Drift-kinetic Simulation Studies on Neoclassical Toroidal Viscosity in Tokamaks with Small Magnetic Perturbations

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Abstract:
Effect of non-axisymmetric magnetic perturbations and $E \times B$ rotation on neoclassical toroidal viscosity (NTV) is investigated by using a drift-kinetic $\delta f$ Monte-Carlo simulation code, FORTEC-3D, and the simulation is benchmarked with an analytic formula which uses the bounce-average approximation. Although the $\delta f$ code agrees with the analytic formula if the $E \times B$ velocity is slow or the radial position is far from the resonant rational flux surface, a clear difference appears when $E \times B$ velocity becomes large. A double-peak profile of NTV amplitude appears around the resonant flux surface only in the $\delta f$ simulation. The double-peak is supposed to be created as the result of resonance of $E \times B$ drift and the bounce motion of trapped particles. The benchmark result suggests that the precise drift-kinetic simulation which follows the exact guiding-center motion around the resonant flux surface is essential to evaluate the rotation damping rate by neoclassical viscosity when the $E \times B$ rotation is not slow.

1 Introduction

Control and prediction of the toroidal rotation is one of the important subjects in tokamaks such as ITER and future DEMO reactors in order to improve the stability of the confined plasmas. Recent studies have shown that the non-axisymmetry in tokamak magnetic field as small as $B/B_0 \sim 10^{-4}$ can induce significant damping in toroidal rotation. Such a small level of non-axisymmetry is possible due to imperfect magnets, MHD activities, or externally applied magnetic field for the purpose of mitigating the Edge Localized Modes (ELMs)\cite{1}, for example. In either case, it is critical to predict the effects of non-axisymmetry in tokamaks on the toroidal rotation.

When the axisymmetry of tokamak magnetic field is broken, the toroidal canonical momentum $P_\psi = \psi - I v_\parallel / \Omega$ is no longer a constant of motion, and guiding centers of trapped particles drift across the $\psi = \text{const.}$ surfaces. The particle flux associated with the broken symmetry is proportional to the neoclassical toroidal viscosity (NTV),
\[ \Gamma_{TV} = q \left( e \cdot \nabla \cdot P \right) / e[2], \]
where \( \langle \cdots \rangle \) denotes the flux-surface average, \( P \) is the pressure tensor, and \( q \) is the safety factor, respectively. The NTV torque has been observed and studied in many tokamaks experiments \([3, 4, 5]\), since the change of rotations is apparent when a small non-axisymmetric perturbation is applied. However, theoretical predictions are nontrivial due to the complicated particle drift motions under the effect of perturbed magnetic field, electric field, and Coulomb collisions, especially if the given perturbation field has resonant rational flux surfaces. A number of theories have been developed in each limited regime concerning the plasma collisionality \( \nu \) and the \( E \times B \) rotation frequency \( \omega_E \) to simplify or ignore other processes\([6]\). The overlapping between the asymptotic limit regimes are significant in practice, and therefore some combined NTV formula have also been developed\([7, 8]\).

Another approach to evaluate neoclassical viscosity more precisely is to use direct numerical solver for drift-kinetic equation. A \( \delta f \) Monte Carlo code FORTEC-3D \([9]\) which solves drift-kinetic equation in general 3-dimensional magnetic configuration is utilized to evaluate NTV in perturbed tokamaks. Though the simulation is time consuming, the strong points of the \( \delta f \) simulation are: (i) It follows the exact guiding center motions in perturbed tokamak field including the finite radial excursion (finite-orbit-width effect) rather than the bounce-average approximated motion, and (ii) the linearized Fokker-Planck collision operator is used which ensures the conservation of momentum and energy while analytic formulae usually adopt simple pitch-angle scattering or Krook operator.

In the previous papers\([10, 11]\), we have benchmarked FORTEC-3D code with Park’s combined analytic formula and the asymptotic limit theories for the \( 1/\nu \) and superbanana-plateau regimes in the limit \( \omega_E \to 0 \). It has been demonstrated that FORTEC-3D simulation result agrees well with Park’s formula in a wide range of collisionality regime \( 10^{-4} < \nu_s < 10 \), where \( \nu_s = 1 \) is the boundary of the plateau and \( 1/\nu \) regimes. It has also been found that the asymptotic limit formula tends to overestimate the neoclassical viscosity when \( \nu_s \ll 1 \).

Following the previous studies, in this paper we report the benchmark test of FORTEC-3D and Park’s formula in the finite \( E \times B \) rotation cases, which is more practical. To summarize, a new phenomenon is found in FORTEC-3D simulation when \( \omega_E \) is comparable to the bounce frequency of trapped particles \( \omega_b \). When \( \omega_E \sim 0 \), single peak profile of NTV on the resonant rational flux surface is generally seen, and it tends to shrink as \( \omega_E \) increases. When \( \omega_E \sim \omega_b \), a new double peaks of NTV emerges around the resonant surface, and it is found that the distance between the two peaks expands as \( |\omega_E| \) increases. It is also observed that the double-peak structure decays when \( \nu_s > 1 \), and the peak amplitude of the NTV is unrelated to the mean parallel flow velocity. Therefore, it is inferred that the double peaks of NTV is created by resonance of the bounce motion of trapped particles around the resonant rational flux surface with \( E \times B \) rotation. We also compare the total NTV torque, which is integrated in the whole plasma volume, between the two calculation methods. It is observed that the amplitude of the NTV torque increases linearly as the \( E \times B \) speed increases. The total toroidal torque is mainly determined by the contribution from non-resonant flux surface regions, and the dependence is very similar between the two methods. The benchmark results suggest that precise numerical simulation is required to make a quantitative, rather than a qualitative, evaluation
of NTV torque to analyze and explain the experimental observations, especially in the region around the resonant flux surface and when the \( E \times B \) rotation velocity is not small.

2 The Benchmark Model

The numerical simulation method used in FORTEC-3D code to evaluate the neoclassical toroidal viscosity \( \langle \mathbf{e}_\zeta \cdot \nabla \cdot \mathbf{P} \rangle \) is explained in detail in the previous papers[10, 11]. The pressure tensor \( \mathbf{P} \) and the NTV are directly evaluated from information of the perturbed plasma distribution function \( \delta f = f - f_M \) solved numerically, where \( f_M \) is local Maxwellian. Only the distribution function of ion species (hydrogen) is treated and the ion-electron collision is neglected. Magnetic field is given by \( B(r, \theta, \zeta)/B_0 = [1 - \epsilon \cos \theta - \delta_{m,n}(r) \cos(m\theta - n\zeta)] \), where \( \epsilon = r/R_0 \) and \( \delta_{m,n}(r) \) is the amplitude of the perturbation field. To make it easy to compare with analytic formula, circular cross-section, large aspect ratio plasma configuration with small banana width is chosen; \( R_0 = 10m, \quad a = 2.5m, \quad T_i = 0.4keV, \quad \text{and} \quad B_0 = 10T \). As in the previous benchmarks, a single-mode perturbation field model is adopted here, \( \delta_{r,\z}(r) = 0.01(r/a)^2 \). The safety-factor profile is \( q(r) = 1.2 + 9.8(r/a)^3 \), which has the \( q = \frac{m}{n} = 7/3 \) resonant surface at \( r/a = 0.486 \).

To survey the NTV dependence on \( E \times B \) rotation speed, a radial electric field profile needs to be specified. In this paper, we utilize the radial force balance relation written in a magnetic coordinate system as[12]

\[
n_i u_i^\zeta = n_i \mathbf{u}_i \cdot \nabla \zeta = -\frac{1}{e} \left( \frac{dp_i}{d\chi} + en_i \frac{d\Phi}{d\chi} \right) + qn_i u_i^\theta,
\]

where \( \mathbf{u}_i \) is the ion mean flow, \( \chi \) is the poloidal flux, and \( d\Phi/d\chi = \omega_E \) is the radial electric field, respectively. From tokamak neoclassical theory, the poloidal flow is given as \( u_i^\theta = (k/eq)\partial T_i/\partial \chi \), where the factor \( k \) depends on plasma collisionality and the radial position. To simplify the problem, we use \( T_i = \text{const.} \) profile so that the \( nu_i^\theta \) term can be omitted. If the magnetic perturbation is small, it is expected that a steady state flow and radial electric field satisfy Eq. (1). Since \( u_i^\zeta \) is initially zero and remains small in the simulations, it is natural to use the force balance electric field,

\[
E_{r0} \equiv -\frac{d\chi}{dr} \frac{d\Phi}{d\chi} = \frac{T_i}{e} \frac{d}{dr} \ln n_i,
\]

as a reference profile, and the parameter survey is carried out by setting the input \( E_r \) profile by magnifying the reference profile as \( E_r(r) = f_c \times E_{r0}(r) \).

The \( n_i, q, \) and \( E_{r0} \) profiles used in the benchmark is shown in FIG. 1. The plasma density is very low, \( n_i(r = 0.5a) = 1.9 \times 10^{17}m^{-3} \) so that the normalized collisionality at the resonant surface becomes \( \nu_s \simeq 0.06 \), at which we have observed that \( \langle \mathbf{e}_\zeta \cdot \nabla \cdot \mathbf{P} \rangle \) has local maximum in the \( E_r = 0 \) case[11].

FIG. 1: \( n_i, q, \) and \( E_{r0} \) profiles.
Using $T_i = \text{const.}$ profile also serves to simplify the evaluation of the combined analytic formula and comparison with simulation. The formula gives the neoclassical toroidal viscosity in the following form:

$$\Gamma_{TV} \sim \sum_{lmn} u_i^\xi \delta_{mn}^2 \int d\kappa^2 F_{lmn}(\kappa) \int dx R_i(x),$$  \hspace{1cm} (3)

$$R_i(x) \sim \frac{\omega_B \nu_K}{[\omega_b - n(\omega_E + \omega_B)]^2 + \nu_K^2},$$  \hspace{1cm} (4)

$$u_i^\xi = \frac{T_i}{e} \left[ k + \frac{\int dx (x - 5/2) R_i(x)}{\int dx R_i(x)} \right] \frac{dT_i}{d\chi},$$  \hspace{1cm} (5)

where $\kappa^2 = \frac{m_i v^2 - 2\mu B_0 (1 - \epsilon)}{(4\mu B_0 \epsilon)}$ and $x = (v/v_{th})^2$ are the velocity variables, $\omega_B$ is the precession drift frequency, and $\nu_K$ is the collision frequency of Krook operator, respectively. For $\nu_\ast \ll 1$ regime, $k \simeq 1$ approximation is used (theoretically, $k \rightarrow 1.17$ when $\nu_\ast \rightarrow 0$). By setting $dT_i/d\chi = 0$, we do not need to care about the error associated with the approximation in $k$ when comparing the result with the $\delta f$ simulation.

### 3 Simulation Results

In the FORTEC-3D simulations, $3.2 \times 10^7$ markers have been used, and NTV is evaluated on 240 radial mesh points to see its fine structure. Radial electric field amplitude has been varied between $-20$ to $+20 \times E_{r0}$ ($\pm 10$ for the analytic formula). Note that the poloidal Mach number of $E \times B$ flow, $M_p = E_r/(B_p v_{th})$, is at most 0.02, so the nonlinear dependence of neoclassical viscosity on $E \times B$ speed when $M_p \sim 1$, discussed in the biasing experiments in LHD[13], does not matter in this benchmark. The profiles of NTV calculated from FORTEC-3D and Park’s combined analytic formula are shown in FIG. 2. One can see that the single peak at the $q = 7/3$ resonant surface at $r/a = 0.49$ shrinks as $|E_r/E_{r0}|$ increases, but then a double-peak profile emerges only in FORTEC-3D results. The new peaks appearing in the large-$|E_r/E_{r0}|$ are even larger in magnitude than the single peak when $E_r = 0$. On the other hand, the NTV profiles at off-resonant region, $r > 0.6a$, resembles between the two methods even in the finite-$E_r$ cases, of which difference in NTV is within factor $O(1)$ in each case. Therefore, it is anticipated that the different feature between the two methods comes from a resonant condition of drift motion in the existence of $E \times B$ rotation around the $q = 7/3$ resonant surface. It is also confirmed that the calculated NTV becomes almost zero as it is expected from neoclassical theory, if the force balance for $u_i^\xi = 0$ is satisfied, i.e., when $E_r = E_{r0}$.

The total NTV torque, which is integrated in the whole plasma volume, is also compared between two methods as shown in FIG. 3. The total NTV torque amplitude has a local peak at $E_r \simeq 0$ and almost linearly depends on $E_r$ when $|E_r/E_{r0}|$ is finite. Analytic formula agrees well with, but tends to predict smaller torque amplitude than the $\delta f$ simulation. Since the total NTV is determined mainly from the off-resonant region, the trend is similar between the two methods. However, it is expected that the double-peak of NTV, which has not predicted from analytic theory, will strongly affect the local plasma rotation profile around the resonant surface.
To investigate what is the origin of the double peak NTV profile, possible resonant conditions when $E \times B$ rotation is large are considered. FIG. 4 shows the radial profiles of the transit frequency $\omega_{tr}$, $(n$-times) $E \times B$ drift frequency $\omega_E$, precession frequency $\omega_B$, and bounce frequency $\omega_b$ when $E_r = 10 \times E_{r0}$. Here, approximations for $\omega_B$ and $\omega_b$ in [7] are used. From the form of the denominator in Eq. (4) of the analytic formula, large peak of NTV is expected when the resonant condition $l \omega_b - n(\omega_E + \omega_B) \approx 0$ is satisfied by $v \sim v_{th}$ particles. For example, the local maximum of NTV at $E_r \approx 0$ which is seen in FIG. 3 occurs because of the $l = 0$, $\omega_E + \omega_B = 0$ resonant. If $|\omega_E|$ is as large as the value when the double peaks are observed, i.e., $|E_r/E_{r0}| > 5$, one can see from FIG. 4 that the $l = 1$ resonant condition $\omega_b - n\omega_E \approx 0$ can be satisfied for $v \sim v_{th}$ particles at $r/a \approx 0.5$.

FIG. 2: Profiles of NTV, $\langle e_{\zeta} \cdot \nabla \cdot P \rangle$, calculated from FORTEC-3D (lines with “F3D” label) and Park’s analytic formula (lines with “Ana.” label) in several magnitudes of $E_r$, $-20 \leq E_r/E_{r0} \leq +20$. 
Here, $\omega_B$ is too small compared with $\omega_E$ when the double peak appears and can be neglected. The resonance of passing particles with large $E \times B$, which does not considered in the bounce-average theory, is not possible, since $\omega_{tr} \gg \omega_E$. Therefore the $\omega_b - n\omega_E \simeq 0$ resonant is a possible reason for the emergence of the double peaks.

We also checked the dependence of the double peaks on the plasma collisionality and magnitudes of $B_0$ and $E_r$. The results are plotted in FIG. 5. The fact that the double peak structure decays when $\nu_s > 1$ suggests that the physics is related to trapped particle motions. However, the distance between the peaks is not related to the banana width but is determined by the $E \times B$ speed, since it does not change when we vary $B_0$ and $E_r$ with the ratio $E_r/B_0$ kept constant. Actually, the typical banana width at $r = 0.5a$ is estimated as $\Delta_b/a < 10^{-3}$, which is negligible small compared with the peak distance.

**FIG. 3:** Total toroidal viscosity torque obtained from FORTEC-3D and the combined formula.

**FIG. 4:** Profiles of $\omega_{tr}$, $n\omega_E$, $\omega_B$, and $\omega_b$ when $E_r/E_{r0} = +10$ and $n = 3$. $\omega_{tr}$ and $\omega_B$ are for $v = v_{th}$.

**FIG. 5:** Dependence of the double-peak of NTV on the plasma collisionality $\nu_s$ when $E_r = -10 \times E_{r0}$ (left fig.), and on the magnitude of $B_0$ and $E_r$ (right fig.), respectively.
Finally, the parallel flow evolution during the FORTEC-3D simulation is investigated. Since the $E_r$ profile is fixed in time, the parallel flow $V_\parallel$, which is proportional to the toroidal rotation $u_{ti}^t$, evolves so that it satisfies the force balance relation, Eq. (1). As an example, the parallel flow profile in the $E_r = -10E_{r0}$ case is shown in FIG. 6. One can see that the $V_\parallel$ has a sheared profile around the resonant surface, and it gradually approaches the force balance solution.

We observed that the NTV is almost unaffected by the change in $V_\parallel$ after $t > 1\tau_i$, where $\tau_i$ is the ion-ion collision time. The local flow shear is created as a result of the double-peak structure of the neoclassical toroidal viscosity.

4 Conclusions

From the evidences we have found, it is probably because the resonance of the bounce motion of trapped particles with $E \times B$ rotation that makes the double peaks of NTV near the $m/n = 7/3$ resonant flux surface. However, we should note that the calculation results from the analytic formula in FIG. 2 and 3 include both $l = 0$ and $l \geq 1$ contributions, while it does not predict strong resonant when $l \omega_b \sim n\omega_E$ as FORTEC-3D simulation does. Moreover, considering only the condition $l \omega_b \sim n\omega_E$ is not sufficient to explain why the $l = 1$ resonant splits into two peaks and why the distance and the peak amplitude of NTV depends on $\omega_E$.

We should take care of the facts as follows: First, the analytic formula adopts very simplified expressions for $\omega_b$ and $\omega_B$ in practice. Second, the bounce-average is defined from the unperturbed, small-$\omega_E$ limit guiding-center equations of motion. Third, the analytic formula is based on local theory in the sense that it does not take account of magnetic shear and $E \times B$ shear. Therefore, it is possible that the double peaks of NTV is related to a type of drift orbit around the resonant flux surface which is not fully described by approximated bounce-average equations of motion. Since the double peaks of NTV results in creating local parallel flow shear, its effects on the rotation damping, shielding of the perturbation field, and modification of the bootstrap current profile at the resonant rational flux surface are interesting problems. We will further investigate the origin of the double peak using the $\delta f$ code to prove the validity of the code and to reveal what is missing physics in evaluating NTV in large-$\omega_E$ cases.

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