Tritium Production and Cycling in a Fusion Reactor with Lithium Blanket

By Karl O. Hintermann* and Rolf Wideröe†

The $T(d,n)He^4$ reaction has the advantage of a larger reaction cross section and greater energy release than the $D(d,p)T$ and $D(d,n)He^3$ reactions. Tritium cycling means the use of the neutrons for tritium production in a lithium blanket in order to support the $T(d,n)He^4$ reaction in the plasma.

TRITIUM PRODUCTION

The first part of this paper deals with the calculations for neutrons slowing down in lithium. The results show that the neutrons have no chance to become thermal, but are involved in a $Li^6(n,t)He^4$ reaction before reaching energies below 10^4 ev in pure Li^6 and 10 ev in natural lithium.

The spatial distribution of the reaction rates throughout the lithium blanket is then treated for slab geometry, using two methods: (1) Fermi age treatment and (2) application of the fast neutron diffusion equation employing an average value of the Fermi age. The second method gives a good approximation and by a special choice of the average Fermi age, good results may be obtained. Because of the relatively small scattering cross section of lithium, a blanket of natural lithium becomes at least as thick as a blanket of a good moderating material with lithium lining or lithium channels.

Finally, the relationships between the blanket efficiency, the $T(d,n)He^4$ reaction rate and the relative numbers of T and D atoms in the plasma are considered.

Passage of Neutrons through Lithium

Cross Sections

The neutrons of the $T(d,n)He^4$ reaction have an energy of 14.1 Mev, those of the $D(d,n)He^3$ reaction 2.45 Mev.[1] The prevailing reactions of neutrons at 14.1 Mev in lithium are elastic and inelastic scatterings. No measurements were available regarding the relative magnitudes of elastic and inelastic scattering cross sections in lithium at this energy. The comparison with other elements (Be, Al) leads to the assumption that these two cross sections have the same order of magnitude, so that an inelastic scattering will occur among the first few collisions. Therefore, other reactions, for instance (n,t), (n,d) and (n,p), which occur at high energy but with much lower cross sections than the scatterings, can be neglected. The energy region of these reactions will be largely bypassed by the energy loss in an inelastic scattering collision.

Initial Energy for Elastic Slowing Down

Let us estimate the energy lost in an inelastic collision. The spacing between energy levels of Li^7 in the region of 14 Mev is approximately 1 to 2 Mev.[2] The cross sections for excitation of the different levels are not known, but it seems reasonable for us to assume that the neutron energy after an inelastic scattering collision is somewhat more than these level spacings. Two Mev is assumed as initial energy for the elastic slowing down. The neutrons which arrive at the blanket with 2.45 Mev from the (d,n) reaction in the plasma have a good chance of making an inelastic collision during their first 5 collisions,[3] leaving 0.48 Mev for excitation. For these neutrons, 2 Mev is also a reasonable value for their initial energy.

In the case of Li^6, no measurements of the excitation levels at 14 Mev are available, but comparison of the energy level schemes[2] of Li^6 and Li^7 leads to the conclusion that the assumption of 2 Mev as initial energy for elastic slowing is also reasonable for pure Li^6.

The influence of the choice of the initial energy on the reaction rate distribution is discussed later.

First Collision Correction

As most neutrons enter from the direction of the plasma pinch, the prevailing flight direction is orthogonal to the blanket. The influence of the direction component parallel to the pinch is neglected. The first collision density resulting from these assumptions is taken as neutron source and the total source strength is standardized to one neutron per second:

$$\text{First collision source} = \Sigma_0 e^{-\Sigma_0 x}$$

Σ_0 is the macroscopic cross section for elastic and inelastic scattering at 14.1 Mev.[4] The dimension of the source comes out as cm^{-1}, corresponding to unit slab thickness.

* Brown Boveri Co. Ltd. and Reactor Ltd., Würenlingen.
† Brown Boveri Co. Ltd.

Because most neutrons are absorbed before Σ_a reaches the same order of magnitude as Σ_s,‡ diffusion theory for weak capture is applied.

Boundary Conditions

It is assumed that no neutrons are absorbed in the space enclosed by the lithium blanket. The reflected neutrons strike the opposite blanket, from which an equal number of neutrons are reflected. Thus the neutron flux and, therefore, the slowing down density have a horizontal tangent on the inner side of the blanket ($x = 0$). The reaction rate is calculated for a blanket of infinite thickness, so that the outer boundary condition is zero flux at infinity. At zero lethargy or Fermi age (neutrons of 2 Mev), the slowing down density is identical with the first collision source distribution.

All calculations are made in slab geometry, and for the total source strength (integrated throughout the slab thickness from 1 to ∞) of 1 neutron per second.

Reaction Rate per Unit Lethargy Interval

The reaction rate per unit lethargy interval is:

$$R(u) = \Sigma_a(u)\phi(u), \tag{1}$$

where the lethargy, $u = \ln(E_0/E)$, E_0 being the initial energy.

The relationship between the flux per unit lethargy interval and the slowing down density with absorption is:[5]

$$\phi(u) = \frac{q'(u)}{\xi\Sigma_s+\gamma\Sigma_a}, \tag{2}$$

where: $\gamma = \dfrac{1-\alpha-\alpha\epsilon-\frac{1}{2}\alpha\epsilon^2}{1-\alpha-\alpha\epsilon}$; $\xi = \langle\ln(E_1/E_2)\rangle$, average logarithmic energy decrement per elastic collision; $\alpha = \left(\dfrac{A-1}{A+1}\right)^2$, minimum value of E_2/E_1 (head-on collision); and E_1 and E_2 are the energies before and after the collision, respectively.

The slowing down density with absorption,

$$q'(u) = q(u)\exp\left(-\int_0^u \frac{\Sigma_a}{\xi\Sigma_s+\gamma\Sigma_a}\,du\right), \tag{3}$$

where $q(u)$ is the slowing down density without absorption.

Without considering the spatial distribution of the neutrons ($q(u) = q(0) =$ constant) and standardizing on the source strength of one neutron per second ($q(0) = 1$), the reaction rate per unit lethargy interval becomes:

$$R(u) = \frac{\Sigma_a}{\xi\Sigma_s+\gamma\Sigma_a}\exp\left(-\int_0^u \frac{\Sigma_a}{\xi\Sigma_s+\gamma\Sigma_a}\,du\right). \tag{4}$$

On the substitution

$$w(u) = \int_0^u \frac{\Sigma_a}{\xi\Sigma_s+\gamma\Sigma_a}\,du. \tag{5}$$

‡ See Glossary of Symbols at end of paper.

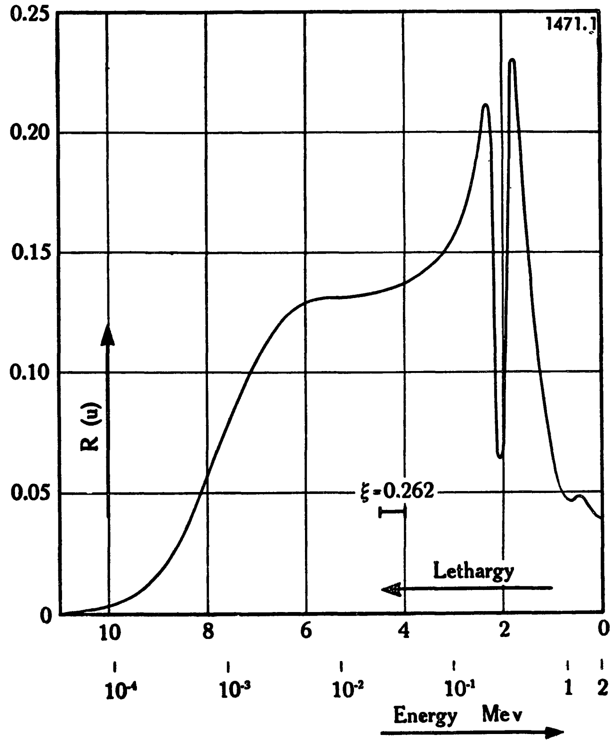

Figure 1. Reaction rate per unit lethargy, for natural lithium

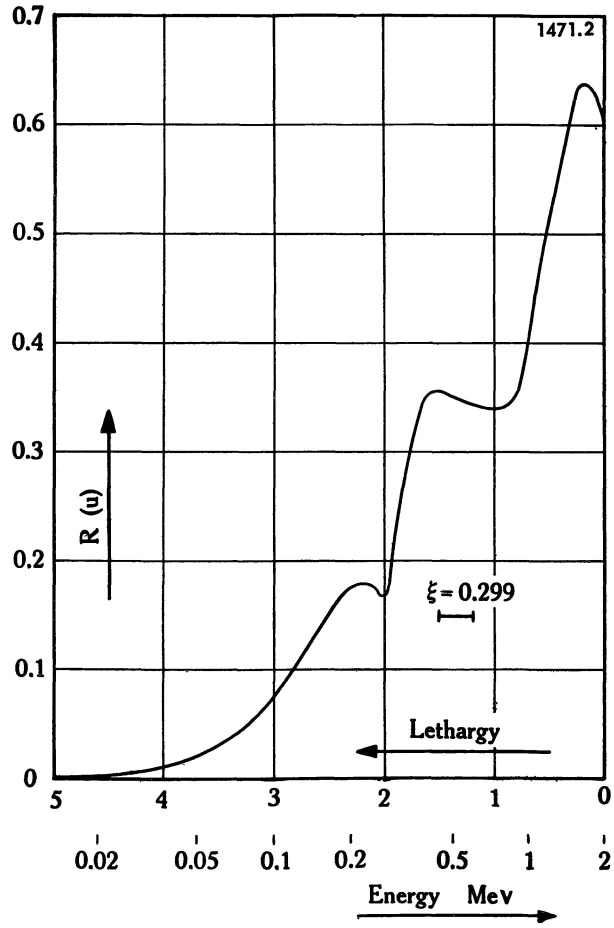

Figure 2. Reaction rate per unit lethargy, for pure Li^6

Equation (4) becomes

$$R(u) = \frac{dw}{du} e^{-w}. \quad (6)$$

Because $q(0) = 1$, the overall absorption must also be unity:

$$\int_0^\infty R(u)\,du = \int_0^\infty e^{-w}\,dw = 1.$$

In Figs. 1 and 2, $R(u)$ is plotted for natural lithium (7.5% Li^6, 92.5% Li^7) and pure Li^6. It will be recognized that a correct value of the initial energy is of relatively minor importance especially in the case of natural lithium. A factor 2 in the initial energy value corresponds to a lethargy interval of ln 2 = 0.693, while the reaction region extends over a lethargy interval of about 10 in the case of natural lithium and about 4 in the case of pure Li^6.

Spatial Distribution of Reaction Rate in the Lithium Blanket

Fermi Age Treatment

The Fermi age equation with absorption is[5]

$$\nabla^2 q(x,\tau') = \frac{\partial q(x,\tau')}{\partial \tau'} \quad (7)$$

with the following definitions for age, diffusion coefficient and average cosine of the scattering angle

$$\tau' = \int_0^u \frac{D'}{\xi\Sigma_s + \gamma\Sigma_a}\,du \quad (8)$$

$$D' = \left[3\Sigma_\tau(1-\bar{\mu}_0)\left(1 - \frac{4}{5}\frac{\Sigma_a}{\Sigma_\tau} + \frac{\Sigma_a}{\Sigma_\tau}\frac{\bar{\mu}_0}{1-\mu_0} + \ldots\right)\right]^{-1}$$

$$\bar{\mu}_0 = \frac{2}{3A}.$$

The general solution of Eq. (7) is[6]

$$q(x,\tau') = \tfrac{1}{2}(\pi\tau')^{-\frac{1}{2}} \int_{-\infty}^{+\infty} q(t,0)\exp[-(x-t)^2/4\tau']dt.$$

Boundary conditions—The slowing down density at age zero is identical with the source distribution of the first collision correction,

$$q(x,0) = \Sigma_0 e^{-\Sigma_0 x}. \quad (9)$$

As it is assumed that no neutrons are absorbed in the space enclosed by the lithium blanket, the reflected neutrons all re-enter the blanket somewhere. The inner boundary condition is then:

$$\left(\frac{dq(x,\tau')}{dx}\right)_{x=0} = 0.$$

This condition can be satisfied by assuming a symmetric source distribution on both sides of the inner boundary, $x = 0$. With this assumption and in view of Eq. (9), the solution of (7) fulfilling the boundary conditions becomes

$$q(x,\tau') = \frac{\Sigma_0}{2(\pi\tau')^{\frac{1}{2}}}\left\{\int_0^\infty \left[\exp\left(-\Sigma_0 t - \frac{(x-t)^2}{4\tau'}\right)\right]dt + \int_0^\infty \left[\exp\left(-\Sigma_0 t - \frac{(x+t)^2}{4\tau'}\right)\right]dt\right\}.$$

After some calculation this becomes

$$q(x,\tau') = \tfrac{1}{2}\Sigma_0 e^{-x^2/4\tau'}\{[1-H(a)]e^{a^2} + [1-H(b)]e^{b^2}\}, \quad (10)$$

where

$$a = \Sigma_0\sqrt{\tau'} - x/(2\sqrt{\tau'}),$$

$$b = \Sigma_0\sqrt{\tau'} - x/(2\sqrt{\tau'}),$$

$$H(a) = \frac{2}{\sqrt{\pi}}\int_0^a e^{-t^2}dt, \quad \text{the probability integral.}$$

Now the spatial distribution of the reaction rate $R(x)$ can be calculated:

$$R(x) = \int_0^\infty R(x,u)du = \int_0^\infty \Sigma_a(u)\phi(x,u)du.$$

Using Eqs. (2) and (3),

$$R(x) = \int_0^\infty \frac{\Sigma_a}{\xi\Sigma_s + \gamma\Sigma_a} q(x,\tau'(u)) \exp\left(-\int_0^u \frac{\Sigma_a}{\xi\Sigma_s+\gamma\Sigma_a}du\right)du. \quad (11)$$

Now again, with substitution (5), Eq. (11) becomes

$$R(x) = \int_0^\infty q(x,\tau'(w))e^{-w}dw \quad (12)$$

where Eq. (5) allows for $\tau'(w) = \tau'(u(w))$. The results of Eq. (12) are plotted as curves 1a and 1b in Fig. 3 for natural lithium and pure Li^6, respectively.

Fast Neutron Diffusion Treatment

Because the evaluation of Eq. (12) includes a numerical integration for each value of x (w is an empirical function), the possibility of an approximation by the fast neutron diffusion equation, using a uniform mean Fermi age, is investigated. This approximation shall be based on the following model:

(1) The neutron source is given by Eq. (9), with the boundary conditions as before.

(2) Because of inelastic scattering at high energies, the Fermi age of the neutrons is counted from 2 Mev as initial energy, as before.

(3) No neutrons are absorbed until they have slowed down to a certain uniform Fermi age, where all of them are absorbed.

Comparing with the two-group diffusion theory, we treat the neutrons considered in the same way as we would for the fast group equation. The thermal neutron diffusion equation of the two-group theory would in our model include the neutrons at the chosen mean Fermi age, where all neutrons are absorbed. As these neutrons are no longer subject to diffusion but are absorbed immediately when they enter the "slow group", the slow group diffusion equation has no more meaning.

The fast group diffusion equation with the first collision neutrons as source term is:

$$D\nabla^2\phi(x) - \Sigma_1\phi(x) + \Sigma_0 e^{-\Sigma_0 x} = 0. \quad (13)$$

In view of the fact that one absorption collision requires all slowing down collisions of one neutron, we obtain the definition of the "slowing down cross section", Σ_1,[5]: $\Sigma_1 = D/\tau$.

The Fermi age, τ, and the diffusion coefficient, D, are now defined without absorption.

With the same boundary conditions as before, the solution of Eq. (13) is

$$\phi(x) = \left[D\left(\frac{1}{\Sigma_0^2\tau}-1\right)\right]^{-1}\left\{\frac{1}{\Sigma_0}e^{-\Sigma_0 x} - \tau^{\frac{1}{2}}e^{-x/\sqrt{\tau}}\right\} \quad (14)$$

and the reaction rate becomes:

$$R(x) = \Sigma_1\phi(x) = \frac{D}{\tau}\phi(x)$$

$$= \left(\frac{1}{\Sigma_0^2}-\tau\right)^{-1}\left\{\frac{1}{\Sigma_0}e^{-\Sigma_0 x} - \tau^{\frac{1}{2}}e^{-x/\sqrt{\tau}}\right\}. \quad (15)$$

Now the question arises of what average Fermi age should be chosen. Two different ways of selecting the average Fermi age are investigated:

(a) In order to obtain a good match to the curve from Fermi age treatment, an average age, designated by $\bar{\tau}_1$ is calculated by postulating coincidence of both reaction rate curves at the inner boundary:

$$R(0)_{\text{Age Eq.}} = R(0)_{\text{Diffusion Eq.}} \quad (16)$$

The slowing down density at $x = 0$ is:

$$q(0_1\tau') = \Sigma_0[1 - H(\Sigma_0\sqrt{\tau'})]e^{\Sigma_0^2\tau'}$$

and from Eqs. (12) and (15) at $x = 0$, Eq. (16) yields:

$$\bar{\tau}_1 = \left\{\left[\int_0^\infty [1-H(\Sigma_0\sqrt{\tau'(w)})] \exp[\Sigma_0\sqrt{\tau'(w)} - w]dw\right]^{-1} - 1\right\}\Big/\Sigma_0. \quad (17)$$

The results of (15), with $\bar{\tau}_1$ as average Fermi age, are plotted in Fig. 3 as curves 2a and 2b. It will be recognized, that with this method of calculating the age, the fast neutron diffusion equation yields a very good approximation to the Fermi age treatment. This is a great advantage because (15) is a handy equation, whereas Eq. (12) is tedious to solve, including a numerical integration over w for every value of x.

(b) Another method is to average the Fermi age in the usual way:

$$\bar{\tau}_2 = \frac{\int_0^\infty \tau(u)R(u)du}{\int_0^\infty R(u)du}.$$

The results of (15) with $\bar{\tau}_2$ for Fermi age are plotted in Fig. 3 as curves 3a and 3b. This approximation, as seen in the case of natural lithium, is much poorer than the first one.

The numerical values of the average Fermi ages calculated by the two methods and the corresponding lethargies are shown in Table 1.

The lack of consistency in the relationship between $\bar{\tau}_1$ and $\bar{\tau}_2$ in the case of natural lithium and Li^6 can be explained by the fact that most neutrons in natural lithium pass the resonance at 255 kev, while in pure Li^6 most neutrons are absorbed before passing the resonance. The resonance at 255 kev[4] appears in both isotopes, Li^6 and Li^7.

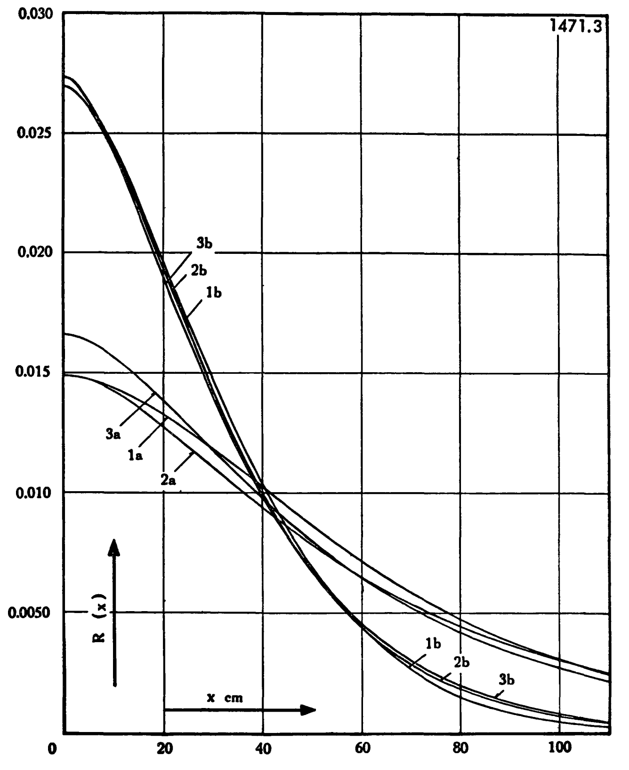

Figure 3. Reaction rates vs. blanket thickness
Curve 1: Fermi age treatment, Curves 2 and 3: fast neutron diffusion equation with $\bar{\tau}_1$ and $\bar{\tau}_2$ respectively; Indices (*a*): natural lithium, (*b*): pure Li^6

Conclusions about Production

Lithium Blanket

The Fermi ages of lithium are much larger than those for the usual moderators for the same lethargy interval. Compared to graphite, lithium has only half the atomic mass, but its average scattering cross section is much smaller. The average slowing down power of lithium is therefore considerably poorer than that of graphite and other usual moderators. A blanket of natural lithium would have to be at least 100 cm thick in order to have a good efficiency for converting the neutrons to tritium. Pure Li^6 would be more advantageous to use, but more expensive too.

Table 1.—Average absorption Fermi ages and their corresponding lethargies[a]

	Natural Li	Li^6
$\bar{\tau}_1$	2595 cm^2	428 cm^2
u_1	5.13	.918
$\bar{\tau}_2$	1965 cm^2	445 cm^2
u_2	4.17	.964

[a] For neutrons with initial energy of 2 Mev in lithium

Moderator Blanket

It seems useful to have a moderator blanket of graphite or beryllium oxide for the purpose of slowing down the fast neutrons before they undergo the $Li^6(n,t)He^4$ reaction. The reaction cross section follows a $1/v$ law below the resonance at 255 kev, and the thermal neutron cross section is more than 10^3 times as large as for the energy region where the neutrons would react in a pure lithium blanket.

If a moderator blanket is lined with lithium both inside and outside, no thermal neutrons can escape. The mean free path for absorption of thermal neutrons is 0.3 cm in natural lithium and 0.023 cm in pure Li^6. Depending on the albedo of the moderator blanket, the inner lining will receive more thermal neutrons than the outer one.

Since lithium may be used as the heat-carrying medium, it would be an advantage as regards the technical feasibility if the lining were replaced by channels in the moderator. The distance of the channels should be of the same order of magnitude as the slowing down length of the moderator, and there should be more than one row of channels in the direction orthogonal to the blanket. Uranium rods for neutron multiplication may also be provided.

The advantage of a moderator blanket is not only the higher efficiency for the same blanket thickness, but also the lower lithium quantity which allows a higher concentration of tritium after the same irradiation dose. This facilitates the recovery of lithium.

As most moderators have a density several times higher than that of lithium, more energy of the inelastic scattering gamma rays will be recovered within the blanket. The energy of the inelastic scattering gamma rays will be more (about 12 Mev for the $T(d,n)He^4$ neutrons) than the energy from slowing down (about 2 Mev) and the $Li^6(n,t)He^4$ exothermic reaction energy (4.8 Mev) together if the efficiency of the tritium cycle exceeds about 50%.

TRITIUM CYCLING

The following reactions take place in the plasma:[1]

(*a*) $D(d,n)He^3$ } denoted as DD reactions
(*b*) $D(d,p)T$ }
(*c*) $T(d,n)He^4$ denoted as DT reaction

The first and the second reaction occur with about the same probability, while the cross section for the DT reaction is about 100 times larger.

We define the symbol r as the number of DT reactions per DD reaction and η as the number of tritium atoms recovered per neutron. With these definitions the following relationship can be established:

$$r = \tfrac{1}{2} + (r + \tfrac{1}{2})\eta.$$

The first term on the right-hand side stands for the tritium atom from reaction (*b*), and the second term takes into account the tritium produced in the lithium by neutrons of the reactions (*c*) and (*a*). The explicit expression for r becomes:

$$r = \frac{1+\eta}{2(1-\eta)} \tag{18}$$

From the definition of r the relationship between the reaction rates becomes:

$$rR_{DD} = R_{DT}.$$

On the other hand, the reaction rates expressed by the average product of cross sections and velocity are:[1]

$$R_{DD} = \tfrac{1}{2}N_D^2\langle\sigma V_{DD}\rangle$$
$$R_{DT} = N_D N_T\langle\sigma V_{DT}\rangle;$$

from this, the ratio of tritons to deuterons in the plasma becomes:

$$\frac{N_T}{N_D} = \frac{r}{2}\frac{\langle\sigma V_{DD}\rangle}{\langle\sigma V_{DT}\rangle}. \tag{19}$$

Numerical example:

kinetic temperature of the plasma = 20 kev
$\eta = 0.72$ (with uranium rods, η may have a value of 1 or more)
$\langle\sigma V_{DD}\rangle/\langle\sigma V_{DT}\rangle = 8.05\times10^{-3}$ Ref. 1.
$N_T/N_D = 0.0121.$

This result demonstrates that it is only necessary to invest a relatively small quantity of tritium in the reaction gas.

ACKNOWLEDGEMENT

The authors express their gratitude to Mr. W. Zünti for useful suggestions, and to Messrs E. Jantsch, W. Lindt, R. Meier, J. M. Patry and W. Winkler for helpful discussions.

GLOSSARY OF SYMBOLS

A	Atomic mass number.
D	Diffusion coefficient without absorption.
D'	Diffusion coefficient with absorption.
E_1; E_2	Energy before and after an elastic collision.
N	Number of atoms per cm³.
H	Probability integral.
R	Reaction rate.
q	Slowing down density without absorption.
q'	Slowing down density with absorption.
r	Number of DT reactions per DD reaction.
u	Lethargy.
v	Velocity.
w	Function of cross sections defined in (5).
x	Coordinate orthogonal to the blanket.
ϕ	Scalar neutron flux.
Σ	Macroscopic cross section.
Σ_1	Slowing down cross section of fast neutron group.
Σ_0	Macroscopic scattering cross section at 14,1 Mev.
Σ_s	Macroscopic scattering cross section.

Σ_a	Macroscopic cross section for the $Li^6(n, t)He^4$ reaction.
Σ_τ	Total macroscopic cross section.
α	Minimum value of E_2/E_1 (head-on collision).
η	Number of tritium atoms recovered per neutron.
$\bar{\mu}_0$	Average cosine of scattering angle.
σ	Microscopic cross section.
τ	Fermi age.
τ'	Modified Fermi age for absorption.
ξ	Average logarithmic energy decrement per collision.

REFERENCES

1. F. Post, *Controlled Fusion Research—An Application of the Physics of High Temperature Plasma*, Revs. Modern Phys., *28*, 338–362 (1956).
2. F. Ajzenberg and T. Lauritzen, *Energy Levels of Light Nuclei V*, Revs. Modern Phys., *27*, 77–166 (1955).
3. J. M. Freeman, A. M. Lane and B. Rose, *Inelastic Neutron Scattering in Lithium Seven*, Phil. Mag., *46*, 17–30 (1955).
4. D. J. Hughes and R. B. Schwartz, *Neutron Cross-Sections*, Brookhaven National Laboratory BNL–325 (1955, Supplement, 1957).
5. S. Glasstone and M. C. Edlund, *The Elements of Nuclear Reactor Thebry*, D. Van Nostrand Co. Inc., New York (1952).
6. A. Sommerfeld, *Partielle Differentialgleichungen der Physik*, Akademische Verlagsgesellschaft Geest u. Portig K.-G., Leipzig (1948).