On the Ionization and Ohmic Heating of a Helium Plasma

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One useful method of heating a plasma confined by a magnetic field is to impose a constant electric field parallel to the magnetic field. Due to the finite resistivity of the plasma, there will be a conversion of electric energy into heat, some of which appears as internal energy of the plasma. In this paper we determine what value of this electric field is needed to ionize and heat the plasma in a reasonably short time.

The plasma is imagined to be at a low temperature and partially ionized when the constant electric field is first imposed. The Joule heat cannot all go to raise the kinetic energy of the ions and electrons since some of the energy is lost in ionization of neutral atoms by electron impact, while some is lost to the plasma through excitation of the neutral atoms which then radiate their energy out of the system. Also, at higher temperatures, bremsstrahlung radiation due to free-free transitions of the electrons in the coulomb fields of the ions becomes a significant loss mechanism. Further, the remaining Joule energy is communicated directly to the electrons which move more readily in the electric field, and is only subsequently given to the ions by electron-ion collisions. Finally, the rate of Joule heating depends on the conductivity of the plasma, which varies with temperature. The number of electrons is continually changing because of the ionization mechanism already mentioned.

During the development of the discharge, the electron distribution function is not expected to be completely Maxwellian since the collision cross section responsible for the approach to equilibrium is a rapidly decreasing function of energy. The distortion is most severe for the energetic particles, those above a certain critical energy. They are essentially unaffected by collisions and gain energy continually from the electric field, acquiring the loop voltage on each transit around the discharge tube. They can be said to “run away” from the body of the electron distribution. The critical energy is roughly that at which an electron gains from electron impact, while some is lost to the plasma through excitation of the neutral atoms which then radiate their energy out of the system. Also, at higher temperatures, bremsstrahlung radiation due to free-free transitions of the electrons in the coulomb fields of the ions becomes a significant loss mechanism. Further, the remaining Joule energy is communicated directly to the electrons which move more readily in the electric field, and is only subsequently given to the ions by electron-ion collisions. Finally, the rate of Joule heating depends on the conductivity of the plasma, which varies with temperature. The number of electrons is continually changing because of the ionization mechanism already mentioned.

If the critical energy is several times the mean thermal energy it is legitimate to assume that the distribution function is Maxwellian for the purpose of computing the conductivity. Also, if the critical energy is much larger than the energy of those electrons contributing most to excitation and ionization, it is legitimate to assume a Maxwell distribution in calculating the average rates at which these processes occur.

However, there must always be some electrons which have run away and are circulating at high energies. If there are enough of these they can well alter the nature of the discharge; for instance, by giving rise to a skin effect, or perhaps collective motions. Throughout the present work such runaways and their associated effects will be consistently ignored.

It will be found that the criteria for achieving a Maxwell distribution are not well satisfied for the densities and electric fields for which the calculations were made, and one therefore should resort to the Boltzmann equation.

One might expect a further distortion of the distribution from Maxwellian to exist in the depletion of electrons from the tail in the inelastic processes of ionization and excitation. However, one finds that in the cases treated, the tail is resupplied by elastic collisions at such a rate that the distortion due to this effect is not serious. However, the assumption of a Maxwell distribution greatly simplifies the calculations and yields results which are qualitatively correct.

If the above assumptions are satisfied, the discharge in helium develops as follows. When the electric field is applied, the current is at first limited by inductive effects which initially are much larger than the resistive effects. After a short time, the inductive effects become negligible compared to the resistive effects, and the current is then given closely by Ohm’s law. The electron temperature rises until the electrons have sufficient energy to excite and ionize the neutral atoms, at which point the temperature stays nearly constant until almost all the atoms are ionized and the energy sink they provide is removed. The temperature then rises again until second ionization and excitation of the singly ionized atoms set in. It pauses again, until second ionization is complete.

The current then starts to rise again since the only remaining energy loss, the bremsstrahlung loss, varies as \( T^2 \) while the power input goes as \( T^1 \). As the current continues to rise, the temperature increases, yielding an increase in conductivity which leads to a skin effect. This current is thus confined to a thin annular region on the discharge boundary. This results in the heating,

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finally, of only the discharge skin. This effect, however, does not enter significantly for the range of temperatures considered in the calculations.

The ion temperature, which is the same for singly and doubly ionized helium because of rapid collisions, lags far behind the electron temperature throughout the entire process, since the transport of energy from electrons to ions is relatively inefficient compared to the rate at which the electrons are heated. Further, the rate of this transport decreases as the electron temperature rises (as $T_e^{-4}$) so that ohmic heating of ions becomes inefficient when the electrons are heated to temperatures of more than several hundred volts. The ions cannot be expected to reach temperatures of more than 100 volts. Consequently, one cannot reach thermonuclear ion temperatures with ohmic heating alone. A more serious limiting effect is the existence of a hydromagnetic instability associated with large currents, as predicted by Kruskal. When the current exceeds a critical value, depending on the size of the confining magnetic field, the instability sets in and the assumptions of this calculation become highly suspect.

EQUATIONS

For simplicity, we consider the gas to be at uniform density, contained in a long cylinder, and in a homogeneous externally maintained axial magnetic field. The electric field is also, of course, in the axial direction, as is the current which it produces.

The equations advanced to describe the development, in time, of the discharge are the following. Let $j$ denote the axial current density, $S$ the cross sectional area of the discharge, $\varepsilon$ the electric field strength, $l$ the length of the discharge, and $\sigma$ the electrical conductivity. Then the equation relating $j$ and $\varepsilon$ is

$$\frac{1}{\sigma} \frac{d(j\varepsilon)}{dt} = I^2,$$

where, in Gaussian units,

$$\frac{1}{\sigma} = \left( 0.582 + \frac{1}{10Z_{eff}} \right) \frac{n_e}{kT_e},$$

and the effective atomic number $Z_{eff}$ is given by

$$Z_{eff} = Z_{++} + \frac{n_{++}}{n_{++} + n_{++}}(Z_{++} - Z_{+}) = 1 + \frac{n_{++}}{n_{++} + n_{++}}.$$

Eqs. (2) and (3) are obtained, for the mixture of positive ions here considered, from Spitzer by linear interpolation between the integral values of the atomic number for which he presents the results.

The equation of electron energy balance is,

$$\frac{2}{\sigma} \frac{d}{dt} \left( \frac{3}{2} n_0 kT_e \right) + n_e n_0 \Sigma_{1} v_1 E_1 + n_e n_{++} \Sigma_{1} v_{1+} E_{1+} + n_e n_{++} h(T_e - T_{++})/m_o + n_{++} n_{++} h(T_e - T_{++})/m_o \right) = \left( \frac{2m_e kT_e}{m_o} \right)^{1/2} m_o \left( \frac{e^2}{m_o} \right)^{1/2} \ln \left[ \frac{12n_{++} h^2 T_e}{\pi e^2} \right] + n_{++} \left( \frac{2m_e kT_e}{m_o} \right)^{1/2} m_o \left( \frac{e^2}{m_o} \right)^{1/2} \left[ \frac{116e^4}{3\pi} \right] \ln \left[ \frac{12n_{++} h^2 T_e}{\pi e^2} \right]$$

The quantities $n_0, n_{++}, n_{++}$ and $N$ are respectively the number densities of electrons, neutral helium, singly ionized helium, doubly ionized helium, and helium nuclei, the last of these being constant in time; $k$ is Boltzmann's constant, $T_e$ the electron temperature; $T_{++}$ is the temperature of the $\text{He}^{++}$ ions which is taken to be the same as $T_{++}$, the temperature of the $\text{He}^{++}$ ions. The left-hand side of Eq. (4) represents the rate per unit volume at which energy is fed into the plasma by the current. The first term on the right expresses the rate at which the internal energy of the electrons increases, the second term the rate at which energy is expended in exciting and ionizing neutral helium, and the third term the rate at which energy is
expended in exciting and ionizing He+. The quantities $E_i$ and $E_i^+$ stand for the energy required for a particular excitation, while the $v_i$ and $v_i^+$ stand for the product of the associated collision cross sections and the electron speed, averaged over the Maxwell distribution of the electrons. The expressions for $\Sigma_i v_i E_i$ and $\Sigma_i v_i^+ E_i^+$ used in the computations were the following analytic fits to results obtained employing experimental cross sections:

$$v_{\text{ion}} = a + b(kT_e) + A - B(kT_e) + c(kT_e)^2$$

The constants for the various fits are given in Table 1.

<table>
<thead>
<tr>
<th>Cross</th>
<th>$E_i$</th>
<th>$E_i^+$</th>
<th>$\nu_{\text{ion}}$</th>
<th>$\nu_{\text{ion}}^+$</th>
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</thead>
<tbody>
<tr>
<td>a</td>
<td>$-9.80288 \times 10^{-18}$</td>
<td>$-1.43009 \times 10^{-18}$</td>
<td>$-3.57046 \times 10^{-9}$</td>
<td>$-9.07731 \times 10^{-10}$</td>
</tr>
<tr>
<td>b</td>
<td>$1.17248 \times 10^{-18}$</td>
<td>$7.54995 \times 10^{-21}$</td>
<td>$3.36217 \times 10^{-10}$</td>
<td>$4.53767 \times 10^{-11}$</td>
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<tr>
<td>c</td>
<td>$1.4740 \times 10^{-4}$</td>
<td>$3.07369 \times 10^{-5}$</td>
<td>$1.21018 \times 10^{-4}$</td>
<td>$1.17600 \times 10^{-5}$</td>
</tr>
<tr>
<td>A</td>
<td>$9.80288 \times 10^{-18}$</td>
<td>$1.43009 \times 10^{-18}$</td>
<td>$3.57046 \times 10^{-9}$</td>
<td>$9.07731 \times 10^{-10}$</td>
</tr>
<tr>
<td>B</td>
<td>$1.10027 \times 10^{-18}$</td>
<td>$3.07369 \times 10^{-5}$</td>
<td>$3.77845 \times 10^{-10}$</td>
<td>$4.17675 \times 10^{-11}$</td>
</tr>
<tr>
<td>C</td>
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<td>$1.63000 \times 10^{-22}$</td>
<td>$1.15091 \times 10^{-11}$</td>
<td>$8.51818 \times 10^{-13}$</td>
</tr>
<tr>
<td>D</td>
<td>$8.95259 \times 10^{-3}$</td>
<td>$2.37593 \times 10^{-3}$</td>
<td>$6.01543 \times 10^{-3}$</td>
<td>$6.49858 \times 10^{-3}$</td>
</tr>
<tr>
<td>E</td>
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<td>$6.00339 \times 10^{-6}$</td>
<td>$5.74003 \times 10^{-6}$</td>
<td>$6.4858 \times 10^{-6}$</td>
</tr>
</tbody>
</table>

where, in addition, there appear separately the constants for the ionization rates. The units have been so chosen that the results are in cgs units if $kT_e$ is expressed in electron volts.

The third and fourth terms on the right-hand side of Eq. (4) represent respectively the rates at which He$^+$ and He$^{++}$ gain energy per unit volume via collisions with electrons, while the last term gives the energy loss per unit volume via bremsstrahlung.

In addition to the preceding equations, there are those expressing the ion energy balance, and the various number and charge conservation laws, viz:

$$\frac{d}{dt} \left[ \frac{3}{2} (n_+ + n_{++}) kT_+ \right] = n_0 n_+ k(T_e - T_+) \left( \frac{2 \pi kT_e}{m_+} \right)^{3/2} \left( \frac{m_0}{m_+} \right)^{1/2} \frac{e^2}{kT_+} \ln \left[ \frac{12 \pi n_0 (kT_e)^4}{4 \pi n_0 e^2} \right]$$

The last term on the right-hand side of Eq. (6) represents the effects of charge exchange collisions. In such an encounter it is assumed that the "fast" ion is converted to an ion at the temperature, $T_0$, of the neutral atoms. $T_0$ is taken to be the temperature of the external walls and is constant in time. The charge transfer cross section, $Q$, was computed to be $5 \times 10^{-18}$ cm$^2$.

The values of the parameters used in the calculations are those for the B-1 Stellarator:

$$i = 450 \text{ cm}$$

$$L = 5.6 \times 10^{-6} \text{ h}$$

$$S = 10 \text{ cm}^2$$

Experimental results on ohmic heating in the B-1 Stellarator are reported in Reference 7.3

RESULTS

The equations were integrated on the N.Y.U. Univac for a variety of electric fields and densities. The results are given in Figs. 1 and 2 for an electric field of 0.11 volts/cm, which corresponds to 50 $V$ around the stellarator, and an initial neutral density of $2.4 \times 10^{13}$ helium atoms/cm$^3$. The levelling off of the electron temperature during the main part of the first ionization phase is prominent, and further there is a detectible decrease in the rate of temperature rise during the second ionization phase. It is clear that the ion temperature lags seriously behind the electron temperature. Note that the current density follows the electron temperature quite well except during the initial stages of the discharge. The intensity curves for the neutral helium line $\lambda 4921$ $\AA$ and the singly ionized...
helium line $\lambda 4686$ Å were also calculated and are presented in Fig. 2. For similar cases, in which the integration was carried out to longer times than in this case, it was found that the ion temperature reached 100 eV in approximately four milliseconds. At this time the electron temperature was approximately 1,000 eV. As mentioned previously, the conditions necessary for the electron distribution to be considered Maxwellian for the purpose of the calculation were not well satisfied. The critical energy $\varepsilon_{\text{crit}}$ in electron volts, for electrons to run away, is given by

$$\varepsilon_{\text{crit}} \approx 4.5 \times 10^{-13} \frac{n_e}{\varepsilon}$$  \hspace{1cm} (11)

if $\varepsilon$ is in volts/cm and $n_e$ in cm$^{-3}$. For full ionization, in the case considered, $\varepsilon_{\text{crit}}$ is approximately 80 eV. Since the electrons which contribute most to the conductivity are those whose energies are several times thermal energy, one cannot expect the conductivity to be given accurately for $T_e$ much above 10 eV. Further, the electrons responsible for first ionization and excitation lie in the range 30–40 eV and therefore the calculated rates for these processes are not accurate. For the case of second ionization the situation is worse in this regard but it should be noted that this does not affect the course of the discharge seriously. A detailed comparison of the calculated and experimental results is carried out in Reference 7.

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REFERENCES


