The Stellarator Concept

By L. Spitzer Jr.*

The confinement of a very hot ionized gas under laboratory conditions would appear to require the use of a strong magnetic field. While an enormous variety of magnetic configurations is possible, two relatively simple geometries may be set apart at the outset, both involving infinite cylinders with axial symmetry. In the first of these, the magnetic field is produced by an axial current flowing through the gas. This configuration is the so-called "pinch effect", which has been extensively discussed in the recent literature. In the second simple geometry, the magnetic field is parallel to the axis, and is produced by external currents, flowing in solenoidal windings encircling the plasma. Such a straight cylinder, with an externally produced field, forms the basis of the "pyrotron" proposed by R. Post.

The stellarator, like the pyrotron, utilizes an external magnetic field, produced by coils encircling a tube containing the heated gas. However, instead of a finite cylindrical tube, the stellarator employs a tube bent into a configuration topologically like a torus, without ends. Such a tube will be referred to as "toroidal". Relative to the pyrotron, this configuration has the advantage, in principle, of permitting more complete confinement, since end losses are eliminated. Relative to the pinch discharge, the stellarator offers the advantage, again in principle, of permitting equilibrium in a steady state. Both these advantages might be of importance in a controlled thermonuclear reactor. The present paper outlines the basic concepts involved in the confinement and heating of a gas in a stellarator.

One basic complexity in the stellarator results from the fact that the simple torus, in which the magnetic lines of force are circles centered at the axis of symmetry, does not permit equilibrium confinement of a plasma in a straightforward manner. Microscopically this result follows at once from the particle drifts associated with the inhomogeneity of the magnetic field. These drifts, first pointed out by Gunn, and analyzed in detail by Alfvén, produce motions perpendicular both to $\mathbf{B}$ and to $\nabla B$; these motions are in opposite directions for electrons and positive ions. The resultant separation of charges produces electric fields which sweep the ionized gas towards the wall. Macro-

* Project Matterhorn, Princeton University, Princeton, New Jersey.
that the magnetic field is non-zero at all points in the cross-sectional plane. Then through any point, such as $P_1$, for example, a line of force will pass. This line of force may now be followed, in the direction of the magnetic field, along the stellarator tube, until it has completed one circuit of the toroidal tube and intersects the cross-sectional plane at a point $P_2$, as shown in Fig. 1. In the ideal torus, $P_2$ coincides with $P_1$ and every line of force is closed after one circuit. If the degeneracy of the torus is removed, $P_2$ no longer coincides with $P_1$.

One might raise some question as to whether any of the points $P_2$ lie inside the tube wall. It is not difficult to construct a toroidal tube with a solenoidal winding such that most of the lines of force stay within the tube for at least one circuit. The interesting question is rather which lines intersect the tube wall after many circuits, and we may safely assume that for most points $P_1$, $P_2$ will exist and will lie inside the tube wall. If we exclude, for the moment, those areas for which $P_2$ does not lie within the tube wall, then for every point $P_1$, we have a point $P_2$; the transformation of the set of points $P_1$ into the set of points $P_2$ is called a “magnetic transform” of the cross-sectional plane.

A magnetic transform of this type is characterized by an important property. Let us take the density of points $P_1$ to be proportional to the component of $\mathbf{B}$ normal to the cross-sectional plane. Since the plane is transformed into itself by the magnetic transform, the density of these points must be the same function of position before the transform as after. A transform with this property has been called “measure preserving” by Kruskal.

Let us now assume that the magnetic transform is also primarily rotational. This condition states that at least the outer portions of the plane rotate in the same direction in a single transformation. As we shall see subsequently, there are several ways of achieving this result in a toroidal system. According to Kruskal, it follows from this assumption and from the Brouwer fixed-point theorem that there must be at least one point in the plane which is transformed into itself. In some types of stellarator, the magnetic transform involves only small deformations of the plane, in addition to a general rotation. In such systems there will be only one point that transforms into itself, and only one line of force that is closed after a single circuit around the stellarator. This line is called the “magnetic axis”.

A second basic result on measure-preserving rotational transforms, also established by Kruskal, is that any point, other than one of the fixed points, when followed through successive transformations, will not move far from a single closed curve. This result is illustrated in Fig. 1, where the points $P_3$, $P_9$, $P_4$ generated by successive magnetic transforms of the point $P_1$, all lie close to a single closed curve. Thus a single line of force, after many circuits around the tube, generates a magnetic surface.

More precisely, let us introduce coordinates $r$, $\theta$ in the cross-sectional plane depicted in Fig. 1; $\tau$ may be measured from the magnetic axis, denoted by the point $O$. The value of $\Delta\theta$ between $P_1$ and $P_2$ is denoted by $\tau$, and is called the “rotational transform angle”. Let $\theta$ equal 0 at $P_1$, and let us assume that $\theta$ equals $2\pi$ for the point $P_9$. The distance $\Delta r$ from $P_1$ to $P_9$ is called the “deviation from closure” of the point $n$. Evidently $\Delta r$ measures how far the line of force has strayed from a closed curve. Kruskal has shown that $\Delta r$ decreases more rapidly than any power of $1/n$. Hence one may surmise that $\Delta r$ varies as about as $\exp(-Kn)$, where $K$ is some dimensionless constant.

The physical reason for this result can best be understood in the special case when the normal component of the magnetic field is constant over the cross-section plane. The analysis of more general systems may be reduced mathematically to a consideration of this special situation. In this case, the density of points in the plane must remain constant in successive transformations. Let us now draw, in the cross-section plane, a closed curve connecting point 1 and its successive transformed points as smoothly as possible. Since the H-transform now preserves areas, the total area enclosed within this curve must remain constant in successive transformations. Hence all points on the curve cannot move inwards with successive transformations. If some move in, others must move out.

In the special case in which the $\theta$ coordinate of every point returns to its original value after $n$ transforms, it is possible for some points on the curve, together with all their transformed points, to move steadily in, while the points between move steadily out. Thus the closed curve develops wrinkles in successive transformations; this rate of wrinkling decreases very rapidly with increasing $n$. In the more general case in which the $\theta$ coordinate of a point never returns exactly to its initial value (to within a multiple of $2\pi$), one would expect a further averaging out of these radial motions to occur. Even if the transform angle, $\tau$, is not small, the deviation of a line of force from a magnetic surface should be small; if necessary, a group of transforms, which give an $\tau$ very nearly a multiple of $2\pi$, can be taken as the basic transform, and Kruskal's theorem used to demonstrate that a line of force never departs very far from a single closed surface. We
shall therefore assume in the following discussions that each line of force does in fact generate a single magnetic surface, and that a single line, if followed sufficiently far, comes arbitrarily close to any point on this surface.

Methods for Producing a Rotational Transform

To produce a rotational transform in a vacuum field it suffices to twist a torus out of a single plane. Virtually any such distortion will remove the degeneracy of the ideal torus and produce a rotational transform.

The simplest such system is the figure-eight, historically the first geometry proposed for a stellarator. The topography is indicated in Fig. 2. The 180° curvature sections LM and KN are in planes each tilted at an angle, α, to the parallel planes in which the reverse-curvature sections IK and MN are placed. Figure 3 indicates that a rotational transform is present. This figure represents cross-sectional planes at K, L, M, and N, all as seen from one end of the device. The point O represents the magnetic axis, while P1 and P2 denote the successive intersections of a single line of force with a cross-sectional plane at K. Intersections of this line of force with the other three planes are denoted by crosses. The solid lines represent the path followed by the magnetic axis. From K to L and from M to N there occur simple translations of the cross-sectional plane, while from L to M and from N to K, a rotation about an axis, inclined at an angle α to the vertical, is involved.

Evidently, the line of force which passes through P1 in plane K, and is then followed through one circuit, through planes L, M, and N, intersects plane K again in a point P3, rotated by a rotational transform angle, η. Examination of the figure shows that for this geometry

$$\eta = 4\alpha$$

(1)

Moreover, η is independent both of distance, r, from the magnetic axis, and of angle, θ. In an actual system, mutual interference between the stray fields of the curving sections LM and MN will modify these results slightly, but the general features remain unchanged.

A rotational transform angle may be produced in a variety of other ways. When a plasma current is flowing around the simple torus, a rotational transform appears, despite its absence in the vacuum field. If steady-state confinement is envisaged, however, a rotational transform must be present in the vacuum field. The most important alternative method for producing such a rotational transform is the use of a transverse magnetic field, whose direction rotates with distance along the magnetic axis. We shall follow a line of force and show that a transform angle appears.

Let us consider an infinite cylinder, with coordinates r, θ, and z. We consider the r and θ coordinates of a single line of force as z increases. The coordinates of points along such a line of force are related by the differential expression

$$\frac{dz}{dr} = \frac{r d\theta}{B_z}$$

(2)

Suppose now that $B_r$ and $B_\theta$ are produced by 2ℓ wires, wound helically, on the outside of the cylinder, so that currents flow in opposite directions in adjacent wires, and with a pitch 2π/ℓ. If we denote the tube radius by r0, and if $r_0/ℓ$ is small compared to unity, then for small r/r0 we have, from the appropriate solutions of Laplace's equation

$$B_r = A r^{1-\ell} \sin(\ell \theta - \ell z)$$

(3)

$$B_\theta = A r^{1-\ell} \cos(\ell \theta - \ell z)$$

(4)

where $A$ is a constant characterizing the strength of the transverse field. There is also a component of $B_z$ associated with the current in the helical wires, but its magnitude is less than $B_r$ by a factor kr. We assume that $B_{z0}$, the component of $B_z$ produced by a separate solenoidal winding, is the dominant axial field.

Let us follow a line of force whose coordinates are $r_0$ and $\theta_0$ in the absence of the transverse field. Equation (2) may now be integrated by means of a power-series expansion in $A$, the coefficient in Eqs. (3) and (4). To first order in A, r = r0 and $\theta = \theta_0$ vary as the cosine and sine, respectively, of $\ell \theta - \ell z$; to this order, the line of force is a helix, and its intersection with a plane moving along the z direction is a circle. Solving to second order in A, we must take into account that for $\ell$ equal to 2 or more, $B_{z0}$ is larger on the inside ($r$ less than $r_0$), where $B_z$ is positive, than on the outside ($r$ greater than $r_0$), where $B_z$ is negative. As a result, the positive values of $d\theta/dz$ in Eq. (2) more than offset the negative ones, and $\theta$ increases systematically with increasing z. A detailed integration by Johnson and Oberman shows that

![Figure 2. Top and end views of a figure-eight stellarator](image-url)
$\omega$, the increase of $\theta$ in one period $2\pi/l$ of the helical field, is given by

$$\omega = \frac{nA^2}{kB_0^2} \left[ 2(l - 1) + h^2 + O(h^4) \right].$$

The term in $(h^4)$ is included to give results for $l$ equal to unity; in this case a rotational transform arises from the variation of $B_0$ with $r$. The configuration for which $l$ is unity, with a helical magnetic axis, and its use for confining a plasma were proposed by Koenig, who first studied the use of helical fields in connection with the stellarator. For small $kr$, an appreciable $l$ is more readily obtained with transverse fields of higher multiplicity for which the transverse field vanishes at the magnetic axis. The properties of such fields, and of the magnetic surfaces associated with them, have been extensively studied by the Matterhorn theoretical group, under E. Frieman.

In the experimental program at Matterhorn, described in the subsequent papers, rotational transforms have been produced both with the figure-eight geometry and with transverse fields, with $l$ equal to 3. In either case, the existence of a rotational transform is readily confirmed experimentally by observation of a narrow electron beam, which follows the line of force. As pointed out subsequently, the chief advantage of the multipolar transverse field over the figure eight is the greater hydromagnetic stability which, in theory, it should yield.

**CONFINEMENT**

The objective of confinement theory is to demonstrate that the number of particles striking the tube wall is negligibly small. An exact proof would presumably require a detailed solution of the Boltzmann equation, together with the field equations. For the complex magnetic configuration of the stellarator this would be a difficult task indeed. An approximate treatment will be followed here.

First we shall use the macroscopic equations for the plasma, based on a number of simplifying assumptions. With these equations, together with the field equations, we can show that an equilibrium situation is possible, in which the plasma is macroscopically confined. Since particles moving with some particular velocity might conceivably escape, even though the plasma as a whole were confined, we shall next consider the trajectories of single particles, in the electric and magnetic fields determined from the macroscopic equations. These two approximate methods, taken together, indicate nearly perfect confinement, if collision and cooperative phenomena are ignored. The hydromagnetic stability of these equilibria has been extensively analyzed by Frieman, Kru skal and their collaborators. The results of this analysis, summarized briefly below, indicate stable equilibrium under certain conditions. All these results, taken together, encourage the belief that magnetic confinement in a stellarator may be adequate for a controlled thermonuclear reactor.

**Macroscopic Equations**

The two-fluid equations for the velocity and electric current of a fully ionized gas are well known. For the derivation of these equations and their application here the following non-trivial assumptions are made:

(a) The electrical resistivity $\eta$ is negligibly small, and the mean free path very long.

(b) Over a distance of one radius of gyration the relative change of all macroscopic quantities is small.

(c) The transverse and longitudinal pressures, $p_t$ and $p_l$, are equal.

(d) All macroscopic quantities are independent of time at each position.

(e) The mean macroscopic velocity, $v$, vanishes.

Assumption (a) is clearly a good approximation for a rarefied gas at very high temperatures. Collisions between particles will produce some diffusion across the lines of magnetic force, but this rate is so slow, compared both with the diffusion rate observed and with the diffusion rate that could be tolerated, that we may neglect collisions entirely in most discussion of confinement. Once this first assumption has been made, assumption (b) is required for use of the macroscopic equations. This second assumption has a number of important consequences. Since the sheath thickness is generally much less than the radius of the gyration, the plasma must be characterized by approximate electrical neutrality, with the electron density $n_e$ closely equal to $Z$ times the ion density $n_i$, where $Z$ is the mean ionic charge. In addition, this assumption leads to the result that the material stress tensor is diagonal, provided that the principal axis is parallel to the magnetic field; the three components are then $p_1$ parallel to the field and $p_t$ in the two directions perpendicular to the field. In any device much larger than the radius of gyration, assumption (b) should be approximately valid in a steady state, except in a thin layer near the wall where this assumption must break down. Assumption (c) should also be valid in a system where any changes are even slower than the time between collisions, since collisions will clearly tend to equalize $p_t$ and $p_1$.

Assumption (d) is of critical importance in the analysis. The assumption of a steady state immediately implies that the plasma is quiescent, and excludes turbulence, oscillations, instabilities, and other cooperative phenomena of the sort normally present in gaseous discharges. Any equilibria obtained on the basis of this assumption may not necessarily be stable against various types of disturbances. The last assumption replaces the more usual one, which partly results from assumption (b), that quadratic terms in $v$ and $J$ are negligible. This more stringent condition is not so arbitrary as might first appear. In fact, the near vanishing of $v$ is a simple consequence (see Ref. 16, pp. 44-45) of the equation of motion. The argument is that in most heating methods there are no appreciable forces tending to produce any
momentum, and hence the macroscopic velocity must be vanishingly small. A fuller analysis of effects associated with macroscopic velocities would be desirable.

On the basis of these assumptions, the equations of equilibrium become, in emu.

\[ \mathbf{j} \times \mathbf{B} = \nabla \mathbf{p} \]  
\[ \nabla \times \mathbf{B} = 4\pi \mathbf{j} \]  
\[ \mathbf{V} \cdot \mathbf{B} = 0. \]

The generalized Ohm's Law determines the electric field, in terms of the pressure gradient for the positive ions, while Poisson's Law then gives the charge density. Since neither of these quantities is of particular significance in the present analysis these equations may be omitted.

On the basis of these equations it was shown several years ago that no simple equilibrium is possible in a torus if the lines of force are assumed to be circles centered at the axis of symmetry. We introduce cylindrical coordinates \( R, \phi, \) and \( z, \) with \( z \) taken along the axis of symmetry of the torus; we assume that only \( B_\phi \) differs from zero, and that all quantities are independent of \( \phi. \) If we now take the curl of Eq. (6), eliminating \( j \) by means of Eq. (7), we obtain

\[ \frac{1}{R} \frac{\partial B_\phi}{\partial z} = 0. \]

Thus, for equilibrium, either \( R \) must be infinite or the magnetic field (and pressure) must be independent of \( z. \) Equation (9) for toroidal fields \( (B_R = B_z = 0) \) corresponds to a theorem by Ferraro in poloidal fields \( (B_z = 0) \) in connection with the theory of magnetic stars. It may be remarked that the left-hand side of Eq. (9) is simply \( 4\pi \nabla \cdot j; \) i.e., the currents required by Eq. (6) for the simple torus possess a non-vanishing divergence.

Plasma Equilibrium in the Stellarator

We next apply these equations to a stellarator, characterized by the existence of toroidal magnetic surfaces. From Eq. (6) it is evident that \( \nabla \mathbf{p} \) is perpendicular to \( \mathbf{B} \) and hence \( \mathbf{p} \) is constant along a line of force and must therefore be constant over an entire magnetic surface. Similarly \( j \) is perpendicular to \( \nabla \mathbf{p} \), and therefore \( j \) must be parallel to the isobaric surfaces, and hence to the magnetic surfaces. We denote by \( j_\perp \) the component of \( j \) perpendicular to \( \mathbf{B} \) (and to \( \nabla \mathbf{p} \)) and by \( j_\parallel \) the component of \( j \) parallel to \( \mathbf{B} \).

An important result, which permits plasma equilibrium in the stellarator, will now be established. Equation (6) determines \( j_\perp \) in terms of \( \mathbf{B} \) and \( \mathbf{p}. \) In general, \( \nabla \cdot j_\perp \) will not be zero. In the torus, the divergence of this current gives rise to charge separation which destroys equilibrium, since conditions are uniform along each line of force and an accumulation of electric charge cannot readily leak out across the lines of force. In the stellarator, currents along the lines of force are possible, and charges may flow from a region where \( \nabla \cdot j_\perp \) is positive to another where \( \nabla \cdot j_\perp \) is negative. The total divergence of \( j_\perp \), integrated over the volume element between two adjacent magnetic surfaces, must vanish, as may be seen by use of Gauss's Theorem, together with the fact that \( j_\perp \) is parallel to the magnetic surface. Hence currents flowing along the lines of force can cancel out the divergences in \( j_\perp \), and will lead to a current system in which \( \nabla \cdot j \) vanishes.

The existence of solutions to Eqs. (6), (7), and (8), in a system characterized by magnetic surfaces, has been analyzed in an elegant manner by Kruskal and Kulsrud, who also take diffusion into account. Here we follow an earlier and simpler treatment, and demonstrate the existence of solutions to Eqs. (6)-(8) by the simple artifice of showing how to construct such solutions. We assume that \( \mathbf{p} \) is a small quantity, and obtain a solution by successive iteration. The zero order solution is taken as the vacuum solution, with \( j_0 \) the current in the external coils, and \( B_0 \) the vacuum field, with its magnetic surfaces. The solution of order \( n \) is then defined by the equations

\[ j_n \times B_{n-1} = \nabla \mathbf{p}_n \]  
\[ \nabla \times B_n = 4\pi j_n \]  
\[ \mathbf{V} \cdot B_n = 0. \]

Evidently, \( \mathbf{p}_n \) must be assumed constant on the magnetic surface obtained in the previous iteration, but is otherwise arbitrary. It may generally be assumed that \( \mathbf{p}_n \) is in each case a monotonically decreasing function of distance from the magnetic axis. If the solution converges, it must evidently yield a solution of Eqs. (6)-(8).

In principle, this iteration scheme is straightforward, but in practice the algebra is cumbersome. It turns out that the chief obstacle to convergence is the distortion of the magnetic surfaces by the magnetic fields associated with the plasma current \( j_0 \) along the magnetic field. Even though this current density is low, the currents must travel an appreciable distance, and even the weak magnetic field associated with these currents may distort the magnetic surfaces out of all recognition.

The approximate condition for convergence in a simple figure-eight device may be derived in the simplest manner. The transverse current \( j_\perp \) is evidently about equal to \( j/Br \), where \( B_0 \) is the axial vacuum field and \( r \) is the distance of the tube wall, or plasma boundary, from the magnetic axis. The divergence of \( j_\perp \) results from the inverse proportionality, between \( B_0 \) and \( R \), and is about equal in magnitude to \( j/Br^2 \). Where we take \( R \) to be the radius of curvature of the magnetic axis. The divergence of \( j_\parallel \) is also equal to this quantity, and over a tube length equal to \( r \), \( j_\parallel \) will build up to about \( j/Br \). Hence \( j_\parallel \) is of the same order of magnitude as \( j_\perp \). Over a tube cross-section, \( j_\parallel \) will have opposite directions on opposite sides. The magnetic field on the axis, due to this plasma current, will be of the order of \( 2\pi j_\perp \) or \( 2\pi j_\parallel /B_0 \). The new magnetic axis will therefore be inclined at an angle \( 2\pi j_\parallel /B_0 \) relative to its direction in vacuo. The condition for negligible distortion of the
magnetic surfaces is that the deviation of the magnetic axis be small compared to $r$. If the currents $j_{i}$ flow along half the axial length, $L$, of the machine, on the average, before cancellation, this condition yields

$$\beta \ll 8r/R;$$  

(13)

where $\beta$ is defined by

$$\beta = 8\eta p/B_{0}^{2},$$  

(14)

and $p$ is evaluated on the magnetic axis. A more precise discussion $^{19}$ taking into account the detailed variation of $j_{i}$ over the cross-section, yields a coefficient $16\eta r$ instead of $8$ in (13); this computation assumes that $L$ much exceeds $2\pi R$ and that the transform angle $4\alpha$ (see Fig. 3) is small. If inequality (13) is not satisfied, the method of iteration fails, and it is not known what type of solution, if any, may exist.

In an infinite cylinder, values of $\beta$ as great as unity might be envisaged. In the figure-eight stellarator, of the type shown in Fig. 2, $r/L$ can scarcely exceed $0.02$, and $\beta$ must therefore be small compared to $0.1$. If the rotational transform is produced by transverse fields, however, the transform angle, $\alpha$, for the device in $r/R$, must therefore be small compared to $0.1$. Inequality (13) is not satisfied, the method of iteration fails, and it is not known what type of solution, if any, may exist.

The question of the hydromagnetic stability of such configurations has been extensively studied by the Matterhorn theoretical group, under E. Frieman. Basic concepts and methods of analysis have been published by Bernstein, Frieman, Kruskal and Kulsrud, $^{23}$ with application to the stellarator in the paper by Johnson, Oberman, Kulsrud and Frieman. $^{8}$ Because of the importance of this work, the results will be summarized briefly here.

Instabilities tend to be most marked if the lines of force can interchange positions with the least possible bending. In the case of an axially symmetric field, if $B_{0}$ vanishes everywhere, the lines of force can interchange positions without bending, and if bulges are present in the field the plasma is unstable. However, if a $B_{0}$ component is present, so that lines of force are helices about the cylinder axis, and if $B_{0}/r$ increases with $r$ so that the pitch of the helices decreases with increasing $r$, then the outer and inner lines of force cannot be interchanged without appreciable bending, and the plasma tends to be stable. In the same way, if the transform angle, $\alpha$, varies with $r$ in a stellarator, the outer lines of force are topologically different from the inner ones, and the plasma is stable against all hydromagnetic disturbances, provided that $\beta$ is less than some critical value, $\beta_{c}$. Computations of $\beta_{c}$ for a cylinder, with helical transverse fields, with $l = 3$, added to an axial confining field, indicate $^{8}$ that values of $\beta_{c}$ as great as $0.1$ could be obtained if the approximate theory could be trusted somewhat beyond its range of validity. There is some reason to believe that corrections for finite radius of gyration may increase $\beta_{c}$ by a factor of about $2$, although the theory is still very incomplete in this respect. An experimental test of this theory has not yet been obtained, although the corresponding theory applied to kink instability in the stellarator (see below) is apparently in close agreement with the observations. The maximum value of $\beta$ for which a stellarator plasma is stable can probably best be determined by experiment.

### Single Particles

In the absence of collisions, confinement of single particles will be shown to follow quite generally from the existence of a rotational transform and from the asymptotic behavior of a gyrating particle in a strong magnetic field. It has been shown by Kruskal $^{23}$ that the magnetic moment, $\mu$ (about equal to $m\omega r^{3}/2$), of a gyrating particle is constant to all orders of $ak$, where $a$ is the gyration radius and $k$ is about $|\nabla|ln B|$. Constancy of $\mu$ to first order in $ak$ had previously been demonstrated by Alfvén, $^{3}$ and to second order by Hellwig. $^{24}$ Similarly, Kruskal has also shown $^{23}$ that the motion of the guiding center is independent of the phase of gyration to all orders of $ak$. Kruskal’s theory does not yield either a simple definition of $\mu$, to all orders in $ak$, nor yet a simple definition of the guiding center, but shows that such definitions must exist and how, in principle, to construct them. In consequence of these results, we may assume that for each particle the total energy, $W$, is not the only integral of motion, but there exists also a second integral, the magnetic moment, $\mu$.

Thanks to these results, it may now be shown that successive intersections of particles with a particular cross-section plane, similar to that discussed above, produce a transformation similar in its properties to the magnetic transform generated by successive intersections of a line of force with this same plane. We restrict consideration to particles within ranges $dW$ and $d\mu$ centered at some energy, $W$, and some magnetic moment, $\mu$, and let the density in phase space, within these narrow ranges of $W$ and $\mu$, be constant everywhere. Within an interval of time $\Delta t$, the guiding centers of these particles will intersect the cross-sectional plane at a number of points, $P_{1}$. The density of such points will be a known function of position in the plane, depending on the magnetic field $B$, and the electric potential $\Phi$, through the two integrals of motion.

Each particle whose guiding center has intersected the plane at a point $P_{1}$ will ultimately cross the plane again, at a point $P_{2}$. Normally, if a particle passes through a point, the three components of its velocity, $w$, are arbitrary, and its subsequent trajectory is not determined. In the present case, the two integrals of motion determine $w_{\perp}$ and $w_{||}$, and the third velocity component, the phase of gyration, has no effect on the motion. Hence to each point $P_{1}$ there corresponds one and only one point $P_{2}$. Moreover, the particles which have produced all the points $P_{1}$ in the time $\Delta t$ will all...
produce points $P_0$ within the same time interval, and hence the density of intersection points in the transformed plane will be the same function of position as in the original plane. Thus this "particle transform" is measure-preserving in the same sense as is the magnetic transform discussed earlier.

From the same arguments as before it follows that free particles are confined in a stellarator to a very high approximation, provided that the particle transform is primarily rotational. Such a transform will be assured for particles whose velocity is mostly parallel to $B$ ($w_\parallel > w_\perp$), so that no reflection can occur from regions of relatively high field. The rotational magnetic transform guarantees a rotational particle transform for such particles. Qualitative experimental confirmation of this prediction is obtained from observations of runaway electrons reported by the Matterhorn experimental group$^{10}$ under M. Gottlieb. Electrons travelling at speeds near the velocity of light are observed to make about $5 \times 10^6$ circuits around a stellarator, during the ten milliseconds or so after the applied voltage is reduced to zero, but the confining field is still moderately high.

For particles which are trapped between two regions of relatively high field, or which are moving at a relatively very slow rate along the magnetic field, further arguments must be invoked to guarantee a primarily rotational particle transform. Two separate mechanisms are important. Firstly, the diamagnetic effect of the plasma produces a radial gradient of the axial field, and this inhomogeneity produces a drift of guiding centers about the magnetic axis. Secondly, a radial electric field will produce a similar rotation. Such an electric field is required by the assumption that the macroscopic velocity vanish, since only a radial electric field can cancel out, in a steady state, the velocity associated with a radial pressure gradient. It has been shown by Spitzer$^{28, 56}$ that such a radial field arises naturally when the gas is ionized and heated. A more detailed analysis of these effects$^{27}$ indicates that these two effects produce adequate rotation about the magnetic axis to guarantee confinement for most particles with relatively low $w_\parallel$.

One important exception should be noted. For some particles, the different effects producing rotation may cancel out, leaving only the unidirectional drift produced by the curvature of the field. The seriousness of this effect is reduced by two factors. As distance from the magnetic axis changes, the different mechanisms producing rotation will change in different ways, and the cancellation will disappear. Moreover, the cancellation will be exact only for a particular particle energy, $W$, and a particular magnetic moment, $\mu$; collisions will change these quantities, and restrict the time during which a unidirectional drift will occur. These effects have been discussed elsewhere$^{5, 27}$; the analysis, while admittedly approximate and incomplete, indicates that this process is not of great importance, although it may increase the diffusion rate somewhat above the value given by electron-ion collisions.
increases so rapidly with increasing energy that electrons which are in the tail of the Maxwellian distribution and which are sufficiently energetic to excite and ionize atoms may gain very appreciable energies in one free path.

Before any experiments had been carried out on ohmic heating, it was pointed out by Kruskal\(^\text{25}\) that discharges in the stellarator should be subject to kink instability. This hydromagnetic instability was predicted for heating currents greater than the critical current that will reduce the transform angle to 0 (or increase it to \(2\pi\)). This critical current is now generally known as the "Kruskal limit".

Extensive observations of ohmic heating have been carried out by the Matterhorn experimental group, under M. Gottlieb, and are reported in several subsequent experimental papers.\(^{10-13}\) The data indicate clearly that nearly complete ionization is attained, with electron and ion temperatures in the neighborhood of \(5 \times 10^5\) °K. The occurrence of the predicted kink instability, at currents above the Kruskal limit, is fully verified experimentally. However, the detailed predictions of the ohmic heating theory are not substantiated, presumably because of the non-Maxwellian distribution of electron velocities. In support of this hypothesis, intense X-rays from runaway electrons are observed, with energies up to \(10^8\) ev.

These data emphasize the very great importance of impurities from the walls streaming into the discharge. In the early observations, the carbon and oxygen ions presumably outnumbered the helium ions during the later stages of the discharge, and sharply reduced the electron temperature. With the use of ultra-high vacuum techniques, resulting in base pressures below \(10^{-9}\) mm of Hg and relatively clean surfaces, the efflux of wall impurities has been reduced by more than an order of magnitude.

Another method of reducing the impurity level has been the use of a divertor. This device was proposed relatively early\(^\text{4}\) to take away from the discharge the particles nearest the wall and to avert bombardment of the discharge tube by charged particles. In the divertor, an outer shell of flux is diverted or bent away from the main discharge into a large auxiliary chamber. Any impurities produced by wall bombardment in the divertor chamber return relatively slowly into the main discharge tube. A schematic diagram of a divertor is shown in Fig. 4. The theory of this device, together with observations on its effectiveness in reducing the impurity level, without an ultra-high vacuum, is reported in the subsequent paper by Burnett, Grove, Palladino, Stix and Wakefield.\(^{18}\) Apparently the divertor reduces the ratio of impurity ions to helium ions by a factor of about 1/5.

The most important new observational result that has emerged from these ohmic heating studies is the evidence on cooperative phenomena. During ohmic heating, the plasma is anything but quiescent. Runaway particles start abruptly to hit the tube wall, producing X-rays, sometimes in short bursts, at times dependent on the magnitude of the confining field. The plasma in the stellarator is not well confined by the magnetic field, and reaches the wall in about \(10^{-4}\) seconds when voltage is applied around the stellarator. After ohmic heating, a current of runaway electrons, amounting to some 10 amp/cm\(^2\) may persist for several milliseconds after the voltage is turned off, and then abruptly disappear, producing a burst of X-rays and additional ionization and excitation in the plasma.

The detailed study of these phenomena should increase our understanding of plasma physics and enable one to predict how an ionized gas might behave in a full-scale thermonuclear reactor.

**Magnetic Pumping**

Pulsation of an axial confining field produces an oscillating electrical field, encircling the tube axis. This electric field can increase the energy of gyrating charged particles. If the pulsation frequency is much less than cyclotron frequency, however, this increase in energy is computed most simply from the magnetic moment, \(\mu\), which, according to the results by Kruskal\(^\text{24}\) should be very accurately constant. Under these conditions, the increase in kinetic energy of motion, transverse to the field, is given by the usual formula for adiabatic compression of a gas, provided \(\gamma\) is set equal to 2, corresponding to the presence of two degrees of freedom. Instead of thinking about the electric fields induced by the pulsating magnetic field, we may think of the external lines of force as constituting a piston, and the pulsation of the external field as providing a pumping action.

Evidently if the plasma were entirely adiabatic, or isothermal, the work done on the gas during the compression would just equal the work done on the piston during expansion. No heating would result. To obtain net heating in a pumping cycle there must be a phase lag between temperature and density. Such a phase
lag may be produced in a variety of different ways, and hence there are many frequencies at which magnetic pumping can be effective.

One mechanism for producing such a phase lag is the effect of collisions in exchanging energy between motions parallel and perpendicular to the lines of force. This effect was analyzed early at Project Matterhorn, as a possible substitute for ohmic heating, but was dropped because it was not an effective method for completing the ionization of a gas. More recently, this mechanism has been analyzed by Berger and Newcomb, and, independently, by Schlüter, with identical results. The analysis at Princeton is given in the subsequent paper by the Matterhorn theoretical group. It is clear that magnetic pumping at the positive ion collision frequency can, in principle, heat a fully ionized gas to very high temperatures; however, the rate of heating falls off as $T^{-4}$ with increasing temperature, an inconvenient drop.

Another method of producing the desired phase lag is to pump in a short section of tube, with a pulsation period about equal to the time required for a positive ion to travel through the pumping section. In this situation the temperature lags because of loss of heat out the ends. If the mean free path is short compared to the length of the pumping section, magnetic pumping at this frequency produces acoustic waves, which travel along the magnetic field. For long mean free paths, the particles may be treated as free, and the energy is effectively thermalized by fine-scale mixing. The analysis of this “transit-time heating”, reported in a subsequent Matterhorn paper, shows that this method should be an effective means for heating a plasma to very high temperatures, particularly since the rate of energy input increases as $T^{-4}$ with increasing temperature, if the frequency is optimized at each temperature.

Experimental verification of heating by magnetic pumping has not yet been possible at Project Matterhorn, since the radio-frequency power available was insufficient to balance plasma losses, due to inadequate confinement and too high an impurity level.

**Ion-cyclotron Resonance Heating**

Pulsation of the confining field at the cyclotron frequency of the positive ions should give very rapid heating at very low ion densities. The effect is most conveniently understood not as a macroscopic pumping but as a microscopic resonance between the oscillating electric field and the gyration of the positive ions. However, this type of heating produces separation of charges, and at appreciable plasma densities the resultant electrostatic fields prevent any appreciable heating in this way. It was pointed out by Stix that use of two adjacent heating sections, with identical pulsation frequencies, but differing in phase by 180°, would make it possible for electrons to cancel out this separation of charges by flowing back and forth along the lines of force, and thus permit heating at the ion cyclotron frequency even at substantial plasma densities.

The axisymmetric free oscillations of a cylindrical plasma column, in a strong axial field, were analyzed in detail by Stix, who found that indeed for sufficiently short wave lengths a plasma resonance existed close to the ion cyclotron frequency. Later analyses by both Stix and by Kulsrud and Lenard have considered the input of energy into the gas both at the exact ion cyclotron frequency and at the adjacent plasma resonance frequency. It appears that a substantial amount of energy can be fed into the gas at the plasma resonance frequency, and that thermalization of the energy can readily be achieved in a small system. At low $\beta$ (low ratio of material pressure to magnetic pressure in the vacuum field) this technique offers the great advantage, in principle, over magnetic pumping that the coupling between the external circuits and the plasma is very much better. For a large system, at moderate $\beta$, it is not obvious from theory alone which system would be most effective.

Detailed observational results on plasma heating at frequencies near ion-cyclotron resonance are reported in the associated experimental paper by Stix. Measures on the external circuits at low power indicate that, in fact, resonant loading is observed at frequencies at and below the cyclotron frequencies both for hydrogen and helium. The energy fed into the gas exceeds the energy dissipated in the external circuit, a prerequisite for efficient heating. Measures at high power appear to be generally consistent with expectations, although direct evidence of plasma heating has not, as yet, been obtained.

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1. R. Post, in press.
4. M. D. Kruskal, informal communication.
13. C. R. Burnett, D. J. Grove, R. W. Palladino, T. Stix and
In presenting Paper P/2170, above, at the Conference, Mr. Spitzer summarized Papers P/358, 359, 362, 363, 364 and 2170, from this Session, and P/357, 1875 and 1876 from Session A-5 (Vol. 31) under the title “Survey of the Stellarator Program”, as follows.†

The controlled thermonuclear program at Princeton University has concentrated on plasma confinement in an endless toroidal tube, in which a strong magnetic field is produced by external coils. Such a system we have called a stellarator. This paper reviews very briefly the theoretical foundations of the stellarator program, and then summarizes the observational results which our experimental group has obtained.

Equilibrium

Our theoretical work has been chiefly concentrated on three main topics—first, the equilibrium of a hot plasma within a stellarator; second, the stability of this equilibrium; third, the initial ionization and heating of a gas to establish this equilibrium. Our work on each of these three subjects will be summarized briefly here. Early studies of equilibrium showed that in the ideal torus, where the lines of force are circles centered at the axis of symmetry, equilibrium plasma confinement was not possible, in any simple way. This result follows at once from the well known particle drifts across an inhomogeneous field. These drifts produce a separation of electrical charge. The resultant electric field is not neutralized by currents, since an electric field transverse to a magnetic field produces no current in a steady state, and this electric field sweeps the plasma to the wall. This catastrophe can be averted, in principle, if the degeneracy of the torus is removed, and if the field lines are effectively twisted, and no longer close on themselves after one circuit around the tube. The situation is shown in Fig. 1. which portrays a cross-section of the stellarator tube, perpendicular to the magnetic lines of force. Point \( P_1 \) represents an intersection of some line of force with this plane. The same line of force, when followed around the stellarator, intersects this plane again at a point \( P_2 \). In a Stellarator, the successive intersection points are rotated about the central regions of the tube. One line of force, the magnetic axis, returns on itself after one circuit around the tube. The other lines rotate about the magnetic axis in successive circuits; the angle of rotation in a single circuit—that is, the angle between \( P_1 \) and \( P_2 \) subtended at the magnetic axis—is called the rotational transform angle.

One might wonder whether successive intersections of a line of force with this cross-section plane might spiral away from the magnetic axis, finally leaving the tube altogether. Fortunately this does not happen. Kruskal has shown that the successive intersection points in this cross-section plane, produced by a single line of magnetic force, generate a closed curve to a very high approximation. Thus inside the stellarator tube a single line of force generates a closed toroidal surface, which we call a magnetic surface. This family of nested magnetic surfaces forms the basis of confinement in the stellarator.

The time available permits only a very brief summary of how a rotational transform angle can be produced. Evidently a current along the magnetic lines
of force will cause such an effect. However, our goal in
the stellarator is to produce confinement in a steady
state and to achieve this goal we must produce a
rotational transform in the vacuum field. We have
found two ways of doing this. In the first, the vacuum
tube and the surrounding solenoidal windings are
twist out of a common plane into a form resembling
a figure eight, for example. The rotational transform
angle is then simply the integral of the torsion around
the magnetic axis. In the second method, the tube
remains in one plane and additional helical windings
are added outside the tube. Four or six groups of con-
ductors are used, with the current directed oppositely
in adjacent groups. Either system can produce a
rotational transform angle as great as 180°. These two
systems have been fully illustrated in the U.S. fusion
exhibit,† and will not be described further here.

The equilibrium of a plasma in a magnetic field
possessing magnetic surfaces has been analyzed in
some detail by the Matterhorn theoretical group, under
the supervision of Edward Frieman. The results of
these studies are, in brief, that a steady equilibrium is
possible. The separation of electric charges, which
destroys equilibrium in the simple torus, is now no
problem, since any electric charges accumulating on
one side of the tube can flow around to the other side
by simply moving along the lines of magnetic force.
The properties of the equilibrium state have been
shown by Kruskal and Kulsrud to be related directly
to the natural invariant functions of the problem—
the total mass enclosed within a magnetic surface, and
the rotational transform angle on this surface—as func-
tions of the total enclosed flux through a cross-sec-
tional plane. In addition, a study of the trajectories
of single particles indicates that these should be confined
within a stellarator to a very high approximation,
provided, of course, that the plasma remains quies-
cent; that is, provided that cooperative phenomena do
not destroy the confinement.

Stability

Since we may conclude that a plasma confined in a
stellarator should possess a steady equilibrium state,
we now pass on to the second area for investigation,
the stability of such equilibria. This has been a major
field of investigation by the Matterhorn theoretical
group. A general method has been developed by
Bernstein, Frieman, Kruskal and Kulsrud for com-
puting whether a plasma is unstable for any hydromag-
netic disturbance. This method is based on a
computation of \( \delta W \), the change in energy for any
arbitrary perturbation; the computation is based on
the fluid equations, with the assumption that the
macroscopic velocity vanishes in the equilibrium state.
Convenient methods have been developed for finding
the perturbation which gives the least \( \delta W \) in any
system. If this least \( \delta W \) is negative, the system is
unstable. This method has been applied to the stel-
larator by Johnson, Oberman, Kulsrud and Frieman.

The results indicate that the figure-eight stellarator
tends to be generally unstable, but that the helical
windings give a system that is completely stable
hydromagnetically, provided that \( \beta \), the ratio of
material to magnetic pressure, is not too great.

The stability produced by these windings results
from the fact that the rotational transform angle
increases markedly with distance from the magnetic
axis. In any hydromagnetic instability, within a
toroidal tube, some lines of force move toward the wall,
while others move inwards. The system is most un-
stable if the lines of force can interchange positions
with very little bending. When the rotational trans-
form angle varies with distance from the magnetic
axis, different magnetic surfaces are topologically
different; considerable bending of the lines is required
if one line is to move inwards and another, outwards.

Our detailed stability calculations have all been
based on the fluid equations, which are not always
realistic in a hot, fully ionized gas. However, Kulsrud
has shown that the more complicated single-particle
analyses of Kruskal and Oberman, and of Rosenbluth,
give essentially the same stability criterion as does the
simple \( \delta W \) formulation. We conclude that in principle
the stellarator, with helical windings, is completely
stable against hydromagnetic disturbances, for \( \beta \)
less than a certain critical value. This upper limit on \( \beta \)
is somewhat uncertain, since the theory has only been
carried through to first order in \( \beta \), and since corrections
for finite Larmor radius may be important. This
critical value of \( \beta \) probably lies between 0.1 and 0.4.

Heating

The various methods used for heating plasma in
stellarators have also been analyzed in considerable
detail. Two general methods have so far been con-
sidered, both based on the use of an electric field to
ionize and heat the gas initially present in the tube.
First, the electric field has been assumed parallel to
the magnetic field. In this situation, where an electric
field around the stellarator is induced by transformer
action, current flows along the lines of force, and the
ohmic losses heat the gas. This method is called ohmic
heating. Second, the electric field has been assumed
perpendicular to the magnetic field. Since external
electrostatic fields tend to be shielded by the plasma,
we have restricted ourselves to induced electric fields
produced by the pulsation of the confining magnetic
field in one section of the stellarator. Since the lines of
force move in and out, compressing and rarefying the
gas as the magnetic field changes, this method is called
magnetic pumping. A variant of this method, proposed
by Stix of Matterhorn, and described in a previous
session, utilizes a magnetic field oscillating at about the
cyclotron frequency of the positive ions.

The analysis of ohmic heating in the stellarator is
simpler than in the pinched discharge because there
are no dynamical effects to worry about; the magnetic
field produced by the ohmic heating current is small
compared to the confining field, and the lines of force
† In the U.N. National Scientific Exhibition.
do not move much. However, the physical processes occurring are many. Detailed numerical computations on the rates of ionization and heating have been carried through on an electronic computer by Berger, Bernstein, Frieman and Kulsrud for helium and for hydrogen. These analyses assume perfect magnetic confinement and a Maxwellian velocity distribution for the electrons. While these two assumptions are not entirely precise, and while the computations take into account only the main excitation and ionization processes, it is believed that the results should provide a reasonably accurate prediction of ohmic heating effectiveness, provided that other phenomena, such as hydromagnetic instabilities, electrostatic oscillations and other types of cooperative activity do not carry plasma and energy to the wall too rapidly.

When this method of heating was first being analyzed at Matterhorn, Kruskal pointed out that the ohmic heating current would produce a kink instability, similar to that observed in the pinched discharge, for a current density greater than a certain limiting value, which we now call the Kruskal limit. In this simplest mode of hydromagnetic instability, all the lines of force within the current channel move uniformly toward the wall. Fortunately the Kruskal limit is rather high, exceeding some 200 amp/cm² in our devices, and currents well below this limit can achieve effective heating at temperatures below a million degrees. In addition to the kink instability, the theory also predicts higher modes, in which the perturbation varies over the plasma cross-section.

Heating of a fully ionized gas to very high temperatures can, in principle, be achieved by magnetic pumping, provided, as usual, that unforeseen cooperative effects do not introduce excessive losses to the wall. The theory indicates that the positive ions are heated most effectively if the period of magnetic field pulsation is about equal to one of the two natural periods of these ions. One of these periods is the time required for Coulomb collisions to produce a deflection of 90°.

Heating at this frequency has been discussed independently by Schliiter. The second natural period is the time required for a positive ion, moving at thermal velocity, to travel through the heating section, in which the magnetic field is pulsating. This interval is called the transit time, and has the practically convenient property that it changes relatively slowly as the temperature increases. The detailed analyses carried out by our theoretical group indicate that magnetic pumping at the transit-time frequency should be an effective way, in principle, to heat a gas to thermonuclear temperatures. Unfortunately, the rate of heating by this technique is relatively slow, and losses from the plasma must be kept very low if very high temperatures are to be reached by this means. Heating at plasma resonance, near the ion cyclotron frequency, is a promising method, especially since Stix has shown that the resultant rate of heating may be relatively rapid.

**Results**

After this brief review of theory, we may now pass on to observational results. To test these ideas, a half-dozen small research stellarators, with auxiliary equipment, have been designed and built at Princeton by engineering groups under Norman Mather and Robert Mills. These devices, referred to as our B series, have discharge tubes some 5 to 10 cm in diameter, and between 250 and 600 cm in axial length. Solenoidal coils surrounding each tube produce axial confining fields up to 50,000 gauss; operation is generally at fields between 10,000 and 40,000 gauss. Some of these devices utilize a figure-eight shape to obtain a rotational transform; others employ the helical windings.

Figure 5 indicates the normal sequence of events with one of these devices. The solenoidal coils are energized from a capacitor bank of ~ 10⁶ joule, whose terminals are shorted when the voltage approaches zero. Near the peak of the field, breakdown is obtained with a radio-frequency electric field, typically at 250 kc, and ohmic heating is produced with a pulse of 30 to 300 v applied from another capacitor bank.

A schematic diagram of the tube, showing the various experimental techniques we have employed in these stellarators, appears in Fig. 6. The solenoidal coils are omitted from this diagram. The rf breakdown voltage is applied across an insulating section in the stainless steel discharge tube. The ohmic heating voltage is applied to the primary winding of the ohmic heating transformer. The plasma current is measured by its magnetic field, in the usual way, and electron densities are determined from the phase shift of 8 mm and 4 mm electromagnetic waves traveling across the discharge. We have made extensive use of spectroscopic methods, and have measured the X-rays emitted when energetic electrons strike the inner edge of a tungsten aperture limiter, a circular disc with a hole centered at the magnetic axis. The most striking result obtained with this technique is the observation of X-rays, with energies of several hundred kilovolts, for an interval of some ten milliseconds after
the ohmic heating pulse. Evidently, energetic single particles are confined for more than a hundred thousand circuits around the stellarator, an indication of very good confinement for single particles.

Figure 7 shows typical results obtained with an ohmic heating discharge in helium; the initial pressure, $3 \times 10^{-4}$ mm Hg, confining field, 27 kilogauss, and applied electric field, 0.067 v/cm. It is readily shown from the high ratio of He$^+$ to He$^0$ light, at about two-thirds of a millisecond after the voltage is applied, that the ratio of ions to neutral atoms at this time is very high, probably greater than a hundred to one. At about this time the electron density starts to drop. This decay of the electron density during ohmic heating, which we have now measured in considerable detail, is called "pump-out". A short time after maximum density, the current peak is reached, and the current starts downwards. Meanwhile substantial amounts of impurity light appear in the discharge.

**Analysis**

Extensive data of this type have been obtained and analyzed by the Matterhorn experimental group, under the supervision of Melvin Gottlieb. The primary problem in which we have been interested is the general area of cooperative phenomena, including all types of plasma activity such as oscillations, instabilities and turbulence. It is generally acknowledged that if such phenomena did not occur and the plasma were quiescent, a thermonuclear reactor would certainly be possible, at least in principle. This basic investigation of plasma physics forms the central part of our program. A secondary problem is the investigation of impurities, which stream off the walls into the discharge. Since purity of the gas is of the greatest importance in a thermonuclear reactor, the investigation of this area is of considerable practical importance.

We are working in three areas of plasma physics. One, which we think we understand, is the existence of a limiting current, which corresponds closely with the current predicted by Kruskal for the onset of kink instability. Another is the rate at which ohmic heating proceeds, and the associated problem of energy balance. A third is pump-out, the disappearance of plasma from the discharge. Finally, a brief discussion will be included of impurities, specially on the practical problem of how these may be reduced in abundance.

**Kink Instability**

Let us first, then, look at the observational evidence for kink instability, for currents greater than the Kruskal limit. In Fig. 8, taken from a paper by Kruskal, Johnson, Gottlieb and Goldman, is shown the maximum current reached in a helium discharge, as a function of the ohmic heating field applied. For voltages only slightly greater than are needed for breakdown, the current is observed to rise to a maximum value that is nearly independent of initial pressure, in the range between $10^{13}$ and $10^{14}$ He atoms per cm$^3$. This maximum current in a single pulse rises only
Figure 9. Observed limiting plasma current as a function of confining field.

The two sets of points are for plasma current in opposite directions.

Figure 10. Comparison of observed plasma current and light with theoretical values.

The applied electric field is 0.11 V/cm; the initial pressure of helium gas is $8 \times 10^{-4}$ mm Hg.

slowly with further increase in the pulsed heating field. Moreover, when the current is at or above the Kruskal limit the voltage is observed to fluctuate rapidly and X-rays disappear; evidently the plasma develops violent activity under these conditions.

If this observed limiting current is plotted against the confining field, we find a closely linear relationship, as shown in Fig. 9. This result agrees with theoretical predictions. The solid circles give results for the current in one direction, the open circles for reverse current. The ratio of the two slopes is 1.19, in very close agreement with the theoretical value 1.22 obtained from the Kruskal theory for the geometry of this particular machine. While it is not clear just what contortions the plasma performs when the Kruskal limit is exceeded, we are fairly confident that the hydromagnetic kink instability does in fact appear under the conditions predicted theoretically. The higher modes of instability, also predicted by the theory, have not as yet been observed.

Ohmic Heating

We next look at the comparison between the ohmic heating theory and the observed rate at which the discharge develops. Figure 10, taken from a paper by Coor, Cunningham, Ellis, Heald and Kranz, shows this comparison for a heating field of 0.11 V/cm, and an initial helium pressure of $8 \times 10^{-4}$ mm Hg. During the first 250 µsec, the current, the electron temperature found from the resistivity, and the intensity of HeII light behave about as expected, but at later times these quantities level off for more than a millisecond, as compared with a continuing rise predicted by theory. Evidently some loss mechanism becomes important after about 250 µsec and prevents the further heating of the plasma. The nature of this loss mechanism is still obscure.

Pump-out

We pass on now to the third topic, about which we are more ignorant—the disappearance of electrons from the discharge, or pump-out. It is clear that the electrons and positive ions in the gas are somehow reaching the wall and sticking there. The effect appears to be identical in hydrogen and deuterium. Extensive measures of pump-out rates in hydrogen discharges have been made by Gorman, Ellis and Goldberg.

Over the limited range of densities measured, the observed decay appears to be roughly exponential; the decay rate in reciprocal seconds is plotted in Fig. 11 as a function of initial pressure, for a confining field of 30 kilogauss. The decay rate does not seem to vary...
appreciably as the ohmic heating field is changed, although the electron temperature (as determined from the measured resistivity) changes by a factor of two as this electric field varies. It is apparent from the figure that the observed decay rate decreases linearly with increasing initial pressure. This result is very plausible if the number of neutral atoms returning to the discharge, per atom reaching the wall, varies linearly with the initial pressure. Since the number of adsorbed hydrogen molecules on the stainless steel wall might well be proportional to the initial pressure, this behavior is entirely reasonable.

If we extrapolate this line back to zero initial pressure, and assume that a hydrogen atom sticks to a metal wall, in the absence of adsorbed hydrogen atoms to be knocked off, we obtain the intrinsic rate at which ions leave the discharge. In Fig. 12 these decay rates, extrapolated to zero pressure, are plotted against the confining field, on a logarithmic scale. Evidently these intrinsic electron decay rates, under the conditions of these particular measures, vary as the inverse square root of the confining field. We have no theoretical explanation of this dependence. These measures have not as yet been carried out in a theoretically stable system, with helical windings; possibly some mode of magnetic instability is responsible. At any rate, pump-out is evidently a very important phenomenon in a stellarator, and its explanation provides a fascinating problem in basic physics.

Impurities

We turn now to the fourth major topic, the impurity level in a stellarator. With the millisecond time scale and small radius of our devices, the inflow of impurities can be relatively high, especially as a result of the intense particle bombardment of the tube walls produced by pump-out.

Several steps have been taken to keep down the impurity level. One very effective method is to clean the walls, by baking for several hours at 450°C. This treatment reduces the base pressure to about 10⁻¹⁰ mm Hg, and does indeed reduce the impurity level by about two orders of magnitude as compared with unbaked systems. To obtain such high vacua in systems with volumes of some ten liters and with many observation ports and gasketed joints, has required the development of a substantial technology by our vacuum group under Don Grove. Prolonged pulsing with ohmic heating reduces the impurity level but has not yet proved to be an adequate substitute for prolonged baking.

Another successful technique for reducing the impurity level involves use of a divertor. In the divertor, an outer shell of magnetic flux is bent away from the discharge tube into a separate chamber. Particles diffusing toward the wall enter the divertor and strike the wall far from the main discharge. Any impurities released in the divertor do not readily find their way back into the discharge.

Figure 13 shows a quarter cross-section of a divertor used on an unbaked stellarator designed by Stix. The divertor is rotationally symmetric about the horizontal axis shown; the vertical axis is a plane of symmetry. The thin curves represent magnetic lines of force. Devices of this type are readily designed on a magnetic analogue board devised by Wakefield.

A series of spectroscopic measurements was made by Burnett, Grove, Palladino and Stix with and without divertor. When the divertor was not activated, that is, when the negative coil was not energized, a limiter disc with a hole in the center was used to confine the discharge to the same central region of the tube as with the divertor. Figure 14 shows a comparison of the intensities of the Ov line, with and without divertor. The curves represent mean intensities observed photoelectrically as a function of time after the beginning of the ohmic heating pulse. If the divertor was used in place of the limiter, the intensity of this impurity line...
Figure 14. Observed intensity of OV line with and without the divertor. The axial confining field is 20,000 gauss, the applied electric field, $0.24 \text{ v/cm}$, the initial helium pressure, $1 \times 10^{-4} \text{ mm Hg}$.

Figure 15. Observed Doppler width of OV line with and without the divertor. Experimental conditions are as in Fig. 11.

was reduced to about a tenth of its former value. Figure 15 shows that the kinetic temperature, obtained from the Doppler width of this same line, is materially increased by use of the divertor. The measurements are not yet sufficiently complete to separate effects of temperature from those of abundance, nor to unravel effects produced by radial variations of both temperature and impurity level. However, they support the belief that the divertor produces a marked reduction in the impurity level, with a large consequent increase in temperature.

**Conclusion**

To conclude, our theoretical work has shown that (1) an equilibrium state for a confined quiescent plasma can exist in a stellarator; (2) this equilibrium is hydro-magnetically stable; and (3) heating to thermonuclear temperatures should be possible by ohmic heating, followed by magnetic pumping, provided the plasma is sufficiently quiescent. The observational research has shown that confinement of high energy electrons is excellent, and that ohmic heating produces a hot, virtually fully ionized plasma. However, the gas is definitely not quiescent. The onset of the gross hydro-magnetic kink instability, predicted by Kruskal, has been fully confirmed observationally. In addition, the evidence on pump-out indicates that there may be other types of activity occurring that we do not as yet understand. The study of these effects will form a challenging and major part of our program during the next few years. A secondary objective of our program is the further reduction of the impurity level. If pump-out can be controlled, and the impurity level kept sufficiently low, we might hope to achieve much higher temperatures by use of magnetic pumping, an objective which has not yet been possible with the present high loss rate from the plasma.

These experimental objectives should be materially aided by the construction of the C Stellarator, a research tool considerably larger than our B devices. With a tube diameter of $20 \text{ cm}$, a field of $80,000 \text{ gauss}$ pulsed for times up to one second, and extensive facilities for diagnostics and for heating, this device should be a powerful tool for analyzing the complex phenomena occurring in hot plasma.