On the kink formation and stability of impurity-generated snakes in Alcator C-Mod

1 Introduction

One of the most interesting and cited examples of 3D magnetohydrodynamic (MHD) activity in the core of a magnetically confined fusion plasma is the long-lived helical mode found at the Joint European Torus [1]-[2] more than 25 years ago. The typical “snake”-like helical patterns observed are characterized by a small region of localized and enhanced plasma density that rotates within the field of view of various diagnostics and is radially concentrated on, or inside the $q = 1$ surface. These long-lived modes are closely associated with sawtooth oscillations and understanding them is potentially important for a burning plasma like ITER where the $q = 1$ radius can be as large as half the minor radius. The standard model [3] to describe the snake formation is based on the physics of tearing modes and suggests that the localized cooling of the $q = 1$ surface is responsible for an increase of the local plasma resistivity, which in turn causes a drop in the current density that leads to the formation of a magnetic island with poloidal and toroidal harmonics $m = 1$, $n = 1$. This island is assumed to trap the particles from the pellet and form the snake.
FIG. 1: a) Vertical and horizontal sightlines of SXR arrays on Alcator C-Mod. Figures b) and c) depict the brightness profiles of two impurity snakes. The dotted lines indicate the time of their formation while the arrows in c) indicate the first two sawtooth crashes.

FIG. 2: Radiated power density profiles before and during the formation (stages 1 and 2) of the two snakes b) and c) shown in Fig. 1.

2 Impurity snakes

Snakes have been a common feature in every major tokamak fusion experiment, as well as in spherical tori and reversed field pinches. A second type, produced by an accumulation of impurity ions rather than the deuterium ions from injected fueling pellets, is also observed. Both types of snakes possess surprisingly good MHD stability and particle confinement, since they can survive tens to hundreds of sawtooth cycles. Helical snakes like the two depicted in Fig. 1 have been observed in Alcator C-Mod [4] ohmic discharges during the plasma current ramp-up phase or early in the plasma current flattop, where the high edge temperature increases the high-Z impurity erosion from the inner wall (e.g. molybdenum limiter), and the absence
FIG. 3: Data from McPherson spectrometer shows the result of impurity accumulation before the formation of the snakes b) and c) shown in Fig. 1.

of sawtooth crashes allows on-axis impurity peaking. An example of the formation of such - apparently - spontaneous phenomena [2] is depicted in Fig. 1b) using the time-history of the soft x-ray (SXR) brightness profiles. The second snake shown in Fig. 1c) had nearly identical radiation profiles before the snake formation until an inadvertent high-Z impurity injection at 0.324 s - 10 ms before the snake formation - nearly tripled its radiated power density ($P_{\text{rad}}$) at mid-radius [compare Figs. 2-a) and -b)]. A common feature between snakes is thus the presence of a peak radiated power density and SXR emissivity due to the strong core accumulation of impurities.

A first glimpse of the identification of the impurity species responsible for snake formation was done using the McPherson spectrometer data shown in Figure 3; these data suggests that molybdenum is the main high-Z impurity before snake formation with no contributions from the stainless steel constituents. Further identification and localization of the main impurity species was possible using the High Resolution X-ray crystal imaging spectrometer with Spatial resolution (HiReX-Sr) [5]. Although this diagnostic was designed primarily for extracting temporally and spatially-resolved spectra from argon, it can also monitor intrinsic impurities such as molybdenum; the latter is the main intrinsic high-Z impurity in C-Mod, with an average ion charge $\langle Z_{\text{Mo}} \rangle = 32$ in the plasma core. The intensity of the line-integrated Mo$^{32+}$ brightness profiles - obtained with a reduced integration time of 10 ms - are shown in Figure 4-a), and indicate a slow peaking of molybdenum density before the snake formation. The normalized intensity of the central Mo$^{32+}$ brightness is over-plotted in Fig. 4-b), and shows a remarkable agreement with the normalized central soft x-ray (SXR) brightness signatures from the tomographic array. It is thus safe to assume that the net core emission in the time-interval $t \in [0.2, 0.5]$, is strictly due to the presence of molybdenum charge states, and that the snake-like pattern in the SXR data is formed by a small region of localized and enhanced molybdenum density on, or inside the $q = 1$ surface [5, 6].
3 C-Mod snake formation

The C-Mod snake formation can be described by a multi-step process corresponding to the numbers in Fig. 1. The first condition (stage one), before the snake formation, is characterized by a plasma with centrally peaked SXR and radiated power density profiles [see grey profiles for \( t \approx 0.285 \, \text{s} \) and \( t \approx 0.334 \, \text{s} \) in Figures 2(a) and -b), respectively]. For snake (b) shown in Fig. 4(b), the core electron density before the impurity accumulation (\( t_0 \approx 0.140 \, \text{s} \), at the end of the current ramp-up phase) is flat and \( n_e(0) \approx 1.2 \times 10^{20} \, \text{m}^{-3} \), while just before the snake formation (\( t_1 \approx 0.285 \, \text{s} \)) the density peaks to \( n_e(0) \approx 2.2 \times 10^{20} \, \text{m}^{-3} \). The net core \( P_{\text{rad}} \approx 2.1 \, \text{MW/m}^3 \) then corresponds to a core Mo density \( n_{\text{Mo}}(0) \approx 1.4 \times 10^{17} \, \text{m}^{-3} \) and a peaked impurity concentration \( n_{\text{Mo}}/n_e \approx 6.3 \times 10^{-4} \); the latter calculation uses a Mo cooling factor \( L_{\text{Mo}} \approx 7 \times 10^{-32} \, \text{W-m}^3 \). Such an impurity density can increase the core \( Z_{\text{eff}} \), as well as the collision frequency \( \nu_{ei} \) and resistivity \( \eta \), by more than 50% over the molybdenum-free state. Central values of radiated power density of up to \( \approx 3.6 \, \text{MW/m}^3 \) have been observed in these discharges and thus even larger increments of \( Z_{\text{eff}}, \nu_{ei} \) and \( \eta \) could be expected when compared to Mo-free plasmas.

During the second stage the snake forms as a growing and rotating kink-like \((m, n) = (1, 1)\) helical impurity density concentration with a nearly circular cross section, as shown in the \( P_{\text{rad}} \) profiles in Fig. 2 and the SXR reconstructions in Fig. 5. This kinked snake precedes any sawtooth onset. The peak SXR and \( P_{\text{rad}} \) emissivity associated to the kinked-core increases by an additional \( \approx 5 - 10\% \) (see Fig. 2), since most of the impurity emission is now confined to a smaller volume. From the net radiated power density and assumed quasi-neutrality it is possible to infer the impurity density at the center of the snake \( \delta n_{\text{Mo}} \approx \delta P_{\text{rad}}/n_e L_{\text{Mo}} \) and its associated electron density perturbation \( \delta n_e \approx Z \delta n_{\text{Mo}} \); a net \( 2.7 \, \text{MW/m}^3 \) with a flat core \( n_e \approx 1.7 \times 10^{20} \, \text{m}^{-3} \) will correspond to an increased snake density of \( \delta n_{\text{Mo}} \approx 2.2 \times 10^{17} \, \text{m}^{-3} \) with \( \delta n_e \approx 7 \times 10^{18} \, \text{m}^{-3} \). The perturbations in density and charge can be as high as \( \delta n_e/n_e \approx 4\% \) and \( \delta Z_{\text{eff}}/Z_{\text{eff},0} \approx 60 - 70\% \) (assuming \( Z_{\text{eff},0} \approx 2 \)). Frequency spectrograms of the SXR brightness, \( P_{\text{rad}} \) (\( \propto n_{\text{Mo}} \)) and electron temperature during the formation of the kinked snake are depicted in Figs. 6(a), -b) and c). The \( n = 1 \) nature of the density perturbation has been measured for the first time using using two toroidally displaced AXUV arrays as shown in Fig. 6(d). The phase between the two signals is nearly the same as the geometric angle between the arrays. Typical \( T_{i0} \) oscillations measured with an electron cyclotron emission

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**FIG. 4:** a) Time history profiles of the Mo\(^{32+}\) line before the snake formation. b) Good agreement is found between the Mo\(^{32+}\) data and the central sightlines from the tomographic system.
FIG. 5: Time history of SXR midplane emissivity a) before and b) during the snake formation; each successive frame is 100 μs apart. 2D tomographic reconstructions of b) are shown in c).

(ECE) radiometer are shown in Fig. 6-e) and are strongest during the kink phase of the snake formation.

4 Challenging the Wesson model with 3D nonlinear effects.

The snake formation departs strongly from the nonlinear island model based on a modified Rutherford equation (MRE), proposed by Wesson [3] and used by others [10, 11]. First, no island is observed during the early stages one and two of the snake. The MRE model postulates that injecting an excess of ions will induce a “nearly-immediate” cooling of the \( q = 1 \) surface which will increase the local plasma resistivity, leading to the formation of a magnetic island. As demonstrated in Fig. 1, the accumulation of an excess of ions and the subsequent cooling of the flux surfaces does not constitute a necessary condition for the impurity-snake formation. Snakes can also occur in plasmas with a slow impurity accumulation in the core (from constant sources at the edge) without any evidence core cooling. The MRE model, based on \( m \geq 2 \) tearing mode reconnection, considers only conditions at the \( q = m/n \) rational surface that affect the local current density layer, but the \( m = 1 \) snake comprises the entire volume inside \( q < 1 \). The model therefore greatly over-emphasizes the local effects of cooling of the \( q = 1 \) layer by line and continuum radiation (\( P_{rad} \)) as well as the changes of ion charge (\( Z_{eff} \)) [10, 11], compared to the \( m = 2 \) radiation-induced NTM magnetic islands [12]. The saturation of the mode, needed to create a sustained snake, would also require that the equilibrium jump in the magnetic flux \( \psi \), measured by \( \Delta \), be large and negative (stabilizing) to counterbalance the strong destabilizing terms. It must remain so during the snake lifetime, which seems to rule out the possibility of
FIG. 6: Frequency spectrograms of a) SXR, b) \( \tilde{P}_{\text{rad}} \) and b) \( \tilde{T}_{e,0} \) core fluctuations during the formation of the snake shown in Figure 4a). d) The AXUV signals during the second kink stage are from two arrays displaced 108° in toroidal angle. Fig. d) shows values of \( T_e \) in three midplane positions.

sawtooth oscillations.

The likely explanation for the persistence of the snake is that it embodies a quasi-steady-state magnetic structure. Its 1/1 helical configuration and close association with the \( q = 1 \) surface and ongoing sawtooth crashes suggest that it is related to the 1/1 internal kink mode \([13]\) with magnetic reconnection. Recent numerical simulations of static ideal MHD equilibria with a 3D helical core based on pressure balance reproduce some of the features of the original pellet snakes observed at JET \([14]\). The helical equilibrium states also resemble the kinked plasma during the early sawtooth-free stage of the C-Mod snake formation. However, the C-Mod observations of the snake’s gradual formation from zero amplitude, its smooth transition from a broadly kinked core to a crescent shape with no evidence of a phase shift, and the details of the periodic sawtooth crashes rule out an explanation based on steady state force balance. Models of the snake based solely on pressure have a fundamental difficulty in explaining its stable co-existence with periodic sawtooth crashes \([6, 7]\).

Nonlinear MHD simulations of the early snake phase in C-Mod-like plasmas with separate temperature and density evolution \([6, 7]\), suggest that 1/1 helical density perturbations near \( q = 1 \) are compatible with the observed early snake formation and also with ongoing sawteeth. For a C-Mod shaped plasma with a small \( q < 1 \) region and a small background toroidal rotation, similar to the early stages of the impurity snake, results from the M3D initial value code show that a positive helical density (\( \delta n/n \)) of a few percent in an annulus peaked around or just outside the \( q = 1 \) radius drives an internal kink-like mode inside \( q < 1 \) (see Fig. 7), in such a fashion as to minimize the perturbed 1/1 pressure gradient. The \( n = 1 \) temperature perturbation tends to becomes positive where the \( n = 1 \) perturbed density is negative and vice versa. The central perturbed pressure and poloidal magnetic flux are closely aligned over \( q < 1 \) \([7]\). The total
FIG. 7: Nonlinear MHD simulation shows a slowly growing $1/1$ kink over $q \lesssim 1$ with an external helical density peaked at or just outside the $q = 1$ (shown by red(+)/blue(-) shading). Contour-lines show poloidal magnetic flux $\tilde{\psi}$ typical of a $1/1$ kink; $q = 1$ falls at the outer edge of the closely nested “D” contours [6] [7].

FIG. 8: SXR reconstructions during third phase of snake formation [6].

density perturbation on the other hand, has a broad helically kinked shape, extending beyond $q = 1$, like the early C-Mod snake. It forms a long-lived quasi-steady-state structure linked to the very slowly growing kink inside $q < 1$ resembling Fig. 5-c). The decoupled pressure and density distributions could allow sawteeth driven by pressure that do not disrupt the density snake. In Fig. 8 the kink moves towards positive $\tilde{p}$ or $\psi$ (red line-contours on the right, low-field side). The sawtooth crash over $q < 1$ has therefore, relatively little effect on the peak snake density that is concentrated near $q \gtrsim 1$ on the top and bottom in the figure. The snake is also consistent with an eventual transition to a crescent-shaped snake trapped in the 1/1 island [see Fig. 8], as the $q = 1$ radius grows steadily and axisymmetrically due to the resistive current.
5 Conclusion

A new suite of imaging diagnostics on Alcator C-Mod enables estimates of the $n = 1$ helical structure of $P_{rad}$, $n_{Mo}$, $n_e$, $T_e$, $Z_{eff}$, and $\eta$ inside the $q \leq 1$ region with adequate temporal and spatial resolution. High-resolution observations of heavy-impurity 1/1 snakes show details of their evolution and the accompanying sawtooth oscillations that suggest important differences between the density and temperature dynamics, ruling out purely pressure-driven processes. The observed differences are consistent with nonlinear 3D MHD simulations that evolve plasma density and temperature separately. This work was performed under US DoE contracts including DE-FC02-99ER54512 and others at MIT and DE-AC02-09CH11466 at PPPL. Computational support was provided by the National Energy Research Scientific Computing Center under DE-AC02-05CH11231.

References