Abstract:
The impact of plasma toroidal rotation on turbulence and transport is studied by means of flux-driven gyrokinetic simulations with the GYSELA code. A volume source of parallel momentum can efficiently drive plasma rotation in either directions, co- and counter-current, up to parallel Mach numbers of several tens of percents. When $M_\parallel$ typically exceeds 20%, the energy confinement time is found to degrade with increasing rotation, as a result of both a reduction of the $E \times B$ shearing rate via the radial force balance and the triggering of the parallel velocity gradient instability. The same conclusion holds when rotation is modified through boundary condition effects, mimicking the coupling to the scrape-off layer. From the various constitutive terms of the momentum flux, namely diffusion, convection and residual stress, the diffusion coefficient has been successfully extracted in these simulations.

1 Introduction. Toroidal rotation is generally thought to be beneficial for tokamak performance. It stabilizes deleterious MHD modes such as resistive wall modes [1], and its shear, when coupled to $E \times B$ sheared flow, may contribute to the saturation of turbulent transport [2]. Conversely, large rotation can trigger PVG (Parallel Velocity Gradient) turbulence and increase the transport. In most present tokamaks, toroidal rotation is largely controlled by external means, mainly by neutral beam injection and possibly by radio frequency heating, among others (see e.g. [3,4]). In ITER however, the injected torque will be weak [5], and is thus not expected to lead to significant toroidal Mach numbers. There are alternative mechanisms leading to plasma spin-up. On the one hand, the breaking of plasma axi-symmetry can generate toroidal rotation. This leads to the so-called intrinsic rotation, which is self-generated by turbulence [6]. This mechanism
depends on the aspect ratio. The evidence of momentum transport independent of both toroidal flow and toroidal flow gradient has also been reported experimentally, by means of unbalanced co and counter neutral beams [7]. On the other hand, scrape-off layer flows may also drive rotation in the plasma core through boundary condition effects [8]. Indeed, they can reach large parallel Mach numbers, up to $M_{\parallel} = 1$ due to plasma-surface interaction.

The present paper reports on simulations performed with the GYSELA code (section 2). By means of finite injected torque, the mean toroidal flow is shown to be counter balanced. The amount of required torque to reach almost vanishing rotation is a measure of both the residual stress term and of the losses/gains of momentum through the boundaries (section 3). When the parallel Mach number typically exceeds a few tens of percents in the co-current direction, it is found to degrade the confinement. Both the PVG instability and the modification of the staircase structure of the radial electric field are likely responsible for this degradation (sections 4 and 5).

2 Detail of the simulations. Momentum transport is studied with the GYSELA code in electrostatic Ion Temperature Gradient (ITG) driven turbulence. Electrons are taken adiabatic, so that the electron density fluctuations $\tilde{n}_e$ are assumed to follow a Boltzman response $\tilde{n}_e/n_{eq} = e (\phi - \langle \phi \rangle)/T_{e,eq}$, with $\langle \phi \rangle$ the flux surface average of the electric potential. As a result, particle transport vanishes. The code is full-$f$, in the sense that gyrokinetic equation is solved for the entire ion guiding-center distribution function $f(r, \theta, \varphi, v_{G\parallel}, \mu, t)$ in global geometry [9,10]:

$$B^*_\parallel \partial_t f + \nabla \cdot (B^*_\parallel \vec{x}_G f) + \partial_{\nu_{G\parallel}} (B^*_\parallel \vec{v}_{G\parallel} f) = C(f) + S$$

with $\partial_{\nu_{G\parallel}} = \nabla_{\parallel} \cdot (B^*_\parallel \vec{v}_{G\parallel} f) = -B^* \cdot \nabla \vec{\Xi}/m_i$ (2)

where $\vec{x}_G = v_{G\parallel} B^* + b \times \vec{\Xi}$ and $B^* = B + (m_i/e) v_{G\parallel} \nabla \times b$. The collision operator $C(f)$ is of Fokker-Planck type and preserves density, momentum and energy, as it should [11]. $\phi$ is the gyro-averaged electric potential. The scalar $B^*_\parallel$ is $B^*_\parallel = B^* \cdot b$, with $b = B/B$ and $B = B_0 R_0/R$. So far, we use circular concentric equilibrium magnetic surfaces, with $b = [1 + (\epsilon/q)^2]^{-1/2} (e_\varphi + (\epsilon/q) e_\theta)$ and $e \approx r/R_0$. The safety factor profile is defined by $q(r) = q_0 + \delta_q(r/a)^{\alpha_q}$, where $q_0$, $\delta_q$, and $\alpha_q$ are free parameters. The system is closed by the quasi-neutrality condition which relates the electric potential $\phi(r, \theta, \varphi)$ to $f$:

$$\frac{e}{T_{e,eq}} (\phi - \langle \phi \rangle) - \frac{1}{n_{eq}} \nabla_\perp \cdot \left( \frac{m_i n_{eq}}{e B^2} \nabla_\perp \phi \right) = \frac{n_G - n_{G,eq}}{n_{eq}}$$

with $\nabla_\perp = (\partial_r, \frac{1}{r} \partial_\varphi, \partial_\theta)$ and $\nabla_\parallel = \frac{1}{\kappa} (\partial_\varphi, \frac{1}{\kappa} \partial_\theta)$. Here, $\varphi$ is the toroidal angle and $\theta$ the straight-field-line poloidal coordinate. The polarization density (second term on the left hand side of Eq. 3) is approximate, valid only in the limit of long wavelengths $k_\perp \rho_i \ll 1$. It is consistent with the Padé approximation used for the gyro-average operator. The guiding-center density is defined by: $n_G = \int J_c \ d\mu \ dv_{G\parallel} (J_f)$, with $J$ the gyro-average operator and $J_c = 2\pi B^*_\parallel/m_i$ the Jacobian in the velocity space. $f$ is replaced by the equilibrium component $f_{eq}$ when computing $n_{eq}$. In the code, all variables are dimensionless. The electric potential is normalized to $T_0/e$, time and length scales to $\omega_i^{-1} = m/e B_0$ and
\( \rho_s = m v_{th}/eB_0 \), respectively. Velocities are normalized to \( v_{th} = \rho_s \omega_c = \sqrt{T_0/m} \). Finally, the initial state consists in an equilibrium Maxwellian distribution function, which is either local or canonical (both choices are relevant at finite collisionality), plus a bath of poloidal and toroidal modes of small amplitude.

One important characteristic of the reported simulations is that they are flux-driven [10,12]. In GYSELA, the volume source term \( S \) in eq. (1) allows one to inject separately momentum, heat and vorticity. It consists of kinetic expressions multiplied by a radial envelope. In the simulations reported in this paper, an isotropic heat source is considered (injecting some amount of vorticity), on top of which momentum can be injected. Up to small terms proportional to the prescribed parallel current, the dimensionless expression of \( S \) takes then the following form [13]:

\[
S = S_p \left\{ \bar{v}^2_G + \bar{\mu} - \frac{3}{2} \right\} \frac{S_0^E}{3\sqrt{2\pi^{3/2} T_s^{5/2}}} + 2\bar{v}_G(2 - \bar{\mu})\frac{S_0^{v_G}}{4\pi^{3/2} T_s^{2}} e^{-\bar{v}_G^2 - \bar{\mu}} \tag{4}
\]

with \( \bar{\mu} = \mu B/T_s \) and \( \bar{v}_G = v_G/\sqrt{2T_s} \). The free parameter \( T_s \) corresponds to the normalized temperature of the source, equal to \( T_s = 1.5 \) here. The source radial profile is \( S_r \propto -\frac{1}{2} \{ \tanh[(\rho - 3L_s)/L_s] + \tanh[(-\rho + 3L_s)/L_s] \} \), with \( \rho = r/a \) and \( L_s = 0.06 \) (resp. \( L_s = 0.1 \)) for \( \rho_\ast = 1/75 \) (resp. \( \rho_\ast = 1/150 \)). The amount of injected heat power and parallel momentum are controlled by the \( S_0^{E} \) and \( S_0^{v_G} \) coefficients, respectively.

**FIG. 1**: Cases \( \rho_\ast = 1/75 \). (a) Parallel Mach number for 3 magnitudes of the toroidal momentum source. (b) Corresponding energy confinement times (thermal energy divided by injected heating power).

Two sets of 3 simulations each are analyzed in sections 3-4. The main differences between these 2 sets are first the \( \rho_\ast \) value, either equal to \( \rho_\ast = 1/75 \) or to \( 1/150 \) (at mid-radius at \( t = 0 \)), and second the inner radial boundary condition (see below). The low collisionality \( \nu_\ast \approx 10^{-1} \) is in the banana regime. The number of grid points in the 5D phase space is set to \( N_r \times N_\theta \times N_\phi \times N_{v_G} \times N_\mu = 128 \times 128 \times 64 \times 128 \times 16 \) (resp. \( 256 \times 256 \times 64 \times 128 \times 16 \) for half a torus), and the time step is equal to \( \omega_c \Delta t = 8 \) (resp. \( 20 \)) for the \( \rho_\ast = 1/75 \) (resp. \( 1/150 \)) cases.

Small \( \rho_\ast \) simulations have not reached steady state yet. Indeed, in the pressure balance which is fulfilled up to \( 1 - 2\% \), most of both parallel and perpendicular energies are
used to build up the pressure profiles \( \partial_t P_{\parallel,\perp} \) terms).

3 Structure of the momentum flux. In the absence of injected torque \( S_{0}^{G\parallel} = 0 \), it has already been shown that the toroidal kinetic momentum is conserved in GYSELA [8]. The \((r,\varphi)\) component of the Reynolds’ stress tensor reads as follows:

\[
\pi_{r\varphi} = \left\langle \int d^3v \, v_{E_r} v_{G\parallel} \hat{f} \right\rangle = \langle \hat{n} v_{E_r} \rangle \langle v_{r} \rangle + \langle \hat{v}_r v_{E_r} \rangle \langle n \rangle + \langle \hat{v}_r \hat{v}_{E_r} \hat{n} \rangle \tag{5}
\]

The first term in the right hand side of eq.(5) accounts for radial particle transport. It is vanishing in GYSELA simulations with adiabatic electrons. The third term accounts for triadic nonlinear interactions. Because of its complex structure, it is often assumed to be negligible. The second term is the momentum flux, usually expressed as follows [6]:

\[
\Gamma_{r\varphi} = \langle \hat{v}_r \hat{v}_{E_r} \rangle = -\chi_{r} \nabla_r \langle v_{\varphi} \rangle + V \langle v_{\varphi} \rangle + \pi_{r\varphi}^{\text{res}} \tag{6}
\]

\(\pi_{r\varphi}^{\text{res}}\) is the residual stress, which neither depends on the mean toroidal velocity nor on its gradient. It is driven by turbulence.

**FIG. 2: Cases \( \rho_* = 1/150 \). Same as Fig.1.**

For the cases with a fairly large value of \( \rho_* = 1/75 \), Dirichlet boundary conditions are considered for all components of the electric potential (i.e. all \((m,n)\) modes, including the \(\hat{\phi}_{00}\) component) at the inner radial boundary, which is located at \(\rho_{\text{min}} = r_{\text{min}}/a = 0.15\). Notice that such annular poloidal cross-sections possess a large inner hole. The heat source magnitude is \(S_{0}^{E} = 1.6 \times 10^{-2}\) and 3 cases are considered with different magnitudes of the momentum source: \(S_{0}^{G\parallel} = \{ -3.1 \times 10^{-3}; 0; 3.1 \times 10^{-3}\}\).

Even in the absence of injected torque, a mean velocity profile can still develop, provided the volume integral remains unchanged. Conversely, when exchanges with the exterior become effective through the radial boundary conditions, where a vanishing parallel velocity is usually imposed in GYSELA (so-called no-slip boundary condition), a net plasma spin-up can appear. Among others, those trajectories which cross the inner hole of the simulation domain contribute to the loss/gain of momentum. As shown in Fig.1, this net rotation is co-current, and reaches a parallel Mach number \(M_{\parallel} \approx 0.36\) in the core.
Fig. 1a also shows that the intrinsic plasma toroidal flow can be either increased or reduced, depending on whether the injected torque is co or counter current, respectively. Although the source of momentum is localized in the deep core (0.15 < ρ < 0.3), it modifies the entire flow profile, eventually leading to a vanishing flow for $S_0^{νC||} = −3.10^{-3}$. This particular case is qualitatively analogous to experimental results obtained with unbalanced co and counter neutral beams [7]. In these experiments, it was understood as a measure of the residual stress $π^{res}$ (cf. eq.(6)). In the present case, it amounts to the combined effect of $π^{res}$ and of the loss or gain of particles with trajectories crossing the inner hole at $ρ_{min}$.

![Graph](image)

**Fig. 3:** Left: momentum flux as a function of the gradient of the toroidal flow. Each point is in the $0.68 < ρ < 0.7$ region, and $95.10^{3} < ω_c t < 10^{5}$. Right: profiles of toroidal flow shear and $E × B$ shear ($ρ_∗ = 1/150$, $S_0^{νC||} = 0$)

So as to get rid of these exchanges with the exterior, a new set of simulations has been performed reaching the very core down to $ρ_{min} = 0.01$, and at smaller $ρ_∗$ value ($ρ_∗ = 1/150$). In this case, Neumann boundary conditions are considered for the equilibrium component of the electric potential $ϕ_00$ at $ρ_{min}$ [14]. The final toroidal velocity profiles are plotted in Fig.[2]a for three values of injected torque. Steady-state has been reached for $⟨v_ϕ⟩$ well before an energy confinement time, the pressure profile still evolving at the end of the simulation. Conversely to the $ρ_∗ = 1/75$ cases characterized by a large inner hole, minimum rotation is achieved for vanishing $S_0^{νC||}$ source. In this case, $M_∥$ remains below 2%. It reaches about −14% (resp. 18%) in the case of counter- (resp. co-) current injected torque. In both cases, again, the velocity profile is modified well beyond the location of the source.

Attempts to recover the usual structure of the momentum flux eq.(6) have revealed particularly difficult. When plotting $Γ_v/⟨v_ϕ⟩$ as a function of the velocity gradient for all radial positions and during the few last thousands of time units of the simulation $S_0^{νC||}$, it readily appears that there is no obvious correlation. Several reasons can be advocated. The first one is that the diffusivity $χ_v$ varies a lot with the plasma radius. The second one is that the residual stress $π^{res}_v$ significantly contributes to the overall momentum transport. The third one will be explored using advanced diagnostic tools recently implemented in GYSELA. It could well be that the usually neglected nonlinear
FIG. 4: Cases $\rho_\ast = 1/75$. Maximum linear growth rates with (red) or without (blue) accounting for the toroidal shear for 2 magnitudes of $S_0^{vG\parallel}$. Simulations performed with the GKW code [15], using the equilibrium profiles of GYSELA simulations.

The $\langle \tilde{v}_\phi \tilde{v}_E \tilde{n} \rangle$ term actually plays a significant role in the total stress tensor eq.(5). The last one is suggested by previous analyses showing that heat and momentum transports are correlated, both exhibiting large scale avalanche-like events [8].

Yet, focusing on a short time slice ($\sim 2.10^3 \omega_c^{-1}$) and a reduced plasma region characterized by an almost vanishing toroidal rotation allows one to extract $\chi_v$. This is the case of the simulation $S_0^{vG\parallel} = 0$ for $t_{end} - 2.10^3 < \omega_c^{-1} \Delta \tau < t_{end}$ and in between $0.68 < \rho < 0.7$. There, a clear line appears in the plot $\Gamma_v / \langle v_\phi \rangle$ vs. $d \log(\langle v_\phi \rangle) / dr$, from which $\chi_v$ is found to be of the order of 1 in Bohm units. Such a large magnitude certainly reflects the fact that these $\rho_\ast = 1/150$ simulations are still far from the gyro-Bohm regime, found to be below $\rho_\ast \approx 1/300$ in GENE, ORB5 [16] and GYSELA [13]. Finally, the line almost crosses the origin, indicating that the pinch term is negligible at this location.

4 Impact of toroidal rotation on energy confinement. Linear calculations teach us that, when the shear of the toroidal rotation acts in combination with the $E \times B$ shear, it can stabilize ITG modes [2]. More precisely, the linear growth rate is reduced by $k_\theta \rho_i d\langle v_\phi \rangle / dr (q_\omega E / s)^2$ to be compared to the $E \times B$ shear effect $(q_\omega E / s)^2$, with $k_\theta$ the poloidal wave number and $s$ the magnetic shear. Conversely, for large Mach numbers of several tens of percents, the PVG (or parallel Kelvin-Helmholtz) instability leads to an increase of the growth rate.

As shown in Fig.1b, the best energy confinement time is achieved for the smallest toroidal rotation in the $\rho_\ast = 1/75$ cases. This result is in qualitative agreement with other gyrokinetic simulations, which only report improved confinement when accounting for the combined effect of fast particles and electromagnetic physics [17]. A detailed analysis reveals that the confinement degradation results from 2 factors here. First, the linear threshold for the PVG instability is reached for the large flow cases Fig.4. Second, the $E \times B$ staircase resulting from turbulence self-organization [18] appears to be eroded when rotation increases in the co-current direction, leading to a complex re-organization of the radial force balance, as evident in Fig.5. The case of large counter-current $M_\parallel$
remains to be studied.

FIG. 5: Cases \( \rho_* = 1/75 \). Radial force balance in the cases \( S_0^{\parallel} = -3 \times 10^{-3} \) (left) and \( +3 \times 10^{-3} \) (right). The former has an almost vanishing contribution of \( \langle v_\phi \rangle \).

Conversely, the situation is different for the \( \rho_* = 1/150 \) cases, where \( M_\parallel \) is smaller. In these cases, the confinement time seems to hardly depend on the magnitude of the toroidal flow. Consistently, the structure of the force balance is not much modified, the toroidal velocity playing in all cases a minor role. Interestingly, in these simulations, the toroidal flow also develops staircase-like structures, i.e. well localized regions of large toroidal flow shear. However, there is no strong evidence that these structures are correlated with those of the \( E \times B \) poloidal shear flow, as seen in Fig.3.

5 Impact of boundary conditions. In GYSELA, the distribution function is kept fixed at \( \rho_{\text{max}} \) (Dirichlet boundary condition or b.c.), equal to its initial value. By default, a centered Maxwellian is used, corresponding to a no-slip b.c., but any other Maxwellian can be chose. Partly due to the inhomogeneity of the particle source in the scrape-off layer (SOL), the parallel Mach number is not expected to be symmetric in \( \theta \), so that there is a priori a net momentum input from the SOL into the edge plasma. So as to explore its possible impact on core plasma confinement, we have implemented such a solution mimicking the L-mode, and provided by the SOLEDGE-2D code [19], in the external b.c. of GYSELA.

As shown in Fig.6a, the rotation profile is modified up to the core, although by a weak amount (few percents only), leading to a slightly smaller gradient. This change is sufficient to reduce a bit the stored energy in the system, leading to a degradation of the confinement. This is exemplified by the reduction, by about 5%, of the temperature gradient in the center of the simulation domain (Fig.6b). More interestingly, the turbulent transport dynamics is significantly modified. Fig.6c shows a color-plot of the \( E \times B \) shearing rate. Staircases are clearly apparent around \( \rho \approx 0.6 \) and \( \rho \approx 0.7 \). Once the radial b.c. is changed to the L-mode like case, from \( \omega_c t \sim 7.65 \times 10^5 \) onwards, the outer structure of the staircase appear to be weakened, through a modification of the radial force balance. The result is a propagation of avalanche-like transport events inward and over very large distances, almost reaching the inner boundary of the simulation domain
\textbf{FIG. 6:} (a) Profiles of toroidal flow with no-slip and SOL-like b.c. (b) $R/L_T$ at $\rho = 0.5$, the SOL-like b.c. is switched on at $\omega_c t \sim 7.65 \times 10^5$. (c) $E \times B$ shearing rate ($\rho_v = 1/150$).

($\rho_{\text{min}} = 0.15$ here). This can in turn explain the observed degradation of the confinement.

\textbf{6 Conclusion.} The impact of toroidal rotation on turbulence and confinement has been studied with the flux-driven gyrokinetic code GYSELA, in the electrostatic regime with adiabatic electrons. The plasma rotation is efficiently controlled by a volume source that injects toroidal momentum on top of the heat source, or by modifying the outer radial boundary condition mimicking SOL flows. In each case, for parallel Mach numbers exceeding a few tens of percents, the confinement has been found to degrade with increasing rotation. This results from both a modification of the $E \times B$ shearing rate through the radial force balance, and to the triggering of the parallel velocity gradient instability. No significant change is observed when $M_\parallel$ typically remains below 20\%. Finally, the attempt to extract the diffusion coefficient, the pinch velocity and the residual stress have revealed particularly challenging. Possible explanations for this difficulty have been discussed.