Nonlinear Modeling for Helical Configurations in Toroidal Pinch Systems

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Abstract. This contribution focuses on 3D nonlinear MHD modeling and nonlinear gyrokinetic tools for studies of helical configurations in the Reversed Field Pinch regime. We present first 3D nonlinear visco-resistive MHD toroidal simulations with the PIXIE3D code. Stationary helical RFP regimes are found to spontaneously develop in toroidal geometry, similarly to the cylindrical case. In parallel, still in cylindrical approximation, we address the issue of stimulating -by suitable helical magnetic boundary condition- a helical QSH configuration whose toroidal periodicity (in particular a non resonant one) differs from the self-organized one. The magnetic topology is studied by NEMATO and ORBIT codes, whose first successful benchmarking tests are presented. First studies employing hybrid numerical approach shows the linear stabilization of Resistive Wall Modes in the RFP. Concerning microturbulence studies in the RFP, we address the nonlinear problem for Ion Temperature Gradient modes, though being much more stable then in Tokamak configurations, due to more effective Landau damping effect. A large set of 2-species gyrokinetic turbulence simulations with GS2 is presented, in order to compare the linear and nonlinear stability thresholds. An up-shift in a/LT_{\text{Ti,c}} is found, which recalls the the Dimits shift in tokamaks. Recent results on Trapped Electron Modes in linear regime are also briefly reported.

1. Introduction

Among the challenges facing ITER, the development of accurate, predictive modeling tools for fusion plasmas is presently recognized as one of the hardest and most urgent ones. Indeed, current modeling capabilities have achieved good qualitative agreement with experiments in several important physical aspects. Examples include the prediction of Tokamak transport properties via microturbulence studies, and the global description of Reversed Field Pinches via 3D MHD modeling. However, moving toward quantitative agreement will require strong efforts, first in the verification of numerical models, and then in their validation with experiments. We present here the current research status of the nonlinear modeling of the RFX-mod experiment. In RFX-mod, several plasma configurations (ranging from self-organized helical RFPs, to circular Tokamaks, passing through Ultra Low q configurations) can be achieved, thus providing a flexible experimental environment, useful for validating numerical modeling tools. We focus our presentation on two main modeling efforts of helical configurations in the Reversed Field Pinch regime: 3D nonlinear MHD modeling, and nonlinear gyrokinetic studies, respectively in sections 2 and 3.

2. MHD modeling for RFP

The theory of helical states in the RFP has been developed so far essentially on the basis of MHD simulations. In the last years, a strong effort has been put in upgrading 3D nonlinear MHD modeling capabilities to deal with more realistic geometry and boundary conditions, besides of extended-MHD effects (see for example [1] and references therein). In this section, we present most recent achievements in the study of effects related to toroidal geometry, to
application of suitable edge magnetic perturbations (in particular non resonant ones), and first applications of hybrid numerical approach for RWM stability in the RFP.

2.1 Spontaneous Helical regimes in toroidal geometry simulations.

In the quest for predictive capability, toroidal geometry effects in the RFP are being investigated with the nonlinear 3D MHD code PIXIE3D [2,3]. A preliminary low resolution (64 radial × 16 poloidal ×128 toroidal mesh points) toroidal MHD simulation of the RFP was reported in Ref. [1]. The simulation was performed under the same conditions (aspect ratio R/a=4, pinch parameter \( \Theta =1.5 \), radially increasing resistivity profile, on-axis Lundquist number \( S=3 \times 10^4 \) and Prandtl number \( P=10^3 \)) that provide a stationary helical equilibrium with \( n=-10 \) mode in the cylindrical approximation [3]. A helical state with same dominant mode (preferred helicity) was found in the toroidal simulation, too. However, after some thousand Alfvén times, a back transition to the multiple helicity regime was observed (Fig. 1).

Here, we consider a simulation with same physical conditions as before but higher numerical resolution (128 radial × 32 poloidal ×512 toroidal mesh points). This simulation results in a stationary helical state like in the cylindrical case (Fig. 2), thus stressing the importance of using a proper numerical resolution in PIXIE3D simulations, as already discussed in the nonlinear verification benchmark study of Ref. [3]. The toroidal geometry induces the outward axisymmetric shift of the magnetic surfaces (Shafranov-type shift), which (mostly) consists of the \( m=1, n=0 \) harmonic. Thus, by nonlinear 3-wave coupling with the dominant helicity, a hierarchy of additional harmonics (with respect to cylindrical geometry simulations) populates the spectrum. In particular, as shown in Fig. 1-2, \( m=0 \) toroidal harmonics like the \( (m=0, n=-10) \) appear, which would vanish to negligible values in cylindrical geometry. The presence of toroidal harmonics modifies the topology of the magnetic field by introducing secondary island chains at the resonant surfaces of each toroidal
harmonic. In particular, the $m=0$, $n=n_{SH}$ harmonic may give rise to an $m=0$ island chain located at the reversal surface of the toroidal magnetic field, if that surface is present at the plasma edge. An example is shown in Fig. 3 (corresponding to a toroidal helical state with dominant $m=1$, $n=-9$ mode). The Poincaré plot, obtained with the field-line tracing code NEMATO [4,5], shows the presence of an $m=0$ island chain at the reversal surface of the edge toroidal field. The radial position and shape of the $m=0$ flux tubes are modulated along the poloidal direction according to the local phase relation with respect to the helical $m=1$ deformation of the reversal surface: $m=0$ islands are toroidally elongated and pushed outward when they lie on top of a $m=1$ hill (external equator) while they are wedge-shaped and inward shifted when they fall in a $m=1$ deep (internal equator). This $m=1$ dependence of the edge topology in helical states is in agreement with the experimental observation of the $m=1$ dependence of edge measurements in the RFX-mod device [6]. We finally note that the toroidal couplings may also produce chaos in toroidal helical states from the PIXIE3D code if the width of secondary islands is so large that overlap of different toroidally-induced island chains occurs. The possibility of chaotic regions to develop in toroidal helical simulations of the RFP was already discussed in Ref. [7].

2.2. Verification benchmark of Orbit and NEMATO codes for magnetic topology studies

![Equatorial Poincaré plots obtained in the verification benchmark between NEMATO and Orbit on the same snapshot of a Multiple Helicity (MH) SpeCyl simulation. (Left) NEMATO. (Right) Orbit. The conserved $m=0$ structures (islands) at the reversal are correctly diagnosed by both the codes.]

A verification study of Orbit and NEMATO codes has been recently performed. The two codes are routinely used to assess the reconstruction of the magnetic topology in the RFX-mod device and in 3D nonlinear MHD simulations. We report here the first successful tests. A snapshot from multiple helicity, MH, chaotic regime from a SpeCyl simulation is used as input for both the codes, similar to the one used in the study of Ref. [8]. It relates a case with Pinch Parameter $\Theta=1.6$, Lundquist number $S=3\times10^4$ and magnetic Prandtl number $P=500$. Orbit is a Hamiltonian guiding-center (GC) code which describes test-particle motion in an electromagnetic field [9]. In the limit of very low-energy ions, Orbit can be used to trace the magnetic field topology of the reversed-field pinch, and in this sense has been used several times since 2001 both on experimental magnetic spectra [10 and references therein] and on outputs of SpeCyl simulations [8,11]. In Orbit the perturbations (modes) enter in terms of vector potential, $A$, with the gauge $A=\alpha(r) B_0$, which is a common expression for tearing modes and shear Alfvén waves, while NEMATO makes no assumption on the form of the 3D field (equilibrium plus perturbation). Both codes preserve the solenoidal nature of $B$, NEMATO in a direct way, Orbit by adapting the time step when too large steps do not allow for automatically conserving “energy” (=magnetic moment in the low-energy limit). The benchmark between codes has initially been performed on the simplest case, the Poincaré plot of a single mode $m=1$, $n=-11$, where the helical flux and island width can be described by an analytical formula in terms of equilibrium fields and $\alpha$-profile [12]. After this preliminary
test, the benchmark has been extended to the chaotic case, by including 25 \( m=0 \) and 55 \( m=1 \)\( n \)-harmonics in the Poincaré, as shown in Fig.4: the plots have been generated with the same number of field lines (\( N=52 \)), deposited at the same point \( (r=\text{the reversal}, \theta=0,\phi \text{ equally spaced in } [0,2\pi]) \), and integrating field lines for a total parallel length \( L=1.8 \) km. Both codes show the same number of conserved \( m=0 \) structures (islands) at the reversal [8], with differences arising only for minor details, which can be ascribed to the mismatch between the original SpeCyl perturbations \( \delta B_\theta, \delta B_\phi \) and the corresponding ORBIT representations \( \text{rot}(\alpha B)_\theta, \text{rot}(\alpha B)_\phi \). Numerical errors have been ruled out by comparing both codes on the \( \alpha \)-profiles computed by ORBIT. This means that, for the RFP dominant MHD modes, the representation in terms of \( \alpha \) is good enough to express the component perpendicular to the flux surfaces, which is the one that generates the island topology of the tearing mode: in the case of a perturbation \( \delta \mathbf{B} \) parallel to the equilibrium field \( \mathbf{B}_0 \), the ORBIT gauge can introduce significant errors. The comparison has been completed by a more quantitative analysis, by calculating the correlation length [13] of field lines \( (10^3 \text{ lines}, L=1.8 \) km), which resulted to differ within less than 1%.

2.3. Stimulated Helical regimes in cylindrical geometry simulations

Hitherto the study of helical regimes in the RFP has been developed mainly in the cylindrical approximation with ideal boundary conditions, for example with the SpeCyl code, which solves the visco-resistive MHD model using a spectral approach and a semi-implicit numerical method (see for example [1] and therein references). Recent work [14] highlighted a possible role of magnetic field penetration at the plasma boundary in favoring the formation of helical ohmic equilibrium. Consequently, an extensive numerical study has been performed to analyze the effect of edge magnetic perturbations (MP), i.e. a finite value of the radial component of the magnetic field at the edge, on a wide class of MHD simulations [15]. A scan of the physical parameters defining the MHD simulations has been performed: the level of visco-resistive dissipation, the MHD mode on which MP has been applied and their intensity has been changed. The main result is that the RFP plasma can be driven to a selected QSH state using a proper value of MPs that depend on the helicity of the final required state. This is true regardless the initial state of the plasma, i.e. a spontaneous QSH state, if dissipation is high (dissipation is represented by the Hartmann number \( H=(\nu\eta)^{-1/2} \), with \( \nu, \eta \) representing viscosity and resistivity), and a Multiple Helicity (MH) state if dissipation is low. When no MP is used on the latter set of simulations, an experimental-like MH state is recovered (as observed in RFX-mod at currents lower than 1MA) with the typical oscillatory behaviour of the reversal parameter and of the MHD modes. A series of quasi-periodical

\[ \text{Fig.5. SpeCyl simulation: Helical regime driven by edge non-resonant Magnetic Perturbation (1,-6). RFX-mod discharge 30675: similar action is successfully applied by magnetic coils control system in order to stimulate helical regime different from the spontaneous one (the (1,-7) in RFX-mod).} \]
crashes of all these quantities are observed. The application of MP allowed long lasting QSH regimes to be obtained, deeply resembling the experimentally observed ones (at current higher than 1MA): it has to be noted, in particular, the presence of long sequences of QSH states suddenly interrupted by strong bursts of MHD activity, bringing the system back to a MH state. A sample case from this wide set of simulations is presented in Fig. 5. For comparison, in the same figure, is reported the successful result of the experimental stimulation of a helical regime based upon the (1,-6) non-resonant mode, different from the spontaneous one, which is the (1,-7) mode in RFX-mod. Many indications can be obtained from the numerical study: on one hand, changing the intensity of the MP applied to a selected MHD mode causes its energy to grow, while secondary modes do not change their behaviour; on the other hand, a diminution of secondary modes energy is recovered lowering the value of visco-resistive dissipation. The combined effect of dissipation diminution and of the application of MP seems very promising toward the aim of secondary modes’ reduction. This reduction would particularly influence energy transport, which is mainly due to a chaotic behaviour of the magnetic field lines in the RFP.

The field-line tracing code NEMATO [4] is employed for magnetic topology diagnosis. Magnetic topology strongly depends on the secondary modes energy. Large conserved structures are obtained reducing the secondary modes energy to experimental values. We recall that less obvious dependences of magnetic topology on secondary modes energy can be obtained in cases in which QSH states are built upon a non-resonant MHD mode [16]. The analysis of these states is ongoing, both from the theoretical and the experimental points of view.

2.4. Hybrid numerical approach for RFP: About kinetic effects on Resistive Wall Mode.

Kinetic effects on the Resistive Wall Mode (RWM) stability are studied for both toroidal Reversed Field Pinch (RFP) and Tokamak configurations by the upgraded kinetic-MHD hybrid stability code MARS-K [17]. A high-beta RFP regime is discovered, where the current driven RWM can be fully stabilized by the transit resonance of passing ions [18]. The critical plasma rotation frequency required for stabilization is in the ion acoustic frequency range, much below the prediction by the fluid theory [19]. By comparing the mode stability in RFP plasmas with that in tokamak ones, it turns out that the kinetic physics is significantly different in the different magnetic configurations. In fact, in tokamak plasmas it is the precession resonance from trapped particles that can stabilize the (pressure driven) Resistive Wall Mode (at very slow or even vanishing plasma flow). A careful analysis further clarifies that to achieve kinetic stabilization (both in Tokamak and RFP) a small fluid energy component (either in $\delta W_b$ or in $\delta W_\infty$) is crucial [20].

3. Mictroturbulence studies

Hitherto attention towards RFP microturbulence has mainly focussed on linear studies of increasing complexity. At a first stage, electrostatic ion temperature gradient (ITG) and trapped electron mode (TEM) instabilities have been considered. Extensive analyses on these instabilities can be found in [21] for ITG modes, in [22] for ITG modes with the inclusion of impurities, in [23] for the first nonlinear cases with impurities, in [24] for the early assessment on TEM stability in the RFP. Such studies have been carried out with gyrokinetic and gyrofluid codes, as well as with an eigenmode equation solver. We present here nonlinear gyrokinetic and gyrofluid studies of 2-species ITG turbulence, and an additional investigation on linear TEM stability. Effects associated with electromagnetic turbulence, in particular due
to microtearing modes, first assessed for the RFP in [25], appear more relevant to present experimental regimes and are discussed in [26].

3.1. Ion Temperature Gradient turbulence in the RFP

Ion temperature gradient (ITG) modes have been studied in the last years as a possible source of ion heat transport in the RFP. Such instabilities have revealed to be strongly stabilized compared to tokamak plasmas, due to the significant Landau damping acting in low-q configurations, such as the RFP. The threshold \( \alpha/L_{T_i,c} \) (with \( \alpha/L_{T_i} = -a(d\log T_i)/dr \) logarithmic ion temperature gradient) is about a factor \( R/a \) larger than for a tokamak with the same aspect ratio and circular geometry [21]. However, in the presence of strongly outwardly peaked impurity profiles – which is the case for typical RFX-mod plasmas [27] – ITG modes can be more easily destabilized [22,23]. In some cases, for peaked enough electron density profiles, even impurity mode destabilization may occur.

While nonlinear ITG turbulence simulations have been already tackled for 3-species RFP plasmas to compare the predicted and experimental impurity pinch [23], in the following we are going to discuss some aspects of ITG turbulence in a more tractable 2-species context. A set of simulations has been carried out with the gyrokinetic flux-tube code GS2 [28], with the aim to compare linear and nonlinear ITG stability thresholds. This is done for the experimental case discussed in [23] for a few mid-radius positions in the interval \( 0.5 \leq r/a \leq 0.7 \). Linear growth rate spectra and magnetic shear differ from a radial location to another. For example, at \( r/a=0.5 \) the magnetic shear is \( s=-0.64 \); the growth rate is peaked around \( k_y\rho_i \approx 0.3 \); the wavenumber grid for nonlinear simulations is chosen to be \( \Delta k_x\rho_i = 0.1 \), \( \max(k_y\rho_i) = 0.6 \), box size \( L_x \approx 64 \rho_i \), \( L_y/L_x \approx 1 \).

The time series of the ion conductivity \( \chi_i = -Q_i/(n_iT_i/\partial r) \), with \( Q_i \) ion heat flux (in gyro-Bohm units \( \chi_{GB} = \rho_i^3 v_{th,i}/a \)) are shown in Fig. 6a for different values of \( \alpha/L_{T_i} \), exactly at mid radius. In Fig. 6b the ion conductivity is shown to increase with \( \alpha/L_{T_i} \), especially for the innermost radius, resulting in a scaling \( \chi_i \sim \rho_i^3 v_{th,i}/L_{T_i} \) when the gradient is largely above the stability threshold. This increasing trend of \( \chi_i \) closely follows the dependence of the maximum linear growth rate \( \gamma \) on \( \alpha/L_{T_i} \) (right scale of the same frame). By comparing linear and nonlinear threshold, we see that an up-shift of the nonlinear \( \alpha/L_{T_i,c} \) is generally found, reminiscent of the so-called Dimits shift occurring in tokamak plasmas [29]; this behaviour, almost absent for \( r/a=0.5 \), i.e., for the lowest \( |s| \) case, is more evident for the outer radial positions, where the
role of zonal flows could be more significant. This topic has to be carefully investigated in the future. GS2 results have been compared with gyrofluid full-radius simulations obtained with TRB [30]. We recall that Landau damping is included in the code by means of a Hammett-Perkins closure. In Fig. 7 we report simulations of saturated ITG turbulence. The steady state profiles obtained for two different input power depositions provide peak values of saturated conductivity $\chi_i$ which are in agreement with GS2 prediction. In particular, the saturated length scale $a/L_{Ti} \sim 4$ is very close to GS2 threshold value (see Fig. 6).

3.2. Trapped Electron Modes

The features of the Trapped Electron Mode (TEM) instabilities in RFP plasmas have been studied extensively in various parameter regions by solving the gyrokinetic integral eigenmode equation; and compared with the circular tokamak cases. The characteristics of TEM instability in RFP plasmas in some aspects are similar with that of the Tokamak plasmas, however, the excitation of the TEM instability in RFPs requires much steeper density gradient than that in Tokamaks. This could be due to the stronger ion Landau damping in RFP, which has been found to play an important role in the ITG mode physics [21]. Fig. 8 shows the TEM growth rates and frequencies as function of the normalized density gradient scale length $\varepsilon_n$ ($\varepsilon_n=L_n/R$, $R$ is the major radius), corresponding to various wave numbers, parameter $\eta_e$ ($\eta_e=L_n/L_{Te}$, $L_{Te}$ is the electron temperature gradient scale length $\eta_e=0$) and effective collisionalities $\nu_{e eff}$. The figure indicates that with respect to the tokamak case, the TEM instability in RFP appears only in a region having a larger density gradient (smaller value of $\varepsilon_n$). A steep electron temperature gradient can enhance the instability. However, it cannot largely influence the requirement on the density gradient for the instability.

4. Summary, final remarks and perspectives

We presented the current research status in nonlinear modeling of the RFX-mod experiment, with focus on 3D nonlinear MHD and nonlinear gyrokinetic tools, and with special attention to verification and validation requirements. Concerning 3D nonlinear MHD, we presented results from the SpeCyl and PIXIE3D benchmarked codes. The PIXIE3D code, which is a highly scalable parallel fully implicit code, showed first toroidal RFP simulations confirming the development of stationary toroidal
helical regimes, as obtained in cylindrical geometry. With the SpeCyl code, which is a cylindrical spectral semi-implicit one, it is shown that edge Magnetic Perturbation (MP) can drive helical regimes different from the spontaneous ones, as successfully obtained in RFX-mod thanks to Feedback Control. Magnetic topology studies are performed with the ORBIT and NEMATO codes, whose successful benchmark tests are reported. Concerning microturbulence in the electrostatic branches, it is confirmed that the RFP requires much steeper temperature, Ti, or density, n, gradients with respect to Tokamak to destabilize ITG or TEM modes (current operation is expected to be free of such turbulence). Gyrokinetic and gyrofluid (GS2 and TRB) codes agree on saturated ITG turbulence levels for potentially unstable cases (upshift of critical gradient observed by GS2).

As a final remark we note that the RFX-mod experiment, where several pinch configurations can be compared ranging from the self-organized helical RFP, to circular Tokamak passing through Ultra Low q ones, provides a flexible experiment in view of a future validation stage for several modeling tools.

References

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