The scaling of confinement in ITER with β and collisionality

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Abstract.

The condition of the latest versions of the ELMy H-mode and L-mode databases have been re-examined in view of their sensitivity to errors in the absorbed heating power and stored energy. It is shown that there is bias in the OLS regression for some of the variables. These short comings are overcome by the use of an error in variables technique and new scalings are derived. These give a very similar performance to existing scalings for ITER at the standard β_n of 1.8, but much improved performance at higher β_n.

1. Introduction

Both the L-mode and ELMy H-mode scaling expressions, used to predict the confinement time in ITER exhibit a strong degradation with β and a weak dependence on collisionality ν*. For example the H-mode scaling IPB98(y, 2) [1] has the form in dimensionless parameters $\omega_c\tau_{IPB98} \sim \rho^{*\ -2.7}\ \beta^{-0.9}\ \nu^{*\ -0.01}$, see [1] (p2203). Single scan experiments of the dimensionless parameters ρ*, β and ν* have been completed on both DIII-D [2,3] and JET [4,5]. These experiments have confirmed the ρ* dependence but disagree with the dependencies on both β and ν*, having virtually no β dependence and a ν* dependence of the form $\omega_c\tau_E \sim \nu^{*\ -0.3}$.

In the paper by Thomsen et al [6] it has been shown that the bias in ordinarily least squares regression due to measurement errors of the ELMy H-mode database is sufficient to explain the discrepancy in the β and ν* dependences provided that the error on the loss power is sufficiently large. In the present paper we examine the condition of the latest version of the

H-mode database DB3v13 and also the L-mode database DB2v9 [7]. We conclude that for both databases the bias related to measurements errors is significant.

Two complementary approaches are used to overcome this problem for the H-mode database. First we use an Error in Variables (EIV) technique on the full database and show how the β and ν^* dependence is sensitively dependent on the errors, notably, in two of the parameters used in the regression, namely the loss power and the thermal stored energy. In the second approach to improve the condition of the data set an 'ITER like' subset is selected, and fewer variables are used in the regression, this gives a very weak β dependence and a degradation with ν^*. For the L-mode database, there is insufficient ITER like data in the database and so the EIV technique is employed on the full database only.

The structure of the paper is as follows: in Section 2 the condition of the H-mode database, the ITER like subset and the L-mode database are examined. Then in Section 3 we employ the EIV technique on these data sets. In Section 4 the results are discussed and predictions are made for ITER operation at standard β_n=1.8 and at higher values of β_n.

2. Condition of the data bases

a) H-mode database

The variables used in the analysis are W_{th} (the thermal stored energy (MJ)), P (the loss power (MW)), R (the major radius (m)), a (the minor radius (m)), A (the cross sectional area (m^2)), n (the central line averaged density (10^{19} m^{-3})), I (the current (MA)), B(toroidal field (T)), M (the isotope mass). In this paper we choose the cross sectional area A and minor radius a, rather than the elongation and aspect ratio, so that the cross correlation between the errors is smaller.

In the analysis the standard extended ELMy H-mode dataset [6] (including ohmic H-modes) is used. The number of observations (N_j) from each tokamak and the weighting factor of each tokamak: $w_j \sim 1/\left(2+\sqrt{N_j}/4\right)$ are as follows: ASDEX (N_j = 431) w_j = 0.125; AUG (526) 0.125; CMOD (45) 0.25; COMPASS (16) O.333; DIII-D (300) 0.142; JET (1413) 0.083; JFT2M (70) 0.2; JT60U (87) 0.2; MAST (9) 0.333; NSTX (5) 0.333; PBXM (59) 0.25; PDX (97) 0.2; START (8) 0.333; TCV (11) 0.333; TFTR (13) 0.333; TdeV (3) 0.333. The measurement errors, taken from Thomsen et al. [6], are reproduced in Appendix 1.

The bias in standard OLS regression related to these errors is estimated as ~ $(\lambda_e/\lambda_{pc})^2$, where λ_{pc} is the standard deviation (STD) of a Principal Component (PC) and λ_e is the STD of the measurement error along that PC. If $\lambda_{pc} > 4\,\lambda_e$ the bias is estimated to be less than 6%, and considered negligible [8]. From the PC analysis the estimate of the STD of a PC is an estimate of $\lambda_{pc} + \lambda_e$, if errors are present. The ratio ERR = $\lambda_e/(\lambda_{pc} + \lambda_e)$ can be estimated, if the errors are known or assumed. Hence, if ERR < 0.2 the bias is less than 6%, and is negligible. The PC's for this dataset are listed in Table 1 together with the estimates of $\lambda_e + \lambda_{pc}$ and ERR.

It is clear that the bias related to errors is certainly not negligible for PC6 and PC8, which are linked with the limited data range in the mass M and elongation ($\kappa \cong A/\pi a^2$). To avoid the problem with PC6 a deuterium only data set has been identified and to avoid the problem with PC8 we selected an ITER like data set with a limited range in elongation and in addition a limited range in q which eliminates the lesser problem with PC7.

	ln(I)	ln(n)	ln(B)	ln(P)	ln(R)	ln(A)	ln(a)	ln(M)	λ_e+λ_{pc}	ERR
PC 1	0.463	-0.026	0.079	0.532	0.217	0.609	0.274	0.049	1.632	0.05
PC 2	0.246	0.724	0.358	0.313	-0.218	-0.317	-0.190	0.071	0.664	0.09
PC 3	-0.277	-0.386	0.645	0.340	0.368	-0.307	-0.004	-0.112	0.380	0.15
PC 4	**0.499**	-0.084	**0.531**	**-0.665**	0.045	0.065	0.092	0.066	0.330	**0.29**
PC 5	-0.314	**0.548**	0.002	-0.243	**0.672**	0.205	0.102	-0.193	0.176	**0.28**
PC 6	-0.207	0.022	0.042	-0.018	0.144	0.049	-0.021	**0.965**	0.157	**0.52**
PC 7	**-0.487**	0.124	0.358	-0.040	**-0.537**	0.360	**0.445**	-0.052	0.105	**0.25**
PC 8	0.143	0.050	-0.193	0.023	0.085	**-0.506**	**0.819**	0.069	0.060	**0.57**

Table 1 *Principal components of ELMy H-mode subset of DB3v13 with estimates of* $\lambda_e + \lambda_{pc}$ *and ERR as explained in the text. Values of ERR > 0.25 are shown in bold face along with the leading terms in these PC's.*

b) ITER like data set

Restricting the data to the following ranges in κ, q_{cyl} and M

$$1.4 < \kappa < 1.93 \quad (1)$$
$$1.6 < q_{cyl} < 2.8 \quad (2)$$
$$1.833 < M < 2.167 \quad (3)$$

We find that the number of observations from each machine is as follows:
AUG (185) 0.2; CMOD (25) 0.3; COMPASS (12) 0.4. DIII-D (132) 0.2; JET (1045) 0.1; JT60U (36) 0.3; MAST (9) 0.4; NSTX (5) 0.4; TCV (11) 0.4. The weights which are given after the observations are chosen using the same algorithm $w_j \sim 1/\left(2+\sqrt{N_j}/4\right)$. The average errors are also slightly modified since these are a different set of machines. They are as follows: δI = 1.25%, δR = 1.5%, δa = 2.4%, δn = 5.9%, δP = 12%, δW_{th} = 12%. One further advantage of this data set is that it contains mainly present generation tokamaks that are still in operation. The principal components are listed in Table 2 along with the STD's and ERR.

	ln(I)	ln(n)	ln(P)	ln(R)	ln(a)	λ_e+λ_{pe}	ERR
PC 1	0.467	-0.131	0.729	0.347	0.336	1.194	0.074
PC 2	0.079	0.852	0.353	-0.269	-0.266	0.600	0.110
PC 3	0.800	0.203	-0.550	0.038	0.120	0.262	0.259
PC 4	-0.322	0.435	-0.200	0.779	0.245	0.162	0.226
PC 5	-0.178	0.162	-0.042	-0.445	0.861	0.099	0.242

Table 2 *Principal components of the ITER like data set with estimates of* $\lambda_e + \lambda_{pc}$ *and ERR.*

The highest value of ERR is 0.26 indicating that this database is reasonably well conditioned for regression wrt to the 5 variables R, I, a, P and n.

c) L-mode database

The L-mode database LDB2v9 (including HL-1M) is used with selection DB2STD = 1 or seldb2 = 1110111 with the Helium data excluded. This gives 1108 observations from the following machines: ASDEX (192) w_j = 0.2, CMOD (114) 0.2, DIII (93) 0.2, FTU (4) 0.4, HL-1M(4) 0.4, JET (123) 0.2, JFT-2M (104) 0.2, JT-60U (152) 0.2, PBX-M (26) 0.3, PDX (32) 0.3, RTP (9) 0.4, T-10 (26) 0.3, TdeV (6) 0.4, TEXTOR (14) 0.3, TFTR (162) 0.2, TOR SUPRA (47) 0.3. Since the variation in the numbers of observations from each machine, which is given in brackets, is quite large, weights w_j are used which are also given.

Using the same errors on R, I etc as for the full H-mode dataset we see from Table 3 that this database is not particularly well conditioned either. Once again the data range in the mass M and elongation κ is too narrow.

	ln (I)	ln(n)	ln (B)	ln (P)	ln (R)	ln (A)	ln (a)	ln (M)	$\lambda_e+\lambda_{pe}$	ERR
PC 1	0.467	-0.028	0.131	0.551	0.203	0.579	0.291	-0.007	1.803	0.046
PC 2	0.235	0.643	0.375	0.356	-0.265	-0.378	-0.221	0.042	0.806	0.078
PC 3	0.469	0.245	0.104	-0.677	-0.134	0.280	0.051	0.389	0.494	0.208
PC 4	0.026	-0.525	0.793	-0.060	0.101	-0.223	0.043	0.172	0.336	0.104
PC 5	-0.180	-0.079	-0.233	0.303	-0.018	-0.066	-0.022	**0.900**	0.280	**0.311**
PC 6	-0.440	0.488	0.235	-0.120	0.637	0.073	0.285	0.081	0.202	0.166
PC 7	0.525	-0.076	-0.285	-0.036	0.618	-0.495	-0.099	0.010	0.136	0.197
PC 8	0.073	0.014	-0.108	-0.008	-0.269	-0.374	**0.878**	-0.021	0.058	**0.543**

Table 3 *Principal components of standard subset of the L-mode database* DB2v9*, with estimates of* $\lambda_e + \lambda_{pe}$ *and ERR. Values of ERR > 0.25 are in bold face, along with the leading terms in these PC's.*

3. Regression analysis using the errors in variable (EIV) technique

The general method employed here is to normalise the variables with respect to their errors and then use a principal component analysis [9] to determine the regression plane.

a) Extended ELMy H-mode standard set

For this data set (with the same weights as in Section 2), the ordinary least squares log-linear regression with I, B, R, n, a, A, M and P as regressors gives,

$$\tau_E = 0.0228\ I^{0.86}\ B^{0.21}\ R^{1.31}\ n^{0.40}\ a^{-0.99}\ A^{0.84}\ M^{0.08}\ P^{-0.65} \qquad (4)$$

In non-dimensional variables Eq (4) has the form

$$\omega_c\tau_E \sim \beta^{-0.66}\ \rho*^{-2.8}\ \nu*^{-0.09} \qquad (5)$$

Eq. (4) is very similar to IPB98(y, 1) [1], which is not too surprising since the data selection is very similar, although the dataset now contains more than twice the number of observations.

Repeating the regression with W_{th} as one of the regressors rather than the loss power P gives a very different scaling

$$\tau_E \cong 0.0322\ I^{1.03}\ B^{-0.14}\ R^{1.11}\ n^{0.02}\ a^{-0.82}\ A^{0.20}\ M^{0.16}\ W_{th}^{-0.01} \quad (6)$$

this in non-dimensional from is approximately,

$$\omega_c\tau_E \sim \beta^{0.48}\ \rho*^{-2.83}\ \nu*^{-0.42} \quad (7)$$

when small R dependent terms are ignored.

Thus we see that the degradation of τ_E with β in equation (5) has been replaced by an improvement of τ_E with β, and the weak dependence on $\nu*$ has been replaced by a much stronger one.

The results from the EIV analysis span these two extremes. With the average errors of Appendix 1 i.e. $\delta P = 14.2\%$, $\delta W = 14.1\%$ etc. the EIV method gives,

$$\tau_E = 0.0150\ I^{0.75}\ B^{0.32}\ R^{1.23}\ n^{0.35}\ a^{-1.53}\ A^{1.14}\ M^{0.06}\ P^{-0.61} \quad (8)$$

which in dimensionless form is

$$\omega_c\tau_E \sim \beta^{-0.51}\ \rho*^{-2.7}\ \nu*^{-0.15} \quad (9)$$

Thus although the β degradation has been reduced from the conventional regression result of Eq. (4), there is still substantial degradation of τ_E with β. By increasing the error on the loss power, the degradation with β can however be further reduced. In Fig. 1. the indexes of β and $\nu*$ are plotted against the assumed error on the loss power and we see that with an error of 25% the β degradation is reduced to zero, whilst the $\nu*$ index is -0.31. This latter form in engineering variables is

$$\tau_E = 0.0198\ I^{0.85}\ B^{0.17}\ R^{1.21}\ n^{0.26}\ a^{-1.25}\ A^{0.82}\ M^{0.11}\ P^{-0.45} \quad (10)$$

b) ITER like data set

Applying the EIV method to the ITER like data set using the errors of Section 2, namely $\delta P = 12\%$, $\delta W = 12\%$ etc.

gives
$$\tau_E = 0.0933\ I^{1.00}\ R^{1.17}\ n^{0.37}\ a^{0.56}\ P^{-0.55} \quad (11)$$

which in dimensionless form is $\omega_c\tau_E \sim \beta^{-0.20}\ \rho*^{-2.78}\ \nu*^{-0.20}$ (12)

Comparing this expression with Eq. (9), we see that the ITER like data set has a much weaker β dependence than the full data set. By increasing the error on the loss power to 18% or reducing the error on the thermal stored energy to 8% one can reduce the β index $-\alpha_\beta$ to zero.

c) L-mode database DB2v9

The log-linear fit to the standard selection from DB2v9 is

$$\tau_E = 7.53\times10^{-3}\ I^{0.87}\ B^{0.08}\ R^{1.96}\ n^{0.45}\ a^{-1.59}\ A^{0.79}\ M^{0.24}\ P^{-0.73} \qquad (13)$$

This expression is very similar indeed to ITERL-97P derived by Kaye et al [10]. In non dimensional parameters it has the form

$$\omega_c\tau_E \sim \beta^{-1.26}\ \rho*^{-2.00}\ \nu*^{0.22} \qquad (14)$$

Employing the EIV method to this data set and using the same error set as for the H-mode data set of Appendix 1, i.e. δP = 14.2%, δW_{th} = 14.1% etc. we find that Eq. (13) is only changed marginally $\omega_c\tau_E \sim \beta^{-0.90}\ \rho*^{-2.2}\ \nu*^{0.1}$. A dramatic change in both δW_{th} and δP is required to reduce the β dependence. For example with δW_{th} = 7% and δP = 25%, the dimensionless scaling is $\omega_c\tau_E \sim \beta^{-0.33}\ \rho*^{-2.5}\ \nu*^{-0.09}$

4. ITER predictions and discussion

From the foregoing results we see that the β and $\nu*$ dependence is very sensitive to the condition of the database. The better conditioned ITER-like dataset (for five rather than eight variables), has a weaker β dependence and a stronger $\nu*$ degradation. The β and $\nu*$ scaling is also very sensitive to the assumptions made concerning the errors on the loss power P and thermal stored energy W_{th}. There is the additional consideration in the case of δP that no account is taken of the additional loss terms such as radiation and charge exchange. The omission of these terms is equivalent to the true power which is lost by the transport processes alone, having an even larger error than that given in Appendix 1.

Thus, with this uncertainty, it is difficult to be precise concerning which of the scaling expressions derived in this paper should be used for ITER predictions. Hence we suggest that the range of expressions given in Table 4 for ITER should be used in any study, which contributes to an interval estimate [11]. Fortunately, for standard operation of ITER at β_n = 1.8, the range of τ_E's is very narrow, however at higher β_n = 2.8 the range of τ_E is very much larger. Note that in computing this Table the loss power is adjusted to obtain the appropriate thermal stored energy and β_n. In Fig. 2 POPCON plots comparing the operational range for ITER with the conventional IPB98(y,2) scaling and that of Eq. (10), the scaling with zero β dependence, are presented. The main difference is that high Q's are accessible at high β with the β independent scaling. The optimum operational point for ITER is also at a slightly higher value of β_n (~ 2.5). To strengthen these results, which have important implications for reactors also, the condition of the ELMy H-mode database will need to be improved by the addition of further high β data with the ITER geometry, and a reassessment of the accuracy of the absorbed heating power would also be most useful.

Scaling	Eq. No.	$\tau_{E\,1}$ (s) $\beta_n = 1.8$	$\tau_{E\,2}$ (s) $\beta_n = 2.8$
IPB98 (y, 2)		3.66	1.38
Standard selection conventional regression	(4)	3.61	1.57
Standard selection no β dependence i.e. with $\delta P = 25\%$	(10)	3.24	2.25
ITER like data set standard errors $\delta P = 12\%$	(11)	3.34	1.94

Table 4. *Confinement time in ITER for β_n = 1.8 and β_n = 2.8, I = 15MA, B = 5.3T, R = 6.2m, A = 22m^2, n = 10^{20} m^{-3}, a = 2m, M = 2.5, W_{th} = 318 MJ for τ_{E1} (β_n = 1.8) and W_{th} = 495 MJ for $\tau_{E\,2}$ (β_n = 2.8).*

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Appendix 1

Estimates of the measurements errors on each Tokamaks with ELMy data in the ITPA global H-mode Confinement Database version DB3v13. The estimates δP and δW_{th} are based on the errors in the variables used to compute PLTH and WTH in the database.

	δR	δa	δA	δB	δI	δn	δM	δP	δW_{th}
ASDEX	1.0%	1.5%	2.0%	1.0%	1.0%	3.0%	10.0%	19.1%	14.2%
AUG	0.2%	1.1%	3.0%	1.0%	1.0%	3.0%	10.0%	12.0%	11.5%
CMOD	0.6%	2.0%	3.0%	1.0%	2.0%	5.0%	3.0%	12.7%	17.4%
COMPASS	2.0%	6.0%	6.0%	2.0%	1.0%	5.0%	10.0%	13.3%	15.0%
D3D	0.6%	0.5%	3.0%	1.0%	1.0%	4.0%	1.0%	11.5%	11.6%
JET	1.0%	3.0%	6.0%	1.0%	1.0%	8.0%	20.0%	15.0%	7.2%
JFTM	0.8%	3.0%	5.0%	1.0%	1.0%	2.0%	20.0%	12.0%	18.1%
JT60U	0.5%	1.0%	5.0%	1.0%	0.5%	10.0%	5.0%	12.8%	15.0%
PBXM	0.7%	3.0%	10.0%	1.0%	1.0%	5.0%	5.0%	16.3%	13.8%
PDX	0.8%	3.0%	5.0%	1.0%	1.0%	5.0%	5.0%	25.5%	16.2%
TCV	1.0%	2.0%	1.0%	1.0%	1.0%	5.0%	1.0%	10.2%	10.0%
TFTR	0.4%	1.3%	5.0%	2.0%	2.0%	5.0%	10.0%	21.9%	23.7%
TDEV	0.6%	5.0%	5.0%	1.0%	2.0%	2.0%	10.0%	7.0%	10.0%
START	6.9%	9.1%	10.0%	6.0%	2.0%	5.0%	15.0%	17.9%	16.5%
MAST	1.4%	2.6%	2.8%	1.5%	1.0%	7.0%	5.0%	10.0%	15.0%
NSTX	2.0%	3.0%	5.0%	1.0%	2.0%	6.0%	5.0%	10.0%	10.05
Average	1.3%	2.9%	4.7%	1.5%	1.3%	5.0%	8.4%	14.2%	14.1%

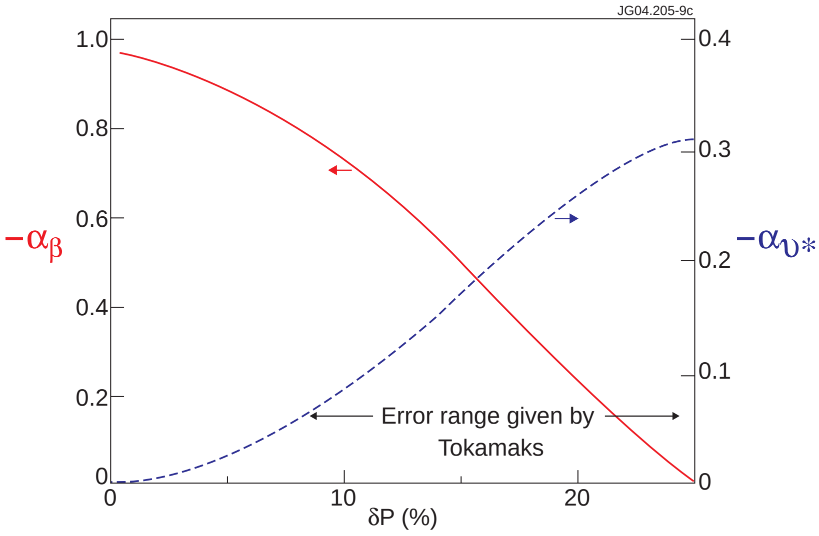


Fig 1. The indices of β (α_β) and ν^* ($\alpha_{\nu*}$) of the confinement expression versus the assumed error on the loss power δP.

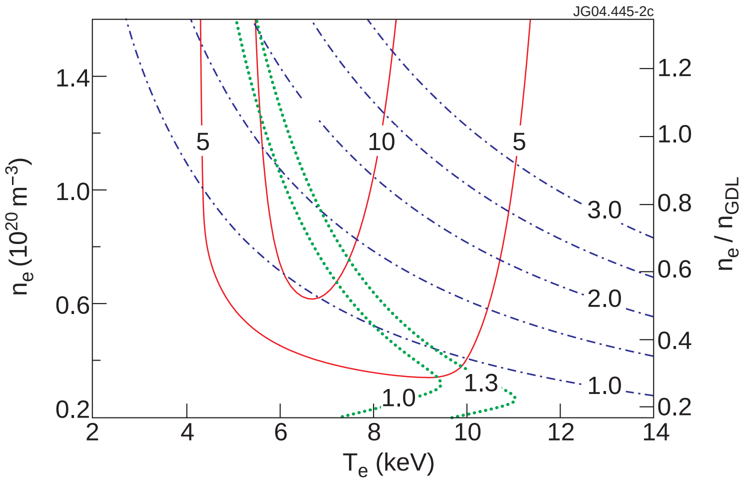
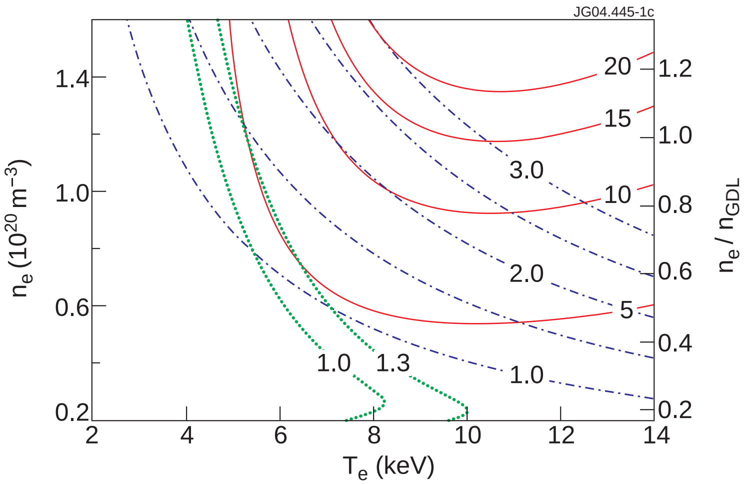


Fig 2. Popcon plot showing the operational regime for ITER. The red curves are constant Q, the blue curves constant β_n, and the green curves the ratio of the loss power to the threshold power

a) The IPB98 (y, 2) scaling



b)The zero β scaling of Eq. (10).