Gyrokinetic Theory and Simulation of Zonal Flows and Turbulence in Helical Systems

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Introduction

- Gyrokinetic Simulations of the ITG Turbulence
  - Zonal flow is a key ingredient to regulate turbulent transport in magnetically confined plasmas.

- In the LHD experiments, better confinement is observed in the inward-shifted magnetic configurations, where the pressure-gradient drives instability stronger while better neoclassical ripple transport.

- Anomalous transport is also improved in the inward shifted configuration.
Outline

- This work deals with gyrokinetic theory and simulations of turbulent transport and related zonal flow dynamics in helical systems.
  - Theoretical analysis and numerical simulations of the linear response of zonal flows in helical systems.
  - Gyrokinetic-Vlasov (GKV) simulations of the ITG turbulence in helical systems.
Collisionless Damping of Zonal Flow and GAM in Tokamaks

- Initial value problem for $n=0$ mode with $\delta f(t=0)=F_M$
- The residual zonal flow is considered to be important in regulating turbulent transport.

Residual Zonal Flow (response kernel)

\[ K = \frac{\langle \phi_{k_x} (t = \infty) \rangle}{\langle \phi_{k_x} (t = 0) \rangle} \approx \frac{1}{1+1.6q^2/\varepsilon^{1/2}} \]

Cyclone DIII-D base case

Distribution Function

Positive
Negative
The result is useful to optimize configurations for enhancing zonal-flow generation and accordingly reducing turbulent transport.

It is suggested that reduction of ripple-trapped particles’ drift not only improves the neoclassical transport but also enhances the zonal flows.

Zonal-Flow Potential

\[
\frac{\psi_{k,1}(t)}{T_i} = \mathcal{K}(t) \frac{\psi_{k,1}(0)}{T_i} + \frac{1}{n_0(k_{\perp}^2 a_i^2)} \int_0^t dt' \mathcal{K}(t-t') \left(1 - \frac{2}{\pi} \left\langle (2\epsilon_H)^{1/2} \{1 - g_{i1}(t-t', \theta)\} \right\rangle \right)^{-1} 
\]

\[
\times \left\langle \int_{k^2<1} d^3 v e^{-ik_{\perp} \vec{v}_d(t-t')} F_{i0} S_{k,1}(t') + \int_{k^2>1} d^3 v F_{i0} S_{k,1}(t') \{1 + ik_{\perp} (\Delta_r - \langle \Delta_r \rangle_{p0})\} \right\rangle
\]

Response Kernel

\[
\mathcal{K}(t) = \mathcal{K}_{GAM}(t)[1 - \mathcal{K}_L(t)] + \mathcal{K}_L(t)
\]

\[
\mathcal{K}(t = 0) = 1 \quad \mathcal{K}(t) \rightarrow \mathcal{K}_L(t) \text{ as } \mathcal{K}_{GAM}(t) \rightarrow 0
\]

Long-time Response Function

\[
\mathcal{K}_{L}(t) \equiv \frac{1 - (2/\pi) \left\langle (2\epsilon_H)^{1/2} \{1 - g_{i1}(t, \theta)\} \right\rangle}{1 + G + \mathcal{E}(t) / \langle n_0 \langle k_{\perp}^2 a_i^2 \rangle \rangle}
\]
Simulation of Zonal Flow Damping in Helical Systems

\( q = 1.5, \quad \varepsilon_t = 0.1, \quad k_r a_i = 0.131 \)

\( \varepsilon_h = 0.1 \)

- Long-Time Response Kernel \( K_L(t) \)
- Simulation

Validity of the theoretical analysis is verified by GKV code.

Radial drift of helical-ripple-trapped particles is identified.

Velocity distribution function for \( \theta = 8\pi/13 \) at \( t = 6.23 R_0/v_{ti} \).

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Limiting Form of the Long-Time Response Kernel $K_L(t)$

$$K_\geq \equiv \lim_{t/\tau_e \to +\infty} K_L(t)$$

$$= \langle k^2_\perp a_{i_r}^2 \rangle \left[ 1 - \frac{2}{\pi} \langle 2\epsilon_H \rangle^{1/2} \right]$$

$$\times \left\{ \langle k^2_\perp a_{i_r}^2 \rangle \left[ 1 - \frac{3}{\pi} \langle 2\epsilon_H \rangle^{1/2} \right] + G \right\}^{-1}$$

\[ \langle k^2_\perp a_{i_r}^2 \rangle \left[ 1 + \frac{2}{\pi} \frac{T_i}{T_e} \langle 2\epsilon_H \rangle^{1/2} \right] \]

The long-time limit of $K_L(t)$ depends on the depth of helical ripples $\epsilon_H$ as well as on the radial wave number $k_{r_r}$.

Lower residual flow (smaller $K_\geq$) is obtained for lower $k_{r_r}$ (longer radial wavelength).
Gyrokinetic Simulations of ITG Turbulence in Helical Systems

- **GK ordering + Flux tube model + Periodic \((x,y)\)**

\[
\frac{\partial}{\partial t} + v_\parallel \hat{b} \cdot \nabla + v_d \cdot \nabla - \mu (\hat{b} \cdot \nabla \Omega) \frac{\partial}{\partial v_\parallel} \delta f + \frac{C}{B_0} \{\psi, \delta f\} = \left( v_* - v_d - v_\parallel \hat{b} \right) \frac{e \nabla \psi}{T_i} F_M + C(\delta f)
\]

- **Co-centric Flux Surface with Constant Shear and Gradients**

\[
v_* = -\frac{c T_i}{e L_n B_0} \left[ 1 + \eta_i \left( \frac{m v^2}{2T_i} - \frac{3}{2} \right) \right] \hat{y}, \mu = \frac{v_\perp^2}{2\Omega}
\]

- **Quasi-Neutrality + Adiabatic Electron**

\[
\int J_0(k_\perp v_\perp / \Omega) \delta f \, d^3 v - \left[ 1 - \Gamma_0(k_\perp^2) \right] \frac{e \phi}{T_i} = \frac{e}{T_e} (\phi - \langle \phi \rangle), \quad k_\perp^2 = (k_x + \hat{s} z k_y)^2 + k_y^2
\]
Model of the ITG Turbulence Simulation in Helical Systems

- Effects of the helical field are introduced through $|\mathbf{B}|$.

\[ B = B_0 \left[ 1 - \varepsilon_{00}(r) - \varepsilon_{0}(r) \cos z - \sum_{l=L-1}^{l=L+1} \varepsilon_{l}(r) \cos ((l - Mq)z - M\alpha_0) \right] \]

\[ \mathbf{v}_d \cdot \nabla = -\frac{\mathbf{v}_{\parallel}^2 + \Omega_0 \mu}{\Omega_0 R_0} \left[ \left\{ R_0 \varepsilon'_{00} + \cos z + \sum_{l=L-1}^{l=L+1} (\varepsilon_{l}/\varepsilon_{0}) l \cos ((l - Mq)z - M\alpha_0) \right\} \frac{\partial}{\partial y} \right. \]

\[ + \left\{ \sin z + \sum_{l=L-1}^{l=L+1} (\varepsilon_{l}/\varepsilon_{0}) l \sin ((l - Mq)z - M\alpha_0) \right\} \left[ \frac{\partial}{\partial x} + \hat{z} \frac{\partial}{\partial y} \right] \]

- The mirror force term also involves the helical components of $|\mathbf{B}|$.

- The simulation is done on a torus with an effective minor radius $r_0$, where $\Psi_T = \pi B_0 r_0^2$. 

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ITG Instability in Helical Systems

Standard Configuration

Inward-Shifted Configuration

\[ \varepsilon_t = \varepsilon_h = 0.1, \quad \varepsilon_{1,10} = -0.2 \varepsilon_t, \quad \varepsilon_{3,10} = 0 \]

- ITG mode is more unstable in the inward-shifted configuration while slower radial drift of helical-ripple-trapped particles.

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ITG Turbulence Simulation in Helical Systems

**Standard Configuration**

**Inward-Shifted Configuration**
Observed transport coefficients are comparable between the two cases, while ~60% differences in their linear growth rates and their different saturation levels in the turbulence energy.

\[
\chi_i \sim 2.5 \frac{\rho^2 v_t}{L_n}
\]

\[
\chi_i \sim 2.9 \frac{\rho^2 v_t}{L_n}
\]
The stronger zonal flows generated in the inward-shifted model configuration regulate the turbulent transport with $\chi_i$ comparable to the standard model case.
The zonal flow spectra in helical systems have relatively smaller amplitude on the low-$k_r$ side than that in tokamaks, as expected from the $k_r$-dependence of the zonal flow response kernel, $K_\phi$. 
Summary

- The response kernel of zonal flows in helical systems is analytically derived from the gyrokinetic theory by taking account of helical geometry and FOW effects.
- The GKV simulations on the ITG turbulence in helical systems show stronger instability in the inward-shifted configuration. Because of the larger zonal flows, however, the resultant transport is found in comparable magnitude to the standard configuration.
- The zonal flows with longer radial wavelengths are observed with relatively smaller amplitudes than those in tokamaks as expected from the analytical theory on the zonal flow response.
The present theoretical and numerical studies confirm that the stronger zonal flows in the inward-shifted configuration regulate the ITG turbulent transport.

The obtained result encourages us to intensively promote the gyrokinetic simulation activities.

In order to investigate the turbulent transport physics further, the GKV simulations will be extended, step by step, so as to include the detailed equilibrium parameters, global profiles ($\omega_*$-shear, variation of rotational transform etc.) as well as multi-physics and multi-scale effects.
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- Gyrokinetic-Vlasov simulations are carried out by utilizing the Earth Simulator under the support by JAMSTEC and by using the Plasma Simulator at NIFS.
Classification of Particle Orbits

Tokamak

\[ B = B_0 (1 - \varepsilon_t \cos \theta) \]

Helical Systems

\[ B = B_0 \left[ 1 - \varepsilon_t \cos \theta - \varepsilon_h \cos (L\theta - M\zeta) \right] \]
GAM dispersion relation

\[ \omega = \omega_G + i \gamma \]

\[
\frac{1}{\mathcal{K}_{GAM}(\omega)} \equiv -i \dot{\omega} - i \frac{q^2}{2} \left[ \left( \frac{R_0 \varepsilon_{10}}{r} \right)^2 \{ J(\dot{\omega}) + J_{FOW}(\dot{\omega}) \} + L \left( \frac{R_0 \varepsilon_{L0}}{r} \right)^2 J \left( \frac{\dot{\omega}}{L} \right) \right]
+ \sum_{|n| \leq n_{\text{max}}} \frac{(L + n)^2}{L + n - qM} \left( \frac{R_0 \varepsilon_h(n)}{r} \right)^2 J \left( \frac{\dot{\omega}}{L + n - qM} \right) = 0
\]

approximate solution

\[ \omega_G^2 = \left( \frac{7 + 4 \tau_e}{4} \right) q^2 \left( \frac{v_{Ti}}{R_0 q} \right)^2 (1 + L^2 c_{L0}^2) \left[ 1 + \frac{2(23 + 16 \tau_e + 4 \tau_e^2)(1 + L^4 c_{L0}^2)}{q^2 (7 + 4 \tau_e)^2 (1 + L^2 c_{L0}^2)^2} \right] \]

\[ \times \left[ 1 + \frac{q^2}{2} \left( 1 + \frac{\pi \tau_e}{2(1 + \tau_e)} \right) \sum_{|n| \leq n_{\text{max}}} \frac{(L + n)^2 (c_h(n))^2}{(L + n - qM)^2} \right]^{-1}
\]

\[ \gamma = -\frac{\sqrt{3}}{2} q^2 \left( \frac{v_{Ti}}{R_0 q} \right) \left[ 1 + \frac{2(23 + 16 \tau_e + 4 \tau_e^2)(1 + L^4 c_{L0}^2)}{q^2 (7 + 4 \tau_e)^2 (1 + L^2 c_{L0}^2)^2} + \frac{q^2}{2} \left( 1 + \frac{\pi \tau_e}{2(1 + \tau_e)} \right) \sum_{|n| \leq n_{\text{max}}} \frac{(L + n)^2 (c_h(n))^2}{(L + n - qM)^2} \right]^{-1}
\]

\[ \times \left\{ \exp(-\dot{\omega}_G^2) \left\{ \dot{\omega}_G^2 + (1 + 2 \tau_e) \omega_G^2 \right\} + \frac{1}{4} \left( \frac{k_{\tau e} v_{Ti} q}{\Omega_i} \right)^2 \exp(-\dot{\omega}_G^2/4) \left\{ \frac{\dot{\omega}_G^2}{64} + \left( 1 + \frac{3}{8} \tau_e \right) \left( \frac{3 \omega_G^2}{8} + \frac{3 \dot{\omega}_G^2}{4} \right) \right\} \right. \]

\[ \left. + \exp(-\dot{\omega}_G^2/L^2) \left( c_{L0}^2 / L^3 \right) \left\{ \dot{\omega}_G^2 + (1 + 2 \tau_e) L^2 \dot{\omega}_G^2 \right\} + \sum_{|n| \leq n_{\text{max}}} \frac{(L + n)^2 (c_h(n))^2}{2 L + n - qM} \left\{ 1 + \frac{\dot{\omega}_G^2}{(L + n - qM)^2} \left( 1 - \frac{\pi \tau_e}{2(1 + \tau_e)^2} \right) \right\} \right\} \]

GAM response function

\[ \mathcal{K}_{GAM}(t) = \cos(\omega_G t) \exp(\gamma t) \]

(Sugama & Watanabe, 2005, 2006)
Simulation Model

- Toroidal Flux Tube Model
- GKV code
  - Directly solving GK eq. in **5-D phase space**
  - Zonal flow and GAM in tokamak and helical systems
  - Entropy balance in GK turbulent transport

\[
\begin{align*}
  x &= r - r_0 \\
  y &= \frac{r_0}{q_0} [q(r)\theta - \zeta] \\
  z &= \theta
\end{align*}
\]
GKV Turbulence Simulation on Earth Simulator

- Large-scale and high-speed computation
  - 192 nodes (1536PEs)
  - Memory ~ 2.6TBytes
  - Speed ~ 4.8-5.0TFlops
- Highly optimized code for Earth Simulator
  - 3D domain decomposition
  - Hybrid parallelization …
ITG Turbulence Simulation in Helical Systems (1)

Color contour of potential perturbations plotted on a flux surface and an elliptic poloidal cross-section.

Model Parameters for the LHD Standard Configuration:

- \( \frac{r_0}{R_0} = 0.1 \), \( q_0 = 1.5 \), \( s = -1 \),
- \( \frac{R_0}{L_n} = 3.333 \), \( \eta_i = 4 \),
- \( \tau_e = 1 \), \( \nu L_n/\nu_t = 0.002 \),
- \( \varepsilon_t = \varepsilon_n = 0.1 \),
- \( \varepsilon_{L-1} = -0.2 \varepsilon_t \), \( \varepsilon_{L+1} = 0 \)
Color contour of potential perturbations plotted on a flux surface and an elliptic poloidal cross-section.

Model Parameters for the LHD inward-shifted Configuration:

\[
\begin{align*}
r_0/R_0 &= 0.1, \quad q_0 = 1.5, \quad s = -1, \\
R_0/L_n &= 3.333, \quad \eta_i = 4, \\
\tau_e &= 1, \quad \nu L_n/\nu_t = 0.002, \\
\varepsilon_l &= \varepsilon_n = 0.1, \\
\varepsilon_{L^{-1}} &= -0.8 \varepsilon_t, \quad \varepsilon_{L^{+1}} = -0.2 \varepsilon_t
\end{align*}
\]