Recent Work on Controlled Thermonuclear Fusion in Germany (Federal Republic)

By L. Biermann [*]

Before presenting a summary of the work on controlled thermonuclear fusion in the Federal Republic of Germany, I should like to make some general comments about the scope and background of this work

Soon after the first Geneva Conference on the Peaceful Uses of Atomic Energy, physicists in a number of scientific institutions in Western Germany began to give serious attention to the potential possibilities of controlled fusion The advantages of thermonuclear devices and of using magnetic fields for containing the hot charged particles were realized very early It was natural, therefore, that both experimental physicists with experience in the physics of gaseous discharges, plasmas and theoretical astrophysicists with their acquaintance with magnetohydrodynamics and with the propagation of cosmic rays in cosmic magnetic fields, started to think about how a plasma with the required high temperature could be controlled

I may say that the closest connections to our work are to be found, perhaps, in the developments at Princeton and at Harwell Experiments on high current discharges at Gottingen [†] and Aachen [‡] are as yet on a fairly small scale as compared with some of the other experiments described at this conference, although experiments with toroidal discharges, suitable exterior magnetic fields, and large diameters ranging between 40 cm and 1 m, have also been considered at Aachen and Gottingen

In addition to the work on toroidal discharges, high current linear discharges are or will be used to investigate controlled fusion problems at Aachen, Munich, Hanover, Kiel and Stuttgart The work at Kiel and Hanover is especially related to spectroscopic problems The Aachen group has also studied the magnetic compression of plasmas by means of fast electrodeless discharges

The theoretical work, most of which has been done in Gottingen, was started there in 1956 in advance of the experiments and pertains to a number of separate subjects The subject of stability has also been considered by the Aachen group

EXPERIMENTS

Linear Discharges

In the Munich experiments,[1] [§] a cylindrical glass vessel 20 cm in diameter and 50-cm long, filled with very pure deuterium at some 10^{-2} mm Hg, is used The electrodes are formed by copper plates at the ends A concentric copper cylinder conducts the current at the exterior of the glass cylinder The capacitor bank stores approximately 30,000 j when charged to 40 kv Measurements show that the current rises to some 100,000 amp in 1–2 μsec and then fluctuates during several μsec Beginning with the second maximum of the current, neutrons are emitted for 2 or 3 μsec in a continuous manner, their integrated number being of the order of 10^6 to 10^7 per discharge with considerable differences between the separate experiments The fluctuations of the current are apparently produced by compressions of the plasma column (by a factor of ~ 25 in the density) owing to the pinch effect The maximum associated radial mass velocities are of the order of 10^7 cm/sec High energy quanta (in the range 50 to some 100 kv) were looked for, but none was observed The number of neutrons depended critically on the purity of the deuterium From the collective evidence mentioned it is concluded that approximate thermal equilibrium and a temperature of the order of several million degrees is present in the later phase of the discharge In the Aachen experiments, [||] currents of the order of a million amperes and temperatures of the order of 10^6°K appear to have been reached, but further

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[*] Gottingen

[†] The Gottingen Max Planck Institute has just been transferred to Munich where facilities will allow work on a much larger scale than was possible in Gottingen The experiments were carried out in the Physics Division of the Institute whereas the theoretical work was performed in the Astrophysics Division

[‡] The Aachen group will move shortly to a Fusion Institute at the Nuclear Center now under construction near Julich

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[||] W Fucks and H L Jordan *et al*, Physikalisches Institut Techn Hochschule details have not been given. In the Stuttgart work,¶ the corresponding values are 10,000 amp and 10^5°K.

The investigations in Hanover ** and in Kiel †† are related especially to the spectroscopic aspects of the problem, and pertain up to now mainly to the temperature range around or below 100,000°K. Those in Kiel, where special attention is given to transition probabilities, may be characterized as " experimental atrophysics " related to the physics of stellar atmospheres. In the experiments in Hanover, the exploding wire technique has also been used; methods of measuring the temperature in the plasma column have been studied.

Toroidal Discharges

The present experiments in Göttingen ‡‡ are performed with a pyrex torus of 100-cm major diameter and 4.5-cm bore. The coaxial electric field, of the order of several v/cm, is induced by means of a set of copper wires along the surface; a large number of ring-shaped magnetic cores serve to improve the coupling with the plasma column. The capacitor bank stores up to 10^4 j when charged to 0.1–30 kv, the potential used depending on the pressure. A number of coils are distributed around the torus in order to provide a coaxial magnetic field up to 20,000 gauss. The initial pressure of the gas is of the order 10^{-4}–10^{-2} mm Hg and the pre-ionization is effected by RF excitation.

The measurements, which are still in progress, show that during approximately 100 μsec, current densities of the order of some 10^3 amp/cm are reached. The temperature should be of the order of 10^6°K. Spectroscopic measurements and photographic and X-ray emission observations are being carried out. Experiments with larger toruses of 30-cm bore and much larger energies are being prepared, but these can be made only after the transfer of the institute to Munich.

THEORY

Basic Considerations

The Göttingen work originated in 1956 in connection with researches in theoretical astrophysics, in particular on astrophysical applications of plasma physics and magnetohydrodynamics and on the origin of cosmic radiation.[2]

The dependence of the ionization state of added heavier elements on the physical conditions is of interest in connection with experimental techniques of measuring the temperature in a discharge by observing the contours of spectral-line ions, e.g., of oxygen (to be added in a small amount or present as impurities), and the state of ionization of the heavier elements. Given sufficient time, for temperatures around 10^6°K and electron densities below 10^{18} cm^{-3}, the ionization depends only on the temperature, precisely as in the solar corona,[3, 4] since radiative recombination is balanced by collisional ionization (in contrast to the conditions in perfect thermal equilibrium). It has been evaluated for oxygen by Knorr [5] who found that at (and above) $kT = 50$ ev ($\approx$ 500,000°K), there should be $< 0.1\%$ of O^{V} left, the prevailing state of ionization being O^{VII} at that temperature. The results depend critically on the cross sections in question, for which the earlier work of Elwert [4] was used. The time scale in which stationary conditions are approximately reached is of the order of a fraction of a millisecond for this temperature and an electron density of 10^{14} cm^{-3}, but much shorter for higher temperatures. The radiative energy losses by impurities of heavy elements in that temperature range are due mainly to the excitation of discrete levels.[5] At lower temperatures other mechanisms dominate,[6] e.g., excitation of molecules around 20,000°K, where the energy losses have a first maximum, and recombination radiation and loss of non-ionized particles around 10^5 to 10^6°K.

The results of the work of Knorr [5] are compatible with the more recent experiments with ZETA [7] if it is assumed that the electron temperature is much lower than the value of some 10^6°K which apparently pertains to the ions. The radiation losses due to impurities were found to be relatively large in the temperature range 10^5–10^6°K even for small amounts of heavier elements.

The work on the equilibrium of dissociation and ionization and the energy losses of a high temperature discharge in pure hydrogen, begun by Schmidt and Stodiek, has been extended by Schmidt.[8] The equilibrium of dissociation and ionization has been calculated by studying the different elementary processes as a function of the electron temperature for a density interval 10^{12} to 10^{15} H/cm^3. It is found that there are deviations from thermodynamic equilibrium, since the photo processes are not important because of the small optical thickness of a typical gas discharge. Molecular recombination on the walls and the cooling of the neutral particles by impacts with the walls reinforce these deviations. (The interactions of the charged particles with the wall, which might be important under certain conditions, have not been included in this discussion because of the assumption of strong magnetic fields.) The losses of energy by radiation and by the transport of the kinetic energy of neutral particles to the walls due to recombination and charge exchange have been estimated. The maximum losses occur around 15,000°K.

Methods of Heating

Three methods have been studied in some detail: Joule heating by means of azimuthal currents induced in the plasma ring; high frequency radiation, in particular near the eigenfrequencies of the plasma; and, finally, forced oscillations of the magnetic field.

The possibilities of the mechanism first mentioned have been discussed by Grossmann-Doerth,[9] who

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reached the conclusion that it should be effective with reasonable technical means up to temperatures of the order of one million degrees. Heating by high frequency radiation, and in particular, the dependence of the refractive index on the parameters involved, has been investigated by Körper.[10] He found that its effectiveness depends critically on the physical parameters which enter. The third mechanism was recognized to be particularly effective if the oscillation frequency of the magnetic field is near to or larger than the collision frequency of the ions. This heating effect is due to the constancy of the magnetic moment between collisions and to the redistribution of the energy by collisions (" gyro-relaxation "); [11] it has been studied in some detail by numerical integrations,[12] in which attention was paid also to the energy losses of electrons.

The equations for gyro-relaxation given by Schlüter have been integrated by Schmidt [12] for a simple model taking into account the free-free radiation of the electrons. Periodic variations of the magnetic field with a frequency of the order of the ion collision frequency lead to an increase of the temperature with the two-thirds power of the time. Finally, one can reach temperatures of the order of 10^9°K asymptotically if the amplitudes are large enough. Here the time scale for the increase is somewhat larger than the collision time of the final state.

Körper [13] has investigated the problem of heating by high frequency radiation in more detail. From the behavior of the refractive index it follows that one can use frequencies below the geometric mean of the gyro-frequency of the ions and the electrons. The efficiency depends very critically on the phase of the oscillations of the plasma at the surface. Since the wave-length in the plasma is very small compared to the linear dimensions of a normal plasma cylinder, a small variation of the radius, of the frequency, or of the physical parameters will change the phase drastically. Thus, averages over all phases have to be taken into account. The input impedance of the heating coils varies in the same manner with the physical parameters and therefore it is not possible to use the customary high frequency methods for matching the generator to the coil. Variational methods are to be used to find the optimum matching.

Configurations in Magnetohydrostatic Equilibrium

The aim of these investigations is to find the properties of exact solutions, of appropriate general character, of the equation of magnetohydrostatic equilibrium. This equation expresses the exact balance of the gas pressure gradient (or pressure difference) by the magnetic volume (or surface) force given by the vector product of the electric current density and the magnetic field intensity. The majority of those investigations pertain to the case of surface currents and to ring-shaped (toroidal) configurations of the plasma surrounded by a vacuum. The special case of axial symmetry and circular cross section was discussed with potential theory methods in 1957: [14] the properties of the solution with exclusively azimuthal surface currents, i.e., parallel to the generating circle, and the geometry of the magnetic field of exterior origin, which have to be superposed, were given in detail. Jörgens [15] extended the analysis to the more general case of non-circular meridianal cross section, using a stream function related to the magnetic flux, and indicated the locations of the singularities, outside the torus, which correspond to electric currents in exterior conductors. Kippenhahn [16] established necessary and sufficient differential geometrical conditions for the surface of a plasma which must be fulfilled if the plasma is to be in equilibrium with a surrounding magnetic vacuum field and discussed, in particular, the case of azimuth-dependent circular cross sections and currents spiralling around the torus surface.

In all these cases it was found that a component of the electric current parallel to the generating circle is necessary for equilibrium. An attempt, made by Meyer and Schmidt,[17] to prove this property to be true for still more general cases, revealed, however, that, in fact, solutions can be constructed, of torus-like geometry but with neither axial symmetry nor circular cross section, which are characterized by electric currents flowing everywhere with equal surface density along closed lines of meridianal character, but not in meridianal planes. The cross section appears to have at least two maxima along the generating circle. If there are only a few maxima, the relative variation of the cross section along the generating circle appears to be fairly large; if, in contrast, there are many extrema, the surface must have a rippled structure at least on the " inner " side (as seen from the centre of the aperture of the torus). The magnetic lines of force, in this case, are everywhere perpendicular to the current if there is no magnetic field in the plasma.

Axially symmetric solutions with volume currents have also been considered, using the stream-function method.[18] For cylindrical symmetry, without dependence on z, the general solution has been given for the case of vanishing volume-forces; i.e., " force-free " magnetic fields.[19]

The investigations of Meyer and Schmidt [17] on configurations of torus-like character with surface currents with vanishing line integral of the total current across closed meridianal lines have been completed. In addition to the analytic treatment by means of a special coordinate system, a simple model of the surface currents and fields has been constructed, using paper strips of suitable shapes, folded and joined together in a special way; this model is a true analogue to the physical situation for the case of vanishing magnetic field in the plasma, and it allows one to survey existing solutions and their general properties rather easily.

Stability Problems

In 1957, general conditions for the stability of hydromagnetic configurations were investigated in Göttingen, by Hain, Lüst and Schlüter,[20] on lines largely analogous to those along which the simultaneous work in Princeton has been done. For the special case of cylindrical symmetry the stability of certain specific configurations has been treated by Hain *et al.*[21] by studying the various possible modes of disturbances. For $j \sim (r^2 + a^2)^{-2}$ and homogeneous coaxial magnetic fields, instability was found, the rates of growth decreasing with increasing coaxial magnetic field. More general cases for the current distributions and for the coaxial magnetic field are still being investigated. Since the emphasis in this work is to compare the properties of various current distributions, no regard is being taken of the possible stabilizing influence of external conductors. The stability of a linear discharge in an exterior longitudinal magnetic field with surface currents has been investigated theoretically by Jordan[22] in Aachen.

Of the work done by Hain, Lüst and de Vries,[21] a first paper by Hain and Lüst[23] has been completed which contains the discussion of the more special case.

To study the problem of the so-called exchange stability, Meyer[24] has extended an investigation by Kruskal and Schwarzschild[34] in such a manner that a plasma with gravity and horizontal magnetic field is supported by a vacuum magnetic field, which is also horizontal but is skew to the inner one. With the approximation used by Kruskal and Schwarzschild the unstable perturbations found by them can be stabilized. The stabilization depends on the angle between the two fields and on their relative strengths. The calculations are extended to the case of large wave-length. It is shown that here unstable perturbations with very large wave-length exist for arbitrary angles and strengths of the magnetic field.

Particle Losses by Drift Motions

Since the macroscopic equations of plasma physics give information only on the net balance of mass, momentum and energy, their discussion must be supplemented by studying, from the microscopic point of view, the motion and, in particular, the containment of individual charged particles in suitable electromagnetic fields. This can be done with the help of methods analogous to those developed earlier in connection with the theory of aurorae and of cosmic radiation.

For axially symmetric magnetic fields, Lüst and Schlüter[25] derived an integral for the azimuthal component of the velocity which shows that this component depends only on the meridianal component of the magnetic field. It may be seen, in addition, that it is not affected by a meridianal electric field (as produced, e.g., by pressure gradients). Also, the influence of an azimuthal electric field (as, e.g., generated by means of a transformer) can be included. This leads (by the energy integral) to the existence of so-called forbidden regions which show that all particles within a certain range of initial conditions can, indeed, be contained in such fields with a suitable meridianal component of the magnetic field; that is to say, with azimuthal electric currents. For the simplest cases of toroidal symmetry, the structure of the forbidden regions and the geometrical properties of a number of individual trajectories have been studied by Fisser and Kippenhahn.[26] The more special case of the motion in an azimuthal curl-free magnetic field of cylindrical symmetry (without dependence on the z coordinate) has been discussed in detail by Hertweck.[27] The particles show, in general, a drift parallel to the z axis. The magnitude of this drift has been calculated using the approximation method of Alfvén and by numerical solutions of the equations of motion. A comparison of the results indicates the range of validity of the approximation. The constancy (" adiabatic invariance ") of the magnetic moment M or of the angular momentum of the gyrating motion of a charged particle in a magnetic field has been the subject of an investigation of Hertweck and Schlüter,[28] who established that, for a homogeneous field varying in time, the relative variation of M goes to zero at least exponentially with the rate of change of the magnetic field strength.

Fisser and Kippenhahn[26] have developed methods by which the properties of given trajectories in relation to the geometry of the allowed and the forbidden regions can be surveyed easily. These methods can also be used to discuss problems of particle loss. The authors have applied these methods to the special case of the magnetic field of an ideal circular ring current.

Magnetohydrodynamic Shock Waves

In connection with the theory of controlled fusion, as well as in that of the acceleration of cosmic rays in cosmic magnetic fields, the structure of hydromagnetic shock waves, in particular their thickness as compared to the kinetic mean path and the gyro-radius, is of importance. For this reason, the special case of a stationary plane wave, propagating in the absence of collisions with unchanging form and speed perpendicular to a magnetic field, in a completely ionized gas (plasma) of " zero " temperature, has been investigated rigorously by Davis, Lüst and Schlüter.[29] In spite of these severe restrictions (in addition, the Debye length has been assumed to be small compared with the gyro-radius), the results seem to make it possible to draw certain general conclusions regarding the structure of hydromagnetic shocks under more general circumstances.

The macroscopic equations for this case are identical with the microscopic ones and both ions and electrons have the same velocity $U(x)$ and the same particle flux F in the direction of propagation x. Furthermore, as in the hydromagnetic analogue to the Rankine–Hugoniot equations of ordinary gas dynamics,[30, 31] $FU(x) + B^2(x)/8\pi$ is an integral of the motion. If one includes in the equations of motion the terms arising from the inertia of the current-carrying particles, one finds that $B(x)$ is determined by the second order differential equation of a one-dimensional, undamped, anharmonic oscillator. Thus, the oscillatory character of the solutions is obvious, and the wave-length turns out to be of the order of the geometric mean of the gyro-radii of the ions and the electrons when moving with the velocity of the wave. The assumption that $U(x)$ is everywhere positive on all trajectories limits the Mach number $\overline{U}/B(4\pi\varrho)^{-1/2}$, ϱ being the mean mass density, to values below 2. Numerical integration of the differential equation has been carried out for a systematic array of values of the parameters. For the limiting case of an oscillatory wave train when the frequency goes to zero, one gets solitary waves with Mach numbers between 1 and 2. In the complete absence of collisions, all particle trajectories are symmetrical about their extrema and no stationary shock waves can be found. But if a few collisions are admitted, both the trajectories and the variation of B with x become asymmetrical. It appears that a shock will start with a wave that is almost a solitary wave followed by some kind of wave train, and that after the particles have travelled a distance of the order of a mean-free path there will be a fairly smooth flow of gas of higher temperature. Thus the total shock thickness will be of the order of a few mean-free paths while its detailed structure should be related to the waves found here. In the limit in which the time between collisions goes to infinity, the only solutions connecting regions of different properties must be non-stationary solutions; thus the stationary waves treated in this work are completely inadequate for the discussion of such cases.

The case of non-zero temperature, both with isotropic and non-isotropic pressure in the plane perpendicular to the magnetic field, has been studied by the same authors and Hain,[32] using mainly the macroscopic formulations of the equations. For the first case, the results do not seem to differ essentially from those obtained for the case of zero pressure. The second case is still being studied; for this, new equations have been derived by discussing suitable moments of the Boltzmann equation.

The investigations of Davis, Hain, Lüst and Schlüter [33] on the structure of hydromagnetic shock waves with non-zero pressure (isotropic perpendicular but not parallel to the magnetic field) have been completed. The results are very similar to the case of zero pressure. To get shock waves, dissipative mechanisms such as viscosity, heat conduction, electrical resistivity and gyro-relaxation were taken into account. A discussion showed that the last two are the most important mechanisms. The dissipation has the consequence that a solitary wave, which one obtains under certain initial conditions and which is symmetrical without dissipation, is somewhat asymmetrical. This solitary wave is followed by a train of waves with decreasing amplitude until finally the homogeneous subsonic solution is reached. The whole thickness of this region will be of the order of the mean-free path and may be very large compared to the thickness of the solitary wave.

That concludes my very incomplete and brief survey of the work that has been done, mainly in the last two years, in the Federal Republic of Germany. You will have noticed that hitherto there has been very much duplication of effort, but I should like to say that I share very much the satisfaction that has been expressed by earlier speakers that now the period of duplication and non-communication has apparently come to an end and that international cooperation gives better promise for the future of physics.

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