Effect of resonant and non-resonant magnetic braking on error field tolerance in high beta plasmas

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New understanding of tokamak plasma response to 3D magnetic field

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With
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Non-axisymmetric magnetic fields can stop the plasma rotation, drive locked modes and cause disruptions

1. Plasma response to external non-axisymmetric perturbations is key in understanding the $n=1$ error field tolerance:
   a) In high $\beta$, H-mode plasmas
   b) In low $\beta$, L-mode plasmas

2. Magnetic braking of the plasma rotation is caused by two effects:
   a) By shielding of resonant magnetic fields at rational $q$-surfaces
   b) By distortion of magnetic flux surfaces enhancing the neoclassical toroidal viscosity (NTV)
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Error field tolerance in NBI heated H-modes is determined by resonant braking leading to a loss of torque balance

- Increase the amplitude of an external $n = 1$ “error” field $\delta B^\text{ext} \propto I_\text{coil}$
- Magnetic probes measure total $\delta B_p$ including the plasma response $\delta B_p^\text{plas}$ (due to perturbed plasma currents)
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- Magnetic probes measure total $\delta B_p$ including the plasma response $\delta B_p^{\text{plas}}$ (due to perturbed plasma currents)
  - At high rotation external resonant field is shielded, but exerts a torque
  - Rotation decrease is followed by a loss of torque balance
  - Magnetic island opens after rotation collapses
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- Magnetic probes measure total $\delta B_p$ including the plasma response $\delta B_p^{\text{plas}}$ (due to perturbed plasma currents)
- Rotation evolution is described by resonant braking [Fitzpatrick, Nucl. Fusion (1993), Garofalo, Nucl. Fusion (2007)]
  - At high rotation external resonant field is shielded, but exerts a torque
  - Rotation decrease is followed by a loss of torque balance
  - Magnetic island opens after rotation collapses
Tolerance to external $n=1$ perturbations decreases with increasing $\beta_N$ due to plasma amplification

- Decrease of critical external field $\delta B_{21,\text{crit}}^{\text{ext}}$ is particularly strong above the no-wall limit
  - Amplification increases when ideal MHD stable $n=1$ kink mode converts to kinetically stabilized RWM [see Okabayashi, EX/P9-5]

- Rotation collapse occurs at a fixed plasma response $\delta B_{p,\text{crit}}^{\text{plas}}$

- Critical plasma response $\delta B_{p,\text{crit}}^{\text{plas}}$ increases with NBI torque $T_{\text{NBI}}$
Plasma is very sensitive to the poloidal spectrum (pitch angle) of the external perturbation.

- Vary poloidal spectrum of external \( n=1 \) perturbations applied with I-coil.

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**Graphs**

- **Top Graph**
  - Toroidal angle \( \phi \) vs. Poloidal angle \( \gamma \)
  - Projected field line
  - 240 deg phase difference
  - 120 deg
  - 180 deg

- **Bottom Graph**
  - Amplification vs. I-coil phase difference
  - MARS-F \( (\kappa_{||}=1) \)

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Plasma is very sensitive to the poloidal spectrum (pitch angle) of the external perturbation

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Plasma is very sensitive to the poloidal spectrum (pitch angle) of the external perturbation

- Vary poloidal spectrum of external $n=1$ perturbations applied with I-coil

- Rotation collapse occurs at a fixed plasma response $\delta B_{p,\text{crit}}$

- Amplification largest for external perturbation with a lower pitch than the equilibrium field at the outboard midplane

Described by coupling to stable $n=1$ kink mode (MARS-F code)
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Ignoring the plasma response even at low $\beta$ leads to paradoxical results in error field correction experiments

- External resonant field: $\delta B_{21}^{\text{ext}} = \delta B_{21}^{\text{intrinsic}} + \delta B_{21}^{\text{correction}}$ at $q=2$ surface
  - Experiments in many tokamaks have shown that the plasma density at locking increases proportionally with the external field
- However, the external resonant field …

... shows no correlation with locking density in DIII-D

... is largest when the locking density is lowest (optimal phase) in NSTX

→ Self-consistent resonant field including plasma response (perturbed plasma current) is necessary
Plasma response given by Ideal Perturbed Equilibrium Code (IPEC)

- IPEC calculates free-boundary 3D tokamak equilibria while preserving $p(\psi)$ and $q(\psi)$ profiles

[IPEC is based on DCON and VACUUM stability codes] [Park, Phys. Plasmas (2007)]

1) Islands are shielded by rotation before locking, so plasma remains ideal
   → Shielding currents at the rational surfaces give the total resonant field

2) Magnetic surfaces are not destroyed, but deformed
   → Important variation of the field strength is along the perturbed field lines, not at fixed points in space (as used in vacuum superposition method)

Example: $n=1$ from NSTX EF/RWM coils

2D Equilibrium
Superposition (equilibrium + $n=1$ vacuum)
IPEC

Islands Flux surface destruction
No islands Flux surface deformation
Total resonant field including plasma response explains paradoxical NSTX and DIII-D low $\beta$ experiments

- **Total resonant field** $\delta B_{21}$:
  - restores the linear density scaling (DIII-D)
  - is consistent with the optimal performance (NSTX) [Park, Phys. Rev. Lett. (2007)]

- **External field that maximizes the total resonant field is**:
  1) Similar to a kink-type distribution (consistent with MARS-F code)
  2) Almost independent of plasma parameters [Park, Nucl. Fusion (2008)]
Plasma response (IPEC) connects error field tolerance at high $\beta$ with Ohmic plasmas via the linear density scaling

- Critical resonant field (IPEC) at $\beta_N=1.5$ and low NBI torque in good agreement with the low-$\beta$ density scaling
  - NSTX $n=1$ resonant field amplification experiments validate IPEC up to the ideal MHD no-wall limit
  [see Park, EX/5-3Rb poster]
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Measured $n=1$ braking torque reveals importance of a non-resonant magnetic braking component

- Measured angular momentum evolution yields magnetic braking torque $T_{MB}$
  
  $$T_{MB} = T_{NBI} - \frac{L}{\tau_{L,0}} - \frac{dL}{dt}$$

  - Assume $T_{MB} \propto (\delta B^{\text{plas}})^2$ to reveal rotation dependence

- At low rotation $T_{MB}$ increases with decreasing $\Omega$ consistent with a resonant torque

- At high rotation $T_{MB}$ increases with $\Omega \rightarrow$ typical for a non-resonant torque

[Shaing, Phys. Plasmas (2003)]
Non-resonant braking reduces the beneficial effect of additional torque input on the error field tolerance

- Torque balance with a resonant torque only yields
  \[ \delta B_{\text{crit}} \propto T_{\text{in}} + T_{\text{NBI}} \]
  with \( T_{\text{in}} \) being the intrinsic torque [see Solomon, EX/3-4 for \( T_{\text{in}} \)]

- Adding a non-resonant torque reduces the dependence of \( \delta B_{\text{crit}} \) on \( T_{\text{NBI}} \) to
  \[ \delta B_{\text{crit}} \propto (T_{\text{in}} + T_{\text{NBI}})^{0.5} \]

- Observed increase of the \( n=1 \) error field tolerance with NBI torque is consistent with a significant contribution of non-resonant braking
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Neoclassical Toroidal Viscosity (NTV) theory gives the toroidal torque for non-resonant braking

• Using the external field (vacuum) and neglecting precession rates result in $O(1)$ agreement with experiment

• Important new physics in NTV theory:
  a) Toroidal precession rates ($\omega_p$) are often faster than the collisional rates ($\nu$)
  b) Trapped particle bounce rates ($\omega_b$) can resonate with the precession ($\omega_p$)
  c) Variation of field strength along the perturbed magnetic field lines, which include plasma response

(1) (a), (b) and (c) are all ignored
(2) (a) is included
(3) (a) and (b) are included
(4) (a), (b) and (c) are all included
[see Park, EX/5-3Rb poster & Becoulet, TH/2-1Rb]
Resonant Magnetic Perturbation (RMP) control of ELMs on ITER can be optimized using IPEC and NTV theory

- Three requirements for optimization:
  1) Islands overlap for $\psi_N > 0.85$ [see Evans, Ex/4-1]
  2) Minimize $\sum (\delta B_{mn})^2 / \sum (\delta B_{mn}^{ext})^2_{\text{boundary}}$ for $\psi_N < 0.8$
  3) Maximize $\sum (\delta B_{mn})^2 / \sum (\delta B_{mn}^{ext})^2_{\text{boundary}}$ for $\psi_N > 0.8$

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Summary

• Plasma response to external non-axisymmetric perturbations is key in understanding the $n=1$ error field tolerance in high $\beta$, H-mode as well as in low $\beta$, L-mode plasmas
  – Plasma response in rotating plasmas with values of $\beta$ up to the ideal MHD stability limit is described by ideal perturbed equilibrium theory (IPEC code)
  – Measurements and calculations show that plasmas are most sensitive to a kink and ballooning-type external perturbation rather than external resonant perturbations

• Magnetic braking of the plasma rotation is caused by shielding of resonant perturbations and by the distortion of magnetic flux surfaces enhancing neoclassical toroidal viscosity (NTV)
  – Non-resonant braking reduces the benefit of additional torque input
  – Description of non-resonant braking has to include the variations of the field strength on deformed magnetic surfaces and particle bounce/precession resonances