Physics of Non-Diffusive Turbulent Transport of Momentum and Origins of Spontaneous Rotation in Tokamaks


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Acknowledgements:
Outline

i. Motivation from Experiments

ii. Some Basic Physics Ideas

iii. Physics of Off-Diagonal Momentum Flux
   a) Residual Stress
   b) Momentum Pinch
   c) Towards a Tractable Model

New Findings on Diffusive Flux:
   • Role of Wave-Particle Resonance ---> Intrinsic Prandtl Number <1
   • Renormalized Diffusive Flux due to Residual Stress Contribution

iv. Importance of Boundary Physics

i. More Theoretical Ideas for Future Work

ii. Suggestions to Experimentalists

vii. Conclusions
Phenomenology of Momentum Transport and Intrinsic Rotation

• Historically, \( \chi_\phi \approx \chi_i \) has been predicted \([\text{Mattor, Diamond, PF '88}]\) observed \([\text{Scott et al., PRL '90}]\)
\( \chi_\phi / \chi_i \) deviates from unity \( \implies \) off-diagonal contribution to momentum flux ?!

Need for Momentum Pinch indicated by many experiments, including modulation experiments
\([\text{Ida, NF '01, Yoshida et al., PPCF '06}]\)
Fluctuation studies indicate \( \langle \delta v_r \delta v_\parallel \rangle \) linked to shear flows \([\text{Hidalgo et al., '06}]\)

• Intrinsic (spontaneous) toroidal rotation observed in nearly all tokamaks
L-mode rotation: complex, closely tied to SOL flows
H-mode plasmas demonstrate simple clear empirical trends \([\text{Rice et al., NF '04}]\)

  – rotation typically co-current
  – \( \Delta v_\phi \sim W/I_p, \ M_A \sim \beta_N \) \( \implies \) no apparent scalings with \( \rho^* \ nu^* \)
  – offset in torque scan matches intrinsic rotation \([\text{Solomon et al., PPCF '08}]\)

• Observations \( \implies \) Rotation initiates at edge, inverts at L-H transition
  Co-rotation builds inward

Intrinsic rotation also suggested in Core:
Modulation \( \chi_\phi, V_{\text{pinch}} \) cannot fit measured \( \langle v_\phi \rangle \) profile \([\text{Yoshida et al., PRL '08}]\).

Momentum transport bifurcations observed at torque-free core
\([\text{Duval et al., PoP '08, Ince-Cushman '08}]\)
Issues on Theory of Momentum Transport

- structure of momentum transport: diffusion, pinch, residual stress
- Physics of resonant and non-resonant contributions
- origins of inward flux, intrinsic rotation
- physics of symmetry breaking?

Momentum Flux (electrostatic turbulence) [Diamond et al., PoP '08]

\[ \langle \tilde{v}_r \tilde{v}_\phi \rangle = -\chi_\phi \frac{\partial}{\partial r} \langle v_\phi \rangle + V \langle v_\phi \rangle + \Pi_{r,\phi}^{\text{resid}} \quad \text{with} \quad \Pi_{r,\phi}^{\text{resid}} \approx \langle n \rangle \langle \tilde{v}_r \tilde{v}_\phi \rangle + \langle v_\phi \rangle \langle \tilde{v}_r n \rangle \]

Waves each contribute to all of \( \chi_\phi V_{\text{pinch}}, \Pi_{\text{resid}} \)

Resonant Particles

- Wave momentum flux \( \Pi_{r,\parallel}^{\text{wave}} \) crucial for fluid-like DWT, ITG

Radial Flux of Parallel Mom. \( \Pi_{\parallel}^{W} \equiv \sum_k v_{g r x} k_{\parallel} N_k \)

Finite momentum flux requires:
  - symmetry breaking \( \langle k_{\parallel} \rangle \neq 0 \)
  - radial wave quanta flux \( \langle v_{g r x} \rangle \neq 0 \)

- Resonant Particle Flux: \( \Pi_{\text{particle}}^{\text{Resonant}} \propto \langle \omega / k_{\parallel} V_{Th} \rangle \text{Spectrum} \)
Wave Momentum Flux and Residual Stress

- Wave momentum flux from radiation hydrodynamics for short mean free path:

\[ \Pi^\text{wave} = \int d\mathbf{k} k_{||} \left\{ -\tau_{c,k} V_{gr} \frac{\partial \langle N_k \rangle}{\partial r} + \tau_{c,k} V_{gr} k \theta \langle V_E \rangle' \frac{\partial \langle N_k \rangle}{\partial r} \right\} \]

- First term ↔ radiative diffusion of quanta
  - \( \frac{\partial}{\partial r} \langle N_k \rangle > 0 \), universally increasing
  - inward scattering from edge

- Second term ↔ refraction induced wave quanta population imbalance:
  - important for regimes of strong shear, sharp \( \nabla P \)
  - mode dependence, via \( \nu^* \) convection
  - can flip direction \([TCV, C-Mod]\)

Symmetry Breaking

- Growth asymmetry \([Coppi, NF '02]\)
  - Bias in \( \gamma(k_{||}) \) from \( \langle V_{||} \rangle' \)
  - unlikely in realistic regime

  - Wind-up by shearing packet
  - akin spiral arm formation:
    - requires \( \langle V_E \rangle \) and magnetic shear \( \partial k_{||}/\partial k_r \neq 0 \)

- Refractive Force due to GAMs
  - relevant near edge
  - polarization effects \([McDevitt et al., PoP '08]\)
Residual Stress

\[ \langle \tilde{v}_r \tilde{v}_\phi \rangle = -\chi_\phi \frac{\partial \langle v_\phi \rangle}{\partial r} + V \langle v_\phi \rangle + \Pi_{r,\phi}^{\text{resid}} \]

Piece of Reynolds Stress without Explicit Dependence on \( \langle v_\phi \rangle \), \( V \), \( \nabla \phi \), etc.

- **Beyond Diffusion and Pinch**
- particle number conserved → \( \Gamma_n = -D \frac{d \langle n \rangle}{d r} + V \langle n \rangle \)
- pinch is only “off-diagonal” for particles
- but: wave-particle momentum exchange possible

\[ \Pi_{r,\phi} \approx \langle n \rangle \langle \tilde{v}_r \tilde{v}_\phi \rangle + \langle v_\phi \rangle \langle \tilde{v}_r \tilde{n} \rangle \]

Can accelerate resting plasma:

\[ \partial_t \int_0^a dr \langle V_\phi \rangle = \partial_t \overline{V_\phi} \approx -\Pi_{r,\phi}^{\text{resid}} |_{0}^{a} \]

\( \nabla p_i, \nabla n \rightarrow \Pi_{\phi}^{\text{resid}} \)

→ residual stress acts with boundary condition to generate intrinsic rotation [Gurcan, Diamond, Hahm, Singh, PoP 2007]

**How?** Broken Symmetry in Turbulence
- akin to \( \alpha \) -effect in dynamo theory
Physics of Residual Stress

- Key Point: $\langle V_E \rangle'$ converts poloidal flow shear into toroidal flow shear via: asymmetry in wave to particle momentum deposition
- finite $\langle V_E \rangle'$ + generic drift-acoustic coupling
  $\rightarrow$ shifted spectral envelope (persists in torus)

$\langle k_{||} \rangle \neq 0$

[S. Itoh, PF ’92, Dominguez et al., PFB ‘93, Diamond et al., IAEA ‘94]

$\rightarrow$ special case of refraction-induced “winding” asymmetry

- directional imbalance in
  $\rightarrow$ acoustic wave populations
  $\rightarrow$ profile of momentum deposition by ion Landau damping
Toroidal Momentum Flux strongly correlated with Zonal Flow Shear in GTS ITG Simulations

Self-generated zonal flow is quasi-stationary in global ITG simulations
Conversion: (no input $V_\phi$ )

Poloidal $\rightarrow$ Toroidal Zonal Flow

Strong correlation among Zonal flow shear, $k_\parallel$ spectra,
Inward Momentum Flux

Mechanism:
Generation of Residual Stress due to $k_\parallel$ symmetry breaking induced by ZF shear:
(extend the mechanism due to mean ExB shear)

$[\text{Gurcan et al., PoP '07}]$

$[\text{W. Wang et al., Paper TH/P8-44, IAEA '08}]$
Physics of Momentum Pinch in Torus

Pinch can come from various physical mechanisms including wave-particle resonance.
Curvature driven toroidal momentum pinch has two parts: \[\text{[Hahm et al., PoP '07]}\]

\(V_{\text{TEP}}\): Turbulent Equipartition Pinch: mode-independent,

due to \(\nabla \cdot \mathbf{v}_E \times B \neq 0\) in torus, simple and robust part.

\(V_{\text{TH}}\): Thermo-electric pinch: mode-dependent, related to thermodynamic (sensitive to turbulence character, as for density)

Physics of TEP pinch:

Particle Pinch in Slab with nonuniform \(B\) \(\text{[Yankov '94, Isichenko, Diamond '97,}\)

\[
\partial_t n + \nabla \cdot (n \mathbf{v}_E) = 0 \quad \nabla \cdot \mathbf{v}_E \neq 0 \quad (\partial_t + \mathbf{v}_E \cdot \nabla) \left(\frac{n}{B}\right) = 0
\]

Mixing of magnetically weighted, locally conserved quantity \(n/B \rightarrow\)

Effective Pinch in Observed Physical Quantity \(n\)
Turbulent Equipartition Pinch of Angular Momentum

[Hahm, Diamond, Gurcan, Rewoldt, PoP 2007, 2008, Gurcan et al., PRL 2008]

From angular momentum conservation and compressible ExB flow

\[
\partial_t \left( nU || R \right) + \nabla \cdot \left( nU || R v_E \right) \approx 0 \quad \nabla \cdot v_E \neq 0 \quad (\partial_t + v_E \cdot \nabla) \left( \frac{nU || R}{B^2} \right) \approx 0
\]

**Mixing/Diffusion** of Magnetically weighted angular momentum \( nU || R/B \)

\[
\Pi_{\text{MWA}} = \left[ \delta_r \left( \delta_r nU || R / B^2 \right) \right] = -\chi_{MWA} \frac{d}{dr} \left( nU || R / B^2 \right)
\]

\( \rightarrow \) Inward Pinch in observed quantity \( nU \)

\[
\Pi_{\text{MWA}} = \left[ -\chi_{\phi} \frac{d}{dr} (nU \parallel) + V_{\text{TEP}} (nU \parallel) \right] R / B^2 \]

\( \rightarrow \) quasilinear calculation \( \rightarrow \)=

\[
V_{\text{TEP}} = -B^2 \frac{d}{dr} \frac{R}{B^2} \approx -\frac{3}{R}
\]

\[
\omega_{\phi} \equiv \frac{U \parallel}{R} \quad \frac{V_{\text{TEP}}}{\chi_{\phi}} \approx -\frac{4}{R}
\]

(with definitions w.r.t. angular rotation freq: \( \omega_{\phi} \equiv \frac{U \parallel}{R} \))

also from derivation based on conservative gyrokinetic equation [Hahm, PF 1988]
Thermo-electric Pinch

\( V_{TH} \): Thermo-dynamic, piece of Reynolds stress related to

\[
V_{TH} = 4 \left( \frac{c}{B} \sum_k k \theta \Re \left( \tau_c \omega_{di} \delta \phi^* \left( \delta T_i / T_i \right) \right) \right)
\]

dependent on mode-characteristics (e.g., phase angle between \( \delta T_i \) and \( \delta \phi \))
sensitive to gyrofluid approximation, dispersion relation, and frequency broadening/shift etc.

\( V_{TH} < 0 \) always (inward) for \( V^*e \) direction mode (TEM)
\( V_{TH} > 0 \) typically (outward) for \( V^*i \) direction mode (ITG)

c.f. ITG-specific calculation by [Peeters et.al., PRL '07]

Convective Momentum Pinch also exists

\[
\Pi_{r, \phi} \equiv \langle n \rangle \langle \tilde{v}_r \tilde{v}_\phi \rangle + \langle v_\phi \rangle \langle \tilde{v}_r \tilde{n} \rangle
\]

originates from particle pinch, contains both TEP and thermoelectric pieces
GTC Simulations indicate Pinch and Wave-Particle Resonance

- Off-diagonal (inward pinch-like) flux is observed in rigid rotation case;
- Diffusive flux is separated by subtracting pinch contribution from the total flux;
- Intrinsic Prandtl number is calculated using diffusive momentum flux only \( \Pr \equiv \gamma_{\text{diff}} / \gamma_{\phi} \approx 0.2 - 0.7 \)

Existence of resonance at \( \nu / v_i > 1 \) explains

Intrinsic Pr < 1: energy flux has larger velocity weight than momentum flux

Quasilinear calculations of Pr based on obtained fluctuation spectra show good agreement with simulation results \( \Pr^{\text{QL}} \equiv \chi^{\text{QL}}_{\phi} / \chi^{\text{QL}}_{\iota} \approx 0.7 \)

\[
\chi^{\text{QL}}_{\phi} = \frac{1}{n_0 \langle Rv \rangle} \int dv R_v D(v) f(v)
\]

\[
\chi^{\text{QL}}_{\iota} = \frac{1}{n_0 T} \int dv \frac{mv^2}{2} D(v) f(v)
\]

\[
D(v) = \frac{1}{2} \frac{\langle \Delta x^2 \rangle}{\tau} = \frac{\pi c^2}{2B^2V} \sum_k \frac{d\phi}{2\tau} \left[ \langle |\phi|^2 \rangle \right]_{\theta k} k^2 J^2_0(k_i \rho_i) \delta (k_p^\rho - \theta)
\]

[Holod, Lin, PoP 2008]
Towards a simple illustrative model of Intrinsic Rotation in H-mode

Key Elements:

→ \( \langle v_{\phi} \rangle \) profile

\[ \Pi_{r,\phi}^R \quad \text{(Residual Stress) due} \quad \langle V_E \rangle \quad \text{and B.C.} \]

\[ \Delta (\nabla P) \leftarrow \Delta W_P \]

\[ V_{TEP} \quad \text{due} \quad \nabla \cdot v_E \neq 0 \quad \text{peaking on axis} \]

→ couple to simple ExB shear-based L-H transition model

N.B. \( \Pi_{r,\phi}^R \) decays with \( \langle V_E \rangle \) slower than \( \chi_\phi, \chi_i, D \ldots \)

→ fix \( \langle v_{\phi} \rangle \) at boundary
Simple Illustrative Model

Conservation Laws:

\[
\frac{\partial n}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (r \Omega_n) = S_n
\]

\[
\frac{\partial P}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (r Q) = H
\]

\[
\frac{\partial L_\phi}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (r \Pi_\phi) = \tau_\phi
\]

Angular Momentum

Radial Force Balance:

\[
E_r \equiv \frac{1}{n e} \frac{\partial P}{\partial r} - u_{\theta,Ne_0} B_\phi + u_\phi B_\theta
\]

\[
\epsilon = \frac{\epsilon_0}{1 + \beta \left( \frac{E_y}{\partial r} \right)^2}
\]

ExB Shear Reduction [Hinton, PF-B ’91]

Fluxes:

\[
\Gamma_n = - D_0 \frac{\partial n}{\partial r} - D_1 \varepsilon \left( \frac{\partial n}{\partial r} + V_n \dot{\Pi}_n \right)
\]

\[
\Pi_\phi = - \nu_0 \frac{\partial L_\phi}{\partial r} - \nu_1 \varepsilon \left[ \frac{\partial L_\phi}{\partial r} + V_r L_\phi \right] + S
\]

where

\[
S = - \varepsilon \alpha (r) \left( 1 - \frac{\sigma}{P_0} \frac{\partial P}{\partial r} \right) \frac{\partial u_{E_y}}{\partial r}
\]

\[
Q = - \chi_0 \frac{\partial P}{\partial r} - \chi_1 \frac{\partial P}{\partial r}
\]

Residual Stress due to ExB Shear

TEP Pinch for Momentum and Density

L-mode Turbulence Intensity \( \epsilon_0 \)

only adjustable parameter

B.C.’s: \( L_\phi(a), V_\phi(a), n(a) : \) given
\( \langle \vec{V}_E \rangle \) driven Residual Stress modifies \( \chi_\phi \)

\[
S_{\text{Resid}} = -\alpha(r, a/L_p) \langle \vec{V}_E \rangle' \quad \text{using} \quad \frac{\partial}{\partial r} (E_r/B_\theta) = \frac{\partial}{\partial r} u_\phi + \frac{\partial}{\partial r} \left( \frac{1}{n B_\theta^2} \frac{\partial P}{\partial r} \right) - \frac{\partial}{\partial r} (u_\theta B_\phi)
\]

\[
= S_{\text{Resid}}^{(0)} - \delta \chi_\phi \frac{\partial u_\phi}{\partial r} \quad \Rightarrow \quad \chi_\phi \quad \text{renomalization}
\]

Lesson: as intrinsic rotation develops, it feeds back on driving \( \langle \delta V_r \delta V_\phi \rangle \) via

- Renormalization of \( \chi_\phi \), i.e., \( \chi_\phi = \chi_{\phi}^{\text{bare}} + \delta \chi_\phi \) (new feedback loop)

  n.b. not a pinch!

- Fluctuation Reduction (usual feedback loop)

- Since

\[
S_{\text{Resid}}^{(0)} / \delta \chi_\phi = - \frac{\partial}{\partial r} \left( \frac{1}{n e B_\theta} \frac{\partial P}{\partial r} \right) \equiv \frac{1}{e B_\theta} \left( \frac{\partial P}{\partial r} \frac{\partial n}{\partial r} - \frac{\partial^2 P}{\partial n \partial r^2} \right) \quad \text{ignoring} \quad \| \theta
\]

and \( S_{\text{Resid}}^{(0)} < 0 \) for Co-rotation, \( \delta \chi_\phi < 0 \)

- Momentum diffusivity reduced for Co-rotation, enhanced for Ctr-rotation.
Scaling Trends manifested by Model

Dimensional analysis for pedestal flow velocity suggests scaling with width:

$$\frac{v_\phi}{v_{Ti}} \propto \left( \frac{\Delta r_{Turb}}{a} \right) \left( \frac{\Delta_{ped}}{a} \right) \propto (\rho^*)^\alpha \left( \frac{\Delta_{ped}}{a} \right)$$

With the simple model linking width to height,

$$\Delta_{ped} \propto P_{ped} \quad \Delta v_\phi \propto \left( \frac{\rho^*}{\Delta_{ped}} \right)^\alpha \Delta W_p$$

where $$\Delta W_p$$: Incremental Stored Energy

$$/p$$ scaling not recovered for GB model → pedestal/edge turbulence issues?

Model is not quantitatively accurate,

- predicts a scaling of the pedestal toroidal velocity with the pedestal width.

$$V_{\phi,ped} \propto \Delta_{ped} \propto P_{ped}$$

- but recovers qualitative behavior
Physics of Boundary Condition Effects

- **SOL Flows:** [LaBombard et al., NF `04]
  
  "ballooning" particle flux produced by outboard source and SOL symmetry breaking (LSN vs USN)

  Influence core $\Delta u_\phi$ in L-mode

  LSN $\to$ $V_B \nabla B$ toward X-point $\to \Delta u_\phi$ co
  USN $\to$ $V_B \nabla B$ from X-point $\to \Delta u_\phi$ counter

  But, in H-mode, $\Delta u_\phi$ is always CO

- **Key question:** How can SOL flow influence core plasma?

- For $S_\parallel(r) =$ speed profile of SOL flow

  \[ \frac{d S_\parallel(r)}{dr} > 0 \text{ in SOL } \rightarrow \text{Inward viscous stress of SOL flow on core} \]

  Key: SOL symmetry breaking sets $\Delta u_\phi$ direction

  Strong for parallel shear flow instability for $\nabla V_\parallel > \nabla n$

  Alternative: Recoil from Blob Ejection (Myra)
Ongoing and Future Theoretical Work

• Electromagnetics:
  – saturation of intrinsic rotation with $\beta$  \[McDevitt, Diamond, PoP ‘08\]
  – Alfvénic waves in burning plasmas --> field momentum

Alternative symmetry breaking mechanism
  Polarization effects
  GAMs and refractive force
  SOL effects

• Poloidal Rotation
  – correct neoclassical theory for ITB, ETB regimes
  – anomaly: off-diagonal elements
  – Charney-Drazin theorem constraints

SOL-Core coupling Dynamics
  SOL Stress on Core
  symmetrization at L-H transition --> source for SOL?
  blobs and wave breaking

Detailed Modelling Work --> Specific Phenomenology
Conclusions: what have we learned?

- Momentum Transport ---> Off-diagonal Flux: \( \left\{ \right. \)
  - Residual Stress
  - Pinch

\[ \Pi^{\text{resid}} \]
 requires \( \left\langle k_{||} \right\rangle \neq 0 \) symmetry breaking
which can come from \( \left\langle V_{E} \right\rangle \)

with B.C., can accelerate \( \left\langle V_{\phi} \right\rangle \)

\[ V^{\text{pinch}} \left\{ \right. \]
- \( V_{\text{TEP}} \)
- \( V_{TH} \)

robust, universal, related to particle pinch
mode-dependent

- **Intrinsic Prandtl Number** in Stiff Profiles
- Off-diagonal \( \Pi^{\text{resid}} \) + B.C. ---> Intrinsic Rotation
  - Model recovers essentials

\[ V_{\phi,ped} \propto \Delta_{ped} \propto P_{ped} \propto \Delta W_{p} \]

- Physics of B.C. is a major unknown.
Challenges to Experimentalists

• Boundary Condition ↔ SOL Flow-Core interaction?
  – dynamics/evolution during slow transitions
  – Low vs high neutral opacity regime comparisons
  – SN vs DN comparisons
  – Poloidal Rotation: is it neoclassical?

• Core Transport Dynamics
  – \( \Pi_{R,t,\phi} \) in Core (c.f. JT-60U) --- Residual Rotation?
  – Compare momentum pinch and particle pinch
  – ITB regime intrinsic rotation --- \( \langle \hat{V}_E \rangle \), magnetic shear sensitivity
  – Intrinsic Prandtl Number in Stiff Profiles?
  – Corrugated \( \langle V_\phi \rangle \) Profile due Zonal Flows?

• Intrinsic Rotation in Electron-dominated Plasmas (ITER relevant!)
  – CTEM as transport agent

• Relative Stiffness of ion channels ?
  electron toroidal momentum
Constant angular velocity (rigid rotation case):

- inward momentum flux (pinch);
- redistribution of momentum (spinning up towards the center)

Angular velocity: \( \omega_{\phi} = (\omega_0 + \omega_1 r/a) v_i / R_0 \)

Momentum flux for various rigid rotation cases

Sheared rotation case

**Flux separation: subtracting pinch contribution from the total flux gives diffusive flux**

with **Intrinsic Prandtl Number**

\[ \text{Pr} \equiv \frac{\chi_{\phi}^{\text{diff}}}{\chi_1} \approx 0.2 \sim 0.7 \]
Residual Stress causes Rotation Buildup in Pedestal

\[ \nabla \langle p \rangle \rightarrow \langle v_E \rangle' \]

→ asymmetry → residual stress

- Sharp gradients cause a “torque density” \( \nabla \langle p \rangle \rightarrow \langle v_E \rangle' \)

- Leading to a net integrated momentum due to the residual influx from boundary i.e., effective local source

\[
\frac{\partial}{\partial t} \hat{V}_\phi + \Pi_{r,\phi}(a_-) = 0
\]

- Pinch can only change the rotation profile.

- H-mode spin-up with no slip B.C. which is appropriate for high neutral friction.

But B.C. not clear for ITER, and other plasmas with high neutral opacity.

- Must face stress exerted by SOL