Non-ideal Modifications of 3D Equilibrium and Resistive Wall Mode Stability Models in DIII-D

By
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In collaboration with
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Main results

• Linear ideal MHD describes $n=1$ equilibria as long as
  - Plasma rotation is sufficiently fast
  - Beta is sufficiently low

• Kinetic effects explain resistive wall mode (RWM) stability

→ Opens possibility of passive RWM stabilization even at low plasma rotation, i.e. under reactor conditions
Three Dimensional Tokamak Equilibria and RWM Stability Share the Same Physics Basis

1. **I-coil only**: 3D equilibrium (usually static)
2. **I-coil + stable plasma**: 3D equilibrium (usually static)
3. **Unstable plasma (no I-coil)**: Unstable RWM (growth, rotation ≤ τ_w^{-1})

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Three Dimensional Tokamak Equilibria and RWM Stability Share the Same Physics Basis

Both are a quasi-static global perturbation

- External 3D field
- 3D equilibrium (usually static)
- Unstable RWM (growth, rotation $\leq \tau_w^{-1}$)

$\Delta B$ magnetic field

$n=1$ magnetic field

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Extend Ideal MHD 2D Equilibrium Model to 3D

- Ideal MHD force balance:
  \[ \hat{\mathbf{J}} \times \hat{\mathbf{B}} = \nabla \mathcal{P} \]

- Axisymmetry (2D)
  - Grad-Shafranov equation solved by various codes

- Non-axisymmetric equilibrium (3D)
  - Linearize force balance
    \[ \delta \hat{\mathbf{J}} \times \hat{\mathbf{B}} + \hat{\mathbf{J}} \times \delta \hat{\mathbf{B}} = \nabla \delta \mathcal{P} \]

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Comparison with Magnetic Measurements Shows that Ideal MHD Can Quantitatively Describe 3D Equilibria


- Perturb plasma with an externally applied $n=1$ field ($\delta B/B_T \leq 10^{-3}$)

![Diagram showing perturbations in magnetic fields with I-coils](image)
Comparison with Magnetic Measurements Shows that Ideal MHD Can Quantitatively Describe 3D Equilibria


- Perturb plasma with an externally applied n=1 field ($\delta B / B_T \leq 10^{-3}$)

\[
\begin{align*}
\delta B_{r,up} & (\star) \\
\delta B_{r,mid} & (\star) \\
\delta B_{r,low} & (\star) \\
\delta B_{p,mid} & (\star)
\end{align*}
\]

\[
\begin{array}{c}
\text{Toroidal angle } \phi \text{ (Deg.)} \\
\text{Poloidal angle } \theta \text{ (Deg.)}
\end{array}
\]

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- Perturb plasma with an externally applied \( n=1 \) field (\( \delta B / B_\text{T} \leq 10^{-3} \))

\[ \delta B_{\perp} \text{ at wall} \]

\[ \text{Poloidal angle } \theta \text{ (Deg.)} \]

\[ \text{Toroidal angle } \phi \text{ (Deg.)} \]

- Upper I-coil
- Lower I-coil

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Comparison with Magnetic Measurements Shows that Ideal MHD Can Quantitatively Describe 3D Equilibria


- Perturb plasma with an externally applied $n=1$ field ($\delta B/B_T \leq 10^{-3}$)

- Toroidal arrays of $B_p$ and $B_r$ sensors measure amplitude and toroidal phase of the $n>0$ plasma response
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Ideal MHD 3D Equilibrium Assumes Perfect Shielding of Resonant Fields

- Resonant components $\delta B_{mn}$ with $m = nq$ of the perturbed field are zero
  - A finite resonant component would lead to an island

$\Rightarrow$ Magnetic topology of nested flux surfaces is preserved

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Linear Ideal MHD Can Describe 3D Equilibria as Long as the Plasma Rotation is Sufficiently Large

- Measure response to $n=1$ I-coil field in magnetic braking experiment

- For “large” rotation
  - $\delta B^{\text{plas}}$ is independent of rotation
  - $\delta B^{\text{plas}}$ is consistent with ideal MHD

- After the rotation has collapsed
  - $\delta B^{\text{plas}}$ deviates from ideal MHD
  - A magnetic island forms

- Consistent with shielding as long as $\Omega \tau_{\text{rec}} \gg 1$ [Fitzpatrick, Nucl. Fusion 1993]

- Resonant braking torque indicates a local deviation from ideal MHD

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Linear Ideal MHD Can Describe 3D Equilibria as Long as Beta is Well Below the Ideal MHD No-wall Limit


• Ideal MHD starts to overestimate $\delta B$ at ~80% of the no-wall limit $\beta_{N,nw}$
  - Diverges for $\beta_N = \beta_{N,nw}$
  - Predicts instability for $\beta_N > \beta_{N,nw}$
Observed RWM Stability Above the No-wall Limit has Long Shown the Importance of Non-ideal Effects

- Ideal MHD RWM unstable when $\beta > \beta_{nw}$

$$\gamma \tau_w = -\frac{\delta W_{nw}}{\delta W_{iw}}$$

RWM growth rate normalized with inverse wall time

Perturbed energy assuming an ideal wall

Perturbed energy assuming no wall

- Tokamaks routinely exceed the ideal MHD no-wall stability limit
  - Originally associated with fast toroidal plasma rotation
DIII-D Discharges Exceed the No-wall Limit with a Wide Range of Rotation Profiles

- Vary neutral beam torque $T_{NBI}$ from 1.5 to 8.0 Nm while keeping $\beta_N \approx 2.3 (> \beta_{N,nw})$

$\omega_E$: Toroidal rotation of the $E_r=0$ reference frame
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- Vary neutral beam torque $T_{\text{NBI}}$ from 1.5 to 8.0 Nm while keeping $\beta_N \approx 2.3$ ($>\beta_{N,nw}$)

- In NSTX the RWM becomes unstable at “intermediate” rotation values ➜ S.A. Sabbagh, et al, next talk

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Wave-particle Interaction Can Lead to an Exchange of Energy Between the RWM and Particles

- **Important particle frequencies are**
  - **Transit frequency** of passing particles:  
    \[ \omega_t \sim \frac{V_{th}}{qR} \]  
    [Bondeson, Chu, Phys. Plasmas 1996]
  - **Bounce frequency** of trapped particles:  
    \[ \omega_b \sim \sqrt{\frac{r}{2R}} \frac{V_{th}}{qR} < \omega_t \]  
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  - **Precession drift frequency** of trapped particles:  
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Perturbed Kinetic Energy Can Be Calculated with the MISK Code


- Energy principle has been extended to include kinetic effects [Hu, Betti, Phys. Rev. Lett. 2004]

\[ \gamma \tau_w = - \frac{\delta W_{nw} + \delta W_K}{\delta W_{iw} + \delta W_K} \]

- The perturbed kinetic energy \( \delta W_K \) has the form (for trapped particles)

\[ \delta W_K^T \propto \sum_{l=-\infty}^{+\infty} \left[ \omega_* N + \left( \frac{1}{2} - 3/2 \right) \omega_* T + \omega_E - \omega_{RWM} \right] \]

- Precession drift
- Bounce frequency \( \propto \) Plasma rotation
- Mode rotation
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\]

Small when \( \omega_E = -\langle \omega_D \rangle \) or \( \omega_E = -l\omega_b \)

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Small when \( \omega_E = -\langle \omega_D \rangle \) or \( \omega_E = -1 \omega_b \)

- MISK assumes structure of a marginally stable RWM (perturbative approach)
Kinetic Stability Model Can Explain the Stability Over the Entire Range of Rotation Profiles

- Thermal particles alone are not sufficient to explain RWM stability
Kinetic Stability Model Can Explain the Stability Over the Entire Range of Rotation Profiles

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  - Fast ions constitute ~20% of the kinetic energy
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Use Plasma Response to an External $n=1$ field, i.e. 3D Equilibrium, to Probe the Damping Rate

- **Amplitude** of plasma response largest at intermediate plasma rotation
  \[ \omega_E \tau_A (q = 2) \approx 0.9\% \]

- **Phase shift** of plasma response with respect to external field largest at
  \[ \omega_E \tau_A (q = 2) \approx 0.6\% \]

- **Single mode model** links $\gamma_{RWM}$ and $\omega_{RWM}$ (e.g. from MISK) to amplitude and phase of $\delta B_{\text{plas}}$
Measured Plasma Response Reveals the Characteristics of Kinetic Stabilization

- MISK modeling reproduces the characteristics of the measured dependence of $\delta B_{\text{plas}}$ on plasma rotation
  - Uncertainty in the single mode coupling can lead to systematic shift of amplitude and phase shift

$\Rightarrow$ Increased stability at low rotation is a direct effect of resonance with the precession drift of trapped ions
Recent Results are an Important Step Towards a Quantitative Understanding of 3D Equilibria and RWM Stabilization

- A linear ideal model is adequate to describe 3D equilibria resulting from externally applied 3D fields \((\delta B/B_T \leq 10^{-3})\) as long as
  - Plasma rotation maintains the shielding currents at resonant surfaces
  - Beta is well below the ideal MHD no-wall stability limit

- Kinetic models explain the observed RWM stability above the ideal MHD no-wall limit provided that fast ions are taken into account

- Measured rotation dependence of the \(n=1\) plasma response reveals the interaction of a quasi-static perturbation with the precession and bounce frequencies of trapped thermal ions
  → Direct evidence for the relevance of kinetic effects for RWM stability
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- Measured rotation dependence of the $n=1$ plasma response reveals the interaction of a quasi-static perturbation with the precession and bounce frequencies of trapped thermal ions
  - Direct evidence for the relevance of kinetic effects for RWM stability

- Quantitative validation of stability models is needed before relying on predictions of passive RWM stabilization in ITER
• Apply external $n=3$ field in fast rotating H-modes to suppress ELMs
  - Magnetic response agrees with ideal MHD* [M. Lanctot, APS invited '10]

*Before ELM suppressed phase

• Splitting of strike point on the divertor target reveals open field lines
  → Breaking of the magnetic topology (conserved in ideal MHD)
Experiments are Carried Out in H-mode Plasmas with $T_e \sim T_i$ and a ~20% Fast Ion Content

- Increased density for a lower than typical fast ion content
  - Fast ion population calculated with NUBEAM including an anomalous fast ion diffusion of $D=2m^2/s$ to match neutron measurement
Plasma Response Measurements Reveal Rotation Dependence – Same for $\delta B_r$ and $\delta B_p$ Measurements

- **Plasma:** $2.3 < \beta_N < 2.4$, $4.0 < n_{e,19} < 4.6$, $4.0 < q_{95} < 4.4$

**Radial field**

**Poloidal field**

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Plasma Response to External $n=1$ Field is Determined by the RWM Damping Rate $\gamma_{RWM}$ and Mode Rotation Frequency $\omega_{RWM}$

• Dependence of the plasma response $\delta B_{\text{plas}}$ on the frequency $\omega_{\text{ext}}$ of an externally applied $n=1$ field described by single mode model


– Perturbed field at wall:

\[
\delta B_j = \frac{M_{sc}^*}{i \omega_{\text{ext}} \tau_W - \gamma_0 \tau_W} I_c
\]

Coupling coefficient

Rotation frequency of external field

Complex RWM growth rate $\gamma_0 = \gamma_{RWM} + i \omega_{RWM}$

– Plasma response at wall:

\[
\delta B_{j \text{plas}} = \frac{M_{sc}^* \left( \gamma_0 \tau_W + 1 \right)}{\left( i \omega_{\text{ext}} \tau_W - \gamma_0 \tau_W \right) \left( i \omega_{\text{ext}} \tau_W + 1 \right)} I_c
\]

from MISK

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