Alfvén eigenmode evolution in ITER steady-state scenario

M.Yu. Isaev¹, P.B. Aleynikov², S.V. Konovalov¹ and S.Yu. Medvedev³

¹National Research Centre Kurchatov Institute, Moscow, Russia
²ITER Organization, Route de Vinon sur Verdon - CS 90 046 - 13067 St Paul Lez Durance Cedex – France
³Keldysh Institute RAS, Moscow, Russia

E-mail contact of main author: isaev_my@nrcki.ru

Abstract. Alfvén eigenmode instability analysis in ITER steady-state plasma scenarios with reversed magnetic shear was performed with the NOVA and TAEFL codes [1]. In our work for this scenario we explore the stability of Alfvén eigenmodes calculated with the KINX code [2]. Both isotropic fusion alphas and beam ions can contribute into the mode drive. Fast particle dynamics, linear growth rate, mode amplitude evolution and the wave nonlinear saturation level are computed with the VENUS+δf [3] orbit following code. Anisotropic beam particle distribution is computed from realistic geometry of ITER NBI. Calculation results give the estimations of the fast particle driven linear growth rates and nonlinear saturation level of the mode amplitude for ITER steady state scenario.

1. Introduction

ITER steady-state plasma scenarios with reversed magnetic shear and both beam ion and alpha populations show Alfvén eigenmode instability [1]. Both reversed shear Alfvén eigenmodes (RSAEs) and toroidicity induced Alfvén eigenmodes (TAE) are found to be unstable with maximum growth rates occurring for toroidal mode number \( n = 6 \). Computations in [1] were performed with the NOVA and TAEFL codes.

In our work for the standard ITER steady state scenario-4 with plasma current of 9 MA and minimal value of the safety factor 1.6 in the middle of plasma radius we explore the stability of RSAEs with toroidal mode number \( n = 4 \). Alfvén continuum and fixed spatial structure of the mode with two dominating poloidal harmonics and small satellites near the plasma edge are calculated with the KINX code [2] (Section 2). Fast particle dynamics, linear growth rate, mode amplitude evolution and the wave nonlinear saturation level are computed with the VENUS+δf [3] orbit following code (Section 3). Both isotropic fusion alphas and beam ions contribute into the mode drive. Anisotropic beam particle distribution is computed with the use of Monte Carlo module accounting for realistic geometry of ITER NBI (Section 4). Nonlinear RSAE evolutions driven by NBI fast ions with anisotropic and uniform pitch angle distributions calculated with the VENUS+δf code are presented in Section 5.

Our simulation work extends the benchmark for the linear stage of the fast particle driven TAE dynamics with a broad variation in the physical and numerical models for the ITPA-EP (Energetic Particle Topical Group) test case with toroidal Alfvén eigenmode number \( n = 6 \) described in [4], as well as the simulation of nonlinear saturation of the TAE mode for the JET#40214 discharge with toroidal eigenmode number \( n = 4 \) from [5].
Calculation results give the estimations of the fast particle driven linear growth rates with toroidal eigenmode number $n = 4$, fusion alphas and beam ions induced mode drive and nonlinear saturation level of the mode amplitude for ITER steady state scenario.

2. **Alfvén continuum and spatial structure of the mode calculated with the KINX code**

We compute the Alfvén continuum and spatial structure of eigenmodes for the standard ITER steady state scenario-4 with plasma current of 9 MA, reversed shear and minimal value of the safety factor $q_{\text{min}} \approx 1.6$ in the middle of plasma radius the with the KINX code. RSAEs with a toroidal index $n = 4$ and the mode frequency $\omega = 2.86138 \times 10^5 \text{s}^{-1}$ (with the ratio to the Alfvén frequency $\omega/\omega_A = 0.1887$) is localized near the magnetic surface with $F_p^{1/2} = 0.45$ ($F_p$ is a normalized poloidal flux) with a safety factor $q = (m+1/2)/n = 1.625$. ITER plasma cross-section with this RSAE perturbations and its main components with poloidal numbers $m = 5, 6, 7, 8, 9$ as a functions of radial coordinate are shown in Figs. 1(a) and 1(b).

![ ITER plasma cross-section with n = 4 reversed shear Alfvén eigenmode perturbations (a) and its main components with poloidal numbers m=5, 6, 7, 8, 9 as functions of radial coordinate (b) computed with the KINX code for ITER steady-state scenario.](image)

Main differences of our standard ITER IDM scenario-4 from the scenario described in the Ref. [1] are: larger total current 9MA, larger alpha particle density with $\beta = 1.257\%$, smaller safety factor $q_{\text{min}} = 1.60$ and on-axis NBI beam injection (see FIG. 2(a)). KINX calculated Alfvén continuum with the TAE gap between blue and green curves, black line – $q$ profile,
dotted line – RSAE normalised frequency $\omega/\omega_A$, upper dashed – normalised bulk density are given in figure 2(b).

**FIG. 2.** ASTRA alpha and beam ion density profiles (a) and KINX calculated Alfvén continuum (b), eigenvalues of 1D shear Alfvén spectral problems at each magnetic surface, TAE gap between blue and green curves, black line – $q$ profile, dotted line – RSAE normalized frequency $\omega/\omega_A$, upper dashed – normalised bulk density for ITER IDM steady state scenario-4.

3. **Nonlinear evolution driven by alpha particles, computed with the VENUS+δf code**

The fast particle dynamics, TAE growth rates and wave saturation levels are computed with the VENUS+δf [3] orbit following codes. Fourier decomposition in Boozer coordinates $(s, \theta, \zeta)$ of the TAE mode has a form

$$\xi^s = A(t) \sum \eta_{mn}(s) \cos(m \theta - n \zeta - \omega t).$$  

(1)

The main equation for the TAE amplitude $A(t)$ evolution due to the fast particle-wave interaction without plasma damping is

$$\frac{dA}{dt} = -<\int Z e \delta f \cdot E \, d\tau>/ (2K \omega^2 A),$$  

(2)

here $K = \int \rho_b \xi^2 \, dV$ – the kinetic energy of the plasma perturbation, $\rho_b = m_b n_b$ – mass density of the bulk plasma, volume unit, $d\tau = d^2x \, d^2V$ – phase volume unit, $Z e$ and $e$ – particle charge and energy, $f = f_0 + \delta f$ – distribution function of fast particles, $V$ – particle velocity vector, $E$ - wave electric field. The TAE growth rate $\gamma$ is computed from $\gamma = dA/(Adt)$, and the radial component of the TAE perturbation $\delta B = rot \alpha B$, normalized to the central magnetic field $B_0$, is defined as

$$\frac{\delta B_r}{B_0} = \delta B \nabla s \cdot \nabla s |B_0| = -\mu_0 (I \zeta \alpha / \partial \theta + J \zeta \alpha / \partial \zeta) \frac{1}{|\nabla s|} |\nabla s|^{1/2} B_0, $$

(3)
where $J$ and $I$ are the toroidal and poloidal current flux functions, $g^{1/2}$ is a Boozer jacobian, components $a_{mn} = \xi_{mn} (m \chi' - n \psi')/ (n \mu_0 J - m \mu_0 I)$, $\psi, \chi$ are the toroidal and poloidal fluxes. The change of the particle weight along the drift trajectory in equilibrium and perturbed fields is computed from

$$
\dot{\mathcal{J}} = -[s_{\text{pert}} \frac{\partial f_0}{\partial S} + Z(\dot{s}_{\text{eq}} E_z + \dot{\theta}_{\text{eq}} E_z + \dot{\xi}_{\text{eq}} E_z) \frac{\partial f_0}{\partial E}] 
$$

The fast $\alpha$-particles have a density profile shown in Fig. 2(a) and a slowing down distribution function in energy, $f(E) = C(1 - \text{erf}(x))/(E^{3/2} + (E_c)^{3/2})$ with $E_c = 4.942 \times 10^5$ eV, $E_0 = 3.5 \times 10^6$ eV, $dE = 4.105 \times 10^5$ eV, $x = (E - E_0)/dE$. The constant $C$ has been defined to provide a central $\alpha$-particle density of $n_{f0} = 8.527 \times 10^{17}$ m$^{-3}$.

The nonlinear saturation of the RSAE mode is measured by the maximal value of function $\delta B_r/B_0$, computed with the VENUS+$\delta f$ code with the 960000 markers, representing the alpha particles distribution, with the initial value of the perturbation equals to $10^{-8}$, is shown in Fig. 3(a) in blue. Saturation is achieved after 40 wave periods. Evolution of the normalized growth rate is shown in red. For the comparison, Fig. 3(b) shows the evolution, computed with 1200000 markers and with the initial value of the perturbation $10^{-6}$, which has the same saturation level of the perturbation after about 25 wave periods and the same linear growth rate.

![FIG.3. Nonlinear saturation of n = 4 RSAE mode (blue) driven by α-particles computed with the VENUS+δf code with initial values δB_r/B_0 = 10^{-8} (a) and δB_r/B_0 = 10^{-6} (b) for ITER steady-state scenario 4. Evolution of the RSAE growth rate is shown in red.](image-url)
4. ITER NBI beam distribution function, calculated with the NBSOURCE code

Both isotropic fusion alphas and heating beam ions contribute into the mode drive. In this section an anisotropic beam particle distribution is computed analytically with the use of the realistic beam deposition profile calculated by Monte Carlo code NBSOURCE [6] accounting for realistic geometry of ITER NBI. Distribution of the beam ions in velocity space, $V$, $\chi$, where $\chi = V_{\parallel}/V$ is a pitch angle, can be approximated by Gauss or Legendre polynomials. Gauss beam distribution function has a form [7]:

$$f(F_p, V, \lambda) = C n(F_p) f_V \exp[-(\lambda - \lambda_0)^2/\Delta_\lambda^2],$$

(5)

where $\lambda = 1 - \chi^2$, $\chi_0(F_p) = 0.75 - 0.15 F_p^{0.5}$ - initial pitch angle dependence on normalized poloidal flux $F_p$ along the central line of injection.

$\Delta_\lambda^2 = \Delta_0^2 - (1-\lambda_0)\ln[(1 + V_c^3/V_0^3)/(1 + V_c^3/V^3)]$ – dispersion, $\Delta_0^2 = 0.10$.

The constant C has been defined to provide a central beam ions density of $n_f0 = 1.529 \times 10^{18}$ m$^{-3}$. Energy dependence has a form $f_V = (V^{1.5} + V_c^{1.5})^4$, where $V_c$ and $V_0$ are “critical” and beam injection velocities. Initial spreading of the beam, $\Delta_0$, was estimated from the NBSOURCE results.

Beam Legendre polynomial distribution function with $S = Q_{nbi}/E_{nbi}$, total beam power $Q_{nbi} = 33$MW, $E_{nbi} = 1$MeV, $\tau$ is slowing down time, $\Theta$ - step function, takes the form [8]:

$$f(F_p, V, \chi) = S \tau \Theta(V_0 - V) \sum_{l=0}^{\infty} \frac{2l+1}{2} P_l(\chi) P_l(\chi_0) \left( \frac{V_c}{V_0} \right)^l \left( \frac{V_c^{3}V_0^{3}}{V_c^{3}V_0^{3}} \right)^{l/2} Z_{d+1,l}$$

(6)

Figure 4 shows ITER NBI distribution profile for the electron temperatures $T_e = 5$ keV (a), $15$ keV (b) and $30$ keV(c) as a function of ion energy $E$ in MeV units and pitch angle $\chi$.

![Figure 4](image)

**FIG.4.** ITER NBI distribution function for $T_e = 5$ keV (a), $15$ keV (b) and $30$ keV(c), computed with the NBSOURCE code, here energy $E$ is in MeV.

Figure 5 shows 3D ITER NBI source calculated by NBSOURCE code as a function of pitch angle $\chi$ in the range $[0.5 1.0]$ and a function of normalized poloidal flux $F_p$ in the range $[0.0 1.0]$. 
FIG. 5. ITER NBI 1MeV beam deposition profile as a function of normalized poloidal flux $F_p$ and pitch angle $\chi$, calculated by the NBSOURCE code.

5. Nonlinear evolution driven by NBI beam ions, computed with the VENUS+δf code

For the RSAE mode the nonlinear saturated value $\delta B_r/B_0$, computed with the VENUS+δf code with the 250000 markers, representing 1MeV NBI beam ions with an anisotropic pitch distribution, is shown in Fig. 6(a) in blue. Evolution of the normalized growth rate is shown in red. Saturation is achieved after about 20 wave periods. For the comparison, Fig. 6(b) shows the evolution, computed with 432000 markers with a uniform pitch angle distribution, which has a lower linear growth rate.

FIG. 6. TAE evolution driven by ITER-9MA NBI 1MeV beam ions with anisotropic (a) and uniform (b) pitch
Linear normalized growth rate of RSAE driven by centrally injected NBI 1MeV fast ions with anisotropic pitch angle distribution of \( \gamma/\omega = 0.065 \) (see Fig. 6(a)) is close to the normalized growth rate of RSAE driven by 3.5MeV alpha particles with uniform pitch angle (see Fig. 3). Mode saturation level of \( \delta B_r/B_0 = 8.6 \times 10^{-3} \) is computed without any dampings. Linear growth rate driven by NBI 1MeV fast ions with a uniform pitch angle distribution of \( \gamma/\omega = 0.045 \) (see Fig. 6(b)) is smaller than the growth rate of the mode driven by the same ions with anisotropic pitch angle distribution.

6. Results and future plans

Fixed structure of the reversed shear Alfvén eigenmodes with the toroidal index \( n = 4 \) for the conventional ITER steady state scenario-4 with a total current of 9MA and the safety factor \( q_{\text{min}} = 1.60 \) have been computed with the KINX code.

Linear normalized growth rate and the nonlinear saturation level for the given RSAE \( n = 4 \) mode has been computed with the VENUS+δf code. Large linear growth rate driven by 3.5 MeV alpha particles of \( \gamma/\omega = 0.07 \) is connected with a large alpha particle fraction with \( \beta = 1.257\% \). Large mode saturation level of \( \delta B_r/B_0 = 10^{-3} \) is computed without radiative, thermal Landau or collisional dampings.

Fast ions distribution function with the anisotropic pitch angle distribution function from the real ITER NBI beam has been computed with the Monte Carlo NBSSOURCE code as an input for the TAE mode evolution VENUS+δf code.

Linear growth rate of TAE \( \gamma/\omega = 0.065 \) driven by the centrally injected NBI 1MeV fast ions with anisotropic pitch angle distribution is close to the growth rate driven by 3.5MeV alpha particles and larger than the growth rate \( \gamma/\omega = 0.04 \) driven by 1MeV fast NBI ions with the uniform pitch angle distribution.

Radiative, thermal Landau or collisional dampings can decrease the TAE growth rate and the nonlinear saturation level - next task for the VENUS code. TAE drive from off-axis NBI ions will be considered in the near future.

Acknowledgements

The authors thank ITPA-EP colleagues for the collaboration, S.P. Hirshman for providing the DESCUR and VMEC codes, W.A. Cooper, T.M. Tran, O. Sauter for the help with the simulations.

The work was supported by the RosAtom Contract H.4A.52.9B.14.1008. The computations were performed on the Kurchatov, LSPP-ITER and CRPP clusters.

Disclaimer: The views and opinions expressed herein do not necessarily reflect those of the ITER Organization.
References


