the second case an infinite number of models can be fitted, considering the accuracy range of the experimental data. However, the models shown are not inconsistent because if a given model yields a lower age of the $^3$H component, a larger fraction ($\beta$) of the $^3$H-free water is obtained. The total mean ages are given by models with $\beta = 0$. There is no doubt that for the spring in Szczawina the
Chapter 2

total mean age is of the order of 150 years and for the spring in Łomnica Nowa this age is of the order of 1 ka. In both cases the final selection of the model was performed on the basis of the geology of the area and isotopic altitude effect as discussed in next section.

2.7 DETERMINATION OF HYDROGEOLOGIC PARAMETERS FROM TRACER AGES

Principles of the interpretation of $^3$H data, especially in combinations with other environmental tracer data, can be found in a number of textbooks, manuals, and reports. However, the hydrologic meaning of the tracer age in double porosity rocks (fractured rocks), or triple porosity rocks (karstic rocks) differs from that in granular rocks where it is related directly to the flow rate. The difference in the meaning of the tracer age between single porosity and double porosity rocks is schematically shown in Fig. 2.11. For fractured rocks, due to diffusion exchange between the mobile water in fractures and stagnant or quasi stagnant water in the micropores of matrix, the tracer transport at large scales can be regarded as if it were flowing through the total open porosity (Neretnieks 1980, Maloszewski and Zuber 1985). Unfortunately, that problem is tacitly omitted in a number of research papers and textbooks. Therefore, basic formulas relating the mean tracer ages obtained from lumped-parameter models with hydrologic parameters are recalled below. However, it should be remembered that these simple relations for the fractured rocks are of approximate character, and they are valid only at large scales and for dense fracture networks.

![Fig. 2.9](image)

$^3$H data and fitted models, a spring in Szczawina, Sudetes Mts., Poland.
Lumped parameter models

Fig. 2.10 $^3$H data and fitted models, a spring in Łomnica Nowa, Sudetes Mts., Poland.

The volume of water ($V_w$) in the part of a given system discharged by a spring is given as:

$$V_w = Q \times t_i$$  \hspace{1cm} (2.22)

where $Q$ is the outflow rate. That volume of water in granular systems is practically equal to the volume of mobile water because fraction of water in the micropores of grains is negligibly low. For fractured rocks, that volume is equal to the total volume (mobile water in fractures and stagnant or quasi stagnant water in matrix (Fig.2.11). Consequently, for a single porosity rock, the rock volume ($V_r$) occupied by $V_w$ is given as:

$$V_r = V_w/n_e$$  \hspace{1cm} (2.23)

where $n_e$ is the effective porosity, which is close to the open porosity and total porosity ($n$). For fractured rocks, the following equation applies:

$$V_r = V_w/(n_f + n_p) \cong V_w/n_p$$  \hspace{1cm} (2.23a)

where $n_f$ and $n_p$ are the fracture and matrix porosities, respectively. The approximate form of Eq.2.23a is the result of the fracture porosity being usually negligibly low in comparison with the matrix porosity ($n_f << n_p$). A similar equation applies for triple porosity rocks (karstic-fractured-porous), where the karstic porosity is usually low in comparison with the fracture porosity (Zuber and Motyka 1998). The approximate form of Eq.2.23a and the following similar approximations are of great practical importance because the fracture porosity (plus karstic porosity in triple porosity rocks) usually remains unknown whereas $n_p$ is easily
measurable on rock samples (taken from unweathered rock at the outcrops, or from drill cores). Matrix porosity based on literature data can be used when no samples are available. If the dimensions of the investigated system are known from the geological map and cross-sections, the rock volume is also known, and it can serve for the verification of the age by comparison with the volume found from Eq.2.23a.

![Diagram of tracer transport in fractured rocks](Image)

**Fig.2.11** Schematic presentation of the tracer transport in fractured rocks at large scales when the tracer is able to penetrate fully into the stagnant water in the matrix.

When the mean distance \( x \) from recharge area to the sampling site is known, the following relation applies for single porosity rocks:

\[
T_t = T_w = x/v_w = x/v_t \tag{2.25}
\]

which means that the tracer age (travel time) and velocity are equal to those of water.

For fractured rocks (double porosity), instead of Eq.2.25 the following relations are applicable (Fig.2.11):

\[
T_t = x/v_t = T_w(n_f + n_p)/n_f = (x/v_w)(n_f + n_p)/n_f \tag{2.25a}
\]

In consequence, the tracer travel time is \( 1 + n_p/n_f \) times longer than the travel time of water (i.e., tracer velocity is \( 1 + n_p/n_f \) times slower than water velocity). The fracture porosity is difficult to estimate, and, therefore, if the tracer velocity is known, the water velocity remains unknown, and vice versa, if the water velocity is known, the tracer velocity remains unknown.
For single porosity rocks, Darcy’s velocity \( (v_f) \) is related to water and tracer velocity by effective porosity:

\[
v_f = n_e v_w = n_e v_t \tag{2.26}
\]

For fractured rocks, Darcy’s velocity is related in a good approximation to water velocity by fracture porosity, because that porosity is usually close to the effective porosity:

\[
v_f = n_e v_w \approx n_f v_w = n_f v_t (n_f + n_p) / n_f = v_t (n_f + n_p) \approx v_t n_p = (x/t_t) n_p \tag{2.26a}
\]

The approximate form of Eq.2.26a means that from tracer velocity (or age) it is easy to calculate Darcy’s velocity without any knowledge on the fracture system. The hydraulic conductivity \( (K) \) is defined by Darcy’s law, i.e., \( v_f = (\Delta H / \Delta x) \times K \), where \( (\Delta H / \Delta x) \) is the hydraulic gradient. In consequence, Darcy’s law yields the following relations:

\[
K = n_e x / [(\Delta H / \Delta x) t_t] \tag{2.27}
\]

for a single porosity rock, and

\[
K \cong (n_p + n_f) x / [(\Delta H / \Delta x) t_t] \cong n_p x / [(\Delta H / \Delta x) t_t] \tag{2.27a}
\]

for a fractured rock.

In both Eq.2.27 and 2.27a, the hydraulic gradient represents the mean value along the flow distance. The simplified form of Eq.2.27a is of great practical importance because it allows estimation of the regional hydraulic conductivity from the tracer age without any knowledge on the fracture network (Zuber and Motyka 1994). A number of examples of different applications of the lumped parameter approach can be found in references given above as well as in other works. Two selected examples related to fractured rocks are given below.

The \(^{3}H\) ages shown in Figs.2.9 and 2.10 are related to two springs discharging at the foot of the same morphological unit of a gneiss formation (Zuber and Ciężkowski in press). The matrix porosity was assumed to be 0.007, i.e., similar to the values measured on rock samples taken from another gneiss formation of the same geological age in the Sudetes. The mean \(^{3}H\) ages estimated from the simplest models are about 160 and 1000 years for Szczawina and Łomnica, respectively. However, according to the stable isotope data, the recharge takes place mainly on a plateau at the top of the unit. Therefore, the models should be selected in accordance with that information. It seems that most adequate are the exponential models (EM) for the local component recharged directly above the springs on the slope of the unit, with a dominance of the \(^{3}H\)-free component recharged at the plateau. The following equation holds for a two-component mixing:
where \( \beta \) is the fraction of the \(^3\)H-free component, and subscripts young and old correspond to the \(^3\)H and \(^3\)H-free components, respectively. From Eq.2.28 the age of the \(^3\)H-free (old) component can be estimated, if the mean and \(^3\)H (young) component ages have been correctly determined.

The parameters of the two flow systems are summarised in Table 2.1. For these calculations, the flow distances were estimated from the morphology map. For the \(^3\)H components, they were taken as the half slope distance, and for the \(^3\)H-free component they were taken as the distance from the adequate part of the plateau (for details see Zuber and Ciężkowski, in press). The mean hydraulic gradients were assumed to follow the morphology.

Considering approximate character of the age, distance and hydraulic gradient estimations, the accuracy of the parameters given in Table 2.1 is probably not better than about 50%. In any case a comparison with other crystalline rock systems suggests their correctness. They are also internally consistent because the slope above the Łomnica spring is distinctly shorter than that above the Szczawina spring, which results in a lower fraction of the \(^3\)H component in the former. It is difficult to say if the difference between the hydraulic conductivity of both \(^3\)H systems is significant. However, a distinctly lower value of the hydraulic conductivity of the \(^3\)H-free system in Łomnica than in Szczawina most probably results from a larger part of the plateau covered by less permeable Cretaceous sediments.

**Table 2.1** Parameters of the Szczawina and Łomnica systems (Zuber and Ciężkowski in press).

<table>
<thead>
<tr>
<th>Site</th>
<th>Age</th>
<th>Q</th>
<th>( V_w )</th>
<th>( V_r )</th>
<th>( K )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[a]</td>
<td>[m(^3)/hour]</td>
<td>( 10^6 ) m(^3)</td>
<td>( 10^8 ) m(^3)</td>
<td>[10(^{-8}) m/s]</td>
</tr>
<tr>
<td>Szczawina</td>
<td>158</td>
<td>0.72</td>
<td>1.0</td>
<td>1.4</td>
<td>0.9</td>
</tr>
<tr>
<td>Young fraction</td>
<td>65</td>
<td>0.32</td>
<td>0.2</td>
<td>0.3</td>
<td>1.0</td>
</tr>
<tr>
<td>Old fraction</td>
<td>230</td>
<td>0.40</td>
<td>0.8</td>
<td>1.1</td>
<td>0.8</td>
</tr>
<tr>
<td>Łomnica</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Young fraction</td>
<td>70</td>
<td>a)</td>
<td></td>
<td></td>
<td>0.5</td>
</tr>
<tr>
<td>Old fraction</td>
<td>1000</td>
<td></td>
<td></td>
<td></td>
<td>0.1</td>
</tr>
</tbody>
</table>

Remark: a) unmeasurable due to a partial discharge of the spring in a stream.
In the urbanised area of Lublin city, eastern Poland, groundwater is exploited from Cretaceous marls which are fractured to the depth of about 100-200 m. In spite of dense urbanisation, the water is of a good quality. A number of wells and springs were sampled twice for $^3$H determinations at the beginning of 1995 and end of 1997. All the concentrations were below 10 TU with slow declines. Models fitted to the data were similar to those shown in Fig.2.10, and the greatest mean $^3$H ages were in the range of 250-500 years. The regional hydraulic conductivity in the range of 4-15 m/d was obtained from Eq.2.27a for the matrix porosity of 0.40 and estimated distances of flow yielded. Hydrodynamic modelling and pumping tests yielded the hydraulic conductivity of 2.5-10 m/d in most of the watershed, and 50-300 m/d at the tectonic zones of the valley axes. Therefore, in spite of a low accuracy of age determinations resulting from a low number of $^3$H determinations, the hydraulic conductivity obtained is in a general agreement with that derived from the conventional methods. Large values of regional $^3$H ages, which result from the matrix diffusion (Eq.2.25a), explain the good quality of water in that densely urbanised area. However, when some non-decaying pollutants appear in the groundwater, their removal will also take a very long time.

2.8 THE LUMPED-PARAMETER APPROACH VERSUS OTHER APPROACHES

The multi-cell approach has been introduced to the tracer method in hydrology by Simpson and Duckstein (1976), and Przewłocki and Yurtsever (1974). When uni-dimensional arrangement of cells is applied the method can be regarded as a less versatile version of the lumped-parameter approach. For a single cell, it is equivalent to the EM, and for a very large number of cells, it approaches the PFM. However, when more complicated arrangements are applied (e.g., different volumes of cells, two- and three-dimensional cell arrangements) the number of sought (fitted) parameters increases and unique solutions are not available. Therefore, the multi-cell models can be regarded as a distributed parameter approach with lumping. When interrelated tracer data distributed in time and space are available, the multi-cell modelling is definitely advantageous over the lumped parameter approach. Unfortunately, quite frequently publications appear in which a single $^3$H determination, or a mean value of several samples taken in a short period of time, is interpreted either with the aid of the EM or the multi-cell approach. Such publications should be regarded as examples of incorrect interpretation.

As mentioned, the lumped parameter models are particularly useful when no sufficient data exist to justify the use of multi-cell models, multi-tracer multi-cell models (Adar 1996), or numerical solutions to the transport equation. They are also very useful in early investigations of little known systems. For a separate sampling site (e.g., a spring, or a withdrawal well), only the use of the lumped parameter models is sufficiently justified. Some investigators express opinions that in the era of numerical models, the use of a lumped-parameter approach is out of date. However, it is like trying to kill a fly with a cannon, which is neither effective
nor economic. Experience shows that a number of representative hydrologic parameters can be obtained from the lumped-parameter approach to the interpretation of environmental tracer data in a cheap and effective way.

### 2.9 CONCLUDING REMARKS

The lumped parameter approach is particularly useful for the interpretation of $^3$H data in groundwater systems with separate sampling sites as, for instance, in investigations of the dynamics of small catchments (Kendall and McDonell 1998). $^{18}$O has also been shown to be applicable in investigations of small retention basins and bank filtration from rivers and lakes. As mentioned, the $^3$H-$^3$He method is advantageous over the $^3$H method for recharge rate measurements. The use of $^{85}$Kr is still troublesome and costly, and its advantages have not been proved so far. Measurements of freons have become routinely used in some countries (especially in the USA), though, most probably, due to a lower accuracy inherent to their character, they cannot so far compete with the $^3$H method.

As mentioned a user-friendly computer-programme for the interpretation of environmental tracer data by the lumped-parameter approach is available free from the IAEA (FLOWPC). In that programme the PFM, EM, EPM, LM (linear model), LPM (combined linear-piston flow model), and DM are included, and it contains options for the applications of stable isotopes and other tracers (excluding $^3$He), as well as for using any $\alpha$ value, and any $\beta$ value with a chosen constant tracer concentration. In addition, ASCII files of the response, input and output functions are yielded. Curves shown in Figs.2.2 to 2.6, 2.9 and 2.10 were calculated with the aid of the FLOWPC.

When solving the inverse problem it should be remembered that in general the lower the number of fitted (sought) parameters, the more reliable the results of modelling (Himmelblau and Bischoff 1968). A better fit obtained with a larger number of parameters does not necessarily mean that a more adequate model was found. The modelling procedure should always start with the simplest models. More sophisticated models with additional parameters should be introduced only if it is not possible to obtain a good fit with a simple model, or if other information excludes a simpler model. However, it should be remembered that if a single parameter model yields a good fit, an infinite number of two parameter models also yield equally good fits. Therefore, in such situations other available information should be used for the final selection of the most adequate model. As the inverse solutions belong to the category of ill-posed problems, and the record of the experimental data is usually very short, exact and unique solutions are in general not available. However, even non-unique and/or non-exact solutions are better than a lack of any quantitative, or semi-quantitative, information.

An additional difficulty results from heterogeneity of groundwater systems. As shown by Varni and Carrera (1998), and Małoszewski and Seiler (1999), in highly heterogeneous systems, the mean tracer age may considerably differ from the mean water age. In some cases
the tracer age practically represents the upper, more active part of the system, whereas in strongly stratified systems, dispersive losses of tracer to deeper layers may result in an apparent value of the $\beta$ coefficient. Similarly other parameters do not necessarily represent properly the system investigated. However, in spite of all the limitations, experience shows the lumped-parameter approach to the interpretation of environmental tracer data is of practical importance and usually yields representative results. Experience also shows that even such heterogeneous systems as karstic rocks can effectively be interpreted by that approach (e.g., Małoszewski et al. 1992, Rank et al. 1992).

**REFERENCES**


Lumped parameter models


Lumped parameter models


