A Global Particle Model for Density Feedback Control In KSTAR

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Abstract:
A 0D global particle model has been developed mainly for PID tuning of density feedback control system. The model is based on Maddison’s multi-reservoir model which is one of the most intensive work of global particle modeling. The new model supplements outgassing-like term $-N_w/\tau_w$ to wall particle balance with the original term proportional to particle influx from the plasmas. This simple alteration of the original model enables to reproduce the low density limit which is frequently observed not only in KSTAR but also in many devices. The self-sustaining density without any external fueling acts as initial condition or offset in density feedback control system. Since the dynamics of plasma density is not linear, transient responses are affected by initial conditions. Hence the proper handling of the initial condition plays an appreciable role in addition to the dynamics of gas puffing and plasma itself.

1 Introduction

Global particle models have been widely used in describing plasma density evolutions [1, 2, 3, 4, 5, 6]. Some are of single equation but with many parameters mostly representing plasma-wall interactions [2, 7]. They are focused on the wall-relevant issues such as hydrogen recycling and retention. Others or most cases are composed of at least two equations describing primary participants as plasmas and wall containments, respectively [3, 4, 5, 6]. They are sometimes referred to as multi-reservoir models since they have different segments interrelated but still no spatial transport equations are included. Those models successfully envisage plasma wall interactions helping people stay outside the integrated fusion machines while substantial efforts are ongoing for direct measurements of plasma-wall activities.

On the other hand, plasma density control, another primary application is not extensively studied except a few work such as by Wong [8] and Brelé [9]. Reasons for the lack of study may be: (1) The global density control is single-input-single-output (SISO) system which can be attempted by manual tuning of involved controllers. (2) The previous global models miss some points that becomes important in feedback control.
The former may be more practical because in magnetic fusion machines e.g. DIII-D, JET and LHD where density feedback control is routinely working, tens of thousands experiments have been carried out for a few decades. The latter seems subsequent since the previous work did not emphasize feedback control probably due to the former reason. However, in KSTAR, time and resources are critically limited whereas the demands on density feedback control is even more urgent. Consequently model based PID tuning is indispensable in KSTAR to save resources. This work develops a global particle model for KSTAR experiments and the model is based on Maddison’s work. The modification of the model will be briefly introduced in the next section and compared with the original one by examples. The comparison with analytic solutions of each model will be also given in section 3 and concluded in section 4.

2 The Global Particle Model

Numerous global particle models have been established with their own concepts and strategies for individual purposes. Most are quite descriptive but solutions of density evolution may not be unique because of a number of parameters which can be hardly determined exactly. Surprisingly however, they still have been sufficiently accurate in reproducing plasma density evolution since all individual terms are canceled eventually as summed in overall\cite{5, 6}, such that inaccuracy of individual parameters can be compensated by each other.

However, more demands on accuracy rises corresponding to particular purpose such as PID tuning of density feedback control. In order to keep the advantages of global models, there can be some basic guidelines of the modeling:

- Simple to exclude uncertainty of parameters.
- Self-consistent to predict the density behavior with minimum initial inputs or if possible other diagnostic information can be used explicitly.
- Again compact for less computing power which is a big advantage of global models. A global model will deliver result even in between discharges of subsequent fusion experiments.

2.1 Modification of Maddison’s model

The Maddison’s work is one of the most intensive study on global particle modeling and validated well with experiments in MAST. The model is dealing with four different participants: \(N_i\) ion contents in plasma volume, \(N_{D2}\) and \(N_D\) the number of deuterium molecules and atoms in the space between plasma and the wall, and \(N_w\) the number of deuterium atoms residing in the wall inventory. They are strongly interrelated each other with their own characteristic times, \(\tau_i\), \(\tau_{D2}\) and \(\tau_D\), respectively. One can notice immediately that no characteristic time of the particles in the wall appears in the
original model. The new concept of the modified model thus come from here.

\[
\frac{dN_w}{dt} = (1 - r_i - d_i) \Gamma_i + (1 - r_D - d_D) \Gamma_D - \frac{N_w}{\tau_w}
\]

where \(\Gamma_i\) represents ion influx from the plasma and \(\Gamma_D\), similarly atomic flux that is originated from charge-exchange interactions and Frank-Condon dissociation in the original model. \(r\) and \(d\) with subscript \(i\), \(D\) for ions and atoms, respectively, are coefficients of reflection and desorption or sometimes re-emission by authors and are constant or can be explicitly determined if necessary. Although eq. [1] still remains simple, it conceives distinctive concept of wall recycling: Some of particle influx (the first two terms on RHS) are immediately re-emitted both by reflection and desorption which is proportional to the influx but others (the last term) are rather spontaneous with their own characteristic time \(\tau_w\). The latter can be referred to as 'outgassing-like' which is evident from its form, but should be distinguished from the conventional outgassing since the surface of the wall is in high-retention or even saturated.

Most previous models contains each term separately: some the former, others the latter. Detail effect of this simple combination will be discussed hereafter.

### 2.2 Effect of the modification

An example of the new model is given in figure \([1]\) with fueling modulation experiment in open-loop control. Additional lines just for some guide is drawn together. The dotted black lines indicate the conventional decay time \(\tau^*_i\) that is defined in the equation as

\[
\tau^*_i = \frac{\tau_i}{1 + \frac{r_i}{d_i} + \frac{d_i}{r_i}}
\]

*FIG. 1: Experiment (solid magenta, thin) and model (solid blue, thick) density evolution caused by the modulated voltage of piezo-electric valve, PV (broken orange). The specific plasma is 300kA ohmic circular discharge in 2.0T.*
below.

\[
\frac{dN_i}{dt} = -(1 - R) \frac{N_i}{\tau_i} + f_{ex} \Phi \quad (2)
\]

\[
\equiv -\frac{N_i}{\tau_i^*} + f_{ex} \Phi \quad (3)
\]

where \( R \) represents total recycling coefficient, \( \tau_i \) global ion confinement time in seconds, \( f_{ex} \) fueling efficiency of external gas, and \( \Phi \) fueling rate in \( s^{-1} \). Hence \( \tau_i^* \) is constant if total recycling remains same and the decay should look like the black dot lines. However, every decay seems saturated in different levels indicated by red solid arrows in figure impose \( R \to 1 \). This is frequently observed not only in KSTAR but in other devices. For instance, recycling in TRIAM-1M increased from 0.98 to higher than 0.995 as \( t \to 50s \) \[7\]. In order to have those density sustainment, \( r_i + r_D + d_i + d_D \to 2 \) in Maddison’s model which means all induced particles into the wall are re-emitted again regardless of emission type. As Maddison also pointed out, those recycling parameters obviously cannot be constant particularly in long-term machines such as KSTAR. They can be manually tuned of course corresponding to actual time-evolution but it is not such good idea for further application, of particular simulation of density feedback experiment since the prediction of the density will be quite limited.

By combining those two re-emission phenomena, proportional to particle flux and and to wall inventory itself respectively, effective self-evolution of those parameters can be obtained. To illustrate the alteration, total recycling coefficient \( R \) is defined as below as Eq.(2) is compared with the Maddison’s model.

\[
R(t) = f_{ex} \frac{\tau_i}{N_i} \left( \rho \frac{N_{D2}}{\tau_{D2}} + \frac{V_p}{V_i} \frac{N_D}{\tau_D} \right), \quad (4)
\]

where \( \rho \) denotes traveling probability of molecules overcoming the strong efflux of atoms out of plasmas \[6, 10\]. \( V_p \) and \( V_i \) are volumes of plasma and the vacuum vessel, respectively. The results of Eq.(5) are drawn in figure 2. The blue line is derived from the new model as given in figure 1 and the green line is from the original model that was the best fitting of the experiment. The circles help to roughly recall the decaying period and the \( R(t) \) in the original model decreases while it grows in the modified one. This is due to the new term \( N_w/\tau_w \) is added into \( N_{D2} \). The effect of modification is apparent when more fuels are injected since the more fuels refused by the plasma at the first fueling moves into the wall leading to more density growing after a while. This is true in general because of low fueling efficiency of gas puffing. An example #5780 which was an experiment of density feedback control is illustrated in figure 3 with the two models’ results. The original model cannot sustain such a high density growing after about 2.2s due to the lack of the contribution from the new term. The primary tuning parameter \( d_D \) both in Maddison’s and the modified model, can be intentionally increased for more density growing, but it always lead to early density increase together. This means again the constant recycling parameters alone cannot reproduce the long pulse discharges satisfactorily.

It is worthwhile to stress the distinctive difference of the beginning time in each modeling work. Since the modified model can virtually make any waveforms, it can start
FIG. 2: $R(t)$ for #5557 from the new model (blue) and from the original model (green).

modeling very early particularly where no fuels are injected. Many discharges were involved with early density sustainment or bottom saturation without any fueling as seen in figure 1 and figure 3. To keep those sustainment, $R \rightarrow 1$ in those periods which requires very high values of the original parameters. Then the rest of waveform is not possible to meet the experiments. On the other hand, the new model can keep the original parameters as low as possible by adjusting $N_w/\tau_w$. Specifically $N_w$ is in order of $10^{21}$ atoms, and $t_w$ extends from $\times 100 \text{ms}$ to $\times 1 \text{s}$.

It can be immediately noticed that initial $N_w/\tau_w$ can be kept same with different combinations of individual $N_{w0}$ and $\tau_w$ where subscript 0 denotes the initial time of modeling. However, the overall behavior of density waveform still varies by them because of different degree of influences from the fuels and plasma particles. With smaller $N_{w0}$, the overall waveform is more likely exponential as accumulated particles in the wall distort waveform a lot whereas it looks like logarithm as $N_w$ is increased substantially. Therefore if all other parameters are accurately determined, the uniqueness of the model result can be guaranteed with the chosen $N_w$ and $\tau_w$.

FIG. 3: #5780 and modeling results from the new model (blue) and from the original model (green).
3 Analytic solutions

Both the analytic solution of original and the new model are obtained for more detailed comparison. Since the atomic temporal behaviors are overwhelming faster than those of others, \( dN_D/dt \sim 0 \) \cite{3, 4}. Furthermore, in the original model no contribution from \( N_w \) exists to other terms. Hence the rest of equations can be elaborated to be expressed simply as below when fueling is switched off (\( \Phi \sim 0 \)).

\[
N_i' = aN_i + bN_{D2} \\
N_{D2}' = cN_i + dN_{D2}
\]

(5) (6)

where \( ' \) denotes time-derivative. By solving these equations, the general solutions become

\[
N_i(t) = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t},
\]

(7)

\[
N_{D2}(t) = C_1 \frac{a - \lambda_1}{b} e^{\lambda_1 t} + C_2 \frac{a - \lambda_2}{b} e^{\lambda_2 t}.
\]

(8)

Therefore the plasma and surrounding molecules turn out to be involved with two characteristic times \( \lambda_1 \) and \( \lambda_2 \). Similarly the new model can be also written in time-domain solutions. The new contribution from the \( N_w \) makes the differential equation more complicated.

\[
N_i(t) = C_3 e^{\lambda_1 t} + C_4 e^{\lambda_2 t} + C_5 e^{\lambda_3 t},
\]

(9)

\[
N_{D2}(t) = C_6 e^{\lambda_1 t} + C_7 e^{\lambda_2 t} + C_8 e^{\lambda_3 t},
\]

(10)

\[
N_w(t) = C_9 e^{\lambda_1 t} + C_{10} e^{\lambda_2 t} + C_{11} e^{\lambda_3 t},
\]

(11)

FIG. 4: Lines are drawn by the analytic solutions. Circles indicate the results from the integrated model. (a) Reproduced Maddison’s calculation\cite{6} (b) With same parameters in (a) but calculated by Eq. (8) with no \( N_{D2}(0) \). \( \tau_i = 10 \text{ms} \). (c) Effect of \( N_D \) is restored from (a) as \( V_p = 0 \rightarrow 10 \). (d) \( \tau_i = 75 \text{ms} \) (e) All other parameters are restored. \( N_{D2}(0) = 1.1 \times 10^{20} \text{m}^{-3} \) (f) \( d_i \) differs from \( d_D \). \( d_i \) is fixed as 0.5. (g) \( N_{D2}(0) \) is increased to \( 1.1 \times 10^{21} \text{m}^{-3} \). Effect of \( \rho \) now appears by different model and solution results. (h) The result from Eq. (8) which is evaluated by the modified model.
With those participants of each model and the initial values of \( N_i \), \( N_{D2} \) and \( N_w \), analytic solutions delivers prompt waveforms from each parameters. It should be mentioned that \( \lambda_3 \) is almost zero by calculation so that each particle converges to \( C_5 \), \( C_8 \) and \( C_{11} \) respectively. This is as expected apparently when \( N_w/\tau_w \) is introduced. Therefore they are still with two characteristic times, but slightly changed due to third constants. The decay time \( t_{\text{decay}} \) is defined as the first moment in density’s reaching \( N_{i0}/e \) and each \( t_{\text{decay}} \) is drawn in figure 4 for each circumstances. The effect of new model is clearly observed as \( t_{\text{decay}} \) abruptly increasing only with \( d_D \) of 2.5. This is mainly due to the constant \( C_5 \) in Eq. (9). As some portion of the density is sustained spontaneously in the new model, it will be able to be used for feedback control simulation more accurately which is always involved with some initial density.

4 Conclusion and future work

The new global model has been developed based on Maddison’s global model. Two different characteristic time were introduced for the wall inventory. The one is the instant emission of the particles either by reflection and by desorption as described in the original model. This term is proportional to the incident particle flux from the plasma. The outgassing-like term is added to the original model so that it works as if there are two different time scale of plasma-wall interaction which is much slower than the former one. This treatment enables the demonstration of bottom limit of density which is not clearly described in previous models. Therefore it will be able to predict the controllable density limit or in other words, self-sustaining density for long-pulse discharges. This is due to the self-consistent evolution of wall recycling which means alteration of recycling-relevant terms in the original model.

The physical meaning of this alteration is simply envisaged. The wall is already saturated in the beginning of modeling as Maddison pointed out. However, because of ‘flooding bathtub’ phenomena, the concentration also diffuses out along the surface of plasma facing components. Therefore only a part of the incident particles are compensated promptly by the saturated wall concentration. The other parts are involved with transverse transport so that it takes time to re-emitted much slowly than those of saturated region. \( \tau_w \) is only responsible for this under-saturated region and it might be also changed in time for even longer discharges.

The modified model can be also used for simulation of PID tuning for density feedback control. The initial condition of plasma density without forcing action i.e. gas puffing can be fairly described so that the transient responses and steady-state level of density in feedback control is expected to be simulated in more accuracy.

References


