Analytic Theory of a Matching Problem Generalized for Stability Analysis of Resistive Wall Modes in Rotating Plasmas

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Abstract. New matching theory is proposed for stability analysis of Resistive Wall Modes (RWMs) in rotating plasmas. It is found that for rotating plasmas, the classical asymptotic matching theory becomes invalid in principle. The matching problem should be generalized to use inner layers with finite width to overcome the difficulty that a resonant surface, which exists somewhere in finite distance apart from the rational surface depending on the unknown rotation frequency of the RWM, should be included in the layers. For the first time, numerical analysis based on the generalized matching problem shows that the location of resonance determined by rotation in the inner layer significantly affects the stability of RWMs. Analytic theory for the generalized matching problem is developed to interpret the results by generalized matching problem and to search the unknown location of the resonant surface.

1. Introduction

Study of rotational effects on magnetohydrodynamic (MHD) modes such as Resistive Wall Modes (RWMs) is acquiring scientific and engineering importance in the tokamak physics community. Concerning the advanced tokamak regime, stabilization of RWM is indispensable, and the plasma rotation is one of the most promising methods [1]. To clarify the stabilization mechanism of plasma rotation on MHD mode requires solving a matching problem [2]. The matching problem divides the whole plasma region into two parts: inner layers and outer regions. The inner layer is a thin region that contains the essential physics for stability (in the ideal MHD, resonance), and the outer regions are regions except for the inner layers. An implicit assumption in the matching problem is that the location of resonance is known *a priori*. The assumption holds only for static plasmas since the resonance inevitably occurs at the rational surface. Then, the asymptotic matching theory works well by identifying the rational surface as the inner layer. In addition, in the static case, the singular point of the Newcomb equation governing the outer regions also degenerates into the rational surface.

2. Basic equations for rotating plasmas

2.1 Generalized Hain-Lust equation

When the rotation exists, the degeneracy among the resonant surface, the rational surface, and the singular points is resolved. To see this, we start from the Frieman-Rotenberg equation [3] that is a linearlized ideal MHD equation,

$$\partial_t \boldsymbol{\xi} + \mathbf{V}\cdot\nabla\boldsymbol{\xi} = \boldsymbol{\Pi},\quad \partial_t \boldsymbol{\Pi} + \mathbf{V}\cdot\nabla\boldsymbol{\Pi} = F\boldsymbol{\xi},\quad (1)$$

where $\boldsymbol{\xi}$ is the Lagrange displacement defined by $\tilde{\mathbf{V}} =: \partial_t \boldsymbol{\xi} + \mathbf{V}\cdot\nabla\boldsymbol{\xi} - \boldsymbol{\xi}\cdot\nabla\mathbf{V}$ with $\mathbf{V}(\tilde{\mathbf{V}})$ the equilibrium (perturbed) rotation and $\boldsymbol{\Pi} = \partial_t \boldsymbol{\xi} + \mathbf{V}\cdot\nabla\boldsymbol{\xi}$ the specific momentum. The operator F is defined by $F\boldsymbol{\xi} = F_s\boldsymbol{\xi} + \nabla\cdot\left[\boldsymbol{\xi}\left(\mathbf{V}\cdot\nabla\mathbf{V}\right)\right]$ where F_s is the well-known force operator for static plasmas. Note that F is a self-adjoint operator, however the problem (1) is non-self-adjoint due to the convective derivatives.

Throughout this paper, the cylindrical plasma is examined in coordinates (r,θ,z). We assume that perturbations are proportional to $\exp(-i\omega t + im\theta + ikz)$, where m is the poloidal mode number and k is the wave number in z direction. The eigenvalue $\omega = \omega_r + i\gamma$ (ω_r is the mode real frequency and γ is the growth rate) is complex due to non-self-adjointness. With appropriate normalization, the Frieman-Rotenberg equation (1) reduces to the generalized Hain-Lust equation [4]:

$$\frac{\partial}{\partial r}\left[f(r,\omega)\frac{\partial y}{\partial r}\right] - g(r,\omega) y = 0, \quad (2)$$

where we have defined some notations as $y = r\xi_r$, $f(r,\omega) = A(r,\omega)S(r,\omega)/C_2(r,\omega)$ and $g(r,\omega) = \partial[C_1(r,\omega)/C_2]/\partial r + r^{-1}C_5(r,\omega)/C_2 - C_4(r,\omega)/r$. Assuming toroidal rigid rotation, the coefficient related with shear Alfven waves reads $A(r,\omega) = (\omega - \Omega)^2 - F^2(r)$ where $\Omega = -nR_0V_z$ is the plasma rotation frequency where $n = -k/R_0$ is the toroidal mode number, R_0 the major radius, V_z the equilibrium toroidal rotation. $F = \mathbf{k}\cdot\mathbf{B} = mB_\theta / r + kB_z$ is related to the parallel wave number with wave number vector $k = m\hat{\theta}/r + k\hat{z}$, and equilibrium magnetic field $\mathbf{B} = B_\theta\hat{\theta} + B_z\hat{z}$. The coefficient associated with the slow wave reads $S(r,\omega) = (\omega-\Omega)^2(\Gamma p + B^2) - \Gamma F^2 p$ where Γ is the ratio of specific heats and p is the equilibrium pressure. Also, we have $C_2(r,\omega) = r(\omega-\Omega)^4 - r(\varepsilon^2k^2 + m^2/r^2)S(r,\omega)$ where $\varepsilon = a/R_0$ the inverse aspect ratio with the minor radius a, $C_1(r,\omega) = 2B_\theta^2(\omega-\Omega)^4/r - 2mFB_\theta S(r,\omega)/r^2$, $C_4(r,\omega) = A + rd/dr(B_\theta^2/r^2)$, and $C_5(r,\omega) = 4\varepsilon^2k^2B_\theta^2[2B_\theta^2(\omega-\Omega)^2 - F^2\Gamma p]/r$.

2.2 Generalized Newcomb equation

Firstly, we examine the singularity of Eq. (2), i.e., $f(r,\omega) = 0$. As a reference, we consider the case without rotation $\Omega \equiv 0$. In this case, the singularity of the marginal ($\omega = 0$) equation (i.e., conventional Newcomb equation [5]) locates where $F(r_{m/n}) = 0$. This singularity corresponds to the rational surface since it exists at $F = r^{-1}B_\theta[m - nq(r_{m/n})] = 0$ where $q = rB_z / R_0 B_\theta$ is the safety factor. Without rotation, the self-adjointness warrant the unstable mode does not have a real frequency, thus the mode inevitably resonates at the rational surface. Hence, in the framework of the matching problem, the rational surface can be identified with the inner layer and asymptotic matching works well [2]. In other words, in the static case, the singular point of the conventional Newcomb equation and the resonant surface degenerate into the rational surface.

However, when the rotation exists, the degeneration among the singular point, the resonant surface, and the rational surface breaks down, and the situation becomes rather complex. To see this, we focus on the resonance with the forward and backward Alfven waves with frequencies $\pm F$. The rational surface exists at $F(r_{m/n}) = 0$. If a uniform rotation ($\Omega = const.$) introduced, due to the Doppler-shift, the Newcomb equation is generalized. In the marginal case ($\omega = 0$), the singular points with $\Omega \pm F = 0$ split from the rational surface. At first glance, it seems that these singular points can be identified with the inner layers, however, it is not true. This is because in rotating plasmas, the unstable mode can have a finite real frequency ω_r due to the non-self-adjointness. Then the resonant surface can deviate from the singular points of the generalized Newcomb equation. The situation is summarized in Fig.1. Since the location of the resonant surface depends on unknown ω_r, it cannot be determined *a priori*. Then, the implicit assumption that the location of the inner layer is known before solving the problem does not hold, and hence, the asymptotic matching theory for RWM [6], which identify the rational surface with the inner layer, overlooks the resonant surface and cannot calculate the eigenfunction and the growth rate correctly.

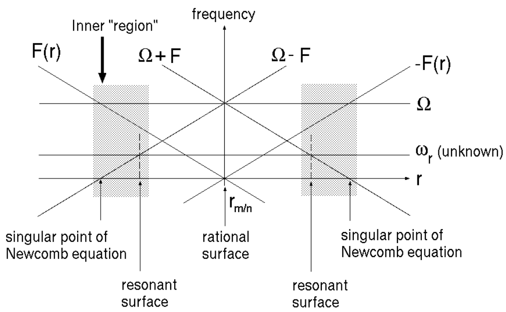

FIG. 1.Schematic view of locations of the rational surface, the singular points of the generalized Newcomb equation and the resonant surface for the shear Alfven waves. The singular points split from the rational surface due to the Doppler shift. The resonant surface deviates from the singular points since the unstable mode has a finite real frequency in rotating plasmas.

3. Generalized matching problem

3.1 Concept of generalized matching problem

It has been shown that the asymptotic matching theory becomes invalid in principle when rotation exits since the location of resonance cannot be determined a priori. To overcome the difficulty, we have proposed a generalized matching problem by invoking inner “regions” with finite width [7,8]. This method was originally suggested for internal modes in static plasmas [9], but has wide application to RWM [7] and tearing mode [10] in rotating plasmas. With the aid of finite width, the inner regions can capture the resonant surfaces even if their locations cannot be prescribed (see Fig. 1).

3.2 Numerical analysis by generalized matching problem

Figure 2 shows the schematic view of the generalized matching problem with $N+2$ inner (hatched) regions and $N+1$ outer (white) regions. The inner regions are allocated to include the singular point of the generalized Newcomb equation, hence the outer regions do not contain any singularity. In the inner layers, the generalized Hain-Lust equation (2) is solved. The matching condition at x_j is that the solutions are continuous and smooth.

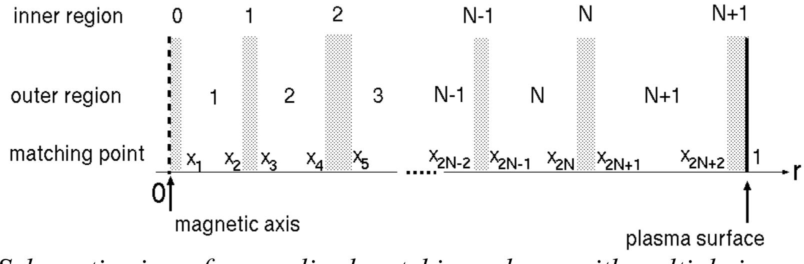

FIG. 2 Schematic view of generalized matching scheme with multiple inner regions.

An example of numerical analysis based on the generalized matching problem is discussed, which analyzes the marginal $m/n = 2/1$ RWM in the normal shear configuration. Figure 3 (a) shows the safety factor profile with a rational surface ($q = 2$) in the vacuum, and Fig. 3 (b) indicates $\Omega \pm F$. The singular points of the generalized Newcomb equation exists at $\Omega - F = 0$. Due to the rotation, the rational surface in the vacuum changes to the singular point near the plasma surface, hence one inner layer is introduced. Figure 4 (a) shows real parts of eigenfunctions by generalized matching scheme and global solution (the term “global” means the solution in the full range), which proves that the generalized inner region can capture the resonance and that the eigenfunction agrees with the global one. We note that if the inner “layer” at the singular point is used, the location of resonance cannot be identified correctly. The growth rate and the rotation frequency (normalized by Alfven time) of the new matching problem are $\gamma = 1.12 \times 10^{-3}$ and $\omega_r = 3.01 \times 10^{-3}$, which agree with global ones $\gamma = 1.12 \times 10^{-3}$ and $\omega_r = 3.44 \times 10^{-3}$. Parameters are chosen as experimentally relevant ones. The present method can reveal new results by extracting the physical effects in the inner region by changing the model. We study the rotation effect on RWM by solving the problem with increasing the rotation in the inner region. Fig. 4 (b) shows the eigenfunction by with artificially increasing the rotation in the inner region by a factor 1.2. To study the effect of resonant location, it has been arranged that the resonances occur inside the region. The rotation effect enters through the location of resonance. It turns out that when the location of resonance moves left, the growth rate increases by a factor of 2.28. Thus, for the first time, we have clarified that the rotation effect (i.e., the location of resonance) inside the inner region significantly affects the RWM stability.

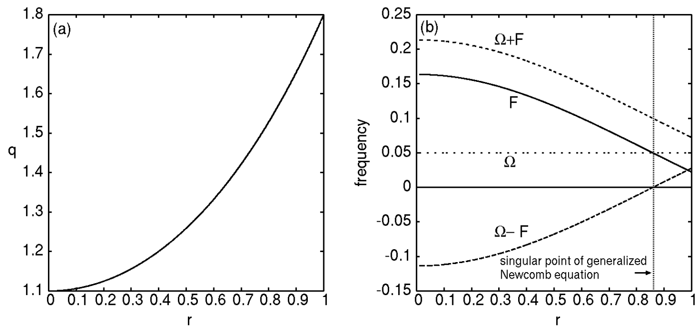

FIG. 3 (a) Safety factor with normal shear configuration. (b) Profiles of plasma rotation frequency Ω, Alfven frequency F, and the Doppler-shifted Alfven frequency $\Omega \pm F$. The rational surface outside the plasma surface becomes the singular point inside the plasma.

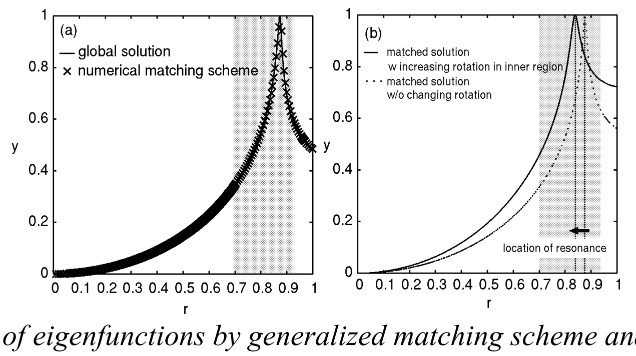

FIG. 4. (a) Real parts of eigenfunctions by generalized matching scheme and the global solutions, which agree well with each other. (b) Real parts of eigenfunction by generalized matching scheme artificially increasing rotation effect in the inner region and by global solutions. The location of resonance due to the plasma rotation is changed.

4. Analysis of the generalized matching problem

4.1. New inner layer equation

To study the generalized matching problem analytically, we derive a reduced equation governing the inner region, which we call the inner layer equation. Let us assume some complex eigenvalue ω_0 is given. Then, the location of singularity can be denoted by $z_0 = z_0(\omega_0)$ in the complex plane. Note that when ω_0 is complex (real), z_0 becomes complex (real). Hence, we make analytic continuation as $f(r,\omega) \to f(z,\omega)$ and $g(r,\omega) \to g(z,\omega)$.

The inner layer equation can be derived by expanding Eq. (2) to the lowest order around the singular point of the generalized Newcomb equation as $f(z,\omega) \sim ax + b\omega$ and $g(z,\omega) \sim g_0$ where $x = z - r_0$, $a = \partial_r f(r_0,0)$, $b = \partial_\omega f(r_0,0)$, and $g_0 = g(r_0,0)$. Then Eq.(2) reduces to

$$\frac{d}{dx}\left[(x+\Lambda)\frac{dy}{dx}\right] - Dy = 0, \quad (3)$$

where $\Lambda = b\omega / a$ $D = g_0 / a$. To see the asymptotic behavior, we derive the standard equation from Eq. (3) by introducing a stretched variable as $X = x / \Lambda$. Then, we get

$$\frac{d}{dX}\left[(1+X)\frac{dy}{dX}\right] - \Lambda Dy = 0, \quad (4)$$

which has analytic basic solutions $y_1(X) = I_0(2[\Lambda D(1+X)]^{1/2})$ and $y_2(X) = K_0(2[\Lambda D(1+X)]^{1/2}) + \gamma y_1$ where $\gamma = 0.57721\cdots$ is the Euler's constant. When considering the asymptotic behavior ($\Lambda \to 0$), the principal parts of basic solutions read $y_1 \sim I_0(2(Dx)^{1/2}) \sim const.$ and $y_2 = K_0(2(Dx)^{1/2}) + \gamma y_1 \sim \log x$, which describes principal parts of Frobenius solutions to generalized Newcomb equation.

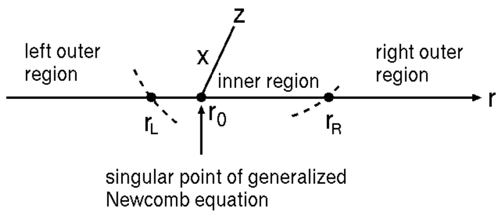

FIG. 5. Matching the inner region and the outer regions governed by the generalized Newcomb equation on the real axis. Continuity and smoothness conditions are imposed at $r = r_L$ and $r = r_R$.

We consider the matching problem on the real axis as is shown in Fig. 5. The inner region is allocated to include the singular point of the generalized Newcomb equation governing the outer regions. In the left outer region, we impose the boundary condition $y|_{r=0} = 0$ and $y|_{r=r_L} = 1$ to obtain the solution $G_{out,L}(r)$. Then the solution in the left outer region reads $y_{out,L}(r) = c_L G_{out,L}(r)$ where c_L is a constant determined by matching condition. Similarly, in the right outer region, we get the solution $G_{out,R}(r)$ under the boundary condition $y|_{r=r_R} = 1$. The boundary condition at $r = 1$ is that the normal magnetic field is continuous when crossing the vacuum. Since the vacuum interacts with the resistive wall governed by the diffusion equation, the solution reads $y_{out,R}(r) = c_R G_{out,R}(r;\omega)$. The continuity condition indicates

$$c_L = c_1 y_1(r_L) + c_2 y_2(r_L),\ \ c_R = c_1 y_1(r_R) + c_2 y_2(r_R)\,. \quad (5)$$

Additionally, we impose the smoothness condition, i.e., the first derivative is also continuous, then, we obtain

$$c_L \Delta_L = c_1 y_1^{'}(r_L) + c_2 y_2^{'}(r_L),\ \ c_R \Delta_R = c_1 y_1^{'}(r_R) + c_2 y_2^{'}(r_R)\,, \quad (6)$$

where the prime indicates derivative with respect to r or z, and $\Delta_{L(R)} = G^{'}_{out,L(R)}\,|_{r=r_{L(R)}}$ corresponds to the matching data. From Eqs. (5) and (6), we obtain the matrix dispersion relation as

$$\det G = \det\begin{pmatrix} \Delta_L y_1(r_L) - y_1^{'}(r_L) & \Delta_L y_2(r_L) - y_2^{'}(r_L) \\ \Delta_R y_1(r_R) - y_1^{'}(r_R) & \Delta_R y_2(r_R) - y_2^{'}(r_R) \end{pmatrix} = 0\,, \quad (7)$$

where $y_j(r_{L,R})$ and $y_j^{'}(r_{L,R})$ can be calculated by analytic solutions. Equation (7) seems rather complex, however, assuming $|\Lambda| << 1$ and taking only lowest order terms, i.e., $y_1 \sim 1$ and $y_2 \sim -\log(D\Lambda + Dx)/2$, equation (7) yields the analytic dispersion relation for γ,

$$\gamma = (a/b)\,\mathrm{Im}[F(\Delta_L, \Delta_R(\omega), x_L, x_R)]\,, \quad (8)$$

where F is an algebraic equation, and $x_{L(R)} = r_{L(R)} - r_0$. Equation (8) can be easily solved for given matching data [$\Delta_R(\omega)$ is calculated by prescribing ω as true eigenvalue]. With the equilibrium and matching data used in Sec. 3.2, we can calculate the growth rate as $\gamma = 1.21 \times 10^{-3}$ from Eq. (8), which is similar with ones by the generalized matching problem $\gamma = 1.12 \times 10^{-3}$. Equation (8) can be also utilized to interpret the numerical results by the generalized matching problem shown in Fig. 4 (b). Equation (8) indicates that the equilibrium profile effects enter through the factor $a/b \sim FF'/\Omega$ for rigid toroidal rotation. The absolute value of the growth rate is affected by combination of the rotation with magnetic field and its shear at the singular point of the generalized Newcomb equation. In fact, equation (8) for artificially increasing rotation in the inner region gives a larger factor a/b by a factor 1.3, which can partially explain the numerical results by the generalized matching [note that the matching data $\Delta_R(\omega)$ also affects the growth rate].

4.2. Application of the new inner layer equation to numerical computation

In the previous subsection, to interpret the numerical results by the generalized matching problem, we have obtained an approximated growth rate with the aid of the lowest order expansion of the analytic solutions to the inner layer equation. To study the singular point in more detail, we utilize the new inner layer equation for numerical computation.

For given ω_0, by introducing notations $\bar{f}(z) = f(z,\omega_0)$ and $\bar{g}(z) = g(z,\omega_0)$, we expand these functions around $z = z_0$ as $\bar{f}(z) = \bar{f}_1(z - z_0) + \bar{f}_2(z - z_0)^2 + \cdots$ and $\bar{g}(z) = \bar{g}_0 + \bar{g}_1(z - z_0) + \cdots$. Note that due to the existence of the rotation, the function $\bar{f}$ starts from the first order. Then the Frobenius solutions describing the behavior of solution around the singular point are obtained as $y_1(z) = 1 + \sum_{n=1}^{\infty} a_n (z - z_0)^n$ and $y_2(z) = y_1(z)\log(z - z_0) + \sum_{n=1}^{\infty} b_n (z - z_0)^n$. As is well known, the logarithmic singular behavior arises. Local equilibrium quantities determine $a_1 = \bar{g}_0 / \bar{f}_1$ and $b_1 = -2a_1$. When the assumed ω_0 is a true eigenvalue, the dispersion relation (7) is satisfied, but in general, the eigenvalue is different from ω_0 so that it should be corrected.

To calculate the correction to the eigenvalue and the dispersion relation, we perturb the eigenvalue and Frobenius solutions as $\omega \to \omega_0 + \lambda$ and $y_j \to y_j + \lambda\psi_j$. Then, in the lowest order, we obtain the equation

$$\frac{d}{dx}\left[x\frac{d\psi_j}{dx}\right] - D^*\psi_j = -\alpha\frac{d^2 y_j}{dz^2} + \beta y_j, \quad (9)$$

where $x = z - z_0$, and $D^* = \bar{g}_0 / \bar{f}_1$ $\alpha = \partial_\omega f(z_0,\omega_0) / \bar{f}_1$ and $\beta = \partial_\omega g(z_0,\omega_0) / \bar{f}_1$ are determined by the local profile of equilibrium. The homogeneous version of Eq. (9) can be derived by the inner layer equation (3) by taking into account of the perturbed location of the singular point. As was shown in the previous subsection, the inner layer equation has analytic solutions as $I_0(2(D^*x)^{1/2})$ and $K_0(2(D^*x)^{1/2})$. Thus, for $y_1 = 1 + a_1 x$ and $y_2 = y_1 \log x + b_1 x$, the analytic solution to Eq. (9) reads

$$\begin{aligned} \psi_1(x) &= -\frac{2\beta}{D^*} - \beta x + c_{11} I_0(2(D^*x)^{1/2}) + c_{12} K_0(2(D^*x)^{1/2}), \\ \psi_2(x) &= -\frac{2\beta}{D^*} + \frac{\alpha}{x} - \beta x \log x + c_{21} I_0(2(D^*x)^{1/2}) + c_{22} K_0(2(D^*x)^{1/2}) \end{aligned}, \quad (10)$$

where the coefficients c_{ij} are determined to satisfy the boundary conditions $\psi_j(r_{L,R}) = 0$. Finally, the solution in the inner layer is corrected as $y_{in}(x) = c_1[y_1(x) + \lambda\psi_1(x)] + c_2[y_2(x) + \lambda\psi_2(x)]$. Note that due to the boundary condition for ψ_j, the continuity condition is not violated. Imposing the smoothness condition yields the correction to the dispersion relation (7) as $\det(G + \lambda\Psi) = 0$ where Ψ is a matrix composed by analytic $\psi_j'(r_{L,R})$. Then finally, we obtain the correction to the eigenvalue as

$$\lambda = -\frac{1}{\mathrm{Tr}(G^{-1}\Psi)}, \quad (11)$$

where Tr indicates the trace operator. We calculate the dispersion relation (7) numerically until the eigenvalue is converged with correcting eigenvalue according to Eq. (11). Equilibrium analyzed in Sec. 3 is employed. We have confirmed that the present method reproduces the same eigenvalue by generalized matching scheme. Figure 6 shows the convergence property of the eigenvalue by new analytic matching scheme for some initial guess, which indicates the good convergence property. Since no mesh is needed inside the inner layer, the computation time is much reduced compared with the calculation in the previous section.

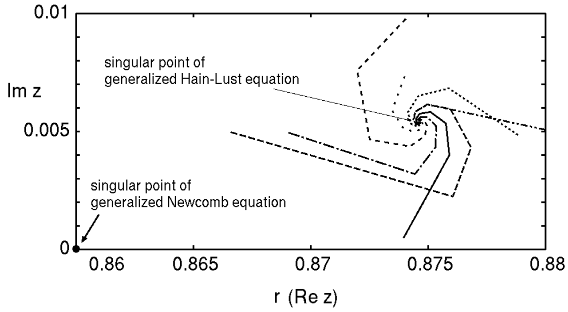

FIG. 6. Convergence of the singular point by analytic matching scheme from several initial guess.

5. Conclusion

In summary, it is pointed out that the classical asymptotic matching theory becomes invalid in principle for rotating plasmas. The essential difficulty is that the location of the resonant surface depends on the RWM rotation frequency, and cannot be determined a priori. This implies that the inner layer, which should contain the resonant surface, cannot be allocated.

To overcome the difficulty, we have developed a generalized matching scheme to use inner "regions" with finite width. Even if we do not know the location of the resonant surface, we can set the region to capture the resonant surface. We have confirmed that the generalized matching scheme is plausible by comparing with the global solution. Also, this scheme can extract the rotation effect in the inner regions. Numerical analysis that the rotation effect is artificially changed in the inner region has revealed that the rotation in the resonant surface significantly stabilizes the RWM by changing the location of resonant surface.

Analysis of the inner regions is conducted by deriving a new inner layer equation. The analytic solution in the inner region is connected to the outer regions described by numerical solution. The analytic dispersion relation has been obtained, which can partly explain the numerical result by the generalized matching problem that the rotation in the inner region has the significant impact on RWM stability. Also, the application of the inner layer equation to the numerical calculation is discussed. Since the inner region has analytic solutions, there exists no mesh. This fact leads to the reduction of computation time and numerical resources.

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