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NUCLEAR FUSION SUPPLEMENT 1991

PLASMA PHYSICS
AND CONTROLLED
NUCLEAR FUSION RESEARCH
1990

PROCEEDINGS OF THE
THIRTEENTH INTERNATIONAL CONFERENCE ON PLASMA PHYSICS
AND CONTROLLED NUCLEAR FUSION RESEARCH
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VOLUME 2

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VIENNA, 1991
FOREWORD

The 1990 International Atomic Energy Agency Conference on Plasma Physics and Controlled Nuclear Fusion Research was characterized by reports of steady technical progress in research on both magnetic and inertial confinement fusion, leading towards the long term goal of producing commercial energy from controlled fusion power generators. Also, major results were reported from completion of the Conceptual Design Activities of the International Thermonuclear Experimental Reactor (ITER) project, which has been conducted since 1988 under the auspices of the IAEA.

This conference, the thirteenth in a series held biennially, was organized in cooperation with the United States Department of Energy, to whom the IAEA wishes to express its gratitude. Over 640 participants and observers from thirty countries and two international organizations attended the conference.

Over two hundred technical papers were presented in thirty technical sessions, including eight poster sessions. There were contributions on tokamak experiments; inertial confinement; non-tokamak confinement systems; magnetic confinement theory and modelling; plasma heating and current drive; ITER; technology and reactor concepts; and the economic, safety and environmental aspects of fusion. The opening session of the conference included an address by Admiral James D. Watkins, Secretary, United States Department of Energy; a round table discussion entitled Why Fusion?; a summary talk on the ITER project; and the traditional Artsimovich Memorial Lecture.

These proceedings, which include all the technical papers and five conference summaries, are published as a supplement to the IAEA journal Nuclear Fusion.

The IAEA contributes to international collaboration and exchange of information in the fields of plasma physics and controlled nuclear fusion by organizing these biennial conferences and by sponsoring technical committee meetings, workshops, consultants meetings and advisory groups on relevant topics. Through these and other activities, the IAEA hopes to contribute significantly towards bringing forward the day whose night will be lit by fusion generated power.
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Session D-III  B.B. KADOMTSEV  Union of Soviet Socialist Republics

Session D-IV
(Posters)
NONLINEAR KINETIC ANALYSIS OF FLUCTUATIONS AND TURBULENT TRANSPORT DUE TO TOKAMAK MICROINSTABILITIES*


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Abstract

NONLINEAR KINETIC ANALYSIS OF FLUCTUATIONS AND TURBULENT TRANSPORT DUE TO TOKAMAK MICROINSTABILITIES.

Results from comprehensive kinetic eigenmode numerical studies indicate that, for typical TFTR supershot parameters, the dominant microinstabilities are the collisionless trapped electron modes (CTEM). A nonlinear kinetic theory is developed for resonant CTEM in relatively peaked density profile toroidal plasmas with ion and trapped electron E x B nonlinearities treated on an equal footing. The results show that, in the nonlinearly saturated state, the density fluctuation level and the anomalous particle and heat fluxes are smaller than the usual mixing length estimates. Confinement is predicted to improve with higher T/T_e, more peaked density profile, and larger aspect ratio. These trends are in qualitative agreement with experimental results from TFTR. For flat density profile plasmas, ion temperature gradient acoustic waves are considered which propagate in the electron diamagnetic drift direction and are destabilized by trapped electron inverse dissipation in the presence of electron temperature gradients. A nonlinear analysis of such modes focusing on the ion Compton scattering yields results showing that \chi_0 > \chi for L_{Te} \gg L_{Te}, in rough agreement with DIII-D H-mode results. A set of fluid equations which accurately models ion Landau damping has been derived and used to improve fluid simulations of ion temperature gradient (ITG) driven turbulence. New analytical insight into the nature of the ITG-driven turbulence is gained by analyzing the nonlinear evolution of twisted eddies with very broad radial scale. It is found that, instead of the linear Fourier radial mode width used in most "mixing length" estimates of saturation, the appropriate radial scale should actually vary as k_n^{-1}.

* Work supported by the US Department of Energy under contract DE-AC02-76-CHO-3073.

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1. INTRODUCTION

Transport properties observed in TFTR supershot plasmas with peaked density profiles\cite{1} and DIII-D H-mode plasmas with flat density profiles\cite{2} exhibit a number of features which require a more precise theory than the conventional "mixing length estimates" (e.g., $\chi \approx \gamma/k^2$) of the trapped electron drift mode (TEM) and/or of the ion temperature gradient mode (ITGM) turbulence. In the present studies involving both analytical and numerical approaches, we show that for specific equilibrium scenarios, the characteristics of electrostatic microturbulence (such as the dominant nonlinear saturation mechanism, the shape of the k-spectrum at saturation, and the relative magnitudes of various fluctuating quantities and the anomalous fluxes) have important scaling trends with nondimensional parameters such as $T_i/T_e$, $L_n/R$, $r/R$, $L_{Te}/L_{Ti}$, and $q$ that are compatible with recent experimental observations in the improved confinement regimes in large tokamaks.

2. COLLISIONLESS TEM-TURBULENCE IN PEAKED DENSITY HOT-ION PLASMAS

A comprehensive kinetic eigenmode code\cite{3}, developed to systematically analyze the kinetic stability properties of high toroidal mode number drift-type instabilities, can be used to evaluate the relative ratios of the quasilinear fluxes of particles and energy for each plasma species. In the present studies, this code is applied to realistic TFTR equilibrium profiles for the investigation of toroidal drift modes destabilized by the combined influence of trapped-electron dynamics and ion temperature gradient effects. In studying the influence of the $\eta_i$-parameter on the single most unstable mode in the spectrum, $\eta_i \equiv (d\ln T_i/dr)/(d\ln n/dr)$ is artificially varied, while the total pressure gradient ($\nabla p = \sum_j n_j \nabla T_j + T_j \nabla n_j$) and therefore the free energy drive) is held constant. It is found that the character of this most unstable mode goes through a smooth transition around $\eta_i \approx 1.3$. Below this value, trapped-electron mode characteristics are prevalent (i.e., $T_e/T_i > \tilde{T}_e/T_i$, $Q_e \geq T_i \Gamma$, $Q_i$, etc.), while for larger values of $\eta_i$, the ion temperature gradient mode features dominate (i.e., $\tilde{T}_i/T_i > \tilde{T}_e/T_e$, $Q_i \gg T_i \Gamma$, $Q_e$, etc.). When compared with data from representative TFTR supershot cases, the theoretical quasilinear flux ratios, $Q_e/Q_i$ and $Q_j/T_i \Gamma'$, are usually found to be within about a factor of three or less of the corresponding experimental values. The results of these studies also indicate that for typical TFTR supershot confinement zone parameters\cite{1} (e.g., very low collisionality, high $T_i/T_e$ ratio, peaked pressure profiles, etc.), the fastest growing drift instabilities are the collisionless trapped electron modes. In addressing the important issue of the possible existence of a local current and/or $q$-dependence for collisionless electrostatic modes, attention is focused here on a careful examination of the linear growth rate ($\gamma$) of these collisionless trapped electron modes. It is found that $\gamma$ scales roughly as $[q(a)]^\alpha$ with $\alpha \approx 0.5$ to 1.5 for representative TFTR parameters in peaked-density collisionless regimes. This
is likely due to the fact that the magnitude of the trapped-electron precession drift frequency (which drives these instabilities) is reduced as the shear parameter, \( \hat{s} = r q' / q \), is reduced. For common equilibria, \( \hat{s} \) decreases as \( q(a) \) decreases. Moreover, as \( q \) decreases, the stabilizing influence of ion Landau damping is also enhanced. Since the strength of the instability consequently decreases for lower values of \( q \), this new trend is suggestive of a favorable local transport scaling with current (frequently observed in global confinement time scaling of auxiliary-heated plasmas but absent in previous studies of CTEM).

In order to determine the nonlinear consequences of CTEM, the present work reports on a systematic development of a weak turbulence theory of resonant CTEM in toroidal geometry in the presence of both ion and electron temperature gradients\[^4\]. The theoretical model consists of the Boltzmann response for the untrapped electrons, the collisionless bounce-averaged nonlinear drift-kinetic equation governing the dynamics of the trapped electrons, and the collisionless nonlinear gyrokinetic equation determining the ion response. The relevant frequencies in the linear analysis are ordered as \( \omega_{te} > \omega_{de} > \omega, \omega_{se} > \omega_{de} > \nu_{eff}, \) and \( \omega, \omega_{se} > \omega_{di}, \omega_{ni} \). Here, the orbit-averaged trapped electron precession drift frequency is \( \omega_{de} = G(c T_e k_{fe} / e BR) \), and the other notations are standard\[^4\]. In the linear regime, a dispersion relation derived for CTEM driven unstable by wave trapped-electron precession resonances, with a fluid approximation (\( \omega > \omega_{ii} \)) for ions, is used. We also assume a simple gaussian-shaped radial structure with a mode-width characteristic of results obtained from numerical eigenmode studies in toroidal geometry. Both ion and trapped electron \( E \times B \) nonlinearities are treated on an equal footing in the analysis. Following standard weak turbulence expansion procedures, we retain the dominant nonlinear wave-particle-wave interactions and obtain the following wave-kinetic equation which can be formally expressed as:

\[
\left( \frac{1}{2} \frac{\partial \text{Re} \epsilon^{(1)} }{\partial \omega} + \text{Im} \epsilon^{(1)} \right) |\Phi_k|^2 = \text{Im} \left( - \sum_{k' = k + k''} (C_s \rho_s \hat{k} \times \hat{k'} \cdot \hat{b})^2 \int d^3v \right. \\
\times (\omega'' - \omega_{de} + i 0^+)^{-1} \omega^{-1} \left( \frac{\omega_{se}}{\omega_k} - \frac{\omega_{se}}{\omega_{k'}} \right) \left[ 1 + \left( \frac{E}{T_e} - \frac{3}{2} \right) \eta_e \right] \\
\times F_{e0} |\Phi_{k'}|^2 |\Phi_k|^2 + \sum_{k' = k + k''} (C_s \rho_s \hat{k} \times \hat{k'} \cdot \hat{b})^2 \int d^3v \\
\times (\omega'' - \omega_{di} - k'' v_{||} + i 0^+)^{-1} \omega^{-1} \left( \frac{\omega_{se}}{\omega_k} - \frac{\omega_{se}}{\omega_{k'}} \right) \\
\times \left[ 1 + (u^2 - 3) \eta_i / 2 \right] J_0^2 J_0^2 F_{i0} |\Phi_{k'}|^2 |\Phi_k|^2, \\
\right)
\]

where \( \epsilon^{(1)} \) is the linear dielectric function and the other notations are standard. From the sign of each nonlinear term on the right hand side, we can show that trapped electron scattering (the first term) transfers fluctuation energy to the shorter wavelength modes, while ion Compton scattering
(the second term) transfers fluctuation energy to the longer wavelength modes (in agreement with the local theory prediction).

In the nonlinearly saturated state, $\partial/\partial t |\Phi_k|^2 = 0$, the expected shape of the radially averaged spectral intensity, $I(k_\theta) \equiv \langle (e\phi/T_e)^2 \rangle \approx \langle (n/n_0)^2 \rangle$, can be characterized by identifying three different asymptotic regions in $k_\theta$. In the very long wavelength regime ($k_\theta^2 \rho_s^2 \ll L_n/R$), fluctuations are suppressed due to the trapped electron scattering. In the long wavelength regime ($1 > k_\theta^2 \rho_s^2 > L_n/R$) where most of the fluctuation energy is populated, $I(k_\theta)$ decays according to a power law as a result of the balance between ion Compton scattering and trapped electron linear driving,

$$I(k_\theta) \simeq (2^5/3\pi)(q^3/\delta^2)(T_e/T_i)(1 + 5\eta_i/4)^{-1}[1 + (T_i/T_e)(1 + \eta_i)]^{-1} \times (R/L_n)^2(R/GL_n)^3/[R/GL_n - 3/2] \exp(-R/GL_n)$$

$$\times r^{-1} L_n^{-1} L_T^{-1} k_\theta^{-3}. \quad (2)$$

Finally, in the relatively shorter wavelength regime, $k_\theta \rho_s > 1$, $I(k_\theta)$ decays more steeply. We note that (i) the saturated amplitude of the fluctuations is smaller than the usual rough mixing length estimates, (ii) the appropriate gradient scaling length here involves $(L_n L_T)^{1/2}$ instead of the conventional dependence on only $L_n$ for electron drift modes, and (iii) the spectral shape exhibits a $k_\theta$-dependence which is steeper than the mixing length prediction.

The expressions for various anomalous fluxes due to CTEM are estimated to be:

$$\Gamma_e = - (2^7 e/15)(q^3/\delta^2)(T_e/T_i)\eta_e(1 + 5\eta_i/4)^{-1} \left[\sqrt{R/L_n} - \ln(\sqrt{R/L_n} - 1)\right]$$

$$\times [1 + (T_i/T_e)(1 + \eta_i)]^{-1/2}(R/L_n)^2(R/GL_n)^3/[R/GL_n - 3/2]^2$$

$$\times \exp(-2R/GL_n)\rho_s L_T^{-1} (cT_e/eB) d\eta_0/d\xi, \quad (3)$$

$$Q_e = (R/GL_n)T_e \Gamma_e, \quad (4)$$

$$Q_i = 2.75 (1 + 1.93\eta_i)(1 + 1.25\eta_i)^{-1} T_i \Gamma_e. \quad (5)$$

The anomalous electron heating rate is given by $Q_{ei} \approx -T_e \Gamma_e / L_n$. As indicated by Equations (3), (4) and (5), the various fluxes obey similar scalings. The plasma confinement is predicted to improve with i) higher $T_i/T_e$, ii) more peaked density profile, iii) larger aspect ratio, and iv) higher plasma current. In addition, the results indicate a significant dependence of transport on the electron temperature gradient. This might be relevant to the rigidity of the electron temperature profiles observed in many tokamak plasmas. The favorable dependence on $T_i/T_e$ [1,5], which is almost universally observed in hot-ion mode operation of the large tokamaks, is a consequence of the fact that ion Compton scattering, which is stronger for higher $T_i$, is an important nonlinear saturation mechanism. The calculated confinement improvement with density peaking is consistent with the particular destabilizing mechanism considered in this paper; i.e., the precession drift resonance of the trapped electrons with waves. Specifically, the driving rate becomes exponentially weaker as $L_n$ decreases. Experi-
mentally, in TFTR auxiliarily heated plasmas, the confinement improvement from L-mode to Supershot can in fact be characterized by the density peakedness[1]. The inverse-aspect ratio ($e$) dependence found in our weak turbulence analysis of the anomalous particle and heat fluxes comes from the fraction of trapped electrons being proportional to $e^{-1/2}$. This is also in rough agreement with the recent results from TFTR. It is also important to note that the results of our present analysis of collisionless trapped-electron modes do indeed exhibit a significant local $q(r)$ dependence. Such behaviour can be explained in part by the dependence of the trapped-electron orbit-averaged magnetic precession drifts on $q(r)$. Moreover, the favorable transit ion nonlinear (as well as linear) Landau damping effects are likewise enhanced for lower $q(r)$ values. The absence of appreciable current dependence in TFTR supershot confinement could be due to the fact that the density profiles in these discharges were observed to steepen more as $I_p$ decreases for fixed beam power[1]. Hence, the unfavorable influence of decreasing $I_p$ might be offset by the aforementioned predicted improvement of confinement with density peaking.

Preliminary 1-1/2D BALDUR simulations of a TFTR supershot with the new CTEM model[4] without adjustable coefficients in a multimode formulation[6] are found to give good agreement for $T_e(r)$. However, this model underestimates the ion thermal losses at the plasma edge.

3. ION TEMPERATURE GRADIENT DRIVEN TURBULENCE

In contrast to CTEM, where $\omega_r \gg \gamma$ is satisfied for a wide range of $k_z$, the ITGM for normal density profiles ($0 < \eta_i < L_s/L_n, R/L_n$) is characterized by very small values for the real frequency $\omega_r \ll \omega_T$. This makes a weak turbulence analysis justifiable only when $\eta_i$ is very close to $\eta_i^{crit}$, and computational studies are needed for larger $\eta_i$ values. Parallel ion Landau damping, which plays a crucial role in both linear and nonlinear ITGM instabilities, ordinarily calls for a kinetic (phase-space) approach. We have found[7] that kinetic phenomena can be successfully modeled by a fluid approach, using the usual first three moments (density, parallel momentum, and parallel pressure) of the drift-kinetic equation, and closing the hierarchy by writing the parallel heat flux as $q_{k||} = -2\sqrt{2/\pi n_0(v_{ti}/|k||)i |k|| T_{k||}}$ in wave number space. In real space, this corresponds to a principal part integral along the magnetic field line. It is also important to avoid any artificial momentum viscosity. This form, which is equivalent to a 3-pole approximation to the plasma dispersion Z-function, indeed reproduces the kinetic result, $\eta_i^{crit} = 2$ in the small $k_{||} \rho_i$ limit, and will be particularly useful for spectral numerical algorithms.

Most analytical theories of ITGM in the strong turbulence regime have used the radial width of the linear Fourier mode as a "mixing length" for the relevant transport process. However our recent investigations of ITGM indicate that twisted eddies with very small $k_r$ can grow to a large amplitude because these are least affected by the $E \times B$ nonlinearity. Therefore, the spatial structure of the turbulence is characterized by the balance between the growth of twisted eddies localized in the bad curvature region and their break-up by secondary instabilities which isotropize the $k$-spectrum. As a consequence, the appropriate radial "mixing length"
would scale as $k_g^{-1}$, rather than the linear Fourier mode radial width, and
the resulting ion thermal diffusivity would be quite different from the usual
mixing length estimates based on the linear Fourier mode radial width.

4. TRAPPED ELECTRON TEMPERATURE GRADIENT DRIVEN
   TURBULENCE IN FLAT DENSITY H-MODE PLASMAS

   Usual ITG-turbulence theories have difficulties reconciling recent DIII-
   D H-mode results where the density profile is flat, but $\chi_e > \chi_i$ in the core
   region where $L_{Te} \gg L_{Ti}$[2]. As a possible candidate, we consider the
   ITG-acoustic wave propagating in the electron diamagnetic direction[8].
   This wave can be destabilized by trapped electron inverse dissipation in
   the presence of an electron temperature gradient. Since the real fre-
   quency is given by a weighted geometric mean of the sound wave frequency
   and the ITG-diamagnetic frequency ($\omega = (C_s/(2qR)^2) \omega_{ITG}^{1/3}$), this wave is
   strongly dispersive even at long wavelengths where it is linearly unstable
   ($(L_{Te}/R)^{1/4} \leq k_e \rho_s > L_{Te}/qR$). Therefore, three-wave resonant interac-
   tion is rendered ineffective, and ion Compton scattering plays a crucial
   role in determining the spectral shape at nonlinear saturation. Nonlinear
   analysis[9] shows that the anomalous fluxes are (i) only weakly dependent
   upon $L_{Ti}$; (ii) $\chi_e > \chi_i$ for $L_{Te} \gg L_{Ti}$; and (iii) $\chi_i \geq \chi_e$ for $L_{Te} \sim L_{Ti}$.
   These trends are in rough agreement with the hot-ion H-mode results from
   DIII-D[2].

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DEVELOPMENTS IN THE THEORY OF TRAPPED PARTICLE PRESSURE GRADIENT DRIVEN TURBULENCE IN TOKAMAKS AND STELLARATORS

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Abstract

DEVELOPMENTS IN THE THEORY OF TRAPPED PARTICLE PRESSURE GRADIENT DRIVEN TURBULENCE IN TOKAMAKS AND STELLARATORS.

Recent advances in the theory of trapped particle pressure gradient driven turbulence are summarized. A novel theory of trapped ion convective cell turbulence is presented. It is shown that non-linear transfer to small scales occurs, and that saturation levels are not unphysically large, as previously thought. As the virulent saturation mechanism of ion Compton scattering is shown to result in weak turbulence at higher frequencies, it is thus likely that trapped ion convective cells are the major agent of tokamak transport. Fluid like trapped electron modes at short wavelengths ($k_i \rho_i > 1$) are shown to

1 CIEMAT, Spain.
1. INTRODUCTION

Trapped particle modes are a crucial element in any theoretical model of tokamak confinement. Trapped particle modes are ubiquitous and universal in that they span the spectrum from \( k_x^p \ll \epsilon^{3/2}/q \) to \( k_x^p > 1 \) and occur in virtually all relevant tokamak operating regimes. This paper surveys recent developments in the theory of trapped particle pressure gradient driven turbulence (TPPGDT). In particular, Section 2 discusses the theory of trapped particle pressure gradient driven turbulence. Developments in the theory of long wavelength convective cells, conventional trapped electron driven drift waves and \( \nabla T_e \) driven microturbulence are presented in Sections 2.1, 2.2 and 2.3, respectively. Section 3 is concerned with related special topics in fusion theory, namely trapped particle modes in flat-density discharges and in devices with flexible magnetic configurations (i.e. stellarators). Section 4 deals with developments in the basic theory of multifield drift wave turbulence models, while Section 5 contains discussions and conclusions.

2. DEVELOPMENTS IN THE THEORY OF TRAPPED PARTICLE PRESSURE GRADIENT DRIVEN TURBULENCE

2.1. Long wavelength convective cells \((k_x^p \ll \epsilon^{3/2}/q)\)

The increasing relevance of high temperature, collisionless regimes and the nearly universal observation that measured \( S(\tilde{k}, \omega) \) spectra attributed to fluctuations in the core tend to increase at long wavelength \((k_x^p \ll 1)\) together suggest that long wavelength convective cells, arising from trapped ion modes with \( \omega < \omega_{ci} \), merit renewed theoretical and experimental interest. Indeed, given the size of predicted fluctuation levels and the insensitivity of the underlying mechanisms to parallel dynamics effects (i.e. shear damping, etc.), it is a mystery that trapped ion convective cells have not received more attention!

The basic physics of long wavelength convective cells is best elucidated in the context of dissipative trapped ion convective cells (DTICC) [1]. Dissipative trapped ion modes have the dispersion relation:
Thus, DTICCs are electron drift waves, with the frequency offset from \( \omega_e \) by \( \sqrt{\varepsilon} \), the fraction of trapped particles, which are destabilized by inverse electron collisional dissipation and damped by ion dissipation. Note that \( \omega < \omega_{\text{bi}} \) wavelengths are long so that the non-adiabatic electron response is dissipative for all relevant parameter regimes. Spatially, the mode amplitude is maximal between adjacent rational surfaces and balloons toward the region of unfavourable curvature. Thus, \( k_r = k_{\text{ps}} \) for DTICC. For \( \eta_i > 2/3 \), the DTICC goes over into the ion temperature gradient driven convective cell to be discussed below.

A simple model of the non-linear dynamics of DTICC is that proposed by Kadomtsev and Pogutse (KP) [1]. The KP model exploits quasi-neutrality and the dissipative character of the non-adiabatic electron response to derive a simple, non-linear equation for the relative density perturbation \( \tilde{n} \):

\[
\frac{\partial \tilde{n}}{\partial t} + \frac{v_{t_i}^2}{2} \frac{\partial \tilde{n}}{\partial y} + v_{\text{eff},i} \tilde{n} + \frac{v_{t_i}^2}{4v_{\text{eff},e}} \frac{\partial^2 \tilde{n}}{\partial y^2} + \frac{v_{t_i}}{e^{1/2}} (1 - \eta) \frac{\partial \tilde{n}^2}{\partial y} - \frac{L_\eta v_{t_i}^2}{e^{1/2} v_{\text{eff},e}} \tilde{\varepsilon}_i \cdot \nabla \frac{\partial \tilde{n}}{\partial y} \times \nabla \tilde{n} = 0
\]  

(2)

An additional, shock non-linearity [2] which is inconsequential in comparison to the advective non-linearity retained above has been neglected. The direction in k-space and mechanism of non-linear transfer clearly are the crucial issues in the theory of DTICC turbulence. As trapped ion modes are basically non-dispersive (modes with \( k_r \rho_i > 1 \) are excluded from the KP model), ion Compton scattering vanishes to all orders, wave-wave interactions are irrelevant and three-mode resonance occurs for all triads. Thus, eddy interaction (i.e. strong turbulence mode coupling) is the dominant non-linear mechanism. With this in mind, the equilibrium statistical mechanics of the KP model were investigated in the limit of vanishing linear growth and damping. As the model supports only a single, non-trivial quadratic invariant in that limit (namely, \( \int d^2r \tilde{n}^2 \)), a statistical equilibrium spectrum corresponding to equipartition results. This indicates that spectral transfer to small scales occurs, in contrast to the conventional wisdom of 2-D turbulence. This is, of course, a direct consequence of the fact that the KP model supports a single invariant, in contrast
to the 2-D Navier–Stokes equation, which supports two. These conclusions are supported by the results of an EDQNM closure analysis of the KP model, which determines spectral transfer by the equation

\[
\frac{\partial \langle |\tilde{n}|^2 \rangle_k}{\partial t} - \gamma_k \langle |\tilde{n}|^2 \rangle_k + T(\bar{k}) = 0
\]  

(3a)

Here,

\[
T(k) = \left( \frac{L_n v_f^2}{\varepsilon^{1/2} \rho_{eff,e}^2} \right)^2 \left[ \sum_{k'} |\tilde{e}_k \cdot \bar{k} \times \bar{k}|^2 k_y^2 g_{k,k',k} \langle |\tilde{n}|^2 \rangle_{k'} \langle |\tilde{n}|^2 \rangle_k \right]
\]

(3b)

\[
+ \sum_{k'} |\tilde{e}_k \cdot \bar{k} \times \bar{k}'| \cdot 2 (k_y^2 - k_y' - \bar{k}_y^2) g_{k,k',k} \langle |\tilde{n}|^2 \rangle_{k'} \langle |\tilde{n}|^2 \rangle_k
\]

\[
- \sum_{\bar{p} + \bar{q} = \bar{k}} |\tilde{e}_k \cdot \bar{p} \times \bar{q} |^2 \rho_2 g_{k,k',k} \langle |\tilde{n}|^2 \rangle_p \langle |\tilde{n}|^2 \rangle_q \right].
\]

is the non-linear transfer rate, and

\[
g_{k,k',k}^{-1} = -i(\omega_k + \omega_{k'} - \omega_{k''}) + (\Delta \omega_k + \Delta \omega_{k'} + \Delta \omega_{k''})
\]

(3c)

\[
- (\gamma_k + \gamma_{k'} + \gamma_{k''})
\]

\[
= (\Delta \omega_k + \Delta \omega_{k'} + \Delta \omega_{k''}) - (\gamma_k + \gamma_{k'} + \gamma_{k''})
\]

is the memory function for three-mode interaction. Here, \( \gamma_k \) is the linear growth/damping rate and

\[
\Delta \omega_k = \left( \frac{L_n v_f^2}{\varepsilon^{1/2} \rho_{eff,e}^2} \right)^2 \sum_{k'} |\tilde{e}_k \cdot \bar{k} \times \bar{k}'| \cdot 2 k_y^2 g_{k,k',k} \langle |\tilde{n}|^2 \rangle_{k'}
\]

(3d)

is the eddy decorrelation rate. It is clear that \( \sum_k T(\bar{k}) = 0 \) and that \( T(k) < 0 \) (\( T(k) > 0 \)) for long wavelength (short wavelength) modes, indicative of local and non-local transfer to small scales [3]. The analytical predictions are supported by the results of a 168-mode numerical relaxation experiment using a 2-D spectral code. In particular, the rate of energy transfer from low \( k \) to higher wavenumbers is plotted versus time in Fig. 1. The negative transfer rate evident during the first 50 eddy turnover times suggests strong transfer to higher wavenumbers. Thereafter, the turbulence reaches a steady state, about which ‘sloshing’ in \( k \)-space produces a fluctuating transfer rate with zero mean.
Energy flow from large scales

\[ \text{Energy flow from large scales as measured in relaxation experiments using the Kadomtsev-Pogutse model. Time is expressed in eddy turnover periods.} \]

Having understood the statistical dynamics of the KP model, we can estimate saturation levels and transport coefficients. In particular, imposition of the stationarity condition \( \gamma_k + T(k) = 0 \) yields density and potential fluctuation levels \( \bar{n}/n_0 = c\dot{\phi}/T_e = 1/k_\phi \delta L_n \). The 'mixing length' result recovered here is a direct consequence of the balance between growth and non-linear transfer, rather than a balance between non-linear decorrelation and frequency mismatch, as is the case for dispersive drift waves [4]. Note also that while density and potential fluctuations are large, i.e. \( \bar{n}/n < (v_n/2, v_T)(L_n/L_m) \), radial velocity fluctuations are no larger than those expected for conventional short wavelength drift waves, i.e. \( \dot{v}_r \approx c_s(\rho/L_m,s) \). Thus, DTICCs are radially broad, but slow moving. The particle and electron thermal diffusivities may be calculated directly if we note that, since \( \nu_{\text{eff}} > \omega, \Delta \omega_k \), the electron-cell decorrelation time is determined by \( \nu_{\text{eff}} \).

Thus,

\[ \chi_e = (3/2)D = (3/8)\varepsilon \rho^2 \frac{v_T^2}{\nu_{\text{eff},e}} L_{\text{m}}^2 \delta^2 \]  

(4)
As trapped ion diffusion is non-resonant, \( \chi_i = \sum |q_k^2|_k \Delta \omega_k / \omega_k^2 = \chi_e \). Thus, it is readily apparent that DTICC transport is not 'catastrophic' [5], as conventionally believed, but rather comparable to predictions based on strong turbulence drift wave models [6]. This is a consequence of the aforementioned fact that the DTICCs are broad, but slow. Finally, the validity of all these results is inexorably tied to the demonstration of non-linear transfer to small scales and the absence of long wavelength 'condensation'.

We now discuss two rather interesting, special features of DTICC turbulence. First, since energy non-linearly coupled to small scales eventually arrives at \( k \) such that \( \omega > \omega_{bi} \), the KP model cannot and need not account for the ultimate disposition of energy. In other words, the 'long wavelength', shear damped part of the electron driftwave spectrum acts as the 'dissipation range' for DTICC turbulence. In the vein of Prandtl mixing length theory, estimates of the spectral 'flow rate' yield \( \Pi_{k_{max}} = \epsilon D/L_e^2 \), which is comparable to the rate of energy dissipation by radial transport. Noting that \( \Pi_{k_{max}} \sim 1/\tau_p \) (where \( \tau_p = D/L_e^2 \)) it follows that transfer from DTICCs is a significant 'source' for shorter wavelengths and should be included in future models thereof. Second, since DTICC fluctuation energy is eventually dissipated by ion Landau damping after transfer to small scales, DTICC growth is, ultimately, in balance with ion heating. Thus, the turbulent electron–ion heating \( Q_{ei}^x \) may be estimated from

\[
Q_{ei}^x = \sum_k \frac{n_0 T_e}{2} \gamma_k \omega_k \frac{\partial \varepsilon(k, \omega)}{\partial \omega_k} \left| \frac{\varepsilon \phi_k}{T_e} \right|^2
\]

Here, \( \varepsilon(k, \omega) \) is the DTICC dielectric. Straightforward calculations yield \( Q_{ei}^x = (\epsilon D/2L_e^2) \sim n_0 T_e / \tau_p \), which is, not surprisingly, closely related to the spectral flow rate and the 'dissipation rate' due to transport.

For \( \eta_i \geq 2/3 \), long wavelength fluctuations are better described as ion temperature gradient driven convective cells (ITGDCCs). While a quantitatively precise calculation of \( \eta_i \) instability thresholds requires a detailed kinetic analysis [7, 8], much useful insight may be gleaned from a generalization of the KP model to one in which density and trapped ion temperature fluctuations \( \delta n, \delta T \) are advanced by the equations:

\[
\begin{align*}
\frac{\partial \delta n}{\partial t} &= -v_{ti}^1 \frac{\partial \delta n}{\partial y} + v_{hi}^2 \frac{\partial^2 \delta n}{\partial y^2} + \bar{v}_d \frac{\partial \delta T}{\partial y} \\
&\quad - \epsilon^{1/2} \frac{D_b}{\nu_{eff,i}} \epsilon_i \cdot \nabla \frac{\partial \delta n}{\partial y} \times \nabla \delta n = 0 \quad (6a)
\end{align*}
\]

\[
\begin{align*}
\frac{\partial \delta T}{\partial t} &= -v_{ti}^1 (1 + \eta_i) \frac{\partial \delta n}{\partial y} + D_R \epsilon_i \cdot \nabla \delta n \times \nabla \delta T = 0 \quad (6b)
\end{align*}
\]
Note that $\delta n$ and $\delta T$ are coupled by curvature, and that the two advective non-linearities are of different strength [9]. Linearization of Eqs (6a) and (6b) yields, for large $\eta_i (\gamma \gg \omega_i \sim \omega_n)$,  
\[ \gamma = \epsilon^{1/4} [\omega_n \bar{\omega}_d (1 + \eta_i)]^{1/2} \]  \[ (7a) \]
while for $\eta_i \gg \eta_{\text{crit}}$ and $\omega_i / \nu_{\text{eff}, e} \ll 1$  
\[ \omega = \omega_{ie} + i \omega_{ie} (\omega_i / \nu_{\text{eff}, e})^{1/2} \]  \[ (7b) \]
Hence, near marginal stability, the mode is similar to a (non-dispersive) dissipative trapped ion mode (electron direction), but with enhanced growth rate! The non-linear dynamics of ITGDCCs are similar to DTICCs. This is a consequence of the fact that only two quadratic invariants of the system exist in the zero growth/damping limit, namely  
\[ \int d^2r \, \bar{n}^2 \text{ and } \int d^2r \, \bar{T}^2 \]
(one for each equation!). Note that, unlike the familiar case of collisional electron drift turbulence, the cross-correlation $\langle \bar{n} \bar{T} \rangle$ is not an invariant [10]. Thus, sample equipartition spectra for $\langle \bar{n}^2 \rangle_k$ and $\langle \bar{T}^2 \rangle_k$ are predicted for statistical equilibrium, and non-linear transfer to small scale can be expected. These conclusions are supported by a closure analysis similar to that implemented for DTICC turbulence.

Since non-linear transfer to small scale is clearly indicated, saturated fluctuation levels and transport may be calculated by using the methods applied to DTICC turbulence. In particular, in the large $\eta_i$ limit ($\eta_i \gg \eta_{\text{crit}}$) results indicate that $\bar{n}/n = 1/k_\theta \delta (L_{T_\theta} R)^{1/2}$, $\bar{T}/T = 1/k_\theta \delta L_{T_\theta}$ and  
\[ \chi_i = \epsilon^{1/2} (v_{T_i}/k_\theta \delta) (\rho_i / (L_{T_i} R)^{1/2}) < (\epsilon^{1/2}/\delta^2) (v_{T_i}^2/\nu_{\text{eff}, i}) (\rho_i^2 / L_{T_i}^2) R^{3/2} \]  \[ (8a) \]
where the upper bound on $\chi_i$ follows from the lower bound on $k_\theta$ imposed by $\bar{\omega}_d > \nu_{\text{eff}, i}$. Similarly,  
\[ D \sim \chi_e \sim (\epsilon^{1/2}/\delta^2) (v_{T_e}^2/\nu_{\text{eff}, e}) (\rho_i^2 / L_{T_i} R) \]  \[ (8b) \]
Note that $\chi_i > \chi_e$, $D$ and $\bar{T}/T > \bar{n}/n_0$. Near marginal stability, however, $\bar{n}/n \sim (\epsilon^{1/4} / k_\theta \delta \ L_{n}) (\nu_{\text{eff}, e} / \omega_n)^{1/2}$ and $\chi_e \sim \chi_i \sim D \sim \epsilon^{3/2} \rho_i v_{T_i} / \delta^2 k_\theta L_{n}$. Again, $\bar{\omega}_d > \nu_{\text{eff}}$, so $\chi_e \leq \epsilon^2 \rho_i^2 v_{T_i}^2 / \delta^2 \nu_{\text{eff}, i} L_{n}^2$. In contrast to the case of $\eta_i \gg \eta_{\text{crit}}$, here $\bar{n}/n_0 > \bar{T}/T_0$ and $\chi_e \sim \chi_i \sim D$, as with DTICCs. In both cases, transport coefficients are moderate in magnitude and comparable to estimates derived from strong turbulence drift wave models. The unfavourable $T^{3/2}$ scaling of transport coefficients is not unreasonable, in the light of recent perturbative transport studies on TFTR [11].
It is interesting to note that because of their large dimensions, long wavelength cells are susceptible to shearing by differential toroidal rotation induced by neutral beam injection. Indeed, suppression can be expected when the shearing frequency \( \omega_s = (n/R)v_\phi^s d \approx (e/qs)v_\phi^s \) exceeds the decorrelation frequency \( \Delta \omega_k \). For DTICCs this occurs when \( v_\phi^s \gg v_{\parallel}^s/4\nu_{\text{eff}}e \approx L_n^2 \). Here again, the observation that the cells are large but slow moving is crucial to the viability of this prediction.

2.2. Trapped electron drift wave turbulence (\( \epsilon^{3/2}/q < k_\perp \rho_i < 1 \))

The rather heavily trodden path of trapped electron drift wave turbulence merits revisitation on account of the rather dubious ‘conventional wisdom’ which abounds in this topic. In particular, in collisionless trapped electron regimes the ever popular ‘i\delta’ model is severely flawed, and non-linear electron dynamics must, a priori, be treated on an equal footing with those of ions. Also, since ion and trapped electron Compton scattering are stronger non-linear transfer mechanisms than is mode coupling, ‘mixing length’ estimates of fluctuation levels and transport coefficients are likely to be overestimates [13]. For this reason, we revisit the theory of slab-like and toroidal [14] trapped electron mode turbulence. In the interest of physics understanding, emphasis is placed on the simpler slab-like case. However, it is interesting to note that standard methods for estimating diffusion, coefficients, etc. indicate that the transport driven by the slab-like mode exceeds that driven by its toroidal counterpart, despite the more robust linear instability of the latter.

The essential physics of slab-like trapped electron driven drift wave turbulence (TEDDWT) is determined by the ratios of the three fundamental scales \( \Delta = 1/k_\parallel \delta, \quad x_T = \rho_s \sqrt{L_n}, \quad x_i = (L_s/L_n)\rho_s \sqrt{T_e/T_i} \) (Fig. 2). These correspond, respectively, to the width of the trapped electron layer [15], the width of the convection cell, and the ion Landau damping point, where the outgoing wave is absorbed. The inequality \( x_T < x_i \) validates the fluid approximation to test mode dynamics, while at the same time rendering the spectrum averaged ballistic frequency comparable to the wave frequency, i.e. \( \langle k_\parallel v_T \rangle \sim k_\parallel x_T v_T \sim \omega \). As a result, the spectrum averaged beat wave resonance \( (\omega + \omega' = (k_\parallel + k_\parallel')v_T) \) is broad for TEEDWT, so that ion Compton

![FIG. 2. Typical scales in a slab-like drift mode.](image)
scattering is strong and can occur for infinitesimal amplitudes. This is in contrast to mode coupling, which requires finite broadening of the three-wave resonance to overcome the dispersion induced frequency mismatch at \( k_0 \rho_i \sim O(1) \). Also, the broad spectral extent enhances non-linear coupling coefficients. Finally, since non-linear transfer is determined by spatial spectrum averages, the inequality \( \Delta < \chi_i \) guarantees that non-linear electron effects (i.e. non-local electron Compton scattering to small scales) are subdominant to ion Compton scattering in the case of slab-like TEDDWT.

A detailed weak turbulence analysis [16, 17] of slab-like TEDDWT has been implemented for both dissipative and collisionless regimes. Detailed analysis indicates that the turbulence induced modifications to the eigenfunction structure are small [18]. Electron and ion non-linearity have been treated on an equal footing at the outset, but the observations that \( \Delta \ll \chi_i \) and that (shear damped) modes with \( k_0 \rho_i > 1 \) are feebly excited underly the conclusion that local ion Compton scattering to long wavelength is the dominant non-linear process. A detailed, quantitative summary of results is encapsulated in Table I. The resulting spectra have the form (Fig. 3) \( \langle \phi(k) \rangle = k_\alpha \), where \( \alpha = 2 \) and 3 for dissipative and collisionless regimes, respectively, decay rapidly for \( k_0 \rho_i > 1 \) and are cut off at long wavelength by shear damping at \( k_0 . \) Fluctuation levels and transport coefficients are considerably smaller than predictions derived from naive mixing length estimates, on account of the 'weak turbulence correction factors' (i.e. \( \gamma / \omega_i \)) and the \( k_\rho_i \sim 1 \) contributions to the coupling coefficients. As usual, \( \bar{n}/n \sim e \bar{\phi}/T < \bar{T}_e/T \) and \( \chi_e \sim \chi_i \geq D \). Interestingly, robust favourable major radius scaling appears, owing to the trapped electron layer localization effect. Also, favourable isotope scaling [19] is manifested. However, the latter occurs only in the deeply dissipative regime (\( \omega_r < \nu_{eff} \)) and is accompanied by a transition from \( \chi_e \sim 1/n \) to \( \chi_e \sim 1/n^2 \) scaling. Hence, its significance is unclear. Finally, as a direct consequence of the \( \exp(-R/L_n) \) dependence of the collisionless TEDDWT growth rate, transport is dramatically reduced in collisionless, peaked profile discharges.

Investigations of toroidicity induced TEDDWT are ongoing. Toroidicity induced modes differ from their slab-like counterparts in that poloidal sub-harmonics are standing waves, with radial extent of the spectrum \( \Delta_k \sim \frac{1}{k_0} \delta \Delta \theta_k \sim \Delta \). Moreover, ion Landau damping is completely negligible since radial wave energy flux is trapped and since \( k_0 V_{TI} \sim \omega_{TI} \ll \omega \). As a consequence, weak turbulence ion Compton scattering is not a strong process in toroidicity induced TEDDWT, and either renormalization of the beat wave resonance (since \( \Delta \omega \sim \gamma_n \gg k_0 V_{TI} \)) or investigation of mode coupling processes is called for. Moreover, since \( \Delta_k \sim \Delta \), electron Compton scattering effects are potentially important in collisionless regimes [20]. As a theoretical treatment of the collisionless regime entails analysis of a three-field turbulence problem (i.e. for \( \tilde{g} \), the non-adiabatic electron fluctuation along with \( \tilde{\phi} \) and \( \bar{\phi}_0 \)), including cross-correlations, we focus first on a simple, hydrodynamic model of the dissipative regime, where the non-adiabatic electron response is laminar. The model equations are:
TABLE I. PRINCIPAL RESULTS FOR SLAB-LIKE TRAPPED ELECTRON DRIFT WAVE TURBULENCE

<table>
<thead>
<tr>
<th>Quantities</th>
<th>Collisionless regime ((\omega &gt; \omega_{de} &gt; \nu_{\text{eff}}))</th>
<th>Dissipative regime ((\nu_{\text{eff}} &gt; \omega &gt; \omega_{de}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\bar{\gamma})</td>
<td>(\varepsilon_n^{3/2} \exp \left( -\frac{1}{\varepsilon_n} \right))</td>
<td>(\frac{c_s}{\nu_{\text{eff}} L_n})</td>
</tr>
<tr>
<td>(</td>
<td>\phi</td>
<td>^2(k_\parallel))</td>
</tr>
<tr>
<td>(\left(\frac{n}{n_0}\right)^2)</td>
<td>0.6 (\frac{T_i}{T_e} \left(A_{e0} \frac{\bar{\gamma}}{s} \right) G(k_\parallel) \left(\rho_i \right)^2 L_n)</td>
<td>same</td>
</tr>
<tr>
<td>(D)</td>
<td>0.4 (\left(\frac{T_i}{T_e}\right)^{3/2} \left(\frac{L_n}{L_\parallel}\right)^{1/2} \left(A_{e0} \frac{\bar{\gamma}}{s}\right)^2 F(k_\parallel) \frac{c \rho_i^2}{L_n})</td>
<td>same</td>
</tr>
<tr>
<td>(\chi_i)</td>
<td>0.8 (\left(\frac{T_i}{T_e}\right)^{3/2} \left(\frac{L_n}{L_\parallel}\right)^{1/2} \left(A_{e0} \frac{\bar{\gamma}}{s}\right)^2 G(k_\parallel) \frac{c \rho_i^2}{L_n})</td>
<td>same</td>
</tr>
<tr>
<td>(\chi_e)</td>
<td>0.8 (\left(\frac{T_i}{T_e}\right)^{3/2} \left(\frac{L_n}{L_\parallel}\right)^{1/2} \left(A_{e0} \frac{\bar{\gamma}}{s}\right)^2 H(k_\parallel) \frac{c \rho_i^2}{L_n})</td>
<td>same</td>
</tr>
<tr>
<td>(I(k_\parallel))</td>
<td>1 - (k_\parallel^3)</td>
<td>1 - (k_\parallel^2)</td>
</tr>
<tr>
<td>(G(k_\parallel))</td>
<td>(\frac{1}{k_\parallel^6} + \frac{5}{3} + \frac{\bar{\epsilon}}{3} \bar{k}_\parallel^{3/2})</td>
<td>(\ln \frac{1}{k_\parallel} - \frac{2}{3} + \frac{2}{3} \bar{\epsilon}_\parallel^{3/2})</td>
</tr>
<tr>
<td>(F(k_\parallel))</td>
<td>(\frac{5}{7} - \bar{k}<em>\parallel^3 + \frac{2}{7} \bar{\epsilon}</em>\parallel^{3/2})</td>
<td>(\frac{1}{15} - \frac{1}{5} \bar{k}<em>\parallel^3 + \frac{2}{15} \bar{\epsilon}</em>\parallel^{3/2})</td>
</tr>
<tr>
<td>(H(k_\parallel))</td>
<td>(\frac{1}{\bar{\epsilon}<em>n} \left[ \left(\frac{1}{\bar{\epsilon}<em>n} - \frac{3}{2}\right) G(k</em>\parallel) - \frac{3}{2} F(k</em>\parallel) \right])</td>
<td>(\frac{1}{\pi} \left[ 25 F(k_\parallel) + 9 \bar{k}<em>\parallel G(k</em>\parallel) \right])</td>
</tr>
<tr>
<td>(\bar{k}_\parallel)</td>
<td>(\frac{L_n}{L_\parallel} \frac{\bar{\gamma}}{A_{e0}^1} \propto \left(\frac{L_n}{L_\parallel}\right)^{1/2} \left(\frac{L_n}{L_\parallel} \frac{\bar{\gamma}}{A_{e0}^1}\right)^{1/4})</td>
<td></td>
</tr>
</tbody>
</table>
FIG. 3. Saturated spectra for slab-like trapped electron driven driftwave turbulence. The spectrum decays exponentially for $b_e < b^c_e$ and as a strong power law for $b_e > 1$.

\[ \frac{\partial \bar{n}}{\partial t} + v_x \frac{\partial \bar{n}}{\partial y} + \nabla_D \cdot \nabla \bar{n} + D_0 \frac{\partial^2 \bar{n}}{\partial y^2} \bar{n} \]
\[ + L_0 D_0 \nabla \left( \frac{\partial \bar{n}}{\partial y} \right) \times \vec{z} \cdot \nabla \bar{n} - \frac{d}{dt} \rho_2^2 \nabla^2 \bar{n} - \mu_\perp \left[ \nabla \cdot \bar{n} \right] + \nabla \bar{v}_1 = 0 \]
\[ \frac{\partial \bar{v}_1}{\partial t} - \rho_s c_s \nabla \bar{n} \times \vec{z} \cdot \nabla \bar{v}_1 - \mu_\parallel \left[ \bar{v}_1 \right] = - \frac{T_e}{m_i} \nabla \bar{n} \]

Here, $\mu_\perp$ and $\mu_\parallel$ are 'viscosity' operators which schematically account for the effects of ion Landau damping, and $D_0 = 1.5 \eta_e v^2/\nu_{eff}$. An interesting feature of this model is that spectral transfer is controlled by two non-linearities, namely the $\vec{E} \times \vec{B}_0$ advection non-linearity, similar to that encountered in the study of DTICCs, as well as the more familiar polarization drift non-linearity. Furthermore, the former tends to transfer energy to small scales, while the latter yields the well known inverse cascade. Dimensional analysis suggests that the advective non-linearity dominates spectral transfer for $k_{min} \rho_3 < k_0 \rho_3 < k_0 \rho_s$, where $k_0 \rho_s = 1.5 \delta^{1/2} c_s / \Gamma T_e \nu_{eff}$, while transfer for $k_0 \rho_3 < k_0 \rho_s$ is determined by the polarization drift. Thus, a very complex overall spectral flow is predicted. While analytical and computational investigations are as yet ongoing, preliminary results indicate that $\langle \phi(k_0)^2 \rangle \sim k_0^{-3}$ and that fluctuation levels are close to the predictions of mixing length theory.

2.3. Electron temperature gradient driven microturbulence

$(1 < k_\parallel \rho_i < (e^{3/2}/q) (m_i/m_0)^{1/2})$

There has recently been heightened interest in ultra-short wavelength ($k_\parallel \rho_i \gg 1$) microturbulence. In particular, the $\eta_e$-mode [21] which is similar to the $\eta$ mode but with ions adiabatic and electrons hydrodynamic, has been discovered and shown to
drive electrostatic fluctuations with $k_p \rho_e \leq 1$. However, practical interest in this class of instability is motivated by the expectation that fluctuation energy will inverse cascade to larger scales and thus drive magnetic fluctuations with correlation lengths $\Delta \sim c/\omega_{pe}$. As the viability of this scenario and the validity of the estimates of associated magnetic fluctuation induced transport remain highly controversial, it is natural to investigate other models of short wavelength turbulence. In particular, the observation that trapped ion modes are broader and more robust than short wavelength $\eta_i$ modes naturally suggests that short wavelength $\nabla T_e$ driven trapped electron modes will be more robust than $\eta_i$ modes. Such modes are possible when $1 < k_p \rho_i < (e^{3/2}/q) (m_i/m_e)^{1/2}$ and $\bar{\omega}_{de} > \nu_{eff,e}$, so that the trapped electron response is hydrodynamic. Note that, for $k_p \rho_i > 1$, access to the collisionless regime is easier than for standard, long wavelength, trapped electron modes. Also, such modes are not tied to miniscule scales where $k_p \rho_e \sim 1$. Rather, $\nabla T_e$ driven microturbulence has $\Delta \sim 1/k_d \delta$, with $k_d \rho_i > 1$. These modes are insensitive to parallel electron dynamics, since the non-adiabatic circulating electron response is small.

A theory of $\nabla T_e$ driven microturbulence has been developed [22], in which hydrodynamic trapped electron fluctuations are neutralized by adiabatic ions. The basic equations and theoretical model of this system are very similar in structure to that of the ITGDCCs, discussed in Section 2.1, with the roles of electrons and ions reversed. In particular, short wavelength $\nabla T_e$ driven trapped electron modes have the dispersion relation

$$\Omega^2 \left( \frac{1 + \tau}{\sqrt{2\epsilon}} - 1 \right) + \Omega \left( \frac{1}{\eta_e \epsilon_T} - \frac{3}{2} \right) + \frac{3}{2\epsilon_T} (1 + 1/\eta_e) = 0$$

where $\Omega = \omega/\omega_d$ and $\epsilon_T = L_T/R$. Thus, $\omega = F(\epsilon, \tau)\omega_d$, while $\gamma = G(\epsilon, \tau, \epsilon_T)\omega_d$, where $F$ and $G$ are functions determined from Eq. (10). Of course, the validity of the hydrodynamic approximation requires $\omega > \omega_d$, which is true for $\epsilon \geq \epsilon_{crit}$ and $\eta_e > \eta_{e crit} \sim 1-2$, for typical parameters. From the previous results one can predict that spectral transfer via mode coupling so small scales will occur, and that saturation levels are reasonably well approximated by mixing length estimates (i.e. $T_e/T_0 \sim 1/k_d \delta L_{Te}$). Hence, the electron thermal diffusivity due to $\nabla T_e$ driven microturbulence is given by

$$\chi_e = \frac{G}{F} \frac{\bar{\omega}_d}{(k_d \delta)^2} = \frac{G}{F} \frac{\rho_{de}^2 \nu_{Ti}}{s^2 R k_d \rho_i}$$

As the non-adiabatic ion response is smaller than that of the trapped electrons, $\chi_e \gg \chi_i$, etc. it should be noted that these results are all straightforward consequences of the simple physics of fluid-like electrostatic modes. Also, since trapped electrons are ubiquitous and since $\omega_d > \nu_{eff,e}$ is relatively easily satisfied at.
$k_{\perp} \rho_i \gg 1$, $\nabla T_e$ driven microturbulence can be expected in a wide range of tokamak operation regimes.

A particularly interesting feature of $\nabla T_e$ driven microturbulence may be noted by retaining the non-adiabatic ion response, which is dynamically irrelevant but provides the phase shift necessary for a non-zero particle flux. Straightforward quasi-linear calculations indicate that

$$\Gamma = - \frac{n_0 \nu_{Ti}}{2\sqrt{\pi}} \sum_k \frac{k^2 \rho_c c_s}{|k_{\perp}| \Delta \omega_k} \left( \frac{1}{L_n} - \frac{1}{L_{Ti}} \right) \left| \frac{e \vec{\phi}_k}{T_i} \right|^2$$

Here $\Delta \omega_k \approx \gamma_k$ is understood, and $e \vec{\phi}_k / T$ is given by mixing length theory. In particular, note that an inward particle pinch is driven by $\nabla n$. Also, since $\eta_e > \eta_{e\text{ crit}}$ is required for instability, peaked profiles quench the turbulence. Thus, a ‘negative diffusion’ catastrophe cannot occur. In view of the fact that other inward pinch mechanisms are limited to highly specialized parameter regimes [23], the pinch caused by $\nabla T_e$ driven microturbulence appears to be the most general model of this ubiquitous phenomenon.

3. SPECIAL TOPICS IN CONFINEMENT

3.1. Flat density discharges

Discharges with nearly flat density profiles are known to occur in H-mode plasmas [24]. Such discharges present a challenge to conventional microinstability theory in that most electron drift modes are quasi-marginally stable waves, with ion response dominated by $\nabla n$ driven diamagnetic drifts, which are then perturbatively destabilized by inverse electron dissipation. Thus, as $\nabla n \to 0$, the characteristic drift wave frequency changes from $\omega_e$ to $\omega_{Ti}$. Similarly, as $\nabla n \to 0$, $\eta_i$ modes with stability boundary $\eta_i = \eta_{i\text{ crit}}$, pass over to $\nabla T_i$ driven modes, with $\nabla T_{i\text{ crit}} = \nabla T_i(R, \delta, q_i, \tau \ldots)$.

Long wavelength convective cells have the generic dispersion relation

$$1 + \tau^{-1} - i \epsilon^{1/2} \frac{\omega_e}{\nu_{\text{eff},e}} \left( 1 + \frac{3}{2} \eta_e \right)$$

$$+ \epsilon^{1/2} \frac{\omega_{T_i}}{\omega} \left[ 1 - \eta_i \frac{k_{\perp}^2 \rho_i^2 \eta_i}{2} - i \frac{\nu_{\text{eff},i}}{\omega} \left( 1 - \frac{3}{2} \eta_i \right) \right] = 0 \quad (13a)$$

In the limit of flat density, long wavelength DTICCs, $(k_{\parallel} \rho_i) < \epsilon^{-1/8} (\nu_{\text{eff},i} / |\omega_{Ti}|)^{1/4} < 1$, still propagate in the electron diamagnetic direction, but are now destabilized by trapped ion collisional dissipation, i.e.
\( \omega \approx \exp(i\pi/4)e^{\frac{1}{14}} \left( \frac{3}{2} \left| \omega_{*T_i} \nu_{eff,i} \right| \right)^{1/2} \)  

At shorter wavelengths \( (e^{-1/8} (\nu_{eff,i}/|\omega_{*T_i}|)^{1/4} < k_\theta \rho_{bi} < 1) \), DTICCs propagate in the ion diamagnetic direction and are purely stable, i.e.

\[
\omega \approx e^{1/2} \frac{k_\perp^2 \rho_{bi}^2}{2(1 + \tau^{-1})} \omega_{*T_i}
- 3i \left[ \frac{\nu_{eff,i}}{k_\perp^2 \rho_{bi}^2} - \frac{\epsilon}{4(1 + \tau)(1 + \tau^{-1})} \frac{\omega_{*T_i}(\omega_{*T_i})}{\nu_{eff,e}} k_\perp^2 \rho_{bi}^2 \right]
\]  

Noting that unstable DTICCs must have \( \Delta \nu \leq e^{3/2}(\rho/L_T)n_{T_i}/(\nu_{bi}L_T) \) in the flat density regime, mixing length estimates indicate that

\[
\chi_i \sim \frac{\epsilon^3}{S^2 \nu_{bi} L_{ki}^2} \frac{c^2 T_i^2}{\epsilon^2 B^2}
\]  

for DTICC turbulence. As usual, for DTICCs \( \chi_i \sim \chi_e \sim 1/D \). In the case of ITGDCCs with \( \bar{\omega}_{bi} > \nu_{eff,i} \), the growth rate is given by \( \gamma \sim e^{1/4}(\omega_{*T_i}(1 + \bar{\eta}))^{1/2} \). Thus, as \( \nabla n \rightarrow 0 \), only \( \nabla T_i \) contributes to instability so that

\[
\gamma \sim e^{1/4}(\omega_{*T_i}(\bar{\omega}_{bi}))^{1/2}
\]  

and, from mixing length estimates,

\[
\chi_i \sim \frac{\epsilon^{9/4}}{S^2 \nu_{bi} (L_T R_3)^{1/2}} \frac{c^2 T_i^2}{\epsilon^2 B^2}
\]  

For ITGDCCs, \( \chi_i > \chi_e \sim 1/D \). At shorter wavelengths, flat-density \( \nabla T_i \) driven turbulence [25] can be enhanced by dissipative trapped electron \( \nabla T_e \) drive. For such modes, \( \chi_e > \chi_i \sim 1/D \). Similarly, \( \nabla T_e \) driven microturbulence, discussed in Section 2.3, is relatively insensitive to \( n \). Indeed, for half-radius DIII-D hot ion H-mode parameters \( (T_e \sim 2.6 \text{ keV}, S \sim 1, K_{\theta \rho_i} \sim 5) \), Eq. (12) predicts \( \chi_e \approx 1.1 \text{ m}^2/\text{s} \). Of course, for \( \nabla T_e \) driven microturbulence, \( \chi_e \gg \chi_i, D \). However, the observation that \( \chi_e > \chi_i, D \) in DIII-D hot-ion H-mode reveals that one must explain the apparent \textit{absence} of \( \nabla T_i \) driven modes as well as the nature of the electron transport mechanism. Apart from a suggestion that conditions of low \( T_e/T_i \) and high shear may decrease threshold values of \( L_{ki}/R \) [26], this mystery has not yet been quantitatively resolved.
Finally, it should be noted that interesting candidates for the electron transport mechanism exist outside the realm of drift wave models [27].

3.2. Dissipative trapped electron modes in stellarators

A stellarator is an ideal venue for experimental tests of the basic elements of trapped electron mode theory. In particular, by varying the vertical field on the ATF torsation, the value of $|B|$ along field lines, the trapped particle fraction, and the average curvature and magnetic shear can all be changed in a controlled fashion. Moreover, the connection length $L_q = R/M$, where $M$ is the number of field periods, so that $V_{Te}/L > v_e$ is easily satisfied for helically trapped particles. For this reason, and the fact that $T_i < T_e$ in ECH heated discharges, dissipative trapped electron modes are expected to be the dominant agent for transport in the core of the ATF torsatron.

Dissipative trapped electron mode (DTEM) stability is determined by two factors, namely the structure of the drift wave (i.e. ignoring non-adiabatic electrons) and the value of the bounce averaged potential $\langle \phi \rangle$. Generically, stellarator drift waves come in two varieties [28]. The first type of eigenfunction is helically extended (radially narrow), with $\langle \phi \rangle$ indeterminate since the periodicity of $\phi$ is different from the periodicity of $|B|$ (Fig. 4). The second type is helically localized, but radially broad, with $\langle \phi \rangle = \phi$ (Fig. 5). Thus, the latter can be expected to drive significant transport, and is of considerable interest. Detailed calculations [29] for ATF equilibria indicate that negative quadrupole field results in relatively flat rotational transform (i.e. $\epsilon' = 0$), so that shear is weak and modes are radially broad and helically localized (Fig. 6(a)). DTEMs localized at half-period and $\theta = 0$ are marginally stable in the absence of non-adiabatic electrons, and are strongly destabilized by retaining the latter (i.e. noting $\langle \phi \rangle = \phi$). On the other hand, for positive quadrupole fields,
\( t' \approx 0 \), so that shear is strong and DTEMs are helically extended and radially narrow, with \( \langle \phi \rangle \ll \phi \) (Fig. 6(b)). Moreover, the fraction of trapped particles confined is high for negative quadrupole field, but decreases as the field increases (Fig. 7). Thus, if the widely held hypothesis that trapped electron modes control electron-channel transport (in the absence of \( \eta_i \) modes) is valid, a significant change in core fluctuation levels and transport in ATF should be apparent upon variation of the quadrupole field. Related experimental studies are in progress.

4. BASIC THEORY OF MULTIFIELD MODELS OF DRIFT WAVE TURBULENCE

The aforementioned difficulty in constructing analytical models of collisionless trapped electron drift wave turbulence is due to the fact that both electron and ion non-linearity compete on an equal footing in that system. Moreover, cross-correlations such as \( \langle \tilde{\phi} \tilde{g} \rangle \) are non-negligible and may be dynamically significant. With this in mind, we have undertaken a research programme aimed at the study of the basic theory of multifield models of drift wave turbulence. In particular, the Hasegawa-Wakatani [30] model of drift wave turbulence is used as a simple paradigm for the basic physics issues involved in multifield plasma turbulence. We focus on the use of integral invariants of a dissipationless system as a means of predicting trends, on the dynamical effects of the cross-correlation between the multifield quantities, and on the role of coherent structures and their use for characterizing the system. A pseudo-spectral numerical simulation code solves the model equations, which consists of separate evolution equations for the density fluctuation and the potential fluctuation \( \phi \), where the non-linear terms arise from the \( \vec{E} \times \vec{B} \) con-
vection. The term $\alpha = \chi_e k^2 / |\omega_k + i\Delta\omega_k|$ is the adiabaticity parameter which describes the degree of linear coupling between the $n$ and $\phi$ equations with a limiting case of $\alpha \rightarrow \infty$ for adiabatic electron response.

In the absence of forcing and dissipation, this system has four dynamical invariants — energy in either of the $n$ and $\phi$ fields, entropy for the $\phi$ field, and cross-spectra $\langle \tilde{n} \nabla^2 \tilde{\phi} \rangle$. However, in contrast to the Hasegawa–Mima system, which closely resembles a two-dimensional fluid (with $\tilde{n}/n = e\tilde{\phi}/T$), energy and entropy are not simply related (as in a 2-D fluid). The methods of statistical mathematics may be used
to analytically determine the equilibrium spectra [31]. These results are reproduced by the numerical simulations. Furthermore, conservation of $\langle \nabla \mathbf{\nabla}^2 \phi \rangle$ constrains spectral evolution of the $n$ field. This effect was predicted theoretically by a closure analysis [32]. Figure 8 shows how the timescale $T_n$ of $n$-field entropy relaxation varies as a function of the initial cross-correlation $\eta$ between the $n$ and $\phi$ fields.

Simulations of the driven/damped Hasegawa-Wakatani system show varying character as a function of the adiabaticity parameter $\alpha$. In particular, for small values of $\alpha$, the density field contains many dipole vortex structures in the steady state. The structures disappear as $\alpha$ is increased.

5. CONCLUSIONS

The principal conclusions of this paper, i.e.

(i) the two results that

(a) long wavelength convective cell transport is not predicted to be 'catastrophic', and

(b) trapped electron drift wave turbulence is weak, with transport considerably lower than mixing length predictions,

together suggest that trapped ion convective cells are a significant agent in tokamak transport and thus merit reconsideration. This conclusion underscores the need for beam emission spectroscopy studies of long wavelength fluctuations;
(ii) non-linear transfer from saturated, long wavelength convection cells is likely to be a significant drive for short wavelength fluctuations and, ultimately, results in non-negligible ion heating;

(iii) short wavelength $\nabla T_e$ driven trapped electron modes are a promising, novel mechanism for explaining the inward particle pinch;

(iv) the flexible magnetic configuration offered by stellarators allows useful critical tests of drift wave turbulence theory by experimental control of the trapped particle fraction and the mode structure (through the magnetic shear, manipulated by the quadrupole field);

(v) cross-correlations play a crucial role in the dynamics of multifield models of drift wave turbulence and are an essential element in any theory thereof.

In addition, some shortcomings of trapped particle mode theory remain unresolved.
These include:

(i) the radial profiles of the predicted $\chi_c$, $\chi_i$ (which should increase with radius, but do not) and the nearly universal observation of favourable $I_p$ scaling (which is not manifested in the theory);

(ii) an understanding of ion thermal transport in flat density, hot ion H-mode plasmas.

The resolution to (i) may result from a better understanding of edge plasma turbulence. In particular, neoclassical drift waves and MHD modes, which are the plateau regime 'relatives' of trapped ion modes, offer significant promise in this context [27]. The resolution to (ii) requires a much better understanding of $\nabla T$, driven trapped ion mode dynamics at and near threshold.

ACKNOWLEDGEMENT

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REFERENCES

M.N. ROSENBLUTH: It is true that, while not 'catastrophic' in the mathematical sense, nevertheless the long wavelength TPM (trapped particle mode) predicts a $T^{7/2}$ variation of diffusivity which might be catastrophic for fusion confinement?

H. BIGLARI: It is true that the diffusion coefficients associated with long wavelength trapped ion convective cells generically exhibit an unfavourable $T^{7/2}$ scaling. While this is hardly comforting, I would not go so far as to characterize it as 'catastrophic' for fusion confinement. Let me draw your attention to the recent controlled perturbative transport studies carried out by Efthimion and colleagues on TFTR where, for the first time, care was taken to hold density profiles similar for different input powers in order to determine the temperature scaling of the diffusion coefficient. In these discharges, for which $\omega_{ce} \sim \nu_{\text{eff,e}}$, it was found that the diffusion coefficient scaled as $T^{5/2}$. Thus, a $T^{7/2}$ prediction, particularly in the dissipative trapped electron regime, i.e. $\nu_{\text{eff,e}} < \omega_{ce}$, is not unreasonable.
COMPARISON OF SIMULATIONS AND THEORY OF LOW-FREQUENCY PLASMA TURBULENCE

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Abstract

COMPARISON OF SIMULATIONS AND THEORY OF LOW-FREQUENCY PLASMA TURBULENCE.

A combination of computational and analytic methods is used to study low-frequency turbulence and turbulent transport in a strongly magnetized plasma. Two major computational efforts, one based on gyrokinetic-particle simulation and the second on numerical solution of closure approximations to

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fluid equations, are described. These codes are used to study instabilities on the drift timescale and to assess the validity of qualitative predictions of energy-transport scalings based on analytic versions of closure approximations. Preliminary analysis of the closure code results indicates that the off-diagonal components of both the correlation matrix and the Green function can have non-negligible effects. However, a Markovian simplification may be useful provided that these off-diagonal terms are consistently incorporated. Several dimensional-analysis-based scaling laws have been obtained with Hasegawa-Wakatani turbulent transport decreasing with increasing electron adiabaticity (in agreement with direct solution of the fluid equations), and with $\eta$-driven transport decreasing with shear.

The chaotic nature of self-consistent turbulence and turbulent transport precludes direct analytic solutions and suggests instead the use of statistical approaches, which require a "closure" theory. The error associated with the various closure theories has defied theoretical analysis, however, and this leads to a demand for direct simulation. Here the kinetic nature of the low-frequency plasma instabilities, requiring description in five-dimensional phase space, together with the disparate timescales $(\Omega_i / m_i / t_e k_i / k_L \gg \omega_e \gg \tau_{\text{trans}}^{-1})$ also presents formidable difficulties. In this paper, we describe two major computational efforts, one based on gyrokinetic-particle simulation and the second on numerical solution of closure approximations to fluid equations. The codes are applied to problems of drift-wave and ion-temperature-gradient-driven (ITG) turbulence and used to assess the validity of the closures and of qualitative predictions of energy-transport scalings based on analytic approximate versions of the closures. We also report advances in dimensional-analysis-based scaling-law construction and compare to code results.

We first examine the application of the gyrokinetic formalism for the Vlasov-Poisson system to particle simulation [1,2]. We note that it is sufficient to solve the gyrokinetic equations of motion and the self-consistent field equations through leading order in $\epsilon \equiv \rho_i / L_n$. This coupled set admits an energy conservation law through second order in $\epsilon$. Higher-order-in-$\epsilon$ effects, such as the "non-linear polarizability" and the orbital ponderomotive term, included for exact energy conservation of the truncated-in-$\epsilon$
system [3], are of little use in simulations and we generally omit them. We have, in addition, extended the formalism to the $k_L L_n \sim 1$, $e\phi \sim T$ regime and demonstrated that the fundamental small dynamical quantity is $v_{x,B}/v_{th} \sim \epsilon$.

We next describe our two-dimensional, slab (sheared or tilted field), electrostatic, particle-in-cell code. A significant number of improvements have been made over Ref. 2: a five- to ten-fold increase in speed has been achieved by carefully vectorizing [4] all important subroutines and applying electron subcycling [5] (the more costly gyrokinetic ions are advanced less often with a bigger time-step than are the electrons and the field solve). Two other multiple-time-scale methods, orbit averaging [6] and semi-implicit (to ensure the numerical stability of extending the field-solve time-step) particle methods [7], have been devised for gyrokinetics and analyzed and are being implemented for electrostatic and finite-beta models. Orbit averaging moves the field solve from the fast, particle time-scales to the low-frequency time-scale of the drift waves; also, by time-averaging the charge and current densities from the electrons or both ions and electrons, orbit averaging can reduce the particle statistical requirements. To reduce code run-times further, a three-dimensional electrostatic gyrokinetic code is being ported to two massively parallel computers with differing architectures (the Connection Machine CM-2 and the BBN TC2000). Physics enhancements of the code include a heat-source routine for maintaining temperature gradients in a bounded simulation (the heat source leaves the local particle density unchanged); multi-component ion distribution functions in the particle loader and heat source to address ITG stability in non-Maxwellian plasmas; the formulation of a momentum- and energy-conserving binary-collision algorithm for finite ion gyroradius to study the effects of collisions on linear stability and steady-state transport; and gyrokinetic-fluid-moment diagnostics to check the assumptions made in fluid theories and the further approximations made in the course of applying statistical closures to the fluid equations.
We have begun a suite of electrostatic gyrokinetic simulations of universal modes and ITG instabilities. Our first simulations have replicated earlier simulation work [8,9] with the same physical parameters, but with improved grid resolution, smaller time-steps, and more particles per cell. The results of the simulations differ very little quantitatively from the earlier published results. These first simulations addressed small systems with relatively strong gradients, e.g., $\rho_i/L_m = 0.05$. We are extending the $\eta_i$ simulations to a range of $\eta_i$ (2 $\leq \eta_i \leq$ 4), larger systems, and systems with weaker gradients. These simulations of $\eta_i$ modes in larger systems differ qualitatively from the earlier work in small systems [9]: instead of mode coupling to long-wavelength, large-scale coherent structures with nearly stationary velocity vortices at saturation, the simulations with larger box sizes exhibit a turbulent saturation with a few coherent structures which move freely in the simulation box and transport that saturates but does not relax to a lower value as do the smaller systems [9].

An electrostatic model in three-dimensional toroidal geometry [10] has been used to study the nonlinear evolution of collisionless trapped-electron drift instabilities in the regime $\nu_{eff} < \tau_{ce}^{-1}$, $\tau_{bi}^{-1} < \omega$. Here the effective collision frequency $\nu_{eff} \sim \nu_{el}(R_o/r)$, and the bounce time $\tau_{be} \sim (R_o/r)^{1/2}(qR_o/v)$. For the case $\omega, \omega_D \gg \nu_{eff}$, the imaginary part of the frequency arises from the resonance $\omega \simeq \omega_D$, where $\omega_D = \omega_{ce}L_m/R$ is the electron curvature-drift frequency. Finite temperature gradients are found to have a significant effect on both the trapped-electron destabilization and on the mode structure. Extensive linear analysis has been done on these instabilities in both slab and toroidal geometry and these calculations are used as a guide to the parameter regime used for the simulation [11-16]. The simulation model has been benchmarked against previous simulation attempts using a full-ion-dynamics model and drift-kinetic electrons [17] and excellent agreement is found for the same parameters.
For this study, a cylindrical plasma model is used for the equilibrium magnetic field and the initial plasma profiles. The initial plasma density and electron temperature go to zero at $r = a$, and the initial ion temperature is uniform in space; the gradients are not maintained. The $q$-profile is parabolic in $r$, and a $1/R$ variation in $B_z$ provides the mirror field for the trapped particles. The linear growth phase of the collisionless trapped-electron drift instability shows semi-quantitative agreement with linear theory for the real and imaginary parts of the frequency and the mode structure. The radial mode structure in this weak-toroidal-coupling regime is predominantly slab-like, but as the inverse aspect ratio and number of toroidal harmonics is increased the normal modes become more toroidal (standing wave type along the magnetic field). The modes are localized toward the outer edge of the torus, where the trapped electrons predominate. The instability mechanism is inverse Landau damping via the toroidal drift precession of trapped electrons, and saturation occurs principally through quasi-linear profile-modification as well as mode coupling. The electron thermal diffusion is about 5-6 times the particle diffusion; the measurements were made both by using cross correlations of the density and electrostatic potential and by following the radial guiding-center displacements of test electrons in the simulation.

A series of simulations employing a hybrid electron model is used to investigate quantitatively the effects of scattering between passing and trapped particles which arise from the suprathermal level of fluctuations excited by the drift instability. Also, simulation time-step restrictions due to the circulating electrons are removed with this model. The circulating electrons are taken to be adiabatic and the trapped electron population is advanced with perpendicular drifts including a bounce-averaged toroidal drift with the appropriate magnetic-moment and shear dependence. Comparisons between the hybrid and the full-electron-dynamics model indicate that velocity-space
scattering across the trapped/passing separatrix has only a weak effect on the saturation and post-saturation phase of this instability.

The second computational effort centers on statistical closure approximations. Simple models of drift-wave turbulence in two dimensions can reproduce many of the characteristics of turbulent fluctuations. We study a two-field model of drift-wave turbulence that allows for a nonadiabatic electron response [18,19]. The model consists of evolution equations for the density fluctuation \( n \) and the potential fluctuation \( \phi \):

\[
\begin{align*}
    \left( \frac{\partial}{\partial t} + v_{exB} \cdot \nabla \right) \nabla^2 \phi &= -\chi_s \nabla^2 (\phi - n) + \mu \nabla^4 \phi \tag{1a} \\
    \left( \frac{\partial}{\partial t} + v_{exB} \cdot \nabla \right) n &= -\chi_s \nabla^2 (\phi - n) + \nu \nabla^2 n - \kappa \frac{\partial \phi}{\partial y} \tag{1b}
\end{align*}
\]

where \( v_{exB} \equiv \vec{E} \times \nabla \phi \), \( \mu \) and \( \nu \) are damping coefficients, \( \kappa \) is an inverse scale length for the unperturbed density profile, and \( \chi_s \) is the parallel electron conductivity [18].

Two-point closure models of Eqs. (1) provide us with a means for directly estimating statistical observables of the turbulence. A primary purpose of the present work is to evaluate the full (i.e. unabridged) Direct Interaction Approximation (DIA) as a means for obtaining spectral quantities. Previous studies of this type have been limited to closure models that were much simpler than full DIA and to simpler physics models such as a one-field limit of Eqs. (1) [20]. We evaluate the closure predictions by comparison with statistics from the direct simulation of Eqs. (1).

The closure code solves the full, anisotropic DIA equations for the two-point correlation functions \( U \) and the response functions \( G \). The equations for \( t > t' \) are:

\[
\begin{align*}
    \frac{\partial}{\partial t} U^k_{ij}(t, t') &= L^k_{ij} U^k_{ij}(t, t') + \int_0^t S^k_{ikj}(t, s) G^k_{kj}(t', s) ds + \int_t^\prime R^k_{ikj}(t, s) U^k_{kj}(s, t') ds \\
    \frac{\partial}{\partial t} G^k_{ij}(t, t') &= L^k_{ij} G^k_{ij}(t, t') + \int_t^\prime R^k_{ikj}(t, s) G^k_{kj}(s, t') ds
\end{align*}
\]

where \( L \) is a matrix of linear terms and \( i, j \) index the field species (e.g. \( U^k_{11}(t, t') = \))
\[ \langle \phi_k(t)\phi^*_k(t') \rangle \], with subscript summation implied. The DIA form for triple correlations is framed in terms of the convolutions

\[ R^k_{ij} = 4\sum_{p,q} M^{kpq}_{ijkl} M^{qpk}_{abj} G^p_{la} U^p_{kb}, \]

\[ S^k_{ij} = 2\sum_{p,q} M^{kpq}_{ijkl} M^{qpk}_{jik} U^p_{ka} U^q_{lb}, \]

where the \( M \)'s are symmetrized coupling coefficients for Eq. (1). The convolutions are efficiently computed using the Fast Fourier Transform. Time stepping is organized to avoid the \( N_t^2 \) scaling of work traditionally encountered in solving such non-Markovian problems (where \( N_t \) is the number of time-steps).

We first focus on the nonlinear coupling by removing driving and damping from the dynamical equations and studying their relaxation. Recent theoretical predictions indicate that relaxation of the density spectrum is inhibited by the presence of a finite cross-correlation \( \langle \nabla^2 \phi \rangle \), an invariant of the equilibrium system [23, 22]. This result is confirmed by both the direct numerical simulations and by the DIA code results. Figure 1 shows relaxation of the density spectrum \( \langle |n_k|^2 \rangle \) as predicted by the DIA model for a particular band centered on wavenumber \( k = 7 \). The parameter \( \eta \) specifies

![Figure 1. Relaxation of density spectrum for modes with \( k = 7 \), showing the effects of varying the cross-correlation, as predicted by the DIA.](image-url)
FIG. 2. Comparison of two-time correlation function $\langle n(t) n(t') \rangle$ with prediction of fluctuation/dissipation relation. (a) $\alpha = 2$ and (b) $\alpha = 0.03$. 

(a) DIA Prediction of $U(t,t')$

F-D Prediction of $U(t,t')$, based on full DIA Response Functions

Off-Diagonal Contribution

Diagonal Contribution (simple Markovian prediction)

(alpha = 2.00)

Normalized Time-Lagged Density Correlation

Lag Time

(b) DIA Prediction of $U(t,t')$

F-D Prediction of $U(t,t')$, based on DIA Response Functions

Diagonal Contribution (simple Markovian prediction)

Off-Diagonal Contribution

(alpha = 0.03)

Lag Time
the degree of cross-correlation. One important consequence of this result is that closure approximations based on simple extensions of one-field models require modification to include the additional physics associated with cross-correlation effects.

A common device for construction of Markovian closure models is use of a fluctuation/dissipation relation: \( U^k(t, t') = G^k_{ij}(t, t') U^j(t', t') \). This relation holds for the equilibrium system and may approximately hold for driven/damped systems as well [21]. Our studies indicate that such a relation does indeed hold to a good approximation in the energy-containing wavenumber subrange. This is illustrated for a particular \( k \)-vector in Fig. 2. Note that neglect of the off-diagonal contribution [i.e. cross-correlation effects: for example \( G^k_{01}(t, t') U^j(t', t') \)] can lead to a gross error. In such a case, one expects that Markovian models based on exponential or single-pole approximations for relaxation times are inadequate.

Figure 3 shows the rapid growth of the density/vorticity angle-averaged cross-correlation spectrum, initially set at zero. There is good agreement between DIA

![Graph showing growth of density/vorticity angle-averaged cross-correlation spectrum](image)

**FIG. 3.** Growth from zero of density/vorticity angle-averaged cross-correlation spectrum. The simulation results have been averaged over an ensemble of 100 runs.
predictions and the ensemble-averaged results of the simulation code. Particularly for this case of initially vanishing correlation, the ensemble-averaging is essential, since the early-period growth in any one realization is dominated by detailed phase relations in the random initial density and potential fields.

Finally we discuss the application of transformation-based scaling laws to ITG and Hasegawa-Wakatani turbulence. Connor and Taylor [24] described a procedure for determining transport-coefficient scalings from invariance properties of nonlinear equations. The procedure is equivalent to finding variable transformations that remove the maximum number of parameters from the system. Connor and Taylor assume the equations to be invariant to transformations of the form \( y_j \rightarrow y_j \lambda^{p_j} \), and the transport coefficients are assumed to depend on the parameters \( \alpha_j \) as sums of products \( \Pi \alpha_j^{q_j} \). The determination of the transformations and scalings reduces to linear algebraic equations for the \( p \)'s and \( q \)'s. Hence the entire procedure can be automated, as we have done using Mathematica. Note that simplifying the nonlinear equation set by neglecting terms tends to reduce the number of unknown parametric dependences. The automated procedure allows us to quickly scan such simplifications and, in the case of a completely determined scaling, determine \textit{a posteriori} validity criteria for the approximations.

We have applied this program to the fluid \( \eta_i \) equations studied by Connor [25], who obtained scalings by approximating \( \phi \ll p \), neglecting parallel compression in the pressure equation, and dropping either \( \partial \phi / \partial t \) or \( \partial \phi / \partial y \) in the continuity equation, corresponding, respectively, to \( Ks \gg 1 \) or \( Ks \ll 1 \); here \( K = (1 + \eta_i)T_i/T_s \), \( s = L_n/L_s \), \( L_s \) is the shear scale-length, and \( \phi \) and \( p \) are normalized density and pressure fluctuations. The resultant scalings for the thermal diffusivity \( \chi_i \) are \( \chi_i \propto (\rho_s^2 c_s/L_n)K^{3/2}s \) and \( (\rho_s^2 c_s/L_n)K^{3/4}s^{1/4} \), respectively, where \( c_s \) is the sound speed and \( \rho_s \equiv c_s/\Omega_i \). As noted by Connor, the former scaling agrees with that obtained by Lee and Dis-
mond [26]. Motivated by the linear dispersion relation and by observations of radially extended structures ("streamers") in fluid simulations [27], we have examined a different set of approximations: \( \partial^2 / \partial x^2 \ll \partial^2 / \partial y^2 \) (\( x \) and \( y \) denote lengths in the radial and poloidal directions, respectively), and either (a) \( \partial \phi / \partial y \ll \partial \phi / \partial t, \phi \sim p \), or (b) \( \phi \ll p \) and \( \nabla_y \psi \ll \partial p / \partial t \). The scalings are \( \chi_i \propto (\rho_s^2 c_s / L_n)K^3 s^{-2} \) and \( (\rho_s^2 c_s / L_n)K^{-3/2} s^{-2} \), respectively, and the \textit{a posteriori} validity criteria are \( K > 1 \) and \( K/s \gg 1 \) for (a), and \( K \gg 1 \) and \( K s \ll 1 \) for (b). Note that both scalings decrease with increasing shear, in agreement with the arguments of Hamaguchi and Horton [28]. The significance of the overlap of the validity criteria for approximations (a), (b) and Connor’s \( K s \ll 1 \) limit, all of which appear to be reasonable in the context of the linear dispersion relation, is not completely resolved. However, it suggests the possibility of multiple solutions of the nonlinear equations, and raises questions about the sensitivity of renormalized turbulence treatments to simplifying assumptions which entail dropping seemingly innocuous terms.

We have also applied the scaling procedure directly to the electrostatic, slab gyrokinetic equations. Without further approximation, we find that \( L_s \) times the perturbed gyrokinetic distribution function scales with \( s, \eta_i \) and \( \eta_e \) only through the combinations \( \kappa_i / s, \kappa_e / s \), where \( \kappa_j = 1 + [(m_j v^2 / 2T_j) - 3/2] \eta_j \); the \( \eta_e \) dependence of course disappears in the adiabatic-electron limit. In the large-\( \eta_i \) limit, we further find that the scaling of all quantities is determined up to unknown functions \( F \) of \( \eta_i / s = L_s / L_T \) and \( T_e / T_i \); \( L_n \) drops out, as is well known. [The case (a) streamer scaling described in the previous paragraph is recovered from the gyrokinetic equations when the additional assumption \( \partial^2 / \partial x^2 \ll \partial^2 / \partial y^2 \) is made.] In the 2D shearless limit, \( s \) is replaced by \( L_n b / \rho_s \), where \( b = B_y / B \) is the tilt of the magnetic field relative to the symmetry direction; for example, the potential at saturation is predicted to scale as \( bF(\rho_s / L_T b, T_e / T_i) \). We
FIG. 4. Saturated root mean square amplitudes of the density \( n \) and nonadiabatic portion \( g \). Solid lines illustrate small- and large-\( \alpha \) predictions of scaling theory.

have tested this scaling with the shearless gyrokinetic code, establishing that easier-to-simulate large-gradient results can be scaled to obtain solutions for the smaller gradients of interest. We hold \( \rho_s/L_T b \) fixed while varying \( L_T \) and \( b \) by a factor of two, and find, for those mode amplitudes that are above the noise, for the power spectrum, and for the time-evolution of the run, agreement with the predicted scalings to within the simulation error bars.

To apply the scaling procedure to the Hasegawa-Wakatani equations (1), we change variables from \((n, \phi)\) to \((g \equiv n - \phi, \phi)\). After dropping \( \mu \) and \( \nu \), without further approximation one obtains scalings up to arbitrary functions of \( \alpha \), where \( \alpha \equiv \chi \alpha^2 / \kappa \); e.g., the flux \( \Gamma \propto \kappa^2 \alpha^2 F(\alpha) \). Then we consider two approximations: (a) \( \kappa \alpha g \ll \partial g / \partial t \), or (b) \( g / \phi \ll 1 \) in Eq. (1b). Approximation (a) is valid for \( \alpha \ll 1 \), \( \mu \alpha^{1/3} / \kappa \ll 1 \) and \( \nu \alpha^{1/3} / \kappa \ll 1 \), and leads to the prediction that \( g / \kappa, \phi / \kappa, n / \kappa \) (r.m.s. saturated...
amplitudes), $x$, $y$ (fluctuation scale-lengths), and $\Gamma/\kappa^2$ all scale as $\alpha^{-1/3}$, while (b) is valid for $\alpha \gg 1$, $\mu/\kappa \ll 1$ and $\nu/\kappa \ll 1$, and predicts that $x$, $y$, and $\phi/\kappa$ do not scale ($\propto \alpha^0$), while $\Gamma/\kappa^2$ and $g/\kappa \propto \alpha^{-1}$. We have numerically integrated Eqs. (1) for $\alpha$ spanning the range 0.01 to 2, and find (see Fig. 4) reasonably good agreement between the numerical and analytic [regime (a)] scaling, particularly for the flux and the saturated amplitudes, with indications of a transition to regime (b), particularly in the r.m.s. level of $\phi$, $g$, and so on, for $\alpha \geq 0.5$. The numerically obtained fluctuation lengths scale more weakly than predicted; furthermore, the density and potential fluctuation lengths differ, a result not predictable at this level of scaling analysis.

REFERENCES

SELF-CONSISTENT MAGNETIC CHAOS
INDUCED BY ELECTRON TEMPERATURE GRADIENT

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Abstract

SELF-CONSISTENT MAGNETIC CHAOS INDUCED BY ELECTRON TEMPERATURE GRADIENT.

Two mechanisms for the self-sustainment of magnetic islands are studied in cylindrical geometry. The first one is based on the different behaviour of electrons and ions in the presence of the islands as a result of their different Larmor radii. This mechanism could maintain magnetic turbulence resulting from a mixture between islands of the size of the ion Larmor radius and chaotic regions. The second mechanism is a pseudo-gravity, used here as a simple analogue for pressure gradient/field curvature modes. It could sustain islands much larger than the ion Larmor radius.

1. INTRODUCTION

There is no agreed explanation of the observed particle and energy losses in tokamaks. One possible cause for these losses is magnetic turbulence, which allows transport along chaotic field lines linking different regions of the plasma [1,2]. The magnetic turbulence could result from a mixture of small islands and chaotic regions [3].

This paper investigates two mechanisms for the self-sustainment of magnetic islands in a collisionless plasma in the cylindrical case. The first one results from the different response of electrons and ions to the islands due to their different Larmor radii. This effect requires a minimum threshold, since it is dependent on the presence of magnetic chaos [4]. The second one is a
pseudo-gravity, which is here used as a simple analogue for pressure gradient/field curvature modes. The electron and ion drift velocities, associated with the pseudo-gravity, combine to produce the current sustaining the island. As this mechanism does not depend on the existence of magnetic chaos, it has no threshold. These effects have been also studied for non-linear microtearing modes with a transverse electron diffusion [5].

The magnetic topology is first defined: it consists of islands separated by nested magnetic surfaces or by a chaotic region. Quasi-neutrality is imposed both inside and outside an island to determine the perturbed potential. This potential produces a net diamagnetic current and the divergence of this current drives an electron current along the field lines thus maintaining the island. Finally, Ampère's law for the island leads to the self-sustainment condition.

2. MAGNETIC TOPOLOGY

The calculation is made in a slab geometry with the coordinates \( x = r - r_s \) and \( y = r_s (\Theta - z/q_s R) = \Theta/k_y \), where \( r, \Theta \) and \( z \) are the cylindrical coordinates. \( r_s \) is the radius of the resonant surface, \( R \) is the major radius of the plasma and \( q_s \) is the safety factor at \( r = r_s \). The poloidal wave number is \( k_y = m/r_s \), where \( m \) is the poloidal mode number.

The overlapping parameter \( \gamma \) is the ratio of the virtual island width \( 2\varepsilon \) to the distance between two island chains \( \Delta = 1.5q^2/q'\delta m \), where \( q' \) is the shear and \( \delta m \) is the range of
(a) Poincaré map computed for an overlapping parameter $\gamma = 1.05$, showing magnetic islands in equilibrium in a chaotic region; the island is defined by its poloidal extension $2\Theta_0/k_y \leq 2\pi/k_y$ and its radial width $2b_0$. $\Delta$ is the distance between two island chains. (b) The chaotic zone modelled by nested magnetic surfaces to define the vector potential outside the island.

For $\gamma < 0.75$ the topology consists in islands separated by nested magnetic surfaces, for $0.75 < \gamma < 1.50$ the islands are embedded in a chaotic zone and for $\gamma \geq 1.50$ the system is fully chaotic [6]. Fig.1-a shows part of a Poincaré map computed by integrating the field line equations with $\gamma = 1.05$. The island is defined by its poloidal extension $2\Theta_0/k_y \leq 2\pi/k_y$ and its radial width $2b_0 \leq 2c$. It is assumed to be thin, that is $k_y b_0 \ll 1$.

A first integral does not exist in the chaotic zone shown in Fig.1-a. However, the vector potential needs to be defined in this
region to calculate the effect of both mechanisms investigated here. This is done by modelling the region outside the island by nested surfaces as represented in Fig.1-b. This defines an approximate magnetic flux $\mathcal{A}^*_z$, which is only used to calculate the perturbed electric potential imposed by the presence of the island:

$$
\begin{align*}
\mathcal{A}^*_z(x, y) &= \frac{B'}{2} x^2 + \frac{B}{k_y} A(x, y) = \left(-\frac{B}{k_y}\right) A(x, y) \\
A(x, y) &= 2 \frac{x^2}{b_0^2} - A(x, y)
\end{align*}
$$

where $B' = (r_s q'(r_s)/R q_s^2) B_z$ is the shear factor, $B_z$ being the toroidal field, and $\tilde{B}$ is the amplitude of the perturbing radial field. From symmetry, $A(x, y)$ is an even function of $y$ with a period $2\pi/k_y$. It is independent of $x$ and equal to $a(y)$ inside the island. The last closed surface of the island is defined by $\mathcal{A}^*_z(x, y) = 0$ and its radial coordinates are given by:

$$
b(y) = b_0 \sqrt{a(y)/2}
$$

3. QUASI-NEUTRALITY CONDITION

The particles have a charge $q_j$ ($j = e, i$) and are assumed to experience a fictitious gravitational potential $\phi_{Gj}$, which is given by:

$$
\phi_{Gj} = -G_j x \quad \text{with} \quad q_j G_j > 0
$$

For $\gamma > 0.75$, quasi-neutrality is ensured in the chaotic region by equating the electron to the ion flow. This leads to the
radial electric field $E_0$ in the reference frame rotating with the islands:

$$E_0 = \frac{Q}{K T_e} \left( \frac{n_e'}{n_e} + \frac{1}{2} \frac{T_e'}{T_e} \right) - G_e$$

(4)

$K$ is the Boltzmann constant. $n_e'$ and $T_e'$ are the average gradients of electron density $n_e$ and temperature $T_e$.

When the ion Larmor radius $\rho_i = \sqrt{m_i K T_i / q_i B_z}$ is of the order of the island half-width $b_0$, the potential $\phi_i$ experienced by the ions can be expressed as:

$$\phi_i = -E_0 x + b_0 (E_0 + G_i) \int_{-\infty}^{+\infty} G(x-x') \tilde{\varphi}(x',y) dx'$$

(5)

The perturbed potential $\tilde{\varphi}(x,y)$ is dimensionless and the integral containing $G(x-x')$ is the finite ion Larmor radius operator. This operator is derived by averaging the Fourier components of the perturbed electric field over a gyroperiod and the phase of the motion of a single ion and over a maxwellian distribution of velocities.

As the electron Larmor radius is much smaller than $b_0$, the electric potential $\phi_e$ felt by the electrons is given by:

$$\phi_e = -E_0 x + b_0 (E_0 + G_i) \tilde{\varphi}(x,y)$$

(6)

The ion density $n_i$ is a function of the potential $\phi_i + \phi_{Gi}$ and the electron density $n_e$ depends on $\phi_e + \phi_{Ge}$ and on the approximate magnetic flux $A_z^*$. $A_z^*$ is an even function of $x$ (see Eq.(1)) and only the part of $\tilde{\varphi}(x,y)$ which is odd with respect to $x$. 
contributes to the current sustaining the island (see Section 4). A function of \( A_z^* \), odd with respect to \( x \), is formed by taking the square root of \( A(x,y) \) defined by Eq.(1). As \( A(x,y) \) is negative inside the island and positive outside, \( n_e \) is assumed to be independent of \( A_z^* \) inside the island, but to depend on the odd function \( \pm \sqrt{A} \) outside, where the upper symbol refers to \( x \geq b(y) \) and the lower symbol to \( x \leq -b(y) \).

The perturbed potential \( \tilde{\phi}(x,y) \) is determined by imposing quasi-neutrality \( n_e = n_i \) inside and outside the island. \( n_e \) and \( n_i \) are expanded at first order as a function of \( \phi_e + \phi_{Ge} \) and \( \pm \sqrt{A} \) and of \( \phi_i + \phi_{Gi} \) respectively in the vicinity of \( 0 \). Replacing these quantities by their definitions given by Eqs.(1), (3), (5) and (6) leads to two integral equations satisfied by \( \tilde{\phi}(x,y) \). A good approximation for the solution of these equations writes:

\[
\tilde{\phi}(x,y) = \frac{1}{2} \frac{T_e}{T_e} \left[ q_e \frac{G_e}{T_e} + q_i \frac{G_i}{T_e} \right] \frac{x}{b_0} \left( 1 - \mathcal{P} \sqrt{1 - \frac{b_0^2 a(y)}{2x^2}} \right)
\]

with: \( \mathcal{P} = \begin{cases} 
0 & \text{for } -b(y) \leq x \leq b(y) \\
1 & \text{for } x \leq -b(y) \text{ and } x \geq b(y)
\end{cases} \)

\( \bar{J}_0^2 \) is an approximation for the finite ion Larmor radius operator, which is defined by:

\[
\bar{J}_0^2 = e^{-\left(\frac{\rho_i}{b_0}\right)^2} I_0 \left[ \left(\frac{\rho_i}{b_0}\right)^2 \right]
\]
FIG. 2. Result of a computation of the perturbed potential, $\mathbf{\phi}(x, 0)$, and normalized current density, $\mathbf{\delta J}_I(x, 0)$, sustaining the island versus $x/b_0$. The island lies between $x = -b_0$ and $x = b_0$. For this case, the ion Larmor radius $\rho_I$ is equal to the island half-width $b_0$, $n'_e/n_e = 1\ \text{m}^{-1}$, $T'_e/T_e = 2\ \text{m}^{-1}$ and $G_e = G_i = 0$. $\mathbf{\delta J}_I(x, 0)$ is normalized to $-q_i E_i \rho'_I / B_0$.

where $I_0(z)$ is one of the modified Bessel functions of zeroth order. $J_0^2$ is equal to 1 for very small Larmor radii ($\rho_i < b_0$) and tends toward zero for very large $\rho_i$ ($\rho_i > b_0$).

Eq.(7) shows that the perturbed potential $\tilde{\mathbf{\phi}}(x,y)$ is a function which is odd with respect to $x$ and which is independent of $y$ inside the island.

The system of integral equations has been solved by numerical iteration. The result for $\tilde{\mathbf{\phi}}(x,y)$ is plotted versus $x/b_0$ in Fig.2 in the case of $y = 0$, $\rho_I/b_0 = 1$, $n'_e/n_e = 1\ \text{m}^{-1}$, $T'_e/T_e = 2\ \text{m}^{-1}$ and $G_e = G_i = 0$. 
4. CURRENT DENSITY SUSTAINING THE ISLAND

The perturbed current density sustaining the island \(\delta J(\text{x},\text{y})\) is obtained from current conservation in steady state:

\[
\nabla \cdot (\text{J}_\parallel e \text{I} + \text{n}_1 \text{q}_1 \text{v}_\text{Di} + \text{n}_\text{e} \text{q}_\text{e} \text{v}_\text{De}) = 0
\]  \(9\)

\(\text{J}_\parallel\) is the electron current density along the field lines, being the sum of the constant plasma current density and \(\delta J(\text{x},\text{y})\). \(e\) is equal to B/B. \(\text{v}_\text{Di} = \{-\nabla (\phi_1 + \phi_{\text{Gi}}) \times \text{B} \} / \text{B}^2\) is the ion drift velocity and \(\text{v}_\text{De}\), the electron drift velocity, is given by a similar expression.

The leading term of \(\nabla \cdot (\text{J}_\parallel e \text{I})\) is \((-B'x/B)\nabla \delta J_\parallel(\text{x},\text{y})\). In the limit of low \(\beta\), \(\nabla \cdot (\text{n}_1 \text{q}_1 \text{v}_\text{Di})\) is zero, because \(n_1\) is a function of \(\phi_1 + \phi_{\text{Gi}}\). Taking account of quasi-neutrality \(n_e = n_1\), it follows that:

\[
\nabla \cdot (\text{n}_e \text{q}_e \text{v}_\text{De}) = -\frac{\text{q}_e}{\text{B}^2} \cdot \left\{ \nabla \text{n}_1 \text{x} \nabla (\phi_e + \phi_{\text{Ge}}) \right\}
\]  \(10\)

where, since \(n_1 = n_1(\phi_1 + \phi_{\text{Gi}})\), \(\nabla n_1\) is equal to \(\text{dn}_1/\text{d}(\phi_1 + \phi_{\text{Gi}}) \nabla (\phi_1 + \phi_{\text{Gi}})\) with \(\text{dn}_1/\text{d}(\phi_1 + \phi_{\text{Gi}}) = -n_1'/\text{E}_0\). From the expressions for the potentials given by Eqs.(3), (5) and (6), it can be shown that the right-hand side of Eq.(10) depends on \(\nabla_y \tilde{\phi}(\text{x},\text{y})\). Eq.(9) is integrated with respect to \(\text{y}\). The perturbed current density sustaining the island \(\delta J_\parallel(\text{x},\text{y})\) is then given by:

\[
\delta J_\parallel(\text{x},\text{y}) = \frac{\text{n}_e}{\text{B}^2} \left\{ -\text{KT}_e \left[ \frac{\text{n}_e'}{\text{n}_e} + \frac{1}{2} \frac{T_e'}{T_e} \left( \frac{\text{n}_e'}{\text{n}_e} \right) \right] + \frac{\text{n}_e'}{\text{n}_e} \left( \frac{\text{q}_e \text{G}_e + \text{q}_i \text{G}_i}{\text{n}_e} \right) \right\} \frac{\text{b}_0}{\text{x}} \tilde{\phi}(\text{x},\text{y})
\]  \(11\)

where an additive function of \(A_Z\) due to integration is omitted. This
function is determined by taking into account other effects such as collisions: the resistivity is assumed to be the same inside and outside the island. Finally, $\delta J_\parallel$ has been computed using the data of $\tilde{\varphi}(x,0)$ shown in Fig. 2 and normalised values are plotted as a function of $x/b_0$ in Fig. 2. $\delta J_\parallel(x,y)$ is an even function of $x$.

5. MAGNETIC ISLAND SELF-SUSTAINMENT

Ampère's law for thin islands ($k_y b_0 \ll 1$) can be written as:

$$\frac{B}{k_y} \nabla^2 A(x,y) = -\mu_0 \delta J_\parallel(x,y) \quad (12)$$

The jump in the derivative of the vector potential across the region associated with the island is equal to the integral of $\delta J_\parallel(x,y)$ with respect to $x$. This leads to an equation in $y$ and $\Theta_0$, which is solved by computing the Fourier components of the vector potential with respect to $y$ for different values of $\Theta_0$ [4]. Eq.(12) then becomes:

$$\mu_0 \frac{n_e eKT_e}{B_0^2} \frac{T_e}{2} \frac{1}{T_e} \left(\frac{n_e}{n_e} \frac{1}{n_e} \frac{q_e G_e + q_i G_i}{KT_e} \right) \frac{1}{k_y} = F(\Theta_0)$$

The left-hand side of Eq.(13) is proportional to the poloidal $\beta$ and is independent of the sign of the shear $q'$. The numerator of the large fraction contains two terms. The first one is due to the finite ion Larmor radius effect and is maximum when $\bar{J}_0^2$ is
zero for island widths much smaller than $\rho_1$. Note that $\frac{J^2_0}{2}$ is close to $1/2$ for an island width equal to $2\rho_1$. This effect has a threshold, because it is switched off when $n'_e/n_e + (1/2) T'_e/T_e$ is zero, that is when islands are separated by nested magnetic surfaces for $\gamma \leq 0.75$ (see Eq.(4)). The second term results from the pseudo-gravity effect. It is generally smaller in modulus than the absolute values of $T'_e/T_e$ and $n'_e/n_e$. This effect is therefore dominant, when $\frac{J^2_0}{2}$ is close to $1$, that is when the island is much larger than $\rho_1$. It can sustain islands even when they are separated by nested surfaces for $\gamma < 0.75$, since it has no threshold. An example of pseudo-gravity is given by the interchange instabilities in the cylindrical approximation: $\frac{q_s G_e + q_i G_i}{n_e}$ is identified with $2rP'_s/(q_s R)^2$, where $r/(q_s R)^2$ is the inverse of the radius of curvature of an helix on the resonant surface at $r = r_s$ in the limit $r < q_s R$ and $P'_s$ is the gradient of plasma pressure at $r = r_s$. In this case, Eq.(11) with $\frac{J^2_0}{2} = 1$ gives the same perturbed current outside the island as the MHD equations [7].

$F(\theta_0)$ is determined from Poincaré map computation using the Fourier components of the vector potential calculated above. Table I shows that $F(\theta_0)$ increases, as $\theta_0$ decreases, that is as the island is destroyed. The minimum threshold for turbulence self-sustainment by the finite ion Larmor radius effect is $F(\theta_0) = 0.20$, which corresponds to an overlapping parameter $\gamma \approx 0.75$.

The above calculation is valid in the limit $T_i \gg T_e$. In general, the ion density $n_i$ is a function of $\phi_i + \phi_{Gi}$ and $\phi_e + \phi_{Gi}$
TABLE I. NUMERICAL RESULTS FOR $F(\Theta_0)$ OBTAINED FROM THE POINCARE MAP COMPUTATION AS A FUNCTION OF $\Theta_0$ AND THE OVERLAPPING PARAMETER $\gamma$

<table>
<thead>
<tr>
<th>$\gamma$</th>
<th>0.4</th>
<th>0.7</th>
<th>1.0</th>
<th>1.2</th>
<th>1.4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Theta_0$</td>
<td>0.91$\pi$</td>
<td>0.70$\pi$</td>
<td>0.44$\pi$</td>
<td>0.32$\pi$</td>
<td>0.13$\pi$</td>
</tr>
<tr>
<td>$F(\Theta_0)$</td>
<td>0.16</td>
<td>0.20</td>
<td>0.30</td>
<td>0.46</td>
<td>2.04</td>
</tr>
</tbody>
</table>

and the term $(n'_e/n_e)(1-J^2_0)$ in Eqs.(7), (11) and (13) has to be replaced by:

$$\left(\frac{n'_e}{n_e} + \frac{T_e}{T_i} \left(\frac{n'_e}{n_e} + \frac{T'_e}{T_e} \left(\frac{1}{2} T_e - \frac{q_e G_e + q_i G_i}{KT_e}\right)\right)\right)(1-J^2_0)$$

6. CONCLUSION

Two mechanisms have been proposed for the self-sustainment of magnetic islands. Both produce a net diamagnetic current in the region associated with the island.

The first effect is due to the difference between the electron and ion drift velocities as a result of their different Larmor radii. It could maintain a magnetic turbulence in which islands with a width of the order of the ion Larmor radius are embedded in a chaotic region. The turbulence is self-sustained when the left-hand side of Eq.(13) is larger than 0.20.

In the second mechanism, the current results from the addition of electron and ion drift velocities associated with the
pseudo-gravity. This effect has no threshold and could sustain islands much larger than the ion Larmor radius. This result can be also directly demonstrated from the MHD equations in cylindrical geometry [7] and could be extended to ballooning modes in toroidal geometry.

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REFERENCES


DISCUSSION

K. ITOH: I have a question concerning the modelling of the artificial magnetic surfaces, which allow a sharp potential gradient, $\partial \Phi/\partial x$. Where the field becomes chaotic two points with different $x$ are connected by the field line. The parallel conductivity does not allow the potential structure you obtained.

Secondly, the finite gyroradius effect outside the island increases the $\vec{\mu}$ felt by the ions (not $\phi''/\phi < 0$). This leads to $J_0^2 < 1$ and changes the sign of $1 - J_0^2$. Does this not contradict your conclusion?

M. HUGON: This model corresponds to a steady state of particle and energy flows in the chaotic region which define $\nabla n$ and $\nabla T$. The current maintaining the islands results from the external product $\nabla n \times \nabla \Phi$ being different from zero.
Therefore, the dependence of the electron density on $A_z$ in the chaotic region must have the same 'symmetry' as $A_z$ defined outside a single island chain.

To answer your second question, $J_0$ is an approximation for the finite ion Larmor radius operator given by Eq. (5), which is used in the calculation. Far from the island, the potential experienced by the ions can be slightly larger than that experienced by the electrons because of the concavity of $\phi$. This leads to a negligible negative current, which decreases as $1/x$.

R.J. GOLDSTON: You and I have had a long standing discussion on whether these islands can produce $\chi_i \sim \chi_e \gg \chi_i^{\text{spec}}$. Have you found a magnetic structure in which this experimentally observed relationship is seen theoretically?

M. HUGON: The number of chaotic field lines which join the plasma bulk to the wall is very small. The resulting energy losses must be a combination of parallel transport in the radial direction and perpendicular diffusion between adjacent flux tubes.

L.J. PERKINS: In previous work you showed that transport depended on $n^{-5}$, where $n$ is the toroidal mode number. Is there now a physics formula to determine $n$, and how does the resulting transport scale with $T$, $B$, etc.?

M. HUGON: The number of island chains is proportional to $n^2$. For the values of $n$ reported previously, the distance between two island chains corresponds to a ratio $\rho/b_0 \approx 0.5-1$. This calculation does not, however, take into account toroidal effects and collisions, which must be included in a full transport model.
ENERGY CONFINEMENT AND THE COUPLING OF TRANSPORT PROCESSES WITH MHD MODES

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Abstract
ENERGY CONFINEMENT AND THE COUPLING OF TRANSPORT PROCESSES WITH MHD MODES.
Starting from the tearing mode equation, the growth rate equations of magnetic islands, the energy and particle balance equations with large effective thermoconduction and diffusion in magnetic islands, it has been shown that the widths of magnetic islands of different tearing modes and the temperature profile all oscillate with time. In some cases the oscillation frequency is the same as that of the sawteeth, but in other cases it is in the range of observed Mirnov oscillations. From these equations the energy confinement time of the plasma has been calculated and the result compared with that obtained purely from the anomalous thermoconductivity in the $q > 1$ region. The scaling laws of these two kinds of confinement time are quite different. This result shows clearly that it is impermissible to compare the experimental results directly with anomalous transport analysis, as has usually been done so far.

1. INTRODUCTION

Many tokamak experiments (experiments on profile consistency [1], helical winding experiments on HT-6B [2], electrical potential experiments at the University of California at Los Angeles, experiments promoting the L–H mode transition by elevating the boundary temperature, improved confinement with peaked density profile, supershots, pellet injection, experiments on the dependence of confinement time on minor radius in the L mode scaling law, etc.) have shown that the energy confinement process in the $q > 1$ region of a tokamak plasma should be closely related to the macroscopic plasma processes. But perhaps more experiments have shown that energy confinement is mainly determined by the local anomalous transport processes. If we consider that all of these experiments only reflect different aspects of plasma behaviour, the energy confinement must be related to both the anomalous transport processes and MHD fluctuations in a combined way.

It is well known that the instability of tearing modes is mainly determined by the plasma current gradient at the corresponding resonant surfaces. Inside the magnetic islands, energy and particle transport processes are very fast owing to convection. In the discussions related to profile consistency, temperature profiles or current profiles have been found which could stabilize all the tearing modes. If the
profiles formed by the transport processes are unstable against some MHD modes, then these modes will develop and the corresponding magnetic islands will grow. These growing modes will act to change the temperature profile along with the transport processes, and gradually become stable. In this process, other modes could become unstable and new magnetic islands appear, which will change the profile further. These considerations make the picture of strong coupling between MHD modes and anomalous transport processes clear. This should be taken into account in any analysis of the tokamak plasma behaviour [3].

2. THEORETICAL MODEL

To investigate the coupling of MHD modes with transport processes, neglecting the toroidal effect, we start from cylindrical geometry and use the following model.

2.1. Transport processes

\[
\frac{\partial n_e}{\partial t} = - \frac{1}{r} \frac{\partial}{\partial r} (r \Gamma_e) + S
\]

Diffusion: \( \Gamma_e = -D \frac{\partial n_e}{\partial r} \)

Source: \( S = n_e n_0 \langle \sigma_{\text{ion}} v_e \rangle \)

\[
\frac{3}{2} \frac{\partial (n_e T_e)}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} r \left( k_e \frac{\partial T_e}{\partial r} - \frac{3}{2} T_e \Gamma_e \right)
\]

Thermoconductivity and diffusion

\[
+ \frac{\eta c^2}{(4\pi)^2} \left( \frac{\partial (r B_e)}{\partial r} \right)^2
\]

Ohmic heating

\[
- \frac{3m_e n_e}{m_i \tau_{ei}} (T_e - T_i)
\]

\[
- W_R
\]

Radiation loss: \( W_R = W_{BR}A \)

\[
- W_i
\]

Ionization loss: \( W_i = S e_i \)

\[
\frac{3}{2} \frac{\partial (n_i T_i)}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} r \left( k_i \frac{\partial T_i}{\partial r} - \frac{3}{2} T_i \Gamma_i \right)
\]

Thermoconductivity and diffusion

\[
+ \frac{3m_e n_e}{m_i \tau_{ei}} (T_e - T_i)
\]

\[
- W_{\text{ch}}
\]

Charge exchange loss,

\( W_{\text{ch}} = \langle \sigma_{\text{ex}} v_i \rangle \frac{3}{2} n_i T_i n_0 \)
\[
\frac{\partial}{\partial t} B_0 = \frac{c^2}{4\pi} \frac{\partial}{\partial r} \left( \frac{\eta}{r} \frac{\partial (r B_0)}{\partial r} \right)
\]

Magnetic diffusion

where

\(n_0\) is the neutral hydrogen density,

\((\sigma_{\text{ion}} \nu_e)\) is the ionization rate,

\(W_{\text{BR}}\) is the bremsstrahlung power density,

\(\epsilon_H\) is the ionization potential,

\(A = 1 + (37.9 Z^2 \text{im}/T_e) + (860 Z^4 \text{im}/T_e^2),\)

\(\tau_{\text{ei}}\) is the electron-ion heat exchange time,

and \(n_i = n_e, \Gamma_e = \Gamma_i\) (neglecting impurities).

2.2. Transport coefficients

\(\eta: \) Spitzer resistivity with impurity effect in \(Z_{\text{eff}}\). The effective resistivity (due to the \(m = 1\) internal mode) [4]:

\[\eta_i = c_0 (1 - q_0) \left[ 1 - 3 \left( \frac{r}{r_i} \right)^2 \right]^2 \quad \text{(for } r^2 \leq \frac{3}{4} r_i^2)\]

where \(r_i\) is the radius of the \(q = 1\) surface.

\(K_e: \) Pseudoclassical electron thermoconductivity, phenomenologically enhanced by:

\[k = k_0 \left( 1 + c \left[ 1 - \left( \frac{x_s}{h} \right)^2 \right] \right) \quad (x_s \leq h)\]

In a magnetic island with width \(h, c \sim 100\). In the \(q < 1\) region,

\[k = k_0 \left[ 1 + c \left[ 1 - \frac{3}{4} \left( \frac{r}{r_1} \right)^2 \right]^2 \right] \quad \text{(for } r^2 \leq \frac{4}{3} r_1^2)\]

\(K_i: \) Neoclassical ion thermoconductivity, anomalously enhanced in magnetic islands and the \(q \leq 1\) region.

\(D: \) Diffusion coefficient, Alcator scaling, enhanced in magnetic islands and the \(q \leq 1\) region.

These transport coefficients (neoclassical ion thermoconductivity, pseudoclassical electron thermoconductivity, etc.) are useful only as a starting point, to begin the concrete calculations more easily.
2.3. Tearing mode equation [5]

The equation of the perturbed radial magnetic field $B_r$ of the $m/n$ mode:

$$\frac{d}{dr} \left[ r \frac{d}{dr} (r B_r) \right] - \left[ m^2 + \frac{dj/dr}{(B_0/r^2) (1 - nq/m)} \right] B_r = 0$$

and $j = \frac{1}{\mu_0} \frac{1}{r} \frac{\partial}{\partial r} (r B_0)$.

The evolution equation of the width of magnetic islands:

$$\frac{d}{dt} w = 1.66 \frac{n(r_s)}{\mu_0} \frac{\Delta(w)}{r_s}$$

where

$$\Delta(w) = \frac{\psi'[r_s + (w/2)] - \psi'[r_s - (w/2)]}{\psi'/r_s}$$

$r_s$ is the radius of the resonant surfaces, $w$ the magnetic island width and $\Delta$ the stability factor.

2.4. Parameters for calculation

The following plasma parameters are used in our calculations: $R = 45$ cm, $a = 12$ cm, $B_z = 7$ kG, $q_a = 2.8-5.0$, $n_e = (1-3) \times 10^{13}$ cm$^{-3}$. Three MHD modes (2/1, 3/2 and 4/3 or 3/1) have been taken into account. Because all the equations could be scaled by the length, the following results could also be applied to the processes in large machines.

3. PLASMA BEHAVIOUR WITH DIFFERENT DISCHARGE PARAMETERS

Plasma behaviour with different discharge parameters was calculated. For a higher density, lower q discharge ($n_e = 3 \times 10^{13}$ cm$^{-3}$, $q_a = 3.2$), Fig. 1(a) shows the time evolution of the positions and amplitudes of the 2/1, 3/2 and 4/3 modes. The average width ($w$) of the 2/1 magnetic island remains around 0.2a, but oscillates with an amplitude of 0.15w. It is a real oscillation of the magnetic island (i.e. expanding and shrinking) due to the strong coupling of transport processes with MHD modes, and its frequency is the same as that of sawteeth. Such oscillation was observed in
m = 2 and m = 3 Mirnov signals but has so far been thought to be due to the propagation of sawteeth. The behaviours of 3/2 and 4/3 tearing modes are different from that of the 2/1 mode. The magnetic islands appear only in part of the sawtooth period; in other words the magnetic islands alternately emerge and vanish. When they reach the maximum, the islands of the three modes overlap and cause energy to transfer very quickly through a wide region. Figure 1(b) shows the time variation of the energy confinement time; the deep valley in each sawtooth period corresponds to such overlapping of the magnetic islands. The movement of the q = 1 surface in Fig. 1(a) represents the sawtooth oscillation in our model. In Fig. 1(c) the time
variation of the temperature shows a typical sawtooth behaviour, and the phase shifts between sawtooth waves at different places could determine a formal thermoconductivity.

The results in Fig. 2 were obtained for a lower density and lower q discharge \((n_e = 1.8 \times 10^{13} \text{ cm}^{-3}, q_a = 3.4)\); 3/1, 2/1 and 3/2 modes have been considered. The sawtooth-like oscillation of the magnetic islands is very obvious. The only difference from Fig. 1 is that the 2/1 island appears and vanishes alternately within each sawtooth period. The lack of a lasting 2/1 magnetic island, of course, should make energy confinement better than that of Fig. 1.

For a lower density and higher q case \((n_e = 1.8 \times 10^{13} \text{ cm}^{-3}, q_a = 4.1)\), the 2/1 mode grows and then saturates to form a stable magnetic island with a width of about 0.1a (Fig. 3). The behaviour of the 3/2 mode is quite different. It is seen that a high frequency oscillation of the 3/2 mode appears. The frequency of the 3/2 mode is in the range of Mirnov oscillation observed on small tokamaks, but the amplitude is very small. There is no oscillation on the temperature. This shows the possibility that the Mirnov oscillation observed in magnetic measurements could be caused by the coupling of tearing modes with transport processes.

The above results show a general trend, which has been confirmed by many experiments on small devices, that the sawtooth oscillation will disappear and the m = 1 mode will arise gradually when the q value and/or electron density increases.

4. MODELLING OF ENERGY CONFINEMENT

Energy confinement has been one of the central problems in fusion research since the 1960s. For a tokamak plasma, the prevailing physical picture is as follows.
1. In the central (q < 1) region the sawtooth process is dominant. In the outer region (1 < q < 2 or 3) anomalous thermoconductivity is dominant for electrons and neoclassical thermoconductivity with some anomaly for ions. In the edge region energy loss is due to anomalous transport and radiation. So far, almost all the transport analyses have been based on this picture and compared with experimental results to determine the anomalous transport coefficients and the corresponding micro-instability. But since the end of the 1970s, more and more experiments on large as well as on small tokamaks have shown a very strong dependence of energy confinement on macroscopic plasma processes, as mentioned above. It has been found that the plasma behaviour sensitively depends on MHD fluctuation, the boundary condition and even some weak external disturbances. Many researchers have suggested that the instability of tearing modes plays the major role in the outer region, others have considered the ergodic magnetic configuration. Such efforts have had no significant influence, because only pure, simple macroscopic factors were considered. If we recheck the sawtooth process, it could be found that the transport processes and MHD motion should be considered together. Thus it is valuable to try to analyse the energy confinement in the major part of the plasma (q < 2 or 3).

\( q_a = 4.1, n_e = 1.8 \times 10^{13} \text{ cm}^{-3} \) and temperatures at different positions.

**FIG. 3.** Time evolution of (a) 3/1, 2/1 and 3/2 modes and (b) temperatures at different positions.
starting from the coupling of MHD modes with transport processes. This could present a new physical picture.

It is the main task here to show the necessity to introduce this new picture, or the significant differences between these two pictures, for which our simple model mentioned above could be used. The energy confinement time has been defined as:

\[ \tau_{ge} = \frac{W}{P_{in} - (dW/dt)} \]

where \( W \) is the total stored energy and \( P_{in} \) is the input power. Figure 4(a) shows the energy confinement times obtained by considering the coupling of tearing modes with pseudoclassical transport processes for different \( n_e \) and \( q_a \). Figure 4(b) shows the usual pseudoclassical results, i.e. considering only the pseudoclassical transport.
in the outer region, as a comparison. For a fixed $q_a$ value the pseudoclassical confinement time decreases with increasing electron density. But the MHD effect totally changes the scaling. In Fig. 4(a) the confinement time increases with electron density, which is in qualitative agreement with experiments. For fixed $n_e$ these two dependences of $\tau_{ge}$ on $q_a$ are also totally different. Figure 5 shows the sensitive dependence of $\tau_{ge}$ on the boundary temperature of plasma in our new physical picture. Such sensitivity is due to the inclusion of MHD effects. The MHD processes coupled with anomalous transport not only change the value of the global confinement time but also, perhaps more importantly, change the scaling law.

It should be noted that the usual way to analyse transport processes is to relate the usual transport calculation to experimental measurements in order to obtain the
local thermoconductivity or diffusivity, which will be further compared with the results of turbulence theory for different microinstabilities. But owing to the strong effects of MHD processes as mentioned above, these transport coefficients have only a formal meaning and could be quite different from the real ones. They could not directly determine the mechanism of the microturbulence. Figure 6 shows the propagation of a heat pulse in a tokamak plasma in the new physical picture; it can easily be simulated by a formal thermoconductivity.

We have not made any attempt to simulate the real plasma processes but only to emphasize the necessity of considering the coupling of transport processes with MHD modes and to renew our point of view to analyse the problem of energy confinement.

REFERENCES

ENHANCED PARTICLE AND HEAT TRANSPORT DUE TO HELICITY TRANSPORT

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Abstract

ENHANCED PARTICLE AND HEAT TRANSPORT DUE TO HELICITY TRANSPORT.

The helicity transport in a current-carrying plasma results in heat and particle transport in the direction opposite to the helicity flux. This result applies whether the helicity is injected externally by means of oscillating fields or is generated internally in the plasma. Self-consistent relations among the helicity, heat and particle fluxes are derived for specific cases of helicity transport in sustained plasma equilibria.

1. INTRODUCTION

To sustain a current-carrying plasma equilibrium, the helicity of the magnetic field should be kept around a constant value for every toroidal volume. Let \( \Omega \) be a certain fixed toroidal volume in a plasma. The helicity in \( \Omega \) is \( \int_\Omega \vec{A} \cdot \vec{B} \, dx \), where \( \vec{B} \) is the magnetic flux density, and \( \nabla \times \vec{A} = \vec{B} \). Here and hereafter we take the Coulomb gauge for the vector potential. We call \( \vec{h} = \vec{A} \cdot \vec{B} \) the helicity density. Maxwell's equations yield

\[
\frac{d}{dt} \int_\Omega \vec{A} \cdot \vec{B} \, dx = - \int_{\partial \Omega} \vec{n} \cdot \left[ -\left( \partial_t \vec{A} \right) \times \vec{A} + 2\vec{B} \phi \right] \, ds - 2 \int_\Omega \vec{E} \cdot \vec{B} \, dx,
\]

where \( \partial \Omega \) is the boundary of \( \Omega \), \( \vec{n} \) is the unit normal vector onto \( \partial \Omega \), \( \phi \) is the scalar potential. The first term in the right-hand-side of Eq. (1) represents the helicity transport; we call \( \vec{F}_h = -\left( \partial_t \vec{A} \right) \times \vec{A} + 2\vec{B} \phi \) the helicity flux.

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density. We note that the first term in $\mathbf{F}_h$ is finite if $\mathbf{A}$ has a circularly polarized component. The second term gives the helicity dissipation. Using (generalized) Ohm's law, we observe $\int_{\Omega} \mathbf{E} \cdot \mathbf{B} \, dx = \int_{\Omega} \eta \mathbf{j} \cdot \mathbf{B} \, dx$, where $\eta$ is the resistivity, and $\mathbf{j} = \mu_0^{-1} \nabla \times \mathbf{B}$ is the current density. Here we neglected the electron pressure gradient. In the MHD relaxation, the helicity of the total plasma volume is approximately conserved, while the excess of the magnetic energy is released through instabilities [1-3]. There is, therefore, a spatial transport of the helicity density $h$ related to the change of the current profile. Also in a sustained equilibrium, to compensate for the helicity dissipation, there should be a balanced helicity flux. There are possibilities to drive the plasma current by actively injecting the helicity by means of externally excited wave fields or through spontaneous fluctuations to transport the helicity from the edge into the plasma [4-9].

In the present work, we calculate the particle and heat fluxes which are correlated with the helicity flux transported through electromagnetic fluctuations. Fluctuations with circularly polarized components induce finite average in cross-field parallel current, $<\mathbf{j}_{//}>_z$, which leads to generation of frictional electron heat flux as well as ion nonlinear polarization current which produces particle flux. The circular polarization of the perturbed electric field thus correlates the helicity flux and the particle flux. Naturally, there are many other possibilities of anomalous transports which are not directly correlated with the helicity transport. Typically they are characterized by the $\mathbf{E} \times \mathbf{B}$ drifts, which contribute to transport in addition to the process considered here.

We assume fluctuations with an average phase velocity (wave momentum) directed parallel to the electron flow velocity of the equilibrium electron current. This assumption is consistent with a case where the helicity transport provides a parallel electron current. We then find that the electromagnetic fluctuations that transport the helicity also transport heat and particles in the direction opposite to the helicity transport. We do not specify the fluctuation to obtain a general relation between the helicity flux and the particle and heat fluxes. The basic relation turns out to include a term that depends on the
dispersion of the relevant fluctuation. When we apply this theory to a specific fluctuation, we can quantify the particle and heat fluxes which appear as an unavoidable consequence of sustaining the helicity of an equilibrium. In this paper, we consider two simple examples for the phase velocity of the fluctuation; one that corresponds to the electron flow velocity, and the other to the Alfvén velocity.

2. TRANSPORTS CORRELATED WITH HELICITY FLUX

We write $f_0 = \langle f \rangle$ (temporal average of $f$) and $f_1 = f - f_0$. In what follows, we consider a slab plasma for simplicity. We use a three dimensional orthogonal $(x, y, z)$ coordinate system, where each $x$-$y$ plane is a magnetic surface of the average equilibrium field, the minus side of $z$ is the inside of the plasma, and $y$ is an ignorable coordinate for the fluctuations. Perturbations are periodic in time $t$ and $x$. Propagation in $z$ is assumed to be small compared with that in $x$. The Coulomb gauge condition allows an expression

$$\vec{A}_1 = \nabla u \times \nabla y + a_y \nabla y,$$

where $u$ and $a_y$ are scalar functions of $x$, $z$, and $t$. We may restrict $\phi_1 = 0$ at $z = 0$ [10]; the plane $z = 0$ is a certain magnetic surface which is not necessarily the outer-most plasma boundary. Functions $u$ and $\phi_1$ are in the same phase of the periodicity in $x$ and $t$. We obtain

$$\langle F^x_h \rangle_z = \langle \partial_t \vec{A}_1 \times \vec{A}_1 \rangle \cdot \nabla u = \langle a_y (\partial_{t,z} u) - (\partial_t a_y)(\partial_z u) \rangle,$$

(2)

$$\langle \phi^x_h \rangle_z = \langle 2 \vec{B}_1 \phi_1 \rangle \cdot \nabla z = \langle 2 (\partial_x a_y) \phi_1 \rangle = 0 \quad \text{at } z = 0.$$

(3)

For $\langle F^x_h \rangle_z$ to be finite, the phase of $a_y$ should be shifted from that of $u$, viz., the perturbation field should have a circularly polarized component [11]. Under the gauge and boundary conditions we use here, $\phi_1$ does not contribute to the helicity flux at $z = 0$, however, the fluctuation is not necessarily
transverse. It is also shown that the parallel electric field \( E_{//} = (\partial_t \vec{A}_1 - \nabla \phi_1) \cdot \vec{b}_0 \) should be finite for \( F_h \) to be finite, where \( \vec{b}_0 = \vec{B}_0 \) / \( \vec{B}_0 \).

We now show that the helicity flux, heat flux and particle flux are related to each other through the parallel current density. We first calculate the current density related to the fluctuation. By writing \( \vec{b} = \vec{B} / B_0 \), the \( z \)-component of the average parallel current is given by

\[
<j_{//}>_z = \langle (\vec{j} \cdot \vec{b}) \vec{b} \rangle \cdot \nabla z
\]

\[
= \left( \frac{\langle \vec{b}_0 \cdot \nabla \rangle}{\mu_0 B_0} \right) \langle (\partial_z \nabla^2 u)(\partial_x a_y) \rangle - \left( \frac{\langle \vec{j}_0 \cdot \nabla \rangle}{B_0^2} \right) \langle (\nabla^2 u)(\partial_x a_y) \rangle
\]

\[
= \left( \frac{\langle \vec{j}_0 \cdot \nabla \rangle}{B_0^2} \right) \langle (\partial_z a_y)(\partial_x a_y) \rangle.
\] (4)

Here we neglected the displacement current. In the low-beta limit, the average equilibrium field satisfies

\[
\nabla \times \vec{B}_0 = \lambda \vec{B}_0,
\] (5)

with a scalar function \( \lambda \) such that \( \langle \vec{b}_0 \cdot \nabla \rangle \lambda = 0 \). Here \( \lambda^{-1} \) is on the order of the size of the plasma, which we assume is much larger than the scale of the fluctuation fields. We thus may neglect the last term in Eq. (4). Furthermore, since we assume isotropic fluctuation in the perpendicular direction, the second term does not contribute either. Hence from Eq. (2), we obtain

\[
<j_{//}>_z = (\mu_0 B_0)^{-1} (\vec{b}_0 \cdot \nabla \langle (\partial_z \nabla^2 u)(\partial_x a_y) \rangle)
\]

\[
= -(2\mu_0 B_0)^{-1} (k_{//} / \omega) k^2 \langle F_h \rangle_z,
\] (4')

where \(-k^2\) is the spectrum of \( \nabla^2 \) for the fluctuation, \((k_{//} / \omega)^{-1}\) is the (average) phase velocity of the fluctuation field.
Now we discuss self-consistent relations among the particle, heat, and helicity fluxes. Equation (4') immediately relates the helicity flux with the frictional electron heat flux, \( \rho_u \),

\[
<q_u^e>_z = -C_Z T_e <j_{\parallel}>_z / e = C_Z T_e (2\mu_0 B_0 e)^{-1} (k_{\parallel} / \omega) k^2 <F_h>_{\parallel}^z
\]

(6)

where \( T_e \) is the electron temperature, \( C_Z \) is a positive constant (= 0.71, if \( Z = 1 \)) [12].

Equation (4') shows that, for the helicity flux \( <F_h>_{\parallel}^z \) to be finite, the perturbed parallel current \( <j_{\parallel}>_{\parallel} \) should be finite. We thus require a balanced finite average in the perturbed perpendicular current to retain the average charge neutrality. The parallel component of the perturbed current is primarily carried by electrons, while the perpendicular component is by ions. Therefore a net particle flux results. The ion current is primarily caused by the nonlinear polarization drift;

\[
<j_p>_z = \frac{\rho_m}{B_0^2} (\nabla \cdot \nabla) \vec{E}_1 \cdot \nabla z,
\]

where \( \rho_m \) is the ion mass density and \( \vec{v} \) is the fluid velocity which is dominated by the \( \vec{E} \times \vec{B} \) drift. The nonlinearity in \( \vec{E}_1 \) causes a finite average of \( <j_p>_z \). By equalizing \( <j_p>_z \) with \(-<j_{\parallel}>_{\parallel} \), we obtain a relation (ambipolar condition) between \( \phi_1 \) and \( u \);

\[
\partial_{z,z} \phi_1 = [1 - (v_A / \omega) k^2] \partial_{t,x,z} u
\]

This is a necessary condition for the fluctuation that can transport the helicity under the charge neutrality criterion. We now have the particle flux by Eq. (4');

\[
<\Gamma>_z = -<j_{\parallel}>_{\parallel} / e = -(\mu_0 B_0 e)^{-1} <(\vec{b}_0 \cdot \nabla x) [\nabla^2 (\partial_x u)] (\partial_x a_y)>
\]

\[
= (2\mu_0 B_0 e)^{-1} (k_{\parallel} / \omega) k^2 <F_h>_{\parallel}^z.
\]

(7)
We note that the sign of \(<F_h>_z\) depends on the sense of polarization, consequently on the sign of \(k_\parallel / \omega\).

Equations (6) and (7) describe basic relations among \(<F_h>_z\), \(<q^e>^z\) and \(<\Gamma>^z\); which are related through \(<j_\parallel>_z\) in the fluctuations. The particle and heat fluxes are quantified by estimating the helicity flux necessary to sustain the equilibrium parallel current. We also need dispersion relations for the fluctuating fields. In what follows we consider two simple cases of electromagnetic fluctuations with helicity fluxes. The helicity being a quantity whose sign depends on the orientation (the direction of the equilibrium current with respect to the magnetic field), the helicity flux should be carried by anisotropic fluctuations (waves) with respect to the parallel wave numbers.

3. APPLICATIONS

First, we consider fluctuations which are isotropic on the frame moving with the flow velocity of electrons (equilibrium current). Then, an anisotropy in the fluctuation spectrum is caused by the Doppler shift due to the Hall effect [13]. This situation is considered to be the lowest anisotropy that an MHD system permits. We obtain

\[
\frac{\omega / k_\parallel}{\omega} = -\frac{\vec{b}_0 \cdot \vec{b}_0}{(en_e)} (:= v_d)
\]

\[
= -\frac{\lambda B_0}{(\mu_0 en_e)},
\]

where \(n_e\) is the electron density, and we used Eq. (5). Equations (6) and (7) now read

\[
<q^e>^z = -C_Z T_e n_e (2\lambda B_0^2)^{-1} k^2 <F_h>_z
\]

\[
<\Gamma>^z = -n_e (2\lambda B_0^2)^{-1} k^2 <F_h>_z,
\]

For a sustained equilibrium, we may calculate \(<F_h>_z\) by the helicity balance condition. For simplicity, we assume that the transformer induction is zero,
and the total helicity is transported by fluctuations. The helicity balance condition then gives

$$\langle F_h \rangle_z = -2d \eta \vec{\gamma} \cdot \vec{B}_0 = -2d \eta \lambda B_0^2 / \mu_0,$$

(10)

where $d$ is the depth of the plasma. Here we assumed that $\lambda$ is constant for simplicity [1]. Plugging (10) into (8) and (9), we observe that $\lambda$ cancels out for the particle and heat fluxes, and obtain

$$\langle q_u^e \rangle_z = \frac{3}{2} T_e n_e d / \tau_p^d,$$

with $\tau_p^d = \frac{3}{2C_Z \eta k^2 / \mu_0}$, (11)

$$\langle \gamma \rangle_z = n_e d / \tau_p^d,$$

with $\tau_p^d = \frac{1}{\eta k^2 / \mu_0}$, (12)

The factor $(\eta k^2 / \mu_0)^{-1}$ corresponds to the MHD relaxation time, which is much smaller than the classical field diffusion time, so that the field distribution is dominated by the MHD relaxation instead of the classical diffusion [1-3].

In the next example, we consider MHD fluctuations which are usually oriented in the direction of $-\vec{j}$. Then the momentum (Poynting vector) of the waves is parallel to the electron current, and it can be most efficiently transferred to the electrons. In this case we obtain smaller particle and heat fluxes. Taking $\omega / k_{II} = -\text{sgn}(\lambda) v_A$, we obtain, instead of $\tau_p^d$ and $\tau_p^E$ in (11) and (12),

$$\tau_p^a = \frac{1}{\eta k_A^2 / \mu_0} \times \frac{1}{\xi} \left[ \frac{2m_e}{\beta m_i} \right]^{1/2},$$

and $\tau_p^E = 3/(2C_Z) \tau_p^a$, where $\xi (= v_d / v_T)$ is the streaming factor, $\beta$ is the beta ratio, $m_e$ and $m_i$ are the mass of an electron and an ion, respectively. The factor $\eta k_A^2 / \mu_0$ represents the resistive absorption rate for the considered Alfvénic fluctuation.

1) J. B. Taylor [9] pointed out that, for a certain fluctuating field, the helicity dissipation in the fluctuating part is comparable to the dissipation in the mean field.
4. DISCUSSION

In summary, we have obtained the heat and particle fluxes that are correlated with the helicity flux; Eqs. (6) and (7). The electron heat flux is given by the nonlinear term in the perturbed parallel current, and the ion particle flux is given by the nonlinear term in the polarization current. For two examples of \( (\alpha / k_{||}) \), we calculated the particle and the frictional heat fluxes. One is the case where the MHD fluctuations are isotropic on the frame Doppler shifted by \( v_d \). The other is the current drive by an oriented Alfvén wave. These two give respectively the upper and the lower bounds of the estimates of the transports through MHD modes. The helicity influx to drive parallel electric current is universally accompanied by outward particle and heat fluxes. These results are compared with the classical \( B^{-2} \) formula for the sustainment of the diamagnetic (perpendicular) current, which reads

\[
\tau_p^{cl} = d^2 / (\eta / \mu_0) \times (2 / \beta).
\]

When \( (k \cdot d)^2 > 1 \), the sustainment mechanism of the parallel current by the helicity transport dominates the particle flux.

Acknowledgment

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REFERENCES


DISCUSSION

M.N. ROSENBLUTH: How is it possible to have classical conductivity and anomalous transport with your model?

Z. YOSHIDA: This paper studies a specific transport channel that is correlated with the helicity flux; it appears in addition to conduction and other E x B types of transport, and is an inevitable consequence of sustaining the helicity of an equilibrium. The heat and particle fluxes estimated by the present model are large.

K. ITOH: I have a question on parity with respect to the sign of $k_B$. When the sign of $k_B$ changes (we choose $\omega$ positive definite), $F_{hB}$ changes sign. This being so, the flux seems to be sign definite. (Isotropy is assumed in the directions perpendicular to $\mathbf{B}$.) How the sign of the fluxes could be independent is hard to understand.

Z. YOSHIDA: The helicity flux $\langle F_h \rangle$ does not depend on $k_B$; this is clear because $\langle F_h \rangle = \langle (\delta_{t} \mathbf{A}) \mathbf{A} \rangle$, which does not include a spatial derivative.
SAWTOOTH RECONNECTION

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Abstract

Resistive mhd is inappropriate to describe tokamak sawtooth reconnection. It is found that under usual conditions the electric field resulting from flux reconnection is very large. The resulting acceleration of the electrons is such that reconnection is determined by electron inertia rather than electron collisions.

Introduction

It is well known that sawtooth oscillations are not understood. There are several features which are not in agreement with theoretical predictions based on resistive mhd. One of the discrepancies is that, in large experiments, the sawtooth collapse is an order of magnitude faster than predicted by Kadomtsev's model which is based on Sweet-Parker reconnection.

The natural response to this situation has been to examine the assumptions underlying both the theory and the experimental procedures. This has led to the realisation that use of the resistive form of Ohm's law is incorrect.

In the m=1 instability the core of the plasma is expected to behave as perfectly conducting fluid. The core moves toward the q=1 surface and drives a narrow current layer at this surface. In the layer the perfect conductivity equation is invalid and reconnection takes place. It is the behaviour in this reconnection layer which we shall examine.

Reconnection Layer

Figure 1 shows the basic geometry. The core moves toward the reconnection layer with a velocity v. The plasma enters the narrow layer, of thickness δ, and then flows out into the island region with a much higher velocity u. The pressure driving the flow comes from the perturbed helical field B* and the strong gradient in this field implies the layer current mentioned above.
The electric field which drives this current is determined by the rate of change of the helical flux.

![Diagram of reconnection model and the associated current layer.](image)

**Figure 1.** Diagram of reconnection model and the associated current layer.

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**Helical Flux**

The helical flux which is reconnected is given by the magnetic field which crosses an imaginary sheet for which $d\theta/d\phi = 1$, where $\theta$ and $\phi$ are the poloidal and toroidal angles. Such a sheet is shown in Figure 2.

![Helical sheet having $d\theta/d\phi = 1$.](image)

**Figure 2.** Helical sheet having $d\theta/d\phi = 1$. 
Field lines having $q = 1$ lie in this sheet, and the equilibrium helical flux inside the $q = 1$ surface, at $r = r_1$, is given by

$$
\Phi = \int_{r_o}^{r_1} (1 - q) B_\theta \, dr
$$

For a parabolic $q$ profile

$$
\Phi \equiv \frac{1}{q}(1 - q_o) B_{\theta 1} \eta_1 \quad (1)
$$

where $B_{\theta 1} = B_\theta (r_1)$.

**Electric Field**

The electric field in the layer resulting from the reconnection of the helical flux can be estimated by noting that, in a full reconnection, this flux is removed in the time, $\tau_c$, of the sawtooth collapse. Then, since there is no electric field in the frame of the core, we have

$$
E = \frac{\Phi}{\tau_c} \quad (2)
$$

**Resistive Model**

The current density which would be expected from the resistive model is

$$
j = \sigma \frac{\Phi}{\tau_c} \quad (3)
$$

where $\sigma$ is the electrical conductivity.

Using equations (1) and (3) together with $\sigma = 2 ne^2 \tau_e / m$, where $\tau_e$ is the electron collision frequency, the corresponding electron drift velocity is

$$
v_d \sim (1 - q_o) \frac{r_1 \tau_e}{R \tau_c} \omega_e r_1
$$
For typical JET values (with $(1-q_0) \sim 0.3$) we find

$$v_d \sim 3 \times 10^8 \text{ ms}^{-1} \ (\sim c)$$

It is clear therefore that the resistive model is inappropriate and that under these conditions the electrons would undergo strong runaway.

**Electron Behaviour**

At first sight it would appear that the runaway electron current would be "superconducting" and would prevent reconnection. However the situation is very different as can be seen from Figure 3.

![Figure 3. Behaviour of electron during reconnection.](image)

Before entering the layer an electron is effectively stationary. Once in the layer the electron has to move with the required high drift velocity along the direction of the $q = 1$ field lines. But almost immediately (in $\sim 1 \mu$s) the electron is swept out of the layer into the island. Thus the high current density has to be maintained by the continuous acceleration of electrons entering the layer. It is clear therefore that, rather than presenting a low impedance, this form of reconnection gives a high impedance.
Inertial Response

When electron inertia dominates, the corresponding term in "Ohm's law" is \((m/e) \frac{dv}{dt}\). The dominant term is \((m/e) v \nabla v\) and neglecting the density gradient the resulting equation is

\[
E + v \times B = \frac{m}{ne^2} v \cdot \nabla j
\]  

(4)

The electric field in the layer is given by the rate, \(v B^*\), at which flux is brought into the layer, where \(B^*\) is here the helical field at the edge of the layer. Thus, using Ampere's law,

\[
\nabla j \sim \frac{B^*}{\mu_0 \delta^2}
\]

and equation (4) gives

\[
\delta \sim \frac{c}{\omega_p}
\]

that is the layer thickness of the order of the collisionless skin depth.

Reconnection Time

We use the same calculation of flow continuity and momentum balance as the conventional model (1). This gives the core velocity

\[
v \sim \frac{\delta}{\tau_A}
\]

where \(\tau_A = r_1 / (B^* / \sqrt{\mu_0 \rho})\). Then, defining the reconnection time

\[
\tau = \frac{r_1}{v}
\]

we obtain

\[
\tau \sim \frac{r_1}{\delta} \tau_A
\]  

(5)
In the resistive model $\delta \sim (\tau_A / \tau_R)^{1/3} r_i$, where $\tau_R = \sigma \mu_0 r_i^2$, and substitution into relation (5) gives the Kadomtsev reconnection time

$$\tau_K \sim (\tau_A / \tau_R)^{1/3}$$

In the present model $\delta \sim c/\omega_p$ and the resulting reconnection time is

$$\tau \sim \frac{r_i \omega_p}{c} \tau_A$$

Taking JET as an example of a large tokamak, $\tau_A \sim 1 \mu s$, $\tau_R \sim 10 s$, $c/\omega_p \sim 1 \text{ mm}$, and $r_i \sim 0.3 \text{ m}$. These values give the Kadomtsev reconnection time

$$\tau_K \sim 3 \text{ ms}$$

and the inertial reconnection time

$$\tau \sim 300 \mu s$$

The observed collapse time on JET is $\sim 100 \mu s$ and it is clear that the new model is in closer agreement.

Qualifications

The calculation given above indicates that the inertial effect is predominant. The numerical values, however, are clearly imprecise. Furthermore the calculation assumes that reconnection takes place in a particular way, whereas other types of behaviour are in principle possible. Also, we have not included other physical effects such as finite Larmor radius and a possible anomalous electron viscosity (2).

There are two other features which should also be borne in mind. One is that it appears in some experiments (3, 4) that only a fraction of the flux is reconnected, $q_0$ not being restored to unity. In this case we might expect a corresponding reduction of the 300 $\mu s$ reconnection time calculated above. Another effect is the velocity space instability of the runaway electrons. Simulations (5) indicate that this might increase the effective electron mass, again reducing the calculated reconnection time.
Conclusion

The above analysis should be regarded as a clarification of the conventional reconnection model rather than an attempted description of the experimental behaviour. With so many uncertainties regarding the physics of sawtooth oscillations it is not clear that the reconnection is of this type. However it might be that examination of the consequences of the theory presented here could lead to better understanding of the experimental observations.

References

4. JET Team (presented by D.J. Campbell), IAEA-CN-53/A-VI-3, these Proceedings, Vol. 1.

DISCUSSION

S.M. HAMBERGER: Have you considered including the effect of electrostatic turbulence, and hence 'anomalous resistivity', on the current skin? This would give a skin with a minimum thickness of $c/\omega_p$, as in your purely inertial model, and would almost certainly account for the remaining discrepancy between your new model and the observation. The actual value of resistivity could be estimated from published data.

J.A. WESSON: I agree with you. We should look at that.

K.M. McGUIRE: Is your model the normal magnetic reconnection type or do you have a convective motion as in your previous sawtooth model?

J.A. WESSON: I was describing normal magnetic reconnection.
IDEAL MHD STABILITY OF
VERY HIGH BETA TOKAMAKS

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Abstract

IDEAL MHD STABILITY OF VERY HIGH BETA TOKAMAKS.

Achieving very high $\beta$ and high $\beta_p$ simultaneously in tokamaks generally implies that the second stability region against ballooning modes must be accessed. We describe several approaches for doing this, which are characterized by the choice of constraints imposed on the equilibrium profiles and the cross-sectional shape of the plasma. The combination of high toroidal beta, restricting the current density to vanish at the edge of the plasma and maintaining a monotonic $q$ profile, proves to be the most stringent. Consideration of equilibria with high $\epsilon \beta_p$ but low $\beta$ facilitates accessibility with peaked pressure profiles and high values of $q_0$. Allowing the pressure gradient and hence the current density to be finite at the plasma edge allows all surfaces to lie within the second stability regime. For free boundary plasmas with divertors, the divertor stabilized edge
region remains in the first stability regime while the plasma core reaches into
the second regime. Careful tailoring of the profiles must be used to traverse the
unstable barrier commonly seen near the edge of these plasmas. The CAMINO
code allows us to compute $s-\alpha$ curves for general tokamak geometry. These
diagrams enable us to construct equilibria whose profiles are only constrained,
at worst, to be marginally stable everywhere, but do not necessarily satisfy the
constraints on the current or $\beta$. There are theoretical indications that under cer-
tain conditions the external kinks possess a second region of stability at high $q_0$
that is analogous to that of the ballooning modes. It is found that extremely ac-
curate numerical means must be developed and applied to confidently establish
the validity of these results.

1. INTRODUCTION

Very high beta tokamaks with $\epsilon_\beta p \geq 1$ and with $\beta \gg 3I/aB$ can substan-
tially increase the attractiveness of a DT reactor design, while also offering the
possibility of using advanced fuels. Here we report on a study to find such equi-
libria which are stable to all ideal MHD modes, in particular, the $n = 1$ kink and
the $n = \infty$ ballooning modes. These requirements lead us to consider the second
region of stability. A viable configuration for a high $\beta$ second stability tokamak
reactor should ideally have the following properties: i) vanishing current density
at the edge of the plasma, ii) modest shaping of the cross-section, iii) monotonic $q(\psi)$ profile, and iv) stability against the external kink mode without needing
a nearby conducting wall. Since much of this work is motivated by the design
of the ARIES[1] device, additional constraints are imposed by engineering
considerations, i.e., aspect ratio, divertor, etc.

In this paper, we show that high $\epsilon_\beta p \geq 1$ stable configurations exist satisfying
all constraints, but that these have only modest values of $\beta_t$. Alternatively,
stable configurations are shown to exist with both $\epsilon_\beta p \geq 1$ and with very high
values of $\beta_t$ ($\beta_t \gg 3I/aB$), but these require relaxing either the edge current
density or the monotonic $q$ profile constraint.

The second stability equilibria develop steep gradients at the highest pres-
resses, requiring fine meshes for numerical calculations. In fact, knowledge of
the resolution needed in these studies has forced us not to rely too heavily on
insights gained from our previous results on high $\beta$ studies.

The second stability studies are facilitated by the use of the CAMINO[2] code
which generalizes the familiar $s-\alpha$ stability boundary[3,4] to arbitrary tokamak
geometry in a three dimensional $(s, \alpha, \psi)$ space. The normalizations of the $s-\alpha$
parameters are given by $s = 2q'V/qV'$, and $\alpha = (2p'V'/4\pi^2)(V/2\pi^2R_0)^{1/2}$, both
of which $\to 0$ as $\psi \to 0$. Here, $' = \partial/\partial \psi$, and $\psi = \Psi/2\pi$, $\Psi$ being the poloidal
flux.
An interesting set of profiles are those which are marginally stable with respect to the $s-\alpha$ stability curves at the critical point between the first and second region throughout the radius of the plasma. This technique could determine the lowest $q_0$ compatible with accessibility.

We have also examined the stability against the external kinks. Although theoretical considerations show that there is a second region of stability at high $q_0[5,7]$ which is analogous to the stability of ballooning modes, we have not yet verified this in the cases investigated, presumably because our $q_0$ values are still too low. The result of these studies will be described in the following sections.

2. Access to Second Stability in the $q_0 > 1$ Regime - theoretical considerations

The theoretical understanding of tokamak stability properties against pressure driven MHD modes is facilitated by studying the large-$q$ (both $q_0$ and $\alpha$) regime. We consider only configurations with smooth pressure and current density profiles that vanish at the plasma edge. For these, the scaling transformation $\beta_p \rightarrow \beta_p$, $\beta \rightarrow \lambda^2 \beta$, $q \rightarrow q/\lambda$, is an invariance of the equilibrium equation in the limit $q_0^2/\epsilon \gg 1$. It has been shown[5] that the ideal MHD stability equations are also invariant under such scaling for $q_0^2/\epsilon \gg 1$ and $nq \gg 1$, where $n$ is the toroidal wavenumber. In this limit, and up to corrections of the order of $1/nq$, the stability properties of all $n$-modes become equivalent. This has some important consequences, depending on whether or not a stable access to the second stability region for $n = \infty$ ballooning modes exists at high $q_0[6,7]$.

If no stable path into the second stability regime exists at high $q_0$, then the first stability limit must be expressed in terms of invariant parameters. The only such parameter that reflects naturally the fact that the beta limit results from a competition between the pressure gradient and the magnetic shear is $\beta_p q_0/q_\star$ or equivalently $q_0 \beta a B/I$. Therefore the Troyon form of the beta limit must be generalized to $q_0 \beta a B/I < C_R(q_0)$ where, in the considered case of inaccessible second stability, $C_R(q_0)$ is a weak function of $q_0$ that becomes independent of $q_0$ and $n$ as $nq_0 \rightarrow \infty$.

If, on the other hand, a stable access to the second stability region for $n = \infty$ ballooning modes exists when $q_0 > q_0^{\text{crit}}$, then all $n$-modes are stable or at worst have growth rates that decay as $1/nq_0$ as $nq_0 \rightarrow \infty$.

These theoretical results have been verified numerically with the PEST codes, for tokamak cross sectional geometries with inaccessible second stability region such as a large aspect ratio circle[5] and a small aspect ratio ellipse[7], as well as for the cases of accessible second stability region discussed below in Sec. 3.1. This opens the possibility of considering a new tokamak operating regime with moderate $\beta$, high $\beta_p$ and relatively low current.
3. Numerical Results

3.1. Maximizing Beta

The global MHD stability properties of families of equilibria can be summarized in the space defined by 
\[ (q*/q_0, q_0, \epsilon \beta_p) \],
where \( q_* = \frac{\pi a^2 B_t (1 + \kappa^2) / (\mu_0 R I_p)}{a^2 \kappa / (\mu_0 I_p^2)} \), and 
\[ \epsilon \beta_p = 8 \pi^2 <p> a^2 \kappa / (\mu_0 I_p^2) \]. The ratio \( q*/q_0 \) represents the peakedness of the current density profile. For the configuration \( A = 3.0, \kappa = 1.60, \) and \( \delta = 0.4 \), the diagrams are shown in Figs. 1a and 1b, looking at the \( q_* / q_0 \) versus \( \epsilon \beta_p \) plane. Fig. 1a displays projections of various \( q_0 \) values showing the stability boundaries to \( n = \infty \) ballooning modes. For the cases shown in this figure the current density, pressure, and pressure gradient go to zero at the plasma edge. The diagram indicates that for peaked current density profiles there exists a value of \( q_0 \) with access to a second stable region. The diagram further shows that as the current profile is broadened the value of \( q_0 \) allowing access to, or even the presence of, a second stable region increases.

In the global parameter space, we can express \( \beta_t \) as
\[ \beta_t = \frac{1}{q_0^2 (q_* / q_0)^2} C(\kappa), \]
where \( C(\kappa) \) depends only on the plasma shape. A stable operating point with the largest \( \beta_t \) would be in the lower right corner of Fig. 1a, with \( q_0 \) as low as possible. The instabilities that occur in this region of parameter space are localized near the plasma edge. We will show that one approach to avoid instability in this region is to allow the pressure gradient \( p'(\psi) \) to be non-zero at the plasma edge, thus implying that the current density also must be non-zero there. Figure 1b illustrates how significant gains can be obtained in the maximum stable \( \beta \) value by allowing finite edge gradients. The curve marked \( J(\alpha) = 0 \) on this diagram is the same as the \( q_0 = 2 \) curve in Fig. 1a. The lower stability curve marked \( J(\alpha) \neq 0 \) also has \( q_0 = 2 \), but allows finite edge gradients. We see from the \( \beta_t \) contours also on this graph that the maximum stable \( \beta_t \) value has increased from about 3% to 18% by this modification of the profiles.

By examining the local \( s-\alpha \) diagrams from CAMINO at different surfaces, we see that there is a qualitative change that occurs on these diagrams for surfaces near the edge when going from low to high \( \beta \) configurations as shown in Figs. 2a and 2b. At low \( \beta \), the equilibrium profile traces a trajectory that passes under the unstable region in the \( s-\alpha \) diagram. Most of the plasma is in the second region of stability (towards the right in the diagram), while the edge of the plasma is in the first stable region (towards the left in the diagram). The connecting transition region lies in the stable zone with \( s > 0 \) everywhere. A typical low \( \beta \) stable trajectory is marked “Method I” in Fig. 2a.

In contrast, we show the instability regions at high \( \beta \) in Fig. 2b. The instability region in the \( s-\alpha \) plane for the flux surfaces near the plasma center
FIG. 1. (a) Global parameter space showing ballooning mode stability boundaries for families of equilibria having several values of $q_0$ and zero edge current. Note that $C_T = \beta/(I_aB_q)$. (b) Same space with only the $q_0 = 2$ stability boundaries for families of equilibria with and without the $J(a) = 0$ constraint. $\beta$, contours are also shown.

FIG. 2. ($s-\alpha-\psi$) CAMINO-like diagrams showing the motion of the stability boundaries with increasing $\beta$ and equilibrium trajectories for (a) a low $\beta$, high $\beta_p$ sequence (Method I), and (b) high $\beta$, high $\beta_p$ sequences with finite edge gradients (Method II), and with a negative shear region (Method III).
remains relatively unchanged. However, for surfaces near the plasma edge, the instability region descends downward in the $s-\alpha$ plane, intersecting the $s = 0$ axis. This behavior prevents a stable trajectory connecting the first and second stable regimes, unless the trajectory lies partially in the region where $s < 0$. Thus there are no high $\beta$ stable equilibria that simultaneously satisfy the constraints that $J(\alpha) = 0$ (implying $p'(\alpha) = 0 = \alpha$) with monotonically increasing $q$-profiles ($s > 0$) everywhere.

In the discussion of Fig. 1b we showed the advantage of allowing finite $p'$ at the plasma edge, corresponding to “Method II” in Fig. 2b. In section 3.2 we discuss another possible stable trajectory with $s < 0$ over some region (“Method III”).

3.2. **Divertor D-Shaped Tokamaks**

High beta stability in divertor D-shaped plasmas is of interest because of the steep pressure gradients just inside $\psi_x$, observed in the H-mode plasmas[8]. A recent theoretical work[9] suggested that an X-point inboard to the plasma major radius tends to stabilize the ballooning modes in its vicinity. However, the results in Sec. 3.1 indicate that the edge region of limiter D-shaped plasmas

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**FIG. 3.** Asymptotic coefficient of ballooning modes at large poloidal angles ($10 \times 2\pi$) for the family of FCT equilibria with $A = 4.5$, $k_x = 2.0$, $\delta_x = 0.7$, $q_0 = 1.78$, $q_{95} = 4.3$, and the indicated values of $\epsilon \beta_p$ and $\beta_w$. 

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with vanishing edge currents often tends to remain unstable in otherwise stable plasmas of the second regime. Ideal MHD stability analysis on flux surfaces arbitrarily close to $\psi_X$ is therefore needed to clarify these apparently opposing effects in very high $\beta$ equilibria. For this purpose we developed new formulæ[10] to solve the ballooning coordinates ($\zeta, \beta$) and the ballooning equation simultaneously on any closed flux surface in the $(R, Z)$ coordinates.

The ballooning stability analysis is carried out for families of FCT equilibria (see Sec. 3.3.2). An example with $A = 4.5$, $\kappa_X = 2.0$, $\delta_X = 0.7$, $q_0 = 1.78$, $q_{95} = 4.3$ and $\beta_p = 0.68 - 3.9$ is of particular interest. In this case, relatively broad profiles of $q(\psi)$ and $p(\psi)$ that ensure a zero edge current density are used, giving $\ell_i = 0.65 - 0.47$ and $I/aB = 0.70 - 0.93MA m^{-1} T^{-1}$. The results of ballooning stability for the entire divertor plasma are plotted in Fig. 3.

As can be seen, the region immediately within the divertor edge remains stable despite large increases in the pressure gradient. This confirms the previous analytic results by Bishop[9], and is consistent with the experimental indications of the H-mode plasmas[8]. However, this region remains in the first regime since the stability degrades with an increasing pressure gradient. On the other hand, the plasma core is clearly in the second regime, as is evidenced by an improving stability with the increasing pressure gradient. There nevertheless exists a small zone of relatively unstable flux surfaces between these two regions of strong stability. Instability is seen to set in when $\epsilon\beta_p \geq 0.6$, and migrate towards the edge as $\beta$ increased further. Stable passage to the second regime in D-shaped divertor plasmas with a vanishing edge current therefore must include this first-regime edge plasma, in contrast with the approach discussed above.

The results of Fig. 3 also suggest that some profile modification is needed near the edge to obtain complete ballooning stability for high values of $\epsilon\beta_p$. We have achieved this for divertor plasmas by decreasing the pressure gradient in this region of instability. Combining with our procedure to ‘freeze’ in the $q$-profile in the interior of the plasma while enforcing the condition that the current density vanish at the plasma boundary, this can be achieved by reversing the shear (causing $q' < 0$) in a small region near the plasma boundary. The resulting trajectories in $s$–$\alpha$ space are similar to those marked “Method III” in Fig. 2b. We have verified that with these pedestal profiles, the entire plasma can remain stable to ballooning modes for $\epsilon\beta_p \gtrsim 1$ and for $\beta \gtrsim 10I/aB$.

3.3. Numerical Considerations

3.3.1. Fixed-Boundary Plasmas

We have increased our capability to perform the studies presented here: A fixed boundary flux coordinate equilibrium code[11] now explicitly accepts the pressure $p(\psi)$ and the current density $J_{oh}(\psi) \equiv (J \cdot B)/(B \cdot \nabla \psi)$ as the two prescribed functions in the Grad-Shafranov equation, thus allowing profiles which are consistent with current drive, and in which the current density can go...
smoothly to zero at the plasma edge. Also, fine structures near the plasma edge have necessitated improvements in the accuracy of our numerical algorithms.

A difficulty that has plagued all of the second stability analyses is accuracy of the equilibrium, the mapping, and consequently the stability calculations. The lack of accuracy is primarily at the plasma edge, where gradients are steep for high pressure plasmas. It is typical of previous studies that equilibria used in second stability analyses have about 50 flux surfaces and 50 poloidal “angle” points (0 ≤ Θ ≤ π). This is found to be insufficient to resolve the equilibrium in high pressure plasmas. In order to examine high accuracy equilibria, a flux-coordinate fixed boundary equilibrium code, derived from [11], has been modified. The calculations can be done in ψ, allowing for packing of grid points toward the plasma edge. In addition, the ability to double the number of ψ and Θ zones and begin the calculation from the latest converged equilibrium has been added enabling very efficient, high accuracy equilibria. A “ghost” flux surface just beyond the last plasma flux surface is introduced to allow for central differences to be used to enforce the equilibrium condition on the last surface, further improving the accuracy at the plasma edge. The mapping procedure is only used to determine metric quantities without interpolation and the stability analysis is done on the equilibrium mesh.

\[ Flux = 0.92 \]
\[ \lambda = -0.39 \]
\[ Flux = 1.0 \]
\[ \lambda = -0.35 \]
\[ \lambda = -0.20 \]
\[ \lambda = -0.12 \]
\[ \lambda = -0.10 \]

\( \otimes \) labels most unstable surface

\( \text{FIG. 4. Computed } n = \infty \text{ ballooning instability regions, maximum growth rates and locations of most unstable surfaces for increasing numerical resolution.} \)
As an example of the required resolution for second stability plasmas, a bean shaped plasma case is examined. Shown in Fig. 4 are flux surface numbers for five different $\psi \times \Theta$ grid spacings: $51 \times 51, 101 \times 101, 201 \times 201, 401 \times 401, 601 \times 601$. Indicated on the figure are the band of flux surfaces that are high-$\alpha$ ballooning unstable and the surface with the largest growth rate. It is clear that only after $201 \times 201$ is the unstable region in the plasma converging, i.e., the growth rate as a function of the reciprocal of the square of the number of zones shows linear convergence only after the $201 \times 201$ zone size. The actual zone size required would be a function of the gradients in the specific plasma configuration, however, the high pressure equilibria of interest for reactors require very high plasma edge resolution - better than what is typically used.

3.3.2. Divertor Plasmas

For divertor D-shaped plasmas, we compute free-boundary equilibria in $(R, Z)$ coordinates and exercise precise control of the plasma shape using a control matrix technique[12]. In this approach, the plasma shape is corrected before recomputation of the plasma current density $J_\phi$, which is used to update $\psi$ at the boundary of the rectangular mesh in $(R, Z)$. As a result, the number of updates to the free-boundary equilibrium and the total computational time are reduced by an order of magnitude from the previous approach[13]. This improvement in computational speed made it possible to solve the divertor plasma equilibrium with prescribed $q(\psi)$ and $p(\psi)$ functions, the so-called flux-conserving tokamak (FCT) equilibrium[14] in free-boundary, whilst it was successful previously only for fixed-boundary non-divertor equilibria. Numerical difficulty associated with the singularity of $q$ at the divertor flux surface $\psi_R$ is avoided by requiring that $F(\psi) \equiv RB_\phi$ be put in terms of cubic splines so as to reproduce the input $q(\psi)$ at a number of evenly distributed flux locations (up to 8 locations), and to enforce the condition that the current density vanish at the plasma boundary[14]. The solutions of the ballooning coordinates $(\zeta, \beta)$ and ballooning mode equation are made accurate by using bi-cubic spline fits to the equilibrium solutions of a mesh size up to $129 \times 257$ in the $(R, Z)$ coordinates.

3.4. $n = 1$ Kink Modes

Preliminary examination of the growth rate verses beta (see figure(5), 193 zones case) of the free boundary kink mode with the PEST code indicated that this mode might have a second region of stability like the ballooning mode. However, a convergence study of the growth rate of this mode shows that this is not so at the attempted values of $q_0 \sim 2 - 3$.

These equilibria need a large number of zones to accurately predict the growth rate of the mode because of the existence of large gradients in equilibrium quantities. The magnitude of $q'(\psi)$ becomes very large near the plasma edge and gets larger as the $\beta$ is increased, thus making the large $\beta$ cases harder
to converge. This convergence study has required running the equilibrium, mapping and stability codes with an extremely large number of zones. Cases have been run starting with a (97,97) mesh in ($\psi, \theta$) and doubling until equilibria with $\sim (769,769)$ zones are produced. We have found it important to obtain converged solutions using the same zones for the equilibrium, mapping and stability codes. Since the codes are only second order accurate, we look for the growth rate to vary linearly with the square of the number of zones. Indeed for a large number of zones we find such a behavior.

A determination of the critical wall position for these modes has also been completed. For this study the critical wall position was calculated for each zone size and again a linear fit to the square of the zone size was made for each beta. The critical wall distance for complete stability to the free boundary $n = 1$ mode is found to be about one-half the minor radius and increases with $\beta$.

3.5. Marginally Accessible Profiles

Unless the profiles chosen for the equilibrium calculations are constrained in detail the results of the parameter studies should depend on the functional forms
chosen. In fact, analysis using the CAMINO code shows that the plasmas are usually either all or partially in the second region throughout their radial profile, and detailed changes in the profiles within the stable regions could quantitatively alter the stability picture. One way to make these profiles more unique is to construct them such that the plasma is marginally stable at the accessibility point for every flux surface. This point is chosen to be at the minimum of the stability boundary curve in the $s$-$\alpha$ plane. This procedure gives unique prescriptions for $p'(\psi)$ and $q'(\psi)$ and since we require $p(\alpha) = 0$ then $p(\psi)$ will be unique, but $q(\psi)$ will be determined only to within an arbitrary constant which is taken to be $q(0)$. This, as is well known, strongly influences the stability of the equilibrium. The constraints that the current or the pressure gradient vanish at the plasma edge cannot be imposed on these CAMINO profiles. In practice we seek profiles that are only, at worst, marginally stable everywhere. In particular, since a balloon stable plasma at $\psi = 0$ is generally infinitely stable there we seek marginal stability beginning only at a finite distance away from the axis.

**Table 1.** Results from the calculations of the marginally accessible profiles.

<table>
<thead>
<tr>
<th></th>
<th>Circle</th>
<th></th>
<th>Dee</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_0$</td>
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<td>2.0</td>
<td>1.5</td>
</tr>
<tr>
<td>$q_a$</td>
<td>12.9</td>
<td>6.1</td>
<td>3.2</td>
</tr>
<tr>
<td>$\beta_{av,%}$</td>
<td>0.47</td>
<td>1.4</td>
<td>2.7</td>
</tr>
<tr>
<td>$\beta_I$</td>
<td>1.2</td>
<td>0.95</td>
<td>0.66</td>
</tr>
<tr>
<td>$C_T$</td>
<td>2.0</td>
<td>2.9</td>
<td>3.3</td>
</tr>
<tr>
<td>$q_0C_T$</td>
<td>5.9</td>
<td>5.7</td>
<td>5.0</td>
</tr>
<tr>
<td>Shift/$\alpha$</td>
<td>0.37</td>
<td>0.34</td>
<td>0.29</td>
</tr>
</tbody>
</table>

The results of obtaining these CAMINO profiles for two shapes – a circular, and a D-shape with $\kappa = 1.5$ and $\delta = 0.3$, both with aspect ratio 3.0 are shown in Table 1. For the circular case, as $q_0$ is decreased, $q_a$ decreases sharply, thus flattening the current profile and causing the edge current to increase. The averaged beta, $\beta_{av}$, together with the Troyon factor, $C_T$, increases such that $q_0C_T$ approaches a constant in accordance with the results of Sec. 2, even though $J(\alpha) \neq 0$ for these cases. Except for the vicinity of the origin the entire circular plasmas are all marginally stable, whereas the D-shaped configuration is marginal up to about 0.8 of the flux and stable from then on. Further iterations are needed to make the rest of the latter configuration completely marginal all the way to the edge. This would enhance the shear at the edge even more than that implied from the table.
4. Conclusions

The present study is highly relevant to advanced fuel reactor concepts based on a stable high β tokamak equilibrium which is of interest to the ARIES studies[1]. The operating point discussed in Sec. 3 with β up to 18%, A ~ 3 – 4, and q₀ = 2 should form the basis for such a reactor design. The accessibility paths demonstrated in this work lend further credibility to this approach.

One of the primary results of this study is that extremely fine resolution is required to obtain accurate numerical results for the equilibrium and stability of very high beta plasmas. It is especially important to resolve the boundary layer near the plasma edge if one is to obtain even a qualitative picture of the stability properties of these equilibria.

The CAMINO code has shown that, near the plasma edge, the instability region in the (s, α, ψ) space moves down at sufficiently high beta to intersect the s = 0 axis. This may exclude the possibility of crossing back to α = 0 by passing under this curve, but keeping s > 0. High beta stable plasma equilibria are still possible by staying to the high pressure side of the instability diagram everywhere, implying finite values of p'(ψ) and hence of J at the plasma edge, or by allowing the shear to reverse sign in a small band near the outside surface.

For lower beta equilibria that have the current density going to zero at the plasma boundary and have positive shear everywhere, we have shown that stable paths to high values of εβ near the equilibrium limit exist. The stability properties of these equilibria are well characterized in a “global s-α” space with q*/q₀ as one axis and εβ as the other, and q₀ as the the third. Access to the equilibrium limit is gained for sufficiently large values of q₀ > 2.5 – 3.0, and for sufficiently large values of q*/q₀ > 2 – 3. These equilibria, while being of modest β value, can exceed the conventional β limit, β < 3.0I/aB by factors of 2 or more, and can exceed the rationalized β limit, β < 3.0I/aBq₀ by factors of 6 or more.

For each value of q₀, there is a unique set of CAMINO or “gateway profiles” that form a line in the (s, α, ψ) space lying on the critical point of the s-α subspace everywhere. We have demonstrated that these profiles can be calculated, and may be of interest as they provide a unique prescription for accessing the second stability region on all surfaces simultaneously.

The n = 1 kink mode stability of these very high beta equilibria is exceedingly difficult to calculate accurately. There are indications that the n = 1 kink mode may restabilize at sufficiently high values of εβ and q₀, but limits on the numerical resolution capabilities prevent us from making definitive statements at present.

ACKNOWLEDGEMENTS

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DISCUSSION

H.L. BERK: Has your analysis taken into account the effect of localization off-axis, i.e. finite $\theta_\phi$?

M.S. CHANCE: Yes, the analysis does take this into account.

W.M. NEVINS: It seems that the $n = 1$ kink modes must have a lot of structure at the plasma edge when $J(a) \neq 0$ since you need very good grid resolution in this case. Are you concerned that non-ideal effects at the plasma edge may affect your conclusions in this limit?

M.S. CHANCE: The case shown had $J(a) = 0$, and resolution was needed in the equilibrium calculation because of the large shear at the edge. We agree that the results may be modified by non-ideal effects, as they are for $n = \infty$ ballooning modes.
INFLUENCE OF TRIANGULARITY AND PRESSURE PROFILES ON IDEAL-MHD BETA LIMITS

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Abstract

INFLUENCE OF TRIANGULARITY AND PRESSURE PROFILES ON IDEAL-MHD BETA LIMITS.

Ideal-MHD stability computations are presented to show the effects of triangularity and pressure profiles on β-limits for plasmas of fixed elongation $\kappa = 2$. For broad pressure profiles, triangularity has a weak effect, whereas for peaked pressure profiles, triangularity clearly increases the β-limit. For sufficient triangularity, $\delta \geq 0.4$, peaked pressure profiles give β-limits similar to those of broad ones and give some improvement in $\beta^*$.

Tokamaks of the next generation are designed to take advantage of strongly shaped cross-sections which allow significantly higher plasma current and higher $\beta$ than circular ones. The scaling law

$$g = \frac{\beta[\%]}{I_N} \leq g_{\text{max}} = 2.8, \quad I_N = \frac{\mu_0 I_p}{a B},$$

was originally obtained [1] for D-shaped cross-sections and broad pressure profiles, and it is important to know in detail how the β-limit, as well as other operational limits, depend on plasma shape and equilibrium profiles. Here, we report on ideal-MHD stability calculations to study the dependence of the β-limit on triangularity and pressure profiles. Stability has been determined for free-boundary $n = 1$ (kink) and $n = \infty$ (ballooning and Mercier) modes. We have considered triangularities $\delta = 0.0, 0.2, 0.4$, and 0.6 at fixed aspect ratio $A = 3.7$ and ellipticity $\kappa = 2$, motivated by NET/ITER design.

Frequently, experimental pressure profiles are more peaked than those that give the highest $g$ in (1), and the peaking could become even more pronounced in an ignited plasma by strong on-axis peaking of the $\alpha$-particle production. Consequently, it is important to know whether the β-limit is degraded by peaking of the pressure and also how the confinement of peaked pressure profiles depends on shaping parameters.
To separate the influence of shaping and equilibrium profiles, we have used two different optimisation procedures. In a first set of runs, current profiles were specified and the pressure profiles were optimised. The profiles of toroidal current are flat in a central region and decay monotonically toward the edge, where both $<J_\phi>$ and $d<J_\phi>/d\psi$ vanish. This leads to optimised pressure profiles that are broad, with the pressure gradients concentrated in the outer region of high shear. For these centrally flat current profiles and $q_0$ close to unity, ballooning modes are the most limiting for $\beta$, except at high triangularity, $\delta = 0.6$, and high current, $q_s < 3$. The resulting $\beta$-limits are shown in Fig. 1 as functions of normalised current $I_N$. Here, $q_0$ has been fixed at 1.07. For these profiles, local stability requires that the pressure gradients be small in the central, low-shear region. Some improvement in central confinement occurs for high triangularity $\delta = 0.6$, which explains the slight increase in maximum $\beta$, from about 4.6 % for $0 \leq \delta \leq 0.4$, to 5.6 % at $\delta = 0.6$. Nevertheless, the influence of triangularity on the $\beta$-limit is weak for broad pressure profiles.

![Graph showing $\beta$ vs. current for triangularities $\delta = 0, 0.2, 0.4, and 0.6$. A centrally flat current distribution has been imposed. The optimised pressure profiles are broad.](image)

Figure 1 also shows that the linear dependence of $\beta_{max}$ on current is only an approximation, best valid at high $q$ (for surface $q$, $q_s \geq 4$). Saturation occurs at high current, typically for $q_s = 3$. For $q_s$ between 2 and 3, $\beta_{max}$ even decreases with increasing current. For all triangularities shown in Fig. 1, $g = \beta / I_N$ decreases from about 3.0 at the lowest current ($q_s = 6.5$) to about 1.8 near the current limit at $q_s = 2$. In the range of practical interest, the $\beta$-limit is rather weakly dependent on current, with a broad maximum for $q_s$ near 3.
Figure 1 also shows that $g = 2.2$, as proposed originally in [1], gives a good approximation to the maximum $\beta$ achieved for each triangularity.

To study the $\beta$-limit for strongly peaked pressure profiles, we have applied another optimisation procedure. Here, we fixed the pressure profile so that $dp/dr = 0$ (where $r$ is a generalised minor radius) in a central region $r < r_p$ and $dp/dr \approx \text{constant}$ for $r > r_p$. The current profile was then optimised to compute the $\beta$-limit for two cases: $r_p / a = 0.4$ (moderate peaking) and $r_p / a = 0.1$ (strong peaking). The main problem posed by the peaked pressure profiles is the destabilisation of local modes in the region where the pressure gradient is large close to the magnetic axis (just outside $r = r_p$). These modes may be stabilised in two different ways: either by peaking the current profile in the centre to increase the shear or by globally flattening the current profile to increase $q_0$ and improve on average curvature. For ballooning modes, these two approaches correspond to increasing the stable pressure gradient in the first stability region or to accessing second stability.

Moderate peaking does not significantly degrade the $\beta$-limit from that obtained for the broad profiles. For $q_s > 4$, no change occurs at all, whereas some reduction is found at low $q_s$ and small triangularity. For $q_s = 2.5$ and an elliptical cross-section, the $\beta$-limit falls from 4.5 % for the broad profiles to about 3.7 % for the moderately peaked pressure profile with an optimised high-shear current distribution. For a highly triangular cross-section $\delta = 0.6$, the reduction is insignificant, from 5.6 % to 5.5 %.

In the case of strongly peaked pressure profiles, we find more pronounced differences from the broad profiles. Here, triangularity has a clearly beneficial effect. The $\beta$-limits for strongly peaked pressure and high-shear

![Figure 2](https://example.com/figure2.png)

**FIG. 2.** Optimised $\beta$ vs current for different triangularities. The strongly peaked pressure profile has been imposed and the current profile has been optimised with high central shear.
current profiles are shown in Fig. 2. For sufficient triangularity, $\delta \geq 0.4$, the maximum $\beta$ decreases only slightly, to about 5.2 %, from the 5.6 % obtained for the broad profiles. However, for lower triangularities, the drop is pronounced, in particular, at low $q_s$. For example, for the ellipse, the maximum $\beta$ achieved with the strongly peaked pressure distributions is 3.0 % (at $q_s = 3.3$), while for $q_s < 3$, the limit falls to less than 1.5 %.

The instabilities most easily excited by the peaked pressure profiles are interchange or interchange-like ballooning modes in the central region. The stability of these modes is determined by the Mercier criterion. It is well known [2] that Mercier stability is favoured by triangular shaping, shear, and high $q$. The current profiles used for Fig. 2 have high central shear, obtained by using a step in the current density near $r = r_p$. Examples of such current profiles are shown in Fig. 3. Note that the elliptical case has stronger central current gradients, yet supports less pressure than the case $\delta = 0.6$. Apart from shaping, one beneficial effect of triangularity on central stability is to increase the shear for a given current profile. This also implies that a strongly triangular cross-section requires a broader current profile with lower internal inductance for given values of $q_0$ and $q_s$. Nevertheless, with a suitable current profile, surface kink modes can be avoided, and stable high-$\beta$ equilibria with $q_s$ down

![FIG. 3. High-shear current profiles optimised for strongly peaked pressure. In (a) $\delta = 0.6$, $q_s = 3.65$, $\beta = 5.2$ % and $\beta_0 = 14.7$. In (b) $\delta = 0$, $q_s = 3.31$, $\beta = 2.9$ % and $\beta_0 = 8.0$.](image)
held fixed, the increased central shear is destabilising for the kink modes and that the cases in Fig. 2 are marginal both to kink and ballooning modes. Experimentally, profile control will be necessary and some non-inductively driven current may be required to maintain in steady-state the broad profiles that are optimal for high triangularity. With sufficient triangularity, neoclassical peaking of the current should be sufficient to ensure local stability in the centre.

Results for peaked pressure, similar to those in Fig. 2, were obtained also by using centrally flat current profiles and raising \( q_0 \) well above unity. This improves on average curvature and allows access to the second region of stability for ballooning modes. However, for such equilibria with strong pressure gradients in a central region of low shear, the \( \beta \)-limit is somewhat decreased by intermediate-\( n \) ("infernal") modes [4]. The infernal modes do not affect the \( \beta \)-limit computed for \( n = 1, \infty \) of the high-shear current profiles.

In conclusion, triangularity is clearly favourable for the stability of peaked pressure profiles in an elongated cross-section. The stabilisation is due both to shaping which makes the field lines linger in the region of good curvature, and to the changed relation between the current and \( q \)-profiles. Although the limit in volume average \( \beta \) falls somewhat when the pressure is peaked, these profiles appear advantageous for reaching ignition. For instance, for \( \delta = 0.4 \), the limit in \( \beta^* = <\beta^2>^{1/2} \) increases from 5.5 % for the broad profiles to 6.4% for the strongly peaked profile. The central \( \beta \) increases from about 10 % to 15 %.

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REFERENCES

DISCUSSION

J.L. JOHNSON: To get to your optimum $\beta$'s, you had to peak the current near the axis. We would probably rely on RF current drive techniques for current profile shaping. Do you have any ideas as to how you can provide central peaking, since the current drive is primarily nearer the edge?

A. BONDESON: Peaking in the centre is expected to occur in Ohmic discharges as a result of neoclassical resistivity. Since the required peaking is rather weak for a triangularity larger than about 0.4, the neoclassical peaking should be sufficient.

R.J. GOLDSTON: You couldn't do a full two dimensional scan in peakedness and triangularity, understandably. Could you nonetheless give us some idea of the point in peakedness $\left( p_0/(p) \right)$ where triangularity becomes significant?

A. BONDESON: The 'strongly peaked' profiles, where triangularity is clearly favourable, have a peaking factor $p_0/(p) = 2.7$, and for the 'moderately peaked' profiles, where triangularity has a weak effect, the peaking factor is about 1.9.
TEARING AND RESISTIVE BALLOONING MODES IN TOKAMAKS

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Abstract

TEARING AND RESISTIVE BALLOONING MODES IN TOKAMAKS.

The asymptotic matching approach to the stability of tearing modes and resistive ballooning modes in toroidal geometry separates the calculation into two parts. One involves the calculation of solutions of ideal marginal MHD equations in the regions between resonant surfaces, the other requires the solution of the full plasma equations in the vicinity of the resonant surfaces — the matching of these solutions provides the dispersion relation which determines the stability of the mode. The paper addresses both of these subjects for a realistic hot plasma in a large aspect ratio tokamak.

1. Tearing and Ballooning Mode Structure

In the first section we describe how the information from the ideal MHD solutions can be compactly represented in terms of a so-called E-matrix. The two cases of tearing and ballooning modes must be distinguished. For tearing modes with a given toroidal mode number $n$, the toroidal geometry produces a coupling of different poloidal harmonics. The E-matrix provides a generalization of the $\Delta'$ quantity familiar from the cylindrical theory of tearing modes. For a large aspect ratio, finite pressure tokamak the elements of the E-matrix can be expressed in terms of the cylindrical tearing mode solutions. The dispersion relation shows that the toroidal effects can only have a significant effect on tearing stability when the system is almost marginal to one or more of the poloidal harmonics in the cylindrical limit. Thus, if several of these poloidal harmonics have small values of $\Delta'$ then the toroidal mode can contain a strong admixture of them, and its
stability depends on the properties of all. In particular, the E-matrix can be expressed in terms of coupling integrals—a generalization of the result of Edery et al.\(^3\) to finite pressure. However, whereas the toroidal tearing mode can thus usually be regarded as a perturbation about a cylindrical mode, the resistive ballooning (or twisting) mode\(^4\) is found to be an intrinsically toroidal mode. This is evident from analysis of the high-n limit where, by including the effect of average curvature in the solution of the Strauss model,\(^5\) it can be shown that a transition from a stable tearing \(\Delta^r\) to an unstable twisting \(\Delta^t\) occurs when ballooning effects exceed interchange effects; i.e. when \(\alpha > \delta\), which is the situation in a tokamak. [Here, \(\alpha = -2Rp^2q^2/B^2_0\) is the ballooning parameter, while \(\delta = \alpha\varepsilon(1 - q^2)/q's^2\) measures the effect of average curvature.]

In order to extend the high-n analysis to low-n modes it is necessary to formulate the problem in configuration, rather than ballooning space. When \(\delta \neq 0\) the matching condition for high-n modes in terms of the ‘large’ and ‘small’ solutions can be applied in either space. However, unlike the tearing parity case, the twisting parity possesses a continuous solution (due to toroidal coupling) in addition to the ‘large’ and ‘small’ solution. In fact, the solution for the potential takes the form \(\varphi = 2\pi\delta(x) - 2\Delta_B^{TW} \log|x|\) near a resonant surface \(x = 0\), where \(\Delta_B^{TW}\) is the discontinuity in ballooning space. The high-n model of Ref. 4, where \(\delta = 0\), can then be analysed in configuration space, when it is seen that, by expanding in \(\alpha\), the \(\delta\)-function singularity drives \(O(\alpha)\) discontinuities in the adjacent sidebands at \(x = 0\) which must satisfy vanishing boundary conditions as \(|x| \to \infty\). These in turn generate in \(O(\alpha^2)\) the logarithmic singularity in the fundamental harmonic, leading to the appropriate expression for \(\Delta_B^{TW}\). This prescription can be readily extended to low-n modes by requiring the sidebands to satisfy the appropriate boundary conditions at the magnetic axis and wall. In the limit of small growth rate the result is an E-matrix, depending only on the profiles of \(\alpha(r)\) and \(q(r)\) and the mode number \(n\). Evaluation of the elements of the E-matrix requires computation of the cylindrical tearing mode solutions. For the simple low \(\beta\) resistive MHD model the eigenvalues of this matrix characterize stability and the most unstable one is shown in Fig.1. At large but finite \(n\), if the kink term \(\sim j_0^2/n\) is neglected, analytic expressions for the tearing mode solutions can be used and the resulting approximate eigenvalues are seen to be in close agreement with the exact eigenvalues. As \(n \to \infty\) the results asymptote to the ballooning limit

\[
\Delta_B^{TW} = \frac{\pi\alpha^2}{4s^2}(s + 2)\left[1 + \frac{(s + 2)}{s} \exp\left(-\frac{2}{s}\right)\right]
\]
FIG. 1. Open circles show the variation of the normalized stability parameter $\Delta'/(\beta_0^2(\%))$ as a function of mode number $n$ for a pressure profile $p = p_0(1 - r^4/a^4)$ and a $q$-profile $q = 1.22(1 + 2r^3/a^3)$. These numerical results can be well represented by the approximation (shown as dots) at large but finite $n$. The $n \to \infty$ limit, Eq. (1), for the most unstable mode (which occurs at $r/a = 0.91$) is also shown.

where it should be noted that one must choose the most unstable value $\theta_0 = \pi/2$ for the ballooning phase angle, rather than the value $\theta_0 = 0$ used in Ref.4.

2. Resonant Layer Dynamics

For a high temperature plasma such as occurs in JET the complementary part of the matching problem, i.e. the resonant layer solution, must be based on a realistic kinetic plasma model incorporating diamagnetic, semi-collisional and trapped particle (i.e. neoclassical MHD) effects. Such a description, and its consequences for stability, are described in the following.

The appropriate linear layer equations are derived from an expansion of the gyro-kinetic equations for a low beta, large aspect ratio tokamak. The two limits in which the ion Larmor radius is either much greater than or less than the resonant layer width are considered. Both ions and electrons are assumed to be in the banana regime of collisionality, while the mode frequency is comparable with the diamagnetic frequency, and is such that
FIG. 2. The critical $\Delta'$ [normalized with respect to the length $\rho_{\ast\text{eff}} = (B/B_0)(T_e/2T_0)^{1/2} \rho_0$] for marginal stability of a low-$m$ tearing mode in the presence of $O(1)$ shear, plotted as a function of the collisionality [parameterized by $C_{\text{eff}} = (B_0^2/B^2)(\nu_e / \omega_e, \nu_m / \omega_m)$; $C_{\text{eff}} \ll 1$ corresponds to the semi-collisional regime, $C_{\text{eff}} \gg 1$ to the collisional regime] and the fraction of trapped particles [parameterized by the inverse aspect ratio, $\epsilon = r/R$]. Note that the presence of only a modest fraction of trapped particle strongly destabilizes the mode in the semi-collisional regime.
the semi-collisional regime is appropriate for the electrons. It is also assumed that both species are 'fluid-like', i.e. that their collision frequencies are significantly greater than the mode frequency. These conditions are all relevant to a realistic description of low- \( m \) modes in present tokamaks and are necessary extensions of the so-called 'neoclassical MHD' theory which describes effects of trapped electrons, such as bootstrap currents, and the neoclassical enhancement of ion polarization drift. A shooting code has been used to solve these resonant layer equations and, by matching to the external ideal MHD solutions, provides a linear dispersion relation for tearing modes.

This theory has been applied to long wavelength modes in the small ion Larmor radius limit.\(^9\) The strong semi-collisional stabilization of (constant-\( \psi \)) tearing modes in cylindrical geometry\(^{10} \) is significantly modified by the presence of trapped particles when the magnetic shear at the mode rational surface is \( O(1) \). This effect is illustrated in Fig. 2. Note how, in the semi-collisional regime, the presence of only a relatively modest fraction of trapped particles changes the critical \( \Delta' \) which must be exceeded in order for the tearing mode to grow, from a large positive value (corresponding to a strongly stable mode) to a large negative value (corresponding to a strongly unstable mode). At low shear, the tearing mode is found to be virtually unaffected by trapped particles, and is thus strongly stable in the semi-collisional regime. In the collisional regime, however, a relatively weak trapped particle driven interchange mode becomes unstable. Preliminary results in the large ion Larmor radius limit also show strong destabilization of tearing modes by trapped electrons if the shear at the rational surface is \( O(1) \).

### 3. Conclusions

The tools for discussing the stability of tearing and ballooning modes in terms of asymptotic matching techniques have been developed. The external solution for the tearing parity modes arise essentially as perturbations about the cylindrical modes. However, when the ballooning effects exceed that of the magnetic well (which is indeed the case in a tokamak) the twisting parity mode becomes an intrinsically toroidal structure. Toroidicity is also manifest in the resonant layer equations that have been derived to complete the stability analysis for both of these types of mode. Thus, a consistent realistic treatment of a high temperature plasma is given which includes neoclassical trapped particle effects; these effects are shown to have important consequences for the stability of \( m = n = 1 \) modes.
REFERENCES


DISCUSSION

L.J. PERKINS: What is the physics used to represent electron and ion response in the resonant layer? How does it correspond to the physics models used for trapped electron and ion modes?

J. CONNOR: The electron and ion physics corresponds to that of neoclassical MHD, including diagnostic frequencies and semi-collisional electron dynamics. Although trapped particle effects are included, the equations are developed for a radially localized resonant layer in which ions and electrons are 'fluid-like'. Passing electron responses are not Boltzmann, and terms of the order of $\omega/\nu_{\text{eff}}$ required for trapped particle modes are not retained.
2/1 TEARING MODE STABILIZATION WITH LOCAL CURRENT DRIVE

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Abstract

2/1 TEARING MODE STABILIZATION WITH LOCAL CURRENT DRIVE.

The stabilization conditions for the m/n = 2/1 tearing mode are considered. A suitable definition of the Δ' parameter for toroidal geometry is proposed. A general criterion for tearing mode stabilization by additional local current is derived for a pressureless plasma with arbitrary cross-section. The effect of finite island width on the stabilization is considered in cylindrical geometry. A numerical code for calculating Δ' is developed and applied to the ITER configuration. The 2/1 tearing mode demonstrates the opposite dependence of its stability margin on i and q0 with respect to the ideal modes. The necessary stabilization conditions for ITER plasmas are obtained numerically.

1. 'ENERGY' FUNCTIONAL FOR TOROIDAL TEARING MODE.
DEFINITION OF Δ'

The 'energy' functional for the tearing mode yielding the stability conditions coincides with the ideal potential energy (see, for example, Ref. [1]) if the latter is written in terms of the flux function perturbation instead of the plasma displacement.

In co-ordinates with straight field lines a, θ, ζ, the equilibrium equations have the following form:

\[ B^i = \left( 0, \frac{\Psi'(a)}{2\pi\sqrt{g}}, \frac{\Phi'(a)}{2\pi\sqrt{g}} \right), \quad j^i = \left( 0, \frac{F'(a)}{2\pi\sqrt{g}}, \frac{J'(a) + \partial\nu/\partial\theta}{2\pi\sqrt{g}} \right) \]

\[ \frac{4\pi}{c} F = \frac{g_{33}}{\sqrt{g}} \Phi' \]

\[ \frac{4\pi}{c} \left( J' + \frac{\partial\nu}{\partial\theta} \right) = -\frac{\partial}{\partial a} \frac{g_{22}}{\sqrt{g}} \Psi' + \frac{\partial}{\partial\theta} \frac{g_{22}}{\sqrt{g}} \Psi' \]
where $g_{ik}$ is the metric tensor. We use a Gaussian system of units.

We shall consider pressureless toroidal configurations, i.e. $p = 0$. In this case, the energy principle (Eq. (2) of Ref. [1]) can be reduced to the simple functional written for the test function $\psi$:

$$W_p = \frac{1}{8\pi} \int d\theta d\xi \left\{ \frac{g_{11}}{\sqrt g} \left( \frac{\partial \psi}{\partial \theta} \right)^2 - 2 \frac{g_{12}}{\sqrt g} \frac{\partial \psi}{\partial \theta} \frac{\partial \psi}{\partial \xi} + \frac{g_{22}}{\sqrt g} \left( \frac{\partial \psi}{\partial \xi} \right)^2 + \psi \frac{\partial u}{\partial \theta} \left( \frac{4\pi c}{\Psi'} F' \right) \right\}$$

$$\psi = \mu \frac{\partial u}{\partial \theta} + \frac{\partial u}{\partial \xi}, \quad \mu = -\frac{\Psi'}{\Phi'} \tag{2}$$

inside the plasma and

$$W_{vac} = \frac{1}{8\pi} \int \overline{B^2} dv \tag{3}$$

in the vacuum region. Note that the $m = 0$ harmonic of the perturbation is absent in Eq. (2).

Considering a harmonic representation for test function $\psi$,

$$\Psi = \sum_{m=-\infty}^{\infty} Y_m(a) \cos [m\theta - n\xi], \quad m \neq 0 \tag{4}$$

we obtain the 1-D functional for a set of harmonics $Y_m(a)$:

$$W_p = \frac{\pi}{4} \int da \sum_{m=-\infty}^{\infty} \left\{ \sum_{j=-\infty}^{\infty} \left[ \frac{g_{11}}{\sqrt g} \cos j\theta \right] Y_m' Y_{m+j}' \right. + 2 \left\langle \frac{g_{12}}{\sqrt g} \sin j\theta \right\rangle (m + j) Y_m Y_{m+j}$$
Permitting a discontinuity in the first derivatives at the resonance surface for some particular harmonic m/n and assuming continuity for all other harmonics and their first derivatives, we can write for \( W = W_p + W_{vac} \) in terms of \( \Delta'_m \):

\[
\Delta'_m = \frac{Y_m}{Y_{m+1}} \left| \frac{Y_m}{Y_m} \right|_{a_{m+n+\epsilon}} - \left| \frac{Y_m}{Y_m} \right|_{a_{m+n-\epsilon}}
\]

This definition of \( \Delta'_m \) is the most suitable one for studying stabilization by an additional local current.

### 2. CRITERION OF STABILIZATION BY AN ADDITIONAL LOCAL CURRENT NEAR THE MODE RESONANCE SURFACE

According to our previous cylindrical consideration [2] and adding a stepwise current \( \delta j'_r \) covering the surface \( a = a_{m/n} \),

\[
\delta j'_r(r, \Psi) = \begin{cases} 
0, & a < a_{m/n} - \epsilon, \quad a > a_{m/n} + \epsilon_r \\
\Delta j'_r \equiv \frac{2}{c} \Delta \left( \frac{FP'}{\Psi'} \right), & a_{m/n} - \epsilon < a < a_{m/n} + \epsilon_r
\end{cases}
\]

to the initial current profile, from Eq. (5) we obtain an explicit input of the additional current into \( W \):

\[
W = - \left( \frac{g_{22}}{\sqrt{g}} \right) Y_m^2 \Delta'_m + \Delta \left( \frac{4\pi}{c} \frac{F'}{\Psi'} \right) \left( \frac{1}{\epsilon_l} + \frac{1}{\epsilon_r} \right)
\]

and a stabilization criterion

\[
\frac{4\pi}{c} \frac{\Delta j'_r}{B_{\mu'} \left( \frac{g_{22}}{\sqrt{g}} \right)} \left( \frac{a_{m/n}}{\epsilon_l} + \frac{a_{m/n}}{\epsilon_r} \right) > a_{m/n} \Delta'_m
\]
The generalization of criterion (9) to an arbitrary profile of the additional current $\delta j_\tau$ can, according to Westerhof [3], be performed by the following substitution:

$$\frac{\Delta j_\tau}{B_\tau} \left( \frac{1}{\epsilon_1} + \frac{1}{\epsilon_r} \right) \Rightarrow \int \left( \frac{\delta j_\tau}{B_\tau} \right)' \frac{da}{a-a_{m/n}}$$  \hspace{1cm} (10)

Thus, $\Delta j_\tau$ and $\epsilon_1$, $\epsilon_r$ in Eq. (9) may be regarded as the characteristic amplitude of the additional current and the characteristic semiwidths of the current profile, respectively.

For the additional total current $\Delta I$, we have, from Eq. (9):

$$\frac{4\pi}{c} \Delta I = \frac{4\pi}{c} \int \delta j_\tau \frac{\sqrt{g}}{r} \frac{da d\theta}{r} = 2\pi B_\tau \mu' \left( \frac{\sqrt{g}}{g_{33}} \right) \left( \frac{\mathbb{E}_{22}}{\sqrt{g}} \right) \Delta_{m/n} \epsilon_1 \epsilon_r$$

$$\Delta I = I_{pl, m/n} \mu' \Delta_{m/n} \epsilon_1 \epsilon_r$$  \hspace{1cm} (11)

where $I_{pl, m/n}$ is the plasma current inside the resonance magnetic surface $a = a_{m/n}$.

3. EFFECT OF ADDITIONAL LOCAL CURRENT ON STABILITY IN THE PRESENCE OF A MAGNETIC ISLAND

For simplicity, let us consider a cylindrical geometry of the magnetic configuration. Let the magnetic surfaces be described by the equation

$$(a - a_{m/n})^2 = w^2 \left( t^2 - \sin^2 m\theta^*/2 \right), \quad m\theta^* = m\theta - n\zeta$$  \hspace{1cm} (12)

where $w$ is the island width and $t$ is the label of the magnetic surfaces: $t = 0$ corresponds to the 0-point and $t = 1$ corresponds to the separatrix. Since the sign of $t$ in the Eq. (12) is not significant, let us assume that $t > 0$ in the region $a > a_{m/n}$ and $t < 0$ at $a = a_{m/n}$.

The following odd function (Fig. 1a):

$$F(t) = \frac{8}{\pi} \int_{0}^{\omega_0} \cos \omega \sqrt{t^2 - \sin^2 2d\omega}, \quad \omega_0 = \begin{cases} 2 \sin^{-1} t, & |t| < 1 \\ \pi, & t > 1 \\ -\pi, & t < 1 \end{cases}$$

(13)

determines the contribution of the additional stepwise current to the stabilization...
FIG. 1. (a) Characteristic function $F(t)$ for stabilization criterion (14); (b) location of most stabilizing additional current for quasi-linear tearing mode (shaded region); (c) a destabilizing case of local current drive with hole in current distribution.

Criterion. For the additional current $\Delta j_1$, located between the magnetic surfaces $t_1$ and $t_2$, $t_1 < t_2$, the stabilization criterion assumes the form

$$\frac{4\pi}{c} \frac{\Delta j_1 R}{B_{\mu}^\prime} \left( \frac{F(t_2) - F(t_1)}{w} \right) > a_{m/n} \Delta_{m/n}'$$

(14)

If the current $\Delta j_1$ is located inside the magnetic island $|t_1| < 1$, then it is assumed that $t_1 = -|t_1|$ and $t_2 = |t_1|$ in Eq. (14). For $t \gg 1$, $F(t) \propto 1/t$ and Eq. (14) transforms into Eq. (9) if we write $wt = \epsilon$. 

1.0-
0.8-
0.6-
0.4-
0.2-
0.0-

$\theta^* = \pi/2$

$\theta^* = 0$

$\theta^* = \pi/2$

$t = 0.9$

$\theta^* = 0$
The function $F(t)$ has extrema at $|t| \approx 0.9$. This means that the interior of the island $|t| < 0.9$ plays a stabilizing role (Fig. 1(b)) while the exterior of the island is destabilizing (Fig. 1(c)). In the absence of phase synchronization between local current drive and island rotation, the interior of the island will be filled by the additional current; this current drive will certainly be stabilizing. Most effective is the current drive inside the magnetic surface, $|t| < 0.9$. It is only the current drive outside the island that is destabilizing.

4. STABILIZATION CONDITIONS FOR ITER CONFIGURATION

A numerical code has been developed to calculate the $A_{m/n}$ for D-shape ITER plasma configuration. The set of equilibrium parameters is as follows: $R = 6.0$ m, $a = 2.15$ m, $\kappa = 2$, $\delta = 0.2-0.4$, $I_{pl} = 22$ MA, $B_f = 4.85$ T, and the current density was taken in the form:

$$j_r = j_0 \left[ 1 - \frac{(\Psi - \Psi_b)}{\Psi - \Psi_0} \right]^{n_l} \frac{R}{r} = A(\Psi) \frac{R}{r} \quad (15)$$

where $\Psi_b$ is the boundary value of $\Psi$. Different $q_0$ values, i.e. $q_0 = 1.0, 1.1, 1.2$ and an internal inductance given by $\xi = \int (\Psi - \Psi_b) j_r dS/(2\pi R I_{pl})$ are considered.

In Fig. 2(a), the $\Delta_{2/1}$ values versus $\xi$ are shown. We see that the $2/1$ mode is unstable and $\Delta_{2/1}$ increases with rising $q_0$. This tendency is opposite with respect to the

![FIG. 2. (a) $a_{2/1}\Delta_{2/1}$ parameter for ITER $R = 6$ m, $a = 2.15$ m, $\kappa = 2$, $\delta = 0.4$ reference configuration as a function of internal inductance $\xi$ (single harmonic approximation); (b) necessary additional current for 0.2 m localization width.](image-url)
stability of the ideal mode when the increase in $q_0$ improves the stability. The influence of triangularity on the stability of the 2/1 tearing mode is also negligible.

On the assumption of a symmetric position of the additional current with respect to the 2/1 mode rational surface $\epsilon_1 = \epsilon_r$, $\epsilon_1 + \epsilon_r = h$, the necessary requirements for additional local current $\Delta I_{pl}$ can be written in the form

$$\Delta I_{pl} \{\text{MA}\} = \Delta I_{0.2} \left(\frac{h}{0.2}\right)^2, \quad h \{\text{m}\}$$

Here, the coefficient $\Delta I_{0.2}$ corresponds to the stabilizing current if it is located in the layer of the 0.2 m width. The results of calculation of the current $\Delta I_{0.2}$ for the 2/1 mode in ITER are shown in Fig. 2(b).

5. CONCLUSIONS

The possibility of stabilizing the 2/1 mode by local current drive is confirmed for both linear and quasi-linear tearing modes. Moreover, the next order terms with respect to $\epsilon$ in the stabilization criteria are also stabilizing. For the ITER reference configuration, an additional current of the order of 0.3 MA is necessary to stabilize the 2/1 tearing mode.

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3-D MHD STUDIES OF SAWTOOTH OSCILLATIONS AND PRESSURE DRIVEN RESISTIVE MODES IN TOKAMAKS

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Abstract

3-D MHD STUDIES OF SAWTOOTH OSCILLATIONS AND PRESSURE DRIVEN RESISTIVE MODES IN TOKAMAKS.

Three-dimensional MHD codes are used to study sawtooth oscillations and pressure driven resistive modes in tokamaks. The sawtooth simulations at high temperature agree with the Kadomtsev model, but compare well with experiment only if neoclassical resistivity is used. An analytic formula for $q_o(t)$ and a semi-empirical sawtooth period scaling are presented. At high $\beta$, the helically twisted hot spot has a toroidal bulge at the large major radius side, characteristic of a high $e_o$ instability. This bulge drives various magnetic islands, resulting in a stochastic annular region. This effect, also seen in experiment, can explain the sawtooth heat pulse 'anomaly' and the 'high $\beta$ disruption'. Linear studies of pressure driven resistive modes give results consistent with various MHD effects measured in 'supershots'. The stabilizing effect of the pressure in the layer would be mitigated by finite initial islands in the experiment. This is modeled either by locally flattening the pressure profile, or by setting the ratio of specific heats, $\Gamma = 0$. Nonlinear studies show that the pressure tends to be destabilizing with multiple low mode number singular surfaces. With one low mode number singular surface present, it is often stabilizing. In supershot type discharges, finite pressure induces many islands and stochastic regions, resulting in many spikes on the pressure profile. This would cause anomalous electron heat conductivity, and may also explain the 'beta collapse' phenomenon.

1. Introduction

We have used the three-dimensional nonlinear toroidal MHD code, $MH3D$,[1] and the linear code, $ARES$,[2] to study sawtooth oscillations[3] and pressure driven resistive modes in tokamaks. The $MH3D$ code solves the full MHD equations in toroidal geometry, and can treat a shifted coordinate origin and shaped plasma boundaries using conformal coordinates. The $ARES$ is a linear code which solves the full MHD equations in toroidal geometry, using flux coordinates.
2. Sawtooth Oscillation

We first review the low beta results.[1] Simulation results at high temperature agree with the Kadomtsev reconnection model,[4] but compare well with experiment only if neoclassical resistivity is used instead of classical resistivity. With neoclassical resistivity of high temperature and low $Z_{\text{eff}}$ plasmas, the change of safety factor at the magnetic axis, $\Delta q_0 \simeq 0.2$ is obtained. (This is much larger than $\Delta q_0 \simeq 0.01$ obtained with classical resistivity.) To understand this, a formula for $q_0(t)$ during the sawtooth rise phase is derived as $q_0(t) \simeq 1 - 2\sqrt{\epsilon_s(t/\tau_R)^1/4}$, where $\epsilon_s \equiv \tau_s/R$ is the inverse aspect at the singular surface, and $\tau_R \equiv \tau_s^2/\eta$ is the skin time. This rapid change of $q_0$ in time may explain some experimental results which obtain $q_0$ values much below one. Some proposed explanations of the ‘anomalously’ fast crash time use the fact that $\Delta q_0$ is very small for classical resistivity.[5,6] However, neoclassical resistivity, which is more appropriate in present-day large tokamaks, gives a $\Delta q_0$ an order of magnitude larger than that given by classical resistivity.

Our 3-D simulation results show that a sawtooth cycle consists of two distinct stages. The first phase is nonlinearly stable and does not involve any current singularity, and thus evolves on a resistive time scale $\tau_R$. (This phase does not exist in a 2-D case.) This stage can be linearly unstable, but becomes nonlinearly stable with a finite island, probably due to the rapidly changing location of the $q = 1$ surface (effective flow through the singular surface), and low shear at the singular surface[7]. In the second, crash phase, a current singularity develops and a complete reconnection occurs in the Sweet-Parker time scale, $\tau_R^{1/2} \tau_R^{1/2}$.

An important parameter which impacts the design of future tokamaks is the sawtooth period. Since the first stage discussed above becomes much longer than the second stage as the $S$ value increases, we can equate the sawtooth period to the duration of the first stage, giving $\tau_{ST} \propto \tau_R \propto \tau_s^2 T_{\text{r}}^{3/2} / Z_{\text{eff}}$. Further, from the character of $\eta_{\text{neo}}$, the functional dependence, $\tau_{ST} = \tau_R f[\epsilon_s, \epsilon_5, Z_{\text{eff}}(\epsilon)]$, is deduced, where $\epsilon_s$ is the inverse aspect ratio of the plateau collisionality layer at the axis. The form factor, $f$, is then semi-empirically obtained from various experimental sawtooth periods, and simulation results at low $S$ values, giving numerical values of

$$\tau_{ST} \propto \epsilon_s^{-2} \tau_R \approx 9 R^2 (T)^{3/2} / Z_{\text{eff}},$$

where $\tau_{ST}$ is in msec, the major radius $R$ in meters, and the peak electron temperature $T$ in keV. This scaling agrees well with various tokamak data. Of course, this assumes the absence of active sawtooth stabilization from
energetic particle effects or current profile tailoring, which can lengthen $\tau_{st}$ drastically.

We now study finite pressure effects on sawtooth oscillations.[8] When the pressure is below the ballooning and kink limits, the main detrimental effect of pressure is found to be that it generates a stochastic annular region during the sawtooth crash. The simulated sawtooth period was not shortened by pressure even at $\epsilon\beta_p \equiv 8\pi\epsilon \int ps/\mu_o I_i^2 = 0.5$.

Figure 1 shows a magnetic field puncture plot for a case with $\epsilon\beta_p$ of 0.2. The remaining hot spot at the right is bulged out forming an eyeball-like shape, because the helically twisted hot spot has an additional toroidal bulge in the large major radius (+$R$) direction. The reason for this bulge is that the hot spot, due to its large pressure gradient, shifts toward the direction of negative field line curvature, which is in the +$R$ direction both at the low and the high field side of the torus. This phenomenon is a characteristic finite $\epsilon\beta_p$ effect, similar to the ballooning mode mechanism. The helical and toroidal displacements add up at the low field side as shown in Fig. 1, while they subtract at the high field side. This asymmetry induces various magnetic island growth. The toroidal bulge shown in Fig. 1 pushes into the plasma just outside of the mixing radius, producing X-points at various singular surfaces, and inducing various $m/n$ magnetic islands to grow. These islands eventually overlap to produce an annular stochastic region as shown in Fig. 1. The X-points induced by the bulge are aligned at the $\theta = 0$ line. This effect of the X-points of the islands being in phase on the low field side is often seen in experiment.

The above predictions correlate well with experimental ECE measurements, shown in Fig. 2, where temperature contours are plotted as functions of time. The large oscillation in the middle is directly due to the hot spot which rotates toroidally with the bulk plasma. The small upward spikes at the +$R$ side denoted by ($B$) can result from the magnetic field distortion just outside of the toroidal bulge of the hot spot. This also explains the absence of similar (but downward) spikes at the −$R$ side. The steeper gradients of temperature contours right after the crash phase as denoted by ($Q$) represent a fast heat pulse propagating through an annular region just outside of the mixing radius, consistent with the annular stochastic region shown in Fig. 1. This interpretation can be used to resolve the sawtooth heat pulse ‘anomaly’. [9]

In contrast to Fig. 1, zero $\beta$ cases showed no toroidal bulge of the hot spot and little induced stochasticity. However, a shaped boundary case shown in Fig. 3, having negligible $\epsilon\beta_p$, produced some stochasticity. An
FIG. 1. Puncture plot of magnetic fields, showing a stochastic annular region.

FIG. 2. Experimental temperature contours as functions of time, from ECE data. R denotes the major radius direction, and MR the mixing radius.
FIG. 3. Puncture plot of magnetic fields in a shaped plasma with $\beta = 0$.

Accurate quantitative prediction of stochasticity is difficult because the degree of stochasticity depends on many parameters, e.g., the durations of the precursor oscillation and crash, the stability margins of various $m/n$ modes before the start of the crash, and the Lundquist number used. However, these studies show that a larger stochastic region will be generated as the pressure and/or the shaping factor at the singular surface is increased. At a higher $\epsilon\beta_p$, the stochastic region generated by the sawtooth crash can be so large that it virtually fills the whole outside region of the plasma. A similar drastic loss of confinement will also occur in the case of Fig. 1, if the wall is located near the $q = 2$ surface. In these cases, the electron thermal energy will be lost so quickly that the discharge will be unable to sustain itself without an application of extremely high loop voltage. This scenario can explain a form of 'high $\beta$ disruption' where the discharge is terminated right after a large sawtooth crash. The conditions for this type of major disruption, namely, a high $\epsilon\beta_p$, a strong shaping, and a large $q = 1$ radius,
also describe the present designs of both CIT and ITER. This may put a stringent error margin on a sawtooth stabilization scheme.

In our simulation, the crash phase corresponds to a complete reconnection process with the Sweet-Parker scaling ($\sim \tau_{A}^{1/2}\tau_{R}^{1/2}$, or equivalently, $\sim 1/2\eta_{x}^{-1/2}$), as in the 2-D case of Ref. 10. Here $\eta_{x}$ is the resistivity at the X-point. In TFTR discharges, the sawtooth crash time varies anywhere between a few msec to about one hundred $\mu$sec. The slower time matches the Sweet-Parker time with non-enhanced $\eta_{x}$, whereas the fast time could result from an $\eta_{x}$ enhancement of two orders of magnitude. This enhancement may result from a microinstability in the current sheet at the X-point, due to strong current density and gradient. Another possible source of the enhanced effective $\eta_{x}$ is the electron parallel viscosity at the stochastic X-point region. However, this possibility seems unlikely, because our present numerical simulations have shown that the severity of the stochasticity increases with $\epsilon\beta_{p}$, whereas, we could not find any experimental correlation between the crash time and $\epsilon\beta_{p}$.

The results presented in this paper show that most of the observed sawtooth phenomena can be explained by the Kadomtsev complete reconnection model with variably enhanced effective $\eta$ values at the reconnection layer. The often debated point of the shape of the hot spot, whether round or crescent shaped, is not a crucial point. The hot spot can have various shapes because it is determined solely by the minimum energy state condition for given shear and pressure profiles, and the topology (the relative size of the island and the hot spot), at the instant during the reconnection process. However, measurements which give $q_{a} < 1$ after the crash[7] can not be explained by a complete reconnection model, because a complete reconnection produces $q_{a} = 1$. The mechanism of this type of sawteeth probably is different from the ones studied here. A partial reconnection model is sometimes mentioned for this. Thermal energy and density could crash from a partial reconnection due to a non-MHD drift. However, the magnetic island formed must somehow disappear to have another rise phase of a sawtooth cycle. For this to occur much faster than the resistive time scale without a complete reconnection, the island has to shrink back in a reconnection of the reverse direction. This could happen for a finite $\beta$ case, where the pressure loss can suddenly change the stability of the plasma. However, for a negligible $\beta$ case, it is not plausible.
3. Pressure Driven Resistive Modes

3.1. Linear Studies

An extensive study of the linear resistive stability properties of TFTR 'supershot' discharges has been carried out at realistic $S$ values ($10^8 - 10^9$) using the linear MHD code, \textit{ARES}. It has been observed experimentally that MHD modes, believed to be resistive in character, sometimes mar their confinement properties. A curious feature is that distinct modes occur in nominally very similar discharges. For example, some supershots exhibit a $m/n = 2/1$ mode, others where the instability is $3/2$ and sometimes no mode at all. Computational results show that most supershot equilibria are remarkably resilient to resistive MHD instability at realistic $S$ values due to the favorable average curvature stabilization in the layer.$[11,12]$ This stabilizing effect is mitigated when the pressure at each rational surface is flattened, modeling finite islands due to fluctuations in the experiment. Then, we find that the computational results are mostly consistent with the experimental behavior.

The discharge, number 26606, was observed experimentally to be purely $2/1$ unstable. Computationally, Fig. 4(a) shows that this mode appears when $q_o$ is sufficiently high ($\gtrsim 1.2$), while the $n=2$ mode is very weakly unstable. On the other hand, the discharge 26637 was observed to be purely $3/2$ unstable, although the equilibrium is very similar. Stability analysis, Fig. 4(b), shows that the $n=2$ mode is more robust and well separated from that with $n=1$. In particular, only the $n=2$ mode is excited when $q_o < 1.3$. Similarly, Fig. 4(c) summarizes the analysis of shot 26627 which was experimentally stable. Again, both $n=1$ and $n=2$ modes are found when $q_o$ is sufficiently large. In this case the absence of a mode in the experiment would imply that $q_o$ was closer to unity than in the previous shots of the sequence. The value of $q_o$ is a relevant parameter which is not measured. The absence of sawtooth oscillations in the discharges considered here is, however, indicative of $q_o > 1$. In addition, the evolution of $q$ calculated by \textit{TRANSP} indicates values of $q_o$ up to about 1.3. The results presented here suggest that stable operation requires that $q_o$ should be kept close to unity.

3.2. Nonlinear Studies

The nonlinear \textit{MH3D} code simulation can start with finite islands to model the actual fluctuation in the experiment and thereby mitigate the layer stabilization discussed above. However, this method is cumbersome, and it is hard to satisfy all the required initial islands. Instead, we set
the ratio of specific heats, $\Gamma = 0$ in the calculation to mitigate the layer effect. The final saturated ohmic steady state thus obtained is very close to a $\Gamma \neq 0$ case. This way, a linearly stable state with $\Gamma \neq 0$ often results in a nonlinearly unstable finite island state.
When multiple low mode number singular surfaces exist, finite pressure tends to produce larger islands in the final steady state as shown in Fig. 5. One cause for this is similar to the $m=1$ case of Fig. 1, where the pressure bulge induces other island growth. In the present case, the pressure inside the $m=2$ islands is hollow, and thus, the islands push in the $-R$ direction. All the islands in Fig. 5 match this phase relation. When the pressure inside the islands is peaked enough, the phase of the driven islands changes. The driven islands are smallest usually when the pressure inside the driving islands is slightly peaked. This suggests a stabilization scheme, where the inside of the islands is heated. When only one low mode number surface exists, the pressure is often stabilizing. The case of Fig. 5 uses $(\eta \mu) = 10^{-10}$.
FIG. 6. Pressure profile of a saturated state in the simulation of a 'supershot' plasma with $\epsilon\beta_p = 0.5$. The $q$ value varies from 1.1 to 9.

The final steady state only depends on $(\eta\mu)$, not $\eta$ and the viscosity $\mu$ separately.[13] One interesting character for finite $\beta$ cases is that when they approach the steady state, the damping is much weaker than zero $\beta$ cases, thus producing wider swings of island size before saturation.

For high $\epsilon\beta_p$ 'supershot' type plasmas, pressure driven resistive modes generate many magnetic islands and stochastic regions which fill a large portion of the plasma. These magnetic islands produce many spikes on the pressure profile as shown in Fig. 6, which is similar to those observed on TVTS measurements. This would cause an anomalous electron heat conductivity, and may also explain the 'beta collapse' phenomenon. Another effect found in the simulation is that as $\epsilon\beta_p$ increases, the ratio between the 3/2 and the 2/1 mode amplitudes also increases. This can give an alternate explanation of the 3/2 dominant discharges seen in TFTR, described in the previous section. However, the $n = 2$ mode occurs only when a substantial $n = 1$ mode also exists in the simulation, whereas an almost pure $n = 2$ can be seen in the experiment.

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DISCUSSION

B. COPPI: Since the plasma pressure gradient is the most important driving factor of $m=1$ modes in toroidal geometry, the analysis that your codes have produced is an important contribution to the field. However, the purely resistive model on which this analysis is based is not sufficient to provide an interpretation of experiments in high temperature regimes such as the 'supershots' that you have considered. Can you comment on this?

W. PARK: In this study, we have assumed the existence of experimental fluctuations large enough to mitigate the stabilizing layer effects, non-MHD as well as MHD. On the other hand, global non-MHD effects are more robust, and important in cases such as sawtooth stabilization due to energetic particles. We plan to incorporate these effects in future studies.
MHD MODES IN ROTATING TOKAMAK PLASMAS

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Abstract

MHD MODES IN ROTATING TOKAMAK PLASMAS.

The internal $m = 1$ kink mode is considered for rotating plasmas with a non-thermal particle population. A guiding centre description of the $q = 1$ layer shows that the ideal MHD growth rate is reduced by kinetic damping. Also, a resistive MHD theory describing the interaction of an applied resonant magnetic perturbation with a rotating plasma is presented. It is shown that relative rotation between plasma and applied signal strongly inhibits tearing. Non-linearly, however, the effect is weakened for sufficiently large islands. Results from COMPASS-C are shown to be in agreement with this theory.

1 Introduction

In this paper various aspects of the interaction of MHD modes with both rotating and auxiliary heated plasmas are investigated, including in Section 3 the effects of applied magnetic perturbations.

2 Kinetic treatment of the $m = 1$ kink mode

2.1 Introduction

We consider the internal $m = 1$ kink mode in axisymmetric toroidal plasmas with steady toroidal plasma rotation and non-thermal particle velocity distributions. The guiding centre equations are used to describe both

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1 H.J. De Blank, F. Pegoraro, T.J. Schep.
the mode and the equilibrium, allowing for plasma rotation and pressure anisotropy. The plasma rotation contributes to the Shafranov shift and therefore affects the stability of the mode. In a hot, almost collisionless plasma the fluid equations of state are not valid, and pressure anisotropy and kinetic effects have to be taken into account. The adopted equations are a set of fluid equations in which the pressure components are obtained kinetically. From these fluid equations an energy functional is constructed. This energy is extremised order by order in the inverse aspect ratio $\epsilon_0$. The resulting expression deviates from the ideal MHD energy functional in $O(\epsilon_0^4)$, where rotation and kinetic effects enter. This approach can be applied to equilibria with an arbitrary $q$-profile [1], but here we treat equilibria with a thin $q \approx 1$ layer. We will restrict our treatment to kinetic effects arising from circulating particles. The diamagnetic and trapped ion effects will be briefly discussed from the point of view of the drift–kinetic equation.

We adopt the low–$\beta$ ordering for the equilibrium quantities $p_\perp$, $p_\parallel$, and $\rho U^2$, which all contribute to the Shafranov shift. Plasma rotation and pressure anisotropy give rise to poloidal variations of the mass density and the pressure components over a flux surface, and to a radial component of the current density. The axisymmetric magnetic field and the rotation velocity, which satisfy the equilibrium fluid equations, are $B = \nabla \psi \times \nabla \phi + F \nabla \phi$ and $U = R^2 \Omega(\phi) \nabla \phi$, where $\phi$ is the toroidal angle and where $R = |\nabla \phi|^{-1}$. The flux quantity $\Omega$ represents a rigid toroidal rotation of each flux surface. The guiding centre description of the equilibrium allows for arbitrary (non–thermal) axisymmetric particle velocity distributions.

### 2.2 Ordering of the mode

In a large aspect ratio torus, the poloidal and toroidal mode numbers $m = n = 1$ are special because the associated perturbation of the magnetic energy can be very small, $O(\epsilon_0^4)$. We divide the mode vector $\xi(r, \theta)e^{i(\omega t - \phi)}$ into a component $\alpha$ parallel to $B$, a poloidal component $\chi$ and a radial component $\xi$. Because $\delta B = O(\epsilon_0^2)$, these components are related through $(\partial / \partial r)(r \xi) + \partial \chi / \partial \theta = O(\epsilon_0^2)$. Since $\nabla \cdot \xi = O(\epsilon_0)$ and the perpendicular displacement is dominantly an $m = 1$ harmonic, it follows that the density perturbations and the parallel displacement $\alpha$ are dominantly $m = 0, 2 \text{ harmonics } \alpha_0(r)$ and $\alpha_2(r)e^{2i\theta}$. Further minimisation of the magnetic energy determines the $m = 1$ eigenfunction through $(q - 1)^2 \xi_1 = O(\epsilon_0^2)$. The energy functional can be minimised to $O(\epsilon_0^4)$ in the standard way [2] without specifying the equation of state. In the limit of a thin $q \approx 1$ layer, it follows that $\xi_1(r)$ is approximately a step function. The resulting expression depends on $\xi_1, \alpha_0$ and $\alpha_2$. In order to eliminate $\alpha_0, \alpha_2$ the equations of state have to be obtained from the kinetic equations. An additional $m = 1$ contribution arises when trapped particle effects are taken into account.
2.3 Guiding centre theory

In the guiding centre approximation of the Vlasov equation the pressure tensor is diagonal, \( P = p_z(I-bb) + p_b bb \), where \( b = B/B \). We will formally express \( \dot{p}_\parallel \) in terms of combinations of the parallel and perpendicular plasma compression, introducing four coefficients, together with a relation which expresses the absence of irreversible transport,

\[
\dot{p}_\parallel = \rho \frac{\partial p_{\parallel \perp}}{\partial \rho} + B \frac{\partial p_{\parallel \perp}}{\partial B}, \quad B \frac{\partial p_{\parallel}}{\partial B} = \rho \frac{\partial}{\partial \rho} \left( \frac{p_{\parallel} - p_\perp}{\rho} \right) \tag{1}
\]

Here a dot denotes \( d/dt = \partial/\partial t + U \nabla \). We assume that Eq. (1) holds for the equilibrium motion and for each poloidal harmonic of the mode separately. After minimising the energy functional with respect to \( \alpha_0 \) and \( \alpha_2 \), the functional depends on \( \xi \) only and contains different coefficients \( \partial p_{\parallel \perp} / \partial \rho \) and \( \partial p_{\parallel \perp} / \partial B \) for \( m = 0, 2 \) and for the equilibrium. These coefficients can be derived from the guiding centre equations for each particle species \( s \)[3],

\[
\frac{\partial f_s}{\partial t} + (U_\perp + v_\parallel b) \cdot \nabla f_s + [v_\parallel U_\perp \cdot (b \cdot \nabla b) - b \cdot \nabla E_s] \frac{\partial f_s}{\partial v_\parallel} = 0 \tag{2}
\]

where \( E_s = \mu B + \Phi_\parallel e_s / m_s - \frac{1}{2}U_\perp^2 \), and where \( v_\parallel, \mu, m_s, e_s, f_s(x, t, \mu, v_\parallel) \) are the parallel velocity, magnetic moment, mass, electrical charge, and distribution function of the particle species \( s \), respectively.

In an axisymmetric, rotating equilibrium we have \( d/dt = U_\parallel b \cdot \nabla \), so the distributions can be written \( F_s(\psi, \epsilon_s, \mu) \), with \( \epsilon_s = \frac{1}{2}v_\parallel^2 - v_\parallel U_\parallel + E_s \)[4]. Then, the coefficients in Eq. (1) can be determined from \( \nabla p_\parallel, \nabla v_\parallel \) and \( \nabla \rho \), expressed in the equilibrium distributions \( F_s \)[4]. The linearised version of Eq. (2) is

\[
[i(\omega - n\Omega) + (v_\parallel - U_\parallel)b \cdot \nabla] \left[ \delta f_s - \frac{\partial F_s}{\partial \epsilon_s}(v_\parallel - U_\parallel)\xi \cdot \nabla \rho \right] = \left[ b \cdot \nabla (\delta E_s) - iv_\parallel(\omega - m/q)\Omega \xi \cdot \kappa \right] (v_\parallel - U_\parallel) \frac{\partial F_s}{\partial \epsilon_s} + \mathcal{O}(\epsilon_s^2) \tag{3}
\]

where \( \delta \) denotes the Lagrangian and \( \kappa = b \cdot \nabla b \) the equilibrium curvature. The right-hand side of Eq. (3) is dominated by \( m = 0, 2 \) harmonics, including the contribution \( \delta \Phi_\parallel \) in \( \delta E_s \), as is guaranteed by charge neutrality. For circulating particles, \( v_\parallel \) is almost independent of \( \theta \), so that the operator on the left can be inverted for each poloidal harmonic. From \( \delta f_s \), we compute \( \delta \rho_\parallel \) and \( \delta p_{\parallel \perp} \). We apply quasi-neutrality and eliminate \( \delta \Phi_\parallel \) and \( \xi, \kappa \) in order to obtain the coefficients in (1) for \( m = 0 \) and for \( m = 2 \). These coefficients satisfy the second equation in (1), so that the application of the energy principle is justified.
2.4 The dispersion relation

By minimizing the mode energy over the entire plasma volume, a dispersion relation is obtained for general $q$-profiles [1], which resembles the MHD result in Refs. [5, 6]. In the case of a thin singular layer we have

$$R_0 B_0^2 \left\{ \int \frac{dr}{r^3} \left[ \left( \frac{1}{q} - 1 \right)^2 + \frac{K(r, \omega)}{B_0^2} \right] \right\}^{-1}$$

$$= \frac{2}{r_0} A_- - \frac{4}{r_0^4} s(r_0) + \left[ \frac{4}{r_0^3} A_- - r_0^3 \beta_s(r_0) - r_0^3 s(r_0) \right] \frac{A_+ - A_-}{A_+ - A_-}$$

(4)

where the integration is over the $q \approx 1$ layer and where we have defined

$$\beta_s(r) \equiv -r^{-4} \frac{R_0^2}{B_0^2} \int_0^r dr' r'^2 \frac{d}{dr} \int \frac{d\theta}{2\pi} (p_\perp + p_\parallel + \rho U^2)$$

and

$$s(r) \equiv r^{-4} \int_0^r dr' r'^3 \left( \frac{1}{q^2} - 1 \right)$$

The flux function $\beta_s$ is a generalisation of the poloidal beta. The quantities $A_\pm$ are the boundary values in $r = r_0$ of $A(r) \equiv r^4(1 - a)/(3 + a)$, where $a(r) \equiv (1 - 2/q)^2 r^2 \zeta' / \zeta$ and where $\zeta(r)$ is the solution of the usual "cylindrical" $m = 2$ Euler equation in the regions $[0, r_0)$ and $(r_0, r_2]$, where the constant-$\xi_1$ approximation applies [2, 5]. The right-hand side of Eq. (4) is proportional to the negative energy functional [2] $\delta W$, while all kinetic effects are in the quantity $K$. Introducing $\gamma \equiv i(\omega - \Omega)$, with positive growth rate $\text{Re}(\gamma) > 0$, we can write

$$K(r, \gamma) = -\rho \gamma^2 R_0^2 + \frac{1}{4} C[(\Omega - i\gamma) R_0] + \frac{1}{4} C[(\Omega + \frac{q}{2-q} i\gamma) R_0]$$

$$-\frac{1}{2} C(\Omega R_0)$$

(5)

where $C(V)$ is given by

$$C(V) \equiv \rho(\Omega R_0)^2 - \sum_s m_s C_{2,s} + \left[ \sum_s e_s C_{1,s} \right] / [\sum_s e_s^2 C_{0,s}],$$

$$C_{i,s}(V) \equiv \int d\mu B(\mu B + V^2)^i \int \frac{dv_\parallel}{v_\parallel - V} \frac{\partial f_s}{\partial v_\parallel}$$

The first term in Eq. (5) is the incompressible MHD kinetic energy. The other three terms arise from the kinetic equations related to the $m = 0$, 2 harmonics $\alpha_0$, $\alpha_2$, and the equilibrium rotation, respectively. We will consider the quantity $K$ for equilibrium distribution functions of the form
\[ F_s(\psi) \exp[-\mu/\mu_{s,0}(\psi)-\epsilon_s/\epsilon_{s,0}(\psi)], \] with \( \mu_{s,0} \) and \( \epsilon_{s,0} \) chosen such that \( p_{\parallel} = \frac{1}{2} p_\parallel \) and \( p_{\perp} = \frac{1}{2} p_\perp \) for all particle species. For \( q \approx 1 \), expansion of (5) to powers of \( \gamma \) and \( \Omega \) yields

\[ K = -\gamma^2 R^2 \left[ \frac{3}{2} \frac{p_\perp}{p_\parallel} + \left( \frac{\pi}{8} - 1 \right) \frac{p^2_\perp}{p^2_\parallel} \right] - \frac{1}{2} \gamma R \frac{p^2_\perp}{p^2_\parallel} \sqrt{\pi \rho_\parallel} + \mathcal{O}(\gamma^3, \gamma \Omega^2) \]

Comparison with the value \( K_{\text{MHD}} = -3\gamma^2 R^2 + \mathcal{O}(\gamma^4) \) obtained from compressible ideal MHD shows that, while the stability boundary is unchanged, the growth rate is strongly reduced due to the damping term (linear in \( \gamma \)). This damping is enhanced by pressure anisotropy \( p_\perp > p_\parallel \), even if this anisotropy is caused by a small population of hot ions. The damping originates from the interaction of the ions with the \( m = 0, 2 \) mode displacement parallel to the magnetic field, which arises in toroidal geometry. Since \( \Omega \) contributes only to higher powers in \( K \), the main effect of plasma rotation is its contribution to the kinetic energy of the central plasma (\( \beta_g \)), which tends to destabilise the mode.

### 2.5 Energetic particle contribution

In the above derivation, \( v_\parallel \) has been taken to be independent of \( \theta \), which is valid for frequencies sufficiently large that only circulating particles need be considered. For lower frequencies and/or for higher temperature particles, trapped ion effects and magnetic drifts come into play. Trapped particles give a dominantly \( m = 1 \) contribution to the perturbed pressure tensor. This essentially leads to a volume contribution to the right-hand side of Eq. (4) [7]. In order to include these effects, the equilibrium velocity is disregarded and the Lagrangian perturbation \( \delta f \) is determined from the drift kinetic equation

\[ [\omega - iv_B \cdot \nabla] \delta f = -\frac{\xi_1}{R} (v^2_\parallel + \frac{1}{2} v^2_\perp)(\omega - \omega_\ast^i) \frac{\partial F}{\partial \epsilon} \cos \theta e^{i\theta} \]

where \( \omega_\ast = -(m/ZeBr)(\partial F/\partial \tau)/(\partial F/\partial \epsilon) \) and where \( v_B \) is the magnetic drift due to the normal curvature. For well circulating particles, neglecting \( v_B \), Eq. (6) is the low frequency counterpart of Eq. (3) and yields \( m = 0, 2 \) components for \( \delta f_i \). The main difference arises from the factor \( (1 - \omega_\ast^i/\omega) \) which enters the integrands of the function \( C_{ij} \) given below Eq. (5). Close to marginal stability, for frequencies below the bounce frequencies, the bounce averaged Eq. (6) becomes

\[ (\omega - \omega_D^{(0)}) \delta f^{(0)} = -\frac{\xi_1}{R} (v^2_\parallel + \frac{1}{2} v^2_\perp)(\omega - \omega_\ast^i) \frac{\partial F}{\partial \epsilon} [\cos \theta e^{i(1-q)\theta}]^{(0)} \]

where \( \omega_D^{(0)} \) is the bounce averaged magnetic drift frequency. It can be seen that Eq. (7) leads mainly to an \( m = 1 \) volume contribution to \( \delta W \) and shifts the stability boundary [8]. Due to the \( \omega_D^{(0)} \) resonance this contribution is complex.
3 The effect of rotation on the $m > 1$ tearing mode

In several tokamak experiments, resonant magnetic perturbations (RMP's) have been applied using coils external to the plasma (e.g., PULSATOR [9], and recently COMPASS–C [10]). Such RMP's have been used to control the internal MHD activity. Generally this MHD activity has a finite frequency, due to diamagnetic and $E \times B$ drifts, whilst the applied perturbations are static; though some experiments have employed active magnetic feedback (e.g., DITE [11]). To understand such situations it is thus generally necessary to study the interaction of modes rotating with frequencies significantly different to that of the applied perturbation. Analytic and numerical calculations which describe such interactions are discussed here, and then compared with experimental results from COMPASS–C.

Linearly, it can be demonstrated that the final amount of reconnected flux $\psi$ induced at the resonant surface, after a helical perturbation with an $e^{-i\Omega t}$ dependence is applied at the edge of the plasma, is given by

$$\frac{\psi}{\psi_c} = \frac{1}{1 + \Delta/(\Omega)/(-\Delta')}$$

Here $\Delta'$ (assumed negative) is evaluated for the unperturbed equilibrium, $\Delta(\Omega)$ is the logarithmic change in the perturbed flux across the resistive layer (assuming an $e^{-i\Omega t}$ time dependence of layer quantities), and $\psi_c$ is the reconnected flux induced when there is complete resonance between the applied perturbation and natural plasma modes. For a diamagnetic plasma, the resonances occur at $\Omega' = \omega_{ce}, \omega_{ni},$ and 0, where $\Omega'$ is the Doppler shifted frequency of the applied perturbation as seen in the local guiding centre frame at the rational surface. Between these resonances (which are found to be extremely narrow), $\psi/\psi_c$ typically falls to a value of order $1/Q$, where $Q \sim (2\pi f_{\text{mhd}}\tau_c)^{5/4}/(-\Delta'a)$. Here, $a$ is the minor radius, $f_{\text{mhd}}$ is the typical frequency of MHD activity, $\tau_c = \tau_R^{-2/5}\tau_H^{-2/5}$ is the reconnection time, with $\tau_R$ and $\tau_H$ the local resistive and hydromagnetic times, respectively. For COMPASS–C parameters ($f_{\text{mhd}} \sim 15\text{kHz}$, etc.) $Q \sim 25$, and so islands induced by static field errors are likely to be substantially rotationally suppressed, relative to their vacuum values.

In the nonlinear regime, we have studied the interaction of an island of width $W$ with an applied helical perturbation of the same helicity. Starting from the resistive MHD equations for a high temperature plasma in the limit of large aspect ratio and large island width, we find that the island growth is described by

$$\tau_R \frac{d(W/a)}{dt} = 1.2 \text{Re}(\Delta'a) - I_U(t)(\Omega'\tau_H)^2 \left(\frac{a}{W}\right)^3$$

where $I_U(t) \sim \mathcal{O}(1)$. We obtain a complementary equation describing the evolution of the island frequency under the influence of both perpendicular

\[2\quad R.\ Fitzpatrick,\ T.C.\ Hender,\ A.W.\ Morris,\ D.C.\ Robinson.\]
ion viscosity and the mode-locking torque due to the coil currents. The exact form of the function $I_U$ (and similar functions in the torque equation) has been derived from an asymptotic matching at the island separatrix of the flows inside and outside the island. The first two terms of Eq. (9) are just the normal Rutherford expression [12], while the third term describes a stabilising rotation effect due to the inertia of the flowing plasma in the vicinity of the island. It can be seen that this rotation term ($\propto \Omega^2$) gives a suppression of tearing, as in the linear regime, but that this suppression diminishes rapidly as $W$ increases ($\propto W^{-3}$). In steady state ($\partial / \partial t = 0$), Eq. (9) and the corresponding frequency equation may be solved analytically. The resulting solution shows a sharp threshold in saddle current, below which there is negligible tearing, and above which the island reaches the full width that would be expected in the absence of plasma rotation. If we parameterise the saddle current ($I_c$) in terms of its vacuum island width, then the critical current for tearing to occur corresponds to

$$\frac{W_{crit}}{a} \approx 14.6 \left( \frac{\Omega'(\text{Hz}) \tau_H}{(-\Delta a)^{1/3}} \right)^{2/3}$$

For COMPASS-C parameters (with $q_u \sim 2.9$) Eq. (10) yields $W_{crit}/a \sim 13\%$ [for larger tokamaks with lower MHD frequencies $W_{crit}/a (\propto \Omega'^{2/3})$ will be accordingly smaller]. A further consequence of Eq. (9) is that if the mode locking force from an RMP slows down a naturally unstable rotating mode,

![FIG. 1. $m = 2, n = 1$ island width evolution for various (2, 1) helical currents applied at $t = 0$, showing that there is a sharp threshold in $I_c$, beyond which significant tearing occurs. (Here $I_c$ is normalized by $aB_0/\mu_0$ and $S = \tau_R/\tau_H = 8 \times 10^4$.)](image)
then the resultant relative flow between the plasma and the mode will be stabilising. Modest islands (W/a ~ 5%) should be significantly stabilised by this mechanism.

Our analytic results are borne out by numerical simulations using the nonlinear reduced MHD equations [13] including $\vec{E} \times \vec{B}$ flows. Figure 1 shows the the $m = 2$ island width evolution for various applied $m = 2,$
$n = 1$ helical currents ($I_e$). The equilibrium in this case was tearing stable $(\Delta'_{2,1} = -0.7, \psi = 3)$, with the applied perturbation and the plasma having a relative frequency $\Omega' = 4 \times 10^{-2} r_H^{-1} (\sim 15 \text{kHz for COMPASS-C parameters})$. It can be seen that there is a sharp increase in growth between $I_e = 10^{-3}$ and $2 \times 10^{-3}$, where $W/a \sim 13\%$ [in agreement with Eq. (10)]. This sharp threshold corresponds to the point at which the mode is 'locked', and the rotation suppression of the island growth ($\propto W^{-3}$) becomes negligible.

The effects of RMP’s have been studied experimentally in the COMPASS-C device [10]. These results are detailed in Ref. 14. Here, we examine some aspects of these results which are relevant to our theory. Far from the density limit it is possible to stimulate disruptions in COMPASS-C by applying dominantly $m = 2, n = 1$ RMP fields. For $q \psi \sim 3.0$ ($I_p = 100 \text{kA}, B_T = 0.7 \text{T}$) a saddle current of $\sim 1 \text{kA}$ (corresponding in vacuum to $\delta B_r(2, 1)/B_\theta = 8 \times 10^{-3}$ at $r = a$) is required to stimulate disruptions. Figure 2 shows such a stimulated disruption. In this case the RMP field has stabilised [14] the $m = 2, n = 1$ activity $[\delta B_r(2, 1)/B_\theta \sim 10^{-3}$ prior to stabilisation], an effect which is expected analytically, as described above. For the $\sim 5 \text{ms}$ preceding the disruption there is no coherent $m = 2$ activity. From Fig. 2 it can be seen that the sawteeth cease $\sim 1.5 \text{ms}$ before the disruption and there is a decline in central confinement. The saddle current required to stimulate a disruption (\sim 1kA) corresponds to vacuum island widths $W/a \sim 15\%$. This is slightly above the theoretically predicted sharp island width threshold, given by Eq. (10) and shown in Fig. 1 (which include plasma response), for significant tearing to occur. The observed confinement degradation preceding the disruption also supports the conclusion that significant reconnection has occurred.

To summarise, we have developed a resistive MHD theory for the interaction of applied fields with rotating plasmas, and demonstrated correlation with COMPASS-C results.

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DISCUSSION

A.B. HASSAM: If one were to drive a large toroidal rotation in a tokamak with rotational shear, would it be possible to stabilize, or 'heal', an existing magnetic island?

R. FITZPATRICK: A large sheared toroidal rotation tends to reduce the effect of toroidal coupling between the various rational surfaces in the plasma. On the other hand, it has been shown that sheared rotation makes tearing modes intrinsically more unstable. Thus, a driven sheared rotation field would lead to slightly larger, but far less coupled, islands.

L.J. PERKINS: As a toroidally rotating plasma passes over error fields, the dissipation at forced islands must contribute to the braking of toroidal rotation. Does this phenomenon in fact make a significant contribution to the braking of toroidal rotation?

R. FITZPATRICK: Error fields will certainly tend to bring the plasma at the various more rational surfaces to rest, thereby leading to a braking force on the whole plasma. In practice, however, the braking force due to eddy currents induced in the vacuum vessel is far larger.
EQUILIBRIUM BETA LIMIT AND ANOMALOUS TRANSPORT STUDIES OF HELICAL SYSTEMS

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Abstract

EQUILIBRIUM BETA LIMIT AND ANOMALOUS TRANSPORT STUDIES OF HELICAL SYSTEMS.

Equilibrium magnetic surfaces and anomalous transport of helical systems are studied. The equilibrium beta limit, which is defined by the breaking of magnetic surfaces due to the finite beta effect, is investigated for two types of helical system. The results indicate that the breaking often imposes a more severe limitation on beta than the Shafranov shift. However, if controllable parameters, such as the vertical field $B_v$, are chosen appropriately, a high beta equilibrium, such as $\bar{\beta} \geq 5\%$, can be obtained. Furthermore, a simple method is proposed, by which the breaking can be actively suppressed and fairly high beta equilibria with clearly nested magnetic surfaces can be realized. To study the anomalous transport based on the fluid type turbulence in heliotron/torsatron, six-field equations are derived from the two fluid model. A new type of $\eta$ mode coupled to the $g$ mode is found in the presence of unfavourable magnetic curvature. For the drift resistive interchange turbulence the Boltzmann relation between the density and the potential fluctuation spectrum is observed in the weak collisional regime.
We discuss two physical aspects of toroidal helical systems: one refers to the breaking of magnetic surfaces in finite beta equilibria and to methods of suppressing it (Part A), and the other deals with anomalous transport (Part B).

**PART A: EQUILIBRIUM BETA LIMIT OF HELICAL SYSTEMS AND SUPPRESSION OF MAGNETIC SURFACE BREAKING**
*(T. Hayashi, A. Takei, N. Ohyabu, T. Sato)*

1. **INTRODUCTION**

The equilibrium beta limit of a helical system is conventionally defined by the magnitude of Shafranov shift, such as \( \Delta_{\alpha}(\beta) < \bar{a}_{\alpha}/2 \). We must, however, consider the breaking of magnetic surfaces due to the finite pressure effect since it is well known that a non-axisymmetric, toroidal, finite beta equilibrium does not necessarily lead to a regular nesting of magnetic surfaces. The boundary region of a helical system is ergodic, even in a vacuum field. Therefore, the physical issue in this paper is to investigate the extent to which the boundary ergodic region expands in a finite beta equilibrium and to look for methods of suppressing the ergodic region.

The origin of magnetic islands in a finite beta equilibrium of toroidal helical systems is attributed to the plasma current which is induced to satisfy the equilibrium force balance condition \( \mathbf{j} \times \mathbf{B} = \nabla p \). The resonant field, which can be produced by the plasma current for the case of a non-axisymmetric torus, causes magnetic islands inside the plasma. When an island is induced, the pressure profile is significantly modified near the island. Thus, a consistent analysis of the relationship between the \( \mathbf{j} \times \mathbf{B} = \nabla p \) condition and the island formation is required.

To analyse the total 3-D effect of the plasma current on rational surfaces quantitatively, we have developed a 3-D equilibrium code (HINT) [1, 2]. In the following, we show results if HINT on Heliotron/Torsatron and Helias configurations and propose a simple method of suppressing the breaking.

2. **\( \ell = 2 \) HELIOTRON/TORSATRON**

To understand the general tendency of the 'fragility' of magnetic surfaces in a finite beta equilibrium, we have carried out a parameter survey for several kinds of physical parameter. The \( M \) (pitch period number) dependence of the breaking of magnetic surfaces is shown in Fig. 1(a) for the \( \ell = 2 \) heliotron configuration. This survey was performed with the pitch parameter \( \gamma_c = (M(a_c/R_c)) \), fixed at 1.3, the vacuum magnetic axis at the helical coil centre, the external quadrupole component \( B_q = 0 \), and the helical coil with no modulation. The broken line in Fig. 1 indicates a tentative beta limit at which the outer region of about 30% of the minor radius
becomes ergodic. In general, the breaking has a tendency to be suppressed as $M$ increases. For low $M$ (low aspect ratio) configurations, however, we find that the breaking can be improved by properly choosing several free parameters [2].

Figure 1(b) shows the effect of the external vertical field $B_v$, which controls the radial position of the vacuum magnetic axis. As is shown in Fig. 1(b), the inward shift of the magnetic axis is favourable to suppressing the breaking, and, in fact, we obtain the high beta equilibrium (such as $\bar{\beta} \geq 5\%$) keeping clearly nested surfaces
by a small inward shift for the $M = 10$ configuration. As to the effect of $B_q$, vertically elliptic shaping of the surfaces is favourable to suppressing the breaking. Another survey indicates that the breaking is significantly improved by decreasing $\gamma_c ( = 1.2)$, and is also improved by making the helical coils positively modulated.

3. HELIAS

The Helias configuration [3], in contrast to the heliotron configuration, has a very low shear, but the Pfirsch–Schlüter current is optimized to be small. The island size induced by the finite pressure effect is determined by a competition between these effects. The way islands appear in Helias is different from that in the heliotron case; the number of relevant dangerous rational surfaces is much smaller, but the islands are much larger when they appear. The results indicate that the dangerous rational surface, i.e. the $\iota = 5/6$ surface, comes into the plasma region as beta increases, and $m = 6$ islands are formed on the surface. However, the islands are not that large as to destroy the whole plasma. When beta is further increased, the $5/6$ surface is again moved away from the plasma region, and a high beta equilibrium with clear surfaces can be achieved. In this process, the pressure profile turns out to play an important role: a broader profile is favourable. This is because the magnetic axis shift becomes smaller and the change of the iota profile is weaker, whereby the number of relevant dangerous rational surfaces can be suppressed.

4. SUPPRESSION OF BREAKING BY ADDING A SIMPLE EXTRA COIL

As was stated above, axisymmetric external poloidal fields, such as $B_v$, can be used to suppress the breaking of surfaces. One problem of this method, however, is that such an external field significantly changes the physical properties of the configuration, such as the well depth, and usually the stability is deteriorated when the breaking is improved. Here, we propose another way of suppressing the breaking. Islands which appear on a rational surface, whether in a vacuum field or in a finite beta field of a heliotron configuration, have the following, empirically established properties: (1) the island size is noticeably larger on the outer side of the torus; and (2) islands appear in phase at the outside of the torus when islands are induced on several rational surfaces simultaneously. By taking advantage of these properties, we can devise a set of simple extra coils, sufficient to suppress induced islands. This method is studied by using the Cary–Hanson technique [4] to measure the island size. Figure 2(a) shows an example of induced islands which appear in a finite beta equilibrium ($\beta \sim 4\%$) of an $M = 10/\ell = 2$ heliotron configuration. As is shown in Fig. 2(b), these islands are clearly suppressed by adding an extra coil, so that clear magnetic surfaces are recovered in the outer ergodic region as is seen in Fig. 2(a). It is interesting to note that the required extra coil current is only about 3% of the helical coil current.
One important advantage of this method is that the physical properties, such as the well depth and the i profile, are very slightly changed when the extra coil field is imposed. Thus, this method provides an efficient way of remedying magnetic islands for any reasonable beta value and, simultaneously, improves the equilibrium beta limit.

PART B: FLR–MHD EQUATIONS AND ANOMALOUS TRANSPORT STUDIES OF HELIOTRON/TORSATRON

(M. Wakatani, H. Sugama, M. Yagi, K. Watanabe, B.G. Hong, W. Horton)

We have derived FLR (finite Larmor radius)–MHD equations by retaining the FLR effects through the collisionless viscosity of ions and using the stellarator expansion from the two fluid mode. The resultant six-field equations describe almost all fluid type instabilities in heliotron/torsatron.
The FLR–MHD equations are written in the toroidal co-ordinates \((r, \theta, \xi)\) by using the Poisson bracket:

\[
\frac{n_0 m_i}{\partial t} \left( \frac{\partial}{\partial t} \nabla_\perp^2 F + [F, \nabla_\perp^2 F] \right) - \frac{1}{\omega_{ci}} \nabla_\perp \cdot [P_i, \nabla_\perp F] \\
= \frac{1}{c} B_0 \nabla_\parallel J_\parallel + \nabla (P_e + P_i) \times \nabla \Omega \cdot \vec{\xi},
\]

\[
\frac{1}{c} \frac{\partial A}{\partial t} = - \nabla_\parallel \left( \phi - \frac{P_e}{n_e} \right) - \eta_\parallel J_\parallel,
\]

\[
\frac{n_0 m_i}{\partial t} \left( \frac{\partial v_\parallel}{\partial t} + \frac{c}{B_0} \left[ \phi, v_\parallel \right] \right) = - \nabla_\parallel (P_e + P_i),
\]

where \(F = -\frac{c}{B_0} (\phi + \frac{P_i}{n_e})\), \(P_e + P_i = -B_0 B_\beta / 4\pi\), \(J_\parallel = -c \nabla_\parallel^2 A / 4\pi\),

\[
\nabla_\parallel = \partial / \partial \xi + (\nabla \psi \times \vec{\xi} / B_0) \cdot \nabla, \quad \psi = A + \nabla \Phi \times \nabla \cdot \int_0^\xi \nabla_\parallel^\xi \Phi \cdot \vec{\xi}
\]

\[
\Omega = 2r \cos \theta / R_0 + |\nabla \Phi|^2 / B_0^2. \quad \text{Here } B_h = -\nabla \Phi
\]
denotes external stellarator fields and the bar denotes an average over one field period length in the \(\xi\) direction. \(P_e, P_i, n_0\) and \(B_0\) are also constant, and \(\gamma_e\) and \(\gamma_i\) are specific heat ratios for electrons and ions. Equations (1) to (6) for six variables \(\{F, A, v_\parallel, n, P_e, P_i\}\) obey an energy conservation relation:

\[
\frac{d}{df} \int dV \left\{ \frac{n_0 m_i}{2} (v_\parallel^2 + |\nabla_\perp F|^2) + \frac{1}{8\pi} \left( (B^\theta)^2 + |\nabla_\perp A|^2 \right) + \frac{1}{2} \frac{(P_e + P_i)^2}{\gamma_e P_e + \gamma_i P_i} \right\} \\
= - \int dV \left\{ \eta_\parallel J_\parallel^2 + \eta_\perp \left( \frac{c}{4\pi} |\nabla_\perp B^\theta|^2 \right) \right\}.
\]
We study the following instabilities which are candidates for an explanation of edge turbulence and anomalous transport in Heliotron E:

(i) the drift resistive interchange mode, by using the two-field equations for $\{\phi, n\}$ in the electrostatic limit, with the assumptions $v_{ti} \rightarrow 0$, $P_i \rightarrow 0$, $\beta_e = 8\pi P_e/B_0^2 \rightarrow 0$, $P_e = nT_{e0}$ and $T_{e0} = \text{const}$ [5].

(ii) $\eta_1$ modes by using the five-field equations for $\{\phi, A, n, v_i, P_i\}$ with the assumption of $P_e = nT_{e0}$ or the four-field equations for $\{\phi, n, v_i, P_i\}$ in the electrostatic limit [6, 7].
For the case (i), the unstable modes localized at the mode resonant surface with growth rate $\gamma \propto \eta_1 s^{-2}$ and $\omega \sim \omega_0$ were obtained in the semi-collisional regime, which is different from the $g$ mode in the RMDH, $\gamma \propto \eta_1^{1/2} s^{-2/3}$, where $\omega_0$ is the curvature drift frequency and $s$ is the shear parameter. Figure 3 shows poloidal mode number spectra of density and potential fluctuations for large $\eta_1$ (right) and small $\eta_1$ (left) cases obtained by a non-linear calculation in cylindrical geometry [5]. For the small $\eta_1$ case, $\eta/\eta_0 = e_0/T_e$ or the Boltzmann relation is seen to be valid and the particle transport decreases significantly, compared to the large $\eta_1$ case. This implies that the particle transport increases towards the edge, according to the increase of $\eta_1$ or to a deviation from the Boltzmann relation.

Figure 4 shows the linear growth rate of the $r\ell = 0$ mode in the cylindrical plasma model of Heliotron E [5] as a function of $\eta_1 \equiv d \ln T_j/d\ln n_0$. When $\nu_e$ or $\eta_1$ is finite, the $\ell = 0$ mode is dominant and its growth rate is larger than the collisionless limit, where $\ell$ is a radial mode number. For finite $\nu_e$, the $\eta_1$ mode couples to the $g$ mode and an anomalous ion thermal transport is predicted by the mixing length theory.

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DISCUSSION

J. NUHRENBERG: Could you comment on the dependence of the flux surface restoring control field on beta?

T. HAYASHI: If the shape of the magnetic surfaces remained the same with increasing beta, the control field could be selected so as to be in proportion to beta. In reality, however, the shape of the surfaces is non-linearly deformed as beta increases. In our study, the control field was empirically changed with $\beta^a$, where $0 < a < 1$ (a depends on the configuration). A more detailed study is in progress.

H.L. BERK: How is the pressure profile treated in island and ergodic regions? Is your technique for eliminating resonances similar to that suggested by Cary and Hanson?

T. HAYASHI: The approach is to smooth the pressure along the field lines during the time evolution of the magnetic fields in our 3-D MHD code; in this way, the pressure profile is automatically flattened in the ergodic region. We used the Cary-Hanson technique to measure the island size and did a thorough investigation of methods to eliminate the islands, with particular emphasis on finite beta equilibria.
HOLLOW DENSITY AND TEMPERATURE PROFILES IN STELLARATORS AND ANALYSIS OF THEIR STABILITY, AND TRANSITION PARTICLE DIFFUSION IN STELLARATORS

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Abstract

HOLLOW DENSITY AND TEMPERATURE PROFILES IN STELLARATORS AND ANALYSIS OF THEIR STABILITY, AND TRANSITION PARTICLE DIFFUSION IN STELLARATORS.

It is shown that for a wide range of parameters the solutions of the stationary balance equations for particles and energy give hollow temperature and density profiles. Their stability is analysed and a comparison with experimental data is given. The transport in stellarators which is connected with transition particles, i.e. the particles undergoing the collisionless change of type of motion, is studied. Some analytic results are presented concerning their diffusion coefficient at low collisionality. The numeric data obtained in Monte Carlo simulation are shown to be in good agreement with the theory.

PART I. HOLLOW DENSITY AND TEMPERATURE PROFILES IN STELLARATORS AND ANALYSIS OF THEIR STABILITY

By L.M. Kovrizhnykh

I.1. INTRODUCTION

Many experiments on auxiliary plasma heating made in various stellarators have shown that, in quite a number of cases, the radial profiles of density N(r) and temperature T(r) for one of the plasma components (j = e, i) turn out to be hollow [1–6]. This was also noticed in tokamak experiments.

In NBI experiments on Wendelstein VII-A [1, 2] the density and ion temperature were peaked (i.e. monotonically decreasing to the edge of the plasma), whereas the electron temperature T_e(r) was hollow.
In the ECRH experiments on Wendelstein VII-A [3], Heliotron-E [4] and Wendelstein VII-AS [5], $T_e(r)$ changed from peaked to hollow when the resonance absorption region was displaced from the magnetic axis. At the same time $N(r)$, on the contrary, changed from hollow to monotonically decreasing.

In the central ECRH experiments on Wendelstein VII-AS [5] and the Compact Helical System (CHS) [6], $N(r)$ changed from peaked to hollow with an increase in the introduced HF power, whereas $T_e(r)$ became more and more peaked.

In Ref. [7] (see also Ref. [8]), where the particle and energy balance equations for a stellarator reactor were analysed, it was mentioned that steady state solutions may lead to the appearance of hollow profiles. However, this remark apparently did not draw enough attention to this problem, so we think it useful to again discuss this problem and present a rather general analysis of the stationary balance equations. As we shall see, in the steady state the existence of hollow profiles is rather the rule than the exception and is likely to be limited only by the onset of MHD instabilities caused by unfavourable plasma pressure profiles.

I.2. BALANCE EQUATIONS AND STATIONARY PROFILES

The particle flux $S_j$ and the energy flux $Q_j$ of the species $j = e, i$ in a general case can be represented as:

$$S_j = S_j^{\text{con}} - D_j \{\ln'N - e_jE/T_j + \alpha_j \ln'T_j\}$$

$$Q_j = Q_j^{\text{con}} - \chi_j \{\ln'N - e_jE/T_j + \beta_j \ln'T_j\}$$

where $S_j^{\text{con}}$ and $Q_j^{\text{con}}$ are convective parts of the fluxes which do not depend on the gradients $N'(r)$ and $T'(r)$ and appear, for example, as a result of the neoclassical pinch effect; $D_j/N$ and $\chi_j/N$ are the coefficients of diffusion and thermal conductivity, respectively, which may, generally speaking, depend on the magnitude of the ambipolar electric field $E$; $\alpha_j$ and $\beta_j$ are numeric coefficients determined by the transport regime (hydrodynamic, plateau, l/$\nu$, etc.) and consequently are dependent on the radius $r$; $e$ is the charge of electrons and ions (for the sake of simplicity single charged); and the prime denotes the derivative with respect to $r$.

Let us denote $\delta (\delta N/\delta t)_j$ the density of the particle sources and $P_j^{\text{in}}$ the power density absorbed by the $j$ component (including auxiliary heating, energy exchange between the components, radiative losses, charge exchange, etc.). Then the steady state balance equations for particles and energy take the form:

1. Note that owing to the symmetry of the kinetic coefficients, the quantities $D_j$ and $\chi_j$ should, generally speaking, satisfy the relation $\chi_j = (\alpha_j + 1.5)T_jD_j$. 
\[
\ln'N - e_jE/T_j + \alpha_j \ln'T_j = -S_j^0/D_j
\]

(2)

\[
\ln'T_j - e_jE/T_j + \beta_j \ln'T_j = -Q_j^0/x_j
\]

where

\[
S_j^0 = -S_j^{con} + \frac{1}{r} \int_0^r \text{d}r \left( \frac{\delta N}{\delta t} \right)_j
\]

(3)

\[
Q_j^0 = -Q_j^{con} + \frac{1}{r} \int_0^r \text{d}r (P_j^{in} + e_j(S_j^0 + S_j^{con}))
\]

Solving Eqs (3) with respect to \(\ln'N, \ln'T\) and \(E\), we find

\[
T_j/T = C_j(1 - \gamma_j)
\]

(4)

\[
\ln'N = \sum_j C_j (\alpha_j \gamma_j - \beta_j)
\]

(5)

\[
E/T_c T_j = \sum_j \frac{C_j}{e_j T_j} (\beta_j - \alpha_j \gamma_j)
\]

(6)

where\(^2\)

\[
\gamma_j = \frac{Q_j^0 D_j}{S_j^0 x_j}, \quad C_j = \frac{T_j S_j^0}{(\beta_j - \alpha_j) D_j T}, \quad T = \sum_j T_j
\]

(7)

Adding Eqs (4) and (5), we obtain the expression for the plasma pressure gradient \(P = NT\):

\[
\ln'P = \sum_j C_j [(\alpha_j - 1) \gamma_j + (1 - \beta_j)]
\]

(8)

which determines the range of parameters \(\gamma_j, C_j\), corresponding to the steady state solutions, since at \(P' > 0\) in the central region (small shear, magnetic well) and at

\(^2\) Since \(D_j\) and \(x_j\) may depend on the ambipolar electric field \(E\), relationship (6) is, generally speaking, not a solution but an equation for \(E\). Its solution for particular forms of \(D_j\) and \(x_j\) is given in Ref. [8].
FIG. 1. Regions of positive and negative values of \( T'_i \) and \( T'_e \) as functions of \( \gamma_{e,i} \). Here \( \gamma^{(1)}_{e,i} = \sum C_j / (\beta_j - 1) / (\alpha_{e,i} - 1) C_{e,i} \); \( \gamma^{(2)}_{e,i} = \sum \beta_j C_j / \alpha_{e,i} C_{e,i} \).

fairly high \( -P' > 0 \) in the peripheral region (high shear, magnetic hill) the plasma may turn out to be MHD unstable. We shall not, however, discuss the stability problems here.

From Eqs (4, 5) it is clear that, depending on \( \gamma_j \), i.e. on the ratio between the powers of particle and heat sources, the derivatives \( T'_j \) and \( \ln'N \) in the centre of the plasma column may be both negative (peaked profiles) and positive (hollow profiles).

In this paper we do not pretend to solve completely the problem of stationary profiles but aim only to show the possibility of obtaining hollow profiles as solutions of the stationary balance equations. That is why we restrict ourselves here to only the most general analysis of Eqs (4, 5) in the case where

\[ \beta_j > \alpha_j > 0 \]

(9)

Note in particular that in the framework of neoclassics, relationship (9) is satisfied at least for the central region of the plasma in stellarators. Thus, for the plateau regime \( \alpha_j = 1.5, \beta_j = 2.5 \); for the \( 1/\nu \) regime \( \alpha_j = 3.5, \beta_j = 4.5 \). It is easy to see from Eqs (4, 5) that hollow profiles are possible in a rather wide range of parameters,
but not for all three quantities \( \ln'N, T'_e \) and \( T'_i \) simultaneously. In every case at least one of the three is positive. This is shown in Fig. 1, where the regions corresponding to the positive and negative values of these quantities are represented as functions of \( \gamma_e, \gamma_i \). The power introduced to electrons rises from left to right in the diagram, and the power absorbed by ions increases from bottom to top. The region above the horizontal line \( \gamma_i = 1 \) and to the right of the vertical line \( \gamma_e = 1 \) corresponds to the peaked profiles of the ion and electron temperatures, respectively. The region to the right of and above the line \( \text{AB} \) corresponds to the hollow density profiles.

The broken lines (1) and (2) separate the regions where the pressure gradient \( P' \) has opposite signs. Line (1) corresponds to \( \beta_j > \alpha_j > 1 \) and line (2) to \( 1 > \beta_j > \alpha_j > 0 \). In the case \( \beta_j > \alpha_j > 1 \), \( P' \) is positive above and to the right of line (1) and in the case \( 1 > \beta_j > \alpha_j > 0 \), \( P' \) is positive below and to the left of line (2); at \( \beta_j > 1 \) and \( \alpha_j < 1 \), \( P' < 0 \) at all \( \gamma_j > 0 \). It is interesting to note that the region of peaked profiles (\( \ln'N, T'_e, T'_i < 0 \)) is rather small and corresponds to the region inside the triangle \( \text{ABC} \).

1.3. STABILITY AND STATIONARY SOLUTIONS

We have found above the stationary solutions of the balance equations. However, these equations may not always be realized since in some cases they may turn out to be unstable against small perturbations of equilibrium. Let us consider this problem in more detail, restricting ourselves, for the sake of simplicity, to the case where the characteristic length of the perturbation is much smaller than the length of the inhomogeneity but exceeds essentially the ion gyroradius \( \rho_i \). Supposing in the balance equations \( N = N^0 + N^1, T = T^0 + T^1, E = E^0 + E^1 \), where \( N^0, T^0, E^0 \) are the solutions (4–6) of the stationary Eqs (2) and \( N^1, T^1, E^1 \sim \exp(ikr - \gamma t) \) are the small deviations from them, and assuming \( kr \gg 1 \), we find for \( \gamma \):

\[
\gamma = \frac{\sum_j \frac{(\Gamma - x_j \beta_j)a_j + \alpha_j d_j b_j}{(\Gamma - x_j \beta_j)(\Gamma - d_j) + (\Gamma - x_j) \alpha_j d_j}}{\sum_j \frac{4\pi m_j c^2}{B^2}} \frac{k^2 T}{4\pi e_i^2 N} = k^2 \rho_i^2
\]  

where

\[
\rho_i^2 = \frac{m_i T_i^2}{e_i^2 B^2} \]

\[
a_j = d_j \frac{T}{T_j} \left[ 1 + \frac{T}{T_j} (\beta_j - \alpha_j) C_j f_j \right]
\]
\[ b_j = x_j \frac{T}{T_j} \left[ 1 + \frac{T}{T_j} (\beta_j - \alpha_j) \gamma_j C_j f_j \right] \]

\[ d_j = \frac{D_j}{N \sum_j A_j} \]

\[ x_j = \frac{2x_j}{3NT_j \sum_j A_j} \]

\[ f_j = \frac{\partial \ln D_j}{\partial E_j} = \frac{\partial \ln x_j}{\partial E_j} \]

\[ E_j = \frac{e_j E}{T_j} \]

\[ A_j = \frac{D_j T}{NT_j} \left[ 1 + \frac{T}{T_j} (\beta_j - \alpha_j) C_j f_j \right] \]

\[ \Gamma = \frac{\gamma}{k^2 \sum_j A_j} \]

Here we have omitted the superscript zero.

If we restrict ourselves to not very short wave perturbations \( \epsilon \equiv k^2 \rho_j \ll 1 \), then Eq. (10) splits into two. For the ‘high frequency branch’, \( \Gamma \gg 1 \), the solution is found immediately:

\[ \Gamma = \frac{1}{\epsilon} \sum_j a_j = \frac{1}{\epsilon} \gg 1 \]

\[ \gamma = \frac{k^2}{\epsilon} \sum_j A_j \]

For the ‘low frequency branch’, \( \Gamma \sim 1 \), we find the following cubic equation:

\[ \Gamma^3 - \tilde{A} \Gamma^2 + \tilde{B} \Gamma - \tilde{C} = 0 \]

where
\[ \hat{A} = \sum_j q_j + a_j s_j + \alpha_j s_i \]
\[ \hat{B} = a_e p_e + a_s p_i + q_j s_e + q_s i \]
\[ \hat{C} = p_e q_i + p_i q_e \]
\[ p_j = d_j x_j (\beta_j - \alpha_j) \]
\[ q_j = \beta_j x_j a_j - \alpha_j d_j b_j = p_j \frac{T}{T_j} \left[ 1 + \frac{T}{T_j} C_j \gamma_j \right] \]
\[ s_j = d_j + \alpha_j (x_j - d_j) + (\beta_j - \alpha_j) x_j \]

and \( C_j, \gamma_j \) are determined by Eqs (7).

It is easy to show that the necessary and sufficient condition for stability of the stationary solutions (4-6) (i.e. the condition assuming that the real part of \( \gamma \) is positive) is determined by the inequalities:

\[ \sum_j A_j > 0, \quad \hat{A} > 0, \quad \hat{C} > 0, \quad \hat{A}\hat{B} > \hat{C} \]  \hspace{1cm} (15)

If one of these conditions is not satisfied then the solutions (4-6) turn out to be unstable. We can see this if we express \( \hat{A}, \hat{B}, \hat{C} \) in terms of the roots of Eq. (13):

\[ \hat{A} = \Gamma_1 + \Gamma_2 + \Gamma_3, \quad \hat{B} = \Gamma_1 (\Gamma_2 + \Gamma_3), \quad \hat{C} = \Gamma_1 \Gamma_2 \Gamma_3 \]

and take into account that

\[ \hat{A}\hat{B} = \hat{C} + (\Gamma_2 + \Gamma_3) (\hat{B} \Gamma_1 + \hat{C/T_1}) \] \hspace{1cm} (16)

In fact, the first of inequalities (15) is evident. If \( \hat{A} \) and/or \( \hat{C} \) is negative then at least one of the roots is always negative; if \( \hat{C} > 0 \) and \( \hat{A} > 0 \) then, denoting a real positive root (which in this case always exists) by \( \Gamma_1 \), it follows from (16) that \( \hat{A}\hat{B} \approx \hat{C} \), if \( \Gamma_2 + \Gamma_3 \approx 0 \). Let us consider the stability conditions (15) for some particular cases.

(a) None of the coefficients \( D_j \) and \( \chi_j \) depend on the ambipolar electric field \( E \).

In this case, as follows from (11, 14), the function \( f_j = 0 \) and, consequently, \( a_j = d_j T/T_j, \quad q_j = p_j T/T_j, \quad b_j = x_j T/T_j \), and the quantities \( A_j, d_j, x_j, a_j, b_j, q_j \) are all positive. Hence, when \( x_j > d_j \) and \( \beta_j > \alpha_j \) (as is usually the case), the quantity \( s_j > 0 \) and the first three inequalities in (15) are satisfied. Let us show that the last inequality is also satisfied. Since \( s_j > d_j, p_j/d_j \), then
\[ \hat{A} > q_e + \frac{a_i p_e}{d_e} = \frac{p_e}{d_e} (a_i + a_e) = \frac{p_e}{d_e} q_e \]

\[ \hat{B} > a_e p_i + q_i d_e \]

\[ \hat{A} \hat{B} > q_e p_i + q_i p_e = \hat{C} \]  \hspace{1cm} (17)

Hence, in this case the stationary solutions (4–6) are stable.

(b) The coefficients of diffusion and thermal conductivity for one of the components (i.e. for ions) depend on the ambipolar electric field \( E_j = e_j E / T_j \), so that

\[ \{D_j, x_j\} = \{D_j^0, x_j^0\} (E_j^2 + \epsilon_j^2)^{-1} \]  \hspace{1cm} (18)

\[ f_j C_j = -2E_j C_j^0 \]

where the coefficients \( D_j^0, x_j^0 \) do not depend on \( E \), and \( \epsilon_j \) is a small parameter removing the singularity as \( E \) goes to 0. Thus, let \( f_e = 0, f_i = -2E_i C_i^0 / C_i \). In this case Eq. (6) becomes

\[ E_i^2 - 2\hat{a}_i E_i - \hat{b}_i = 0 \]

\[ \phi_i^\pm = \frac{E_i^\pm}{\hat{a}_i} = 1 \pm \sqrt{1 + \hat{b}_i / \hat{a}_i^2} \]  \hspace{1cm} (19)

\[ 2\hat{a}_i = \left[ \frac{T_e}{T_i} C_i^0 (\beta_i - \alpha_i \gamma_i) \right]^{-1} \]

\[ \hat{b}_i = \frac{C_e (\beta_e - \alpha_e \gamma_e) T_i}{C_i^0 (\beta_i - \alpha_i \gamma_i) T_e} - \epsilon_i^2 \]

and

\[ A_i = \frac{D_i T}{N T_i} \left[ 1 - \frac{T}{T_e} \frac{\beta_i - \alpha_i}{\beta_i - \gamma_i \alpha_i} \phi_i^\pm \right] \]

\[ A_e = \frac{D_e T}{N T_e}, \quad a_j = \frac{A_j}{\sum_j A_j} \]

\textsuperscript{3} In the opposite case, when \( f_i = 0, f_e = -2E_i C_i^0 / C_e \), the subscripts \( i \) and \( e \) should be interchanged.
From the last expression in (20) we can see that the larger root $\phi^+_i$ is unstable since $C < 0$. Thus, only the smaller root $\phi^-_i$, for which $C > 0$, may correspond to the stable solution. However, we failed to prove that when $A_i$, $a_i$, $q_i$ are arbitrary and negative, all the inequalities are fulfilled for $\phi^-_i$. But when these quantities are negative but satisfy the conditions:

\[ A_i + A_c > 0 \]

\[ q_i + a_i \left( s_e - \frac{P_e}{d_e} \right) + a_c s_i > 0 \]  \hspace{1cm} (21)

\[ a_i p_c + q_i (s_e - d_i) + q_c s_i > 0 \]

the inequalities (17) are satisfied as earlier, and hence the root $\phi^-_i$ corresponds to the stable solutions. We do not give the detailed analysis of inequalities (21) here.

(c) The diffusion and thermal conductivity coefficients depend on $E$ according to (18). In this case the equation for electric field $E_0 = E_i T E/T_1 T_e$ also has two solutions:

\[ E_0^2 - 2\hat{a}_0 E_0 - \hat{b}_0 = 0 \]

\[ \varphi^\pm_0 = \frac{E_0^\pm}{\hat{a}_0} = 1 \pm \sqrt{1 + \frac{\hat{b}_0}{\hat{a}_0^2}} \]

\[ 2\hat{a}_0 = \left\{ \frac{T^2 T_i^2}{T} \sum_j C_j^0 (\beta_j - \alpha_j \gamma_j) e_j \right\}^{-1} \]

\[ \frac{\hat{b}_0}{2\hat{a}_0^2} = -\sum_j \frac{T}{T_j} C_j^0 (\beta_j - \alpha_j \gamma_j) e_j^2 \frac{e_i}{e_i} \sim e_j^2 \]

with

\[ \hat{C} = p_c p_i \frac{T^2}{T_e T_i} (1 - \varphi^\pm_0) \]
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i.e., as earlier, only the small root \( \phi_0 \) may be stable. Since \( \epsilon^2 \ll 1 \), then, generally speaking, \( b_0 \ll a_0^2, \phi_0 - (b_0/2a_0^2) \ll 1 \), and hence the term proportional to \( C_j f_j \sim \phi_0 \ll 1 \) can be neglected in (11, 14). In this case \( A_j, a_j, q_j \) are apparently positive and according to (17) all the inequalities are satisfied. In other words, \( \phi_0 \) corresponds to the stable stationary solutions.

1.4. QUALITATIVE COMPARISON WITH EXPERIMENT

A detailed comparison of the formulas (4–8) obtained above with experiment requires knowledge of all the involved functions (theoretically or, better, experimentally found), and that is not the aim of our paper. We try only to show that our results do not contradict the experimental results cited above [1–6].

In the case of NBI experiments, in the centre of the plasma column the power of the particle sources \( S_0^0 \) is fairly high, and the energy absorbed by ions is also high and exceeds (with account taken of radiative losses) the energy absorbed by electrons. In this case we may suppose that the quantity \( \gamma_i \) exceeds unity but does not significantly exceed the ratio \( \beta_i/\alpha_i \), and \( \gamma_e < 1 \). The point corresponding to these \( \gamma \)'s is depicted in Fig. 1 by the cross and corresponds to the hollow electron temperature profile \( T_e > 0 \) and radially falling profiles of the ion temperature \( T_i < 0 \) and density \( N' < 0 \).

With central ECRH, the power of the particle sources is comparatively low and the energy absorbed by electrons is much higher than that absorbed by ions and grows with an increase in the introduced HF power. We may thus suppose that \( \gamma_i > 1 \) and \( \gamma_e > \beta_e/\alpha_e \). The corresponding point is denoted by the solid circle in Fig. 1 and corresponds to the peaked electron temperature and hollow density profiles. It can be seen that with an increase in HF power \( \gamma_e \) grows, and the point is displaced to the right, which corresponds to the deepening of the hollow in the centre of the density profile and more peaked electron temperature.

With non-central ECRH, the power absorbed by the electrons in the centre is comparatively low and consequently \( \gamma_e > 1 \). The corresponding point in Fig. 1 is denoted by an open circle and it can be seen that it corresponds to the hollow electron temperature and peaked density profiles.

We have tried above to explain the appearance of the hollow profiles in the central part of the plasma column. However, to make this explanation more realistic, it is also necessary to answer the questions of why in the periphery of the plasma all three values of \( \ln N, T_i' \) and \( T_e' \) are always negative in the experiments, and why in the experiment \( \gamma_i \) and \( \gamma_e \) always turn out to lie in the triangle ABC. A possible answer, which does not contradict Eqs (4, 5), may be obtained: (a) if we take into account that in fairly pure discharges, when the radiative and charge exchange losses at the plasma periphery are comparatively small, the functions \( \gamma_i(r) \) monotonically grow with the radius;\(^4\) (b) if we make the supposition (which does not contradict the

\(^4\) Since \( \chi_j = (\alpha_j + 1.5)T_jD_j \), then \( \gamma_j \sim 1/T_j \).
experiments) that at the periphery, for example, the electron thermal conductivity is anomalously high, so that \( \beta_e \gg \alpha_e \); in this case, as follows from (5), even at fairly high \( \gamma_1 \) the density profile remains a decreasing function of radius.

Although on the basis of the rough comparison presented above it would be too daring to speak about the agreement between our theory and experimental data, it may nevertheless be considered that these formulas correctly reflect the experimentally observed tendency of the profile change depending on the value of the absorbed power.

PART II. TRANSITION PARTICLE DIFFUSION IN STELLARATORS

By L.M. Kovrizhnykh, S.G. Shasharina and Yu.A. Volkov

II.1. INTRODUCTION

For a long time a problem of transition particle transport in asymmetric tori has been discussed in the neoclassical literature [9-12]. It concerns the particles which undergo collisionless transitions between locally and toroidally trapped states. The existence of this group of particles entails a problem of the boundary condition for the distribution function of the zeroth order in the collision frequency \( \nu \). Thus the distribution function of the locally trapped particles should be continuously matched to that of the toroidally trapped particles. If the kinetic equation for the locally trapped particles is considered apart from the phase space region corresponding to the toroidally trapped state, the boundary condition can be imposed somewhat artificially and, in some sense, ambiguously. This very choice, done a priori, is a starting point for the discrepancies occurring in the literature devoted to the so called \( \nu \) regime [9-12].

In fact, the boundary condition should be found from certain considerations but not imposed. Some attempts to do this were begun in Ref. [11], where the choice of the condition was related with the ratio of two characteristic times, \( A = t'/\tau' \), with \( \tau' \) and \( \tau' \) being the time of the transitional particle stay in the toroidal and the local state, respectively. As is shown in Refs [9, 10], the general approach of Ref. [11] is correct, but the rigorous results are, in some sense, opposite to those in Ref. [11].

The essence of the problem, which is to solve the bounce averaged kinetic equation in all the velocity space, is discussed in Refs [9, 10], so we shall not present the details here (except for some in Section II.2). In Refs [9, 10] certain scalings for the diffusion coefficient of transition particles are presented, and our main aim here is to comment on the analytic theory of Refs [9, 10] and compare the scalings with the numeric results obtained in Monte Carlo simulation.
II.2. THEORY

First of all, let us recall that the scalings for the diffusion coefficients for transition particles were obtained with certain assumptions which made it possible to find the solution of the kinetic equation as an expansion in \( \nu \). This means that they are valid only at very low collisionality. The upper limit of this regime can be estimated from the following simple considerations. As we consider the transition particles, the collisions should be so rare that the layer in \( k^2 \) space corresponding to the collisionless transitions is not destroyed by the pitch scattering. This can be fulfilled if the width of the steady state collision layer \( \Delta_c \) is much smaller than \( \Delta_i, \Delta_L \).

\[
\Delta_c \ll \min \{ \Delta_i, \Delta_L \} \quad (23)
\]

where \( \Delta_i, \Delta_L \) are the widths of the collisionless transition layers below and above the boundary \( k^2 = 1 \), which separates the toroidally and locally trapped states. The quantity \( \Delta_c \) can be estimated from the expression for the collision operator for \( k^2 < 1 \) (the operator for \( k^2 > 1 \) gives the same result):

\[
C'(F) = \frac{2\nu}{\epsilon df/dk^2} \frac{\partial}{\partial k^2} f(k^2) \frac{\partial F}{\partial k^2}
\]

which leads, in the steady state, to

\[
\Delta_c = \rho(2\epsilon \Omega)^{1/2} \quad (24)
\]

Here, \( f(k^2) = E(k^2) - (1 - k^2)K(k^2) \), \( \epsilon \) is the helical modulation of the magnetic field and \( \Omega \) is the poloidal drift frequency, which for a sufficiently large electric potential \( \Phi \) is equal to

\[
\Omega = \frac{\epsilon d\Phi/dr}{r \omega_m} \quad (26)
\]

The parameter \( k^2 \), characterizing the state of a particle:

\[
k^2 = \frac{E - \mu B_0 (1 - \delta \cos \theta - \epsilon)}{2 \mu B_0 \epsilon} \quad (27)
\]

with \( \delta \) the toroidal modulation.

The width of layer \( \Delta_i \) below \( k^2 = 1 \) can be found from the conservation of the bounce action for the locally trapped particle \( J^l = a(r, \theta) f(k^2) \), with the function \( a(r, \theta) \) given for the stellarator case by [9, 10]:
\[ a \equiv \frac{16R_0(1 + 0.5\delta \cos \theta)\sqrt{\mu B_0}}{M} \]  

(28)

Note that the stellarator case implies that \( \alpha_0 = \delta/\epsilon qM \ll 1 \). From \( J^1 = \text{const} \) one can find \( \Delta k^2 = -\Delta a f(k^2)/(df(k^2)/dk^2) \), which gives:

\[ 1 - k^2(r, \theta) = \frac{1}{2} \frac{f(k^2)}{df/dk^2} \rho(\cos \theta - \cos \theta_0) \]  

(29)

with \( \theta_0 \) the detrapment point \( (k^2(\theta_0) = 1) \) and \( \rho \) given by:

\[ \rho = \frac{\delta}{2} \left[ 1 + \frac{E\delta_e/dr}{\epsilon e d\Phi/dr} \right] \]  

(30)

This means that the width of the collisionless layer \( \Delta_i \) depends on \( \theta \) and has an order of toroidicity:

\[ \Delta_i = 2\rho \approx \delta \]  

(31)

Analogously, it can be shown that the width of the layer \( \Delta_t \) is:

\[ \Delta_t = \delta/\epsilon \]  

(32)

Finally, from Eqs (25, 31, 32) we obtain the following region of the \( \nu \) regime for the transition particles in a stellarator:

\[ \nu_{\text{eff}} = \nu/2\epsilon \ll \rho^2\Omega \]  

(33)

Now that the conditions of applicability of the results have been discussed, let us consider the results themselves. The procedure of solving the kinetic equation, with the requirement (33) implied, involves the so-called integrability (or solubility) condition, which means, in some sense, that the solution should be single valued. It can be schematically written as

\[ \int_1^t \frac{C^l(F_0 + G)}{d\theta^l/dt} d\theta + \int_t \frac{C^t(G)}{d\theta^t/dt} d\theta = 0 \]  

(34)

Here, \( C^l, t \) are the collision operators; \( d\theta^l, t/dt \) are the poloidal velocities for the locally and toroidally trapped states, respectively; \( F_0 \) is a Maxwellian function in the detrapment point (see the comment on this below) \( F_0(r) = F_M(r_0) \); \( G \) is a deviation of the distribution of the \( \nu \)-zeroth order from \( F_0 \) on the boundary \( k^2 = 1 \). Integration in Eq. (34) is performed along the local (l) and toroidal (t) legs of the transition orbit.
In order to estimate the contributions from different parts of Eq. (34) in terms of times $\tau'$ and $\tau^1$ (see Section II.1) it is necessary to note that the collision operators have orders $C^{1,1} \sim 1/(\tau^{1,1})^2$, and integration along the paths with weights $d\theta^1/dt$ gives, additionally, factors $\tau^{1,1}$. Finally, one can obtain the following ordering for $G$:

$$\hat{P}G \approx \tau^1/(\tau^1 + \tau^1) \hat{R}F_0$$

(35)

where $\hat{P}$ and $\hat{R}$ are operators of order unity. It can be seen from (35) that if

$$A = \tau^1/\tau' \ll 1$$

(36)

then we cannot neglect $G$ and thus the $\nu$-zeroth order distribution function on the boundary $k^2 = 1$ cannot be considered Maxwellian. Taking into account that for the stellarator [9, 10]

$$\tau^1 \approx 1/(d\theta^1/dt), \quad \tau^1 \approx 1/(P d\theta^1/dt), \quad P \sim \rho \sqrt{\epsilon}/r$$

(37)

with $\rho$ the gyroradius in the poloidal field and $P$ the trapping probability, we obtain the following ordering of $A$ in the stellarator case:

$$A \sim \sqrt{\delta} \max\{\sqrt{\epsilon}, \sqrt{\delta}\} \ll 1$$

(38)

Thus, the $\nu$-zeroth order distribution function on the boundary $k^2 = 1$ in the stellarator configurations is not $F_0$ but $F_0 + G$, where $G$ has the same order as $F_0$. Note that for the rippled tokamak the contribution from $G$ can be neglected ($A \gg 1$).

The accurate taking into account of the $G$ contribution to transport is performed in Refs [9, 10], where the radial fluxes of transition particles are evaluated for both stellarator and rippled tokamak.

It is necessary to note that to obtain all the results, though this is not evident in the speculations presented above, we do not need any particular dependence of $F_0$ on the energy. All we actually require is that this function depend on the radius of the detrapment point but not on its poloidal angle. To some extent, $G$ is a part of the $n-\nu$-zeroth order function on the boundary which includes in itself all the dependence on the poloidal angle $\theta_0$. That is why the results are also applicable to a monoenergetic ensemble of test transition particles, for which the diffusion coefficient in the $\nu$ regime can be obtained from the general equations of Refs [9, 10], where instead of the Maxwellian unperturbed function one should insert the monoenergetic one:

$$D = \nu \left[ \frac{v_d}{\Omega} \right]^2 \frac{2\sqrt{2}}{\pi \delta^2} \begin{cases} \sqrt{\epsilon} & \epsilon \gg \delta \\ \frac{16}{9} \sqrt{\delta} & \delta \gg \epsilon \end{cases}$$

(39)

(40)

where $v_d = \mu B_0/R\omega$ is the drift velocity.
II.3. MONTE CARLO RESULTS

The Monte Carlo technique is quite well known and is described elsewhere [13, 14]. The only difference in our approach here was an initial pitch angle distribution of the particles which was chosen in such a way that only the transition particles were involved:

\[ 1 - \Delta_i < k^2 < 1 + \Delta_i \]

*FIG. 2. Diffusion coefficient $D_0$ versus helical modulation $\epsilon_0$.*

*FIG. 3. Diffusion coefficient $D_0$ versus toroidicity $\delta_0$.***
The total number of monoenergetic ions \( E = 10 \text{ keV} \) was \( N = 500 \), the initial dimensionless radius \( r/a = 0.5 \) and the magnetic field on the axis \( B_0 = 5 \text{ T} \). The electric potential had the form \( e\Phi = -3E(1 - r^2/a^2) \). The magnetic field modulation had the form \( \delta = \delta_0 r/a \), \( \epsilon = \epsilon_0 (r/a)^2 \) (so that \( 1 = 2 \)); \( q = 4 \) (safety factor), \( M = 14 \) (number of periods). In our runs we kept the effective collision frequency constant: \( \nu_{\text{eff}} = 0.02 \).

The results of our simulation are shown by the solid circles in Figs 2 and 3, where the dimensionless diffusion coefficient \( D_0 = D/\Omega a^2 \) is depicted versus the geometrical factors \( \epsilon_0 \) and \( \delta_0 \).

The toroidicity in the first run (Fig. 2) was \( \delta_0 = 0.05 \), whereas \( \epsilon_0 \) was larger and variable. The solid line in Fig. 2 depicts the analytic result (39), valid for \( \epsilon_0 \gg \delta_0 \) (note that with \( \nu_{\text{eff}} \) kept fixed \( D_0 \) is proportional to \( \epsilon_0^{3/2} \)).

In the second run (Fig. 3) we varied the toroidicity \( \delta_0 \), whereas the helical modulation was smaller and constant, \( \epsilon_0 = 0.1 \). The solid line in Fig. 3 depicts the analytic result (40).

In both cases we saw an excellent agreement of numeric and analytic results, so we can draw the conclusion that the transport theory presented in detail in Refs [9, 10] and briefly in Section II.2 is confirmed by the numeric simulation.

**REFERENCES**


DISCUSSION

F.X. SÖLDNER: In tokamak discharges on ASDEX we observed that the electron density profile can become hollow when the toroidal electric DC field is decreased to zero with lower hybrid current drive. Local analysis of the transport coefficients in this case shows that the anomalous diffusion coefficient remains unchanged but that the inward flow velocity reverses sign in the centre. How could you explain such behaviour in your model?

L.M. KOVRIZHNYKH: It is quite difficult to give an immediate answer because I would need to know the experimental conditions in more detail and perform a new analysis of my model, also taking into account the changes in the equation for the radial electric field, and so on.

P.K. KAW: Have you considered the role of radiation due to heavy metal impurities in determining temperature profiles in stellarators? A point to bear in mind is that hollow profiles in tokamaks were earlier thought to arise because of radiation from tungsten impurities in the centre.

L.M. KOVRIZHNYKH: Yes, I have taken radiative losses into account in the theory: they are included in $P_{\text{in}}$ (Eq. 3).
ADVANCES IN REVERSED FIELD PINCH
THEORY AND COMPUTATION

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Abstract

ADVANCES IN REVERSED FIELD PINCH THEORY AND COMPUTATION.
Recent advances in reversed field pinch (RFP) theory and computation are detailed. These include
the self-consistent calculation of radial transport of field aligned current due to electromagnetic turbu-
ence; the analysis of turbulent transport using a method which establishes rigorous bounds; analytical
and numerical studies of resistive edge turbulence; a description of the detailed non-linear interaction
of low m fluctuations in the context of quasi-periodic oscillations; an examination of single helicity
states; and feedback stabilization of helical fluctuations.
1. INTRODUCTION

Theoretical study of the reversed field pinch (RFP) has long been preoccupied with the relaxed state and its associated mean fields, and with identification of the processes driving relaxation. Less has been learned of the consequences of these driving fluctuations, particularly for transport; or of the consequences of fluctuations not directly tied to the dynamo. With extensive fluctuation measurements now being undertaken in RFP plasmas, and given the need to understand fluctuation induced transport in toroidal configurations in general, these issues have become increasingly important. The present work examines a variety of such issues. In Section 2 the radial transport of field aligned momentum by electromagnetic turbulence is considered. Section 3 details calculations of microinstability driven turbulence and transport, including edge turbulence, and Section 4 deals with the characterization and control of low m fluctuations and their non-linear couplings.

2. SELF-CONSISTENT TREATMENT OF MAGNETIC FLUCTUATION INDUCED TRANSPORT

Of key interest to the RFP is the radial transport of field aligned current associated with the streaming of electrons along perturbed magnetic field lines [1]. Given a sufficiently large transport rate, this process could enable the plasma to maintain a reversed toroidal magnetic field by transporting central field aligned current to the edge [1]. Unlike the MHD dynamo, this mechanism does not require finite resistivity or turbulent decorrelation, and hence is viable in a collisionless plasma. We examine this transport process self-consistently [2] so that the perturbed fields are determined from the particle distributions through the constitutive relations. Both wave-like and eddy-like components of the turbulent fluctuations needed to correctly describe the mode coupling process are included in the fluctuation dynamics. Radial transport of field aligned momentum due to collisionless drift Alfvén turbulence [2] is examined. The usual long parallel wavelength, low frequency regime is assumed with \( v_{lh} < \omega/k \leq v_{lh} \). The ions are hydrodynamic and described by a linear response. The electron distribution consists of an adiabatic response and a non-adiabatic contribution governed by a drift kinetic equation incorporating the dynamics of parallel free streaming, magnetic flutter, and \( E \times B \) convection. Inhomogeneities in the temperature and density are present in the equilibrium distribution so that the source depends on gradients in both quantities as well as both potentials (electrostatic and parallel component of magnetic potential). The ion response depends only on the electrostatic potential since ion streaming is small relative to that of electrons and the phase velocity.

The field aligned current is given by the parallel velocity moment of the electron particle distribution, \( \langle v_1 \rangle = \int |d^3v\ v f_e(v) | \). The evolution of current (and hence transport) is, in turn, obtained from the evolution of the average distribution function,
\[ \frac{\partial \langle v_\| \rangle}{\partial t} = \frac{1}{\rho^3} \frac{\partial}{\partial \rho} (\rho \langle v_\| \rangle) / \partial \rho = 0, \text{ where the flux } \Gamma v_\| = \text{Re} \left[ d^2 v \langle v_\| \rangle / \partial t \right] \times \Sigma_{k,\omega} (ic/B_0) k \times b_0 \cdot \hat{\epsilon} \langle e_{\|} - \langle v_\| / c \rangle \hat{A}_1 \rangle \hat{h}_e \chi_{k,\omega} \text{ is governed in the usual way by the correlations of electrostatic potential and parallel component of vector potential with the fluctuations of the non-adiabatic electron distribution function } \hat{h}_e. \]

The fluctuating particle distribution possesses a wave-like character described by a normal mode solution which satisfies a collective resonance condition (dispersion relation). Additionally, however, the fluctuations have a component which represents localized eddy-like or blob-like structures which cannot be incorporated into the normal or collective mode response [3]. Such fluctuations arise as a consequence of the non-linear mode coupling. As these fluctuations propagate ballistically through the plasma, they are shielded by the collective modes and suffer a drag. Both diffusion and drag operators appear in the transport flux when wave-like and eddy-like components are retained in the fluctuating electron distribution. The shielding of electron blobs by the plasma medium is imposed on the flux function by relating the eddy-like component of the distribution function \( \hat{h}_e \chi_{k,\omega} \) to the shielding potentials \( \hat{A}_1 \) and \( \hat{\phi} \) through quasi-neutrality and Ampère’s law. When these relations are imposed on the turbulent collision operator, the drag terms are no longer independent of the diffusion. For collisionless electrons and steady state turbulence with moderate line broadening at most (\( \Delta \omega \leq \omega_0 \)), the electron contribution to the shielding dynamics is governed by the parallel streaming electron Landau resonance. In this case, the electron-electron drag (drag on electron blobs by shielding electrons) cancels the diffusion, both in the flutter and \( E \times B \) components [2]. As a consequence, the radial flux of field aligned current is governed entirely by the electron–ion drag. Moreover, the surviving electron–ion drag is electrostatic (i.e. \( E \times B \) in origin) as the magnetic flutter component represents an ion contribution to current and is smaller than the electrostatic component by the square root of the ratio of electron to ion mass. The transport of field aligned current is therefore regulated by electron–ion dissipation through electrostatic fluctuations.

The physics which underlies this result is analogous to transport processes (representing the role of discreteness) in the Lenard–Balescu equation. Transport and relaxation require a net change of energy and momentum through the resonant interaction of blobs via the intermediary of the fields. These interactions are constrained to satisfy energy and momentum conservation by the self-consistency relations (quasi-neutrality and Ampère’s law). For the essentially 1-D character imposed by the collisionless parallel resonance, interactions involving the same species leave the final state indistinguishable from the initial state. Only interactions with the opposite species are capable of producing a change in state, given these constraints.

The radial flux of field aligned current which emerges for the drift Alfvén model is

\[
\Gamma_{v_\|} = - \sum_k \frac{1}{2} \left( \frac{cT_e}{eB_0} \right) \frac{u_1}{|k_||} k \times \hat{b}_0 \cdot \hat{\epsilon} \frac{T_e}{e^2} \text{Im} d_{\phi\phi}^{(\text{ion})} \left( \frac{\rho_i^2}{L_i^2} \right) \left( \frac{\Delta^2}{\rho_i^2} \right)
\]

(1)
where \( u_B \) is the resonant phase velocity, and the amplitude of the cross-correlation \( \langle \dot{\phi} \bar{h} \rangle_{k,\omega} \) has been approximated by a mixing length expression with \( \Delta \) the mixing length and \( \rho_i \) the ion gyroradius. The dielectric \( d_{\phi}^{(m)} \) is the ion contribution to the quasi-neutrality condition. This flux relation is strikingly different from the quasi-linear flutter result which typifies non-self-consistent calculations. In addition to the fact that it is electrostatic (depending on \( \langle \phi \rangle^2 \) rather than \( \langle A \rangle^2 \)) and proportional to ion dissipation, its amplitude is also strongly reduced relative to the quasi-linear flutter value. Taking the ratio of Eq. (2) with the flux calculated non-self-consistently from quasi-linear theory, and assuming \( \delta B_r / B \sim \sqrt{\beta} \ (k \rho_i \epsilon / T_e) \) (consistent with a non-linear regime) \( \Gamma_{v_b} / \Gamma_{QL} \approx \rho_i / \rho_b \). Noting that \( \beta \approx 10\% \) in the RFP, the flux is sufficiently small to make the kinetic dynamo mechanism ineffective in maintaining a reversed toroidal magnetic field.

Other features of this result are worth mentioning. The field aligned current flux enters Ohm's law as the radial derivative of \( r \) times the flux: \( E_{\phi_0} = \eta \langle J_\phi \rangle - (m_e / e) \rho_i^{-1} \partial / \partial r (\Gamma v_b) \). In computing the bulk resistance by integrating over the surface normal to the current density, the turbulence induced contribution to Ohm's law is a surface term representing the net flux of field aligned current out of the plasma. Hence, transport of electron momentum within the plasma does not contribute to the bulk resistance; rather the resistance is affected solely by momentum loss, either by electron-ion collisions or transport out of the system. It is also worth noting that fast particles, if resonant with collective plasma fluctuations, are subject to the constraints of this calculation. If, on the other hand, the particle velocity is well above the phase velocity of the collective modes, but not so large that the particle drift takes the particles more than a radial correlation length off the mode rational surface, then the particles can behave as test particles. Such particles are not subject to the self-consistency constraints of the present calculation.

Efforts to examine field aligned current transport numerically have been initiated. A Monte Carlo code has been written to study charged particle transport arising from pitch angle scattering, loop voltage and magnetic surface breakup due to MHD modes with a variety of spectra. From an initial cloud of particles (representing runaways, thermal particles, trapped and passing particles) macroscopic particle and energy diffusivities are obtained using the canonical drift Hamiltonian. Preliminary results indicate that test particle transport due to a stochastic magnetic field (non-self-consistent) is governed by Rechester-Rosenbluth type scaling, \( D \sim \langle (\delta B / B)^2 \rangle \). An effort to include the effects of a self-consistent ambipolar potential is under way.

In another test particle model, the roles of electron-ion collisions and cross-field electron currents are considered. Magnetic flutter transport due to a specified stochastic magnetic field is examined using a non-local Ohm's law to solve for the electric field parallel to \( B \). The full electron distribution function is used, ranging from the low velocity, highly collisional electrons to those that are fast and non-resonant. In this model, cross-field currents of collisional electrons relieve the induced electric fields which otherwise would inhibit flutter transport in collisionless, stochastic field models.
A key question concerns the extent to which the self-consistent calculation outlined above might apply to resistive modes. Inasmuch as relaxation is directly tied to the evolution of fluctuations [3], including the instability and growth of collective modes, this issue relates to the stability of resistive modes in the presence of turbulence. Here, the role of turbulent electron viscosity [4] is considered in a self-consistent calculation. The evolution of a long wavelength, hydrodynamic mode (e.g. resistive fluid instabilities) is treated in a collisionless bath of short wavelength electromagnetic fluctuations such as drift Alfvén waves. For typical fluctuation levels, i.e. \( e\hat{\phi}/T_e \lesssim 10^{-3} \), turbulent scattering of fluctuations is found to significantly dominate collisional scattering in inducing a non-zero parallel electric field by field-fluid decoupling. The assumption of collisionless, ambient microturbulence modifies Ohm's law in two crucial respects. First, resonant wave-particle interactions give rise to relaxation of higher order moments (i.e. heat flux and stress) in addition to relaxation of parallel current. Second, the static MHD or dynamo non-linearity (so-called hyperviscosity) is replaced by a dynamic, non-local turbulent collision operator. In order of magnitude, the electrostatic piece of this kinetic hyperviscosity is the dominant non-linear term. The collisionless, fluctuating Ohm's law can then be written as

\[
\hat{E}_\parallel = (2\pi/\omega)^2 [4\pi(u^2/c^2)e b_k - \alpha_1 v_{eo}] \hat{\nu}
\]

(2)

where \( u_\parallel = \omega/k_\parallel \), \( \alpha_1 \) is a constant of order unity, \( b_k = (\pi c/B)^2 |k_\parallel|^{-1} \partial_{k_\parallel}^2 \times |\text{Im} d_{\lambda,\lambda}(k',k'u_\parallel)| \text{Im} d_{\lambda,\lambda}^{(0)}(k',k'u_\parallel)| \langle \hat{\phi} \hat{\phi}(u_\parallel) \rangle \hat{\nu}_{\lambda,\lambda,\lambda} \) is the electrostatic kinetic hyperviscosity, and \( \epsilon \) is the dielectric. The first term of the right hand side is the modification to Ohm's law. When this term dominates the collisional resistivity (i.e. \( e\hat{\phi}/T_e \approx 10^{-3} \)), instabilities are driven by the background fluctuations. The growth rate, mode width and transport coefficient scalings of well established resistive modes are all correspondingly modified. In the case of resistive interchange modes, those are now given by \( \gamma/\omega_A \sim A^{1/4} m^{1/2} (L_v/r_s)^{1/2} [(v_A^2/c^2) \rho_i^2 k_\parallel P_0]^{3/4} \), \( \Delta x/r_s \sim A^{1/2} m^{1/2} (L_v/r_s)^{1/2} \times [(v_A^2/c^2) \rho_i^2 k_\parallel P_0]^{1/4} \), and \( D/\omega_A r_i^2 \sim A^{3/4} m^{1/2} (L_v/r_s)^{3/2} [(v_A^2/c^2) \rho_i^2 k_\parallel P_0]^{5/4} \), where \( A \equiv \pi^3 (k_\parallel \rho_i)^2 \langle e\hat{\phi}/T_e \rangle \), \( \hat{\phi} \) is the fluctuation level of the background turbulence, and \( \kappa \) is the curvature. In the case of tearing modes, \( \gamma/\omega_A \sim A^{1/2} m \Delta \L_t \) and \( \Delta x/r_s \sim (A \Delta') (L_t^2/mr_s)^{1/3} \). An important conclusion of the theory is that hybrid fluid/kinetic models which fail to account for resonant interactions are prone to miss important physics.

3. MICROINSTABILITY DRIVEN TURBULENCE AND TRANSPORT

Despite its importance, the role of anomalous transport in the RFP has received relatively little attention. In addition to the effect on transport of the internally resonant global tearing modes which figure in the dynamo, small scale fluctuations are also present in the RFP, and may in fact control global confinement if there is
an edge confinement zone, as is frequently postulated. Two widely differing approaches to the study of transport in the RFP are described in this section.

3.1. Rigorous bounds

The fundamental problem in the theory of turbulent transport is to find the flux $\Gamma$ of a quantity such as heat. Methods based on statistical closures are subject to controversies and practical difficulties. However, it is possible to bound $\Gamma$ by employing constraints derived rigorously from the equations of motion [5, 6]. Here, we consider anomalous resistivity generated by self-consistent turbulence in an RFP. Application of this method to the RFP [6, 7] begins with the resistive, viscous, and incompressible MHD equations subject to reasonable, statistically sharp boundary conditions. Using the assumption of steady state turbulence driven by a prescribed axial electric field $E_0$, a positive-definite form for the spatial average $\bar{\epsilon}$ of the axial electromotive force is found, where $\epsilon = \hat{z} \cdot \langle \delta B \times \delta u \rangle$, and $\delta B$ and $\delta u$ are the turbulent fluctuations in the magnetic field and flow velocity. An upper bound on $\bar{\epsilon}$ is obtained subject to the constraints of global energy and/or helicity balance by solving the appropriate (complicated and non-linear) Euler–Lagrange equations. From the solution, a variety of quantities are obtained for direct comparison with experiment or simulation. It is speculated that the extent to which the plasma achieves a bound determines how close the calculated quantities are to measured values.

This procedure has been used to predict general features of the RFP including profiles of mean magnetic fields, fluctuation levels, and the degree of toroidal field reversal using solely the global energy constraint. It is found that $\bar{\epsilon} \sim H^{0.1} J^{1.1}$, where $H = (a^2 B_0^2/c^2 \eta \rho)^{1/2}$ and $J$ is the mean axial current density. For small $J$, the dominant mode is $(m,n) = (1,-2)$, field reversal is predicted at a pinch parameter of about 1.5 for $H \sim 10^5$, and the magnetic field fluctuation level is small. Although preliminary, these results are in reasonable agreement with simulation and experiment. The natural but technically involved extension of this calculation is to study the effects of an additional helicity constraint.

3.2. Resistive edge turbulence

The role of edge turbulence in the confinement of plasmas in toroidal devices is increasingly apparent. This is particularly evident in the tokamak H-mode where edge localized poloidal flow shear is identified analytically with suppression of turbulent fluctuations and correlated experimentally with improved confinement. In the RFP, edge confinement is also frequently invoked, though little is known of the confinement properties of modes unstable in the RFP edge. Moreover, theoretical study of edge turbulence and transport is greatly aided and indeed motivated by the availability of extensive experimental data.

Resistive interchange (g-mode) turbulence has long been considered a principal candidate for RFP edge turbulence [8–11]. The physics of saturation is reasonably
well understood and can be described in terms of turbulent power flow [12]. Eddies extract energy from the mean pressure gradient and dissipate energy to the electrons through resistivity and to the ions through viscosity via a cascade of kinetic energy to the dissipation scale. Balancing these sources and sinks allows the properties of the turbulence to be calculated. In particular, the spectrum of \( m = 1 \) modes is given by

\[
\frac{\tilde{B}}{B} \sim (\lambda a)^{-3/2} g^{3/4} S^{-1/2} n^{-1/2},
\]

where \( \lambda = \mu_0 J / B - 2 B_0 B_\phi / r B \) is the inverse shear length, \( g = \frac{-2}{\sqrt{3}} a^2 P / r B \) is the dimensionless interchange drive, \( S = \mu_0 a v_A / \eta \) is the Lundquist number (with \( v_A \) the Alfvén velocity), and \( n \) the mean toroidal wavenumber. These results are in reasonable agreement with recent magnetic fluctuation data from ZT-40M [13].

Turbulence also plays a role in transferring energy between the electron and ion species, consistent with the observations that \( T_i \sim T_e \) holds even when classical equipartition times are long. The interchange drive powers from either species are proportional to their individual temperatures, while the resistive dissipation to the electrons and the viscous dissipation to the ions are comparable. Thus, turbulence will act to equilibrate the ion temperatures on the time-scale of the eddy turnover time (also the energy throughput time). This time is

\[
\tau_{\text{el},B} = a / v_A (R / a)^{-2/3} (\lambda a)^{2/3} g^{2/3} S^{2/3} n^{-2/3}.
\]

This turbulent equilibration rate is faster than the classical rate by the ion/electron mass ratio, leading to nearly equal temperatures in the present experimental regime. As size and current are scaled, however, the turbulent/classical rate ratio decreases as \( a^{-11/3} t^{-4/3} \), so that the species may be less closely coupled in future experiments. A one-dimensional calculation including classical transport, radiation, and these turbulence driven transport effects reproduces the experimentally observed flat temperature profiles, with \( T_i \sim T_e \) throughout the discharge.

Recent probe measurements of fluctuations in the edge of TEXT, ATF and ZT-40M show a striking similarity of numerous salient features, including correlations lengths, spectrum characteristics, and relative levels of \( \phi / T_e \), \( \bar{n} / n \), and \( \bar{T} / T \) [14, 15]. Furthermore, theoretical and numerical modelling suggests the importance of processes intrinsic to the edge, specifically, radiation and ionization [15]. A first step towards introducing such processes into the structure and dynamics of resistive interchange modes is to extend the model to incorporate the driving source and physical mechanisms of resistivity gradient driven turbulence (RGDT) [16]. The linear rippling mode which underlies RGDT is strongly affected by the larger shear and smaller magnetic field strength of the RFP. Hence, its growth rate is small in comparison with that of the g-mode. However, in the non-linear regime, RGDT has an inverse magnetic field scaling which offsets the stronger shear. Indeed, for MST parameters, the fluctuation level and mode width of RGDT are comparable to those of mixing length estimates of g-mode turbulence.

The model for the coupled system consists of equations for vorticity, density, and temperature evolution, and Ohm's law. The non-linearities are \( E \times B \) convection of vorticity, density and temperature. Parallel thermal conductivity and resistivity (line bending) are the energy sinks. Numerical solution of the linear equations shows breaking of the even (g-mode) parity with a shift of the mode structure off the rational
surface. Analysis of the energy evolution equations indicates that decoupling of the current and potential fluctuation structures in the turbulent steady state is unlikely. Saturation occurs through a balance of the line bending term in the energy evolution equation with the resistivity gradient and pressure gradient (curvature) source terms as mediated by a turbulently broadened mode width. The diffusivity is given by

\[ D = \left( \frac{3}{2} \right) \left( \frac{\eta_0 j_0 / B_0}{r/\Delta} \right) \left( \frac{\partial q}{\partial r} \right)^{-1} \left( \frac{\Delta}{L_T} \right) + \kappa (r/R)^2 \left( \frac{\partial \rho}{\partial r} \right)^{-2} \left( \frac{\eta_0 P_0 / L_T B_0^2}{L_{\tau, \kappa}} \right), \]

where \( \Delta \) is the mode width, \( \kappa \) is the curvature, \( q \) is the safety factor, \( L_T \) (\( \sim L_n \)) is the temperature gradient scale length, and \( \eta_0, j_0, B_0, \) and \( P_0 \) are the equilibrium resistivity, current, magnetic field and pressure. This expression indicates that the diffusivity of pure g-mode turbulence is enhanced by a resistivity gradient term which is comparable, assuming the mode width is not significantly altered in the combined system.

4. NON-LINEAR DYNAMICS OF LOW m FLUCTUATIONS

Important aspects of low m fluctuations are explored. These include the possibility of controlling fluctuations associated with a resistive shell, limiting where possible the contribution of low m dynamo fluctuations to anomalous losses, and fully characterizing the details of their dynamics.

4.1. Feedback stabilization of helical fluctuations

Long pulse RFP operation with pulse lengths in excess of the shell time potentially poses a serious stability problem [17]. One possible solution is feedback stabilization of helical fluctuations associated with a resistive shell. To investigate the physics of feedback stabilization, a 3-D non-linear resistive MHD code has been operated with a resistive shell and feedback coils to compensate for the absence of a conducting shell. The feedback is accomplished in the code by setting the radial magnetic field of selected helical modes to zero at the plasma boundary. Extensive results have been obtained on the effect of feedback on the loop voltage, equilibrium quantities and fluctuations, as a function of modes selected for feedback [18].

At an aspect ratio of 2.5, stabilization of a few dominant modes (e.g. \( m = 1 \) and \( n = 5,6,7 \)) reduces the loop voltage drastically from the case without feedback. The loop voltage for RFP sustainment falls to within a factor of two of that obtained with a conducting shell. Field reversal is maintained by adjacent non-stabilized modes in the wavenumber spectrum. These acquire energies which are somewhat larger than the non-stabilized case. When feedback is performed on many additional modes a diminishing effect is produced. The role of feedback stabilization in reducing the loop voltage is understood through the effect of the fluctuations on the induced electric field \( \langle v \times B \rangle \). The velocity fluctuation appears to be most strongly affected by the feedback. Little change was observed in the magnetic fluctuations. This work indicates that feedback stabilization of a few key helical modes is a promising method for dealing with the thin shell problem in the RFP. The number of modes which must
be stabilized for higher aspect ratio and the effect of a larger Lundquist number are issues which are presently being studied.

4.2. Single helicity states

The possibility of producing an RFP plasma with a single helicity state would allow for relaxation and reversal with potentially lower loop voltage and reduced transport. An extensive study of single helicity RFP Ohmic states has also been carried out both analytically and numerically [19]. In the steady state, the 3-D, $\beta = 0$ MHD equations reduce to $J \times B = 0$, $\nabla \times (V \times B - \eta J) = 0$, and $J = \nabla \times B$. Numerically, we have found that these states are universal time asymptotic attractors for the single helicity equations (i.e. the single helicity equations asymptote into these states independent of the initial conditions). These states are characterized by complete reversal on the boundary, although the states have perfect flux surfaces. We have demonstrated numerically that they obey the analytic constraint [20] that the flux surface averaged value of $\lambda$ [$\langle \lambda \rangle = \langle J \cdot B \rangle / \langle B \cdot B \rangle$] reverse sign at the same surface on which the flux surface averaged value of $B_\phi$ changes sign. These solutions also show self-similarity in their profiles and in the analytically predicted scalings that $\delta B / B$ is independent of $S$ ($S = \tau_R / \tau_A$) while the velocities are proportional to $S$.

We have also shown that these solutions are attractors in three dimensions although their basin of attraction is significantly reduced. Bifurcation phenomena have been observed with two stable states (one of which is single helicity and the other one multiple helicity) existing at the same set of plasma parameters. The basin of attraction is largest for high dissipation (either resistivity or viscosity) and may also be widened by inducing resonant field errors on the boundary. The effectiveness of inducing these states with external field errors is still being investigated.

4.3. Quasi-periodic oscillations

A key question in the workings of the MHD dynamo is the relation of the experimentally observed flux jumps (sawteeth) to the detailed non-linear dynamics. A similar and related phenomenon is observed in simulations of non-linearly interacting internally resonant resistive modes and manifests itself as quasi-periodic oscillations of the reversal parameter [21]. This issue has been investigated using a 3-D resistive MHD code [22] to examine the power flux through Fourier space as poloidal magnetic energy is converted to toroidal magnetic energy by the dynamo. It is concluded that the dynamo can be dominated by either quasi-linear [23] or non-linear interactions [24]. They both occur during each cycle of oscillation: the non-linear dynamo dominates the 'crash phase' when fluctuations are large and reversal is increasing; the quasi-linear dynamo maintains reversal during the 'diffusion phase' when fluctuations are small (weak modal interaction) and reversal is decreasing.

The non-linearly sustained $m = 0$ modes (driven by internally resonant, unstable $m = 1$ modes) play a crucial role in the non-linear dynamo mechanism described
here in three stages. First, interaction between \( m = 0 \) and \( m = 1 \) modes non-linearly modifies the \( m = 1 \) velocity and magnetic fluctuation, which then affects the \( m = 1 \) induced electric field \( E_f \). Here, \( E_f \) is given by \(-\langle \mathbf{v} \times \mathbf{b} \rangle\), where \( \langle \rangle \) denotes an average over poloidal and axial directions, and \( E_f \) induced by each mode can be isolated (i.e. \( E_{f,m,n} = -\langle v_{m,n} \times b_{m,n} \rangle \)). This enhances the rate of energy flux (through work done by \( E_f \) on the mean current, i.e. \(-\int d^3r E_{f,m,n} \cdot \langle J \rangle\)) from the poloidal magnetic field to the \( m = 1 \) modes resonant near the axis. Part of the energy flux sustains the \( m = 1 \) modes against dissipation; the remainder is transferred to the \( m = 1 \) modes resonant closer to the reversal surface through back-coupling of \( m = 0 \) with \( m = 1 \) modes [24]. Finally, energy is converted to the toroidal field from modification of the \( m = 1 \) modes resonant close to the reversal surface again via \( m = 0 \) modification of the \( m = 1 \) induced \( E_f \). These interactions produce effects on Alfvénic time-scales, thus inertia plays a significant role. The immediate consequences are that reversal is deepest when \( m = 0 \) peaks, and that non-linear dynamics becomes more important with high \( S \) and high \( \Theta \).
Key aspects of the process of magnetic energy conversion from the poloidal field to the toroidal field are illustrated in Fig. 1. In this example, a single crash in $F$, the reversal parameter, was simulated (Fig. 1(a)) by slowing the increase of $\Theta$ in a low $S (=10^3)$ and low aspect ratio ($R/a = 1$) system. Figures 1(b) and 1(c) show the time history of the instantaneous growth rates for the $(m,n) = (1,-2)$ mode resonant near the axis (Fig. 1(b)), and $(1,-4)$ mode (Fig. 1(c)) resonant near the reversal surface. Here, the actual growth rate, $\gamma$, is a sum of three components: $\gamma_q$, the quasi-linear growth rate (power flux from the mean field to a specified mode normalized to the modal energy); $\gamma_n$, the non-linear growth rate (power flux from the non-linearly coupled modes to a specified mode normalized to the modal energy); and $\gamma_D$, the dissipation rate. After the crash, a dramatic increase in $\gamma_q$ for the $(1,-2)$ mode is observed (due to increase in power influx from the mean poloidal magnetic field), which balances the increased power loss from mode coupling (decrease in $\gamma_n$). On the other hand, the $(1,-4)$ mode becomes a recipient of mode coupling energy (increase in $\gamma_n$); this energy balances the decrease in $\gamma_q$ (because of the increase in power flux to the mean toroidal field).

The non-linear dynamo is stabilizing to the $m = 1$ modes as it enhances energy cascade to small scales [24] and increases shear (more reversal). Owing to the inertial time lag between the appearance of $m = 0$ and $m = 1$ modes, however, the non-linear stabilizing effect always peaks after the $m = 1$ modes have already saturated. This overstabilizes the $m = 1$ modes and causes their crash, thus resulting in the observed quasi-periodic oscillations.

ACKNOWLEDGEMENT

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REFERENCES

DISCUSSION

R.H. COHEN: My question concerns the comparison of rigorous upper bounds with the two point closure calculations. Analytic closure calculations typically involve a number of approximations and do not guarantee the production of fluxes less than, or equal to, the true fluxes. You showed different scalings for the rigorous upper bound and two point closures — so they would, in principle, cross for some parameter values. Does the upper bound result actually bound the closure result for all interesting parameters? Which approximation breaks down when the two point result finally crosses above the upper bound?

P.W. TERRY: There is no general theorem or general understanding of the relation of the bounded flux to those of any two point closure. For one specific case, that of passive scalar advection and the direct interaction approximation (DIA) as the closure, the closure flux was lower than the true flux, which in turn was lower than the upper bounds flux. There were no crossings.

I.H. HUTCHINSON: The self-consistent calculation of electron transport appears to apply primarily to small scale modes. If the perturbations are dominated by large scale, e.g. m = 1, modes, does the flux reduction still occur? In other words, in such a case can there be a substantial fraction of the distribution function that consists of 'test particles' in your sense?

P. TERRY: The large scale fluctuations (m = 1) in an RFP are presumably tearing modes and do not satisfy the collisionless condition assumed for the calculation of field aligned current transport. In any case, a substantial fraction of the distribution will alter the potentials and cannot therefore be treated as test particles.
SIMULATION STUDY OF THE
NON-LINEAR DYNAMICS IN A
REVERSED FIELD PINCH CONFIGURATION

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Abstract

SIMULATION STUDY OF THE NON-LINEAR DYNAMICS IN A REVERSED FIELD PINCH CONFIGURATION.

Significant progress in the understanding of the non-linear dynamics in a reversed field pinch has been achieved through a 3-D MHD simulation study. The paper presents findings on: (1) the details of the dynamo process, (2) the self-sustainment mechanism, and (3) the phase locking of kink modes. The most important finding in these studies is that the non-linear coupling between a few unstable kink modes plays an essential role in the magnetohydrodynamics in a reversed field pinch.

1. INTRODUCTION

The key process in the non-linear dynamics of the reversed field pinch (RFP) is the dynamo action, which has been investigated by many authors. Nevertheless, many questions on the dynamics in the RFP still remain unresolved. In this paper, we consider three subjects relating to the non-linear dynamics in the RFP. The first is the dynamo process. In particular, we resolve the question of what is the essential non-linear coupling in the RFP dynamo process. The second subject is the self-sustainment mechanism and the third is the phase locking process, which was observed experimentally [1]. Then we discuss the common features of the non-linear dynamics in the RFP.
For the investigation of these subjects, we make use of MHD simulations, using a Fourier analysis MHD code. In this code, any physical variable \( f(r, \theta, z) \) is decomposed into the complex Fourier component \( \tilde{f}(r : m; n) \) for the poloidal (m) and the toroidal (n) mode, i.e.

\[
f(r, \theta, z) = \sum_{m,n} \tilde{f}(r : m; n) \exp i(m\theta - nz/R)
\]  

(1)

The details of this code can be found in Ref. [2]. The basic equations are the primitive MHD equations, but the density and the plasma pressure are assumed to be homogeneous, \( \rho = p = 1 \). Hence the pressure driven modes are excluded from the equation system. The focus of our interest is on the non-linear behaviour of the current driven modes.

We use the boundary condition that guarantees the conservation of the total toroidal magnetic flux and of the total toroidal current. For the study of the phase locking process in Section 2.3, however, the perfectly conducting boundary condition is used. The initial condition is always composed of an unstable force free equilibrium and a perturbation. The equilibrium is specified by the profile of the ratio \( \lambda \) between the magnetic field \( B \) and the current \( J \), i.e.

\[
\lambda = \frac{\mathbf{J} \cdot \mathbf{B}}{B^2} = \lambda_0 \{ (\cos \pi r + 1)/2 \} \phi
\]

(2)

The perturbation is given by the linear eigenfunction for the unstable modes.

In Section 2 the simulation results for the dynamo process, self-sustainment and phase locking will be presented. In Section 3 we will discuss the general features of the non-linear dynamics in the RFP, on the basis of the simulation results.

2. SIMULATION RESULTS

2.1. Details of the dynamo process

Although it is widely believed that the \( m = 1 \) kink mode instabilities lead to a field reversal process, the 'dynamo', the question of the dominant non-linear coupling in the dynamo process is still in dispute. Roughly speaking, there are two different models of the non-linear process in the dynamo: the \( m = 1 \) dynamo and the \( m = 0 \) dynamo. The former proposes that the coupling of the unstable kink mode with itself is the dominant dynamo coupling [3], while the latter proposes that the \( m = 0 \) mode driven by the different kink modes generates the dominant dynamo coupling [2, 4]. In this subsection, we focus our interest on the question of how large the contribution of the kink (\( m = 1 \)) mode (linearly unstable) and of the \( m = 0 \) mode (driven by non-linear coupling) is to the dynamo.
Generally speaking, the dynamo process is a kind of flux generation process, and hence the dynamo contribution must be evaluated by the amount of the time integration of the dynamo electric field. In the self-reversal process of an RFP, it should be given by:

\[ \Phi_{\text{dyn}}(m; t_0, t_1) = - \int_{t_0}^{t_1} \sum_n \{ \vec{V}(r_{\text{rev}}, m, n) \times \vec{B}(r_{\text{rev}}, m, n) \} \, dt \]  

where \( r_{\text{rev}} \) is the radius of the reversal surface. The integration \( \Phi_{\text{dyn}} \) corresponds to the reversal flux generated by the dynamo process for the \( m \) mode in the period \( t_0 \) to \( t_1 \). We carry out four different simulations of the self-reversal process, where four different initial equilibria are employed. These equilibria (IC1, IC2, IC3 and IC4) have the same normalized magnetic helicity (\( \alpha = 6.87 \); the normalization in \( \alpha \) is the same as that in Ref. [5]), but have different \( \xi \) in Eq. (2): \( \xi = 0.7, 0.9, 1.1 \) and 1.5 for IC1, IC2, IC3 and IC4, respectively. If the total magnetic energy \( W \) is normalized by the energy of the Taylor Bessel function model state \( W_{\text{BFM}} \) for \( \alpha = 6.87 \) [6], the value \( W/W_{\text{BFM}} \) is 1.059, 1.081, 1.104 and 1.148 for IC1, IC2, IC3 and IC4, respectively. Obviously, a larger \( \xi \) leads to a higher magnetic energy. This means that a larger \( \xi \) generates a more unstable state for kink modes, since the difference \( W - W_{\text{BFM}} \) corresponds to an excess magnetic energy driving the MHD instabilities. Figure 1 shows the minus \( \Phi_{\text{dyn}}(m; 0, 200\tau_\alpha) \) for \( m = 0 \) and 1 in the

**FIG. 1.** Dynamo flux, \( \Phi_{\text{dyn}}(m; 0, 200\tau_\alpha) \), in the simulations using IC1, IC2, IC3 and IC4. Open circles: \( m = 0 \); solid circles: \( m = 1 \).
above four simulations. For IC1, the $m = 0$ dynamo flux, $\Phi_{\text{dyn}}^{m = 0}$, is larger than the flux for $m = 1$. Hence the contribution from the $m = 0$ mode surpasses that from the $m = 1$ kink mode. As the excess energy is further increased, however, the contribution of the $m = 0$ dynamo gradually decreases as that of the $m = 1$ dynamo increases. For IC4, $\Phi_{\text{dyn}}^{m = 0}$ for $m = 0$ becomes positive, and hence the $m = 0$ mode acts as an antidynamo agent.

These results provide an answer to the above question regarding the size of the contributions to the dynamo process. The contribution of the $m = 0$ mode depends on the excess energy accumulated in the system just before the dynamo process. The larger the excess energy, the smaller the contribution of the $m = 0$ mode in comparison with the $m = 1$ mode. Therefore, in order to comprehend more thoroughly the non-linear dynamics in RFP experiments, we have to advance to the next question, that of how much excess energy is spontaneously accumulated in the sustainment phase of the RFP discharge.

2.2. Self-sustainment mechanism

Now we must calculate the dynamo as a spontaneous process in the resistive MHD system rather than as a result of an instability arising from an initially assumed unstable equilibrium. This requires a numerical simulation of long duration (long compared with the resistive time-scale), since the accumulation of the excess energy is due to the resistive evolution in the sustainment phase of the RFP discharge.

Figure 2 shows the long time history of the magnetic energy normalized by $W_{\text{BFM}}$ for two aspect ratios, (a) 1.6 and (b) 4.8. Figures 3(a) and (b) show the evolution of the radial profile of the toroidal magnetic field, $\mathbb{Re}\{B_z(r, m = n = 0)\}$, for

![Figure 2](image-url)

**FIG. 2.** Time history of the total magnetic energy $W$ in the cases of (a) small aspect ratio (1.6) and (b) large aspect ratio (4.8). The energy is normalized by a Taylor minimum energy $W_{\text{BFM}}$ determined from the instantaneous $\alpha$. 
the two aspect ratios. We can clearly see in Fig. 2(a) that the magnetic energy has an oscillation that is composed of a slow ramp-up phase and a fast relaxation phase, like a sawtooth oscillation. The slow ramp is due to the resistive evolution, while the fast relaxation is due to the dynamo. Figure 3(a) shows that at each relaxation phase the reversed field is spontaneously generated and the value $B_z$ on the axis and the wall decreases sharply. It is worth noting that the critical level of the excess energy,
at which the dynamo process starts, is fixed at about 6% of $W_{BFM}$. On the other hand, in Figs 2(b) and 3(b) we can see that the critical excess energy and the oscillation amplitude in the large aspect ratio case are about 1% smaller than those in the small aspect ratio case. These results indicate that the critical excess energy and the amplitude of the energy oscillation depend on the aspect ratio of the device.

In order to explain physically the dependence on the aspect ratio, we analyse the linear stability for ideal kink modes of the $(m; n) = (0; 0)$ component before and after the typical dynamo process, which starts at $270\tau_A$ in the small aspect ratio case. Table I shows the linear growth rate for the two most unstable kink modes, $(m; n) = (1; 3)$ and $(1; 4)$, at four different times, where the dashes denote that the mode is stable at that time. The growth rate of the $(1; 3)$ mode gradually increases with time. Just before the relaxation process starts ($t = 256\tau_A$) the adjacent mode $(1; 4)$ becomes unstable, in addition to the $(1; 3)$ mode. After the relaxation process ends, however, both modes become stable again ($t = 300\tau_A$). This result is consistent with the $m = 0$ dynamo model, in which at least two kink modes must be unstable to drive non-linearly the $m = 0$ mode. So we can explain why the critical energy depends on the aspect ratio by using the $m = 0$ dynamo model. The interval between the neighbouring kink modes in wavelength space,

$$\Delta \lambda \equiv \lambda_n - \lambda_{n+1} = \lambda_n^2 (2\pi R + \lambda_n)^{-1}$$

becomes larger as the aspect ratio $R$ becomes smaller, where $\lambda_n = 2\pi R/n$. Larger $\Delta \lambda$ requires a wider unstable region and also a higher excess energy.

<table>
<thead>
<tr>
<th>$n$</th>
<th>$t = 152\tau_A$</th>
<th>$t = 200\tau_A$</th>
<th>$t = 256\tau_A$</th>
<th>$t = 300\tau_A$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>$1.1 \times 10^{-1}$</td>
<td>$2.7 \times 10^{-1}$</td>
<td>$2.9 \times 10^{-1}$</td>
<td>—</td>
</tr>
<tr>
<td>4</td>
<td>—</td>
<td>—</td>
<td>$1.6 \times 10^{-3}$</td>
<td>—</td>
</tr>
</tbody>
</table>

We can conclude that the RFP configuration is sustained in a cyclic process, where the MHD relaxation (dynamo) phase and the resistive diffusion phase appear cyclically and alternately. When at least two kink ($m = 1$) modes become ideally unstable, MHD relaxation can take place. This is due to the fact that the MHD relaxation progresses through the dynamo process for the $m = 0$ mode. The larger the aspect ratio of the device, the lower the critical excess energy for the dynamo and
the smaller the amplitude of the oscillation. Remember that the critical energy even for the small aspect ratio (1.6) is at most 6% of $W_{BFM}$. This value is lower than the excess energy of the initial equilibrium IC2 in the previous subsection. As expected from this result, the dynamo flux for the $m = 0$ mode is confirmed to be as large as that for the $m = 1$ mode. Therefore, as long as the aspect ratio is as large as those in the usual experimental devices, the critical excess energy cannot be higher than about 6% of the Taylor minimum energy. Namely, it is impossible for the $m = 1$ dynamo to overpower the $m = 0$ dynamo. Through detailed examination, we have further found some other characteristics of this relaxation-diffusion oscillation (see Ref. [2]).

The dependence of the oscillation on the aspect ratio helps to explain the difference in behaviour of the experimental plasmas between the large aspect ratio and the small aspect ratio devices. In a small aspect ratio device like MST a clear sawtooth oscillation is always observed, while in a large aspect ratio device like ZT-40M it is observed only for high $\theta$ discharges [7, 8]. Our results suggest that the oscillation is always present, even in large aspect ratio devices, but that the amplitude of the oscillation becomes as small as the level of background fluctuation for low $\theta$ discharges.

2.3. Phase locking of kink modes

Phase locking of kink modes is numerically observed in simulations of self-reversal and self-sustainment processes. It has characteristics similar to the ‘slinky mode’ observed in the OHTE experiment [1]: (1) it appears when the MHD relaxation process occurs; (2) only for the resonant modes does phase locking occur. Figure 4 shows typical results of phase locking, where the normalized toroidal profiles for $\text{Re}\{\vec{B}, (r = 0.9, m = 1, z)\}$ are plotted for the internal kink modes ($n = 11$ to 20) at three different times. In this simulation, the initial perturbation $\vec{V}_0$ and $\vec{b}_0$ incorporates seven different kink modes ($n = 2$ to 9), i.e.

$$
\left( \frac{\vec{V}_0}{\vec{b}_0} \right) = \sum_{n=2}^{9} \left( V_{\text{eigenfunction}}(r, n) \right) \exp \left( i(\theta - nz/R + \phi_n) \right)
$$

where the original phase distribution $\phi_n$ is given by a random number. Figure 4(a) shows that the phase distribution is completely random at $t = 10\tau_A$, owing to the initial random phase distribution for the unstable modes. At $t = 28\tau_A$, however, the phases of these internal kink modes are locked at a certain toroidal location where $z = 1.1$ (Fig. 4(b)), after which the phase locking is lost and the phase distribution becomes random again (Fig. 4(c)).

Now let us consider the question of what determines the toroidal location where the phase locking takes place. Figure 5 shows the relation between two toroidal locations, $z_{p.1}$ and $z_{0.5}$, for 27 different simulations, which have different phase distributions $\{\phi_n\}$ in Eq. (4). The location $z_{p.1}$ is where the phase locking takes place, but
FIG. 4. Normalized mode profile of $\text{Re}\{\tilde{B}, (r = 0.9, m = 1; n) \exp(-inz)\}$ for $n = 11$ to 20 at three different times ($t = (a) 10\tau_A$, (b) $28\tau_A$ and (c) $40\tau_A$).

FIG. 5. Phase locking points $z_{p,l}$ plotted as a function of the location $z_{A/5}$ where the modes $(m; n) = (1; 4)$ and $(1; 5)$ initially have the same phase. Solid circles show the 27 different simulation results, in which the different phase distributions in the initial perturbation are adopted.
the location $z_{n_1/n_2}$ is where the two kink modes, $(m; n) = (1, n_1)$ and $(1, n_2)$, initially have the same phase, i.e.

$$z_{n_1/n_2} = R \frac{\phi_{n_1} - \phi_{n_2}}{n_1 - n_2} \quad (5)$$

We can see that there is a good correlation between the locations $z_{p,1}$ and $z_{4/5}$. However, for the other modes except $(1; 4)$ and $(1; 5)$ we cannot observe a correlation with $z_{p,1}$. In fact, the modes $(1; 4)$ and $(1; 5)$ are the most unstable modes in the initial perturbation. These results mean that the two most dominant kink modes rule the other modes through the non-linear coupling between them and introduce phase locking. We also confirmed that if the most dominant mode $(1; 5)$ is excluded from the system, the phase locking process becomes more obscure.

It is worth while to point out that in these simulations the perfectly conducting boundary condition is adopted. Therefore, the phase locking process is not a special phenomenon of an RFP with a resistive shell but is a general characteristic in the MHD relaxation processes. This result is consistent with the simulation study in Ref. [9].

3. DISCUSSION

From the simulation study presented we can conclude that the fundamental mechanism in the RFP dynamics is a coherent rather than a turbulent process. Namely, the dynamo, a key agent for the non-linear dynamics in the RFP, is led by the non-linear coupling between a few (at least two) ideally unstable kink modes. Of course, the RFP plasma is a turbulent plasma that usually has a broad Fourier wave spectrum and a large stochastic region of the magnetic field lines of force. However, at least in the MHD regime these two phenomenological characteristics of a turbulent plasma in the RFP are merely the effects of the non-linear coupling in the dynamo process.

REFERENCES

DISCUSSION

S. ORTOLANI: In your simulation the energy variation is computed with respect to the energy of the BFM state with the same helicity as the initial state, but is magnetic helicity conserved in the calculation?

K. KUSANO: In our simulations, the total toroidal current and the total toroidal magnetic flux are conserved. Hence, the magnetic helicity is approximately sustained on a certain level.
INFLUENCE OF SHEARED E×B ROTATION ON EDGE TURBULENCE DYNAMICS AND ACCESS TO ENHANCED CONFINEMENT REGIMES

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Abstract

INFLUENCE OF SHEARED E×B ROTATION ON EDGE TURBULENCE DYNAMICS AND ACCESS TO ENHANCED CONFINEMENT REGIMES.

A theory of the $L-H$ mode transition, premised on the notion of sheared rotational stabilization of ambient turbulence, is presented. The theory addresses in detail how the flow is generated, how it quenches the turbulence and improves confinement, how it results in a bifurcation in thermal confinement, and how hysteresis in the confinement time is accounted for. Evidence is provided from the TEXT tokamak qualitatively corroborating several features of the theory.

1. Introduction

One of the most important problems in current fusion research is to understand the mechanism by which the plasma makes the "phase transition" from degraded to enhanced confinement, the so-called $L-H$ transition. Indeed, in view of the degree to which the viability of future-generation fusion devices, such as CIT, are predicated on the existence of an $H$-mode confinement state, an understanding of the $L-H$ transition takes on added urgency. Although the transition has been explored in depth experimentally and phenomenologically, such an understanding has until recently proven elusive. Recent experiments on a number of tokamaks, however, have been an important catalyst in the search for a theoretical understanding of the $L-H$ transition. Spectroscopic measurements of poloidal and toroidal plasma flows on DIII-D [1] have shown that the poloidal rotation increases suddenly and significantly during the $L-H$ mode transition, and remains large ($\sim 30$ km/sec) in $H$-mode, except during ELM activity. Equally compelling are results from CCT [2], which indicate that the $H$-mode can be actively triggered by injecting electrons into the plasma, thus inducing strong flows and rapid rotation. Similarly, limiter bias experiments on TEXT [3] also provide a clear correlation between plasma flows and rotation, and reduced turbulence. These experiments have led to speculation that poloidal rotation and the radial electric field associated with it may be playing a causal role in the transition [4,5]. A complete theory of the $L-H$ transition must simultaneously address three inter-related issues:

i) What is the mechanism of the transition and how is it generated?
ii) Why is turbulence quenched and confinement improved?

iii) What triggers the bifurcation or phase transition, and how is hysteresis accounted for?

In this work, we present a general theory premised on the incidence of sheared $E \times B$ rotation which resolves all three issues in a qualitatively natural and satisfying manner. In what follows, we shall speak of the $E \times B$ and the poloidal rotation interchangeably, i.e., $v_\theta(r) = -cE_r/B$. This is motivated by experimental results from DIII-D [1] which indicate that at the time of the transition, pressure gradients are relatively small at the edge so that the diamagnetic flow introduces only a small correction to the previous expression. However, we stress here that it is the shear in the $E \times B$ flow that is the fundamental quantity in suppressing turbulence, not the poloidal flow per se. Sections 2 through 4 address each of the three issues discussed above. In Sec. 5, evidence is presented from the TEXT tokamak which qualitatively corroborates the main features of the theory. We conclude in Sec. 6 with a discussion of unresolved theoretical and experimental issues.

2. Generation of Sheared Poloidal Rotation

We begin by discussing the various mechanisms which lead to the generation of sheared poloidal rotation. The time evolution of mean poloidal flow is governed schematically by the following equation:

$$\frac{\partial}{\partial t} \langle v_\theta \rangle \simeq -\alpha v_{ii} \left( \langle v_\theta \rangle + \frac{c}{eB} \mu_0 \frac{dT_i}{dr} \right) + \Omega_i \rho_{pi} v_{ii} n_f n_0 g(\langle v_\theta \rangle)$$

$$-\left( \frac{\partial}{\partial r} \langle \tilde{v}_r \tilde{v}_\theta \rangle \right) + v_A^2 \left( \frac{\partial}{\partial r} \left( \frac{\tilde{B}_r}{\tilde{B}_0} \frac{\tilde{B}_\theta}{\tilde{B}_0} \right) \right),$$

(1)

where $\alpha \sim O(\omega_{ti}^2/\nu_i^2)$ and $\mu_0 = 1.7$ in the Pfirsch-Schlüter regime, $\omega_{ti} = v_{ti}/qR$ is the ion transit frequency, $\rho_{pi}$ is the poloidal ion gyroradius, $\Omega_i = eB/m_i c$ is the ion gyrofrequency, $n_f$ is the fast trapped-ion population at the plasma edge, $g(\langle v_\theta \rangle)$ is a complex function which decreases with increasing poloidal flow [4], and $v_A$ is the Alfvén velocity. The first two terms follow from standard neoclassical theory [6]. A temperature gradient causes a poloidal torque to appear on the ions as a result of the dependence of ion collision frequency on ion energy. This "thermal stress" torque, represented by the second term on the rhs, drives the poloidal flow against rotation damping due to magnetic pumping (given by the first term on the rhs), which represents the tendency of the plasma to remove anisotropy between its parallel and perpendicular degrees of freedom. The third term on the rhs of Eq. (1), as discussed by Shaing and Crume [4], is the torque associated with fast ion
i.e., ions on the tail of the distribution function, \( \nu_n(v) < 1 \), which are still sufficiently energetic to reside on trapped banana orbits] orbit loss. It is best to think of these fast ions, which generate a nonambipolar radial current, as effectively decoupled from the bulk plasma since they are getting lost, i.e., they should be thought of as an "external" current. This external current then induces an opposite (i.e., radially inward) return current, which then generates poloidal rotation through the \( J \times B \) torque. The last two terms, which are less familiar, represent the generation of mean poloidal flow by electromagnetic microturbulence [7,8]. The first of these, which is manifestly identified as the radial divergence of the angular momentum flux or Reynolds stress, is induced by electrostatic turbulence, while the second is associated with magnetic turbulence.

It is instructive to discuss the circumstances under which turbulence can induce poloidal rotation. First, as is clear from the form of the terms in Eq. (1), the fluctuations must be radially propagating since for standing waves, the cross-correlations are identically zero. Thus, for example, electrostatic drift waves, ITG modes, or diamagnetically-modified resistive pressure gradient-driven modes [8] can generate a mean flow, but purely MHD resistive interchange or rippling modes cannot. A second characteristic needed for flow generation is some form of radial asymmetry in the fluctuation spectrum about the radius of which mean flow is generated. Three practical, important situations can be envisioned when this occurs:

a) turbulence is situated within a few spectral widths of the outermost closed flux surface or separatrix;

b) edge turbulence is situated in a steep gradient region where the radial correlation length is comparable to the gradients of global plasma parameters such as temperature, density, and/or the safety factor;

c) \( q \) passes through a low \( (m,n) \) resonance, so that a large-amplitude "convective cell" or magnetic island is situated in a steep gradient region.

In case a), the presence of the boundary breaks the left-right parity symmetry of the spectrum about its midpoint, and the spectral width overlaps the outermost closed flux surface. Regarding case b), it is clear that fluctuation levels are quite large when the turbulence is located in a steep gradient region. This situation is often encountered even for the linear Ohmic confinement regime of TEXT where the density gradient scale length is \( L_n \sim 1-2 \text{ cm} \), while the measured radial correlation length \( \Delta \sim 0.5 \text{ cm} \), and the potential fluctuation amplitudes are \( e\delta/T_e \sim 0.2-0.3 \) (cf. Fig. 1). Finally in case c), the existence of a large-amplitude cell or island induces a region of nonambipolarity which can result in a turbulence-induced torque. Such a situation can exist, for example, in \( Q \)-mode operation seen on TFTR, in ergodic magnetic limiter experiments on TEXT and TORE-SUPRA, and fi-
nally in auxiliary heating schemes such as lower hybrid current drive which modify the global current distribution. Indeed, in 3D numerical simulations of finite-$\omega_{ce}$ resistive interchange turbulence [8], the presence of a low-$q$ resonance results in the $k$-spectrum being dominated by the low-$m$ modes of the resonant helicities. In this case, although the result obtained in the limit of densely-packed fluctuations is no longer applicable, a substantial electric field is generated leading to poloidal rotation. The self-generated flow acts to self-consistently regulate the turbulence levels, and is found to be the dominant

FIG. 1. Radial profiles of (a) fluctuation phase velocity, $E_{r} \times B$ rotation, and diamagnetic drift velocity; (b) density and floating potential fluctuations; and (c) density and velocity shear on TEXT.
effect for the saturation of resistive interchange turbulence with $\omega_{ce}$ effects (cf. also Sec. 3). An important conclusion of this work is that changes in the turbulence levels and poloidal rotation are concomitant. Indeed, it may be difficult to separate them experimentally and settle the question of causality without ambiguity.

In case a), ambipolarity generally breaks down due to ion loss. This is because, e.g., for virtually all members of the drift mode family of fluctuations, the nonadiabatic electron response is localized close to the mode rational surface, and the radially propagating wave is carried by the ions in the form of an ion acoustic wave. Since the particle flux is calculated to be radially outward, this results in an outward nonambipolar radial current, which will partially cancel the return current associated with ion orbit loss, thus yielding only moderate poloidal rotation. Anticipating the developments of Sec. 3, when fluctuations are quenched as a result of the ensuing sheared poloidal rotation, the turbulence-induced radial current is suppressed, so that a net radial current in the inward direction is “uncovered,” spinning up the plasma more rapidly.

For concreteness, we conclude this section by providing two illustrative calculations of the turbulence-induced nonambipolar radial current. As an example of electrostatic turbulence, we consider toroidal (curvature-driven) ion temperature gradient (ITG) modes. These fluctuations are interesting candidates for poloidal flow generation for two reasons: i) they are poloidally asymmetric, maximized on the unfavorable curvature region (only the poloidally asymmetric portion of $J_r$ contributes to poloidal spin-up/down), and ii) they are radially-propagating waves, leading to finite angular momentum flux generation. Analysis shows that

$$J_r^{ITG} \simeq e n_0 v_{ti} \epsilon^{-1/2} q \frac{\rho_s^2}{L_n^{1/2} R^{3/2}} \frac{1 + \eta_i}{\tau^{3/2}},$$

where $\rho_s = c_s/\Omega_i$, $c_s = (2 T_e/m_i)^{1/2}$, and $\tau = T_e/T_i$. For typical DIII-D edge parameters, i.e., $n_0 \simeq 10^{13}$ cm$^{-3}$, $T_i \simeq 100$ eV, $\rho_s/L_n \sim 1/33$, and $B_t \sim 10$ kG, Eq. (2) gives $J_r \simeq 100$ A/m$^2$. Next, as an example of magnetic turbulence, consider diamagnetically-modified neoclassical resistive interchange modes. Again these fluctuations satisfy the right criteria to be practically relevant, and after some involved analysis, the result is

$$J_r^{NRM} \simeq e n_0 v_{ti} \frac{\beta_i}{\beta_{edge}^{11/12} S_M^{1/6}} \frac{1}{\omega_A^{1/2}} \left[ \frac{\epsilon}{q} \frac{\tau}{L_p} \right]^2 \lesssim 10 A/m^2,$$

where $S_M$ is the magnetic Reynolds number, $\hat{s} = r q'/q$, $\omega_A$ is the Alfvén frequency, $L_p^{-1} = d \ln p/dr$, and the number given is for typical DIII-D parameters.
3. Turbulence Suppression by Sheared $E \times B$ Rotation

Once established, sheared poloidal rotation acts to suppress ambient edge turbulence, thus paving the way to an enhanced state of confinement [5]. To understand the physics of the suppression mechanism, consider as a working prototype, a generic fluid model in which fluctuations dynamically evolve according to

$$\left[ \partial / \partial t + (v_0 + \ddot{v}) \cdot \nabla + \mathcal{L}_d \right] \ddot{\xi} = \dot{S},$$  \hspace{1cm} (4)

where $\ddot{\xi}$ is the fluctuating field. In Eq. (4), $v_0 = v_\theta(r)$ is taken to be the equilibrium $E_0 \times B$ flow which, as discussed earlier, we shall identify with the poloidal rotation, i.e., $v_\theta(r) = -cE_r(r)/B$. $v \cdot \nabla$ is the advective nonlinearity, which can be either electrostatic (perpendicular $E \times B$ advection) or magnetostatic (parallel flow along stochastic field lines). $\dot{S}$ in Eq. (4) represents a source of free energy driving the turbulence, and $\mathcal{L}_d$ is an operator responsible for dissipation of that energy. There are two physical processes, represented by the second and third terms of Eq. (4), which physically characterize the fissure of a fluid element: poloidal decorrelation due to rotational shear, and radial decorrelation due to turbulent scattering. To each of these processes, one can associate a rate: $\omega_\tau = (k_{0y} \Delta r_t) |v'_\theta - v_\theta / r_+|$, which can be thought of as the rate at which two fluid elements separated radially by $\Delta r_t$ become separated poloidally by $k_{0y}^{-1}$, and $\Delta \omega_\tau = 4D/\Delta r_t^2$, which is the random, diffusive scattering rate of the ambient turbulence ($\Delta r_t$ and $k_{0y}^{-1}$ characterize the spatial correlation lengths of the ambient turbulence in the radial and poloidal direction, respectively, while $r_+$ is the radial center-of-mass of the fluid element). That $\omega_\tau$ has the form shown is manifested by the fact that there could be no rotational shearing of the fluctuations if the plasma were rotating as a rigid body. Thus, even in the absence of flow shear, i.e., $v'_\theta(r) = 0$, fluctuations can still be sheared apart due to curvature effects associated with curvilinear geometry. As a preliminary observation, however, note that if the shear in the poloidal flow is steeper than the radius of the minor cross section, i.e., $L_v = |d \ln v_\theta / dr|^{-1} < r_+ \sim a$, as can be expected for spontaneously-induced shear layers, then it is the former that dominates rotational shearing. This indeed appears to be the case on both TEXT [3] and DIII-D [1], and so without loss of generality, we focus on this contribution to $\omega_\tau$ for the balance of this section. More importantly, it is neither turbulent radial scattering nor poloidal shearing that determines the physical decorrelation process, but rather a hybrid of the two weighted towards the latter. Physically, this arises because the effect of poloidal flow is to enhance poloidal decorrelation by coupling radial scattering to sheared poloidal streaming, i.e., $r_+^2 \langle \delta \theta^2 \rangle = v_0^2 \langle \delta r^2 \rangle \tau_c$ [5], a feature that finds analogy in the situation encountered in the nonlinear dynamics of the universal instability,
and the stochastic divergence of magnetic field lines [9]. The preceding equation should be thought of as an equation which defines the shear-induced decorrelation time, \( \tau_c \), when the poloidal excursions of two adjacent points in the fluid blob become comparable to blob dimensions, i.e., \( k_0^2 r_+^2 \langle \delta \theta^2 \rangle = 1 \). It then follows that

\[
\tau_c = (k_0^2 v_0^2 D)^{1/3} \equiv (\omega_s^2 \Delta \omega_t)^{1/3}.
\]

Shear-induced decorrelation dominates decorrelation associated with the ambient turbulence when \( \tau_c < (\Delta \omega_t)^{-1} \) or equivalently, \( \omega_s > \Delta \omega_t \), i.e., when the shearing rate for a turbulent eddy of size \( (\delta r_t, k_0^{-1}) \) in a differentially-rotating plasma exceeds the diffusive decorrelation rate of the background turbulence. Two noteworthy features about Eq. (5) are that i) shear figures more prominently than radial diffusion in detuning fluctuations, and ii) the sign of \( \omega_s \) is irrelevant, i.e., the turbulence quench mechanism is insensitive to the sign of the flow shear. The criterion for rotational stabilization can be written in terms of radial scales as \( \Delta r_t / L_v > \Delta \omega_t / \omega_\theta \), where \( \omega_\theta = k_0 v_\theta \) is the rotation frequency, and \( L_v^{-1} = d \ln \nu_\theta / dr \). Further analysis shows that the radial correlation length and the fluctuation amplitudes are both reduced by a factor \( (\Delta \omega_t / \omega_s)^{1/3} \) relative to their ambient values, assuming there is no self-consistent back-reaction of the transport on the flow. If the flow is generated self-consistently, then it can be the main factor in determining the saturation level of the fluctuations, and the scaling of the reduction factors is likely to change [8] (cf. also discussion in Sec. 6). Finally, the extent to which sheared poloidal flow would instigate a Kelvin-Helmholtz (K-H) instability depends on the competition between destabilization associated with flow shear and stabilization associated with magnetic shear [10], or in other words, the extent to which the vorticity maximum (i.e., the free energy source) localizes itself relative to the region of minimum magnetic dissipation (i.e., a \( k \cdot B \) surface). In the case of modes with a definite parity, the relevant comparison is between the shear flow scale length and the width of the resistive layer, i.e., \( L_v \) vs. \( L_J = (\eta v_\theta L_s^2 c^2 / k_0^2 \nu_\lambda^2)^{1/4} \), where \( L_s = qR/\hat{s} \) is the magnetic shear length scale, and \( \eta \) is the resistivity. In general, \( L_v > L_J \) for physically meaningful parameters, and thus a K-H instability is unlikely. It should be borne in mind, however, that the poloidal flow also depends on the diamagnetic flow (\( \propto dp/dr \)), and so during ELM activity when the pressure profile considerably steepens at the edge, the K-H instability needs to be examined as a possible candidate for the consequent degradation in confinement.

We illustrate this general scenario in the context of two concrete, self-consistent models, namely, a) resistivity gradient-driven turbulence [11] and b) resistive pressure gradient-driven turbulence [8] in a differentially-rotating
plasma. In the former case, the balance of the resistivity gradient drive, i.e., \( \nabla \eta \), and thermal conduction in the shearless case and poloidal shearing in the strongly-sheared limit defines the radial correlation lengths \( \delta_r \) and \( \Delta_r \), respectively. Here, \( \delta_r \sim (L_n E_{||}/L_n B_0 k_{||} v'_\parallel)^{1/3} \) and \( \Delta_r \sim (L_n E_{||}/L_n B_0 k_{||} v'_\parallel)^{1/2} \), where \( E_{||} \) is the parallel electric field, \( L_n^{-1} = d \ln \eta/d r \), \( \chi_{||} \) is the parallel thermal conduction, and \( k_{||} = k_{0y}/L_n \). The transition between the two regimes occurs when \( \delta_r \sim \Delta_r \). In the shearless case, the fluctuating field, i.e., \( \bar{\eta} \), is diffusively scattered to finite radius where it is then dissipated by thermal conduction, and saturation is achieved when turbulent diffusion balances the free energy drive. In the strongly-sheared limit, on the other hand, \( \bar{\eta} \) is turbulently diffused to finite radius, and then destroyed by rotational shear. Saturation is achieved when fissure due to flow shear balances the \( \nabla \eta \) drive. Detailed calculations indicate that the thermal diffusivity is given by \( D \sim (L_n E_{||}/L_n B_0)^{3/2} |k_{0y} v'_\parallel|^{-1/2} \) (i.e., decreasing with increasing velocity shear, but independent of its sign) in the strongly-sheared limit. In addition, poloidal rotation induces a real (nonlinear) frequency shift \( \Delta \omega_r \equiv \text{Im} \, D / \Delta_r^2 \), which varies with velocity shear and is thus additive to the trivial Doppler shift. Since both \( D \) and \( \Delta \omega_r \) decrease with increasing \( k_{0y} \), the theory predicts increased suppression of turbulence at high \( k_{0y} \), in spite of the fact that the transition from the weak to the strong shear regime will occur for low-\( m \) modes first.

Turning next to resistive interchange instabilities in the presence of rotational shear [8], the linear picture is conveniently characterized with the aid of two dimensionless parameters: a) \( R^2 = L_n^2 / k_{0y} L_j^4 \), which is a measure of K-H destabilization in the presence of magnetic shear; and b) the Richardson number, \( Ri = (c_s/v_\parallel)^2 (L_n^2 / L_n r_c) \), which is the ratio of buoyancy to shear flow effects (\( r_c \) is the radius of magnetic curvature, and \( L_n^{-1} = d \ln \eta/d r \)). Three regimes of interest can be identified as the flow shear is increased:

i) \( Ri, R \gg 1 \): For low values of shear flow, the resistive interchange mode is unstable and practically independent of the flow;

ii) \( Ri \sim 3/4, R > 1 \): As the sheared flow increases, the growth rate of the mode is reduced until the mode is totally stabilized for \( Ri = 3/4 \).

iii) \( Ri < 3/4, R < 1 \): As the flow continues to increase, the K-H instability can be triggered, and the growth rate becomes proportional to the flow rate jump.

The effect on saturated resistive pressure gradient driven turbulence of adding diamagnetic rotation, which breaks the poloidal symmetry of the turbulence, has also been investigated. For a constant radial electric field, for which \( e \phi_e/T_e = \pm 1 \) and \( \gamma > \omega_e \), the effect on turbulence is weak and no change in the saturation levels is observed. For a sheared electric field, there is clear evidence of a reduction of the fluctuation level for low-\( m \) modes.
However, it is difficult to distinguish between linear stabilization effects and nonlinear turbulence reduction. The coupling between poloidal and radial decorrelation in shearing or detuning fluctuations is quite evident in Fig. 2, which is a snapshot of density contours in the unsheared and strongly-sheared cases.
4. Bifurcation in Thermal Confinement and the L-H Transition

A bifurcation in thermal confinement [12], which shows strong resemblance to the $L - H$ mode transition, follows naturally from the notion of turbulence suppression by sheared poloidal rotation as discussed in the previous section. It is assumed that the thermal conductivity can be expressed as the sum of a neoclassical contribution and an anomalous contribution which decreases with increasing poloidal rotation and is independent of its sign, i.e., $\kappa = \kappa_n + \kappa_a/(1 + \gamma_a v_\theta^2)$. This choice is motivated by reasoning from the previous section that at sufficiently high shear flow, the anomalous contribution will be suppressed and one is left with an irreducible minimum contribution to transport, namely, that associated with neoclassical transport. However, it serves to caution the reader that the shear dependence chosen for the general model is \textit{ad hoc} and intended only to be illustrative. In practice, the shear dependence is model-specific and varies depending on the model of ambient edge turbulence adopted. In contrast to Shaing and Crume [4], the poloidal rotation is assumed to be given by the standard neoclassical expression alone, proportional to the ion temperature gradient [i.e., $v_\theta = -\mu_0 (c/eB) dT/dr$].

This leads to a particular nonlinear dependence of the local heat flux on the temperature gradient $g \equiv -dT/dr$, i.e., $f(g) \equiv \kappa_n g + \kappa_a/(1 + \lambda_a g^4)$, which is plotted in Fig. 3. The equilibrium temperature profile is now determined by

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig3}
\caption{Local heat flux versus temperature gradient.}
\end{figure}
numerical integration of the equation \( Q(r) = -\kappa dT/dr \), where \( Q(r) \) is the heat flux at radius \( r \). The equation is integrated radially inward from \( r = a \), at which point the temperature is assumed to be zero. Assuming uniform heating, so that the heat flux varies linearly with \( r \), i.e., \( Q(r) = Q(a)r/a \), the following results are obtained:

a) \textit{L-mode}: If \( Q(a) < Q_c \), where \( Q_c \) is the threshold heat flux, the edge temperature gradient is small, since the poloidal rotation shear is not yet large enough to suppress the edge turbulence. In this case, the small gradient root must be used over the entire range from \( r = a \) to \( r = 0 \), and a profile like the lower curve in Fig. 4 is obtained.

b) \textit{H-mode}: If \( Q(a) > Q_c \) in a layer near the edge, rotational shear suppresses edge fluctuations and allows the formation of steep gradients. The integration must then start on the large gradient root and stay on this branch from \( r = a \) to \( r = a Q(g_2)/Q(a) \), where this root becomes imaginary. At this point, the solution jumps to the small root and the integration continues to \( r = 0 \). The discontinuous jump in the temperature gradient, which results in a profile like the upper curve in Fig. 4, is rather reminiscent of a first-order phase transition in the theory of critical phenomena. The time for bifurcation to occur corresponds to the time that it takes for the edge transport barrier to be established, which can roughly be written as \( \tau_c (\Delta r_{\text{edge}}/\Delta r_c)^2 \), where \( \Delta r_{\text{edge}} \) is the radial width of the transport barrier.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig4.png}
\caption{Temperature profiles for (a) \( Q(a) = 0.99Q_c \), and (b) \( Q(a) = 1.01Q_c \) (arbitrary units).}
\end{figure}
By integrating under these curves to obtain the energy content, the global energy confinement time is obtained: \( \tau_E = \int_0^a dr \, r n T / Q(a) \). Assuming a constant density for simplicity, the ratio of the confinement times for \( L \) and \( H \)-mode cases with nearly the same heating power shown in Fig. 4 is found to be 3.0. Interestingly, when \( \tau_E \) is plotted vs. heat flux in Fig. 5, a hysteresis curve is obtained by first increasing the heating power through the critical value, and then decreasing it. Hysteresis manifests itself because the sequence of equilibria as heating power is increasing becomes discontinuous when \( Q > Q_c \), yet remains continuous when heating power is decreasing. This, in turn, is due to the fact that the edge region of large gradient shrinks continuously to zero as the power approaches the minimum \( f(g_2) \), so that there is no difference in the profiles just before and just after the jump to the small gradient branch.

Finally, with respect to the scaling of the power threshold with plasma parameters, it is necessary to postulate some dependence of the free parameters \( \kappa_a \) and \( \lambda_a \) on global plasma parameters. This, in turn, requires adopting a specific model of edge turbulence which, in the absence of definitive experimental identification, must be viewed here with caution and considered to be merely illustrative. Assuming curvature-driven, toroidal ITG modes as earlier, it is found that for a fixed edge temperature, \( Q_c \sim B^{10/7} n^{23/7} \).

![FIG. 5. Power hysteresis in the energy confinement time: (a) increasing power, (b) decreasing power.](image-url)
The magnetic field scaling is similar to experimental observations. Although the density scaling is considerably stronger than what is seen experimentally, experimental density scans are difficult to secure while keeping other parameters fixed; hence, any agreement or disagreement may be spurious at this time.

5. Experimental Tests of the Theory on TEXT

The edge region of TEXT, which lends itself to detailed experimental measurements, and where a poloidal shear layer has been observed, provides an ideal environment to qualitatively test aspects of these theoretical ideas [3]. A velocity shear layer due to a peaking plasma potential close to the outermost closed flux surface has been characterized on TEXT. Fluctuation phase velocities are determined to be predominantly associated with $\mathbf{E} \times \mathbf{B}$ drifts, as diamagnetic contributions are negligible in comparison [cf. Figs. 1(a) and (c)]. The density and floating potential fluctuations, $\bar{n}$ and $\bar{\phi}$, are suppressed in the velocity shear region, as shown in Fig. 1(b), while the mean density is slightly steepened in the region of maximal shear, as shown in Fig. 1(c).

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**FIG. 6.** Turbulent decorrelation times due to a constant radial electric field ($\tau_E$) and due to the sheared electric field ($\tau_{sh}$) compared with the autocorrelation time of the fluctuations $\tau^{\text{lab}}_c$ (dashed area indicates confidence limit). Also shown is the diffusive time $\tau_D = \Delta r^2_c/D$. 

To qualitatively test the theoretical ideas of Sec. 3, a shear decorrelation time, defined by
\[ \tau_{sh} = \frac{\sigma_\phi}{\sigma_r} |v'_E - v_{EB}/r|^{-1} \]
can be directly deduced from measured poloidal (\(\sigma_\phi\)) and radial (\(\sigma_r\)) correlation lengths and \(E \times B\) velocities (\(v_{EB}\)). The dominant contribution to the decorrelation process is found to be due to \(v'_E\) and not \(v_{EB}/r\), as is evident from Fig. 6. Furthermore, measurements reveal that the decorrelation times are much shorter inside the velocity shear layer than on either side (cf. Fig. 6). The reduction of decorrelation times in the strong shear region is found to be consistent with the theoretical predictions of Sec. 3. Also, the coupling between poloidal and radial correlation lengths is consistent with a shear decorrelation process. No evidence for the onset of Kelvin-Helmholtz instability is present in the data. On the strength of these observations, it is speculated that the \(H\)-mode may be an exaggerated representation of the same physical mechanism observed on TEXT, but caused by a much larger poloidal rotation velocity and velocity shear than in Ohmic plasmas.

6. Conclusion and Summary

In this work, we have presented a general theory of the \(L - H\) transition which is premised on the sheared rotational stabilization of ambient edge turbulence. A flowchart summary of the theory is presented in Fig. 7. Although the qualitative predictions of the theory appear thus far to be corroborated experimentally, there are important details that still await resolution. Foremost among these is the question of self-consistency relating to flow generation and turbulence suppression. Although appealing from a physical point of view, the \textit{general} theory propounded in Sec. 3 ignores any self-consistent back-reaction of the turbulence on the flow. As discussed in Secs. 2 and 3 however, results from numerical simulations of diamagnetically-modified resistive pressure gradient-driven turbulence indicate that when the mean poloidal flow is allowed to evolve self-consistently with the fluctuations, then the self-generated flow is the main cause of turbulence saturation [8]. Thus, this issue deserves more detailed investigation. Secondly, given the important role played by particle transport during the \(L - H\) transition, it is desirable to extend the bifurcation theory of Sec. 4 to self-consistently treat particle and thermal confinement on the same footing, and progress in this direction is already under way. There are also important experimental issues in need of resolution. Foremost among these is the issue of causality with respect to the incidence of sheared flow rotation, fluctuation reduction, and the \(L - H\) transition. The theory presented here, where all three aspects of the \(L - H\) transition are coupled together in a bootstrap fashion, suggests
that it may be very difficult to settle the question of causality with any measure of confidence. A second important question relates to how the width of the turbulence suppression region scales with global plasma parameters in general, and with the power threshold in particular. It is hoped that the theoretical ideas expounded in this report will spur more detailed experimental studies of the $L - H$ transition physics.
REFERENCES


DISCUSSION

A.K. SEN: We appreciate your thesis concerning the influence of flow shear on turbulence. However, we have had some puzzling and contradictory experimental evidence in the Columbia Linear Machine. We observe a strong curvature driven trapped particle mode which survives a strong rotational shear layer. If a single mode can ignore sheared rotation, presumably so can turbulence. By the way, we do not observe any Kelvin-Helmholtz instability.

H. BIGLARI: It is difficult to comment without actually having seen the data. However, let me make the general comment that the presence of rotational shear does not completely stabilize turbulence, but rather reduces its level. It is already clear from the TEXT results that the fluctuation levels are reduced in the shear layer, but not completely quenched. Thus, what you may be observing is a level of instability or turbulence consistent with the presence of rotational flow, which would rise if you didn't have sheared flow.

R.R. WEYNANTS: One of the features of the CCT experiment is that it is possible to go from low rotation to high rotation in a completely smooth way. Could this feature perhaps be a way of discriminating between the various models?

H. BIGLARI: It is important to recognize that there are differences between spontaneous and externally induced H modes. In particular, bifurcation is induced by power increasing above a threshold value in the spontaneously induced case, and by biasing voltage (or equivalently, radial current) increasing above a threshold value in the externally induced case. The shear in the flow is the key quantity in spontaneously generated shear layers (such as on TEXT), while the curvature contribution to the shearing frequency can be equally important in the externally induced case. The bifurcation scenario outlined in this work attempts to address the spontaneously induced H mode, and so efforts to test it on CCT may not be relevant. Moreover,
it is a bifurcation in thermal confinement, *not* poloidal rotation. On the question of turbulence reduction, we would expect this also to happen slowly as the shear in the flow is increased.

J.A. WESSON: Would it be possible to improve confinement in the bulk of the plasma by an externally induced shear flow?

H. BIGLARI: Yes, if you could externally generate strongly sheared flows, either poloidal or toroidal, in the plasma core, then, assuming the constraints of the theory are met, you could reduce fluctuations. This is particularly interesting with respect to trapped ion convective cells, which are radially large size fluctuations.
ALPHA PARTICLE EFFECTS ON GLOBAL MHD MODES, AND ALPHA PARTICLE TRANSPORT IN IGNITED TOKAMAKS

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Abstract

ALPHA PARTICLE EFFECTS ON GLOBAL MHD MODES, AND ALPHA PARTICLE TRANSPORT IN IGNITED TOKAMAKS.

The high frequency, low mode number toroidicity induced Alfvén eigenmodes (TAE) are shown to be driven unstable primarily by the circulating α-particles through wave particle resonances. To destabilize the TAE modes, the inverse Landau damping associated with the α-particle pressure gradient free energy must overcome the velocity space Landau damping due to both the α-particles and the core electrons and ions, as well as Alfvén continuum damping. Stability criteria are presented for TFTR, CIT, and ITER tokamaks in terms of the α-particle beta $\beta_\alpha$, the α-particle pressure gradient parameter $\omega_a/\omega_\alpha$, where $\omega_a$ is the α-particle diamagnetic drift frequency, and the α-particle velocity ($v_a/v_A$) parameter. Typically, the volume averaged α-particle beta threshold is of the order of $10^{-4}$. Rough estimates of the TAE mode saturation level give $\delta B/B = 10^{-3}$ for typical DT tokamak operations. Significant α-particle losses are found when the amplitude of the global MHD modes is large, of the order of $(\delta B/B) \geq 10^{-4}$. For $(\delta B/B) = 5 \times 10^{-4}$, the α-particle loss time is appreciably shorter than the α-particle slowing-down time.

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1. Introduction

Among the major issues in the $\alpha$-particle physics of ignited tokamaks are the $\alpha$-particle driven global MHD instabilities and the resultant $\alpha$-particle transport. If large amplitude global MHD modes are excited, they can cause anomalous $\alpha$-particle losses, severely affecting tokamak reactor operations. In this paper the effects of $\alpha$-particles on the toroidicity-induced Alfvén eigenmodes (TAE) [1-6] are studied for DT tokamaks such as the Compact Ignition Tokamak (CIT) and the International Thermonuclear Experimental Reactor (ITER). $\alpha$-particles may destabilize the TAE modes through wave particle resonances by tapping the free energy associated with the $\alpha$-particle pressure nonuniformity. To destabilize the TAE mode, the inverse Landau damping associated with the $\alpha$-particle pressure gradient free energy must overcome the dissipation mechanisms in the the system. Typical growth rates of the n=1 TAE mode [2-5] can be on the order of $10^{-2} \omega_A$, where $\omega_A = v_A/qR$. The anomalous $\alpha$-particle losses due to the TAE mode are investigated by a Hamiltonian guiding center orbit Monte Carlo code [7,8]. The non-linear saturation amplitude of the TAE mode is studied by a quasi-linear theory [9]. Finally, $\alpha$-particles can affect the m=n=1 internal kink mode.

2. The toroidicity induced Alfvén eigenmode

The TAE mode [1,10] exists inside gaps, due to toroidal coupling, in the shear Alfvén continuum spectrum. For example, modes (n,m) and (n,m+1) couple at radial location $r_o$, where $q(r_o) = (m + \frac{1}{2})/n$, to form a gap which is bounded by $\omega_\pm^2 = \omega_0^2 \pm 2 \omega_0^2 \left( \frac{r_o}{R} + \Delta'(r_o) \right)$, where the center of the continuum gap is $\omega_0 = V_A/2qR$ at $r = r_o$, and $\Delta(r)$ is the Shafranov shift of the non-concentric flux surfaces with $\Delta' > 0$. The global TAE modes had been shown to exist with discrete frequencies inside the continuum gap. For small (large) magnetic shear the TAE mode frequency is near the lower (upper) continuum gap boundary. For the n=1 TAE mode the m=1 and 2 poloidal harmonics are dominant and peak near the $q=1.5$ surface. The existence of the high-n TAE modes has also been shown previously [10,11].

3. Alpha particle destabilization of the TAE mode

For typical DT parameters the $\alpha$-particle birth velocity $v_\alpha = (2\varepsilon_\alpha/M_\alpha)^{1/2} = 1.29 \times 10^9$ cm/sec for an energy $\varepsilon_\alpha$ of 3.5 MeV is comparable to the Alfvén speed. Thus, the transiting $\alpha$-particles can destabilize shear Alfvén waves by the expansion free energy associated with the spatial gradient of their pressure via inverse Landau damping through the $\omega = k_|| v_\parallel$ wave particle resonance. Here, $k_|| = (m-nq)/qR$ is the parallel wavenumber for linearized waves that are Fourier decomposed as $\exp[i(m\theta-n\zeta-\omega t)]$. 
Consider an axisymmetric toroidal plasma consisting of electron, core ion, and alpha components with $n_{\alpha} \ll n_i$ and $T_{\alpha} \gg T_{e,i}$, so that $\beta_{\alpha} < \beta_{e,i}$. From the linear momentum equation we obtain a quadratic form $\omega^2 \delta K - \delta W_f - \delta W_k = 0$, where the inertial energy $\omega^2 \delta K$ and the fluid potential energy $\delta W_f$ are identical to the ideal MHD energies but with the ratio of specific heat set to zero, and the kinetic potential energy $\delta W_k$, which is derived from the drift kinetic equations, contains the wave particle resonances due to all particle species.

The stability of a global Alfvén wave due to wave particle resonances can be obtained perturbatively from the quadratic form by assuming that the growth rate is small ($|\gamma| \ll |\omega|$). Then we have $\omega^2 = \{ \delta W_f + \text{Prin} [\delta W_k] \} / \delta K$, and $\gamma = \text{Res} [\delta W_k] / 2 i \omega_\gamma \delta K$, where $\text{Prin} [\delta W_k]$ and $\text{Res} [\delta W_k]$ are the principal part and resonance contribution, respectively. In $\delta W_k$, the core electron and ion distributions are taken as Maxwellians, and the $\alpha$-particle distribution function is taken to be isotropic in pitch angle and slowing-down in energy for $\varepsilon \leq \varepsilon_\alpha$ and zero for $\varepsilon > \varepsilon_\alpha$.

3.1. Local stability analysis

If the particle trapping effects and the magnetic drift resonance are ignored, a local instability criterion of the TAE mode can be obtained analytically. Evaluated at the $q=1.5$ surface and $\omega = v_A/2qR$, the instability condition is given by

$$\beta_\alpha \left\{ \frac{4 \omega_\alpha \omega_{\alpha}}{3 \omega_A v_\alpha} \left[ \frac{(1 + 8 v_A^2 + 4 v_A^4)}{(1 + 2 v_A^2)} \right] - 4 v_A^3 \right\} - \left[ \frac{4 v_A^2 + (3 - 4 v_A^2)}{(1 + v_A^2)} v_\alpha^2 (v_{\alpha}^2 + v_A^2) \right] \geq [8 v_\alpha v_{\alpha}/3 \pi 1^{1/2} v_A] \sum_j \beta_j z_j \left[ 1 + 2 z_j^2 + 2 z_j^4 \right] \exp (-z_j^2)$$

(1)

where $v_\alpha = v_\alpha^2 / [v_\alpha^2 - v_A^2]$, $v_{\alpha}^2 = v_A^2 / [v_\alpha^2 - v_A^2]$, $v_\alpha$ is the alpha birth velocity, $\omega_\alpha^{(m)} = m \rho_\alpha v_\alpha / 2r L_\alpha$, $\rho_\alpha$ is the alpha particle gyroradius at birth velocity, $r$ is the minor radius, and $L_\alpha$ is the alpha particle pressure scale length. The summation index $j$ is over the electron and core ion species, $v_{j} = (2T_{j}/m_{j})^{1/2}$, and $z_j = v_A / v_j$. The first term on the left hand side is due to the $\alpha$-particle destabilizing inverse Landau damping associated with its pressure.
gradient and the second term is associated with the \( \alpha \)-particle velocity Landau damping. The right hand side contribution is due to the stabilizing electron and core ion Landau damping. Note that \( \beta_j z_j \) are not free parameters but are proportional to \( \nu_\alpha / \nu_A \). Equation (1) shows that to destabilize the TAE mode the \( \alpha \)-particle free energy drive associated with \( \omega_{*\alpha} \) must be large enough to overcome the usual Landau damping (typically when \( \omega_{*\alpha} / \omega_A > 1 \)) and that above this threshold the growth rate \( \gamma \) will scale linearly with \( \omega_{*\alpha} \). The minimum critical \( \beta_\alpha \) occurs at \( \nu_\alpha / \nu_A = \sqrt{2} \), where the instability condition is roughly given by \( \beta_\alpha (\omega_{*\alpha} / \omega_A - 2) \geq 0.4 (M_\alpha \nu_\alpha / M_i \nu_\alpha) \). For TFTR the DT operation parameters are chosen as \( R = 250 \) cm, \( a = 80 \) cm, \( B = 5 \) T, \( T_e = T_i = 10 \) keV, \( L_\alpha = 15 \) cm, \( r = 25 \) cm, the alpha charge state \( Z_\alpha = 2 \), \( M_\alpha / M_p = 4, M_i / M_p = 2.5 \). For \( n_\alpha = 10^{14} \) cm\(^{-3} \), we have \( \nu_\alpha / \nu_A = 1.88 \) and the critical \( \beta_\alpha = 2.5 \times 10^{-4} \). For CIT the parameters are chosen as \( R = 210 \) cm, \( a = 65 \) cm, \( B = 11 \) T, \( T_e = T_i = 10 \) keV, \( L_\alpha = 20 \) cm, and \( r = 16 \) cm; the critical \( \beta_\alpha = 7.5 \times 10^{-4} \) for \( \nu_\alpha / \nu_A = 1.5 \). For ITER the parameters are chosen as \( R = 600 \) cm, \( a = 215 \) cm, \( B = 4.85 \) T, \( T_e = T_i = 10 \) keV, \( L_\alpha = 50 \) cm, and \( r = 50 \) cm; the critical \( \beta_\alpha = 1.2 \times 10^{-3} \) for \( \nu_\alpha / \nu_A = 1.5 \).

### 3.2. Global stability analysis

A more complete non-local perturbative stability calculation can be performed for realistic equilibria by the use of the zeroth order solutions (eigenfunctions and mode frequency) obtained from the NOVA code[12]. We assume the \( \alpha \)-particle density to be \( n_\alpha = n_\alpha(0) \exp \left[-\left( \frac{r}{L_\alpha} \right)^2 \right] \). The volume averaged \( \alpha \)-particle beta \( \langle \beta_\alpha \rangle \) is related to the central \( \alpha \)-particle beta \( \beta_\alpha(0) \) by \( \langle \beta_\alpha \rangle = (L_\alpha / a)^2 \beta_\alpha(0) \). The \( \alpha \)-particle density scale length is \( L_\alpha^2 / 2r \). The tokamak reactor type equilibria are modeled with non-circular plasma surfaces defined by \( X = R + a \cos [\theta + \delta \sin(\theta)] \), and \( Z = \kappa \sin(\theta) \), where \( \kappa \) is the ellipticity, \( \delta \) is the triangularity, \( a \) is the horizontal minor radius, and \( R \) is the major radius.

Calculations are performed for CIT and ITER parameters. For CIT we study a non-circular tokamak equilibrium with the parameters: \( R = 210 \) cm, \( a = 65 \) cm, \( \kappa = 2, \delta = 0.2, q(0) = 1.01, q(1) = 3.1, q'(0) = 0.9, q'(1) = 13 \). The plasma density is assumed to be constant. The critical volume averaged \( \alpha \)-particle beta \( \langle \beta_\alpha \rangle \) vs. \( \nu_\alpha / \nu_A \) stability curves are shown in Fig. 1(a) for \( T_{\text{eo}} = T_{\text{io}} = 10 \) keV, \( B_0 = 11 \) T, but with several \( L_\alpha / a \) values. The electrons and the ions are assumed to have the same temperature profiles. For \( \nu_\alpha / \nu_A = 1.3 \) and
FIG. 1. Critical volume averaged $\langle \beta_\alpha \rangle$ versus $(v_\alpha/v_A)$ stability curves for (a) a CIT equilibrium and (b) an ITER equilibrium with several $L_\alpha/a$ values. CIT parameters: $T_\alpha = T_\beta = 10$ keV, $R = 210$ cm, $a = 65$ cm, $B_0 = 11$ T. The values for $T_\alpha = T_\beta = 20$ keV are about a factor of three higher than those for $T_\alpha = T_\beta = 10$ keV. ITER parameters: $T_\alpha = T_\beta = 10$ keV, $R = 600$ cm, $a = 215$ cm, $B_0 = 4.85$ T. Cases with $T_\alpha = T_\beta = 20$ keV are also shown.

$L_\alpha/a = 0.2$, the critical value is $\langle \beta_\alpha \rangle = 8 \times 10^{-6}$ with corresponding $\beta_\alpha(0) = 2 \times 10^{-4}$. Local calculation with $r L_\alpha/a^2 = L_\alpha^2 / 2a^2 = 0.02$ and $m=2$ yields the critical $\beta_\alpha = 1.1 \times 10^{-4}$; thus the results from both the global and local calculations are consistent. Higher ion temperature provides higher ion Landau damping, and the critical $\langle \beta_\alpha \rangle$ vs. $(v_\alpha/v_A)$ stability curves for $T_\alpha = T_\beta = 20$ keV are about a factor of three higher than those for $T_\alpha = T_\beta = 10$ keV. As the $\alpha$-particle pressure scale length increases, the TAE mode will be stable. For the $T_\alpha = T_\beta = 10$ keV case, the TAE mode is stable for $L_\alpha/a > 0.4$.

For ITER we study an equilibrium similar to the CIT equilibrium but with its triangularity twice as large. The critical $\langle \beta_\alpha \rangle$ vs. $(v_\alpha/v_A)$ stability curves are shown in Fig. 1(b) for the physical parameters $T_\alpha = T_\beta = 10$ keV, $R = 600$ cm, $a = 215$ cm, $B_0 = 4.85$ T, but with several $L_\alpha/a$ values. The critical $\langle \beta_\alpha \rangle$ is roughly a factor of three higher than that of CIT case shown in Fig. 1(b) due to larger ion Landau damping. For $v_\alpha/v_A = 1.3$, and $L_\alpha/a = 0.2$, $L_\alpha/a = 0.3$,
the critical $<\beta_0> = 2.2 \times 10^{-5}$. The stability results for higher electron and ion temperatures ($T_{e0} = T_{i0} = 20$ keV) are similar to those obtained for the CIT case.

Thus, for typical DT tokamak parameters, the volume averaged $\alpha$-particle beta threshold for TAE instability is very small, of the order of $10^{-4}$. The TAE modes will be robustly unstable in these proposed DT tokamaks with typical growth rates of the order of $10^{-2}\omega_A [2-5]$.

4. Continuum resonant damping effect on the TAE mode

Since the continuum gap frequency $\omega_0$ is roughly proportional to $(1/q_\rho^{1/2})$, the TAE mode may resonate with the Alfvén continuum near the plasma edge, where the density is low, and experience additional damping. The bulk eigenmode structure is assumed only weakly modified by the resonance, and the resonance damping is estimated perturbatively by taking into account the local logarithmic singularity in the eigenmode structure. This method describes the dissipative power transfer to electrons and ions without specifying the detailed damping process which requires a more involved calculation. The zeroth order eigenvalue is determined by the condition that the Wronskian be continuous at the singular resonance point and the eigenfunction be real and vanish at the plasma edge. In the next order the damping is obtained rigorously without the need to calculate the perturbed eigenfunction. For simplicity, consider a nearly cylindrical geometry and an equilibrium density profile that is mostly flat, but decreases rapidly near the plasma edge in a layer of thickness $\Delta$. For the $n = 1, m = 1-2$ TAE mode with $3 < q(a) < 4$, the continuum damping rate, which is proportional to $\Delta$, is obtained to be $\gamma_d / \omega = 0.01$ for $\Delta/a = 0.1$ and $a/R = 0.25$. As $q(a)$ increases, the higher $m$ harmonic will be resonant with the Alfvén continuum and the continuum damping rate decreases.

When continuum resonance occurs, the $\alpha$-particle induced growth rate can be significantly reduced. The damping rate is sensitive to the density gradient and safety factor at the edge; hence experimental control of these parameters could perhaps be used to regulate TAE stability. Further studies with various density and $q(r)$ profiles and more realistic tokamak geometry are needed to examine the spectral gap structure near the edge, which will affect the predicted damping rate.

5. Quasi-linear saturation of the TAE mode

The non-linear behavior of the $\alpha$-particle driven TAE modes has been investigated [9] with a model in which the finite amplitude of a single TAE mode alters the $\alpha$-particle interaction with the mode quasi-linearly. The $\alpha$-particle distribution is flattened locally in phase space by the perturbed magnetic field which reduces the $\alpha$-particle-to-wave energy transfer rate below the
ambient dissipation rate. For typical parameters \((R = 4 \, \text{m}, a = 1 \, \text{m}, B = 5 \, \text{T}, T_e = 15 \, \text{keV}, n_i = 10^{14} \, \text{cm}^{-3})\), the saturation level is estimated to be \(\delta B_r/B = 10^{-5} (\gamma_\alpha/\gamma_d)^{2/3}\), where \(\gamma_\alpha\) is the \(\alpha\)-particle induced growth rate in the absence of dissipation and \(\gamma_d\) is the total damping rate in the absence of \(\alpha\)-particles. With continuum resonance damping, the saturation level can possibly be low enough that \(\alpha\)-particles will not be prematurely lost from the plasma. However, if continuum damping is absent, saturation levels as high as \(\delta B_r/B = 10^{-3}\) are predicted, for which \(\alpha\)-particles will suffer significant loss. Note that, at high excitation level, the \(\alpha\)-particles should spread out radially, which would tend to reduce the destabilizing drive. A more complete calculation of the TAE mode saturation level with self-consistent eigenfunctions is needed to ascertain if the spreading of the \(\alpha\)-particle distribution will allow sufficient heat containment to maintain ignition conditions.

6. Alpha particle transport due to the TAE mode

The anomalous \(\alpha\)-particle losses due to the TAE modes have been investigated with a Hamiltonian guiding center orbit Monte Carlo code \[7,8\]. Whereas the low frequency internal kink (fishbone) modes are responsible for the anomalous losses of the trapped \(\alpha\)-particles, the high frequency TAE modes affect the transport of the untrapped \(\alpha\)-particles. Circulating \(\alpha\)-particles that resonate with the TAE modes will lose energy and increase their radial outward excursions. Near the edge, they can become trapped and enter prompt loss banana orbits. The rate of particle entering the prompt loss orbits is roughly equal to the particle loss rate. By establishing the particle prompt loss region in the phase space of particle energy, pitch angle variable, and toroidal momentum for a given equilibrium, one can compute the rate of particles scattered into the prompt loss domain by the TAE mode. The \(\alpha\)-particle loss rate, computed from the Hamiltonian guiding center orbit code, scales roughly linearly with \((\delta B_r/B)\). Significant \(\alpha\)-particle losses were found when the level of the global MHD modes is large with \((\delta B_r/B) \geq 10^{-4}\). For \((\delta B_r/B) = 5 \times 10^{-4}\) the \(\alpha\)-particle loss time is appreciably shorter than the \(\alpha\)-particle slowing-down time. The \(\alpha\)-particle loss rate also decreases with decreasing \(\alpha\) energy; thus there is not much hope for He ash removal by the TAE mode. If the \(\alpha\)-particles excite both the \(n = 1\) and \(n = 2\) TAE modes, their losses will be enhanced because stochastic particle orbit losses.

7. Alpha particle effects on internal kink modes

Low frequency internal kink modes and ballooning modes can be destabilized by trapped \(\alpha\)-particles through two mechanisms. First, for \(\omega \ll \omega_d\) (the trapped \(\alpha\)-particle bounce averaged magnetic drift frequency), the trapped \(\alpha\)-particles are destabilizing if \((\omega_a/\omega_d) < 0\) and stabilizing if \((\omega_a/\omega_d) > 0\). At a given minor radius, \(\omega_d\) of the barely trapped \(\alpha\)-particles is opposite in
sign from that of the deeply trapped \( \alpha \)-particles. Since plasma shape also affects the value of \( \omega_d \), \( \langle \omega_\phi/\omega_d \rangle \) averaged over the trapped \( \alpha \)-particle pitch angle space will be strongly influenced by the plasma equilibria. For CIT and ITER equilibria, we find that \( \langle \omega_\phi/\omega_d \rangle \) will change from being positive near the magnetic axis to negative as the minor radius increases, i.e. magnetic drift reversal. Therefore, the net effect of trapped \( \alpha \)-particles on the ideal internal kink mode must be integrated over the entire \( \alpha \)-particle population with proper weightings of different poloidal harmonics. Second, for resonant trapped \( \alpha \)-particles with \( \omega = \omega_d \), their dynamics are no longer rigid with respect to the MHD perturbation and wave particle resonances can occur. Resonant "fishbone" internal kink instabilities can be excited by tapping the free energy of the \( \alpha \)-particle pressure non-uniformity. Stability diagrams of the \( n=1 \) internal kink for the parameters \( \beta \) and \( q(0) \) computed from the NOVA-K code indicate that the trapped \( \alpha \)-particles have destabilizing effects and can significantly lower the total beta threshold [4].

Also, a new mechanism [13] is proposed to explain fishbone oscillations, based on the sawtooth model of Wesson, in which the shear is extremely low or the \( q \)-profile has a slight minimum off-axis. This permits the establishment of a global Alfvén wave at a frequency low enough to resonate with the precessional frequency of injected ions and thereby cause mode amplification and subsequent particle loss. The \( q \)-profile is governed by current diffusion, which will be enhanced by the increased spatial gradient that arises as the mode amplitude is excited. This extra diffusion causes a frequency shift of the global Alfvén wave, which reduces the resonant interaction and allows stabilization. Numerical simulations show either (1) a steady saturated wave response with a continuous amplitude when resistivity is the dominant wave damping mechanism or (2) quasi-periodic fishbone-like pulses when viscosity dominates.

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REFERENCES

DISCUSSION

H.L. BERK: I would like to make the comment that the dissipation due to resonance does not require a strict singularity, only a steepening of the eigenfunction derivatives, so that dissipative terms such as Landau damping and viscosity can actually absorb the energy.

C.Z. CHENG: I agree with your comment.

Ya.I. KOLESNICHENKO: Have you any predictions concerning the influence of TAE instability on alpha particle confinement in ITER?

C.Z. CHENG: If the amplitudes of TAE modes are large, significant alpha particle losses are expected in ITER class machines.
MHD STABILITY LIMITS OF TOKAMAK PLASMAS OBEYING NEOCLASSICAL OHM'S LAW, AND RADIATIVE THERMAL INSTABILITIES IN TOKAMAKS

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Abstract

MHD STABILITY LIMITS OF TOKAMAK PLASMAS OBEYING NEOCLASSICAL OHM'S LAW, AND RADIATIVE THERMAL INSTABILITIES IN TOKAMAKS.

In Part A the beta limit is studied for a tokamak reactor plasma (ITER) obeying neoclassical Ohm's law. The plasma is assumed to be in the inductive phase. The non-inductive current necessary to keep the safety factor q near unity at the magnetic axis is about 30% of the total current. The beta limit (Troyon factor) is 2.2 for a peaky temperature profile, which is required to attain a high fusion yield. In Part B radiative thermal instabilities such as MARFE (multifaceted asymmetric radiation from the edge), density limit phenomena and the "snake" perturbation are studied. Simulations of the "snake" are carried out by using reduced MHD equations including temperature evolution. Pellet injection leads to a local temperature perturbation, the "snake", at the q = 1 surface. This local perturbation grows owing to the tearing mode instability, which leads to full reconnection and makes the "snake" structure straight.

Part A: MHD Stability Limits of Tokamak Plasmas Obeying Neoclassical Ohm's Law

(S. Tokuda, T. Tsunematsu, M. Azumi, T. Ozeki ,Y. Kishimoto, T. Takizuka, K. Tani, Y. Nakamura, M. Yamagiwa, T. Takeda)

1. Introduction

The current profile of a tokamak plasma strongly depends on profiles of density, temperature and impurity ($Z_{eff}$) through neoclassical Ohm's law [1,2]. When the temperature profile is made peaky to attain a high fusion yield a plasma with only ohmic current suffers from $m = 1$
MHD activities (m: poloidal mode number) such as sawteeth and fishbone instabilities because of its $q_0 < 1$ profile ($q_0$: safety factor at the magnetic axis). Such MHD activities should be avoided because the plasma in a reactor is seriously deteriorated by the rapid energy loss due to the activities. Therefore, current profile control by a non-inductive method is necessary to keep $q_0$ near unity and suppress the $m = 1$ activities. We compute the quantity and the profile of the non-inductive current necessary to produce a current profile such that $q_0$ is kept near unity in a plasma obeying neoclassical Ohm's law. ITER reference parameters [3] are used in the calculation. We also study the ideal MHD stability limits for the plasma under current profile control.

2. Physical Model

By neoclassical Ohm's law the surface averaged parallel current $\langle J \cdot B \rangle$ is expressed as [1]

$$\langle J \cdot B \rangle = \langle J \cdot B \rangle_E + \langle J \cdot B \rangle_B + \langle J \cdot B \rangle_S,$$

where $\langle J \cdot B \rangle_E$ is the ohmic current, $\langle J \cdot B \rangle_B$ the bootstrap current and $\langle J \cdot B \rangle_S$ the current driven by external sources. The following are assumed. (i) The plasma is in a stationary state. (ii) Only one impurity ion (carbon) is considered. (iii) Thermodynamic forces of the impurity are neglected. Impurity effects are reflected in the neoclassical coefficients of electrons and main ions (deuterons) through Coulomb collisions. The neoclassical coefficients are numerically computed by the velocity space integral (energy space partitioning) according to references [1,4]. The fraction of circulating particles, $f_c$, is obtained by evaluating numerically the integral,

$$f_c = \frac{3\langle B^2 \rangle}{4(B_{max})^2} \int_0^1 \frac{\lambda d\lambda}{\sqrt{1-\lambda B/B_{max}}}. \quad (2)$$

The parameters chosen in the calculation (ITER reference parameters) are: major radius $R_0 = 6$ m, plasma radius $a = 2.2$ m, toroidal field $B_t = 4.85$ T, ellipticity $\kappa = 2.2$, 2.2,
and triangularity $\delta = 0.2$. The profiles of density and temperature are given by

$$y(\psi) = (y_0 - y_f)(1 - \psi)\alpha(\psi) + y_f,$$

$$\alpha(\psi) = \alpha(1 - \psi^4) + 1.5\psi^4,$$

(3.a) (3.b)

where $\alpha = \alpha_n = 0.5$ for density and $\alpha = \alpha_T = 0.7$ for temperature and $\psi$ is the normalized poloidal flux function. A uniform $Z_{\text{eff}}$ profile with $Z_{\text{eff}} = 1.7$ is used.

3. Results of Computation

Figure A1 shows profiles of the safety factor when the plasma obeys neoclassical (solid line) and classical (broken line) Ohm's law without any externally driven currents. Here $T_0 = 15\text{keV}$, $T_f = T_0/10$, $n_0 = 3.6 \times 10^{20} \text{m}^{-3}$, $n_f = n_0/10$ and $I_p = 22\text{MA}$. Neoclassical conductivity causes a strongly peaked current profile to yield $q_0$ much below unity ($q_0 = 0.34$) and a wide region of $q < 1$. To control the current profile we use two kinds of non-inductive current. One is the current in the co-direction to the ohmic current (co-current) driven mainly in the outer region. The other is the current in the counter-direction to the ohmic current (counter-current) to eliminate

![FIG. A1. Safety factor profiles for the cases of neoclassical (solid line) and classical (broken line) Ohm's law. $n_0 = 3.6 \times 10^{20} \text{m}^{-3}$, $T_0 = 15\text{keV}$ ($n_f = n_0/10$, $T_f = T_0/10$) and $I_p = 22\text{MA}$.](chart)
the peakedness of the ohmic current profile at the axis. The profile of the co-current is given by the function

\[
\langle J \cdot B \rangle_{co} \propto \exp\left[-\frac{(x-x_f)^2}{x_w^2}\right],
\]

where \( x = \sqrt{\psi}, x_f = 0.7, x_w = x_{ew} = 0.2 \) for \( x < x_f \) and \( x_w = x_{ow} = 0.15 \) for \( x > x_f \). The profile of the counter-current \( \langle J \cdot B \rangle_{ctr} \) is determined from the ohmic current profile by specifying
FIG. A3. (a) Profiles of $\langle J \cdot B \rangle$ for $I_p = 15$MA, $n_0 = 1.5 \times 10^{20} m^{-3}$ ($n_f = n_0/10$) and $T_0 = 15$keV ($T_f = 1$keV). Solid line: ohmic current; dotted line: bootstrap current ($I_{bs} = 3.04$MA); broken line: externally driven current ($I_{co} = 3$MA, $I_{cr} = 33$kA); bold line: total current. (b) Safety factor $q$, pressure gradient (solid line) and critical pressure gradient (dotted line) against high-n ballooning modes ($\psi_e = 0.1$). $q_0 = 1.31$ and shear is strong near the axis. The Troyon factor $g$ reaches 2.28.

the width of the counter-current and the value of the total current density at the axis ($\langle J \cdot B \rangle_S = \langle J \cdot B \rangle_{CO} + \langle J \cdot B \rangle_{ctr}$). For the case of 6MA co-current and 38kA counter-current, the resultant current profile realizes $q_0$ as high as 0.85 and reveals rather strong shear. It was found that when the co-current is increased further to raise $q_0$, very weak or negative shear appears and the attainable $g$ (Troyon factor)
decreases. Finite pressure gradients were found to have two effects. First they violate the Mercier stability condition near the axis at low beta. Second, even if the bootstrap current, $I_{BS}$, is too small, at 2MA, to change the current profile substantially, this value is still enough to deteriorate the stability near the axis. It should be noted that the region near the axis is in a magnetic hill or a weak magnetic well and the localized modes are stabilized only by the shear. Therefore, $dp/d\psi \sim 0$ is desired near the axis for stability and density and temperature profiles for $\psi < \psi_g = 0.16$ are multiplied by a gaussian function. Combination of a smaller co-current ($I_{co} = 4$MA) and a larger counter-current is favorable for raising $q_0$. However, a counter-current tends to bring about weak shear and a large counter-current is necessary to produce a magnetic well for stability. When the counter-current is 1.68MA, the current profile becomes flat with $q_0 = 1.53$ (Fig.A2a). For this case the magnetic well is produced and the stability can be retained against the changes in shear until $g$ reaches 2.25 (Fig.A2b). For the case of $I_p = 15$MA it is possible to make a current profile with both magnetic shear and magnetic well by small counter- and co- currents ($I_{co} = 3$MA and $I_{ctr} = 33$kA in Fig.A3). The relatively high bootstrap current ($I_{BS} = 3.04$MA) contributes to raise $q_0$ easily. However the attainable value of $g$ is limited to 2.28, which is nearly equal to the value for the $I_p = 22$MA case.

4. Summary

For the ITER plasma the non-inductive current necessary to control the current profile from a peaked to a flat one is about 30% of the total current. A magnetic well should always exist to keep stability for the different current profiles during the control. It is also necessary that the gradients of the pressure profile are null near the axis for stability. For the $I_p = 22$MA case weak shear results when the available non-inductive current is restricted within 30% of the total current. For the $I_p = 15$MA case a current profile with both well and shear can be produced. By this optimization $g = 2.2$ was obtained for both cases.
Part B: Radiative Thermal Instabilities in Tokamaks

(G. Kurita, T. Tuda, M. Azumi, T. Takeda)

1. MARFE and Density Limits

MARFE [1,2] often occurs at the plasma parameter near the Hugill limit \( n_c \sim B/Rq \) and is considered to be a kind of thermal instability [3]. To examine the relation between MARFE and density limit phenomena, the energy balance of the edge plasma layer (its depth is of the order of the radial scale length of the MARFE, \( Rq\sqrt{\kappa_L/\kappa_\parallel} \)) is studied with carbon impurity radiation. With increasing electron density, radiation power from this region becomes large and when \( P_{rad} = n_e n_f L_c(T_e) \geq 0.5P_{in} \) and \( T_{edge} \leq 15\text{eV} \), parallel electron thermal conductivity cannot prevent a poloidally asymmetric perturbation and a MARFE structure is formed. When the radiation loss exceeds the input power, energy balance is no longer satisfied. This is one of the density limits caused by impurity radiation [4]. In Fig.B1, density limits due to carbon impurity radiation (OH plasma with \( R = 3\text{m}, a = 1\text{m}, B_t \leq 4\text{T and } n_f = 1.0\times10^{18}\text{m}^{-3} \) of carbon ions) are shown in a Hugill plot, the zone of appearance of MARFE is shown by closed triangles and stable symmetric equilibria (no MARFE) are shown by open circles.
2. "Snake" and Sawtooth Oscillation

Radiative thermal instabilities with $m \neq 0$ poloidal asymmetry are stabilized by parallel electron heat conduction. When a large perturbation, such as the one in pellet injection experiments, is caused near a rational surface of $q = 1$, perturbation of electron temperature induces current perturbation and an island structure is formed. This island removes the stabilizing effect of $\kappa_{\parallel}$, and the $m/n = 1$ structured local temperature perturbation called "snake" appears [5] and survives near the rational surface due to impurity radiation. We carried out simulations of $m/n = 1$ modes by solving a nonlinear reduced set of resistive MHD equations with an electron temperature transport equation. Time is normalized by the poloidal Alfvén transit time. We assume the following: the function of radiation loss is $L \propto |\rho^2 - \rho_0^2|_{t=0}$, where $\rho$ is the plasma density and is obtained from the convective equation. The initial $q$ profile and plasma density profile are
FIG. B2. Time evolution of central electron temperature. The solid line shows sawtooth oscillation. The effect of pellet injection is shown by the broken line. In this case sawtooth oscillation is suppressed.

FIG. B3. Time evolution of temperature contour. Temperature perturbation, initially resulting from pellet injection ($t = 4100$), becomes a "snake" structure at $q = 1$ ($t = 4150$) but moves to the center and becomes straight after complete reconnection ($t = 8000$).
chosen as \( q(r) = 0.8(1+r^2) \) and \( \rho(r) = 1 \), respectively. The time evolution of the central electron temperature is shown in Fig.B2 (solid line). The result is similar to that of the sawtooth oscillation obtained by using the reduced MHD equation [6]. A pellet is injected into the plasma just before the second sawtooth collapse (solid line in Fig.B2 at \( t = 4100 \)). The time evolution of the electron temperature contour is shown in Fig.B3. The poloidal dip of the electron temperature formed by injection of the pellet \( (t = 4100) \) becomes flat along the magnetic field line instantaneously by large parallel heat conduction except near the rational surface and a "snake" appears \( (t = 4150) \). This electron temperature perturbation survives for a long time, with that of negative plasma current, although it moves to the plasma center by the convection of the \( m/n = 1 \) modes \( (t = 8000) \), and the "snake" structure becomes straight. The simulations show that this result does not depend so much on the time of the pellet injection if there is a rational surface of \( q = 1 \).

Wesson et al. [7] pointed out a possibility of the suppression of full reconnection by quasi interchange modes for a very flat \( q \) profile with \( q_0 \approx 1 \). Their simulation, however, does not contain the transport effect. Such a flat profile cannot be kept due to the heating effect, in our simulation, and the value of \( q_0 \) decreases and the local \( m/n = 1 \) large perturbation grows due to the tearing mode instability, which brings about full reconnection. After the full reconnection, the heating effect removes the central temperature dip and the sawtooth starts again in the case of a small size pellet. On the other hand, when the size of the pellet is large, the central temperature cannot be increased smoothly due to the radiation (see broken line in Fig.B2), and the \( m = 1 \) tearing mode is maintained stable \( (q_0 > 1) \).

The pellet injection causes a local electron temperature perturbation, the "snake". The condition of forming the "snake", the existence of a \( q = 1 \) rational surface, is the same as for the \( m = 1 \) tearing mode instability. In the present simulations, this local \( m/n = 1 \) perturbation easily grows due to the tearing mode instability, which leads to full reconnection and makes the "snake" structure straight. This is not consistent with the experimental results. In order to
explain the experimental results, a new theoretical model of the sawtooth is required, because recent full-MHD 3-dimensional simulations of the $m = 1$ mode with transport effect result in full reconnection [8].

REFERENCES TO PART B

MECHANISMS UNDERLYING SAWTOOTH PRECURSORS, RELAXATIONS AND RAMP PHASES

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Abstract

MECHANISMS UNDERLYING SAWTOOTH PRECURSORS, RELAXATIONS AND RAMP PHASES.

To arrive at a coherent explanation of the sawtooth precursors, two properties are theoretically required of the current density profile which invalidate the assumptions inherent to the conventional calculation of \( \Delta' \) = \( [s_i \text{dln} \psi/\text{d}r]_s \), the jump of the logarithmic derivative of the MHD solution across the \( q = 1 \) singular magnetic surface. In the revised toroidal theory \( \Delta' \) passes through zero for \( s_i = s_i,c \) (\( s_i \) is the shear parameter at the \( q = 1 \) surface \( r = r_1 \)), where \( s_i,c \) depends on \( r_1 \) and the value \( q_0 \). In the course of the sawtooth ramp, \( s_i,c \) is approached from below with \( \Delta' \) < 0, contrary to conventional wisdom, and a dramatic increase of the precursor follows a more moderate one. The magnetic surfaces then lean asymmetrically on the discharge 'confining zone' where drift instabilities act as particle and heat sinks. This situation leads to a sheared flow along the field lines which destabilizes the Kelvin–Helmholtz mode; the latter, in turn, triggers very rapid cross-field transport. Assuming that the \( q = 1 \) surface remains still during the ramp phase, it is demonstrated that the ratio of the rates of rise, on the magnetic axis, of the current density and temperature is about 0.7. The paper compares theoretical predictions and observations for sawtooth precursors, relaxations and ramp phases.

1. SAWTOOTH PRECURSORS

In a pressureless toroidal plasma, conventional calculations of the jump \( [\psi_i']_r \) of the derivative of the MHD solution always yield positive values [1]. According to the tearing mode dispersion relation (which results from resistive layer theory [2, 3]), the instability should thus be weak in a torus so that non-linear effects could quench the mode at low amplitude. However, \( [\psi_i']_r \) is calculated in the limit of infinite aspect ratio, which approximation fails if simultaneously (i) \( s_i \) is small, i.e. the current density has a plateau or shoulder in the neighbourhood of \( r_1 \), and (ii) \( J' \) varies abruptly around \( q = 1 \) so that the jump of the derivative of the usual logarithmic solution no longer vanishes. It is stressed that a rapid variation of \( J' \)

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around \( r_1 \) is expected as \( q = 1 \) is a transition zone where one transport mechanism (drift turbulence) overtakes another (sawtooth relaxations). The revised pressureless toroidal theory, neglecting coupling to the \( m = 2 \) harmonic, yields

\[
[\psi_{\|} / n]_{r_1} = -r_1 t'_1 - \frac{c \pi}{r_1^3 Q_1^{3/2}} \left[ \frac{t'''}{3} - \frac{t''}{2t'} \left( \frac{t'''}{t''} - \frac{t''}{4} \right) \right]
\]

(1)

where \( t'_1, t''_1, t'''_1 \) are the derivatives of the rotational transform \( t \equiv 1/q \) at \( r_1 \), \( \delta_1 = -r_1 t'_1; Q_1 = [(2t'_1 t'''_1 - t''_1^2)/3] \) is assumed positive, whereas \( t'_1 < 0 \) to ensure that \( t \) is a monotonically decreasing function of \( r \); \( c \equiv (13r_1^4/8R^2) \int_0^1 \times [t(r_1 \sqrt{x}) - 1] \times dx \) with \( r_1 \sqrt{x} = r \). The first term in Eq. (1) is the conventional result; the novel contribution will be of opposite sign if \( 0 \leq (t''_1/t'_1)^2 < 0.845 (t'''_1/t'_1)^2 \) and of equal magnitude if \( |r_1 t'_1| = 1.3 \left| t''_1 t'''_1 \right|^{1/2} |c/r_1^2|^{2/5} \) for the case \( t''_1 = 0 \). Thus \( [\psi_{\|} / n]_{r_1} \) passes through zero for \( \delta_1 \sim (c/r_1^2)^{2/5} \sim (r_1/R)^{4/5} \) if \( |r_1 t''_1| \sim 1 \) and \( t_0 - 1 \sim 1 \). The analytical result has been confirmed by a numerical unexpanded solution of the MHD equations obtained by Connor et al. [5] and the corresponding \( \Delta' \) has been inserted in the dispersion relation [3] for a set of parametric profiles \( t(r) \); Fig. 1 summarizes the results. The discontinuous transition of the growth rate \( \gamma \) for \( \delta_1 = \delta_{1,c} \) is associated with a profound modification of the resistive layer.

![FIG. 1](image-url)

(a) Variation of \( 1/\Delta' \) versus \( \delta_1 \). It is assumed that \( t_0 = 1.5, t''_1 = 0, t_a = 1/3, r_1/a = 0.45 \). Open circles pertain to the numerical results, crosses to the analytical calculation. Dashed line corresponds to conventional wisdom. (b) \( \gamma \tau_e \) versus \( \delta_1 \) (\( \tau_e \) is Braginskii's electron collision time for TEXTOR parameters) for the conditions of (a). Solid line is the numerical solution of the dispersion relation, the dashed line its analytical solution in various asymptotic limits. Dash-dotted line corresponds to conventional wisdom.
radial eigenfunction, as can be seen by inspection of its Fourier transform obtained (in compact form) in Ref. [3]. As $-1/\Delta' \gg 1$, the growth rate takes on MHD values: 
\[ \gamma \tau_H = (-\pi Q_s/\tau_r \Delta'), \]
but the width of the singular layer remains small (a few ion Larmor radii): \[ \text{width}'r_i = \left[ (-\pi Q_s/\tau_r \Delta')(\tau_H/\tau_R) \right]^{1/4}, \]
where $\tau_H$ and $\tau_R$ are the Alfvén and resistive times [2].

2. SAWTOOTH COLLAPSE

The current pinches during the ramp phase of the sawteeth; thus $s_i$ approaches $s_{i,c}$ from below and $\Delta' < 0$, contrary to conventional wisdom. In the last phase of the precursor, as $\gamma \rightarrow \infty$ (Fig. 1), the magnetic surfaces lean asymmetrically on the 'confining zone' where drift instabilities are particle and heat sinks. The flow which then builds up along the field lines is radially inhomogeneous and triggers the Kelvin–Helmholtz instability. Owing to Landau resonance, the electrons are not adiabatic and the fluctuations lead to cross-field transport in addition to parallel transport. Consider a pipe of length L along the field lines (co-ordinate $z$) initially filled with a plasma at rest of density $n = n^{(0)}(r) + n^{(1)}(r)$ and temperature $T = T^{(0)}(r) + T^{(1)}(r)$. The pipe is opened at $z = L$ at initial time $t_0 = -L/c_s$ ($c_s = \sqrt{2T_m}$ is the sound speed). We find that the Kelvin–Helmholtz turbulence induces a radial flux of parallel ion momentum \( \langle U^{(0)}_{z,i}, n^{(0)} U^{(0)}_{z,i} \rangle \) which cannot be balanced at the fluctuation level (capital letters) required for competitive radial particle transport unless the instability has relaxed: $\Gamma^{(0)} = 0$ for all modes, which entails [6]

\[ k_z/k_\theta = \frac{\partial u^{(0)}_{z,i}}{2\Omega} \partial r; \quad \frac{\partial u^{(0)}_{z,i}}{c_s} \partial r = -\frac{\partial n^{(1)}}{n^{(0)}} \partial r \]

(2)

where $\Omega$ is the gyrofrequency; note that $u^{(0)}_{z,i}/c_s$ is smaller here than in classical shocks. The ambipolar continuity equation and the parallel ion momentum equation then take the form (Eq. (2) is used to simplify the turbulent transport terms):

\[ \frac{\partial n^{(1)}}{n^{(0)}} + \frac{\partial u^{(0)}_{z,i}}{\partial z} - \frac{cT_e}{eB} \left( 1 - \frac{\eta_e}{2} \right) \text{sign} \left( \frac{\partial n^{(1)}}{n^{(0)}} \right) \]

\[ \times \sqrt{\frac{\pi m_e}{9m_i}} \left( \frac{\partial}{\partial r} + \frac{1}{r} \right) \sum_k |k_\theta| [\hat{N}_k]^2 = 0 \]

(3)

\[ \left( \frac{\partial}{\partial t} - c_s \frac{\partial}{\partial z} \right) \left( \frac{u^{(0)}_{z,i}}{c_s} - \frac{n^{(1)}}{n^{(0)}} \right) + \left( \frac{\partial}{\partial r} + \frac{1}{r} \right) \]

\[ \times \left[ \frac{1}{\partial n^{(1)}/n^{(0)} \partial r} \left( \frac{\partial}{\partial t} - c_s \frac{\partial}{\partial z} \right) \sum_k |\hat{N}_k|^2 \right] = 0 \]

(4)
where the last term corresponds to convection of the relaxed turbulence; \( \hat{N}_F = \hat{N}_F^{(0)}/n^{(0)} \). The solution of these equations consists of an ingoing shock of which the front trajectory is given by \( z + c_s t = 0 \), followed, after the end \( z = 0 \) of the pipe has been reached, by a reflected shock of front trajectory \( z - c_s t = 0 \).

Downstream of the ingoing shock front:

\[
\frac{n^{(1)}}{n^{(0)}} = (1 - \rho^2) c_s \tanh(c_1 \beta z^+/r_1)
\]

\[
u_{z,1}/c_s = \rho^2 c_1 \tanh(c_1 \beta z^+/r_1)
\]

\[
\sum |\hat{N}_F|^2 = \rho^2 (1 - \rho^2) (c_1^2/2) \cosh^2 (c_1 \beta z^+/r_1)
\]

\[
z^+ = z - c_s t
\]

whereas upstream:

\[
\frac{n^{(1)}}{n^{(0)}} = (1 - \rho^2) K
\]

\[
u_{z,1}/c_s = 0
\]

\[
\sum |\hat{N}_F|^2 = 0
\]

\( K \) is the initial \( n^{(1)}(r = 0, z, t_0)/n^{(0)} \), which is a constant, \( \rho = r/r_1 \), and we have assumed that

\[
2 \beta = (\pi m_e/2m_i)^{1/2} (1 - 0.5 \eta_e) \sum |k_\phi| a_e \rho |\hat{N}_F| \sqrt{\sum |\hat{N}_F|^2}
\]

is also constant (note that \( k_\phi \propto 1/\rho \)). We choose \( c_1 \) to avoid discontinuity of \( n^{(1)}/n^{(0)} \) initially, i.e. \( c_1 \tanh(2c_1 \beta L/r_1) = K \). For the reflected shock, the downstream solution identically vanishes, whereas upstream we recover (5). There is an implicit assumption for the above exact solution, namely that the drift turbulence sink can swallow a flux \( n^{(0)}u_{z,1}^{(0)} \propto r^2 \) of the amplitude imposed by Eq. (2). Figure 2 shows the \( z \) dependence of the solution for four different times. The discontinuity of the density profile at the position of the ingoing shock front arises because the radial flux identically vanishes upstream, whereas the parallel flux is finite at the front.

3. SAWTOOTH RAMP PHASE

We linearize \( \sigma_\phi = \sigma_\phi^{(0)}(r) + \delta \sigma_\phi(r, t) \), \( \sigma_1 = \sigma_1^{(0)}(r) + \delta \sigma_1(r, t) \) (\( \sigma_\phi \) is the parallel electrical conductivity) and assume a linear relationship with time: \( \delta \sigma_1(r, t)/\sigma_1^{(0)}(r) \)
FIG. 2. Solutions $n^{(0)}(z)/n^{(0)}$ (dashed line), $u^{(0)}(z)/c$ (dotted line) and $\Sigma |\tilde{N}_q|^2(z)$ (dash-dotted line) at times $t - t_0 = L/3c$, $2L/3c$, $4L/3c$ and $5L/3c$ if tank $x = x$ ($p$ fixed).

$= \alpha \Phi h(r)$ (with $h(0) = 1$, $h(r_1) = 0$, i.e. $\delta T_e(r_1, t) = 0$). The solution of the current diffusion equation subject to the constraint that the $q = 1$ surface does not move is:

$$\delta I_\phi/I^{(0)}_\phi = \alpha \tau_3 \left\{ f(r) + \left(t/\tau_3\right)[h(r) + g_1] \right\}$$

where $\tau_3$ is the sawtooth period and

$$f(r) = \int_0^r (dr'/r') \int_0^{r'} \left[ 4\pi \sigma_1^{(0)}(r'')/c^2 \tau_3 \right] [h(r'') + g_1] r'' \, dr'' + f_0$$

The constant $g_1$ is given by the constraint:

$$\int_0^{r_1} (4\pi \sigma_1^{(0)}/c^2 \tau_3) (h + g_1) \, r \, dr = 0 \quad (6)$$

whereas $f_0$ can be obtained by requiring that $\int_0^{r_1} \delta I_\phi(r, 0) \, r \, dr = 0$. Note that $\delta J_\phi$ has a node where $h(r) + g_1 = 0$. The ratio of the rates of rise of $\delta I_\phi/I^{(0)}_\phi$ and $\delta T_e/T^{(0)}_e$ is, on the magnetic axis:

$$\left[ \frac{\delta I_\phi/I^{(0)}_\phi}{\delta T_e/T^{(0)}_e} \right]_{r=0} = 1.5 \left(1 + g_1\right) \quad (7)$$
We have estimated $g_1$ by assuming that $h = 1 - (r/r_1)^2$ and $\sigma_i^{(0)} \propto 1 - \beta^*(r/r_1)^n$, and have obtained $=0.7$ for the right hand side of Eq. (7), which, it should be stressed, is independent of $r_1$, $\tau$, and $\sigma_i^{(0)} (r = 0)$.

4. COMPARISON WITH EXPERIMENTAL RESULTS AND DISCUSSION

(a) If, through non-linear saturation, the amplitude of the precursor signal $S \propto \gamma_1/\gamma_2$, one can estimate its exponentiation rate from Fig. (1). A realistic theoretical value prior to collapse is $1 \text{ ms}^{-1}$ for TEXTOR, which agrees reasonably well with the value deduced from the ECE traces: $d \ln S_{\text{EXP}}/dt \sim 0.65 \text{ ms}^{-1}$.

(b) Constraints (i) and (ii) in Section 1, theoretically required from the t profile, are fulfilled in a low $q_a$ discharge (Fig. 5 of Ref. [7]). Furthermore, density plateaus or shoulders, easier to detect, are always present [8].

(c) In all cases numerically investigated $0.05 < \delta_1 < 0.15$; since $\delta_1 < \delta_{1,c}$, this conclusion agrees with the low shear $\delta_1$ measured on JET [9] and also on TFTR and JT-60.

(d) It can be shown from the results of Section 2 that radial transport inside of $q = 1$ occurs on the time-scale $\tau \sim L/c_s$. We estimate $L \sim 2 \times 10^3 \text{ cm}$ and $c_s \sim 4 \times 10^7 \text{ cm/s}$ in TEXTOR. Hence $\tau \sim 50 \mu$s; experimentally the crash occurs in less than 200 $\mu$s.

(e) The shock-like solution of Section 2 is irrelevant if $\beta = (1 - 0.5\eta_e) < 0$ since $K > 0$. This could be related to the observation that the period increases dramatically in 'monster sawteeth' when intense auxiliary heating power is deposited inside of $q = 1$ [10]. In TEXTOR sawtoothing Ohmic discharges, $\eta_e = 1.5$ [11].

(f) In these TEXTOR discharges one does indeed find $\Delta T_e/T_e^{(0)} = 0.12$ and $\Delta N/N^{(0)} = \Delta J_e/J_e^{(0)} = 0.08$, in agreement with the prediction of Section 3; the implication is that $\sigma_\parallel$ is neoclassical over most of the ramp phase.

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THE EFFECT OF NEUTRAL PARTICLES ON THE DISSIPATIVE DRIFT INSTABILITY IN THE EDGE PLASMA OF TOKAMAKS

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Abstract

THE EFFECT OF NEUTRAL PARTICLES ON THE DISSIPATIVE DRIFT INSTABILITY IN THE EDGE PLASMA OF TOKAMAKS.

The influence of ionization and charge exchange processes on the dissipative drift wave instability is studied. The solutions of the eigenvalue equation show that the weakly toroidally coupled modes only become unstable through ionization if the neutral density exceeds a critical value. In the case of strong mode coupling, the increment of the drift waves is modified significantly both by ionization and charge exchange processes.

1. INTRODUCTION

It is well known that in the edge plasma of tokamaks strong density and potential fluctuations exist which can lead to anomalous transport phenomena [1]. There are some reasons for assuming that the level of these fluctuations is linked up with the neutral particles penetrating into the plasma as a result of gas puffing or recycling processes. The dissipative drift mode is one of the most important modes with frequencies \( \omega = \omega_\ast \ll \nu_{ei} \) in the collision dominated edge plasma [2–6]. Here, \( \omega_\ast \) is the electron drift frequency and \( \nu_{ei} \) is the electron–ion collision frequency.

Using the linearized MHD equations, we study the effect of neutral particles on the behaviour of these electrostatic modes in the tokamak edge plasma with an arbitrary ion temperature \( T_i \). By taking into account ion viscosity, ionization of neutral particles and charge exchange processes, a fourth order differential equation for the wave potential is derived. We show that ionization and charge exchange can change the stability criteria of these waves significantly. The case of cold ions was considered in Ref. [6].

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It is well known that in the absence of neutrals the magnetic shear $s$ stabilizes the dissipative drift modes in the slab approximation [3]. This mode becomes unstable owing to ionization if the neutral density $N_0$ exceeds the critical value of

$$N_{0c} = \eta L_n \omega_s / L_s k_{ion},$$

where $L_n = |d \ln n_0 / dr|^{-1}$ is the density scale-length, $L_s$ the shear length, $k_{ion} = \langle \sigma_{io} \rangle$ is the rate coefficient for the electron impact ionization, and $\eta = 1 + T_i / T_e$. In toroidal geometry, this weakly localized mode conserves the main properties, but the value of the critical density is modified: $N_{0c} = \eta L_n \omega_s \times (1 + 2q^2 / \pi) / R q k_{ion}$. Here, $R$ is the major tokamak radius and $q$ is the safety factor. The calculations show that the charge exchange process does not affect the stability of the modes.

On the other hand, some unstable solutions of the eigenvalue equation for the drift waves exist in the absence of neutral particles, connected with the strong toroidal mode coupling [4, 5]. The growth rate of these modes may be modified both by ionization and charge exchange. The ionization of the neutral particles always leads to an increase in the growth rate. The influence of the charge exchange processes on the time behaviour of the drift waves depends on their frequencies. The calculations show that the ion viscosity always gives only a small contribution to the wave damping.

2. BASIC EQUATIONS

Using a fluid description for the dynamics of charged ($\alpha = e, i$) and neutral ($\alpha = N$) particles with constant temperatures,

$$\frac{\partial n_\alpha}{\partial t} + \nabla_j (n_\alpha v_{aj}) = c_\alpha k_{ioe} n_N n_e$$

$$\frac{\partial v_{aj}}{\partial t} + v_{am} \nabla_m v_{aj} = -v_{ae}^2 \nabla_j \ln n_\alpha - e_\alpha \nabla_j \Phi / m_\alpha$$

we analyse the eigenvalue problem with the ansatz $A_n(\vec{r}, t) = A_{n0}(\vec{r}) \times \exp(-i \omega t)$. Here, $c_e = c_i = 1$, $c_N = -1$, $v_{ae}^2 = T_e / m_e$, $\omega_{co} = e_\alpha B / m_\alpha c$, $b = B / B$, $e_{jm}$ is the totally asymmetric unit tensor, and $\pi_{ajm}$ is the viscosity tensor, $\nu_{ Nel} = n_i k_{ex} = (n_i / N) \nu_{Nel}$, $\nu_{io} = (m_i n_c / m_n) v_{ei}$ and $k_{ex} = \langle \nu_{io} \rangle$. The functions $A_{n0}$ describe the steady state parameters ($n_{n0} = N_0$, $n_{e0} = n_{i0} \equiv n_0$, $v_{aj0} \equiv \Phi_0 = 0$) and $\bar{A}_n$ their perturbations ($\bar{n}_N$, $\bar{n}_e$, $\bar{n}_i$, $\bar{v}_{aj}$, $\bar{\Phi}$). Assuming the relationships $A_{n0} \gg \bar{A}_n$, $\nabla_j \ln A_{n0} \ll \nabla_j \ln \bar{A}_n$, $\bar{n} = \bar{n}_i \equiv \bar{n}_e$, $\nabla_j v_{aj0} = 0$ to be valid, we obtain from the linearized Eqs (1) and (2) for $n = \bar{n} / n_0$ and $\bar{\Phi} = e \bar{\Phi} / T_e$ the following system of equations:

$$\bar{A}_n = \bar{B}\Phi, \bar{C}_n = \bar{D}\Phi$$

(3)
with the operators

\[
\begin{align*}
\hat{A} &= \omega(1 - ir\hat{e} + \tau k_2^2 q_s^2) - (1 + \tau) k_2^2 \nu_s^2 / (\omega - i\nu_{\text{ion}}) \\
\hat{B} &= \omega_* + \omega(i\hat{e} - k_0^2 q_s^2) \\
\hat{C} &= \omega(1 + i\hat{e}) - (1 + \tau) k_2^2 \nu_s^2 / (\omega - i\nu_{\text{ion}}) \\
\hat{D} &= \omega_* + i\omega\hat{e} + ik_2^2 \nu_s^2 / \nu_{\text{ion}}, \quad \hat{e} = i \nabla \cdot B v q_s [\vec{b}, \vec{k}] / B \omega
\end{align*}
\]

(4)

where \(\omega_* = k_0 v_s q_s / L_n, \quad \nu_s = (T_e/m_e)^{1/2}, \quad \bar{k} = k_y \bar{B} + k_z = -i \nabla, \quad q_s = v_s / \omega_{ci}, \quad \omega = \omega + i\nu_{\text{ex}} + i4k_2^2\nu_s^2 / 3 \nu_{\text{ex}}, \quad \nu_{\text{ion}} = k_{\text{ion}} N_0, \quad \nu_{\text{ex}} = k_{\text{ex}} N_0, \quad \tau = T_e / T_c, \quad n_0 is the density \(n_0\) at the rational surface.

To simplify the situation, in deriving Eq. (3) we have assumed that the relations \(|\omega - k_0^2 q_s^2| > kv_{\text{TN}}, k_{\text{ion}} n_0, k_{\text{ex}} n_0, \) and \(\nu_{\text{ex}} < \omega < \nu_{\text{ci}}\) are fulfilled and we have neglected effects due to the undisturbed radial particle flux and the toroidal plasma rotation [7, 8]. Using the tokamak field representation,

\[
B \approx B_r = B_0(1 - r \cos \theta / R)
\]

(5)

and introducing the ballooning transformation [9],

\[
\Phi(\vec{r}) = \sum_m e^{-i(m\phi - n\theta)} \int_{-\infty}^{\infty} dy \Phi(y)e^{i(m - nq(y))y}
\]

(6)

we finally obtain the equation for the potential \((\omega \approx \omega_*):\)

\[
\partial^4 \Phi / \partial y^4 + \partial^2(2g^2 \Omega^2 P(y) \Phi) / \partial y^2 + i\nu^3 \Omega^3 Q(y) = 0
\]

(7)

where

\[
\begin{align*}
P(y) &= \bar{k}^2(1 + s^2 y^2) + 2 \epsilon_{\text{n}}(\cos y + sy \sin y) / \Omega + \Delta \\
&\quad -i\nu_{\text{ion}} / \Omega + i \Delta \nu_{\text{ex}} / \Omega = P' + iP'' \\
Q(y) &= \bar{k}^2(1 + s^2 y^2) + 2 \epsilon_{\text{n}}(\cos y + sy \sin y) / \Omega^2 \\
g &= (\omega_* R q / v_{\text{si}}), \quad \bar{k} = k_0 \epsilon_{\text{si}}, \quad \nu_{\text{si}} = v_{\text{si}} / \omega_{\text{ci}}, \quad v_s = (1 + \tau)^{1/2} v_s \\
\Delta &= (\Omega - 1) / \Omega, \quad \Omega = \omega / \omega_*, \quad \epsilon_{\text{n}} = (1 + \tau) L_n / R, \quad \nu_{\text{ex}} = (1 + \tau) \nu_{\text{ion}} / \omega_*, \\
\bar{\nu}_{\text{ion}} = \nu_{\text{ion}} / \omega_*, \quad \bar{\nu}_{\text{ex}} = \nu_{\text{ex}} / \omega_* \quad \text{and} \quad s = r \partial \ln q / \partial \tau \text{ is the magnetic shear.}
\end{align*}
\]
In the limiting case $\tau \to 0$, Eq. (7) corresponds to the equation analysed in Ref. [6]. Since the ion viscosity only makes a small contribution to wave damping, we neglect the corresponding term in Eq. (7), which has two types of solution. The first type corresponds to the case of weak toroidal mode coupling if $h = \epsilon_n/k^2s^2$ is a small parameter. For $h \geq 1$, the 'potential' $P'$ becomes strongly modulated (Fig. 1) and the solutions are localized with respect to $y$ and, accordingly, to $\theta$.

3. UNSTABLE SOLUTIONS

3.1. Weak toroidal mode coupling

In this section we analyse the first type of solutions, which reduce, in the limit $r/R \to 0$, to the solutions in the slab geometry. We represent the function $\Phi$ in the form $\Phi = \tilde{\Phi} + \hat{\Phi}$, where $\tilde{\Phi}$ is a strongly oscillating and $\hat{\Phi}$ a weakly oscillating function of $y$, respectively. Substituting $\tilde{\Phi}$ in Eq. (7) and averaging this equation, we obtain an equation for $\Phi(z) = \int_{-\infty}^{\infty} dy \tilde{\Phi}(y)e^{iz\gamma_0}$:

$$p\partial^2\Phi/\partial z^2 + [-\lambda + z^2 + iS/(z^2 - iz^2)]\Phi = 0$$

(9)

with $\lambda = C_iz_0^2 - iz_0^2$, $p = 1$, $S = C_iz_0^4 - z_0^2$$ \lambda$

$$C_1 = g^2(k^2(1 + 2q^2(1 + 2s)) + \Delta - i\nu_{\text{ion}} + 2i\nu_g^4\epsilon_n^2(1 + 4s))$$

$$C_2 = \nu_g^6(\epsilon_n/q)^2(1 + 2q^2(1 + 2s))$$

$$z_0^2 = \tilde{\nu}(gz_0)^6s^2(\epsilon_n/q)^2(1 + 2q^2)$$

$$z_0^4 = (sgk)^2(1 + 2q^2) + 2i\nu_g^4s^2g^6$$

This equation formally corresponds to the equation derived in Ref. [3] for weak three wave coupling in a plasma slab. Fitting the solution of Eq. (9) in the regions $z < z_c$ and $z > z_c$, we obtain for the more unstable even mode:

$$\Omega = \Omega + i\Omega''$$

$$\Omega' = 1 - k^2(1 + 2q^2(1 + 2s))$$

$$\Omega'' = -\nu_{\text{ion}} - \gamma, \quad \gamma = \epsilon_n s(1 + 2q^2)^{1/2}/q$$

(11)
Here, we have neglected the collisional part \( \sim \gamma_1 z_c \) in the shear damping term. For the plasma slab, we obtain \( q = 0, \gamma_1 = L_n(1 + \tau)/L_s \), with a shear length of \( L_s = Rq/s \). These slab-like eigenmodes correspond to unbounded eigenstates with anti-well potential structures. The critical value of the neutral density for the instability is given by

\[
N_{\text{cr}} = \omega \gamma_1 / k_{\text{ion}} \tag{12}
\]

The weakly toroidally coupled waves studied are also weakly localized \( \Phi \sim \exp(-\beta \Omega'' y^2) \) for large \( y, \beta \sim 1 \). The threshold value (12) increases with increasing \( \tau \).

### 3.2. Strong toroidal mode coupling

If \( \epsilon_n \) is large enough, a localized solution of Eq. (7) exists because of the appearance of turning points \( (P'(y) = 0) \). Expanding \( P \) and \( Q \) near \( y_m \), where \( y_m \) must satisfy the conditions \( \partial P/\partial y = 0 \) and \( \partial^2 P/\partial y^2 < 0 \), we again obtain, after a Fourier transformation, from Eq. (7), an equation of type (9), with

\[
p = -1, \quad C_1 = \Omega^2 g^2 P(y_m), \quad C_2 = \Omega^3 g^4 Q(y_m)
\]

\[
z_c^2 = \bar{\nu}(\partial^2 Q/\partial y^2)/(\partial^2 P/\partial y^2)_{y = y_m} g^2 z_0^2
\]

\[
z_0^4 = -(1/2) \Omega^2 g^2 (\partial^2 P/\partial y^2)_{y = y_m}
\]

(13)

Noting that \( S \sim \bar{\nu} \ll 1 \), a perturbative treatment of Eq. (9), with (13), can be done, and we find for the lowest eigenstate:

\[
(1 + k^2(1 + s^2 \gamma_2^2)) \Omega' = (1 - 2s y_m \epsilon_n)
\]

\[
(1 + k^2(1 + s^2 \gamma_2^2)) \Omega'' = \bar{\nu}_{\text{ion}} + (1 - \Omega')(\bar{\nu}_{\text{ex}} + \gamma_2)
\]

(14)

where \( \gamma_2 = (\nu \pi/2)^{1/2} z_0 g \) describes the wave excitation by the ion–electron collisions [4]. These eigenmodes correspond to eigenstates that are quasi-bounded by local potential wells induced by strong toroidal coupling. Damping due to tunnelling leakage is negligible here. We see from Eqs (14) that both ionization and charge exchange lead to an enhancement of the wave growth rate. Finally, we solve Eq. (7) numerically in order to test the analytically obtained dependence of \( \Omega'' \) on \( \bar{\nu}_{\text{ion}} \) in the case of \( \epsilon_n = 0.05; k^2 = 0.01; s = 0.5; q = 1, \bar{\nu} = 0.01 \) and \( \bar{\nu}_{\text{ex}} = 0 \). Figures 1 and 2 plot \( P \) versus \( y \) and \( \Omega'' \) versus \( \bar{\nu}_{\text{ion}} \) for \( \Omega'' = 0.932 \), respectively. The structure of the real (a) and imaginary (b) parts of \( \Phi \) for \( \bar{\nu}_{\text{ion}} = 0.01 \) is shown in Fig. 3.
FIG. 1. $-P'$ as a function of $y$.

FIG. 2. Dependence of the growth rate on $\bar{v}_{\text{ion}}$.

FIG. 3. Real (a) and imaginary (b) part of the wave potential.
4. CONCLUSIONS

In this paper we have investigated the effect of ionization, charge exchange and ion viscosity on the stability of dissipative drift modes with $\omega \leq \omega_i$. It is shown that the ionization process plays the dominant role, i.e. it can destabilize the weakly coupled modes and significantly increase the growth rate of strongly toroidally coupled modes. A numerical treatment of Eq. (7) confirms the linear dependence of $\Omega''$ on $\bar{\nu}_{ion}$ and the wave localization in the case of strong toroidal coupling.

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REFERENCES

MARFES, RADIATIVE CONDENSATION AND BALLOONING INSTABILITIES IN TOKAMAKS

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Abstract

MARFES, RADIATIVE CONDENSATION AND BALLOONING INSTABILITIES IN TOKAMAKS.

An improved linear theory of radiative condensation instabilities and a model nonlinear theory for MARFES are presented. The effect of sheared toroidal flow on the MHD equilibrium and ballooning instabilities is also investigated.

1 Introduction

In this paper we have examined two aspects of the equilibrium and stability properties of tokamaks. In Sec. 2 we investigate the gross features of a two-dimensional axisymmetric thermal equilibrium at the tokamak edge determined by an energy balance between thermal conduction from the core and the local impurity radiation losses. We present a model nonlinear calculation which analytically describes the transition from the poloidally asymmetric MARFE state to the symmetric 'Detached Plasma' state (DP). We have also re-examined linear theories of radiative condensation instabilities in the tokamak edge region retaining noncoronal radiation effects and poloidally asymmetric perpendicular transport coefficients and find significant modifications of the growth rates, eigenmode structure etc. In Sec. 3 we consider the ideal MHD equilibrium and ballooning stability of a plasma with sheared toroidal rotation speed. Using the generalised MHD equilibrium equations for axisymmetric rotation we have carried out a detailed numerical investigation of the form of magnetic surfaces and other equilibrium quantities as a function of the velocity shear parameter. Analytic results are presented in certain simple limits. Turning to the ballooning stability of a plasma with sheared flows we find it more appropriate to solve an initial value problem. Using a modified eikonal ansatz we study the solutions of the perturbed ideal MHD equations and investigate the effect of velocity shear on average growth rate. The solutions display significant stabilization of the ballooning instabilities.
2 MARFE-DP Transition and Radiative Condensation Instabilities

The observation of MARFE and Detached Plasmas as one approaches the density limit in Tokamaks is a well documented phenomenon now [1]. During a MARFE-DP transition (obtained typically by increasing the density or decreasing the current) the total power lost by impurity radiation at the edge approaches the input power to the core, there is less and less heat flux to the limiter and the radiative layer becomes progressively more symmetric in the poloidal direction. There have been attempts to understand MARFES in terms of radiative-condensation instabilities of the edge plasma [2]. Physically, let us imagine a temperature fluctuation which locally cools the plasma in a poloidally asymmetric way in the edge region. Radiation from the locally colder region is enhanced because (a) one is typically in a region where \( \frac{dL}{dT} < 0 \), \( L \) being the radiative loss rate and (b) density \( n \) increases locally to maintain pressure balance along the lines increasing \( L \) as \( L \propto n^2 \).

This cools the local region further and an instability develops; parallel and perpendicular conduction of heat opposes the instability and defines the parameter space of its operation. It is recognized [3] that to understand the final form of the poloidally asymmetric MARFE and its possible relation to the DP state, one must carry out a nonlinear investigation of the saturated state of the above instability. We present such a nonlinear analytic theory here and construct an axisymmetric two-dimensional thermal equilibrium in the radiative layer. We consider the steady state heat balance equation in the edge region which in the slab approximation can be written as

\[
\frac{\partial}{\partial x} f_{\perp}(T) \frac{\partial T}{\partial x} + \frac{\partial}{\partial y} f_{\parallel}(T) \frac{\partial T}{\partial y} = \left[ \frac{\rho}{T} \right]^2 \tilde{g}(T)
\]

(1)

where the functions \( f_{\perp} \) and \( f_{\parallel} \) determine the temperature dependences of the perpendicular and parallel thermal conductivity coefficients. The function \( g = \frac{L(T)}{n^2} \) where \( L(T) \) is the radiation loss term. Explicit \( T \) dependence on the rhs is due to use of pressure balance \( (nT = p) \) in the expression for \( L \). Since the scrape-off layer width (scaling parameter for \( x \)) is typically very small compared to \( a \) the slab approximation is justified. All quantities are normalized to a characteristic temperature \( T_L \) which we take as the temperature where \( L = L_{max} \). Note that \( \rho^2 \) is a crucial physical parameter giving the ratio of maximum radiative power \( L(T_L) \) to \( P_{cond} \), the power conducted to the cold spot by parallel thermal conduction. In the steady state \( P_{cond} \approx P_{in} \), the input power to the core;
therefore $\rho^2 \approx P_{rad}/P_{in}$. This is the critical experimental parameter used in the explanation of MARFE-DP transition [1] and is limited to values $\leq 1$ since no thermal equilibrium would be possible if the radiated power $L(T_L)$ exceeded the input power. An analytic solution for Eqn. (1) can be obtained in the limit where $f_{\perp} = f_{||} = 1$ and modelling $\hat{g}(T)$ ($g$ normalized by its value at $T = 1$) by the expression, $\hat{g}(T) = T^2 \exp[-2(T - 1)]$ which gives a maximum for $\hat{g}(T)$ at $T = 1$ in accordance with the well known coronal radiation curves. Writing $\psi = T - 1$ and $\epsilon^2 = 1 - \rho^2$, Eqn. (1) may be put in the form:

$$\nabla^2 \psi = (1 - \epsilon^2) \exp(-2\psi)$$

(2)

which has well known island solutions $\psi = \ln[\cosh(x + \delta) + \epsilon \cos(y)]$. The minimum temperature $\psi_{\text{min}}$ is at $y = \pi$, i.e., the coldest spot is chosen at $\theta = \pi$. The parameter $\delta$ determines the amount of heat flux leaving the edge plasma at $x = 0$ (which may be interpreted as the cold or 'limiter' boundary). We fix $\delta$ by the physically interesting condition that $\psi = 0$ at the coldest spot $\theta = \pi$; this gives $\delta = \cosh^{-1}(1+\epsilon)$. As the density increases, $L(T_L)$ increases and $\epsilon^2$ decreases. With the decrease of $\epsilon$, the width of the radiative scrape-off diminishes, the temperature fluctuation diminishes and $L_{\text{max}}/L_{\text{min}}$ approaches unity, $\delta$ and hence the heat flux to the limiter diminishes and the plasma approaches a DP state. In Fig.1 we display how the poloidal distribution of radiative emissivity becomes more symmetric as $\epsilon$ is varied from 0.6 to 0.0. These parameters do not correspond to any particular experiment; however, there is a striking similarity between Fig.1 and the zone reconstruction from crossed bolometer arrays in TFTR when the gas feed is on (Art. 4 in [1]). The physical reason for the symmetrization process is clear. As the radiative capacity of the whole region goes up, less heat needs to be transported into the colder region. Consequently, the parallel gradients in $T$ weaken, leading to a poloidally elongated cold region. At $\epsilon^2 = 0$, all the input power is lost by radiation ($\delta = 0$, and hence heat flux to the limiter vanishes) and a poloidally symmetric equilibrium is obtained. This marks the beginning of the Detached Plasma phase.

To investigate the effect of retaining the transport nonlinearities we have next taken $f_{\perp} = T^{1/2}, f_{||} = T^{5/2}$ and also retained the exact coronal radiation function to solve Eqn. (1) for a range of $\epsilon = 0.0$ to 0.6. The choice of $f_{||}$ and $f_{\perp}$ is motivated by the fact that perpendicular heat conduction is dominated by anomalous effects and depends rather weakly [4] on $T$ whereas the classical parallel conduction follows the usual $5/2$ power dependence. A comparison of the two solutions indicates that the qualitative features are not changed by the inclusion of the above physical effects.
We have re-examined several aspects of the linear theories of short and long wavelength radiative condensation instabilities believed to be responsible for MARFES and other edge fluctuations in tokamaks. Firstly, we have looked at the influence of non-coronal radiation effects on the linear theories; we find that the growth rates are severely modified if $\gamma \geq \nu_i$, the ionization collision frequency. Secondly, we have taken account of the effect of density and temperature gradients (i.e. drift-like terms) and the field line curvature on the properties of the linear modes. Noting that the typical mode width is larger than the distance between neighbouring mode rational surfaces, we introduce a ballooning formalism for the study of the instabilities. In contrast to earlier work [2], we have also considered the influence of anomalous electron perpendicular thermal conductivity on the modes. The final dispersion relations show that for $\omega \ll \frac{m}{M} \nu_{ei}$ the curvature effects are negligible and the marginally stable modes are restricted to mode numbers $\leq 150$ in agreement with experiments. For $\omega \gg \frac{m}{M} \nu_{ei}$, the curvature effects are found to be important and lead to a stabilizing effect on the stability. Lastly, we have also considered the effect of a poloidal asymmetry in the electron thermal conductivity $\chi_{le} = \chi_{le}^0 (1 + \Delta \cos \theta)$ on the mode localization of short wavelength condensation instabilities. We
find that the mode localization on the inside can be understood in terms of a weaker $\chi_\perp$ on the inside which may arise due to stronger $B$ and/or weaker fluctuation induced transport mechanisms in the strong $B$ field regions.

3 Ballooning Instabilities

In recent years several tokamak experiments have reported large toroidal plasma rotations induced by neutral beam injection during auxiliary heating. The measured rotation profiles [5] indicate that velocity shear can be significant. We examine the effect of such sheared flows on the equilibrium and the stability of tokamak plasmas. The principal physical effects of plasma flows on the equilibrium of an axisymmetric toroidal plasma are an outward shift of the magnetic axis, relative shift of the pressure surfaces with respect to the magnetic surfaces and a distortion of the magnetic surfaces (e.g. elliptic elongation, triangularity etc). The flow parameter can be characterized by the quantity $\Theta(\psi) = \omega^2/T$ where $\omega$ is the toroidal rotation frequency, $T = T(\psi)$ is the temperature and $\psi$ is the poloidal flux function. Most analytic and numerical investigations in the past have been carried out for $\omega^2/T = $ constant. We consider a more generalized form

$$\frac{R_0^2 \omega^2}{\bar{R} T} = \Omega^2 + \frac{\lambda \psi}{\psi_{\text{ref}}}$$

(3)

where $\bar{R}$ is the gas constant, $R_0$ is the major radius and the constants $\Omega$ and $\lambda$ are respectively the rigid flow and velocity shear parameters. The generalized Grad-Shafranov equation which includes axisymmetric flow is given in cylindrical coordinates $(R,\phi,Z)$ as [6],

$$\Delta^* \psi + I \frac{dI}{d\psi} + R^2 \left[ \frac{dp}{d\psi} + p R^2 \frac{d}{d\psi} \left( \frac{\omega^2}{2\bar{R} T} \right) \right] \exp \left( \frac{R^2 \omega^2}{2\bar{R} T} \right) = 0$$

(4)

where $\Delta^* = \partial^2/\partial R^2 - (1/R) \partial/\partial R + \partial^2/\partial Z^2$ and $p = p_T \exp(-R^2 \omega^2/2\bar{R} T)$, and $I$ are arbitrary functions of $\psi$. We numerically solve Eqn. (4) for the general profiles $p = p_0 \psi_{n_1}$, $I = R_0 B_0(1 - \gamma \psi_{n_2})$ where $B_0$ is the externally imposed vacuum toroidal field at $R = R_0$, $\psi_n = (\psi - \psi_\ast)/(\psi_\ast - \psi_\ast)$ is the normalized $\psi$, $\psi_\ast$ is the flux on the magnetic axis, $\psi_\ast$ is the flux on the plasma surface and $p_0, \alpha_1, \alpha_2$ are input parameters. $\gamma$ is adjusted in each iteration to keep the total toroidal current $I_0$ constant. Care is taken to maintain a constant thermal $\beta$ while varying the rotational contribution.
We find that rotation can induce a significant shift as well as elongate the flux surface, particularly in the high $\beta$ limit. In this limit we have also investigated the flux conserving tokamak equilibrium. For this we have adopted the variational formulation of [7]. In the high $\beta$ ordering ($\beta \sim \epsilon, \beta_p \sim 1/\epsilon, q \sim 1$) the Grad-Shafranov equation reduces to

$$\nabla^2 \psi + \Gamma'(\psi) + \beta'(\psi) r \cos \theta = 0$$  \hspace{1cm} (5)

where $\Gamma(\psi)$ and $\beta(\psi)$ are free functions and the latter incorporates the effect of toroidal flow. Following [7] we reduce Eqn. (4) to a set of coupled second order ordinary nonlinear differential equations for $\sigma$ and $\kappa$ — the shift and the elliptic elongation factors. These equations are solved subject to the boundary conditions $\kappa = 1, \sigma = 0$ at $r = a$ and $\kappa' = \sigma' = 0$ at $r = 0$ and using a shooting method. Fig. 2 shows examples of $\sigma', \kappa'$ as a function of $\lambda$ for the specific profiles given in [7]. The quantities $\kappa', \sigma'$ play a direct role in the ballooning mode stability analysis and their profiles can be used to obtain global stability boundaries for these modes [7,8]. It is interesting to note that $\kappa'$ has a maximum and decreases in the outer flux surfaces. To supplement these numerical results we have also carried out an analytic solution of the generalized Grad-Shafranov equation for the simple special
case $\beta'(\psi) = C$ and $\Gamma'(\psi) = A$, where $A$ and $C$ are constants. The solution for $\psi$ is

$$\psi = (Aa^2/4) \left[(1 - \rho^2) + \nu(1 - \rho^2)\rho \cos \theta\right]$$

(6)

where $\rho = r/a$, $\nu = Ca/2A$.

The shift of the magnetic axis from the geometric centre $A$ and the elongation of the flux surfaces in the neighbourhood of the magnetic axis $\kappa$ are

$$\frac{\Delta}{a} = \frac{\nu}{1 + \sqrt{1 + 3\nu^2}}, \quad \kappa = \left(\frac{1 + 3\nu^2 + \sqrt{1 + 3\nu^2}}{1 + \nu^2 + \sqrt{1 + 3\nu^2}}\right)^{1/2}$$

(7)

Choosing $\psi_{ref} = I_0 R_0$ the parameter $\nu$ can be shown to be given, correct to first order in $\lambda$, by

$$\nu = \epsilon\beta_p \left\{ \left(1 + \frac{\Omega^2}{2}\right) + \frac{\lambda}{12\pi} \left[1 + \frac{e^2\beta^2}{8} \left(1 + \frac{\Omega^2}{2}\right)^2\right]\right\}$$

(8)

The equilibrium limit on $\beta_p$ is given by $\nu = 1$ as in the static case. In this limit we have $\Delta/a = 1/3$, $\kappa = \sqrt{3}/2$.

We now go on to investigate the influence of shear in the toroidal rotation velocity on the ballooning instability. For simplicity in this calculation we ignore the modification of equilibrium introduced by the flow and use the standard shifted circle model of magnetic surfaces. Early work had suggested [9] that in the presence of velocity shear, ballooning instabilities as Weyl sequences do not exist and that the usual WKB formalism breaks down. However, physical arguments suggest that velocity shear should lead to a significant stabilization of the ballooning modes. Several authors have continued [10–12] an investigation of the instabilities. Hameiri and Chun [12] and Cooper [10] in particular set up a modified ballooning formalism in which the eikonal $S$ appearing in the perturbation expression

$$\tilde{\xi}(\psi, \theta, \phi, t) = \xi(\psi, \theta, t) \exp[\imath n S(\psi, \theta, \phi, t)] \text{ (where } \psi, \theta, \phi \text{ are the usual magnetic flux coordinates and } n \gg 1 \text{ is the toroidal mode number)}$$

was assumed to satisfy the two conditions:

$$\frac{dS}{dt} = \frac{\partial S}{\partial t} + (V_0 \cdot \nabla)S = 0; \quad (B_0 \cdot \nabla)S = 0$$

(9)

This produced the desirable result of separating the lowest order equation (in $1/\sqrt{n}$) for $\xi$, as usual, into an equation involving $\theta$-derivatives only. The $\psi$ dependence of $\xi$ is then determined by the next order in the eikonal approximation. However, unlike the usual ballooning problem, the lowest order equation now contains time dependent coefficients and therefore does
not lead to any standard eigenmodes. Cooper [10] therefore resorted to a numerical solution of the resulting partial differential equations and demonstrated that periodic bursts of oscillations arise with an average growth rate much smaller than ballooning modes in static plasmas and with a spatial structure in which the peak in ballooning space is localized at different \( \theta \) at different times. These solutions are best interpreted as amplification of ballooning wave packets in an initial value type of problem. We now provide an analytic description of the above numerical results. We start with the linearised ideal MHD equations for an incompressible fluid and construct an eikonal \( S \) satisfying Eqn. (9) in the form \( S = \phi - q(\psi)\theta + k(\psi) - \Omega(\psi)t \) where \( q, k \) and \( \Omega \) correspond respectively to the tokamak safety factor, radial wave number and the toroidal rotation frequency for the flux surface with label \( \psi \). We derive evolution equations for \( \rho, p, B \) and \( V \times ( \) where the last two terms correspond to \( B_0 \times \nabla S \) components of \( B_1 \) and \( V_1 \). Introducing the displacement variable \( \xi \) by the equation \( \partial \xi / \partial t = V_L \) and eliminating all other dependent variables we get the final equation:

\[
\rho_0 \frac{\partial}{\partial t} \left[ |\nabla S|^2 \frac{\partial \xi}{\partial t} \right] - (B_0 \cdot \nabla) \left[ |\nabla S|^2 (B_0 \cdot \nabla) \xi \right] - (B_0 \times \nabla S) \cdot (V_0 \cdot \nabla) V_0 \frac{B_0 \times \nabla S}{B_0^2} \cdot \nabla \rho \xi - 2 \frac{B_0 \times \nabla S}{B_0^2} \cdot \kappa (B_0 \times \nabla S) \cdot \nabla \rho_0 \xi = 0
\]

(10)

where for the simple shifted circle magnetic surfaces model the flux surface label \( \psi \) is replaced by the minor radius variable \( r \) and \( \nabla S = -\hat{e}_r (q' \theta + \Omega' t - \hat{e}_\psi q(r)/r + \hat{e}_\phi /R \). The second term on the right side of Eqn. (10) is the usual Alfvén wave term, the third term gives the contribution from centrifugal force and the last term is the curvature term which drives the ballooning instability. The main departure from the standard problem arises because \( \nabla S \) is a function of \( t \). Eqn. (10) can be reduced to the following form

\[
\phi_{rr} + 2 \epsilon h(\tau) \phi_{\eta \eta} = \phi_{\eta \eta} + [a(\tau) - c(\tau) \eta^2] \phi
\]

(11)

where \( \eta = \theta + \Omega' t / q' \) and \( \tau = t \) are new independent variables and \( \phi \) is a new dependent variable. In deriving Eqn. (10) we have treated \( \epsilon \equiv \Omega' / q' \), the velocity shear parameter as a small parameter and considered the strong shear limit where the ballooning modes are strongly localised such that the \( \eta \) dependent expressions can be expanded up to order \( \eta^2 \). Eqn. (10) may be solved using a WKB ansatz

\[
\phi = A(\eta, \tau) \exp \left[ -i \int \omega d\tau - \sqrt{c} \eta^2 / 2 \right]
\]

(12)
where we use the zeroth order Hermite function with slow (of order $\epsilon$) modulations determined by $A(\eta, \tau)$ as our basic solution. The basic novelty of the above ansatz lies in seeking the solution to Eqn. (10) with time-dependent coefficients in terms of a slowly modulated Hermite function rather than in terms of an expansion in terms of a complete set of basis functions. Such a modulated Hermite function can be regarded as a partial sum of many terms and is the proper quasi-mode in terms of which the evolution of the ballooning instability may be understood. The slow partial differential equation for $A(\tau, \eta)$ may be exactly solved to finally give

$$\phi(\tau, \eta) = \sqrt{\frac{\omega(0)}{\omega(\tau)}} \exp \left[ -i \int \omega d\tau - \left( \frac{\sqrt{c}}{2} - R(\tau) \right) (\eta - g)^2 + (\eta - g)Q(\tau) \right] \tag{13}$$

where

$$\omega^2(\tau) = \sqrt{c(\tau)} - a(\tau) \tag{14}$$

$$Q(\tau) = \int_0^\tau dt' (\sqrt{c})eh(t') \exp \left[ i \int_\tau^{t'} \frac{\sqrt{c}}{\omega} dt'' \right] \tag{15}$$

$$R(\tau) = -\frac{1}{2} \int_0^\tau dt' (\sqrt{c}) \exp \left[ 2i \int_\tau^{t'} \frac{\sqrt{c}}{\omega} dt'' \right] \tag{16}$$

For time scales of the order of $\pi/\epsilon$ (for which our WKB model is valid) the solution displays strong damping effects due to shear and a shift in the peak of the wave packet as a function of time.

References


THEORETICAL INTERPRETATION OF EDGE FLUCTUATION EXPERIMENTS IN ALCATOR

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Abstract

THEORETICAL INTERPRETATION OF EDGE FLUCTUATION EXPERIMENTS IN ALCATOR.

The Ohmic heating effect is included in the theoretical model for tokamak plasma edge turbulence. The mechanism of Ohmic dissipation, which decreases the saturated fluctuation levels to a substantial extent, is discussed. The fluctuation levels at saturation are calculated. The theoretical results are in good agreement with the experiments on Alcator.

1. INTRODUCTION

Recently, resistivity gradient driven turbulence (RGDT) theory has become very successful in explaining tokamak edge fluctuation and transport characteristics. The linear antecedent of this turbulence is the rippling mode on which the theoretical model for RGDT is based. By including the impurity radiation effect in the model, the thermally driven convective cell turbulence theory was developed [1]. The predictions in Ref. [1] are, to a certain extent, consistent with several experiments.

For high density machines such as Alcator, however, these theoretical results cannot be in agreement with experiments. The tokamak edge turbulence experiments show that the saturated levels of electrostatic potential fluctuation (e$\Phi$/Te) and density fluctuation (\bar{n}/n_0) are lower than, or approximately equal to, 1.0 [2], but the available theoretical values are an order of magnitude higher than the experimental ones. To interpret the experiments, the Ohmic heating effect is taken into account in our theory as a new dissipative effect, which lowers the saturated turbulent levels considerably. This is exactly the starting point of our paper.

2. BASIC THEORETICAL MODEL

As is well known, in the immediate vicinity of the mode rational surface, we have \chi T_k x^2 \tilde{\eta}_{pk} \approx 0. Then it is only in regions relatively far away from the rational surface that the parallel thermal conduction effectively dissipates the fluctuation energy. The effect of radial turbulent diffusion broadens the resistivity fluctuation...
structure and enhances parallel thermal conduction. In particular, the broadened fluctuation structure strengthens the interaction between resistivity fluctuation and parallel current, i.e. the Ohmic heating effects of the fluctuations will effectively dissipate the fluctuation energy and, hence, substantially decrease the saturated levels.

The reduced resistive MHD equations are given by

\[ \rho d \nabla \phi / dt = B_{s0} \nabla I_z \]  

(1)

and

\[ d\psi / dt = -B_{s0} \partial \phi / \partial z - \eta I_z \]  

(2)

Here, \( \psi \) is the poloidal flux function, \( \phi \) the fluid stream function (\( \phi = \Phi / B_{s0} \)), where \( \Phi \) is the electrostatic potential, \( J_z \) the parallel current density (\( J_z = -(1 / \mu_0) \nabla \phi \)), and \( \rho \) the mass density. The total convective derivative is given by \( d/dt = \partial / \partial t - \nabla \cdot (V \times e_z) \cdot \nabla \), with \( \nabla \nabla = \nabla - e_z \partial / \partial z \); \( \nabla_1 = [(1 / B_{s0}) \nabla \phi \times e_z + e_z] \cdot \nabla \).

The temperature and impurity equations are, respectively,

\[ (3/2) n \dot{T} / dt = \kappa T \nabla^2 T - n n_z I_z (T) + \eta I^2 + H \]  

(3)

and

\[ dZ_{\text{eff}} / dt = \chi Z_{\text{eff}} \]  

(4)

Here, \( \eta I^2 \) is the Ohmic heating term, \( H \) represents the auxiliary power source, \( \eta = Z_{\text{eff}} \eta_{sp} \), where the Spitzer resistivity \( \eta_{sp} \propto T^{-3/2} \) and \( Z_{\text{eff}} \) is the effective ion charge number, \( T \) the electron temperature, \( n \) the electron density, \( n_z \) the impurity density, \( \kappa_T \) the parallel thermal conductivity, \( I_z \) the impurity radiation rate, and \( \chi_z \) the parallel impurity conductivity.

For the rippling mode problems we can adopt the electrostatic approximation (i.e. \( \delta \delta A / \delta t \) can be omitted in \( \delta \delta E = - \nabla \Phi - \delta \delta A / \delta t \) compared with \( \nabla \Phi \)) and make use of the Spitzer relation and the effective ion charge relation for a single impurity species with charge state \( Z \), i.e. \( Z_{\text{eff}} = 1 + Z^2 n_z / n_0 \). Then, we obtain the following fluctuation equations from Eqs (1)-(4):

**Parallel Ohm’s law:**

\[ -B_{s0} \nabla \phi \dot{\phi} = \eta_0 \dot{J}_z + J_{s0} \dot{n} \]  

(5)

**Parallel vorticity equation:**

\[ \rho d \nabla \phi / dt = B_{s0} \nabla \phi \dot{I}_z \]  

(6)
Spitzer resistivity evolution equation:
\[
\frac{d\rho}{dt} - \chi_T \nabla^2 \bar{\rho} - \gamma_R - \gamma_z \bar{\eta}_{sp} = \frac{d}{dr} \left( \frac{\partial \rho}{\partial \theta} \right) + \left( \gamma_Z + \gamma_J \right) \left( \eta_{sp0}/Z_{eff0} \right) \bar{Z}_{eff} + 2\gamma_J \left( \eta_{sp0}/J_{z0} \right) \bar{I}_z
\]  
(7)

Impurity equation:
\[
d\bar{Z}_{eff}/dt = (d\bar{Z}_{eff0}/dr) \frac{\partial \bar{\rho}}{\partial \theta} + \chi_z \nabla^2 \bar{Z}_{eff}
\]  
(8)

Here, \( \nabla_\perp = \nabla_{z0} = (1/B_{z0}) (\nabla_\perp \Psi_0 \times e_\perp) \cdot \nabla_\perp + \partial/\partial z \). The quantity \( \chi_T = 2\chi_T/3n_0 \) is the normalized parallel thermal conductivity, the radiation growth rates are \( \gamma_R = (2/3)n_0 \left[ I_s(T_0)/T_0 - dI_s(T_0)/dT_0 \right] \) and \( \gamma_z = Z_{eff0} \left( \eta_0/Z^2 \right) I_s(T_0)/T_0 \); \( \gamma_J = -Z_{eff0} \eta_{sp0}/n_0 T_0 \) is the Ohmic heating dissipation parameter peculiar to our theory. The fluctuation resistivity relation is given by \( \bar{\eta} = Z_{eff0} \eta_{sp} + \eta_{sp0} \bar{Z}_{eff} \), where the subscript 0 denotes the unperturbed and the tilde the perturbed quantities.

In Eq. (7), the three terms with the parameter \( \gamma_J \) take the place of the Ohmic heating term in Eq. (3). For Alcator parameters, we have \( \gamma_J = -4.8 \times 10^5 \, s^{-1} \), \( \gamma_R = 1.5 \times 10^4 \, s^{-1} \), and \( \gamma_z = 1.8 \times 10^4 \, s^{-1} \), which shows that in Eq. (7) \( \gamma_J \) is completely contrary in effect to \( \gamma_R \) and \( \gamma_z \), and its magnitude is much greater than those of \( \gamma_R \) and \( \gamma_z \). In other words, the Ohmic heating plays the role of dissipation with the fluctuations and, relative to the radiation cooling driven effects, a very important role in the problem of edge fluctuations in the tokamaks. Thus, introducing the Ohmic heating effect into edge turbulence theory may be reasonable and necessary.

The set of coupled equations (5)-(8) forms a complete system for potential (\( \phi \)), resistivity (\( \eta_{sp} \)), parallel current (\( J_z \)) and impurity (\( Z_{eff} \)) fluctuations and constitutes our theoretical model.

3. NON-LINEAR ANALYSIS OF TURBULENCE AND DERIVATION OF SATURATION RELATIONS

In the saturated turbulent state, the current fluctuation \( \bar{I}_z \), which is centred about the mode rational surface, decouples from the other fluctuations (\( \phi, \eta_{sp} \) and \( \bar{Z}_{eff} \)), which are shifted away from the rational surface. Thus, we can assume \( \bar{I}_z = 0 \), except at the rational surface, and eliminate the vorticity equation (6). Using the direct interaction approximation (DIA) and the standard iterative method the temperature (7) and impurity (8) equations are renormalized, and the predominant non-linear effects in the random convection are expressed as radial turbulent diffusion effects (\( D_{Rk} \) and \( D_{Zk} \)). Using Eq. (5) with \( \bar{I}_z = 0 \) in order to eliminate \( \bar{\phi} \), we can write the renormalized versions of Eqs (7) and (8) at saturation (\( \partial/\partial t = 0 \)) as follows:
\[ x_{\parallel} \vec{k} \cdot \vec{x}^2 - \gamma_R - \gamma_I - D_{Tk} \partial^2 / \partial x^2 \frac{\eta_{spk}}{\eta_{sp0}} \]

\[ = (L_s \eta_0 / L_\omega B_{\omega 0}) \left( \eta_{spk} / \eta_{sp0} + \tilde{Z}_{eff,k} / Z_{eff,0} \right) + (\gamma_Z + \gamma_I) \tilde{Z}_{eff} / Z_{eff,0} \]  \hspace{1cm} (9)

and

\[ (x_{\parallel} \vec{k} \cdot \vec{x}^2 - D_{zk} \partial^2 / \partial x^2) \tilde{Z}_{eff,k} / Z_{eff,0} \]

\[ = (L_s \eta_0 / L_Z B_{\omega 0}) \left( \eta_{spk} / \eta_{sp0} + \tilde{Z}_{eff,k} / Z_{eff,0} \right) \]  \hspace{1cm} (10)

where \( L_s \) is the shear scale length, and the radial turbulent convective diffusion coefficients \( D_{Tk} \) (for the resistivity) and \( D_{zk} \) (for the impurity) are defined as

\[ D_{Tk} = \sum_{k'} k_{\parallel}^2 |\tilde{\phi}_{k'}|^2 \left[ \gamma_{k+k'} + \chi_T (k_k + k_i)^2 \right]^{-1} \]  \hspace{1cm} (11)

and

\[ D_{zk} = \sum_{k'} k_{\parallel}^2 |\tilde{\phi}_{k'}|^2 \left[ \gamma_{k+k'} + \chi_Z (k_k + k_i)^2 \right]^{-1} \]  \hspace{1cm} (12)

Here, we have written the parallel wavevector \( k_{\parallel} \) as \( k_{\parallel} = k_{\parallel} x \) where \( x = r - r_s \) and \( r_s \) is the mode rational surface position; the resistivity and impurity gradients are expressed as the resistivity and \( Z_{eff} \) scalelengths (\( L_\eta \) and \( L_Z \)), respectively, where \( L_\eta = (d \ln \eta_{sp0} / dr)^{-1} \) and \( L_Z = (d \ln Z_{eff,0} / dr)^{-1} \).

It is easy to see that in Eq. (9) the Ohmic heating effect (\( \gamma_T \)), together with the parallel thermal conduction (\( \chi_T \)), dissipates the fluctuations due to the resistivity gradient (\( L_\eta \)) and the radiation cooling (\( \gamma_R \) and \( \gamma_I \)) and the turbulent diffusion (\( D_{Tk} \)) enhances the dissipation effectively. The asymptotic balance of the turbulent radial diffusion with the parallel thermal and impurity conduction determines the characteristic radial scales for resistivity (\( \Delta_T \)) and impurity (\( \Delta_Z \)) perturbations, respectively. When these scales are adjusted, a saturated turbulent state will be reached. According to Eqs (9) and (10), the radial scales are expressed as

\[ \Delta_{Tk} = \left( D_{Tk} / \chi_T \vec{k}_{\parallel} \right)^{1/4} \]

and

\[ \Delta_{zk} = \left( D_{zk} / \chi_Z \vec{k}_{\parallel} \right)^{1/4} \]

By taking a Markovian approximation, Eqs (11) and (12) yield \( D_T = \Sigma_{k-k} (\chi_T \vec{k}_{\parallel} \Delta_T) \) and \( D_Z = \Sigma_{k-k} (\chi_Z \vec{k}_{\parallel} \Delta_Z) \), where \( \Sigma_{k-k} = \Sigma_{k-k} (\tilde{\phi}_{k})^2 \) is the mean square radial turbulent velocity. So we have \( \Delta_T = (V_T / \chi_T \vec{k}_{\parallel})^{1/3} \) and \( \Delta_Z = (V_Z / \chi_Z \vec{k}_{\parallel})^{1/3} \).
The condition for the non-trivial solution of Eqs (9) and (10) leads to an equation for $V_r$:

$$V_r/V_{r0} = 1 + \eta_z \left[ 1 + (\gamma_Z - \gamma_R + \gamma_f^Z - \gamma_f^R)(V_r/V_{r0})^{-2/3} \right.$$ 
$$\left. + (\gamma_R + \gamma_f^R)(V_r/V_{r0})^{1/3} \right]$$

(13)

Here, $V_{r0} = L_o \eta_0 n_0/L_x B_{z0}$, $\Gamma_R = \gamma_R/\gamma_R$ and $\Gamma_Z = \gamma_Z/\gamma_Z$, where $\gamma_R = V_{r0}^{2/3} (\chi_T \chi_2)^{1/3}$, $\gamma_Z = \gamma_R (\chi_T / \chi_Z^{1/3})$, $\Gamma_f^R = \gamma_f/\gamma_R$, $\Gamma_f^Z = \gamma_f/\gamma_Z$, and $\eta_z = L_n/L_Z$. The parameters $\Gamma_f^R$ and $\Gamma_f^Z$ represent the Ohmic heating dissipation special to the present study. This dissipative effect reduces the turbulent velocity and then affects the saturation levels.

By means of the expression for $V_r$ and Eq. (13), the potential fluctuation $\varepsilon/e$ at the saturated turbulent state can be written as

$$\frac{\varepsilon/e}{(\varepsilon/e)_0} = 1 + \eta_z \left[ 1 + (\Gamma_Z - \Gamma_R + \Gamma_f^Z - \Gamma_f^R) \left( \frac{\varepsilon/e}{(\varepsilon/e)_0} \right)^{-2/3} \right.$$ 
$$\left. + (\Gamma_R + \Gamma_f^R) \left( \frac{\varepsilon/e}{(\varepsilon/e)_0} \right)^{1/3} \right]$$

(14)

where $(\varepsilon/e)_0 = V_{r0}^2/m \rho_s C_s$, with $C_s = (T_e/m_i)^{1/2}$ and $\rho_s = C_s m_i/e B_{z0}$. From the particle continuity equation and Eq. (13), we obtain the saturated density fluctuation:

$$\frac{n/n_0}{(n/n_0)_0} = \left\{ 1 + \eta_z \left[ 1 + (\Gamma_Z - \Gamma_R + \Gamma_f^Z - \Gamma_f^R) \left( \frac{n/n_0}{(n/n_0)_0} \right)^{-2} \right. \right.$$ 
$$\left. + (\Gamma_R + \Gamma_f^R) \left( \frac{n/n_0}{(n/n_0)_0} \right)^{1/3} \right\}$$

(15)

where $(n/n_0)_0 = V_{r0}^{1/2}/L_n (\chi_2 \chi_3)^{1/2}$ and $L_n = (d \ln n_0/dr)^{-1}$.

From Eqs (14) and (15), we see that Ohmic dissipation ($\Gamma_f^R$ and $\Gamma_f^Z$) decreases the saturation levels of the potential and density fluctuations effectively.

4. COMPARISON WITH EXPERIMENTS AND CONCLUDING REMARKS

In terms of Eqs (14) and (15), the theoretical results can be compared with the experiments. The Alcator parameters are as follows: $a = 0.165$ m, $R = 0.64$ m, $T_0 = 15.0$ eV, $B_{z0} = 13.0$ T, $n_0 = 2.0 \times 10^{20}$ m$^{-3}$, $L_n = 0.05$ m, $r_s = 0.13$ m, $L_s = 0.69$ m, $\eta_0/\mu_0 = 17.0$ m$^2$/s, $\mu_0 J_{x0}/B_{z0} = 0.32$ m$^{-1}$, $\chi_T = 5.5 \times 10^4$ m$^2$/s,
FIG. 1. A scheme of $e\Phi/T_e$ versus $I_2/I_{20}$ for previous (dotted curve) and present (solid curve) models. The experimental value of $e\Phi/T_e$ on Alcator is lower than or approximately equal to 1.0 (dash and dotted line).

FIG. 2. $\bar{n}/n_0$ versus $I_2/I_{20}$ for previous (dotted curve) and present (solid curve) models. The experimental value of $\bar{n}/n_0$ on Alcator is less than 1.0.

FIG. 3. $e\Phi/T_e$ (solid curve) and $\bar{n}/n_0$ (dotted curve) versus $I_2/I_{20}$ for the present model.
$C = 3.8 \times 10^4 \text{ m/s}$, $\rho_s = 3.0 \times 10^{-3} \text{ m}$, $L_z/T_0 = 1.2 \times 10^{-15} \text{ m}^2/\text{s}$, $-dL_z/dT_0 = 8.0 \times 10^{-15} \text{ m}^3/\text{s}$, $L_q/L_z = 2.0$, $\eta_z = 0.5$, $\chi_z/\chi_T = 0.020$, $n_{Zo}/n_0 = 0.013$, $Z = 4.0$, $Z_{eff} = 1.2$, $m = 22.0$, $n = 7.0$. We use these parameters to calculate the potential and density fluctuations. The results for the previous [1] and present models are shown as functions of the normalized impurity radiation rate in Figs 1 and 2. Since Ohmic dissipation decreases the linear growth rates strikingly, the nonlinear saturation time is prolonged. During the prolonged period, the strong nonlinear interactions make the Ohmic effect and the other dissipative effects more effectively dissipate the fluctuation energy so that the theoretical values of this paper are much lower than the previous values [1] and are in good agreement with the experimental results [2]. In Fig. 3, we plot the potential and density saturation levels in our theory as functions of the normalized impurity radiation rate. We find that the two fluctuation levels at saturation are different from each other. This is consistent with the current opinion.

Thus, in conclusion, our theoretical model indicates that the instability driven by resistivity and impurity gradients and by impurity radiation cooling is balanced by Ohmic dissipation as well as by turbulently enhanced parallel thermal and impurity conduction. It is the Ohmic dissipation that reduces the saturation levels further and makes the theoretical values close to the experimental results on Alcator. Recently, more edge turbulence theories [3, 4], based on the theory of Ref. [1] or other models, have been developed, but none of them can explain the experiments on Alcator. The theoretical results of this paper are in better agreement with the experiments, thus indicating that the Ohmic heating effect may play an important role in the evolution of edge turbulence in tokamaks, especially in high density devices like Alcator.

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PREDICTIVE MODELLING OF TOKAMAK PLASMAS

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Abstract

PREDICTIVE MODELLING OF TOKAMAK PLASMAS.

Predictive simulations of tokamak plasma parameter profiles have been compared with experimental data using a rapid transport equilibrium calculation. Full time dependent calculations with time dependent boundary conditions calibrated against H-mode discharges have also been used. Many discharges have been adequately simulated with combinations of drift wave and other theoretical transport fluxes, but additional transport mechanisms still need to be included under some conditions. Better analyses are also being developed for long mean free path effects from wall generated neutrals and boundary plasma transport, helium ash transport, toroidally rotating MHD equilibria, and statistical methods for comparing simulations with experimental data.

1 Introduction

In this paper, we describe comparisons of predictions of two theoretical flux-surface-averaged plasma transport model formulations with experimental results. It is evident that substantial progress has been made in obtaining theoretical transport models which predict observed plasma energy confinement and radial profiles. However, some remaining discrepancies between predictions and experimental results point out the need for refinements in the theories tested, as well as in the statistical methodology used for comparing theory with experiment. This becomes particularly evident when one of the models is applied to prediction of data from H-mode temperature and density profiles in four different divertor tokamaks.
H-mode modelling studies also point out that a more complete theory of scrape-off plasma transport is needed for more reliable predictive modelling. A major complication here is that the mean free paths of both neutral and ionized particles always become comparable to the local scale lengths of plasma parameters somewhere near material boundaries.

An improved model of plasma-surface interactions has, therefore, been added to a Monte Carlo neutral transport model to propagate information away from complex structures at the material boundary and into the plasma. Hydrogen recycling, helium pumping, and erosion of walls, limiters, and divertor plates have all been simulated with predictive models. In all of these simulations, an accurate treatment of low energy (1–500 eV) ion reflection and sputtering coefficients is essential. By treating the surfaces as having atomic scale fractal dimension (a quantity measurable by gas adsorption techniques) behavior in agreement with experiments has been found [1]. These coefficients have been applied to neutral Monte Carlo codes, including DEGAS [2], to predict hydrogen isotope recycling and helium removal [3]. In combination with the edge impurity transport code WBC [4], erosion of plasma facing components has been predicted for a proposed engineering test reactor [5].

We have also obtained a set of five independent linear integro-differential ion Fokker-Planck equations linearized to second order in the ratio of the Coulomb mean free path to the scale length for variation along a magnetic field. These equations are presently being solved with the inclusion of a general heating and fuelling source needed to support sonic outflow along magnetic field lines to a material boundary.

Finally, in preparation for application of these various more sophisticated models to reactor design, studies of helium ash buildup in an engineering test reactor have been done. BALDUR code [6] simulations have been run to study sensitivity to various ash transport models as well as the effects of edge recycling, sawtooth disruptions, and operation near a soft beta limit on ash buildup. Thus, for example, with empirical radially dependent thermal diffusivities scaled from the results of an analysis of JET experimental data [7], but assuming $D_{H}/D_{DT} \sim 0.5$, a case was found where ash buildup prevented ignition in an International Thermonuclear Experimental Reactor (ITER) type plasma. However, for $D_{H}/D_{DT} \sim 1.0$, ignition occurred. Active control of ash buildup appears desirable in such cases, and methods are under study to include controlled fishbone pumping and third harmonic ICRF heating. In preparation for study of toroidally rotating plasmas, a rapid Lagrangian minimization method for solving MHD equilibria for rapidly rotating plasmas has also been developed [8].

2 Transport Equilibria

Drift wave turbulence models have recently been incorporated into tokamak transport codes in order to make detailed comparisons between theory and experiment in ohmic discharges as well as beam-heated L-mode discharges [9].
The standard drive wave model employs a combination of trapped electron, circulating electron, and ion temperature gradient (ITG) mode turbulence to model anomalous transport in the bulk plasma. The modeling efforts have been fairly successful in describing neo-Alcator confinement time scaling of low density ohmic discharges, high density saturation of confinement time in ohmic discharges, and the scaling of L-mode confinement time with respect to power, density, size, and magnetic field. The principal deficiency of the drift wave model is its failure to account for the strong plasma current scaling of confinement time in beam-heated tokamaks (both L- and H-mode).

For investigating this problem, a radial "shooting" code has been developed at General Atomics to numerically solve the coupled steady-state power balance equations for both electrons and ions and avoid numerical instabilities often encountered in evolving time dependent transport codes [10] to transport equilibrium. The equations solved in this shooting code are

\[-r_n \chi_e \partial T_e / \partial r + r_n \chi_i \partial T_i / \partial r = A P_e(r) \]

where \( r_n \) is the electron (ion) density, \( A \) is the surface area, and \( P_e(r) \) is the conduction power in the electron (ion) channel flowing through the surface at radius \( r \).

Components of the anomalous thermal conductivities have now been refined. In the electron channel, \( \chi_e \) includes the effects of trapped electron, circulating electron collisional drift wave, and semi-collisional high-\( m \) tearing mode turbulence. The trapped electron loss mechanism includes complicated trapped electron velocity integrals [11]. Previously, this effect was modeled using the asymptotic result for \( \nu_{eff} \gg \omega_e^* \). (Here \( \nu_{eff} \) is the effective trapped electron collisional frequency and \( \omega_e^* \) is the electron diamagnetic drift frequency). This gives the strong electron temperature scaling \( \chi_e \propto T_e^\beta \) with \( \beta = 3.5 \). However, this scaling only obtains for \( \nu_{eff}/\omega_e^* \geq 50 \), which is rarely found in present tokamaks. More typically \( 1.5 \leq \beta \leq 2.5 \) for the usual tokamak operating range of \( \nu_{eff}/\omega_e^*, 1 \leq \nu_{eff}/\omega_e^* \leq 10 \). Plots of the temperature index, \( \beta \), vs. \( \log(\nu_{eff}/\omega_e^*) \) for toroidicity parameters in the range \( 0.1 \leq g \leq 0.4 \) show that \( \beta \) is almost monotonically increasing with \( \nu_{eff}/\omega_e^* \) but sometimes has shallow minima and is not monotonic in \( g \). (Here \( g = -2L_n/R, L_X = X/(\partial X/\partial r) \) for any parameter \( X \), \( n \) is the electron density, and \( R \) is the major radius.) Even though the resulting temperature dependence of \( \chi_e^{TE} \) is quite complicated, the essential features of neo-Alcator confinement time scaling are still preserved in this more realistic model. The additional loss mechanisms in the electron channel, the circulating electron collisional drift wave and high-\( m \) tearing mode, have nearly the same parametric dependence, \( \chi \propto nT^{-0.5}(qR/L_nT_e)^2 \), and provide some favorable current dependence. It is interesting to note that magnetic islands resulting from tokamak error fields [12] could produce the same effect, leading to additional favorable current dependence. In the ion channel, the instability threshold for the ITG mode has been modified to include flat density profile effects [10]. In general, the stability boundary depends on two parameters, \( L_n/R \) and \( \eta_i = L_n/L_T \). Even though \( \eta_i \) may be large (as in H-mode plasmas), the ITG mode may or may not be stable, depending on the value of \( L_n/R \).

Recent experiments on DIII-D [13] have also obtained low density H-mode plasmas with ion temperatures much larger than electron temperatures in the
bulk. Conversely, the inferred ion thermal conductivity was much smaller than the electron thermal conductivity. These observations are not compatible with the components of the anomalous transport model previously described since the ITG mode generally plays a dominant role, even for losses in the electron channel. For the hot ion H-mode, the ITG mode should be weak since $T_i \gg T_e$. Further, the trapped electron mode vanishes in the flat density regime of the H-mode. We conjecture that the source of the observed anomalous transport in the hot ion H-mode is an unstable trapped ion mode. Quasilinear estimates of the heat flux [14] support the observation that $\chi_e \gg \chi_i$. The hot ion trapped ion mode loss mechanism has been added to the previous drift model for $\chi_{e,i}$ [14], and the results have been used to perform simulations with our shooting code [14]. A comparison of experimental and simulated electron and ion temperature profiles for one shot is shown in Fig. 1. The agreement is obviously quite good, but we caution that more comparisons are necessary. Refinements of the theoretical model for the hot ion trapped ion mode are in progress.

3 Time Dependent Transport

In previous work using the BALDUR time dependent transport code, the seven confinement scaling exponents obtained from log-linear regression on a large “L-mode” global confinement data base by Kaye were reproduced to within a root-mean-square deviation of 0.17. This work used quasilinear drift/\eta_i transport fluxes and an older version of the resistive ballooning fluxes, as documented by Ghanem et al. [15]. The only parameter adjustment used to obtain this excellent fit was to multiply the nominal theoretical drift/\eta_i transport fluxes

FIG. 1. Electron and ion temperature profiles for a hot ion H-mode shot plotted versus r/a. The experimental temperature profile plots show typical error bars.
by a factor of 0.3. A better fit to experimental results from a controlled scan of plasma elongation [16] has now been obtained by further multiplying the drift/$\eta_t$ and resistive ballooning transport flux formulas by $\kappa^{-4}$, where $\kappa(r)$ is the local elongation of a flux surface of midplane halfwidth $r$.

Using this elongation scaling (and some minor changes to improve numerical performance of the particle transport fluxes, including a new resistive ballooning transport model [18, 19, 20]), this work has now been extended to simulating the temporal evolution of H-mode discharges in the JET, D-III, ASDEX, and PDX tokamaks. To heuristically emulate the effect of divertor shear on anomalous transport in these new simulations, a factor of $\{[k^2 \ln(4/k)]^{-1} - (\ln 4)^{-1}\}$ was added to $\dot{s}$ in the anomalous transport formulas, where $k = 1 - (r/a)$. Also, time dependent boundary conditions were set at a location $\lambda = 0.05/B_0$ (in SI units) inside the midplane separatrix at $r = a$ as described by Singer, Bate-
man, and Stotler [17]. Using detailed published neutral beam and other input time dependent parameters [21], and accounting for suppression of sawteeth after the H-mode transition, gave the simulated electron temperatures shown by the solid curve in Fig. 2. The long dashed curve in Fig. 2 shows how omitting the above described divertor shear enhancement reduces energy confinement in this model. ($T_e \approx T_i$ for these simulations.) The short dashed curve in Fig. 2 omits the divertor shear enhancement and also uses L-mode boundary temperatures. While Fig. 2 illustrates various possible contributions to confinement degradation in L- vs. H-modes, a detailed comparison would allow for different temporal evolution of the two types of discharge. For example, continuing saw-
teeth throughout auxiliary heating (which often occurs in L-mode discharges) further depresses the central electron temperature after each sawtooth event.

FIG. 2. Simulated electron temperature profiles versus experimental data for an ASDEX H-mode discharge.
Comparably good fits to reported H-mode electron and ion temperature profile shapes were obtained for the discharge examined from the three other above mentioned tokamaks, but central electron temperatures deviated from measurements by a root mean square value of 28%. A significant enhancement of the resistive ballooning particle transport contribution included here improved numerical tractability and allowed sufficiently large inward transport of the H-mode density rise in the JET discharge. As with the energy transport results described above, this suggests that an additional edge plasma transport mechanism needs to be added to the older drift/\eta_i models.

To allow a systematic optimization of the turbulence saturation levels in the type of transport models just described, a Bayesian statistical method has been derived which relies on choosing prior probability distributions that describe previous analyst’s confidence that these parameters lie within given ranges. To allow use of large global energy confinement data bases, statistical models have been included of both random measurement variations on a given machine and of variations which are systematic on a given machine but vary randomly from one machine to another [22]. An efficiently optimized transport code simulator is also being developed to allow multidimensional parameter scans over large data bases [23].

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TOKAMAK IGNITION PROJECTIONS FROM DIMENSIONALLY SIMILAR DISCHARGES*

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Abstract

TOKAMAK IGNITION PROJECTIONS FROM DIMENSIONALLY SIMILAR DISCHARGES.

Dimensionally similar tokamak discharges with all dimensionless parameters the same except the relative gyroradius can be scaled to ignition regime discharges of larger size and/or magnetic field in analogy to the principles of wind tunnel design. The paper describes the first controlled experiments to determine whether the scaling with relative gyroradius corresponds to transport based on shortwave or longwave length turbulence.

INTRODUCTION

Tokamaks operate in a variety of confinement regimes: low density neo-Alcator and high density saturated ohmic heating, L-mode high power heating as well as H-mode. This suggests that there may be several transport mechanisms at work. Even if we understood these mechanisms better, modeling them in sufficient detail to describe even the global confinement time becomes very complex. However we argue that our lack of complete understanding need not prevent us from accurately scaling present tokamaks to ignition devices of larger size (a) and magnetic field (B) provided we apply our most basic knowledge of the dimensional constraints on transport mechanisms.

The transport diffusivity $\chi$ and the global confinement time $\tau$ could depend on a lengthy but finite list of dimensionless parameters. These include those based on geometry (safety factor $q$, aspect ratio $R/a$, and

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elongation $b/a$), and on relative profile lengths. Apart from the atomic mass, $Z_{\text{eff}}$, and the temperature ratio, $T_i/T_e$, the remaining parameters based on pure plasma physics include $\beta \propto n T/B^2$, collisionality $\nu = (\nu_{\text{ei}} c_s) a \propto n a/T^2$ and the relative gyroradius $\rho_* \equiv \rho_s/a \propto T^{1/2}/Ba$ ($c_s = \sqrt{T/m}$, $\rho_s = c_s/\Omega$, $\Omega = eB/Mc$). (The relative Debye length is specifically excluded.) Discharges with all these parameters held fixed could be called "dimensionally identical". It has been shown that dimensionally identical discharges should have their global energy confinement time scaled to the gyrofrequency: $\tau \propto \Omega^{-1} \propto B^{-1}$ [1,2]. While it has been argued [3] that ignition tokamaks will have dimensionless parameters comparable to existing discharges, it is not practical to scale existing dimensionally identical discharges of large $n T \tau$ to ignition. However it is possible to reach ignition by scaling discharges with all dimensionless parameters fixed except the relative gyroradius $\rho_*$.

We can call these "dimensionally similar" discharges.

Theoretically the scaling of dimensionally similar discharges with respect to relative gyroradius should be very simple. All theories fall to two extremes: those characterized by turbulence with short wavelengths scaled to intrinsic plasma parameters like the gyroradius ($\rho_*$) which we call gyroBohm-like; and those with long wavelengths scaled to the plasma size ($a$) which we call Bohm-like. In the gyroBohm case the diffusivity scales as $\chi_{\text{gB}} \propto (c_s/a)\rho^2_{\text{gB}}$ whereas for the Bohm case $\chi_B \propto c_s \rho_a F_B$. The $F$ form factors represent the functional dependence on all the dimensionless parameters held fixed for dimensionally similar discharges. A survey of theoretical models in current use shows that nearly all are gyroBohm-like and thus their form factors may be simply added. For example, neoclassical diffusion, $E \times B$ drift wave diffusion, stochastic magnetic field line transport from microtearing modes or electromagnetic transport from models based on the collisionless skin depth $c/\omega_{pe}$ may be written in the gyroBohm form. A clear exception is the recent version of resistive MHD magnetic transport [4]. In this case numerical simulation of the turbulence suggests that the average poloidal wave number scales to $q/a$ giving an almost Bohm-like diffusion form $\chi \propto c_s \rho_a (\rho_s/a)^{1/3} \ldots$. For dimensionally similar discharges which will have density $n \propto B^4/a^{-1/3}$, temperature $T \propto B^{2/3} a^{1/3}$, and $\rho_s \propto (B^4 a^5)^{-1/6}$, the diffusivity should scale as $\chi_{\text{gB}} \propto B^{-1} a^{-1/2}$ or $\chi_B \propto B^{-1/3} a^{1/3}$ at the extremes.

If experiments can verify that either the gyroBohm or Bohm extreme prevails, then we could have a powerful "wind tunnel" like method for scaling fusion devices to ignition. If net heating profiles are self similar then the global confinement time should scale as $\tau \propto a^2/\chi$. For dimensionally similar discharges we should expect $\tau_{\text{gB}} \propto B a^{5/2}$ or $\tau_B \propto B^{1/3} a^{5/3}$ which implies a substantial difference in the ignition parameter $n T \tau_{\text{gB}} \propto B^3 a^{5/2} \propto I^3/a^{1/2}$ or or $n T \tau_B \propto B^{1/3} a^{5/3} \propto T^{1/3} a^{2/3}$. While the preponderance of theoretical models have a gyroBohm-like scaling,
the standard empirical L-mode scaling often used in ignition projections is worse than Bohm-like. Along a dimensionally similar path the Goldston empirical scaling \([5]\), which appears to give the best statistical characterization of the global confinement time based on diamagnetic stored energy, reduces to \(\tau_{\text{emp}} \propto B^0 a^{1.8}\) with \(n T \tau_{\text{emp}} \propto B^2 a^{1.8} \propto I^2/a^{0.2}\). There have been recent attempts to "correct" the diamagnetic data base for fast ion storage \([6,7]\) and neutral beam penetration effects \([7]\). While these corrections redirect the L-mode scaling from Bohm-like toward gyroBohm-like, they are very approximate and do nothing to address the uncertainties introduced by statistical co-variations of the dimensionless parameters within the data base.

To resolve this apparent discrepancy and uncertainty, we have performed and analyzed a series of controlled \(B\) field scaling experiments on dimensionally similar discharges in DIII-D and in TFTR. The key result is that while the gross confinement time follows the empirical L-mode scaling \(\tau \propto B^0\), the diffusivity is consistent with a gyroBohm-like scaling \(\chi \propto B^{-1}\). The failure of the confinement time to follow \(\tau \propto \chi^{-1}\) is due to the lack of self similarity of the neutral beam heating profile. Dimensionally similar discharges with larger \(B\) have higher density and poorer heating penetration.

**DIII-D EXPERIMENT**

In the DIII-D experiment standard L-mode discharges of fixed size \(a = 65\) cm, aspect ratio \(R/a = 2.7\), elongation \(b/a = 1.70\), and \(q = 3.8\) were compared at \(B = 1.05\) T (1 MA) and \(B = 2.1\) T (2 MA) keeping \(\beta\) and collisionality fixed. The 1 T reference discharge was established at \(n_e = 3.8 \times 10^{13}\) cm\(^{-3}\) with 3.7 MW of total power (0.36 MW OH). It had a reactor relevant \((\beta)_{\text{th}} = 1.9\% \left(\beta/\beta_{\text{crit}} \approx 0.35\right)\) with \(\nu^\text{min} = 0.13\). The 2 T discharge was at \(n_e = 9.6 \times 10^{13}\) cm\(^{-3}\) (2\(^4/3\) times the reference density) with 16 MW of power. For this discharge \((\beta)_{\text{th}}\) and \(\nu^\text{min}\) were unchanged. The \(Z_{\text{eff}}\) profile was nearly constant at 1.5 to 1.6 and nearly the same in each case. The electron and ion temperatures were nearly equilibrated in both discharges and we combined the measurements together in a single temperature. \(T(0)\) increased from 1.89 to 2.75 or 1.46 times. This is close to the similarity ratio \(2^{2/3} = 1.587\). The measured profiles were very similar.

The global total confinement followed the expected L-mode empirical scaling and remained almost constant: 97 to 81 msec or a 17% decrease whereas \(\tau \propto I n^0/P^{1/2}\) predicts only a 4% decrease. The global thermal energy confinement time was nearly constant: 82 to 72 msec. However, despite the near constancy or even degradation of total confinement time, the local heat diffusivity was best described by a \(1/B\) scaling. Standard transport code analysis was used to calculate \(\chi(\tau)\) from the
Fig. 1. DIII-D experiment. Ratio of 2 T and 1 T average heat diffusivity versus normed volume radius (solid line). Ratio of heat diffusivities normed to inverse temperature (dotted line) gradient, and inverse density (dashed line) gradient scale lengths \((\chi \cdot L_T)_2/(\chi \cdot L_T)_1\) and \((\chi \cdot L_n)_2/(\chi_1 \cdot L_n)_1\).

experimental density and temperature profiles and the known profile of transport power flow \(P_{tr}(r) = P_{beam}(r) + P_{OH}(r) - P_{rad}(r) - P_{conv}(r)\):

\[-2 n(r) \chi(r) \partial T/\partial r = \rho P_{tr}(r)/S(r),\]

where \(r\) is the volume radius normalized to the equivalent volume radius \(\rho\) and \(S(r)\) is the surface area. The plasma convection \(P_{conv}\) played a negligible role in these discharges. Figure 1 shows that for the inner 95% of the discharge, \(\chi\) behaves in a nearly gyroBohm fashion, i.e. \(\chi_2/\chi_1 \approx 1/2\). We cannot, however, rule out Bohm-like processes localized to the edge. Since \(\chi\) in most theories is inversely proportional to the plasma gradients and these were not strictly constant, we also show the ratios of \(\chi \cdot L_n\) (dashed line) and \(\chi \cdot L_T\) (dotted line) which do not change our conclusion.

The sawtooth period may give another measure of central diffusivity. The Soler-Callen formula [8] argues that the sawtooth period \(\Delta \tau\) is given by the transport “refilling time” \(8/3 \cdot \tau_s^2/\chi(0)\) where \(\tau_s\) is the singular surface radius. If \(\chi(0) \propto 1/B\) we may expect \(\Delta \tau \propto B\). The DIII-D discharges had \(\Delta \tau_2/\Delta \tau_1 = 130 \text{ msec}/74 \text{ msec} = 1.75\) which is close to 2. This may be only coincidental evidence. The sawtooth skin time also scales as \(T(0)^{3/2} \propto B\) which may be more appropriate if the sawteeth are triggered by the local current rather than pressure profile changes.
Fig. 2. DIII-D experiment. (a) Normalized beam power deposition (solid line) and normalized transport power flow (dashed line) versus volume radius. (b) Experimental transport confinement time versus normed volume radius (solid line). Projected 2 T transport confinement time using one-half the 1 T heat diffusivity (dashed line).
To understand why the global confinement time had the pessimistic $\tau \propto B^0$ scaling whereas the heat diffusivity was consistent with the optimistic scaling $\chi \propto 1/B$, it must be remembered that the global confinement time depends on the heat deposition profile as well as the diffusion. The neutral beam heating is less effective in the higher density 2 T case where it deposits more power near the edge relative to the center. Figure 2(a) shows the normalized profiles of the radially integrated beam power (solid lines) and transport power (dashed lines), $P(r)/P(1)$. Note $P_{\text{beam}}(0.5)/P_{\text{beam}}(1)$ decreased by 2 fold and even more at smaller radii in going from 1 T to 2 T. Figure 2(b) shows the transport confinement time $\tau_{tr}(r)$ as a function of normalized volume radius for the 1 T and 2 T cases (solid lines). $[\tau_{tr}(r): W(r)/P_{tr}(r)$ where $W(r)$ is the thermal energy inside $r$.] The global values for $\tau_{tr}$ at $r = 1$ are nearly the same whereas the central values are considerably larger in the 2 T case since relatively little power is deposited at the center. To verify that the poor global confinement time scaling is due to the poor beam penetration and that the diffusion is actually close to gyroBohm-like, we can solve the transport equation for temperature using the 2 T experimental density $[n(r)]$ and the 2 T experimental transport power $[P_{tr}(r)]$ but with $\chi(r) = 1/2 \chi_1(r)$ i.e., according to perfect gyroBohm scaling. The resulting temperature can be used to compute $\tau_{tr}(r)$. As shown by the dashed line, $\tau_{tr}(r)$ is nearly the same as the 2 T experimental curve and in particular reproduces the poor global transport confinement.

If diffusivity has an optimistic $\chi \propto 1/B$ scaling, it should be possible to recover an optimistic scaling for global confinement $\tau_{tr}(1) \propto B$ by keeping the heating profile self similar over the dimensionally similar scaling. An experiment on DIII-D is planned to do this. With the existing neutral beam equipment it is not practical to increase the penetration with higher voltage for the 2 T higher density discharge to obtain 2 times larger confinement time. However to demonstrate the principle we expect to obtain a factor 2 degradation in confinement time of the 1 T lower density discharge by vertically shifting the equilibrium off-axis to give poor heating penetration. We expect highly localized ECRF heating systems planned for DIII-D to be more suitable for self similar heating profile studies.

**TFTR EXPERIMENTS**

Recent TFTR confinement experiments with dimensionally similar discharges were able to confirm the DIII-D results in part but several unresolved problems appeared. Both neutral beam L-mode and ohmically heated $B$ scaling experiments were performed to scan over $\rho_s$ keeping $q$,
In these circular discharges $a = 80$ cm, $R = 245$ cm, and $q = 3.1$. There were two L-mode scans: a low density 5 shot series with $B = 1.44, 2.16, 2.87, 4.31, 4.88$ T, $\bar{n}_e = 1.2, 1.8, 2.6, 4.4, 5.2 \times 10^{13}$ cm$^{-3}$, and $P_{\text{beam}} = 2.1, 4.7, 7.3, 17.7, 22.7$ MW; and a high density 2 shot series with $B = 2.16, 4.88$ T, $\bar{n}_e = 2.8, 7.9 \times 10^{13}$ cm$^{-3}$, and $P_{\text{beam}} = 4.3, 22.4$ MW respectively. The ohmic discharges included a 4 shot scan at $B = 1.44, 2.87, 3.59, 4.31$ T, and $\bar{n}_e = 0.3, 0.99, 1.23, 1.32 \times 10^{13}$ cm$^{-3}$ respectively.

In contrast to the DIII-D L-mode experiment the TFTR L-mode discharges suffered from lack of temperature equilibration (even in the high density scan). The lack of equilibration made it difficult to keep the $T_i/T_e$ ratio and all species specific $\nu*$ and $\beta$ constant; even after discounting the lowest density discharge, there were factor 2 variations in $T_i/T_e$. In fact the relative equilibration rate $\tau_B/\tau_{\text{eq}}$ can not be held constant in a $\rho*$ scan. Definite conclusions are precluded without good dimensional similarity. Nevertheless we proceeded with a preliminary analysis of the TFTR experiment. In both the low and high density scans the global thermal confinement time followed the empirical scaling $\tau \propto B^0$ closely. The $\chi$ scaling from a density averaged one temperature local transport analysis gave mixed results. In the high density scan with $B = 2.16$ and 4.88 T, which was most similar to the DIII-D cases, the outer half of the discharge showed $\chi \propto 1/B$ whereas the inner half (even after carefully phasing the analysis to the top of the sawteeth) was notably less $B$ dependent. Nevertheless the sawtooth period $\Delta \tau$, which may be an indicator of $1/\chi(0)$ as described above, was roughly proportional to $B$. The discrepancy may be within the error bars on $\chi(r)$ which are difficult to assign. To deal with this uncertainty a radially integrated transport simulation similar to that described above for DIII-D was performed to show that $\chi$ is consistent with $1/B$ scaling on the average: $\chi(r) = (2.16/4.88) \chi_{2.16}(r)$ was able to reproduce the 4.88 T global transport confinement time $\tau_T(a)$; the reverse projecting from 4.88 to 2.16 T was also obtained. Agreement for the central values of $\tau_T(r)$ was less satisfactory. The neutral beam penetration in the high field case was markedly poorer with $P_{\text{beam}}(0.5)/P_{\text{beam}}(1)$ no more than half the value of the low field case. Again as in the DIII-D experiment, this seems to account for the poor global scaling. The low density scan was less clear. The change in heating penetration over the scan was weaker with $P_{\text{beam}}(0.5)/P_{\text{beam}}(1)$ decreasing by about 40%. Although the sawtooth period was again roughly consistent with $\Delta \tau \propto B$, the transport analysis $\chi$ profiles showed no consistently favorable $B$ dependence advancing through the scan. Other methods based on radial integration and simulation gave a similar conclusion.
The ohmically heated $\rho_*$ scan was consistent with gyroBohm scaling. Only in the case of gyroBohm transport can we expect to produce a dimensionally similar discharge series with ohmic heating. For dimensionally similar discharges the ohmic power scales as $P_{OH} \propto B a^{1/2}$. Since we can safely assume that constant $q$ ohmic heating has little profile variation, the power required to maintain fixed $v_*$ and $\beta$ for gyroBohm-like transport also scales as $P_{GB} \propto B a^{1/2}$ whereas for Bohm-like transport $P_B \propto B^{5/3} a^{4/3}$. It was found that the $n \propto B^{4/3}$ constant $q$ scan maintained $v_*$ and $\beta$ roughly constant although there was significant variation in $Z_{eff}$ as well as in $T_i/T_e$. The global confinement scaled as $\tau \propto B$. This is not surprising since projecting neo-Alcator scaling $\tau \propto n a R^2 q$ along a dimensionally similar path gives $\tau \propto B^{1.33}$ close to $B^1$.

To this point we have restricted the discussion to single machine comparisons. While there have as yet been no inter-machine dimensionally similar $\rho_*$ scans to verify the gyroBohm size scaling $\tau \propto B a^{5/2}$, a comparison of dimensionally identical fixed $\rho_*$ ohmic discharges to test the principles of dimensional analysis has been made. This requires that $B^4 a^5$ in addition to $q$, and $R/a$ be held fixed. Irrespective of the $\rho_*$ dependence (i.e. whether transport is gyroBohm or Bohm-like), $B \tau$ for ohmic discharges compared at the same value of $n/B^{8/5}$ should be invariant. Comparison of ohmic density scans from PLT ($a = 40$ cm, $B = 3.25$ T) and Alc-C ($a = 16.5$ cm, $B = 10$ T) have shown a remarkable overlay (see Fig. 1, Ref. 9) in both linear and saturated regimes. However a new low field TFTR ($a = 74$ cm, $B = 1.54$ T) ohmic density scan designed to be dimensionally identical to Alc-C failed to overlay. The confinement in TFTR was at least 2 times larger than expected. In the linear regime radial variations in both Alc-C and TFTR have shown neo-Alcator scaling $\tau = C n a R^2 q^{(0-1)}$ which projects to $\tau \propto B^{-0.8}$ along a dimensionally identical path. However the fit coefficient $C$ appears to be 2 times larger in TFTR than Alc-C. This discrepancy appears to be due to the markedly poorer ion channel confinement in the Alc-C discharges. At the same normalized $\bar{n}_e$, the Alc-C discharges had much less peaked density profiles suggesting that the ion temperature gradient mode has had an earlier onset in the scan. Further experiments to make dimensionally identical ohmic discharges within TFTR by variation of $R$ and $B$ did show $B \tau$ invariance.

**CONCLUSION**

We believe the first steps have been taken to establish a wind tunnel like design principle for magnetic fusion devices based on dimensional
analysis. Tokamaks can be extrapolated to ignition keeping all dimensionless constants fixed except the relative gyroradius. The preponderance of data analyzed to date suggests the transport process is gyroBohm-like. The ignition parameter $n T \tau$ should then have a strongly favorable scaling with current $I^8/a^{0.5}$ rather than the worse than Bohm-like empirical scaling $I^2/a^{0.2}$. However more work is needed to show that this favorable scaling can actually be obtained with discharges having self similar heating profiles. Recent work [7,10] has shown that the proposed Compact Ignition Tokamak (CIT) comfortably ignites with a dimensionally similar gyroBohm projection from DIII-D H-mode discharges.

REFERENCES


THREE-DIMENSIONAL PARTICLE SIMULATION STUDY ON STABILIZATION OF THE FRC TILTING INSTABILITY

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Abstract

THREE-DIMENSIONAL PARTICLE SIMULATION STUDY ON STABILIZATION OF THE FRC TILTING INSTABILITY.

By carrying out a three-dimensional nonlinear particle simulation in the cylindrical coordinates it is shown that ion kinetic effects act to stabilize the FRC tilting instability and to form an anisotropic temperature distribution. For the case of \( \tilde{s} \approx 1 \) a large number of ions execute a large amplitude oscillatory motion around the field-null line and carry most of the ion toroidal current, where \( \tilde{s} \) measures the number of ion gyroradii over the radial distance between the magnetic separatrix line and the field-null line. It is found that this motion, which is called a meandering motion, plays an important role in keeping the FRC plasma stable against tilt disruption and in forming the temperature anisotropy.

1. Introduction

The magnetohydrodynamic (MHD) linear theory\[1\] predicts that the field-reversed configuration (FRC) plasma is unstable against the tilting instability, while no experimental evidence has so far been reported on the tilt disruption. Two possibilities have been considered to explain the discrepancy between the MHD linear theory and the experiment. The first explanation is that the nonlinear saturation mechanism could protect the FRC plasma from the destructive growth of the tilt mode. In this respect Horiuchi and Sato's [2] three-dimensional full MHD simulation found no evidence for the nonlinear saturation of the tilt mode except for a highly spinning case.

An alternative explanation is that the stabilization effect due to the ion finite-Larmor radius (FLR) can operate effectively in the currently operating devices since the plasma confinement scale is comparable to the ion Larmor radius. Barnes et al.\[3\] derived the linear growth rate from the Vlasov fluid dispersion equations and found that the tilt mode could be stabilized for a large gyroradius case of \( \tilde{s} \leq 2 \), where \( \tilde{s} \) is defined by
\[ s = \int_{r_a}^{r_s} \frac{r \, dr}{r_s \lambda_i}, \]  
\( r_s \) is the separatrix radius, \( r_a \) is the radius of the magnetic null, and \( \lambda_i \) is the local ion gyroradius. By solving the MHD equations with the Hall term Ishida et al.\cite{4} and Milroy et al.\cite{5} have shown that the Hall term could reduce the growth rate of the tilt mode for the highly prolate and small \( s \) case. However, they could not satisfactorily explain the discrepancy between the theory and the experiment. This is because the Hall term can represent only a part of the ion FLR effect.

In order to investigate fully the FLR stabilization effect against the tilting instability we carry out a macro-scale particle simulation that can describe both the electron and ion FLR effects and the global behavior over the device scale simultaneously\cite{6}.

2. Simulation model

We study the FRC plasma in a cylindrical conducting vessel in which plasma is confined by a uniform external field. The equations to be solved are the equations of motion

\[ \frac{d(\gamma_j v_j)}{dt} = \frac{q_j}{m_j} [E + \frac{v_j}{c} \times B], \]  

and the Maxwell equations

\[ \frac{1}{c} \frac{\partial B}{\partial t} = -\nabla \times E, \]  

\[ \frac{1}{c} \frac{\partial E}{\partial t} = \nabla \times B - 4\pi j, \]  

\[ \nabla \cdot B = 0, \]  

\[ \nabla \cdot E = 4\pi \rho, \]  

where \( x_j(t), v_j(t), m_j, q_j, \gamma_j, j(x,t) \) and \( \rho(x,t) \) are the position, the velocity, the rest mass, the charge, the relativistic \( \gamma \)-factor of the \( j \)-th particle, the current density and the charge density, respectively. We solve the equations (2)-(5) in the cylindrical coordinates \( (r, \phi, z) \) by assigning the initial conditions \( x_j(0), v_j(0), B(0, 0) \) and \( E(0, 0) \) which satisfy a two-fluid MHD equilibrium. The boundary condition is such that the physical quantities are periodic at two axial edges of the cylindrical vessel and a particle is completely elastically reflected on the conducting wall. The numerical scheme used for the three-dimensional particle simulation relies
on a semi-implicit method\cite{6}. Four simulation runs with different values of $\bar{s}$ are carried out by using a hundred thousand particles. The simulation runs are terminated after one Alfvén transit time $t_A$ where $t_A$ is defined by $r_0/v_A$; $r_0$ and $v_A$ are the device radius and the average Alfvén velocity in the plasma region.

3. Results

One of the characteristic features of the FRC plasma is that a field-null line exists in the central plasma region due to the strong toroidal plasma current. Figure 1 shows the initial profiles of the poloidal magnetic flux, the ion thermal pressure and the toroidal current density in the poloidal plane for $\bar{s} = 1$. It is worth noting in Fig. 1 that the FRC plasma is distributed in a fairly prolate region around the field-null line. Figure 2 shows the top

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig1.png}
\caption{Initial profiles of poloidal magnetic flux, ion thermal pressure and toroidal current density in the poloidal plane for $\bar{s} = 1$.}
\end{figure}
view of the orbits of one hundred ions for the cases of $\bar{s} = 1$ (top) and $\bar{s} = 5$ (bottom) where each curve represents the projection of the ion trajectory onto the midplane ($z = \text{constant}$) during one Alfvén transit time from the start of the simulation. Most of the ions in the vicinity of the field-null line cannot make gyration motions but execute a meandering motion around the field-null line without any self-intersections of orbits. The number of meandering ions increases and the oscillation amplitude of meandering motion becomes larger as $\bar{s}$ decreases.

Figure 3 shows the profiles of the electron distribution (top) and the ion distribution (bottom) in the $(v_{\phi}, v_z)$-plane (right) and in the $(v_r, v_z)$-plane (left) for the case of $\bar{s} = 2$. The meandering ions drift along the field-null line with larger oscillation amplitude along the $z$-direction compared with that along the $r$-direction because the scale height of magnetic field strength in the $z$-direction is larger than that in the $r$-direction. The anisotropy of the meandering motion results in an anisotropic ion temperature, i.e., $T_z > T_r$ and $T_z > T_{\phi}$ (bottom part of Fig. 3). On the other hand, the electron distribution is almost isotropic because the number of meandering electrons is very small (top part of Fig. 3).

The dependence of the average growth rate of the tilt mode on the parameter $\bar{s}$ is plotted in Fig. 4, where the open circles represent the value obtained by the simulation, and the filled triangles show the results of a linear theory[3]. The evolution of the tilt mode is completely suppressed when $\bar{s} \approx 1$. As $\bar{s}$ increases, the tilt mode tends to be more unstable and the growth rate approaches the MHD value. The behavior of the kinetic growth rate is in good agreement with the result of the linear theory. It can be concluded therefore that the stabilization effect due to the finiteness of the ion Larmor radius is very efficient for the FRC tilt mode.
FIG. 3. Electron distribution and ion distribution in the \((v_\varphi, v_z)\)-plane and in the \((v_r, v_z)\)-plane for the case of \(s = 2\).

FIG. 4. The \(s\)-dependence of the average growth rate of the tilting instability. Open circles represent the value obtained by the simulation and the filled triangles show the results of a linear theory [3]. Dashed line represents the average tendency of the growth rate as a function \(1/s\).
4. Conclusion

Here we give a theoretical model to explain the FRC tilt stabilization in connection with the characteristic of a meandering motion. For the kinetic plasma of $s = 1$ most of the ions are free from the constraint of the magnetic field and oscillate around the field-null point with a large amplitude. Suppose that a perturbation of $n = 1$ tilt mode is added to the velocity field of the meandering ions in a two-dimensional (axially symmetric) equilibrium. The ion changes to a new oscillation orbit the amplitude of which varies dependent on the phase difference between the meandering oscillation and the perturbation. However, the oscillation center of the new orbit remains the same. When the orbit is averaged over one oscillation period, therefore, the $n = 1$ tilt perturbation does not appear in the toroidal current carried by the meandering ions on the average. In other words, ions with meandering orbits do not contribute to the growth of the perturbation of the $n = 1$ tilt mode. We thus conclude that the ions with meandering orbits play a key role in keeping the system stable against the tilting perturbation, and that the evolution of tilt mode can be completely suppressed when most of the ions move on the stable meandering orbits, i.e., when $s \approx 1$.

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INSTABILITY EFFECTS CAUSED BY CONDUCTING END WALLS IN A PLASMA ON OPEN FIELD LINES

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Abstract
INSTABILITY EFFECTS CAUSED BY CONDUCTING END WALLS IN A PLASMA ON OPEN FIELD LINES.

Two new mechanisms on how a conducting wall can affect plasma stability are presented. The instability mechanisms are of interest to the general problem of plasma stability on open field lines and they are particularly relevant to the concept of the gas dynamic trap (GDT). It is shown that the stabilizing effect of ion momentum flow through expanders with favorable field line curvature onto an insulating wall disappears in the presence of a conducting wall. With an electron temperature gradient present, a rapidly growing (compared with MHD growth rates) instability arises with end conducting plates present, while this instability is completely absent with insulating end plates.

1. Introduction

A fundamental problem in plasmas is how its plasma wave properties are affected by contact to bounding end walls. This subject was of considerable interest in previous decades [1-6]. Today, it is likely to re-emerge as a topic of interest in both mirror machines and tokamaks. In tokamaks the edge physics of the scrape-off layer appears to have a strong effect on the global confinement characteristics, e.g. the formation of an H-mode. In mirror machines, there is always a steady axial loss of plasma, and in one machine, the gas dynamic trap (GDT), this flow mechanism is supposed to provide the basic mechanism of stabilization.

The prime motivation of this investigation is to understand the basic stability properties of the GDT [7-9], a mirror machine experiment at the Institute of Nuclear Physics in Novosibirsk that is being investigated in order to develop a compact plasma based neutron source [10,11] using the
D-T reaction. A schematic diagram of the machine is shown in Fig. 1 and a detailed discussion of the concept is found in Refs. [7-9]. It consists of a long central cell with a large mirror ratio $R \approx 50-100$ and a short expander region. The mean free path $\lambda < LR$ where $L$ is the axial length. Hence, the mirror loss region is always full and ions exit from the ends at sonic speeds. It was proposed that the momentum flux in this expander region could provide MHD stabilization of this symmetric system. We have found that this stabilization effect depends on the nature of the boundary conditions present at the end walls. If the end walls are insulators, the MHD stabilization mechanism due to the ion momentum outflow is indeed present. If the end walls are conductors the stabilization due to the momentum output flux disappears. Instead, if the central region is MHD unstable, the only remaining stabilization term is due to the electron pressure in the expander region (there most of the electrons are still trapped by the ambipolar potential) which is usually considerably less than the ion momentum flux. As a result the MHD stabilization mechanism for the conducting boundary case is considerably smaller than in the insulation boundary case. When the cross-field
electron temperature gradient in the equilibrium vanishes and when the unstable MHD instability drive of the central cells is larger than the electron pressure stabilization mechanism in the expander, the instability is only slowed down with end conductors by the “line tying” mechanism described by Kunkel-Guillory [2]. However, when an electron temperature gradient is present we find a strong new instability source, giving growth rates that are typically larger than one would predict from MHD theory. This new instability appears generic for any open field line geometry terminated with end conductors. Hence, it should be relevant in the understanding of the fluctuations observed in the scrape-off layer of tokamaks and other interesting plasma systems (e.g. plasma magnetosphere).

2. Transit Particle Stabilization of the Gas Dynamic Trap

Detailed stability analyses of the GDT of Ref. [12] based on MHD equations confirm that in the case of insulating end walls the expanders do indeed stabilize the interchange instability, due to the momentum flux of escaping ions. However, recent studies [11,12] have shown that for perfectly conducting boundaries a stabilizing contribution only comes from the electron pressure in the expanders which is relatively small and is not as efficient in providing overall plasma stability.

To describe both the cases of insulating and perfectly conducting end walls we use a moment equation for a flute mode in a paraxial mirror neglecting finite Larmor radius and finite β effects and find

\[ \delta \varphi \left( \omega^2 M_i \int \frac{ds}{B^3} nk^2_\perp + k^2_\theta \int \frac{ds}{B^2 r} \kappa \frac{\partial}{\partial \varphi} (P_\perp + P_\parallel) \right) \]

\[ = -\frac{i\omega}{c^2} \begin{pmatrix} \delta j_\parallel \left| \right. \text{right wall} \\ \delta j_\parallel \left| \right. \text{left wall} \end{pmatrix} \]

where \( \delta \varphi \) is the perturbed potential in eikonal approximation, \( \delta \varphi = \delta \varphi(s) \exp[-i\omega t + iS(\psi)] \) with \( k_\perp = \nabla \psi (\partial S/\partial \psi) + \nabla \theta \partial S/\partial \theta = k_\theta \nabla \psi + 2k_\theta \nabla \theta \), \( \psi \) is the magnetic flux, \( \theta \) is the azimuthal angle, \( s \) is the distance along a field line, \( \kappa \) is the field line curvature, \( n \) is the plasma density, \( P_\perp \) and \( P_\parallel \) are the perpendicular and parallel “pressures” which includes the directed ion velocity \( (P_\perp + P_\parallel = P_{i\perp} + P_{i\parallel} + 2P_e) \) where the subscripts “i” and “e” refer to ions and electrons respectively; the electron pressure in GDT is isotropic. The integration in Eq. (1) is carried along a field line from one end wall to the other and the perturbed parallel current \( \delta j_\parallel \) refers to the net current flowing from the plasma to the walls.
In the case of insulating boundaries $\delta j_{\parallel} = 0$ and Eq. (1) gives a standard dispersion relation for a flute mode. Applied to GDT it shows that the gradient of total pressure in the expanders contributes to stability, in agreement with the result of Ref. [12].

Proceeding now to the case of conducting end walls we first calculate the electron component of the current to the walls. This current depends on the collisionality of electrons. In the case when the mean free path is larger than the length of the machine, it is given by the Pastukhov formula [15]; in the opposite extreme a simple Boltzmann-factor expression is valid. To cover both cases, we take

$$j_{\parallel e} = nG \left( \frac{q_e(\varphi - \varphi_c)}{T_e} \right)$$

where $n$ is the electron density in the central cell, $\varphi$ is the plasma potential and $\varphi_c$ the potential on the end wall, $G$ is a function whose exact form depends on the collisionality of electrons. From Eq. (2) it follows that the perturbation of $j_{\parallel e}$ is

$$\delta j_{\parallel e} = j_{\parallel e} \left( \frac{\delta n}{n} + \alpha \frac{q_e(\delta \varphi - \delta \varphi_c)}{T_e} \right)$$

where $\alpha \equiv G'/G - 1$ and $\delta n$ denotes the density perturbation on a given field line, $\delta n = -(k_0c\delta \varphi/\psi)\partial n/\partial \psi$. Here, for simplicity, in Eq. (3) we do not take into account the transverse gradient of the electron temperature $T_e$ which can give rise to a specific instability considered in the next section. The perturbed potential is straightforwardly related to the end current by using the charge continuity equation and Ohm's law, if the end plates, with conductivity $\sigma$, are taken as thin with a thickness $b$ and we assume that currents transverse to the magnetic field are uniform over the thickness. We find $\delta \varphi_c = \delta j_{\parallel e}/\sigma b k_B^2$, with "w" referring to evaluation at the wall position.

To find the perturbation of the ion current $\delta j_{\parallel i}$, we use a collisionless drift kinetic equation for the perturbed distribution function $f$ of ions:

$$v_{\parallel} \frac{\partial f}{\partial s} - i(\omega - \omega_{di})f - iq_i\delta \varphi \omega \frac{\partial F}{\partial E} - i k_0c \delta \varphi \frac{\partial F}{\partial \psi} = 0$$

where $v_{\parallel}$ and $E$ are the parallel velocity and total energy of a particle, $\omega_{di}$ is the drift frequency, $F$ is the ion equilibrium distribution function. Neglecting the plasma rotation and finite $\beta$ effects we have $\omega_{di} = (v_{\parallel}^2 + \frac{1}{2} v_{\perp}^2) c k_0 M_i/q_i r B$. Equation (4) is solved separately in the central cell and in the end expanders using the ordering: $L_{ek} \ll L_c \ll v_{Ti}T_0^{-1}$ and matching
both solutions at the mirror throat. Integrating $q_i f_{||}$ over half of the phase space one finds $\delta j_{||}$ at the wall:

$$\frac{\delta j_{||}}{B}_{\text{wall}} = -j_{||} \frac{\delta \varphi_{ec}}{\omega B_w} \frac{\partial n}{\partial \varphi} + i k^2_{||} c \frac{\delta \varphi}{\omega} \int_{L_{ex}} \frac{\kappa ds}{B^2 r} \frac{\partial}{\partial \varphi} (P_{||} + P_{\perp})$$

$$+ i q^2 \delta \varphi \int_{L_{ex}} \frac{ds}{B} \int \frac{d^3 v}{\omega_d} \frac{\partial F}{\partial E} \quad (5)$$

where $j_{||}$ denotes the equilibrium ion current density to the wall and $B_w$ is the magnetic field at the wall. The integration in Eq. (5) is taken over the length of the expander.

Now, we sum the currents (3) and (5) with the use of $j_{||} = j_{||e}$ for the steady state and put the result into Eq. (1). Note that the second term in Eq. (5) combines with the ion contribution arising from the expander regions on the left-hand side of Eq. (1). As a result, the dispersion relation takes the form

$$\omega^2 + A \omega + C = 0 \quad (6)$$

where

$$C = k^2_{e} \left( \int_{L_{ex}} \frac{ds}{B^2 r} \frac{\partial}{\partial \varphi} (P_{\perp} + P_{||}) + \frac{2\lambda}{1 + \lambda} \int_{L_{ex}} \frac{ds}{B^2 r} \frac{\partial}{\partial \varphi} (P_{||} + P_{||}) \right)$$

$$+ 2 \int_{L_{ex}} \frac{ds}{B^2 r} \frac{\partial p_e}{\partial \varphi} \left( M_{t} \int \frac{ds}{B^3} n k^2_{||} \right)^{-1} \quad (7)$$

$$A = \left( \frac{q^2 i}{e^2 T_{e} \tau} \int \frac{ds}{B} - \frac{n_{ec} k^2_{e}}{e B_{max} L_{ex}} \int \frac{\kappa ds}{B r} \right) \left( M_{t} (1 + \lambda) \int \frac{ds}{B^3} n k^2_{||} \right)^{-1} \quad (8)$$

$\lambda = n_{ex}^2 L / 2 \sigma_b T_{e} k^2_{\perp w} \tau$, $\tau$ particle lifetime on flux tube, $B_{max}$ peak magnetic field; note $Br^2 \equiv \text{const}$, $k^2_{||} \tau^2 \equiv \text{const}$. In the derivation of Eq. (8) we have used that the plasma density in the expander is $n_{ex} = n_{0} B / 4 B_{max}$ which is valid when $T_{e} \ll T_{i}$ and included a factor of 2 to the integrals over $L_{ex}$ in Eq. (7) to account for both expanders. For $\sigma \to 0 (\lambda \to \infty)$ the insulating boundary condition is recovered. For $\sigma \to \infty (\lambda \to 0$ i.e. for an ideal conductor) we see that the strong stabilization term due to the momentum flux of escaping ions is lost in the coefficient $C$. Since the curvature of field lines in the expanders is positive and large, $\kappa > 0$, we conclude that the only stabilizing contribution comes from the electrons whose temperature in the expanders is several times lower than that of the ions. Another specific feature of dispersion relation (6) is appearance of the linear term in $\omega$. The imaginary part of the coefficient $A$ is due to the electron current to the wall; in general, it gives rise to a decrease of the
growth rate (the line tying effect) of the instability for low-$m$ modes [2]. The real part of $A$ stems from charge uncovering effect. It can improve the stability similar to finite Larmor radius effect. In addition ion FLR effects, omitted here, can contribute to the charge separation stabilization for $m \neq 1$.

3. Electron Temperature-Gradient Instability Induced by Conducting End walls in Mirror Devices

In this section, in contrast to the previous one, we concentrate on the effect of a radial gradient of the unperturbed electron temperature $T_e$. This introduces a source of instability that produces growth rates that are fast compared to an MHD curvature driven instability. An important feature of the instability is that it is present only in the case of conducting end walls. We consider the geometry of a typical mirror experiment (Fig. 1), using the "long thin approximation" $L_c \gg a$ (for notation see Fig. 1).

In a plasma with open field lines, the unperturbed plasma potential $\varphi$ with respect to the end walls is determined mainly by the electron temperature with $e\varphi/T_e$ typically 3-5 depending on collisionality and $e = -q_e$. When we take into account the radial gradient of $T_e$, the electric potential then also varies and thereby have an equilibrium electric field which introduces the centrifugal instability drive.

The analysis is for a low-beta plasma and we restrict ourselves to electrostatic (curl $E = 0$) flute-like perturbations ($\partial \delta \varphi/\partial s = 0$ where $s$ is the coordinate along the field line). We consider a mirror device like a tandem mirror (TM) or GDT where the particle lifetime $\tau$ in the solenoidal part is much larger than the ion collision time and ion bounce time. Under such conditions the particles in the central cell are Maxwellian and are homogeneously distributed between the end plugs (these plugs would be the strong mirrors in case of GDT and electrostatic barriers in the case of TM). This implies that there is negligible axial equilibrium electric field in the central region, while at the ends (in the plug, expander and sheath regions) the axial electric field accelerates ions to sonic speeds. We assume that the plugs are short enough so that they do not contribute to the inertia of the flute. In the analysis the only characteristic that is required is the response of the current flowing through the inner side of the plug and as a result it is not necessary to describe the detailed structure of the plugs.

We show below that the instability we are studying typically has a growth rate greatly in excess of the ion bounce frequency $v_{Ti}/L_c$. This means that we can neglect the curvature driven terms responsible for the usual flute instability. However, we retain a possible competitive term that is responsible for the centrifugal instability.
To derive the equations describing the instability we first consider the response of the plasma column to the flute-like perturbation of the electrostatic potential. As in Eq. (5) we obtain the relation of \( S_j \) at \( z = Lc/2 \) to the flute response.

Secondly, we write the boundary condition for \( \delta j || \) in terms of \( \delta \phi \). If the end walls are conducting, then this boundary condition is (cf. Ref. 3):

\[
\delta j || = \chi \left( \delta \phi + \frac{\partial \phi}{\partial \psi} \delta \psi \right)
\]

(9)

where \( \psi \) is the flux variable, \( \varphi(\psi) \) is the unperturbed potential distribution, \( \delta \psi \) is displacement of the flute from the initial flux surface, and \( \chi \) is some coefficient that depends on plasma parameters and, generally speaking, depends on the complex frequency of the perturbations. The potential at the conducting wall is assumed to be zero. Equation (9) has the property that the potential perturbation for a rigid displacement (i.e. \( \delta \phi = -\frac{\delta \varphi}{\delta \psi} \delta \psi \)), does not produce current as then the axial loss mechanisms on the tube do not change. Hence the electrical current from the ends remain zero, and the current is the same as the unperturbed state. Of course, this conclusion is valid only for a pure displacement mode when there is no transverse transport. Thirdly, we equate the two expressions for \( \delta j || \) and obtain the desired equation for \( \delta \phi(\psi) \).

Note that in the case of insulating walls the boundary condition is \( \delta j || = 0 \) and the conducting wall \( T_e \) gradient drive disappears as the perturbed end currents entirely disappear.

We consider perturbations of the form \( f(\psi) \exp(-i\omega t + im\theta) \) where \( \theta \) is the azimuthal angle. For such perturbations, from the MHD analysis of the cross-field motion quite similar to the one made in Ref. [16], we obtain in the long thin approximation using \( \delta \psi = 2\pi mc\delta \varphi / \Omega \),

\[
\left. \frac{\delta j ||}{B} \right|_{Lc/2} = iA \left\{ \Omega^2 \left[ \delta \psi - \frac{4\psi}{m^2 n} \frac{\partial}{\partial \psi} \psi n \frac{\partial \delta \psi}{\partial \psi} \right] + \frac{4\Omega}{m} \left[ 2\psi^2 \right. \right.
\]

\[ \left. \left. + \frac{\partial \omega_B}{\partial \psi} \frac{\partial \delta \psi}{\partial \psi} - \frac{\psi}{n} \frac{\partial (n \omega_B)}{\partial \psi} \delta \psi \right] - 2\omega_B^2 \frac{\psi}{n} \frac{\partial n}{\partial \psi} \delta \psi \right\} \]

(10)

where \( n \) is the unperturbed plasma density (homogeneous between the plugs), \( \Omega = \omega - m \omega_B \), \( \omega_B \) is \( E \times B \) rotation frequency, and \( A \) is defined as follows

\[
A = \frac{mM_c n}{2\psi} \int_0^{Lc/2} ds / B^2
\]

(11)
By combining Eqs. (9)–(11), we obtain a single differential equation for the eigenfunction $\delta \psi(\psi)$, from which the eigenfrequency $\omega$ can be determined,

$$
\Omega^2 \left\{ \delta \psi - \frac{4\psi}{m^2n} \frac{\partial}{\partial \psi} \psi \frac{\partial \delta \psi}{\partial \psi} \right\} + \Omega \left\{ \frac{i \nu}{m^2 \psi_\varphi} \delta \psi + \frac{4}{m} \left[ 2\psi^2 \delta \psi \right] \right\} + \frac{\partial \omega_E}{\partial \psi} \frac{\partial \delta \psi}{\partial \psi} - \frac{\delta \psi}{n} \frac{\partial (n \omega_E)}{\partial \psi} \frac{\delta \psi}{\partial \psi} \right\} + i \frac{\Gamma^2}{m} \frac{\psi}{\psi_\varphi} \delta \psi - \frac{2 \omega_E^2 \psi}{n} \frac{\partial \psi}{\partial \psi} \delta \psi = 0 \quad (12)
$$

where

$$
\nu = \frac{2 \chi \omega_B \psi_\varphi}{\pi ecn L_c}, \quad \Gamma^2 = \frac{2 \chi \omega_B \psi_\varphi}{en L_c}, \quad \psi_\varphi^{-1} = \frac{2}{\varphi} \frac{\partial \varphi}{\partial \psi}
$$

$$
\omega_{Bi}^{-1} = \frac{2}{L_c} \omega_{Bi} (L_c/2) \int_0^{L_c/2} dz/\omega_{Bi}^2 \quad (13)
$$

The meaning of $\psi_\varphi$ is that of the "scale length" of the magnetic flux coordinate of the unperturbed plasma potential; the dimension of $\nu$ and $\Gamma$ is that of frequency.

Note that Eq. (12) has a solution $m = 1$, $\omega = 0$, $\delta \psi \propto \sqrt{\psi}$ which corresponds to a displacement of the plasma column as a whole.

For further analysis we consider the perturbations that are sufficiently slow so that the coefficient $X(\omega)$ in Eq. (9) is $X(0)$, independent of $\omega$. For GDT this implies that the time $L_{ex}/v_{T_i}$ in which an ion traverses the expander (see Fig. 1) is much shorter than $\omega^{-1}$.

For the localized perturbations of the type $m > \psi \frac{\partial}{\partial \psi} \gg 1$, Eq. (12) reduces to the dispersion relation

$$
\Omega^2 + \Omega \frac{i \nu}{m^2 \psi_\varphi} \psi + i \frac{\Gamma^2}{m} \frac{\psi_\varphi}{\psi_\varphi} - \omega_E^2 \frac{\psi}{\psi_\varphi} = 0 \quad , \quad \psi_\varphi^{-1} = 2 \frac{\partial \ln n}{\partial \psi} \quad (14)
$$

The last term describes the usual centrifugal instability. The term proportional to $\nu$ appears in the equation because of the terminal ohmic resistance of the end sections of the device, including the ohmic resistance of the Debye sheaths. This term leads to the dissipative response described in Ref. [2]. The term proportional to $\Gamma^2$ is the one that is responsible for the instability we are considering. In mirror machines the potential variation across the plasma column is naturally proportional to the radial variation of the electron temperature, and hence the centrifugal drive is also related to the electron temperature gradient.

Depending on the rotation frequency $\omega_E$, the instability is either purely centrifugal at large $\omega_E$ (if it is assumed that the density is radially
FIG. 2. Normalized growth rates for the conducting wall $T_e$ gradient instability for various parameters of the centrifugal drive.

decreasing) or a purely dissipative electron temperature-gradient drive at small $\omega_E$. The results of the corresponding numerical analysis are presented in Fig. 2. The dimensionless variables used in this figure are

$$\tilde{\omega} = \frac{\nu}{\Gamma^4} \frac{\psi^2}{\psi}$$

One can show [17] that for a collisional plasma typical for GDT, one has

$$\chi = \frac{e^2 n L_c}{2 T_e \tau}, \quad \frac{e \varphi}{T_e} \equiv \Lambda \sim 5$$

For the numerical parameters of the GDT experiment [19] ($T_e = T_i = 100$ ev, $L_c = 700$ cm, $\bar{\omega}_{Bi} = 2 \times 10^7$ s$^{-1}$, $L/a = 50$, $\tau = 30 L_c/v_T$) with the assumption that all scale lengths are equal to the characteristic of the plasma radius ($\psi = \psi_T = \psi$) it turns out that $\tilde{\omega}^2 = 0.2$. Thus the centrifugal effects are small and our electron temperature gradient instability is dominating. The maximum growth rate is 4 times larger than the ion bounce frequency $L_e/v_T$; i.e., the instability is indeed faster than the curvature driven MHD instability. It is interesting to note that the electron temperature gradient instability does exist even in the regions where the plasma density increases radially, and plasma is centrifugally stable ($\tilde{\omega}^2 < 0$).
The detailed analysis of the FLR effects was carried out in Refs. [17,18]. In the limit $m \gg \psi/\partial \psi \gg 1$, they can be taken into account if one adds to the left-hand side of Eq. (14) the term $-m\Omega \Gamma^2 \delta$, where $\delta = \frac{e\eta T_i}{e} \left( \frac{1}{\psi_n} + \frac{1}{\psi_{n_i}} \right) \frac{\nu}{\Gamma^2}, \psi_{n_i}^{-1} = 2 \frac{\partial \ln T_i}{\partial \psi}$. The influence of the FLR effects on the conducting wall electron temperature gradient instability is illustrated by Fig. 3. For the numerical example given above they are still relatively unimportant ($\delta = 0.2$).

The analysis of the finite beta effects has been published elsewhere (see [18]).

4. Conclusion

We have investigated the effect of conducting end walls on the stability of plasmas on open field lines. Two strong destabilizing effects have been obtained. For GDT, it was found that the stabilization mechanism with insulating boundary conditions that arises from momentum flux of escaping ions passing through the favorable curvature of the expander disappears in the conducting boundary limit. Elsewhere [13], the mecha-
nism responsible for stabilization in the insulating boundary condition limit is interpreted as a passive feedback mechanism. With insulating boundaries, the end potentials at the wall can oscillate in just the appropriate way to regulate the electron current to just equal the ion current. The perturbed ion end current is proportional to the curvature and escaping ion momentum flux in the expander region, and hence so is the electron current. The oscillating end potential in turn induces a perturbed electron density that is proportional to the ion MHD stabilization term that arises in the low beta electrostatic dispersion relation. When a conducting boundary is present, the end potential cannot oscillate and this passive "feedback" no longer exists. As a result, in this case the ion momentum flux does not produce an MHD stabilization term from the expander. The ions pass through the expander too quickly to produce a stabilizing density perturbation. However, the electron pressure in the expander does contribute a stabilizing term, since the electrons in the expander are primarily trapped by the ambipolar potential and stay in the expander many bounce times.

Optimal stable operation of the GDT thus requires insulating boundaries. Special insulating end materials that remain insulating when exposed to the plasma flux, or segmented ends to stabilize low-\(m\) numbers would then seem to be required. Alternatively, direct feedback mechanism can be designed if the passive feedback mechanism from insulating boundaries cannot be established.

The second part of the paper discussed a new electron temperature gradient instability that arises with conducting boundary conditions but is absent with insulating boundary conditions. Typically, the growth rate of this instability is much greater than the ion bounce time, and hence this instability may produce strong turbulence effects which can flatten the electron temperature profile. One of the experimental puzzles has been the absence of an expected cross-field electron temperature gradient, and perhaps this instability now affects GDT operation. A competitive instability mechanism drive can be the more conventional centrifugal force drive that comes from plasma rotation. It is interesting to note that the electron temperature gradient implies plasma rotation as \(e \varphi / T_e\) is constant in equilibrium. For present GDT parameters the new conducting wall electron temperature gradient drive appears to be a more important drive than centrifugal or FLR effects.

Finally, we emphasize that we have demonstrated that the plasma wall interaction can have a basic effect on the overall stability of a plasma. This interaction should be of basic interest in a variety of other plasma situations with open field lines. Examples include the open field lines of a toroidal plasma in contact with a limiter or divertor, the plasma in a magnetosphere that is in contact with an ionosphere or the plasma on flux tubes on the surface of the sun.
REFERENCES

PROBLEMS OF GLOBAL PLASMA COLUMN EQUILIBRIUM IN STELLARATORS

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Abstract

PROBLEMS OF GLOBAL PLASMA COLUMN EQUILIBRIUM IN STELLARATORS.

Problems related to the equilibrium of the plasma column as a whole in ordinary stellarators are considered. The problem of an external transverse (vertical) magnetic field $B_\perp$ on the finite $\beta$ plasma equilibrium is analytically solved. The problems of magnetic diagnostics and the effects accompanying a plasma pressure rise in a flux conserving stellarator are discussed.

1. INTRODUCTION

Ensuring the equilibrium of the plasma column as a whole is one of the most important conditions for organizing a plasma discharge in fusion devices. Two classes of problems are related to the general problem of a 'global' plasma equilibrium:

(a) Finding the external (control) magnetic fields necessary for providing the desired shape of the plasma column and maintaining it in a given equilibrium position;
(b) Determination of plasma induced fields (due to equilibrium plasma currents), the measurement of which can provide important information about integral plasma parameters.

For tokamaks these problems were thoroughly studied long ago [1]. The necessity of their solution for stellarators has not been so evident up to the present. The point is that in stellarators the problem of maintaining the plasma column as a whole in equilibrium is partially solved by the very method of magnetic configuration production: vacuum magnetic field lines form a family of nested surfaces with some 'margin of strength' against perturbations produced by the plasma. In the experiments with a low $\beta$ plasma this margin was sufficient. Will it be sufficient for the achievement of those plasma parameters on which stellarators are based? The answer to this question is important in principle and also in direct connection with the startup of the large stellarators ATF and Wendelstein VII-AS and with the design and construction of future devices. The calculations show (see below) that this margin is insufficient: the $\beta$ induced displacement of a plasma column can be intolerably large at high $\beta$ in stellarators. The magnetic field configuration in a stellarator should be
properly adjusted during the discharge to avoid this effect and thereby to make the achievement of $\beta$ close to $\beta_{eq}$ possible. Thus the complementary problems (a) and (b) above become urgent for stellarators, too, at high $\beta$.

2. GENERAL SOLUTION

A strict general solution to these two problems for an ordinary stellarator with a plane circular axis is given in Refs [2, 3]:

$$\oint_{r_p} G \mathbf{B} \cdot d\mathbf{r} - \frac{1}{2\pi} \oint_{r_p} \psi_v \frac{\partial G}{\partial n'} \mathbf{d}s' = \begin{cases} rA_{pl} & \text{in vacuum} \\ -rA_{ext} + \text{const} & \text{in plasma} \end{cases}$$

(1)

The solution relates a toroidal component $A$ of a magnetic vector potential to the equilibrium field $\mathbf{B}$ at the plasma boundary and to the function $\psi_v$ characterizing the vacuum stellarator configuration. Here we consider average characteristics: $\mathbf{B}$ is the field whose components are independent of the longitudinal co-ordinate (toroidal angle) $\xi$; $\Gamma_p$ is the boundary of the transverse plasma column cross-section averaged over $\xi$; $\partial/\partial n = \mathbf{n} \cdot \nabla$, where $\mathbf{n}$ is the normal to $\Gamma_p$; and $G(\mathbf{r}, \mathbf{r}')$ is Green's function for the equation

$$r \, \text{div} \left( \frac{\nabla (rA)}{r^2} \right) = -j$$

(2)

with conditions $rA \to 0$ at $r \to 0$, $r^2 + z^2 \to \infty$ ($2\pi G$ is the poloidal magnetic flux of a ring unit current).

The poloidal field produced by plasma currents is expressed in terms of $A_{pl}$ by the formula

$$\mathbf{B}_{pl} = \nabla (rA_{pl}) \, \nabla \xi$$

(3)

The external poloidal field independent of $\xi$ is expressed in a similar way through $A_{ext}$.

Expression (1) differs from the similar one for a tokamak by a term with $\psi_v$. This difference is related to the fact that in a stellarator the magnetic surfaces of an axisymmetric field

$$\mathbf{B} = \frac{1}{2\pi} \left[ \nabla (\psi - \psi_v) \, \nabla \xi \right] + \frac{1}{2\pi} F \, \nabla \xi$$

(4)

do not coincide with the averaged magnetic surfaces $\psi = \text{const}$. The function $\psi_v$
included in (1) and (4) is determined by the helical stellarator field structure and can be calculated from the known vacuum field to any needed accuracy. In the stellarator approximation

$$\psi_v = \frac{2\pi r^3}{R_0 B_0} \left\{ \mathcal{B}_r \int \, d\zeta \right\}$$  \hspace{1cm} (5)

the brackets $\left\langle \ldots \right\rangle_r$ denote an averaging over $\zeta$. The consequence of (5) is the similar and illustrative relationship

$$\psi_v = \psi_v(\rho) = -2\pi B_0 \int \rho \mu_h(\rho) \, d\rho$$  \hspace{1cm} (6)

which is valid for practically all existing stellarators (except Wendelstein VII-AS). Here, $\mu_h$ is the vacuum rotational transform and $\rho$ is the radius measured in the transverse cross-section from the stellarator geometrical axis $r = R_0$. This expression for $\psi_v$ with the parabolic (at $A_{\text{ext}} = 0$) profile of $\mu_h$,

$$\mu_h = \mu_0 + \mu_2 \rho^2$$  \hspace{1cm} (7)

will be used below in the examination of concrete problems.

We restrict ourselves by consideration of stellarator configurations with a large aspect ratio $R/b$. In this case the function $G$ included in (1) can be replaced by its approximate expression

$$G_t(\vec{r}, \vec{r}') = G_0 - \frac{k(x + x')}{2} \left( G_0 + \frac{R}{2\pi} \right)$$  \hspace{1cm} (8)

where the toroidal corrections are taken into account in a linear approximation. Here, $k = 1/R$ and $R$ is the radius of the circular axis (e.g. the plasma column axis), around which the expansion of $G$ is done, $x = R - r$,

$$G_0 = \frac{R}{2\pi} \left( \ln \frac{8R}{[(x - x')^2 + (z - z')^2]^{1/2}} - 2 \right)$$  \hspace{1cm} (9)

All further calculations are made for a plasma column with a circular average cross-section. With a change in $\beta$ and under the action of an external vertical field the radial position of the plasma column changes. The shift $\Delta_0$ of a circular plasma column relative to the geometrical axis, $r = R_0$, in the $\vec{z}$ direction can be explicitly taken into account, presenting the plasma boundary $\Gamma_p$ by the equation

$$b^2 = \rho^2 + \Delta_0^2 + 2\rho \Delta_0 \cos u$$  \hspace{1cm} (10)
The adopted assumptions allow one to reduce the basic equation (1) to the form

\[ f_0 + f_1 \cos \alpha + \sum_{n=2}^{\infty} f_n \cos n\alpha = \begin{cases} rA_{pl} & \text{in vacuum} \\ -rA_{ext} + \text{const} & \text{in plasma} \end{cases} \]  

(11)

where

\[ f_0 = bRB_j \left( \ln \frac{8R}{r_{max}} - 2 \right) \]  

(12)

\[ f_1 = bRH_1 \frac{\xi}{2} - \psi_1 \frac{b}{4\pi} \frac{\partial \xi}{\partial b} - \frac{b}{2} dB_j \left( \ln \frac{8R}{r_{max}} - 1 + \frac{b}{\ell} \frac{\xi}{2} \right) \]  

(13)

\[ f_n = bRH_n \frac{\xi^n}{2n} - \psi_n \frac{b}{4\pi n} \frac{\partial \xi^n}{\partial b}, \quad n \geq 2 \]  

(14)

Here, \( \xi = \ell_{min}/\ell_{max} \), \( \ell_{min} \) is the lesser distance of \( \ell \) and \( b \), \( \ell_{max} \) is the longer; \( \ell, \alpha \) are the quasi-cylindrical co-ordinates associated with the plasma column centre (with the axis shifted by \( \Delta b \) relative to the geometrical axis); \( B_j = J/2\pi b \) is the field of the longitudinal current \( J \) at the plasma boundary; \( H_n \) and \( \psi_n \) are the coefficients in a Fourier expansion of the quantities \( B_\alpha = \overline{B} \cdot \overline{e}_\alpha \) and \( \psi_\alpha \) at \( \Gamma_p \):

\[ \overline{B} \cdot \overline{e}_\alpha |_{\Gamma_p} = B_j + \sum_{n=1}^{\infty} H_n \cos n\alpha \]  

(15)

\[ \psi_\alpha |_{\Gamma_p} = \sum_{n=0}^{\infty} \psi_n \cos n\alpha \]

3. EFFECT OF AN EXTERNAL VERTICAL FIELD ON THE PLASMA EQUILIBRIUM IN STELLARATORS

The equality (11) for \( rA_{ext} \) completely determines the external magnetic field necessary (in addition to a helical stellarator field) for the circular average cross-section plasma column to be maintained in a given position. As shown in Ref. [3], the direct consequence of Eq. (11) is the relationship
\[ B_\perp = -\frac{b}{2R} \left( B_r \left( \ln \frac{8R}{b} - \frac{3}{2} \right) - B_0 \left[ \mu \Delta' \left( 1 + \frac{\Delta'^2}{2b} \right) \right] \right) + \Delta \left( \frac{a^2 \mu_b}{a^2} \right) \bigg|_{r_p} \]

which binds the external vertical field \( B_\perp \), i.e. \( B_z \) at the geometrical axis \( r = R^0 \) (in Ref. [3] \( B_z \) is calculated at \( r = R_0 + \Delta_b \)), with the plasma column shift. Here, \( \Delta' \) is the derivative of the shift \( \Delta \) over the minor radius \( a \), \( \mu(a) \) is the rotational transform coinciding with \( \mu_\beta \) at \( \Delta = 0 \) and \( B_\perp = 0 \), and \( \mu_1 = RB_1/aB_0 \) is the rotational transform produced by a longitudinal current when the magnetic surfaces are not shifted.

The unknown parameters in (16) implicitly dependent on the pressure and current profiles are \( \Delta_b \) and \( \Delta' \). Finding these quantities requires solving the equilibrium problem, which is reduced in the simplest case to the equation for the shift of magnetic surfaces [4, 5]:

\[ \left( \frac{a^2 \mu_\beta}{a} \right)' + \frac{\Delta}{a^2} [a^3 (\mu_\beta - \mu_1)]' = \frac{2p'(a)R}{\mu B_0^2} \]

Important consequences of (16) can be seen, however, even without its solution: at \( \beta - \beta_{eq} \), in the absence of a vertical field the plasma column should be noticeably shifted outwards. It can be maintained at a fixed position by the vertical field \( |B_\perp| \leq B^*/2 \), where \( B^* = B_0 \mu_\beta(b/R) \). Indeed, \( \Delta'(b) \equiv -1 \) at \( \beta \) close to the equilibrium limit, so that \( \Delta_{eq}/b \equiv 0.5-0.25 \). If \( \Delta_b = 0 \) is required, in this case the necessary value \( B_\perp = -B^*/2 \) will be obtained from Eq. (16).

For a shearless stellarator Eq. (18) can be easily solved analytically. As a result, one obtains for a currentless plasma

\[ \frac{\Delta_b}{b} = \frac{B_\perp}{B^*} + \frac{\beta}{2\beta_{eq}} \]

Here, \( \beta_{eq} = \mu^2 b/R \). It is stated in Ref. [6] that in the \( \ell = 2 \) stellarators magnetic surfaces are not sensitive to finite plasma pressure. Therefore there is no need for a vertical field in these systems, although it is desirable for \( \ell = 3 \) stellarators [6].
study shows that the vertical field is necessary in both systems to achieve high $\beta$. Moreover, from (16) it follows that at $B_\perp = 0$ in the $\ell = 3$ stellarator $\Delta\psi/b \leq 0.25$, and in $\ell = 2$ systems, as seen also from (19), the plasma column shift at $\beta \rightarrow \beta_{eq}$ turns out to be twice as large. This difference is related to a stronger dependence of the magnetic axis shift in the $\ell = 3$ stellarators on $\beta$, which finally determines an equilibrium limit $\beta_{eq}$.

4. PROBLEMS OF PLASMA MAGNETIC DIAGNOSTICS IN STELLARATORS

Magnetic measurements are invariably an important part of the plasma diagnostics in tokamaks and stellarators. There were some attempts to determine the plasma energy content [7, 8], its pressure profile [8, 9] and the magnitudes of Pfirsch-Schlüter currents [9] from the measured poloidal field in stellarators. In these cases certain assumptions on the unknown pressure and plasma current profiles are adopted to obtain a desired result. Therefore the accuracy of these methods is not very high.

The possibilities of magnetic diagnostics in tokamaks and stellarators, based on poloidal measurements, are actually determined by Eq. (1), which has been reduced for the circular plasma column to the form (11). Analytical calculations show, and the numerical calculation [10] confirms, that in this case for representing $rA_p$ it is sufficient to retain only the first two terms in the left hand side of (11). It is clear that the most one can count on in the poloidal field measurements is to find $H_1$ and $\psi_1$ (in addition to $B_1$). Using $H_1$ and $\psi_1$, one can find, for example, $A_b$ and $A_\perp$ at the plasma boundary. Strictly speaking, from the poloidal field measurements, not knowing the current and plasma pressure distributions and without solving the equilibrium problem, one can determine only two independent quantities (besides plasma current) characterizing a plasma with a circular average cross-section.

The measurable values are the total poloidal magnetic flux, $\psi_{pol} = 2\pi r(A_{pl} + A_{ex})$, and the local magnitudes of the magnetic field. One should know the magnitude of the external field $B_\perp$ to separate the plasma contribution to the measured signals. In the general case, it is natural to assume that $B_\perp$ is an unknown (image currents can make a noticeable contribution to $B_\perp$). Thus the three parameters $H_1$, $\psi_1$ and $B_\perp$ are unknown in the problem of magnetic measurements. Relationship (11) gives all three equations necessary for their determination. The first equation is the dependence of $B_\perp$ on $H_1$ and $\psi_1$ [3]. The other two, independent equations can be obtained by calculating the measurable magnetic quantities with the help of (11). Remember that we are speaking here about magnetic field components non-oscillating over $\xi$. A helical magnetic field does not make any contribution to the poloidal flux measured by large ring-like loops ($\psi$ loops) but can affect the results of local measurements. To find the magnetic field components independent of $\xi$, it is sufficient to add the signals from the similar probes located at similar positions at
two cross-sections which are at a distance one from another equal to the half-period of a helical field.

Let us set down the necessary equations in which the plasma shift is taken into account in a linear approximation. The equation for the external vertical field [3] has the form

$$B_\perp = -\frac{b}{2R} B_j \left( \ln \frac{8R}{b} - 1 \right) + B^* \Delta b + C_0$$

(20)

The constant $C_0$ included here can be expressed in terms of $\Delta b$ and $\Delta'$:

$$\frac{R}{b} C_0 = 0.25B_j + 0.5B_0 (\mu \Delta' + 2\mu_2 b \Delta_b)$$

(21)

For the measurable total poloidal flux $\psi_{pol}$ and azimuthal component of the poloidal field $B_u = \bar{B} \cdot \xi_u$ from (11) one obtains

$$\psi_{pol} = \psi_j - 2\pi R \rho \bar{B}_\perp \cos u$$

(22)

$$B_u = B_j \frac{b}{\rho} + B_1 \cos u$$

where $\psi_j = 2\pi f_0(\rho)$,

$$\bar{B}_\perp = B_\perp - C_0 \frac{b^2}{\rho^2} + B_j \frac{b \Delta_b}{\rho^2} + B_j \frac{b}{2R} \left( \ln \frac{8R}{\rho} - 1 \right)$$

(23)

$$B_1 = B_\perp + C_0 \frac{b^2}{\rho^2} - B_j \frac{b \Delta_b}{\rho^2} + B_j \frac{b}{2R} \ln \frac{8R}{\rho}$$

(24)

From the measured quantities $\bar{B}_\perp$ and $B_1$ one can find the plasma column shift relative to the geometrical axis:

$$2(B^* + B_j) \frac{\Delta b}{b} = \bar{B}_\perp + B_1 + \frac{\rho^2}{b^2} (\bar{B}_\perp - B_1)$$

$$+ B_j \frac{b}{R} \left[ \ln \frac{\rho}{b} + \frac{1}{2} \left( \frac{\rho^2}{b^2} - 1 \right) \right]$$

(25)
There was no necessity for plasma column position control in the experiments with \( \beta \ll \beta_{eq} \). When \( \beta \) is high and the plasma's own magnetic field greatly distorts the magnetic configuration, this control is necessary. The practical solution to the problem of high \( \beta \) plasma equilibrium over the major radius requires, in stellarators, the use of equilibrium control systems similar to those used in tokamaks [11]. The measurement of a plasma displacement relative to the desired position is an obligatory feedback element for such a system.

The quantity \( \beta \) does not appear explicitly in (20), (22) and (23); therefore in the general case the measurements of a poloidal field only are not sufficient for its determination. In Refs [7, 8] \( \beta \) is estimated by \( \overline{B} \perp \) measured with the \( \psi \) loops. The lack of information was partially compensated in those experiments by preliminary measurements allowing the chamber and helical winding effect on the measured signal \( \overline{B} \perp \) to be taken into account. Net current was small (\( B_I \ll B^* \)). Thus in fact the quantity \( C_0 \) was determined by measured \( \overline{B} \perp \). At low \( \beta \) and low longitudinal current the second term in the left hand side of Eq. (18) can be neglected. Then for \( C_0 \) one obtains

\[
C_0 \equiv 0.5B_0 \frac{b}{R} (\mu \Delta' + 2 \mu_0 b \Delta_b) = \int_0^b \frac{a^2}{b^2} \frac{p'(a)}{\mu(a) B_0} da
\]

(26)

It is easy to be sure that in the general case

\[
\frac{\overline{\beta}}{\mu_b} \leq - \frac{2}{B_0} \int_0^b \frac{a^2}{b^2} \frac{p'(a)}{\mu(a)} da \leq \frac{\beta_0}{\mu_b}
\]

(27)

where \( \beta_0 \) is the \( \beta \) at the magnetic axis and \( \overline{\beta} \) is the mean value of \( \beta \). The lower limit in (27) is reached when \( \mu = \mu_b \), i.e. in a shearless stellarator; the upper limit is reached in the \( \ell = 3 \) stellarator (\( \mu = \mu_0 a^2/b^2 \)). As seen from the derivation of (26) and from the final result, determination of \( \beta \) from the measurement of \( \overline{B} \perp \) only is not sufficiently reliable. In many respects the result depends on assumptions (circular magnetic surfaces, \( \mu \) profile independent of \( \beta \), etc.) which cannot always be justified. In that sense the discussed method of determining \( \beta \) is less adequate than diamagnetic measurements.

5. FLUX CONSERVING STELLARATOR

During rapid plasma heating the evolution of an equilibrium configuration occurs with frozen magnetic fluxes. In this case the rotational transform \( \mu(a) \) in the plasma remains unchanged, in spite of possible strong changes in the internal geometry, shape and position of the plasma column. Conservation of \( \mu(a) \) (see (17)) is
provided by the generation of a longitudinal current in the plasma. Some features of such stellarator equilibria have been considered in Ref. [5]. Here we consider the effects related to an equilibrium of the plasma column as a whole.

The total poloidal flux $\psi$ linked by a plasma column is

$$\psi = 2\pi r (A_{pl} + A_{ext}) + \psi_v$$  \hspace{1cm} (28)

The last term here characterizes the contribution of a helical field to $\psi$. During rapid plasma heating the magnitude of $\psi$ at the boundary, $\psi(b)$, should remain unchanged. Substituting explicit expressions $\psi_v$ and $rA_{pl}$ into (28), one obtains the equation for this quantity with the right hand side as a Fourier expansion like that in (15). By definition $\psi(b) = \text{const}$; therefore all the terms of the expansion except the first one should vanish. This condition relates the external field components to the equilibrium parameters and results, in particular, in (16). The very condition $d\psi(b)/dt = 0$ results for the circular plasma column in

$$2bRB_j \left( \ln \frac{8R}{b} - 2 \right) + R^2 B_\perp = \Delta_b^2 B_0 (\mu_0 + 2\mu_2 b^2)$$  \hspace{1cm} (29)

The initial current and vertical field (at $\beta \ll \beta_{eq}$, $\Delta_b = 0$) are assumed here to be absent. When $B_\perp = 0$, the relationship (29) gives the magnitude of the total longitudinal current induced in the plasma after its rapid heating. At $B_j = 0$ it gives the value of the uniform vertical field necessary to maintain an equilibrium in a flux conserving stellarator without generation of a longitudinal current. In both cases $B_\perp$ and $B_j$ are small and can be omitted in (16). Then, taking account of (26), one obtains the equation for the plasma column shift $\Delta_b$ after rapid plasma heating:

$$\frac{B^*}{B_0} \frac{\Delta_b}{b} = - \int_0^b \frac{a^2}{b^2} \frac{p'(a)}{\mu_0(a)B_0^2} \, da$$  \hspace{1cm} (30)

Substitution of (30) into (29) allows one to express the longitudinal current induced after rapid plasma heating in terms of the plasma pressure. At any pressure profile, when $B_\perp = 0$ (compare with (27))

$$\frac{1}{8} \frac{\beta^2}{\beta_{eq}^2} \leq \frac{B_j}{B^*} \left( \ln \frac{8R}{b} - 2 \right) \leq \frac{1}{4} \frac{\beta^2}{\beta_{eq}^2}$$  \hspace{1cm} (31)
6. CONCLUSION

Equation (1) turns out to be a suitable and effective means for solving the problems of plasma equilibrium as a whole. Generalizing the known virtual casing principle for tokamaks [1] to more complicated systems, Eq. (1) allows stellarators and tokamaks to be considered within the framework of a common approach. The majority of the results presented here are equally true for both systems.

An analysis of free boundary equilibria shows that the plasma column should be considerably shifted outwards at $B_\perp = 0$ in stellarators with $\beta$ rise. This shift can be experimentally determined from the magnetic measurements. Thus the efficiency of operation of the plasma equilibrium control system can be evaluated. The obtained equations relating the external control field to plasma parameters provide a physical basis for the design of such systems in stellarators.

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SPONTANEOUS POLOIDAL SPIN-UP OF TOKAMAKS AND THE TRANSITION TO H-MODE*

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Abstract

SPONTANEOUS POLOIDAL SPIN-UP OF TOKAMAKS AND THE TRANSITION TO H-MODE. The radial transport of toroidal angular momentum and circulation in a tokamak resulting from diffusion which is poloidally asymmetric is shown to produce an instability of the poloidal rotation. This instability, due to Stringer, sets in where the local particle confinement time is smaller than the damping time of poloidal flow and leads to poloidal velocity shears that may quell microturbulence. The nonlinear interplay between the poloidal spin-up and turbulence-driven anomalous transport is shown to lead to bifurcated equilibria of the type observed in the L to H mode transition in tokamaks.

Poloidal and toroidal rotation driven by a radial current source $J_r$ is considered. Parallel viscosity-driven and convective inertia-driven fluxes develop to cancel $J_r$ and maintain ambipolarity at the steady state. Because the poloidal Mach number $M_p$ is of the order of unity in the L-H transition, a shock appears in the inside of the torus when $1 - M_p \lesssim \sqrt{\epsilon}$, with $\epsilon$ the inverse aspect ratio, and rotates in the direction of poloidal flow as $M_p$ increases. The parallel viscosity associated with the shock is independent of collisionality and is the irreducible minimum to be overcome to have a supersonic poloidal rotation. It is shown that convective poloidal momentum transport weakens the shock viscosity in the edge region and facilitates the L-H transition.

1. INTRODUCTION

Recent experiments have shown that H-mode enhanced confinement [1] is accompanied by and, perhaps, is a result of a large increase in the poloidal rotation of the edge plasma [2,3]. In the CCT device, improved confinement is achieved by applying an external torque to rotate the edge plasma [3]; in DIII-D, the poloidal rotation is seen to grow spontaneously, with improvement in confinement appearing in a concomitant manner [2]. In this report, two aspects of poloidal rotation and its relation to the L-H transition are considered. In Section 2, we report on a mechanism for spontaneous poloidal spin-up leading to bifurcation of equilibria. In Section 3, the formation of a shock at large poloidal Mach numbers in relation to parallel viscosity and convective fluxes is examined.

2. SPONTANEOUS SPIN-UP AND BIFURCATION (Hassam, Antonsen, Dimits, Drake, Guzdar, Lau, Liu)

In this section, we show that tokamaks can spontaneously develop poloidal velocity shear. The velocity shear builds up as a direct consequence of the radial transport of particles and momentum and can occur in a tokamak where the particle diffusivity is poloidally asymmetric and the local particle confinement time is shorter than the damping time of poloidal rotation. We further show that this instability of the poloidal rotation, being intimately connected with particle transport, leads to a bifurcation of the equilibrium density profile with the bifurcated state having steep density gradients at the edge. The bifurcation stems from an interplay of two effects: the fact that the poloidal spin up is controlled by particle diffusion and that, in turn, particle diffusion is influenced by poloidal rotation since shear in the latter is likely to quell the microturbulence that precipitates anomalous particle diffusion.

The fact that tokamaks can spontaneously spin up poloidally was shown by Stringer [4] who found that an initial poloidal rotation in the presence of Pfirsch-Schluter diffusion was unstable [4,5]. It was pointed out, however, that poloidal rotation is strongly damped by magnetic pumping, at a rate somewhat less than $\nu_{ii}$, the ion-ion collision rate [6,7]. The Pfirsch-Schluter transport rate being generally much smaller than $\nu_{ii}$, the Stringer spin-up was considered unimportant. In reality, however, the particle loss rate is much larger than neoclassical, so spin-up is possible. By incorporating a general poloidally dependent particle
loss in the theory, we find that the spin-up occurs if the in-out asymmetry in the particle transport rate, measured by $\delta$, the fractional difference in transport rates inside and outside, is sufficiently large. The condition for spin-up is roughly given by (see also Eqs. (6) and (7))

$$\frac{(\delta/\epsilon)D}{L_n^2} > \gamma_{MP},$$

(1)

where $\epsilon$ is the inverse aspect ratio, $D$ is the particle diffusivity, $L_n$ is a measure of the radial scale of density variation, and $\gamma_{MP}$ is the magnetic pumping rate. For DIII-D type parameters [2] ($D = 10^4$ cgs, $n = 10^{13}$ cgs, $T = 100$ eV, with $\gamma_{MP}$ taken to be $\nu_{ii}$), (1) becomes $L_n < (\delta/\epsilon)^{1/2}$ cm.

The equations governing the poloidal spin-up are the toroidal transport equations of mass, toroidal angular momentum, and parallel flow. They are obtained in the usual manner by assuming an ideal equilibrium with flow on the MHD time scale and computing the slow, transport scale evolution of the plasma upon annihilating the "magnetic differential" operators by taking flux surface averages [7]. This procedure yields three flux surface averaged equations for the density, $\langle n \rangle$, the toroidal angular momentum, $\langle n R v_\phi \rangle$, and the circulation, $\langle v_\| B \rangle$. In the absence of sources, mass and toroidal angular momentum are conserved; circulation, however, is only convected. The resulting equations expressing the two conservation laws and the convection of circulation, expanded to lowest order in $\epsilon$, are: [7]

$$\frac{\partial n}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (rn\bar{v}_r) = 0,$$

(2)

$$\frac{\partial}{\partial t} [nV_T] + \frac{1}{r} \frac{\partial}{\partial r} [nV_T\bar{v}_r - qV_p\bar{v}_r] = 0,$$

(3)

$$\frac{\partial}{\partial t} [V_T + \Theta(1 + 2q^2)V_p] + \bar{v}_r \frac{\partial V_T}{\partial r} - \bar{v}_r \frac{\partial}{\partial r} [qV_p] + \text{magnetic pumping} = 0,$$

(4)

where in (4) damping terms from magnetic pumping [6,7] will be included later.

Equations (2)–(4) describe the evolution of the averaged quantities $\langle n \rangle = n(r)$, $\langle n v_\phi R/R_0 \rangle \equiv nV_T(r)$, $\langle v_\theta R/R_0 \rangle \equiv V_p(r)$ in the usual $(r, \theta, \phi)$ toroidal coordinates assuming circular, concentric flux surfaces with $\vec{B} \equiv [0, \Theta(r), 1] B_\theta(r) R_0/R$ assumed given, $R \equiv R_0 + r \cos \theta,$
\[ q(r) \equiv r/\Theta R_0 \] being the safety factor, and \( \langle f \rangle \equiv \int (d\theta/2\pi)(R/R_0)f \). The toroidal and poloidal flows, \( V_T \) and \( V_p \), are assumed to be subsonic but larger than diamagnetic speeds. The quantities \( \bar{v}_r \) and \( \bar{v}_r \) represent radial diffusive velocities. Diffusive velocities arise as a consequence of electron-ion momentum transfer. In our case, we assumed a general momentum transfer term \( \bar{R}_\perp \) such that \( \bar{R}_\perp \) is parallel to \( B \times \nabla n \). With this form, the radial diffusive flux is \( n\bar{v}_r = \bar{R}_\perp/eB; \bar{v}_r \) and \( \bar{v}_r \) are defined according to \( \bar{v}_r \equiv \langle v_r \rangle \) and \( \bar{v}_r \equiv \langle 2\cos \theta v_r \rangle \).

We now show that poloidal spin-up arises provided the radial flux is poloidally asymmetric, i.e., \( \bar{v}_r \neq 0 \). Equations (2)–(4) can be reworked to obtain an equation for \( V_p(r, t) \); we eliminate \( \partial n/\partial t \) from (3) using (2) and use the resulting equation to eliminate \( \partial V_p/\partial t \) from (4) to obtain

\[ \Theta(1 + 2q^2)\frac{\partial V_p}{\partial t} + \gamma_{MP} V_p + qV_p \frac{1}{nr} \frac{\partial}{\partial r}(nr\bar{v}_r) = 0, \]  

(5)

where we have now introduced the damping due to magnetic pumping as \( \gamma_{MP} V_p \). From (5), we note that poloidal flow may be unstable only if \( \gamma_{MP} \) is small enough. In particular, since the \( \bar{v}_r \) term is a particle transport term, we obtain condition (1) if we assume \( \bar{v}_r \sim \delta D/L_n \). Note that even if (1) is satisfied, the rotation is unstable only if the particle transport term in (5) is negative, i.e., a necessary condition for instability is

\[ (\partial/\partial r)(nr\bar{v}_r) < 0. \]

(6)

The physical mechanism underlying this spin-up can be traced to the so-called "Pfirsch-Schluter" flows of a poloidally rotating tokamak. These flows are harmonic toroidal flows that are necessarily associated with poloidal rotation, i.e., there cannot be a purely poloidally rotating tokamak plasma. As a tokamak plasma rotates poloidally, the flux tubes alternately compress and decompress thus driving parallel flows to keep \( \nabla \cdot \vec{v} \approx 0 \). From the latter equation and the fact that the electric potential is a flux function, it can be shown that in terms of \( V_T(r) \) and \( V_p(r) \), defined earlier, \( v_\theta \) and \( v_\phi \) are given by \( v_\theta = V_p R_0/R \) and \( v_\phi \simeq V_T - 2qV_p \cos \theta \). Thus, if \( v_\theta \neq 0 \), \( v_\phi \) has a harmonic component, independent of \( \epsilon \) even if there is no average toroidal flow. It is then clear that poloidally asymmetric transport (\( \bar{v}_r \neq 0 \)) can affect the transport of angular momentum in a nontrivial manner since the harmonic parts of \( v_\phi \) and \( v_r \) can couple. This coupling results in the "cross terms" in (3) and (4), proportional to \( qV_p \) and \( \bar{v}_r \); the cross terms are of the same order
as the "direct" terms (proportional to $V_T$ and $\bar{v}_r$) and are responsible for the spin up. As the plasma diffuses radially, the energy released from adiabatic expansion ends up in a build-up of poloidal rotation.

The physics of Eq (5), when taken together with the fact that shear in the poloidal rotation may quell microturbulence [8], can be applied to show that a bifurcation in transport equilibria can be obtained. For simplicity, we assume for $v_r$ the form $v_r = -D(1 + \delta \cos \theta)(n'/n) - v_0(r/a)$. Thus, the radial velocity is made up of a poloidally asymmetric diffusive piece and an azimuthally symmetric pinch, in which case $\bar{v}_r = -D(n'/n) - v_0(r/a)$ and $\bar{v}_r = -\delta D(n'/n)$. For this choice of $v_r$, the complete condition for instability becomes

$$\delta D(n''/n + n'/nr) > \epsilon \gamma_{MF} (1 + 2q^2)/q^2.$$  

We further assume that $D$ is a decreasing function of $|\partial V_p/\partial r|$: we use the explicit form $D(V'_p) = D_1 + (D_0 - D_1 \exp(-\alpha V_p^2)$, with $D_1 < D_0$. With these modifications Eqs. (2) and (5) were solved numerically for $n(r,t)$ and $V_p(r,t)$. A nonlinear damping term [7] proportional to $-V_p^3$ was added to the right hand side of (5) to provide saturation of the rotation at large $V_p$; a small amount of perpendicular viscosity was also added.

FIG. 1. Radial profiles of $n$ showing bifurcated states A and B (solid lines) and radial profile of $V_p$ for case B (dotted line). Dashed lines are initial conditions for $n$. Identical parameters were used for A and B: $D_1/D_0 = 0.2$, $\alpha = 10$, $a_v_0/D_0 = 1$, $a^2 \gamma_{MF}/D_0 = 2$; we let $q(r) = 1$ and $\delta(r) = 3\epsilon$. 

Bifurcated equilibria for $n$ were obtained as shown in Fig. 1 (indicated by the solid curves A and B). Identical parameters were used to obtain A and B; the only difference was the initial condition $n(r,0)$, indicated by the dashed curves (the parameters are specified in the caption for Figure 1). $V_p(r)$ for case B is also shown.

The bifurcation occurs as follows. If we start with a discharge where (7) is not satisfied anywhere. Then there is no rotation and the transport is large for all surfaces (profile A). Suppose now we locally modify the profile such that $n''$ is positive and large enough so that (7) is locally satisfied. Rotation then commences, thus depressing $D$, in which case the $n$ profile must steepen to maintain particle flux, thus accentuating the magnitude of $n''$. The system then locks into the profile B.

3. SHOCK FORMATION, PARALLEL VISCOSITY, AND CONVECTIVE FLUX (Shaing, Christenson, Houlberg, Hazeltine)

Since the observation of the L-H transition [1], many theories have been proposed to explain the phenomenon [9-12, Section 2]. Among these theories, those based on the radial electric field $E_r$ seem to be the most promising [11,12]. The thrust of the theories is that anomalous transport fluxes are influenced by $E_r$ and $dE_r/dr$, where $r$ is the minor radius, and that the L-H transition is caused by a sudden change in the profiles of $E_r$.

The difference between these theories is that in Ref. [11] the anomalous particle transport fluxes are assumed to be nonintrinsically ambipolar and are involved in determining $E_r$, while in Ref. [12] the anomalous particle transport fluxes are assumed to be intrinsically ambipolar and are not involved in determining $E_r$. Here, we summarize the concept of the model in Ref. [12] and the new results on the shock formation associated with the sonic poloidal rotation.

The basic logic of the theory is illustrated in Fig. 2. We determine $E_r$ from the nonintrinsically ambipolar processes, which can be either neoclassical or anomalous, in the poloidal and toroidal momentum balance equation. Once the profile of $E_r(r)$ is determined, it can suppress the turbulent fluctuations by modifying the decorrelation time through the shear of the angular velocity, for example. Plasma confinement is thus improved.

The first step in understanding the L-H transition is to determine the $E_r(r)$ from the toroidal and poloidal momentum evolution equations
\[
\frac{\partial}{\partial t} \left( \left\langle R^2 \nabla \cdot N \vec{V} \right\rangle + \frac{1}{4\pi c M} \left\langle \frac{\partial \vec{E}}{\partial t} \cdot \nabla \phi \right\rangle \right) \\
= -\nu_{\text{eff}} \left\langle R^2 \nabla \cdot N \vec{V} \right\rangle - \left\langle R^2 \nabla \cdot \nabla \cdot \pi \right\rangle - \frac{1}{c M} \left\langle R^2 \nabla \cdot \vec{L} \right\rangle \tag{8}
\]

and

\[
M_{\text{eff}} \frac{\partial}{\partial t} \left( \frac{\left\langle N \vec{V} \cdot \vec{B}_p \right\rangle}{\left\langle N \right\rangle} - \frac{I}{(R^2 N) 4\pi c M} \left\langle \frac{\partial \vec{E}}{\partial t} \cdot \nabla \phi \right\rangle \right) \\
= \frac{I}{c M (R^2 N)} \left\langle \vec{J}_r \cdot \nabla \phi \right\rangle - \left\langle \vec{B} \cdot \vec{V} \cdot \nabla \vec{V} \right\rangle - \left\langle \frac{\vec{B} \cdot \nabla \cdot \pi}{NM} \right\rangle - \nu_{\text{eff}} \left\langle \vec{B}_p \cdot \vec{V} \right\rangle \\
+ \frac{I}{(R^2 N)} \left\langle R^2 \nabla \cdot N \vec{V} \cdot \nabla \vec{V} \right\rangle + \left\langle \left( \frac{I^2}{(R^2 N) MB^2} - \frac{1}{NM} \right) \vec{B} \cdot \vec{L} \right\rangle \tag{9}
\]

where angle brackets denote flux surface average, \( M \) is the ion mass, \( N \) is the plasma density, \( \pi \) is the ion viscous stress tensor, \( \vec{V} \) is the plasma flow velocity, \( \vec{E} \) is the electrostatic electric field, and \( \nu_{\text{eff}} \) is the effective damping frequency associated with either the anomalous or charge exchange processes. The radial current \( \left\langle \vec{J}_r \cdot \nabla \phi \right\rangle = c \left\langle \nabla \phi \cdot \vec{B} \times \vec{L} / B^2 \right\rangle \) is

\[\text{FIG. 2. Flow chart of the physical processes involved in the L–H transition. Dotted line indicates that there is no direct relationship between the two processes.}\]
employed to model ion orbit loss current, probe current, and any other radial current sources that are driven by the force $\vec{L}$. We use standard tokamak flux coordinates $\vec{B} = \nabla \zeta \times \nabla \psi + IV \nabla \zeta$, $\vec{B}_p = \nabla \zeta \times \nabla \psi$, and $I = R^2 \vec{B} \cdot \nabla \zeta$. The effective poloidal inertia $M_{\text{eff}} = 1 + Cq^2$, with $C$ a constant ranging from 2 for $M_p \ll 1$ to $(2/e)$ with $e = r/R$ when $M_p \approx 1$. The poloidal Mach number $M_p = V_p B/(v_t B_p)$, where $V_p$ is the poloidal flow speed and $v_t$ is the ion thermal speed. In [12], a bifurcation model for poloidal rotation is developed based on Eq. (9) by neglecting $\langle \vec{B} \cdot \vec{V} \cdot \nabla \vec{V} \rangle$, $\nu_{\text{eff}} \langle \vec{B}_p \cdot \vec{V} \rangle$, and the compressibility effect. The reason for the bifurcation is the existence of a local maximum in $\langle \vec{B} \cdot \nabla \cdot \pi/n M \rangle$ around $M_p \approx 1$. The source of the radial current in the model is the ion orbit loss. However, because $M_p \approx 1$, we must take the compressibility effect into account. These refinements lead to new physics: the formation of a shock as $M_p$ approaches unity [13].

The shock is characterized by a steep gradient in the perturbed density $\tilde{n}$ and electrostatic potential $\tilde{\Phi}$ at shock angle $\theta_0$. If plasma rotates counterclockwise poloidally, the shock first appears in the inside of the torus at $\theta_0 \geq \pi$ when $1 - M_p \leq \sqrt{e}$. As $M_p$ increases, $\theta_0$ moves counterclockwise in the direction of $V_p$. At $M_p = 1$, $\theta_0 = 2\pi$. When $M_p - 1 \leq \sqrt{e}$, the shock angle is at $\theta_0 \leq \pi$. The evolution of the shock angle $\theta_0$ from [13,14] is illustrated in Fig. 3 with the shock solution for $[(e \tilde{\Phi}/T_e) + (1 - 2A)/4A']$ as $M_p$ increases. The parallel viscosity associated with the shock (shock viscosity) is collisionality independent and is valid for all collisionality in contrast to the conventional neo-classical viscosity.

![Fig. 3](image.png)
The shock viscosity has a maximum at $M_p \simeq 1$, which is an irreducible minimum to be overcome to have a poloidal supersonic rotation, and has the form [13,14]

$$\left\langle \frac{\vec{B} \cdot \nabla \vec{v}}{NM} \right\rangle = \frac{4\sqrt{2}}{3\pi} \frac{v_i^2}{\left( \frac{5}{3} + \frac{T_e}{T_i} \right)} \left( \vec{B} \cdot \nabla \theta \right) \frac{V_p}{|V_p|} e^{3/2} \times \left( \frac{A + G}{A'} \right)^{3/2} A'$$

(10)

where $A \simeq M_p^2/2 \simeq G \simeq A'$. Besides the parallel viscosity, we also need to examine the convective momentum transport associated with $\langle \vec{B} \cdot \vec{V} \cdot \nabla \vec{V} \rangle$ and $\langle R^2 \nabla \zeta \cdot \vec{N} \vec{V} \cdot \nabla \vec{V} \rangle$. In the case of $M_p \simeq 1$,

$$\left\langle \vec{B} \cdot \vec{V} \cdot \nabla \vec{V} \right\rangle = 3\frac{cM}{e} \left[ \frac{I}{N} \frac{\partial}{\partial \psi} \left( \frac{\vec{N} \vec{V} \cdot \nabla \theta}{\vec{B} \cdot \nabla \theta} \right) + \frac{1}{2} \frac{\partial}{\partial \psi} \left( \frac{R^2 \vec{V} \cdot \nabla \zeta}{\vec{B} \cdot \nabla \theta} \right) \right]$$

$$\times \left\langle \frac{\vec{B} \cdot \nabla \vec{v}}{NM} \right\rangle$$

(11)

In the edge region $dV_p/dr < 0$, $\langle \vec{B} \cdot \vec{V} \cdot \nabla \vec{V} \rangle$ accelerates poloidal rotation and facilitates the L-H transition. The complete expressions for $\langle \vec{B} \cdot \nabla \vec{v}/nM \rangle$ and $\langle \vec{B} \cdot \vec{V} \cdot \nabla \vec{V} \rangle$ in the range of $0 < |M_p| < B/B_p$ can be found in [14].

Because of the near-cancellation between the shock viscosity and convective momentum transport, the qualitative behavior of the bifurcation model in [12] is still valid. We note that the width of the turbulence stabilization zone, $W_s$, does not have to be the same size as that of the $V_p$ shear layer, $L_v$, and that $W_s$ may scale with plasma current $I_p$. The bifurcation solution in [12] can be expressed as $M_p = f(\nu_*, \rho_{pi}/\Delta r, \sqrt{\epsilon})$. (For simplicity, we neglect the temperature gradient.) Therefore, $V_p$ in the edge region scales as $V_p \propto f(\nu_*, \rho_{pi}/\Delta r, \sqrt{\epsilon}) v_i B_p/B$. We conclude that the value of $V_p$ is roughly proportional to $I_p$ if all other parameters are held fixed, which implies that the potential well is deeper when $I_p$ becomes large. It is emphasized in [12] that $W_s > L_v$ can occur in tokamaks because $E_r$ can change over a wider region than the shear of $V_p$. Detailed profiles can be obtained only by coupling Eqs (8) and (9) to density and temperature evolution equations. We also note that the
onset of the shock can explain the well-known mystery of the location of the MARFE [15]. The first appearance of the MARFE is at $\theta = \pi$, corresponding to the onset of the shock. The subsequent movement of the MARFE corresponds to the movement of the shock angle $\theta_0$. The scaling law for the MARFE threshold, which is proportional to $n/I_p$, corresponds to the scaling law for the onset condition of the shock $1 - \dot{M}_p \simeq \sqrt{\epsilon}$. We suggest that in general the location and the movement of the MARFE are a consequence of the density and temperature variations associated with the plasma mass flows.

REFERENCES

MODELLING OF IMPROVED CONFINEMENT — PEAKED PROFILE MODES AND H-MODE

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Abstract

Theoretical models on improved confinement with a peaked density profile and on the H-mode are discussed. A model theory of inward pinch and peaked density profile of Ohmic discharges in tokamaks is presented. Ion anomalous viscosity in the presence of sheared toroidal rotation causes a drift across the magnetic field. The ratio of viscosity to diffusion coefficient, the Prandtl number, determines the structures of $E_r$ and the density profile in a stationary state. In viscous plasmas, peaked profiles of both density and rotation velocity are possible without central particle momentum sources. The resulting rotation of the core relative to the edge is a direction opposite to that of the plasma current. Reduction of edge neutrals can induce further density peaking. The H-mode transition model is further developed. The radial derivative of $E_r$, $E'_r$, changes the ion loss cone loss through squeezing of the poloidal gyroradius and electron loss by affecting the microinstabilities. The charge neutrality condition predicts a cusp type catastrophe in the relationship between gradient and flux at edge. At the transition, $E'_r$ jumps to negative values and fluctuations and anomalous fluxes are reduced simultaneously. The combined model predicts that the core density profile is flattened during the H-phase.

1. INTRODUCTION

Recently, various improved modes other than the H-mode have been found in tokamaks. In the H-mode, the steep gradient is established only near the edge [1]. Other improved modes are characterized by peaked density/ion temperature profiles [2, 3]. We first present a model of the peaked density profile in the core associated with the radial electric field, $E_r(r)$, showing that anomalous viscosity in the presence of sheared toroidal rotation induces an inward flow of particles. An extended model of the H-mode, based on the bifurcation of $dE_r(r)/dr$ associated with poloidal rotation, is presented. Combining both models, we discuss the effect of the edge pedestal on the core profile.

2. PEAKED PROFILE MODES

Density continuity and charge conservation in the presence of diffusion determine the structure of $E_r(r)$ and the density profile, $n(r)$. To obtain $E_r(r)$, electron and ion fluxes are assumed to be anomalous because of drift type microturbulence of
frequency, \(\omega\), and poloidal wavenumber, \(m\). Applying quasi-linear theory, we have [4]

\[
\Gamma_{s,a} = -D_s n_s \left[ \frac{n'_s}{n_s} (1 + \alpha_s \eta_s) - \left( \frac{e E_f}{T_s} - \frac{r \omega}{m} \right) \left( \frac{c B_i}{c T_s} \right) \right]
\]

(1)

where \(\eta = d \ln T / d \ln n\), \(\alpha\) is a numerical factor and \((\omega / m)\) is the spectrum averaged phase velocity, the subscript \(s\) refers to the particle species. The radial ion flux includes an anomalous viscosity flow, \(F_{iv} = (q R / Z e R B) n_i m_i \nabla \mu_\perp \nabla U_\phi\), \((\mu_\perp\) is the viscosity) [5] and the momentum loss by charge exchange, \(\Gamma_{i,ex} = (q R / Z e R B) \times n_i m_i U_\phi n_0 (\sigma_{ex} v)\), where the toroidal rotation, \(U_\phi = (q R / Z e R B) [n'/n_i (1 + \psi) - Ze E_f / T_f] + q R U_e / \rho\), is derived from the radial momentum balance. The charge conservation equation, \(\Gamma_{e,tot} = \Gamma_{i,a} + \Gamma_{i,v} + \Gamma_{i,ex}\), and the equation of continuity, \(\nabla \cdot \Gamma_{e,tot} = \Gamma_{e,tot}\), are basic equations. \(\Gamma_{e,tot} = \Gamma_{i,ex}\) is assumed.

In the absence of beams or pellet particle sources, \(S_{e,i}\) and \(\Gamma_{i,ex}\) are localized near the edge, and we assume that \(S_e = S_i\) (\(S = n_0 n_e \langle \sigma / v \rangle\) is the ionization rate). We introduce a normalized electric field, \(E (= a^2 e E / r T_i)\) and its shear part \([E] (= E - \text{const})\). We take \(\eta = 0\) with \(T_e = T_i\) and \(U_\rho = 0\). \(D, \mu_\perp\) and \(q\) are numerical quantities. In the core, where \(n_0 \sim 0\), \(\Gamma_{e,tot} = 0\) yields the relationship \(-2 (\ln n)' = [E] + C_1\) in the \(y = r^2 / a^2\) co-ordinate. \(C_1 (\alpha (\omega / m))\) is assumed to be constant; it is determined by the boundary conditions. The condition \(\Gamma_{e,tot} = \Gamma_{i,tot}\) gives

\[
-\nu_\mu [E] + \nu_\mu \frac{d}{y} \frac{d}{dy} y \frac{d}{dy} [E] = 0
\]

(2)

where \(\nu_\mu = (1/Z + 1) q^2 R^2 \mu_\perp / a^4\), \(\nu_\alpha = (1 - Z) D_\alpha / \rho_\alpha^2\) and \(\rho_i\) is the toroidal gyroradius.1

The solution is given by \([E] = A_1 I_0 (\sqrt{\nu_\mu / \nu_\alpha} y), \) and \([E] + C_1 = -2 (\ln n)'\). The profile of \(E\) is dictated by the parameter \(\nu_\mu / \nu_\alpha\), which is the diffusion Prandtl number [6]. When higher order derivatives of the pressure are neglected, \(\nu_\mu / \nu_\alpha\) reduces to \((\mu_\perp / D_\alpha) (q^2 R^2 \rho_i^2 / a^4).\)

Neutrals in the outer region \((a - \Delta < r < a\), where \(\Delta\) is the penetration length of the neutrals\) affect the momentum balance. For the boundary condition at \(r = a\), \((d n / d r) n \big|_{r=a} = 1 / \lambda_n\), we obtain

\[
[E] = \left\{ \frac{a}{\lambda_n} - \frac{a^2}{2 D_e} (S) \bigg|_{r=a} - C_1 \right\} \frac{I_0 (\sqrt{\nu_\mu / \nu_\alpha} r^2 / a^3)}{I_0 (\sqrt{\nu_\mu / \nu_\alpha})}
\]

(3)

1 If we evaluate \((\omega / m)\) by the local dispersion relation, we have \([E] = -2 C_2 (\ln n)' (C_2 = 1 / (1 - A_0), A_0 = I_0^{-b}, b = k_\perp \rho_i^{-2}, I_0\) being the zero order modified Bessel function and \(\nu_\mu\) is redefined as \((C_2 / Z + 1) q^2 R^2 \mu_\perp / a^4\). The qualitative nature of the solution does not change in the following.
in the limit of $\Delta \ll a$, where $\langle S \rangle = \int n_r n_0 (a r) \, dy / n_y$. We plot $n(r)$ for various values of $\nu_p / \nu_a$ in Fig. 1(a). For fixed line averaged density, $\nu_p / \nu_a$ is changed from 0.01 to 100. $A_0 = (a / \lambda_n - C_1 - a^2 \langle S \rangle / D_e)$ is chosen to be 10. The more viscous plasma has the more pronouncedly peaked profile. The viscous frictional force due to the toroidal rotation and the poloidal magnetic field causes the $\mathbf{F} \times \mathbf{B}$ drift of the ions. Only if the rotation flow is opposite to the current direction, an inward drift occurs. The force acts differently on the electrons, and $E_r$ is increased. The electrons also drift inward. The peaked profile can thus be sustained without particle source. Peaking is expected if $\nu_p / \nu_a$ is large, i.e. if $\mu_\perp / D$, $R/a$ and $\rho_i / a$ are large or $I_p$ is small. The radial profile of the toroidal rotation is given by $U_\phi / \nu_{\text{T1}} = (\rho_p / a) A_1 (\sqrt{\nu_p / \nu_a y}) + \text{const}$; it is shown in Fig. 1(b). We plot the value of $(U_\phi - U_\phi (a - \Delta)) / \nu_{\text{T1}} A_0 (\rho_p / a)$. The rigid rotation part is determined by the boundary condition. Owing to the density peaking, the rotation profile also peaks. We show the effect of the boundary condition on $n(r)$ in Fig. 1(c). As $A_0$ becomes large, the density peaks. When the gas puff is reduced, the edge source is expected to become small. The reduction of $\langle S \rangle$ reduces the momentum loss of the rotation, which can increase the density gradient at $r = a - \Delta$. $A_0$ rises if $\lambda_n$ remains similar. This causes the peaking of the core density. The SOC–IOC transition [2] is attributed to this mechanism [7]. When $\nu_p / \nu_a$ is small (less viscous), a reduction of $\langle S \rangle$ is found to have only a little effect.
on density peaking. The actual value of $\nu_{\mu}/\nu_2$ is unknown; if, however, we apply a quasi-linear theory to the drift wave fluctuations [5] this ratio becomes of the order of unity, up to ten. Recent measurements of the toroidal rotation [8] support this estimate.

3. H-MODE

We consider the edge of a circular plasma. The particle flux can be bipolar near the edge [9]. The relation $\Gamma_{e,NA} = \Gamma_{i,NA}$ determines the ambipolar electric field, $E_r$. The bipolar flux of ions, $\Gamma_{i,NA}$, comes from the loss cone loss. In the presence of $E'_r$, the banana width changes by a factor of $G = 1/(1 - u_g/c_e)$, where $u_g = \rho_i E'_r/v_{Ti} B_r$. Taking $u_g$ corrections, we have $\Gamma_{i,NA} = \Gamma_{i,0} = \rho_i e E_r/T_i$ and $F$ is a numerical coefficient ($\sim O(1)$).

The microscopic mode stability is affected by $E'_r$. The collisionless trapped particle drift instability, for example, is stabilized if the condition $u_g < -u_c$ is satisfied, ($u_g^2 = 8\sqrt{2}\varepsilon(4 - u_c)$). We use an analytic estimate of the growth rate and the transport coefficient as $\gamma \sim \gamma_0 \sqrt{1 + u_g/u_c}$ and $D_e = D_{eo} \sqrt{1 + u_g/u_c}$, respectively, where the subscript 0 denotes cases with $E'_r = 0$. The bipolar electron flux near the edge, $\Gamma_{e,NA}$, originating from the convection of excited waves [9], is given by $\Gamma_{e,NA} = n_e D_{e0} \sqrt{1 + u_g/u_c} (\lambda - X)$, $\lambda = -(T_e/T_i) \rho_i (n_e/n_i + c_i T_e/T_i + eBr/T_i \omega/m)$. Equating $\Gamma_{e,NA}$ and $\Gamma_{i,NA}$, we have a refined equation (d = $\sqrt{eD_{eo}/\rho_i F \rho_i^2}$):

$$d \sqrt{1 + u_g/u_c} (\lambda - X) = G \exp[-X^2]$$

The poloidal bulk viscosity yields $U_p/v_{Ti} \propto \rho_i \nabla T_i/T_i$ [12], i.e. $X = -\lambda_i - U_g/v_{Ti}$, where $\lambda_i = -\rho_i (n_i/n_i + c_i T_i/T_i)$; $c_i$ is a numerical coefficient ($\sim O(1)$).

**FIG. 2.** Radial gradient of $E_r(u_g)$, transport coefficient ($D_e/D_{eo}$) and flux ($\Gamma/\Gamma_0$) shown as functions of $\lambda$. In the interval $\lambda_2 < \lambda < \lambda_3$, the solutions become multivalued. Transition takes place at $\lambda = \lambda_3(L - H)$ and $\lambda = \lambda_2(H - I)$. 
Since the toroidal flow in the scrape-off layer is damped [13], we neglect $U_d/v_{ti}$ and have $X = -\lambda_i$. The solution is obtained only in the negative $E_r$ region.

In Fig. 2, the solution of $E'_r$, the relative transport coefficient and the resulting flux are shown for $\lambda = \lambda_i$. At the threshold value of $\lambda$, $\lambda_{st}$, $E'_r$ jumps from positive to negative, and $D_e/D_{e0}$ and $\Gamma/\Gamma_0$ show transitions to smaller values.

Combining the models in Sections 2 and 3, we finally note the effect of the H-mode on the core density profile. The transport barrier effectively increases $\langle S \rangle/D_s$ (i.e. local diffusion is reduced) and reduces $A_0$. The density profile and $U_d$ of the core plasma are flattened, owing to the decrease of $A_0$. The reduced momentum loss increases the rigid toroidal rotation.

4. SUMMARY AND DISCUSSION

We have presented a model of improved confinement with peaked density profile as well as a model of H-mode transition.

The inward pinch induced by the ion viscosity is studied; it is predicted that the peaking occurs by a reduction of the edge source if the value of $v_p/r_s$ is large enough. This mechanism explains the SOC–IOC transition. A bifurcation of $E'_r$ at the edge is predicted. Anomalous diffusivity and the associated flux are reduced simultaneously. $E'_r$ turns negative and the negative $E_r$ becomes still more negative. By combining these models, the core density profile is predicted to be flattened in the H-mode.

The inward pinch is studied in the absence of an external momentum source and $\nabla T_i$. The extension of the present model to various peaked profile modes is left for future studies.

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TOKAMAK FLUIDLIKE EQUATIONS, WITH APPLICATIONS TO TURBULENCE AND TRANSPORT IN H-MODE DISCHARGES*

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Abstract

TOKAMAK FLUIDLIKE EQUATIONS, WITH APPLICATIONS TO TURBULENCE AND TRANSPORT IN H-MODE DISCHARGES.

Significant progress has been made in developing tokamak fluidlike equations which are valid in all collisionality regimes in toroidal devices, and their applications to turbulence and transport in tokamaks. The areas highlighted in the paper include: (1) the rigorous derivation of tokamak fluidlike equations via a generalized Chapman–Enskog procedure in various collisionality regimes and on various time scales; (2) their application to collisionless and collisional drift wave models in a sheared slab geometry; (3) applications to neoclassical drift wave turbulence, i.e. neoclassical ion-temperature-gradient-driven turbulence and neoclassical electron-drift-wave turbulence; (4) applications to neoclassical bootstrap-current-driven turbulence; (5) numerical simulation of nonlinear bootstrap-current-driven turbulence and tearing mode turbulence; (6) transport in Hot-Ion H-mode discharges.

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1 INTRODUCTION

Neoclassical MHD theory was first set forth four years ago at the Kyoto IAEA meeting [1]. Considerable progress was reported two years ago at the Nice IAEA meeting [2]. Since then the theory has been developed in two parallel approaches: One is to develop neoclassical MHD equations (here called “tokamak fluidlike equations” to include kinetic effects) rigorously based on a generalized Chapman-Enskog procedure. Using this procedure, closure relations are being developed for various time scales, mode structures and collisionality regimes, including kinetic effects. The other is to refine the theory of neoclassical bootstrap-current-driven turbulence, develop the theory of neoclassical drift wave turbulence and compare these results with experimental data. The remainder of this brief paper highlights recent developments and progress in tokamak fluidlike theory and its applications since the last Nice IAEA meeting.

2 HYBRID FLUID/KINETIC MOMENT DESCRIPTIONS OF TOKAMAK PLASMAS

Conventional fluid moment descriptions (resistive MHD, Braginskii equations [3], etc.) that are often used to explore macroscopic behavior, microscopic fluidlike instabilities, and transport properties of tokamak plasmas, do not contain a number of important kinetic effects such as Landau damping, magnetic particle trapping, etc. On the other hand, conventional kinetic studies do not include a number of important fluid effects such as neoclassical polarization, poloidal flow damping, bootstrap current, etc. that emerge from fluid moment descriptions. In order to bridge the gap and to facilitate a comprehensive theory of tokamak plasmas that includes both types of effects, we have developed an exact generalized Chapman-Enskog approach for determining the non-Maxwellian part of the distribution function that is needed for calculating the fluid moment closure relations. The needed closure relations are obtained by calculating the viscous stress tensors $\Pi$ and $\Theta$ from solutions of the kinetic equation for the distribution function.

The basic procedure is to divide the total distribution function $f$ into a heat-flow-shifted Maxwellian and the remaining kinetic part of the distribution function, which must be determined from kinetic theory, depending on geometry, collisionality regime, time scale, mode structure, etc. By taking account of exact fluid moment equations for $\partial n/\partial t, \partial T/\partial t, \partial V/\partial t$ and $\partial q/\partial t$, we obtain the recast kinetic equation for the kinetic part of the distribution function $F$ [4,5]:
\[
\frac{dF}{dt} - C^l(F) = -f_m \left[ \frac{2}{3p} \Pi : \nabla \nabla L_1^{(1/2)} + \mathbf{v}' \cdot \mathbf{U} + \left( \mathbf{v}' \mathbf{v}' - \frac{\mathbf{v}'^2}{3} \mathbf{I} \right) : \mathbf{W} \right],
\]  
(1)

where

\[
\begin{align*}
f &= f_m \left[ 1 + \frac{2\mathbf{v}'}{v_t^2} \cdot \left( -\frac{2\mathbf{q}}{5p} \right) L_1^{(3/2)} \right] + F, \\
\mathbf{U} &= -\frac{1}{p} \nabla \cdot \Pi L_0^{(3/2)} - \frac{2}{5p} \nabla \cdot \Theta L_1^{(3/2)}, \\
\mathbf{W} &= \frac{m}{T} \nabla \nabla L_0^{(3/2)} - \frac{2m}{5pT} \nabla \mathbf{q} L_1^{(3/2)}.
\end{align*}
\]

Here, higher order terms (in \( \rho_0/L_\perp \)) on the right of Eq. (1) have been neglected for simplicity. Although Eq. (1) looks more complicated than the kinetic equation for the total distribution function, it has several advantages. It preserves exact conservation properties of \( n, T, V, q \) on various time scales. Hence, it can be used to treat plasma instabilities with \( \omega \sim \nu \), including both collisionless and collisional kinetic effects such as magnetic pumping, Landau damping, etc. It rigorously conserves fluid moments \( (n, T, V, q) \), which the kinetic approach sometimes fails to do. Also, it explicitly includes the irreversible, dissipative processes that are needed for net transport. Once we solve Eq. (1) for \( F \) and then the viscous stress tensors \( \Pi \) and \( \Theta \), we can substitute these into moment equations to determine transport fluxes. In a strongly magnetized plasma, Eq. (1) can be gyroaveraged in a standard way to obtain the drift kinetic equation for \( \mathbf{F} \). The resultant equation has been solved in equilibrium in toroidal geometry in references [6,7]. For \( \partial \mathbf{q}/\partial t = 0, \partial \Pi/\partial t = 0 \) and \( \lambda_{mfp} b \cdot \nabla \ln B \ll 1 \), this formalism exactly reproduces the Braginskii equations [3]. For \( \omega \sim k_\parallel v_t \sim \nu \), we can obtain the collisionless and collisional Landau damping contributions to the viscous stress tensors [5]. Approximate dynamic viscosity coefficients in the banana regime have been derived in Ref. [8]. More accurate dynamic viscosity coefficients in all collisionality regimes are under development [9]. Also, procedures and closure relations are being developed for studying trapped-particle instabilities [10]. Treating trapped particles as separate species, a reduced set of fluidlike equations in the long mean-free-path regime has been proposed [11] to exhibit the coupling of electrostatic trapped-particle modes with resistive ballooning modes in tokamak plasmas.

3 SHEARED SLAB ELECTRON DRIFT WAVES

The generalized Chapman-Enskog procedure has been applied to linearized drift type microinstability problems. Neglecting the \( \partial \mathbf{q}/\partial t \) equation...
and assuming $\nabla T = 0$, Eq. (1) has been solved in a sheared slab geometry for a Krook-like collisional model. The linearized closure relations which include full Landau damping and some collisional effects can be written as [5]

$$b \cdot k \cdot \tilde{v}_\parallel = \frac{Z Z''}{D} n m v_t k_\parallel \tilde{V}_\parallel + \frac{2 Z Z' - Z''}{D} n k_\parallel \tilde{T},$$

(2)

$$k \cdot \tilde{q} = \frac{2 Z Z' - Z''}{D} p v_t k_\parallel \tilde{V}_\parallel + \frac{3 Z'' + 12 (Z')^2}{8 D} n v_t k_\parallel \tilde{T},$$

(3)

where $D \equiv -2 Z Z' - Z''/2$, and $Z$ is the plasma dispersion function with argument $\zeta \equiv (\omega + i \nu_{eff})/k_\parallel v_t$. It is easy to show that Eqs. (2) and (3) are consistent with the usual drift kinetic results. Our results show that Landau damping comes into the fluid equations through both the viscous stress and heat flux contributions, instead of just through the heat flux [12]. Various methods (asymptotic expansions and multipole curve fitting) can be used to simplify the above equations.

4 NEOCLASSICAL DRIFT WAVE TURBULENCE

The theory of collisionless fluid ion-temperature-gradient-driven turbulence has been extended to the banana-plateau collisionality regime [13]. Neoclassical ion nonlinear fluid evolution equations for flute-type modes are developed and utilized to study ion-temperature-gradient-driven modes in the banana-plateau regime. Neoclassical effects modify negative compressibility $\eta_t$-modes by: introducing parallel viscous damping which makes the long wavelength parallel ion flow response dissipative rather than inertial, and enhancing the linear [14] and nonlinear polarization drifts by a factor of $B_t^2/B_p^2$. As a result of these modifications, growth rates become dissipative rather than sonic [i.e., $\gamma \sim k_\parallel^2 c_s^2(\eta_t - 2/3)/\mu_i$] and radial mode widths are broadened [$\Delta_x \sim \rho_s(B_t/B_p)(1 + \eta_t)^{1/2}$]. Neoclassical $\eta_t$-modes are fundamentally three-dimensional excitations. Thus, spectral transfer to small scale dissipation occurs, resulting in saturation. Renormalized turbulence theory is used to calculate the ion thermal diffusivity $\chi_i$ at saturation. For low $k_\parallel$ ($\Delta \omega_{ke} < \mu_i$):

$$\chi_i \sim \frac{(3\pi)^2}{4} \frac{c_s^2}{\mu_i} \rho_s^2 \frac{(k_\parallel \rho_s)}{L_s^2} \frac{B_t^4}{B_p^4} (1 + \eta_t)^2(\eta_t - 2/3) \hbox{ for } \eta_t > \eta_{th} = 2/3,$$

where $k_\parallel$ is restricted to $k_\parallel \rho_s < \mu_i(L_s/c_s)(B_p/B_t)(\eta_t - 2/3)^{-1/2}(1 + \eta_t)^{-1/2}$. For moderate $k_\parallel$ ($\Delta \omega_{ke} > \mu_i$):

$$\chi_i \sim \mu_i \rho_s^2 \frac{B_t^2}{B_p^2} (1 + \eta_t).$$
In both cases, a strong favorable dependence on $B_p$ (and hence $I_p$) is exhibited. Furthermore, the $\chi_i$ for the long wavelength mode exhibits favorable density scaling ($\chi_i \sim 1/\mu \sim 1/n$). Both of these results are in agreement with experimental findings.

Neoclassical $\nabla T_i$-driven turbulence is a natural candidate for modelling strong, moderate collisionality turbulence in L-mode edge plasmas and the L $\rightarrow$ H transition. This claim is motivated by the predominantly electrostatic character of such turbulence, the robust character of neoclassical $\nabla T_i$-driven modes, and the observation that such modes typically have low threshold (i.e., $\eta_i \approx 2/3$). Moreover, the steepening of the edge density gradient which accompanies the L $\rightarrow$ H transition naturally quenches the turbulence, consistent with the notion of an edge transport barrier in H-mode. The quenching of neoclassical $\nabla T_i$-driven turbulence causes simultaneous reduction in $\chi_i$, $\chi_\phi$, $\chi_e$ and $D$, consistent with experimental findings. However the conventional $\nabla T_i$-driven mode theories have difficulties in explaining plasma current scaling and are valid for the rather collisionless core region. These difficulties are resolved by the neoclassical $\nabla T_i$-driven turbulence theory.

Ongoing work in this area is concerned with exploring the effects of sheared rotation on the linear stability and nonlinear dynamics of neoclassical $\nabla T_i$-driven turbulence.

The neoclassical polarization drift also changes the mode structure of the standard shear damped electron drift wave [15] from an outgoing mode to a localized one, and introduces an explicit $B_p$-dependence into the mode width through the neoclassical polarization. Hence, explicit favorable plasma current scaling will appear in neoclassical electron drift wave induced transport, as well [13]. Further progress in this topic awaits the development of a dynamic viscous damping coefficient valid for $\omega \sim \mu_i$ in low collisionality regimes.

5  NEOC-classical bootstrap-current-driven turbulence

The nonlinear evolution and saturation of neoclassical bootstrap-current-driven turbulence (NBCDT), evolving from linear bootstrap-current-driven instabilities [16] described by the neoclassical MHD equations has been studied [17]. For high-$T_i$ discharges (such as DIII-D Hot-Ion H mode), the decorrelation rate $\Delta \omega_k$ usually exceeds the neoclassical viscous damping frequency, so the enhancement factor in the neoclassical polarization is reduced to $(B_i^2/B_p^2)(\mu_i/\Delta \omega_k)$. The calculation of the electron heat transport resulting from stochastic magnetic fields driven by NBCDT is revisited. Taking account of the high frequency modification, we obtain [19]
where
\[ \Lambda^* \equiv \Lambda^{7/3} \left( \frac{1}{\Lambda} - \frac{1}{\Delta} \right) \left( 1 - \frac{\sqrt{2}}{\Lambda + \Delta^3} \right)^{-1/2}, \]
and other notations are explained in Ref. [17]. The magnetic fluctuation levels and associated electron thermal conduction are enhanced by increasing \( \beta_p \) and a steep pressure gradient, but are suppressed by strong shear. While resistive MHD turbulence models [18] are relevant primarily at the edge of the plasma, the region of NBCDT applicability extends over a wide zone between the center and the edge of the plasma. Also, since NBCDT is aggravated as the pressure is increased by additional heating, NBCDT is of particular relevance to regimes of moderate to high plasma \( \beta_p \), such as DIII-D Hot-Ion H-mode and TFTR Supershots.

6 SIMULATION OF BOOTSTRAP-CURRENT-DRIVEN TURBULENCE

Numerical calculations of NBCDT are of importance for: (a) identification of the turbulent saturation mechanism and as a test of the analytic theory, (b) obtaining the \( k_\theta \) spectrum, which has not yet been analytically calculated, and (c) generating detailed 3-D isolated magnetic field structures for use in studies of electron thermal conductivity enhancement by parallel losses along ergodic field lines. The numerical model consists of a neoclassical Ohm's law, neoclassical parallel and perpendicular momentum balance equations, and a continuity equation. This results in four coupled time evolution equations for the poloidal flux function, the fluid vorticity, the density and the parallel ion flow velocity. If the assumption of rapid ion flow damping (\( \mu_i \gg \gamma \)) is invoked then the system can be reduced to three equations (the \( V_{\|} \) equation can be eliminated).

The linear numerical results have been compared to the analytical predictions. Results indicate generally good agreement. The only deviation occurs at high mode numbers where the numerical growth rates are somewhat lower than the analytical ones, due to the breakdown of the rapid parallel ion flow damping assumption used in the analysis. In the nonlinear regime, the numerical model has been investigated using both 3-field and 4-field versions. Although systematic comparisons have not yet been made
between the two calculations, there do not appear to be any significant qualitative differences, in the parameter regimes which have been considered. In Figure 1 the typical time evolution for the 3-field model is shown with the fluctuating potential, density, and radial and poloidal components of magnetic field plotted at a fixed radial point \( \tau/a = 0.6 \), demonstrating the achievement of saturation. This calculation was run with 111 modes and 200 radial grid points. The spectrum includes resonant modes for the rational surfaces over a radial range from \( \tau/a = 0.39 \) to 0.97. A comparison of the saturated radial and poloidal magnetic field fluctuation levels is shown in Fig. 2 where the analytical predictions [17] have been evaluated locally using the profile values and the saturated \( k_\theta \) spectrum of the numerical calculation. As may be seen, there is semi-quantitative agreement between the nonlinear numerical results and analytical theory in the outer half of the plasma. Similar agreement results when the fluctuating densities and potentials are studied.
7 APPLICATIONS TO H MODE DISCHARGES

As an application of the NBCDT model, consider core transport in Hot-Ion H-mode discharges. Such discharges have flat density profiles and are characterized by $\chi_e > \chi_i \sim \chi_\phi$. They thus present a challenge to conventional drift wave turbulence theory. However, the moderately high values of $\beta$ attained in the Hot-Ion H mode, along with the disparity between $\chi_e$ and $\chi_i$, suggests that magnetic turbulence may control transport in such plasmas. We have undertaken detailed comparisons of the NBCDT model electron thermal diffusivity ($\chi_e$) with the effective one obtained by power-balance calculations using profiles from DIII-D Hot-Ion H-mode plasmas [20]. Figure 3 shows the comparison of the theoretical prediction with the experimental results for a $B_t = 2T, I_p = 1.0MA, P = 8.8MW$ Hot-Ion H-mode discharge. The agreement in the confinement zone is clearly good,
both in regard to profile and magnitude of $\chi_e$. Since $\nabla n \to 0$ in such plasmas, diamagnetic corrections to the neoclassical fluid turbulence theory are unimportant. The favorable results obtained in this comparison underscore the need for supporting fluctuation and runaway electron confinement studies, and suggest that NBCDT and other electromagnetic dissipative fluid turbulence models offer considerable promise, particularly in high-$\beta$ regimes, and thus should be considered serious candidates for anomalous electron heat transport in tokamaks.

8 SUMMARY

Tokamak fluidlike equations including kinetic effects (Landau damping, particle trapping, etc.) and geometrical effects (magnetic pumping, neoclassical polarization, bootstrap current, etc.) have been developed rigorously via a generalized Chapman-Enskog approach. These equations are applied to neoclassical drift wave turbulence and neoclassical bootstrap-current-driven turbulence models. The resultant transport fluxes and fluctuation levels seem to agree reasonably well with tokamak experimental results from DIII-D. Further rigorous derivations of tokamak fluidlike equations and detailed comparison of neoclassical MHD turbulence predictions with experimental results are promising and should be encouraged.
REFERENCES


CURRENT DENSITY TRANSPORT,
CONFINEMENT AND FUSION BURN CONDITIONS

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Abstract

CURRENT DENSITY TRANSPORT, CONFINEMENT AND FUSION BURN CONDITIONS.

The coupled evolution of the plasma current density and the electron temperature in toroidal, magnetically confinement plasmas is investigated, special attention being devoted to regimes where D-T ignition can be attained. On the assumption that the electron temperature, $T_e$, and the current density, $J$, diffuse in response to each other’s gradients as well as their own, a matrix equation for both quantities including a new thermal-viscous transport coefficient is introduced. Its symmetry properties are used to establish the form of the electron thermal and momentum balance equations. The forms of the thermal conductivity and the related thermal-viscous coefficient are deduced from the condition that the profiles of $T_e$ and $J$, are well behaved and of the same type as those experimentally observed, while the electrical resistivity is assumed to be collisional. A model for the current density transport that is readily implemented in numerical transport codes has been used, as a supplement to the symmetry based theory, to reproduce existing experiments. — The initial current rise phase of a representative, high field D-T ignition experiment is studied by using a numerical, free boundary transport analysis. This phase is shown to provide an important fraction of the ohmic heating in such experiments and also to impose important constraints on the further evolution of the discharge, in particular, on its macroscopic stability. Macroscopic $m^0 = 1$ modes that depend on magnetic reconnection and finite resistivity are shown to be stable in a large parameter region that can apply to ignition experiments and present advanced experiments. High field D-T experiments with tight aspect ratio can operate within this region since they can maintain low values of beta poloidal up to ignition conditions. Then the ideal MHD stability threshold is approached only after a significant population of alpha particles has been produced, which renders both ideal MHD and resistive modes harder to excite.
I. INTRODUCTION

The understanding of the relations governing the evolution of the plasma current density and plasma temperature is a crucial missing component in the theory of thermonuclear plasmas, where the collisional transport theory is generally not valid. We investigate several aspects of this problem and consider relevant issues confronting ignition in toroidal experiments. First, the anomalous transport of the electron thermal energy can be related, by symmetry considerations, to that of the current density, in a way that can explain the observed, steady state profiles. An ad hoc model that can be easily implemented in a numerical transport code is also described. The time evolution and the effects of the initial current rise phase on a representative high field D-T ignition experiment, the Ignitor, are then investigated numerically, using a free boundary transport analysis. Finally, the problem of the onset of modes with poloidal mode number \( m^o = 1 \), in the sequence of plasma regimes that are encountered when approaching ignition, is discussed. The existence of a broad and accessible region in parameter space that ensures the linear stability of resistive modes of this kind is pointed out, while the edge against the onset of ideal MHD modes that are to be considered when the highest values of \( \beta \)-poloidal are reached at ignition can be reinforced by the presence of a significant population of \( \alpha \)-particles with high energy.

II. SYMMETRIES AND TRANSPORT

A common conclusion, based on the analysis of existing experiments [1], is that, on average, the current that is induced in a plasma column can be accounted for by the collisional (neoclassical) resistivity \( \eta_\parallel \) in regimes where a significant fraction of trapped electrons is present. On the other hand, when only neoclassical resistivity is considered, well-behaved electron temperature radial profiles \( T_e \) result in toroidal current density profiles \( J_\phi \) with a "strange" shape [2] characterized by a cusp at the center of the plasma column. Therefore we have elaborated a "unifying" theory [3] that establishes a relationship between the (anomalous) electron thermal energy transport and that of \( J_\phi \), while producing well-
Referring, for simplicity, to a large aspect ratio toroidal configuration with circular cross section, under steady state conditions and in the absence of injected heating, we consider the matrix equation

\[ \nabla \cdot \left( \frac{D}{J} \cdot \nabla \left| \begin{array}{c} T_e \\ J_\parallel \end{array} \right| \right) = \left| \begin{array}{c} S_1 \\ S_2 \end{array} \right| \]

where \( D \) is the diffusion matrix with \( D_{12} = D_{21} \) and \( D_{11}D_{22} - D_{12}^2 > 0 \). (Here \( J_\parallel \approx J_\phi \)). The three transport coefficients involved are the electron thermal conductivity \( \kappa_e \), the thermal-viscous coefficient \( L_e \), and the collisional resistivity \( \eta_\parallel \). On the left hand side, \( S_1 \) represents the entropy production rate under the condition that this is minimal when \( \frac{d\ln J_\parallel}{dx} = \frac{3}{2} \frac{d\ln T_e}{dx} \), as is required in regimes where \( \eta_\parallel = \eta_{ce} \), the classical resistivity that does not include the effects of trapped electrons.

In particular,

\[ S_1 = \eta_\parallel \frac{J_\parallel^2}{T_e} + \kappa_e \left( \frac{\nabla T_e}{T_e} \right)^2 + \frac{L_e J_\parallel^2}{T_e} \left[ \left( \frac{\nabla J_\parallel}{J_\parallel} \right) - \frac{3}{2} \left( \frac{\nabla T_e}{T_e} \right) \right]^2 \]

and \( S_2 = E_\parallel - \eta_\parallel J_\parallel \). Thus, the electron thermal energy balance is represented by

\[ -\nabla \cdot \left[ \kappa_e \nabla T_e - \frac{1}{2} L_e J_\parallel \left( \nabla J_\parallel - \frac{3}{2} \frac{J_\parallel}{T_e} \nabla T_e \right) \right] = E_\parallel J_\parallel \]

and the electron momentum balance by

\[ -\nabla \cdot \left[ L_e \left( \nabla J_\parallel - \frac{3}{2} \frac{J_\parallel}{T_e} \nabla T_e \right) \right] = E_\parallel - \eta_\parallel J_\parallel \]

with the condition that

\[ E_\parallel = \langle \eta_\parallel J_\parallel \rangle \]

where \( \langle \rangle \) indicates the average over the plasma cross section. We see that the current density gradient has an opposite effect to that of the
temperature gradient. The form of \( L_c \) is related to that of \( \kappa_e \) by the condition that well behaved profiles of \( T_e \) and \( J_\parallel \) be produced. The form of \( \kappa_e \) that corresponds to “canonical” (well behaved) \( T_e(r) \) profiles is [4]

\[
\kappa_e^0 = 4 \frac{a^2}{x T_e(x)} \langle \alpha_T \rangle_{\alpha_T} E_\ast \int_0^x J_\parallel(x') dx'
\]

where \( x = r^2/a^2, \langle \alpha_T \rangle = -\langle (dT_e/dx)/T_e \rangle \), \( E_\ast \) is a weak function of \( x \), \( 2\pi \text{Re} E_\ast = \epsilon \tau_0 C_e(x) \), where \( C_e \) is a dimensionless quantity characterizing the microscopic plasma processes responsible for the anomalous transport described by \( \kappa_e \) such that \( T_e C_e \) is a weak function of \( x \), and \( \epsilon \) is a numerical coefficient obtained by a fit to the loop voltage measured by a consistent set of experiments. In particular, \( \langle E_\ast \rangle \approx E_\parallel \). A form of \( L_e \) consistent with Eq. (6) is \( L_e^0 \approx 2\lambda \kappa_e^0(T_e/J_\parallel^3) \), where \( \lambda \) is a finite numerical coefficient. We note that Eq. (6) is basically the Coppi-Mazzucato-Grüber form of the thermal conductivity that has been used already to reproduce a relatively large variety of experiments with ohmic heating only. Then Eq. (3) implies \( T_e \approx T_0 \exp(-\alpha_T x) F(x) \) and \( J_\parallel \approx J_0 \exp(-3\alpha_T x/2) H(x) \) where \( F(x) = H^{2\lambda/(3+3\lambda)}(x), F(0) = 1 \), and \( \alpha_T \) is a weak function of \( x \). The function \( H \) is obtained by solving, numerically, a first order integro-differential equation [3] derived from Eq. (4) with the condition (5) and produces a well behaved profile for \( J_\parallel \), while complying with the observation that the applied electric field is that required, on average, by the collisional resistivity.

When injected heating is applied, a new source term independent of the plasma parameters is added to the r.h.s. of Eq. (3). As the temperature \( T_e \) rises under the influence of this source, \( E_\parallel \) drops below \( E_\ast \). Equation (4) maintains its form with \( L_e = L_e^0(E_\parallel/\hat{\alpha}_T E_\ast) \), anchored to the value of \( E_\parallel \) there, \( \hat{\alpha}_T = \alpha_T/\langle \alpha_T \rangle \). Then the symmetries mentioned above break and we assume that as \( P_{\text{inj}} \) approaches \( P_H \), \( P_{\text{inj}} \) being the injected power and \( P_H \) the total heating power, \( \kappa_e \) becomes related to an effective thermal conductivity \( \kappa_\mu \) that is no longer associated with the process of current density redistribution, \( \kappa_e = \kappa_e^0(E_\ast \hat{\alpha}_T/E_\parallel) + (P_{\text{inj}}/P_H)\kappa_\mu \). An expression for \( \kappa_\mu \), that assumes the combined excitation of the trapped electron “ubiquitous” mode within the main body of the plasma column, and of impurity driven modes at the edge of it was given in Ref. [4] and
has been included in a transport code in order to reproduce the results of experiments where injected heating is prevalent.

III. APPROACH TO IGNITION

The initial phase of a toroidal discharge, when the plasma current is being ramped to its maximum value, can provide an important part of the total ohmic heating to ignition. It also imposes significant constraints on the evolution of the discharge for all ignition plasmas, including the stability of the plasma to sawtooth oscillations and other macroscopic modes with low mode number, which are related to the evolution of the current density \( J_\phi \).

A free boundary, axisymmetric simulation of a representative D-T plasma has been carried out [5], using the MHD and transport code TSC [6]. The reference parameters were chosen to be those of the Ignitor machine [7], which envisions a level of current density and magnetic field close to those attained in the Alcator experiments in a tight aspect ratio, elongated plasma column (Fig. 1). This is expected to ignite at central densities around \( n_{eo} = 10^{21} m^{-3} \) and temperatures \( T_{eo} < 15 \text{ keV} \). During the ramp, the current was typically increased at 5 MA/sec for the first second and more slowly thereafter. The magnetic field was raised from 80% to full value. The initial conditions were \( I_p = 1 \text{ MA}, T_{eo} = 1 \text{ keV} \) at \( t = 0.2 \text{ sec} \) to avoid the uncertainties associated with modelling the breakdown and startup.

The plasma current was assumed to evolve under neoclassical resistivity alone, so that the detailed evolution may differ from that predicted by the theory of Section II, particularly in the center of the plasma column where it has relatively little effect on the results discussed here. An electron thermal transport model that reproduced clean, ohmically heated plasmas and that degrades with strong additional heating was used, \( \chi_e = \kappa_e / n_e = \chi_e^B f_e (T_{eo} / T_{OH}) \), where \( \chi_e^B \) is closely related to the ohmic coefficient of Eq. (6), \( \chi_e^B = 3.64 \times 10^{13} \left( I_\phi / n_e^{4/5} T_e A_i^{2/5} \right) \left( \pi^{3/2} V_a^2 / A_a^{3/2} \right) \left( 1 / \langle |\nabla V|^2 \rangle \right) \left( 1 + 9 \beta_p \left( (P_\alpha + P_{inj}) / (P_\alpha + P_{inj} + P_{OH}) \right) \right)^2 \) (m²/s) in mks units, with \( T_e \) in keV and \( I_\phi \), the toroidal current inside each flux surface, in kA. Here \( V \) and \( A \) are the volume and cross sectional area of a flux surface,
FIG. 1. Vertical cross-section of the Ignitor-Ult machine. The plasma column has $R_o$ (major radius) = 1.30 m, $a \times b$ (minor radii) = 0.47 x 0.87 m$^2$, triangularity $\delta_o = 0.4$. The toroidal plasma current is $I_p \leq 12$ MA, while the poloidal (paramagnetic) current is $I_e \leq 10$ MA and the toroidal field on axis is $B_T \leq 13$ T, with a further contribution from paramagnetic effects, typically $\sim 13/8$ T for $I_e = 10$ MA. The average toroidal current density is $J_o \leq 0.93$ kA/cm$^2$ and the mean poloidal field $B_p = 3.75$ T.
\( A_i \) is the average atomic mass number of the ions in amu, and \( \langle \rangle \) represents a flux surface average. The temperature parameter \( T_{OH} \) reduces the energy confinement when the fusion \( \alpha \)-particle heating \( P_\alpha \) becomes important relative to the ohmic heating \( P_{OH} \). These parameters were varied and other diffusion coefficients were also considered [5].

The results show that the total ohmic heating \( P_{OH} \) can increase continuously with time during the current ramp, if the plasma density is raised simultaneously, and it can reach large values even as the central temperature \( T_{eo} \) becomes high (Fig. 2 and End of Ramp, Table 1, which shows representative discharges). At high temperatures it can significantly exceed predictions made without considering the current ramp.

![Figure 2](image-url)  
**FIG. 2.** Typical time evolution for an Ignitor discharge that was started from the outside limiter (larger \( R \)), with a 3 sec current and density ramp to \( I_p = 12 \) MA, \( n_{eo} = 10.7 \times 10^{27} \) m\(^{-3} \), \( B_{Te} = 13 \) T, with \( Z_{eff} = 1.2 \) (same as the middle case in Table 1). There is a faster initial current rise, 5 MA/s, to \( t = 1 \) sec and a corresponding rate of growth of the plasma size. Ignition occurs at \( t = 3.9 \) sec (fusion \( \alpha \)-particle heating balances the conduction and radiation losses). The top graph shows the central electron temperature, while the bottom shows the power balance (OH: ohmic heating, \( \alpha \): fusion heating, Tot: ohmic plus fusion heating, B: bremsstrahlung radiation losses indicated as a negative quantity; impurity radiation losses and cyclotron radiation losses are small throughout).
Table 1. Free Boundary Current Ramp and Ignition

<table>
<thead>
<tr>
<th>End of Ramp</th>
<th>Ignition ($P_\alpha = P_{Cond} + P_{Rad}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_R$ (s)</td>
<td>$I_p$ (MA)</td>
</tr>
<tr>
<td>4.5</td>
<td>12</td>
</tr>
<tr>
<td>3*</td>
<td>12</td>
</tr>
<tr>
<td>4.5</td>
<td>11</td>
</tr>
</tbody>
</table>

* Started on outside limiter, others started from inside with same $q(\psi)$; also, lower edge $T_{ea}$ at beginning of ramp.

Note that the 4.5 sec current ramp case has a more penetrated current profile by the end of the ramp than the shorter ramp with the same $\chi_e$. The temperature profile is also more peaked at the end of the ramp and at ignition because of the current profile and because of the increased central fusion heating.

This occurs because the plasma voltage, driven by the current ramp, peaks in the outer half of the plasma radius, where $T_e$ remains relatively small. This spatial distribution, and the resulting ohmic heating, can persist for a relatively long time after the current ramp ends.

One of the most important unknowns in the prediction of ignition is the extent of the degradation of the energy confinement that can be expected when the fusion alpha heating exceeds the ohmic. Good thermal confinement that does not fall off strongly from the ohmic heating level over the range $1 \lesssim P_\alpha/P_{OH} \lesssim 2$ (Table 1, $T_{OH} = 8$ keV) allows ignition soon after the ramp, or even during the longer ramp (not shown). In the L-mode regime, relatively large confinement times $\tau_E$ are allowed at ignition by the most recent scalings (Kaye–Goldston, Rebut–Lallia, Lackner–Gottardi, Coppi [4]), which are similar to the times shown in Table 1 for $T_{OH} = 5$ keV.
Careful control of the plasma current, particle density, and the plasma size and shape can maintain, up to and beyond ignition, sufficient stability to central sawtooth oscillations (a small region where the magnetic field line parameter \( q(\psi) < 1 \)) and to other macroscopic modes associated with magnetic reconnection (monotonically decreasing toroidal current density profiles \( J_\phi(\psi) \)). The plasma must be grown continuously in size while the current is being raised, to control the value of the edge \( q_a \) and to avoid the accumulation of current density in the interior, which can lead to the development of hollow \( J_\phi \) profiles. In addition, an appreciable current density pedestal should develop near the plasma edge during the ramp, to act as a current reservoir to prevent too fast an inward diffusion.

In order to acquire the needed degree of immunity to the onset of the \( m^o = 1 \) mode associated with the occurrence of sawtooth oscillations, the numerical examples follow the strategy outlined at the end of Section V.

IV. AD HOC MODEL FOR CURRENT TRANSPORT

We have also developed an ad hoc approach to current transport which is readily implemented in existing transport codes and which describes the time evolution of the current density toward well-behaved steady state profiles in a wide range of experiments. This model avoids the calculation of the higher derivatives of the current density, introduced by the theory of Section II.

Our remedy is to add an ad hoc “pinch” term \( \Omega_\ast \psi' \) to the usual form of Ohm’s law and to postulate an anomalous resistivity \( \eta_{AN} > \eta_{neo} \), so that the loop voltage becomes

\[
V_L = \frac{\eta_{AN} F}{\mu_0 (1/R^2)} \frac{\partial}{\partial V} \left( \frac{V' \psi'}{F} \left( \frac{|\nabla \rho|^2}{R^2} \right) \right) + \Omega_\ast \psi'
\]

Here \( \rho \) is a flux surface variable, primes denote differentiation with respect to \( \rho, \psi \) is the poloidal flux, \( V \) is the volume inside a flux surface, and \( F \equiv RB_\phi \). The quantity \( \Omega_\ast(\rho) \) is chosen so that the two (large) terms nearly cancel in steady state, leaving a remainder whose magnitude is consistent with neoclassical resistivity, in the sense that \( I_p/V_L \propto \langle \eta^{-1}_{neo} \rangle \). We note that there is no implied association with a macroscopic plasma
flow. Using Ampere's law, $\Omega_*$ can be straightforwardly obtained as a function of $V(\rho)$ and the current density profile characteristics.

To simplify the representation of well-behaved steady state profiles, we assume that, for instance, $(J_\phi/R) \propto (1 - V/V_a)^\gamma$, where $V_a$ is the total plasma volume. This form seems consistent with the polarimetry measurements and equilibrium fits referenced below, but it must be emphasized that the error bars associated with these procedures are large and, in particular, are also consistent with other representations such as the form used in Section II. From Ampere's law and the definition of the magnetic field line parameter $q(\psi)$, it is easy to express the exponent $\gamma$ in terms of $q_a/q_o$ and geometric factors, where the subscript $o$ denotes the magnetic axis.

As a prescription for the steady state value of $q_o$, we propose an approximate relation $q_o \simeq 0.55 + 0.05 q_a$, independent of the flux surface geometry and the electron temperature profile shape. This expression was motivated by consideration of polarimetry measurements in ohmically heated, circular TEXTOR discharges with and without sawteeth [8], and in a non-sawtoothing, elongated JET ICRF discharge [9], in addition to equilibrium fitting results using magnetic data from D-III-D [10]. We again point out the large uncertainties associated with these techniques, including the identification of steady state conditions, and note also that other approaches often yield higher values of $q_o$.

To complete the ad hoc model, the anomalous resistivity $\eta_{AN}$ must be specified. The time evolution of $q_o$ for both the JET discharge referenced previously [9] and a non-sawtoothing TEXTOR discharge [8] with much lower temperature can be reproduced reasonably well by a numerical solution of Faraday's law if we assume that $\eta_{AN}$ is nearly independent of the electron temperature profile and considerably larger than $\eta_{neo}$ near the magnetic axis.

V. GLOBAL PLASMA MODES

"Sawtooth" oscillations of the central part of the plasma column, associated with the instability of global plasma modes with dominant poloidal mode number $m^o=1$, can prevent [11] the attainment of ignition, if their period is considerably shorter than the energy confinement time
and the affected plasma volume is a significant fraction (< 1/10 [12]) of the total, a prescription that remains valid at present.

The ideal MHD stability parameter $\lambda_H$ is of particular importance to assess the properties of these modes. The growth rate is defined as $\gamma_{MHD} = \lambda_H v_A/(R\sqrt{3})$, where $v_A$ is the Alfvén velocity on the surface where $q = 1$, $\lambda_H \sim (r_1/R)^2 \beta_{p*}^2$, $\beta_{p*} = 8\pi[\langle p\rangle_1 - p_1]/B_{p1}^2$, where $r_1$ is the mean radius of this surface, $p_1$ and $B_{p1}$ the pressure and the poloidal magnetic field on it, and $\langle p\rangle_1$ the average pressure within the same surface [13]. The threshold $\beta_{p*}^c$, for which $\gamma_{MHD}$ exceeds zero, deteriorates with increasing vertical elongation of the plasma cross section and improves with its triangularity. However, the triangularity that can realistically be produced is rather limited. Therefore the values of $\beta_{p*}^c$, and consequently of $\beta_{p*}^c$, for which $\lambda_H < 0$ can be relatively low [14]. In particular, a tight aspect ratio (that is, favorable) configuration such as that of Ignitor, where $R/a \approx 2.7$ and the elongation is relatively modest, $\kappa \approx 1.8$, has $\beta_{p*}^c \approx 0.3 - 0.35$ for $q_a \approx 3 - 3.5$ and realistic profiles of the pressure and $q$.

A high energy particle population, created by an injected heating system such as ICRH or by the $\alpha$-particles produced by D-T reactions, can suppress the onset of sawtooth oscillations [15]. Therefore, to eliminate ideal MHD modes, it is advisable that $\beta_p$ remain well below the value of $\beta_{p*}^c$ that is obtained in the absence of a high energy particle population, until substantial D-T burning is produced and a considerable $\alpha$-particle population is present. This is the strategy of Ignitor, that is designed to attain values of $\bar{B}_p = I_\phi(MA)/5\sqrt{ab}$, the average poloidal field ($I_\phi$ in MA), close to 4T while programming the $J_\phi$ profile to maintain the volume where $q \leq 1$ relatively small (see Table 1). If a low $\bar{B}_p$ strategy is to be pursued instead, the total plasma current should be kept sufficiently low to maintain $q \geq 1$. This conflicts with the need to achieve the highest possible current in order to ensure maximum energy confinement under nonohmic conditions, at the high values of $\beta_p$ that are then reached at ignition.

While the ideal MHD modes may be avoided, there remain resistive modes that are also macroscopic in nature, but become more benign as the plasma temperature increases [16]. In particular, there exists a relatively large region in parameter space where these modes are stable.
This corresponds to temperatures where \( \omega_s = \omega_{se} \tau_H / \tau_H^{1/3} > 1 \). Here \( \tau_H \equiv \sqrt{3} R / \nu_A \), \( \tau_H \equiv \sqrt{3} R / \nu_A \), \( \varepsilon \equiv d \ln n / d \ln r \), \( D_m = \eta c^2 / 4\pi \), \( \omega_{se} = -1/(3r_1)(d \ln n / dr)(c T_e / e B) \), and \( n \) is the particle density. When \( \lambda_H < 0 \), the modes [16] that can be excited are the so-called "strong resistive mode" modified by diamagnetic effects and the slower growing drift-tearing mode [17]. The former is an almost purely growing mode that occurs close to ideal MHD marginal stability, corresponding to large values of the jump \( \Delta' \propto 1/|\lambda_H| \) in the derivative of the perturbed transverse magnetic field. The drift-tearing mode [17] is a propagating mode with frequency \( \omega \approx \omega_{se} \) and occurs for smaller values of \( \Delta' \), i.e., further away from ideal marginal stability. Within the two fluid model, both modes are found to be stable [18] in a wide "window" [19,20] of positive values of \( \Delta' \), \( 1/|\omega_{se} \tau_H| \sim \Delta'_1 < \Delta' < \Delta'_2 \sim (\omega_{se} \tau_H / \varepsilon q)^{1/2} \), when the effects of ion–ion collisions, that are stabilizing, are neglected. In this case, the growth rate in the drift tearing domain (\( \Delta' < \Delta'_1 \)) is a nonmonotonic function of \( \Delta' \) and is considerably smaller than that in the strong resistive mode domain (\( \Delta' > \Delta'_2 \)). In reality, since the width of the layer where magnetic reconnection is induced by these modes becomes related to the ion gyroradius, the combined effects of ion–ion collisions and of finite Larmor radius are important [21]. A particle and momentum conserving operator, such as that used in [21], has been employed to derive the mode equation in the reconnection layer and solve it numerically over a wide range of temperatures [22]. Thus, for \( \lambda_H \lesssim 0 \), complete stability can be attained. For parameters typical of the Ignitor, the strong resistive mode is found to be stable when the central temperature exceeds about 5 keV. Therefore the strategy that can be followed is that of programming the current density profile so that the region enclosed by the \( q = 1 \) surface does not exceed about 1/10 of the total volume in order to limit the effects of the expected sawtooth oscillations, until this temperature is reached. From this point on, a moderate expansion of the \( q = 1 \) surface can be tolerated while regimes where a substantial population of \( \alpha \)-particles that add a further factor of stabilization are achieved. In addition, r.f. waves at the ion cyclotron frequency that create an anisotropic high energy population can be injected in order to increase the stability margin against all resistive modes [15].
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EFFECT OF LOW ENERGY IONS AND NEUTRALS ON TOKAMAK TRANSPORT

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Abstract

EFFECT OF LOW ENERGY IONS AND NEUTRALS ON TOKAMAK TRANSPORT.

Part A. Low energy ions from recycling and gas puffing diffuse inwards too rapidly to thermalize. The resulting presence of excess low energy ions leads to simple explanations for many ion transport phenomena. Part B. The kinetic theory of neutral particles in tokamaks is studied analytically. The authors focus on regimes in which the mean-free path for charge exchange, \( \lambda_x \), is short, but allow for temperature variation and three-dimensional geometry. The neutral kinetic equation is analyzed by Chapman-Enskog theory, yielding the full neutral transport matrix. The authors also study the effects of neutrals on ion transport, finding: (i) the effect on ion particle transport is small; (ii) ion diamagnetic rotation is changed by addition of the neutral pressure to the ion pressure; (iii) ion energy transport is enhanced by a new ion heat flux, \( Q_x \). It is related to the neoclassical ion heat flux, \( Q_{NC} \), by \( Q_x/Q_{NC} \sim (n_\rho/n)(r/r_x)(\lambda_x/\rho)^2(R/r)^5 \). Mainly because \( \lambda_x \gg \rho_x \), \( Q_x/Q_{NC} \) need not be small.

Part A. The Effect of Low Energy Ions on Tokamaks by A.A. Ware

1. Introduction

It has been shown that due to the combined effects of electrostatic diffusion, large collision frequency for pitch angle scattering (\( \nu_{PA} \)) and small collision frequency for energy exchange (\( \nu_E \)), low energy ions diffuse inwards in tokamaks too rapidly to thermalize with the outward diffusing hot ions [1]. These effects are summarized in Section 2. Allowing for the presence of excess low energy ions leads to simple explanations for many hydrogen and impurity ion transport phenomena (see Section 3). In particular, there is a large increase in toroidal momentum transport leading to order of magnitude agreement with experiment [2]. The most compelling experimental evidence for excess low energy ions involves the density asymmetry measurements on the PDX tokamak [3]. The in/out density asymmetry caused by toroidal mass motion was found to increase with plasma density. This is
contrary to the predictions from simple momentum balance within a magnetic surface assuming each particle species has a single maxwellian velocity distribution. But, allowance for the presence of a 10-15% concentration of excess low energy ions, gives agreement with experiment [1].

2. The Diffusion of Low Energy Ions

Neoclassical effects generate electrostatic potentials on a magnetic surface $[\Phi(\theta)]$, whose order of magnitude satisfies at least $e\Phi / T \sim (\rho_{i\theta} / L) (r/R)$; but it can be substantially larger if poloidal or toroidal rotation is present [4]. The associated electric drift $(E \times B / B^2)$ is independent of particle energy, causing the low energy ions to have a substantial Pfirsch-Schluter flow. Since $\nu_{PA} \sim v^{-3}$, allowing for impurity ions, there are near singularities at low energies in both the Pfirsch-Schluter friction [5] and the electrostatic diffusion [1]. The dominant "force" driving the inward diffusion of low energy ions will generally be the negative radial electric field but, for discharges with the effective electric field $(E_r - V_{\parallel} B_\theta)$ small, the main "force" will be proportional to $(n'_c / n_c - n'_z / n_z)$, where $n_c, n_z$ are the densities of the excess low energy ions and impurity ions respectively. Assuming, for simplicity, that the excess low energy ions have an approximate maxwellian distribution $(n_c, T_c)$ and taking $n_H, T_H$ to represent the hot ions, a simple energy balance gives

$$\frac{T_H - T_c}{T_H} \sim \frac{\nu_{PA}}{\nu_E} \left( \frac{\rho_{i\theta}}{L} \right)^2 \frac{n_z Z^2}{n_H} \left( \frac{\rho_{i\theta}}{L} \right)^2 \left( \frac{T_H}{T_c} \right)^{3/2}, \quad (1)$$

where $\rho_{i\theta}$ is the poloidal Larmor radius of the hot ions. It is the extra factor $(T_H / T_c)^{3/2}$ which makes it possible for this fraction to be of order unity. Electrostatic turbulence which is capable of causing ion heat conduction will enhance the inward diffusion of low energy ions.

Another effect which reduces the rate of heating of the low energy ions as they diffuse inwards occurs because it is primarily the lower energy hot ions which interact with the excess cold ions. Assuming the temperature of the hot ions is maintained by processes at higher ion energies, steady state requires a distortion of the hot ion distribution function $f_H$ at low energies similar to the distortion of an electron distribution at low energies when there is collisional energy exchange between electrons and ions [6]. For small $n_c T_c / n_H T_H$, the normal Spitzer formula for the rate of energy exchange used in Eq. (1) is multiplied by the factor

$$\left[ 1 - (2\pi^2 3^{-5/4} n_c T_c / n_H T_H)^{2/3} \right],$$

reducing the rate.
3. Ion Transport Phenomena Explained by the Excess Low Energy Ions

As shown in reference [1] the following ion transport effects have simple explanations when allowance is made for the effect of the excess low energy ions. In most cases the explanation results from the ambipolar requirement that if low energy ions are diffusing inwards (or outwards in the case of pellet injection), there must be an equal and opposite diffusion flow involving impurity ions and/or hot hydrogen ions.

(a) Pulsed gas puffing leads to rapid increase of central density but no increase in ion energy content.

(b) Improved energy containment when recycling is reduced — H-mode, pellet injection, supershots and improved ohmic containment in ASDEX.

(c) Anomalous transport of toroidal momentum.

(d) The experimental absence of an effective electric field \((E_r - V_{\parallel} B_\theta)\) in PDX in conflict with that required to explain ion motion near the observed discontinuity in \(\partial f_0/\partial v\).

(e) The increase of the density asymmetry in PDX with increasing density.

(f) No neoclassical peaking of \(n_z(r)\) under normal conditions.

(g) Large decrease in \(n_z\) with pulsed gas puff.

(h) Increase of \(n_z\) in center with pellet injection.

Two other effects which also have simple explanations are:

(a) The observations on the DITE tokamak [7] that the containment time for injected impurities (which do not recycle) is reduced by more than a factor 2 when there is extra gas puffing \((dn/dt > 0)\) compared with the constant density case. The increased inward diffusion of excess low energy ions drives the impurities out.

(b) Similarly, adding strong gas puffing with H-mode operation in the JET tokamak reduces the buildup of carbon impurity [8].

4. Conclusions

The inward diffusion of excess low energy ions associated with recycling or gas puffing has the beneficial effect of driving impurity ions outwards
preventing their neoclassical buildup. The deleterious effect is that in steady state the parallel friction between cold ions and impurities is balanced by extra friction between hot ions and the impurities; the resultant extra outward diffusion of hot ions means increased ion heat conduction. Correspondingly with pellet injection, which reverses the flow of excess low energy ions for radii outside the evaporation region, there is increased inward diffusion (and accumulation) of impurities and reduced ion heat conduction.

REFERENCES


Part B. Neutral Particles in Tokamaks
by R.D. Hazeltine, M. Calvin, P.M. Valanju, and E. Solano

1. Introduction

Neutral particles are subject to three inelastic processes
target, with operator

\[ v_x X(f, g) = \int d^3v' \sigma_x |v' - v| \left\{ |\tilde{f}(v)g(v') - \tilde{f}(v')g(v)| \right\}, \]
where \( g \) is the neutral particle distribution function and \( \hat{f} \) is an ion distribution normalized to have unit particle density; impact ionization, \(-\nu_z g\); and recombination, \( \nu_r f \). Thus the kinetic equation for \( g \) is

\[
\frac{\partial g}{\partial t} + \mathbf{v} \cdot \nabla g = -\nu_z X(g, f) - \nu_z g + \nu_r f .
\]

The corresponding ion kinetic equation is

\[
\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla f + \mathbf{a} \cdot \frac{\partial f}{\partial \mathbf{v}} - C(f) = -\nu_z X(f, g) + \nu_z g - \nu_r f .
\]

Most of our results pertain only in the special case \( \nu_z \ll \nu_x \), in which each neutral suffers many exchanges in its lifetime. The rates \( \nu_x \) and \( \nu_z \) can be measured in terms of the mean free paths, \( \lambda = v_n/\nu \). The third length of interest is the scale length, \( L \), for neutral density variation. A random walk argument shows that \( L \sim (\lambda_x \lambda_z)^{1/2} \), whence

\[
\lambda_x \ll L .
\]

The inequalities are usually satisfied in realistic cases, if not very strongly.

2. Variational Principle

The CX entropy production can be expressed as

\[
\Theta[G, G] = \left( \frac{1}{2} \right) \int d^3 \mathbf{v} d^3 \mathbf{v}' \left( \frac{\sigma_{x}}{n_n} \right) |\mathbf{v}' - \mathbf{v}| \hat{f}(\mathbf{v}) \left( \frac{G}{f} - \frac{G'}{f'} \right)^2
\]

showing that

\[
\Theta[G, G] = 0 \iff G(x, \mathbf{v}) = N(x) \hat{f}(x, \mathbf{v})
\]

for an arbitrary spatial function \( N \). The inequality \( \Theta[G, G] \geq 0 \), for any \( G \), completes the entropy theorem for CX: the neutral distribution relaxes to have the velocity dependence of the ion distribution.

The isothermal, one-dimensional neutral kinetic equation can be solved by singular eigenfunction techniques.\(^2\) A complementary approach, based on a variational principle, allows for arbitrary temperature variation, arbitrary energy dependence of the CX cross section, and three-dimensional geometry. A fully general variational principle, applicable for arbitrary \( \lambda_x \), has been constructed from \( \Theta \). Here we present its short \( \lambda_x \) version.

3. Short Mean-Free-Path Theory

We introduce the small parameter \( \Delta \equiv \lambda_x/L \), and expand \( g = g_0 + g_1 + \cdots, g_n = \mathcal{O}(\Delta^n) \), to obtain a sequence of equations for the \( g_n \):

\[
\Delta^0: X(g_0, f) = 0 ,
\]

\[
\Delta^1: X(g_1, f) = 0 ,
\]

\[
\Delta^2: X(g_2, f) = 0 ,
\]

\[
\vdots
\]

\[
\Delta^n: X(g_n, f) = 0 .
\]
\[ \Delta^1: X(g_1, f) + \left( \frac{1}{\nu_x} \right) \mathbf{v} \cdot \nabla g_0 = 0, \] (5)

and so on. The CX conservation law,
\[ \int d^3v X(G, g) = 0, \] (6)

for any \( G \), provides a solubility condition in each order. In view of (3), (4) has the unique solution
\[ g_0 = n_n(x) \hat{f}, \] (7)

where \( n_n \) is the neutral density. We choose
\[ \frac{V}{v_n} \ll 1. \] (8)

with \( \nabla \cdot (n_n \mathbf{V}) = 0 \) and \( \frac{V}{v_n} \ll 1 \). Equation (8) approximates the observed ion distribution in most confinement experiments. The first order equation has become
\[ X(g_1, f) = Q, \] (9)

where
\[ Q = -\left( \frac{n_n}{\nu_x} \right) F \nabla \ln n_n + \left( \frac{v_{\text{th}}}{v_{\text{th}}} - \frac{5}{2} \right) \left( 1 + \frac{2 \nu \cdot \mathbf{V}}{v_{\text{th}}^2} \right) \nabla \ln T; \]
\[ + \left( \frac{2}{n_n v_{\text{th}}^2} \right) \nu \cdot \nabla (n_n \mathbf{V}) \Bigg\} \]

For a variational solution to (9), we introduce the linear form \( P[G] \equiv \nu_x \int d^3v GQ/(n_n \hat{f}) \), and find that the functional
\[ H[G, G] \equiv \frac{P^2[G]}{\Theta[G, G]} \] (10)

is variational: \( \delta H = 0 \). The extremal value, \( H[g_1, g_1] = P[g_1] = \Theta[g_1, g_1] \), gives the entropy production rate,
\[ H = -\Gamma \cdot \nabla \ln P_n - q \cdot \nabla \ln T; - \left( \frac{1}{p_n} \right) \pi: \nabla (n_n \mathbf{V}) \] (11)

as a familiar product of forces and fluxes.
When $\sigma_x|v-v'|$ is independent of relative velocity, we can calculate the neutral fluxes directly. Thus if $\sigma_x|v-v'|n_i/\nu_x = 1$, we have

$$X(g_1, f) = g_1 - \hat{f} \int d^3v' g'_i$$

and since $\int d^3v Q = 0$, $g_1 = Q$. Thus we obtain the fluxes

$$\Gamma = -\left(\frac{1}{2}\right) n_x \nu_x \lambda_x^2 \nabla \ln P_i,$$
$$q = -\left(\frac{5}{4}\right) n_x \nu_x \lambda_x^2 \nabla \ln T_i,$$

and

$$\pi_n = \frac{\rho_n}{\nu_x} \left\{ \left[ V \nabla \ln T_i + \left(\frac{1}{n_n}\right) \nabla (n_n V) \right] + \text{transpose} \right\}.$$  \hspace{1cm} (12)

The form is transparent: since they acquire an uncorrelated momentum at each exchange of charge, neutral particles execute a random walk with frequency $\nu_x$ and step-size $\lambda_x$.  

In second order we need only the solubility condition $\nabla \cdot \Gamma = \nu_x n_i - \nu_x n_n$, which implies $\Gamma \approx \left(\frac{n_n}{\nu_x}\right) \frac{v_x^2}{L_n} \approx n_n \Delta n$, whence $\Delta n \approx \nu_x \Delta^2 \approx \nu_x$, making small $\Delta$ consistent with $v_x < v_x$.

4. Effect on Ion Transport

First we recall the exact moment equations for ion and neutral evolution. The momentum conservation law for ions is

$$\left(\frac{\partial}{\partial t}\right) m_i n_i V_i + \nabla \cdot P_i - en_i (E + c^{-1} V_i \times \mathbf{B})$$
$$= F_e - F_x + \nu_x m_i n_i V_n - \nu_x m_i n_i V_i.$$ 

Here $P_i \equiv \int d^3v m_i vv' f$ is the ion stress tensor, $F_e$ is the collisional friction force on ions due to Coulomb collisions with electrons, and

$$F_x \equiv \int d^3v m_i \nu_x X(f, g)$$

measures the friction due to charge exchange. We define the neutral friction

$$F_n \equiv -F_x + \nu_x n_n V_n - \nu_x m_i n_i V_i$$

to write
\[
\left( \frac{\partial}{\partial t} \right) (m_i n_i V_i) + \nabla \cdot P_i - en_i(E + c^{-1} V_i \times B) = F_e + F_n. \tag{13}
\]

The corresponding neutral force law is
\[
\left( \frac{\partial}{\partial t} \right) (m_i n_i V_i) + \nabla \cdot P_n = -F_n. \tag{14}
\]

The ion pressure evolves according to
\[
\left( \frac{3}{2} \right) \left( \frac{\partial}{\partial t} \right) [p_i + \left( \frac{1}{2} \right) m_i n_i V_i^2] + \nabla \cdot Q_i = V_i \cdot (F_e + en_i E) + W_e + W_n \tag{15}
\]
where \( Q_i \) is the ion energy flux, \( W_e \) is the Coulomb energy exchange with electrons, and
\[
W_n \equiv \int d^3 \nu m_i v^2 / 2 \nu_c X(f, g) + V_i \cdot F_n + \nu_e \left[ p_n + \left( \frac{1}{2} \right) m_i n_i V_i^2 \right] - \nu_r \left[ p_i + \left( \frac{1}{2} \right) m_i n_i V_i^2 \right], \tag{16}
\]
reflects inelastic ion-neutral interaction. The neutral counterpart to (15) is
\[
\left( \frac{3}{2} \right) \left( \frac{\partial}{\partial t} \right) [p_n + \left( \frac{1}{2} \right) m_n n_n V_n^2] + \nabla \cdot Q_n = -W_n. \tag{17}
\]
Here \( Q_n \) is related to the previously computed neutral heat flux by
\[
Q_n = q_n + \left( \frac{5}{2} \right) \Gamma_n. \tag{18}
\]

We next use these moments to determine neutral effects on ion dynamics. The net effect of momentum exchange is to allow the neutral pressure gradient to act on ions: \( p_i \to p_i + p_n \). This is seen by adding (13) and (14):
\[
\left( \frac{\partial}{\partial t} \right) (m_i n_i V_i) + \nabla \cdot (P_i + P_n) - en_i(E + c^{-1} V_i \times B) = F_e. \tag{19}
\]
Note that neutral viscosity also adds to ion viscosity \( \pi_i \to \pi_i + \pi_n \), a fact that might bear on tokamak rotational relaxation.

In the short \( CX \)-mean-free path regime both stresses are approximately isotropic, whence
\[
V_i = b V || + \left( \frac{c}{e B n_i} \right) b \times [en_i E + \nabla (p_i + p_n)].
\]
Since the neutral and ion pressure gradients are opposed in much of the edge region, the observed effect is diminished ion diamagnetic rotation.

Neutrals do not affect ion particle transport, because the radial particle flux in an axisymmetric system is proportional to the toroidal component of the friction force, \( F_T \). Since \( F_{nT} = -(\nabla p_n)_T = -(1/R)\partial p/\partial \zeta \), the axisymmetric effect of neutrals on \( \Gamma_i \) vanishes exactly. (Even with asymmetry the effect appears small.)

Similar physics applies to energy transport, except that here the CX effect is large. It appears as enhanced ion heat conduction, together with convection. To estimate its importance, we compare it to the neoclassical ion heat loss,

\[
Q_{NC} \sim \nu_i \rho^2 \left( \frac{B}{B_p} \right)^2 \nabla p_i .
\]

The corresponding measure of the new process is \( Q_n \); Eq. (12) provides

\[
\frac{Q_n}{Q_{NC}} \sim \left( \frac{n_n}{n_i} \right) \left( \frac{\nu_x}{\nu_c} \right) \left( \frac{\lambda_x}{\rho_p} \right)^2 .
\]  

(20)

This ratio exceeds unity in typical circumstances because its last factor is large.

The explicit calculation begins with the sum of (15) and (17):

\[
\left( \frac{3}{2} \right) \left( \frac{\partial}{\partial t} \right) \left[ p_i + \left( \frac{1}{2} \right) m_i n_i V_i^2 \right] + \nabla \cdot (Q_i + Q_n) = V_i \cdot (F_e + e n_i E) + W_e .
\]  

(21)

All the terms in (21) are conventional except \( Q_n \). Thus neutral energy transport simply adds, in the ion energy balance equation, to ion energy transport.

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NOVEL COMPUTATIONAL TECHNIQUES TO PREDICT TRANSPORT IN CONFINEMENT DEVICES, AND APPLICATIONS TO ION TEMPERATURE GRADIENT DRIVEN TURBULENCE*

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Abstract

NOVEL COMPUTATIONAL TECHNIQUES TO PREDICT TRANSPORT IN CONFINEMENT DEVICES, AND APPLICATIONS TO ION TEMPERATURE GRADIENT DRIVEN TURBULENCE.

The thermal conductivity $\chi_i$ is computed for realistic experimental parameters by several different 3-d simulation techniques for ion temperature gradient driven modes in a slab. A widely used fluid model is also simulated. Both the kinetic $\chi_i$ magnitudes and the simulation results are inconsistent with the $\chi_i$ profiles seen on TFTR and JET. This indicates that the slab branch of ion temperature gradient driven modes cannot explain experimental transport. Fully kinetic calculations of $\chi_e$ are made possible using two different, new gyrokinetic simulation techniques which are up to two orders of magnitude faster than earlier gyrokinetic particle simulations. Both give $\chi_e$ in rough agreement with mixing length estimates using linear kinetic eigenfunction scales. However, $\chi_e$ is an order of magnitude or more lower than inferred values on TFTR and JET. Ion Landau damping and gyroaveraging effects are important for experimental parameters. The kinetic $\chi_i$ is an order of magnitude lower than the fluid $\chi_i$ but the scaling is similar. The fluid simulation $\chi_i$ agrees with mixing length estimates from linear fluid eigenfunctions. The differences between the fluid and kinetic $\chi_i$ can be explained by the difference in growth rates and scale lengths present in linear theory, which leads to a difference in the mixing length $\chi_i$ of about the same size as that found in the simulations. In a separate study, regression analyses are performed to obtain the local $\chi_i$ between the $q = 1$ and $q = 2$ surfaces in terms of the local dimensionless parameters present in the gyrokinetic equation. The slab $\eta_i$ model does not fit the data well.

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KINETIC SIMULATION TECHNIQUES

The nonlinear gyrokinetic equation in slab geometry to lowest order in the gyrokinetic expansion is

$$\frac{\partial \delta f}{\partial t} \left( x, v_\perp, v_{||} \right) + \dot{z} \times \nabla \langle \phi \rangle \cdot \nabla \delta f + v_{||} \nabla_{||} \delta f = \nu_{||} \nabla_{||} \langle \phi \rangle F_M + \left[ 1 + \eta \left( v^2 - \frac{3}{2} \right) \right] \dot{z} \cdot \nabla \langle \phi \rangle \cdot \hat{z} F_M,$$

where $\delta f = h + \langle q \varphi / T_i \rangle f_M$, time is normalized by $\omega_{\ast n}^{-1}$, $x$ and $y$ by $\rho_i$, and $s = L_n / L_s$, and $\langle \rangle$ is the gyroaverage. Two totally different algorithms are presented.

a) $\delta f$ Particle Algorithm. Previous particle algorithms compute the charge density by accumulating the number of particles in a cell. (Gyroaverage effects will be neglected in this discussion for simplicity.) Statistical fluctuations in the number of particles per cell leads to noise in $\varphi$, which can swamp the part of $\varphi$ from saturated instabilities.

In the $\delta f$ particle algorithm, the nonlinear equation Eq. (1) is solved for $\delta f$ by integrating $\varphi$ along the nonlinear particle orbits (i.e., the method of characteristics). The particle positions are evolved and act as markers for the value of $\delta f$. Note $\delta f$ is related to the full distribution function $f$ and background distribution $f_M$ by $\delta f = \langle f \rangle - f_M + \langle \langle q \varphi / T_i \rangle \rangle - q \varphi / T_i f_M$; thus $\delta f$ is proportional to the fluctuation amplitude, not the background distribution function. The perturbed charge density is computed by accumulating $\delta f$ on the markers to a grid. Since the nonlinear orbit equations preserve phase space volume, no net marker bunching errors arise. Statistical fluctuations in $\varphi$ are smaller than previous codes by roughly the factor $\delta f / f$. Typically $\delta f / f \sim 10^{-2}$; the $\delta f$ algorithm requires orders of magnitude fewer particles to simulate such microinstabilities because of the reduced noise. (Dimitz and Lee independently invented a similar gyrokinetic algorithm, but did not notice the low noise feature.) Note that for 3-d runs, $\delta f$ was damped to zero near the boundary to prevent quasilinear flattening.

b) Spectral Code. Here, $\delta f$ is expanded in basis functions: Fourier modes in $y$ and Hermite functions in $x$, and a grid is used in $v_{||}$ and $v_{\perp}$. Large time steps are possible since the linear terms in the equation are solved implicitly, using analytically derived linear orbit integrals over the source term for given $\varphi$. Further economies accrue since Hermite functions are close to the linear eigenfunctions, so few are needed.

Boundary conditions in $x$ are chosen so that the energy flux out of one side is put in on the other side, thus preventing profile flattening.
The spectral code is expensive for many modes, so 3-d runs use fewer modes than the \( \delta f \) particle code.

c) Tests and Comparisons. Both codes agree with linear fully gyrokinetic eigenvalue codes, typically to within several percent.

Comparisons were made with a standard gyrokinetic particle code for the nonlinear saturation of a 2d \( \eta_i \) mode in sheared slab. Parameters typical of TFTR were used: \( \eta_i = 4 \) and \( L_n/L_s = 0.25 \). However, the standard code needed a large equilibrium gradient scale (\( \rho_i/L_n = 1/40 \), much stronger than experiment) to increase the saturated amplitude above the noise.

The standard particle code was run with 300 K and 3000 K particles. The 300 K simulation was noise dominated; it showed no well defined exponentiating phase and had a large \( \varphi \) amplitudes. The 3000 K simulation gave a saturated amplitude which agreed well with the \( \delta f \) and spectral code, but required roughly 50 hours of Cray CPU time. Both the \( \delta f \) and spectral codes agreed, using roughly 10 min of Cray CPU time. The \( \delta f \) code gave converged results with 32 K particles (1 K = 1024).

All codes need an order of magnitude more time for 3-d runs. The \( \delta f \) and spectral codes run for acceptable expense (~ 4 Cray CPU hours). The cost of standard gyrokinetic algorithms for such low noise levels is many hundreds of hours or more.

**FLUID SIMULATIONS**

A fluid model used by many authors was simulated in sheared slab geometry. The value of the parallel viscosity and parallel pressure diffusion were chosen to mimic the effects of ion Landau damping as well as possible. Extensive parameter scans were done to arrive at the expression

\[
\chi_i = g \left( \frac{\rho_i}{L_n} \right) \left( cT_i/eB \right) (\eta - \eta_i) \exp(-\alpha s),
\]

where \( g \approx 1 \) and \( \alpha \approx 5 \).

**COMPARISONS OF 3D SIMULATIONS**

Figure 1 gives a comparison of linear growth rates and scale lengths (\( \Delta x^2 = \int \varphi^2 dx / \int (d\varphi/dx)^2 dx \)). Over most of the kinetic eigenmodes, \( \omega/\eta \nu_i \approx 1 \) and \( \Delta x \sim \rho_i \). Thus, for experimental parameters kinetic effects such as Landau damping and gyroaveraging are important. This leads to large differences between the fluid model and the kinetic results. Here \( \eta/\eta_{\text{critical}} \approx 2 \) for both fluid and kinetic cases. Comparison of results for \( \eta/\eta_c \approx 4 \) shows similar differences.
The linear mixing length estimates of the fluid and kinetic cases differ by roughly 20. The simulation results for $\chi_i$ differ similarly. Results for the normalized heat conductivity $F$, defined by $\chi_i = (v_i \rho_i^2 / L_n) F (\eta_i, L_s / L_n, T_i / T_e)$, are shown in Fig. 2. Despite the linear discrepancy, one might have hoped that simpler fluid models are closer to more complete kinetic models for nonlinear dynamics. This is not found. Similar results are found for $2 < \eta < 5$.

Also, note that the $\delta f$ code and the spectral code agree to within a factor of 2. The differences may be attributed to the different number of Fourier modes used in the two codes, the somewhat different boundary conditions in $x$, and to the presence of small dissipative terms in the spectral code added for numerical reasons.

For both kinetic codes, most of the transport comes from modes with the highest linear growth rates, at $k \rho_i \sim .35 - .7$. The spectrum drops to much smaller values as $k \rho_i$ increases.
Comparison of Simulation Heat Conductivities vs. Shear Parameter

In Fig. 3 we compare the kinetic and fluid $\chi_i$ with values inferred from experiment. In all cases, the $\chi_i$ from slab $\eta_i$ modes strongly decreases with minor radius, in contradiction with TFTR and JET observations. Also, the kinetic simulation values are much lower than the inferred values. Thus, we conclude that slab $\eta_i$ modes produce insufficient transport (especially at larger minor radii) to explain the experimental $\chi_i$.

Toroidicity induced $\eta_i$ modes and trapped particle modes may produce larger transport. Support for this possibility has been found in fully toroidal particle simulations using a code with full Lorentz ion dynamics and drift kinetic electrons. Toroidal runs with $\eta_i = \eta_e = 1$ have been found to have an order of magnitude more transport than otherwise equivalent cylindrical runs. Fluctuations in the saturated state had coupled poloidal harmonics with the same frequency, indicative of a toroidicity induced mode. Further simulations to test these possibilities, including runs with toroidal versions of the spectral code and with the $\delta f$ particle code, are in progress.
We turn now to the statistical analysis of experimental data. Scaling arguments applied to the gyrokinetic equation show that \( \chi_i = (v_i^2/\rho_i^2) F(\eta_i, L_n/L_s, T_i/T_e) \). Regression was used to obtain the best power law expression for \( F \) for beam heated TFTR shots, for minor radii between \( q = 1 \) and \( q = 3 \) surfaces, to obtain \( F = 0.17\eta_i^{1.0}(L_s/L_n)^3(T_e/T_i)^{1.4} \). The scaling with \( \eta \) and \( L_n/L_s \) is roughly similar to the simulation results. However the best fit did not explain 75% of the variation in the logarithm of \( \chi_i \). The experimental data scatter away from the fit value by a factor of 3 both high and low. This poor fit indicates that variables other than those arising in slab \( \eta \) modes are needed to reproduce the experimental \( \chi_i \).

REFERENCES

MODELS OF ENERGY TRANSPORT 
BASED ON MICROINSTABILITY THEORY

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Abstract

MODELS OF ENERGY TRANSPORT BASED ON MICROINSTABILITY THEORY.

The theory of the $\eta$ mode limit in the long wavelength limit is discussed, and a transport code analysis is carried out for the Frascati Tokamak and TFTR. Numerical simulations of $\eta$ mode turbulence are presented. Finally, the absorption of lower hybrid waves by fast ions and ion Bernstein wave propagation are treated.

1. THEORY OF THE $\eta$ MODE IN THE LONG WAVELENGTH LIMIT

The $\eta$ mode theory has been developed, in the past, in the short wavelength limit ($k_\rho \rho_i \gg \epsilon_T = L_T/R$ and $L_T = |\nabla T_e/T_i|^{-1}$). In this limit, the eigenfunctions have a moderate ballooning structure, and use can be made of the strong coupling approximation [1]. In the long wavelength limit ($k_\rho \rho_i \ll \epsilon_T$), the structure of the eigenfunction along the magnetic field becomes broader, and the strong coupling approximation can no longer be applied. As is shown in Ref. [2], within the framework of a fluid analysis, two different branches exist in this case, a toroidal and a slab branch. Here, we generalize the results of Ref. [2] to include the kinetic effects associated with the ion transit resonance.

The toroidal branch is characterized by eigenfunctions varying over the connection length scale, $\theta_0 \approx 1$, with an envelope varying over a secular scale, $\theta_1 \ll 1$, $\phi = \exp(-\sigma_\rho^2 \cos \theta_1/2$ [2]. Upon balancing parallel compressibility and adiabatic electron response on the scale $\theta_1 = 1$, we obtain $\omega \approx (\omega_\rho^2 \omega_\rho^2)^{1/3}$, with $\omega_\rho^2 = -k_\rho (cT_e/eB) \nabla p_e/p_i$, $\omega_\parallel = v_i/qR$; $q$ is the safety factor and $R$ is the major radius.

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4 NET Team, Garching, Federal Republic of Germany.
The width of the envelope is obtained by balancing parallel compressibility and geodesic curvature on the secular scale, yielding $\theta_i = \left[ e^{\lambda/(k_p\rho_i)} \right]^{1/2}$. The effect of parallel compressibility becomes dominant for $\omega \approx \omega_i$, corresponding to $k_p\rho_i \approx \epsilon_T$. Therefore, the optimum ordering for the toroidal branch is given by $\omega = \omega_{i}^{*} \approx \omega_i$, $k_p\rho_i \approx \epsilon_T$ and $\theta \approx \epsilon_T^{-1/2}$.

To solve the ion drift kinetic equation, the usual two-scales expansion can be used and the following dispersion relation is obtained:

$$1 + \frac{1 + \delta\tau}{\tau} + \Omega^2 \left\{ \frac{2\lambda}{\Omega} + \left[ 1 + \frac{2\lambda}{\Omega} \left( \Omega^2 - \frac{1}{2} + \frac{1}{\eta_1} \right) \right] \frac{1}{\Omega} Z(\Omega) \right\} = 0 \quad (1)$$

with $\tau = T_e/T_i$, $\eta_1 = d\ln T_i/d\ln n$, $\Omega = 2n/\omega_i$ and $\lambda = qk_p\rho_i/2^2\epsilon_T$. The quantity $\delta_T$ accounts for the effect of trapped electrons and is defined in Ref. [3].

Neglecting, first, the trapped electrons, we recover the fluid limit for $\lambda \gg 1$ [2], which yields two physical solutions corresponding, respectively, to an unstable mode propagating in the ion diamagnetic direction, $\Omega = (\lambda\tau)^{1/2}e^{(2\lambda\tau)^{1/2}}$, and a marginally stable mode, propagating in the electron diamagnetic direction, $\Omega = (\lambda\tau)^{1/2}$. When kinetic corrections are retained, the ion mode becomes marginally stable at a critical value of $\lambda$, given by $\lambda = 0.5[2(1 + 1/\tau)/(1 - 2/\eta_1)]^{1/2}$. Note that the critical $\lambda$ value diverges for $\eta_1 \to 2$. For $\eta_1 < 2$ the ion mode is stable for any value of $\lambda$.

The inclusion of the trapped electron response does not significantly alter the results for the ion toroidal mode because $\delta_T = O(\epsilon_T^{1/2})$. In the absence of trapped electrons, the electron mode is stabilized by the ion transit resonance. Meanwhile, in the presence of trapped electrons, the mode becomes unstable for sufficiently large $\lambda$.

The slab branch is characterized by eigenfunctions varying over the secular scale $\theta_i$, $\phi = \exp(-\sigma\theta_i^2)$ [1, 2]. Upon balancing inertia, parallel compressibility, and adiabatic electron response we obtain $\omega = (\omega_{i}^{*}k_p\rho_i\omega_{i})^{1/2}$ and $\theta_i = \left[ e^{\lambda/(k_p\rho_i)} \right]^{1/2}$. Again, the effect of parallel compressibility is important for $\omega \approx \omega_i$ or $k_p\rho_i \approx \epsilon_T^{1/2}$. Therefore, the optimum ordering for the slab branch is $\omega = \omega_i \approx \epsilon_T^{1/2}$ and $\theta \approx \epsilon_T^{-1/4}$.

In the case of the slab branch, we can employ the usual two-scales expansion, and the dispersion relation takes the following form:

$$\left[ \frac{1}{\tau} \left( 1 + \frac{2\lambda}{\pi \delta\tau} \right) \frac{2\lambda}{\eta_2 \Omega} \right] \frac{\Omega^2}{1 + \frac{2\lambda}{\Omega} \left( 1 + \frac{1}{\eta_1} \right)} = \lambda\epsilon_T \frac{s}{q} \left[ \frac{1 + \frac{2\lambda}{\Omega} \left( 1 + \frac{1}{\eta_1} \right) + \Omega \left( \frac{\Omega}{2} \right)}{1 + \frac{2\lambda}{\Omega} \left( 1 + \frac{1}{\eta_1} \right)} \right]^{1/2} \quad (2)$$
The function $F(R/2)$ accounts for the effect of the ion transit resonance and in the interesting limit $\Omega \ll 1$ reduces to

$$F \approx \frac{i\pi^2 q^2}{\Omega} \left\{ 1 + \frac{2\lambda}{\Omega} \left( \frac{3}{2} + \frac{1}{\eta_i} \right) \right. \\
- \left. \frac{1}{2} \left[ 1 + \frac{2\lambda}{\Omega} \left( \frac{1}{2} \right) \left( \frac{1}{\eta_i} - \frac{1}{2} \right) \right]ight\}$$

(3)

The analysis of Eq.(2) shows that an unstable mode always exists for $\eta_i > 0$.

2. TRANSPORT CODE ANALYSIS

A quantitative comparison has been performed between the steady state energy transport, for the Frascati Tokamak and TFTR tokamaks, and the transport model, derived in Ref. [3], which accounts for the combined effect of ion dynamics and trapped electron response. In the transport code analysis, the expressions for the ion flux computed in Ref. [3] have been multiplied by a numerical factor $C_i$ in order to account for deviations from the quasi-linear estimate.

The transport code used is the 1-D version of the JETTO 1.5-D transport code [4]. The density profile has been taken fixed to the experimental value, while evolving the equations for $T_e$, $T_i$ and $j_i$.

2.1. Frascati Tokamak (FT)

The parameters of the Frascati Tokamak are $a = 19$ cm, $R = 83$ cm and $B_T = 6$ T [5]. The density profile is modelled as a generalized parabola, $n = (n_0 - n_a) \times (1 - r^2/a^2)^{2\eta_n} + n_a$. Four discharges have been considered, two low density discharges (shots #20401 and #20457), corresponding to a line average density of $4 \times 10^{13}$ cm$^{-3}$, an intermediate density discharge (shot #19377) with $n_e = 10^{14}$ cm$^{-3}$ and a high density discharge (shot #10151) with $n_e = 2.2 \times 10^{14}$ cm$^{-3}$. At low density, the comparison with the measured neutron yield indicates that the ion transport is neoclassical. This is compatible with the model of Ref. [3], provided the $\eta_i$ threshold at low $\epsilon_n = L_n/R$ is increased from $\eta_{ic} = 2/3$ [3] to $\eta_{ic} = 1$ [1]. The simulation of the high density discharge shows temperature profiles broader than the experimental profiles while the central temperature is in good agreement. This yields values of $V_{loop}$ lower than the experimental values. Upon introducing 200 kW of radiation located at $r/a > 0.7$, the edge electron temperature tends to be broader, and the agreement is better. In Table I the results of the simulations discussed above are presented.
TABLE I. FT TOKAMAK

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<th>$K_{11}$</th>
<th>$V_{\text{loop}}$(V)</th>
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<th>$T_{\text{i,exp}}$(keV)</th>
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*In all cases, $\eta_{ic} = 1$ has been assumed.

2.2. TFTR

The Ohmic discharges considered for the TFTR device (a = 82 cm, R = 256 cm) taken from the MFE database [6] correspond to the shots #11603 (B = 3.9 T, $I_p = 1.79$ MA, $Z_{\text{eff}} = 4.7, n_e = 2.2 \times 10^{13}$ cm$^{-3}$) and #11609 (B = 3.9 T, $I_p = 1.79$ MA, $Z_{\text{eff}} = 2.2, n_e = 3.7 \times 10^{13}$ cm$^{-3}$) [7].

In this case the agreement is also remarkably good. The temperature profiles are slightly broader than the experimental value because the model tends to overestimate the transport in the central region and to underestimate it at the periphery. The ion temperature profile is also depressed in the central region in the shot #11603, with a difference of 450 eV in the central ion temperature. These discharges are characterized by $\epsilon_n$ values less than 0.2 outside the half-radius and of order of 1 inside. Upon increasing the $\eta_i$ threshold at low $\epsilon_n = L_n/R$ from $\eta_{ic} = 2/3$ to $\eta_{ic} = 1$, the ion temperature profile becomes slightly more peaked in the outer part, yielding a central ion temperature which is higher by 150 eV. A reduction of the ion transport by a factor of 2 ($C_i = 0.5$) yields a further increase of 150 eV in the central ion temperature.

Shot #11609 shows better agreement with the measured values. As in the previous case a change in the parameter $\eta_{ic}$ tends to affect the outer part of the discharge more pronouncedly. Upon increasing $Z_{\text{eff}}$ from 2.1 to 3.3, the agreement becomes better. In Table II the results of the above discussed simulations are presented.
TABLE II. TFTR TOKAMAK.

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* The last case of shot #11609 has $Z_{eff} = 3.3$. 

The NBI heated discharge considered [8] is shot #12951 ($B = 3.8$ T, $I_p = 1.79$ MA, $n_e = 4.1 \times 10^{13}$ cm$^{-3}$, $a = 80$ cm, $R = 259$ cm and $Z_{eff} = 1.9$). The total auxiliary power is $P_{aux} = 3.1$ MW with 0.97 MW going to the electrons and 2.13 MW going to the ions. The density profile is almost parabolic. Both electron and ion temperatures tend to be lower than the experimental values in the central part of the discharge. As in the Ohmic cases the ion temperature profile tends to be close to marginal stability. An increase in the parameter $\eta_{ic}$ from 2/3 to 1 raises the central ion temperature by 150 eV. A larger increase in the ion temperature is obtained by decreasing the ion transport by a factor of 2. Because of the proximity of the ion temperature profile to marginal stability it is expected that an increase in the $\eta_i$ threshold for $\epsilon_a > 0.2$ will improve the agreement. The results of the simulations are shown in Table II.

3. NUMERICAL SIMULATIONS OF $\eta_i$ MODE TURBULENCE

Two-dimensional numerical simulations of the $\eta_i$ mode show that long lived, large scale coherent structures exist and considerably affect the magnitude of the anomalous transport. The hydrodynamic ion equations are used to describe the toroidal $\eta_i$ instability, and the electrons are assumed to satisfy the Boltzmann relation,
\( n_0/N_0 \equiv e\Phi/T_e \). Two-dimensional slab geometry is considered, with \( \nabla \equiv x/L_\phi \partial/\partial y \equiv ik_y/L_\phi \), where \( L_\phi = qR/s \) is the shear length, \( s = rq'/q \) and the curvature term is evaluated at \( \theta = 0 \). The perturbed electrostatic potential \( \varphi \), the parallel ion velocity \( v \) and the ion pressure \( p \), are described by the following set of non-linear fluid equations:

\[
(1 - \nabla^2) \frac{\partial \varphi}{\partial t} = -(1 - 2\epsilon_n + \eta_i) \nabla_x^2 \varphi + 2\epsilon_n \frac{\partial p}{\partial y} - \frac{\partial v}{\partial y} + [\varphi, \nabla^2 \varphi] \tag{4}
\]

\[
\frac{\partial v}{\partial t} = -S \frac{\partial}{\partial y}(\varphi + p) - [\varphi, v] \tag{5}
\]

\[
\frac{\partial p}{\partial t} = -(1 + \eta_i + \epsilon_n) \frac{\partial \varphi}{\partial y} - [\varphi, p] \tag{6}
\]

with \( S = L_n/L_\phi \). In Eqs (4) to (6), normalized variables are used, with \((x,y) \rightarrow \rho(x,y), z \rightarrow L_n z \) and \( t \rightarrow L_n/c_s t \) with \( c_s = (T_e/M_i)^{1/2} \), \( r_i = c_s/\Omega_i \), and the fields scale as \((e\Phi/T_e, v_{i0}, c_s, p_i/p_e) = (\varphi, v, p)\rho_e/L_n \). The \( \mathbf{E} \times \mathbf{B} \) convective non-linearities are expressed in terms of the Poisson bracket operator by \( v_{\mathbf{E}} \cdot \nabla f = [\varphi, f] \).

Here, we address the question whether Eqs (4) to (6) admit a stationary solution in the presence of shear, corresponding to a vortex travelling with velocity \( u \) in the \( y \)-direction. Looking for solutions of the form \( \varphi(x,y,t) = \varphi(x,y - ut) \), we obtain

\( p = F(\varphi - ux) + Kx, v = G(\varphi - ux)Sx^2(1 + F')/2 \) and

\( \nabla^2 \varphi = H(\varphi - (u + K)x) + P_0 + xP_1 + x^2P_2 + x^3P_3 \tag{7} \)

where \( F, G \) and \( H \) are arbitrary functions, \( K = (1 + \eta_i)/\tau \) and \( P_j(\varphi - ux), \) with \( j = 0, 1, 2 \) and \( 3 \), are determined by \( K' F_1 = S^2F''/2, K' F_2 = -3P_3, K' F_3 = -2P^2 - 2G' \) and \( K' F_4 = u - 1 + 2\epsilon_n + 2\epsilon_n F' - P_1 \). Outside the vortex boundary the form of the functions \( F, G \) and \( H \) can be explicitly determined by the condition that \( p, v \) and \( \varphi \) vanish as \(|x|, |y| \rightarrow \infty \), giving

\[
\nabla^2 \varphi = C\varphi - \frac{S^2}{u^2} x^2\varphi - 3D(u + K)x\varphi^2 + Dx^3 \tag{8}
\]

with \( A = K/u, B = -S(1 + A)/2u^2, C = (u - 1 + 2\epsilon_n(1 + A))/(u + K) \) and \( D = SB(3u + K)/3(u + K)^3 \). Inside the vortex boundary, the choice of \( F, G \) and \( H \) is arbitrary.

In the shearless case, Eq. (8) reduces to \( p = A\varphi, v = 0 \) and \( \nabla^2 \varphi = C\varphi \), with \( C > 0 \) for a localized solution, giving a condition for the vortex speed, \( u > 0 \) or \( u < -K \), if \((1 - 2\epsilon_n)^2 < 8\epsilon_nK \), which is satisfied by the unstable \( \eta_i \) mode. For \( C < 0 \), the solution reduces to the stable linear mode, \( \exp(ikx - i\omega t) \) with the phase
velocity $u = \omega / k_y$. With a linear dependence for $H$, we obtain the usual dipolar vortex solution $\phi = (u + K) r_0 \xi \cos \theta$ with $\xi = K_1(\alpha ) / K_0(\alpha )$ for $r > r_0$ and $\xi = (1 + k^2 / q^2) J_1(qr_0) / J_1(qr)$ for $r < r_0$. Moreover, $k^2 = C$, $x = r \cos \theta$, $y - ut = rsin \theta$ and $q$ is determined by the condition $K_2(\alpha ) / [kK_1(\alpha )] = -J_2(qr_0) / [qJ_1(qr_0)]$. The solution in the shearless case has two free parameters of the vortex speed $u$ and the vortex radius, $r_0$.

In the case with shear, we use perturbation theory to find a stationary solution for small vortex radii. To lowest order, we obtain the shearless solutions. The magnetic shear yields a second order contribution, and the circular boundary becomes an ellipse because of the effect of shear, $r = r_0 + R \cos \theta$, with $R$ determined by the continuity conditions at the boundary. Coherent solutions thus exist also in the presence of shear, provided the vortex radius is smaller than the ion sound turning point scale. For larger values of $r_0$, it is possible from Eq. (8) to show that the solution for $\phi$ is oscillatory for large $x$ rather than evanescent. Therefore, localized solutions cannot exist in this case.

4. ABSORPTION OF LOWER HYBRID (LH) WAVES BY FAST IONS

Ion tails up to several MeV are generated and confined in the JET plasma during ion cyclotron (IC) heating experiments. During lower hybrid injection at 3.7 GHz, such ions interact with the LH waves, limiting the current drive (CD) efficiency of the LH system. To calculate the fraction of the LH power absorbed by fast ions during CD experiments in IC heated JET plasmas, a 1-D Fokker–Planck model, including the IC and LH quasi-linear terms, has been coupled to a deposition code for LH waves [9]. Since the ion tails generated by IC waves are strongly extended toward high energies, the wave–ion interaction occurs as the LH power reaches the region where the ion tail prevails. Then for given plasma and RF conditions, the percentage of the LH power absorbed by fast ions depends on the degree of overlapping of the IC and LH power deposition profiles. To find the maximum absorption capability by $^2$H and $^3$He ions, two different IC deposition profiles have been considered (on and off axis), while the LH deposition profile has been continuously changed from a peripheral to a central one, by changing the LH power, the electron density and the parallel refractive index. The results are summarized in Table III.

5. ION BERNSTEIN WAVE PROPAGATION

A ray tracing code for the study of ion Bernstein waves (IBW) propagation in tokamaks has been set up. The Hamilton equations for the rays are solved numerically, the Hamiltonian being given by the electrostatic dispersion relation and the effects associated with the ion Landau resonance and finite ion Larmor radius taken
TABLE III. LH POWER ABSORPTION BY FAST IONS

<table>
<thead>
<tr>
<th>Ion</th>
<th>Minority concentration (%)</th>
<th>IC deposition profile</th>
<th>IC power (MW)</th>
<th>( P_{\text{abs}}/P_{\text{LH}} ) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>5-10</td>
<td>on axis</td>
<td>10-20</td>
<td>0-20</td>
</tr>
<tr>
<td>H</td>
<td>5</td>
<td>off axis</td>
<td>10-20</td>
<td>15-35</td>
</tr>
<tr>
<td>(^3)He</td>
<td>5</td>
<td>on axis</td>
<td>10</td>
<td>0-10</td>
</tr>
<tr>
<td>(^3)He</td>
<td>5-10</td>
<td>off axis</td>
<td>10-20</td>
<td>0-30</td>
</tr>
</tbody>
</table>

into account. The behaviour of the complete numerical solution can be understood by expanding the dispersion relation according to the ordering \( k_1 \rho_i = N = (\omega - N \Omega_i)^{-1} = O(e^{-1}) \) and \( (\omega - N \Omega_i) \gg k_1 \nu_i \) (where \( N \) is the selected harmonic number). The dispersion relation then reduces to \( H = b k_1 - c = 0 \), where

\[
b = \frac{2\omega_{pi}^2}{\rho_i^2 \Omega_{io}^2} - \frac{M_i}{m_i} \omega_{pi}^2 k_1^2 \text{ and } c = \frac{1}{\epsilon \Delta x} \frac{\omega_{pi}^2}{\pi \rho_i^2 \Omega_{io}^2} \]

Here, \( \rho_i \) is the Larmor radius, \( \Delta x = R - R^* \), and \( R^* \) is the position of the vertical layer where \( \omega = N \Omega_i \). The ray equations reduce to \( d\theta/dr = -L_1 m_\theta/r \) and \( dm_\theta/dr = L_2 \theta/r \). As the variation of \( \theta \) and \( m_\theta \) with \( r \) is faster than the equilibrium profile variation \( (\nu = (L_1 L_2)^{-1} = (m_i/m_\nu)^{\nu} \Omega/(2 \pi \nu \omega q) \gg 1) \), these equations can be solved easily. On assuming that the resonant layer is at the centre of the discharge, the solution for \( k_1 \approx (n_\nu + m_\nu/q)/R \approx m_\nu/qR \) is \( k_1 = k_{10} \cos[\nu \ln(r/r_0)] \). As can be seen from this expression, \( k_{10} \) has an oscillating behaviour with respect to the radius, which could affect the absorption of the wave in the resonant layer. The absorption can be calculated from the antihermitian part of the dispersion relation, \( k_{10}^{-1} = -[\omega/(\rho_i \nu_i k_1)] \exp\{[-(\omega - N \Omega_i)/(k_1 \nu_i)]^2\} \), with \( k_{10} \) given by the previous expression. The calculation of the linear damping has shown that this oscillation of the parallel wavenumber with increasing frequency near the harmonic location does not affect the power deposition, the ray being totally absorbed near the resonance.

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CONTROL OF ALPHA PARTICLE TRANSPORT BY ICRH IN A TOKAMAK*

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Abstract

CONTROL OF ALPHA PARTICLE TRANSPORT BY ICRH IN A TOKAMAK.
A possible scheme to assist the control of alpha ash accumulations in a tokamak fusion reactor by enhancing their radial losses at the intermediate energy range is introduced. It is shown that electromagnetic waves in the ion cyclotron range of frequencies (ICRF) can induce tail alpha transport in a tokamak geometry.

1. INTRODUCTION

Modeling of burning plasmas has shown that the alpha particle confinement time must not be too much longer than the plasma energy confinement time in a sustained burn [1]. However, efforts to enhance the plasma energy confinement time, which may be necessary to produce a burning plasma, usually lead to a much larger enhancement of the particle confinement time (e.g., H-mode), resulting in too large an alpha accumulation. Hence, the development of active alpha removal schemes is a major concern.

Alphas can be subject to enhanced loss near the edge where the ripple field could be non-negligible, and within the \(q = 1\) surface where sawtooth or fishbone activities may be present. However, between these two regions (we call it the alpha confinement region in the present work), alpha particles are likely to be confined as well as the background plasma components. For alpha confinement shorter than the background plasma, we need to enhance selectively the alpha loss in the confinement region. The scheme presented in the present report is to induce alpha loss while they slow down, and thus to help deteriorate the alpha confinement.

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2. ALPHA TRANSPORT INDUCED BY AN ICRF WAVE

In the case of small \(|k_m v_\parallel| \ll \omega\), where \(\omega\) is the ICRF wave frequency, Ref. [2] obtained the particle flux driven by ICRF heating in the form,
\[
\Gamma_r = \left< f_p d^3v(v_\parallel/\Omega_p)\{Q\}(f_0) \right>,
\]
where \(\{Q\}\) is the bounce averaged ICRF heating operator, \(\Omega_p\) is the gyrofrequency in poloidal field, \(f_p\) indicates that the velocity integral is to be evaluated in the passing regime only [2].

From this expression, we can easily see that we may have a nonzero \(\Gamma_r\), if either \(\{Q\}\) or \(f_0\) is not even in \(v_\parallel\). For alphas produced by fusion reactions, \(f_0\) is characteristically even. There are two ways to make \(\{Q\}\) uneven in \(v_\parallel\) when the ICRF wave is directional in \(k_\parallel\). The first way is to have an in-out asymmetric ICRF-wave power density with zero-\(v_\parallel\) resonance layer through near the magnetic axis. In this case, the Doppler shift effect makes particles with different \(v_\parallel\) resonate at different values of major radius and feel the in-out asymmetric wave power density. The second way is to place the zero-\(v_\parallel\) resonance layer at the far inside of the flux surface. In this case having an in-out asymmetric wave field is not necessary, because alphas traveling in the opposite direction to the parallel wave vector will have their Doppler shifts inward and they cannot find their resonance points within the flux surface. In the present report, we present the first method because of its higher efficiency.

We use Ref. [3] for \(\{Q\}\), introduce normalized velocity variables \(s = \bar{v}/v_0\) and \(\lambda = \mu/w\), where \(\mu = v_\perp^2/2\) and \(w = v^2/2B\), and normalize the alpha distribution function \(f_0 = v_0^3 f_0/\beta_\alpha T_\alpha\). After some algebraic procedures, the alpha particle flux is expressed as

\[
\Gamma_r = \frac{P_{RF}}{\rho_\text{ep}} \frac{I_1}{I_2}
\]

\[
I_1 = -\pi \bar{B} \sum_\ell \sum_\sigma \int_0^{\omega = \Omega + k_m v_\parallel} \sigma d\lambda ds \frac{2^\ell}{\ell!} \frac{2\pi}{\ell!} \int_0^{\omega = \Omega + k_m v_\parallel} d\lambda \tau_0 \int_0^1 ds D_b \left| J_{\ell-1} \left( \frac{k_m v_{\perp R}}{\Omega_R} \right) \right|^2 \frac{\partial \tilde{f}_0}{\partial s}
\]

\[
I_2 = -\sum_\ell \sum_\sigma \frac{2\pi}{\ell!} \frac{2^\ell}{\ell!} \int_0^{\omega = \Omega + k_m v_\parallel} d\lambda \tau_0 \int_0^1 ds D_b \left| J_{\ell-1} \left( \frac{k_m v_{\perp R}}{\Omega_R} \right) \right|^2 \frac{\partial \tilde{f}_0}{\partial s}
\]

where \(\rho_\text{ep} = v_0/\Omega_p\), \(\epsilon_0 = 3.5MeV\) is the alpha birth energy, \(\bar{B}\) is a representative value of \(B\), \(J_{\ell-1}\) is the Bessel function of order \(\ell - 1\), \(v_\parallel R (\Omega_R)\) is the value of \(v_\parallel (\Omega)\) at the resonance location, \(D_b (\propto |E_\perp|^2)\) is the quasilinear ICRF diffusion coefficient, \(l\) is the length along \(\bar{B}\), and \(\tau_0 = \tau_\alpha T_\alpha v_0\) is the normalized bounce time. The resonance condition \(\omega = \Omega + k_m v_\parallel\) requires that the particles must be resonant somewhere along its orbits to have a non-vanishing \(\{Q\}\). We notice here that the present alpha particle flux is purely convective.
FIG. 1. Tail alpha flux as function of \(k_\parallel\) for a few representative values of \(k_\perp\). ITER geometry at half-radius is considered with an absorbed ICRF wave power density of 0.1 MW/m\(^3\), and a constant wave power density localized within the angular half-width of \(\pi/2\) radians at the outside of torus is used.

3. RESULTS AND DISCUSSIONS

Here, a standard form of slowing-down alpha distribution [4] is used, \(\dot{\phi}_0 = [4\pi(v_c^3 + v^3)]^{-1}\) for \(v \leq v_0\), and \(D_b\) is considered to be independent of velocity. The tail alpha flux driven by an ICRF wave is shown in Fig. 1, with \(k\) range appropriate for ITER plasma. ITER geometry at half radius is considered with absorbed ICRF-wave power density of 0.1 MW/m\(^3\) and a constant wave power density localized within the angular half-width of \(\pi/2\) radians at the outside of torus is used. Major contribution is from the third harmonic resonance, while the fourth harmonic resonance makes small contribution at larger \(k\) values. The dependence of alpha flux on the localization angle is shown in Fig. 2.

Figure 3 shows the flux surface averaged alpha flux when the same calculation is done at smaller minor radius (\(r=a/4\)). The third harmonic resonance makes sole contribution in the \(k_\parallel\) ranges shown. It can be seen that the flux-surface averaged alpha flux is larger at \(r=a/4\) than at \(a/2\) for the same amount of RF power density. In fact, the flux surface integrated alpha flux is larger at \(a/4\) than at \(a/2\). Counting the fact that the RF power density is going to be higher when focused into a smaller volume near the plasma center, we can see that the present scheme is more efficient when applied to central plasma region than the larger minor radial region. If we confine the RF power evenly inside of the half radius and use the result of Fig. 3 for an average radial flux within \(r=a/2\), a qualitative estimate can show that we need to have about 20 MW of total absorbed power by the alpha particles to get \(1 \times 10^{20}\) particles removed from the inner half radius.
Even though we get better efficiency when we apply the present scheme to the central burning region only, sometimes it may be necessary to emphasize the alpha removal near the half radius. An example case can arise when the sawtooth activity can remove enough alpha particles from the inner half of the minor radii and the ripple transport is strong enough to remove alphas at outer 1/3 of the minor radii. This leads to about 0.5 meter for the average radial thickness and about 200 m$^3$ for the volume of the confinement region in between the sawtooth and ripple transport regions. To obtain $1 \times 10^{20}$ /sec removal rate from the confinement region, a qualitative estimate from Fig. 1 shows that the total ICRH power absorbed to the alphas must be about 30MW. When the sawtooth activity is not frequent enough for alpha control in the
plasma center, we can put the RF power in the whole $r \leq 2a/3$ radius and obtain $1 \times 10^{20}/\text{sec}$ particle removal rate with about 50 MW of total absorbed power by the tail alpha particles.

The value of $\omega/k_v v_e$ is still very high, but the absorption of fast wave energy by the electrons could be non-negligible in this case because the waves will bounce many times before getting absorbed by the alphas at the third (and fourth) harmonic resonance. Previous study [5] shows that the electron absorption is negligible at $k_v \approx 10 m^{-1}$, but it becomes nonnegligible as $k_v$ becomes above $20 m^{-1}$. Hence, we may be able to drive electron current [6] simultaneously with alpha removal, if we choose large enough $k_v$ values. It is worth noting here that careful study shows the removed alphas are those at intermediate energy range (below several hundred KeV). Hence the associated energy loss is not significant.

REFERENCES

SUPPRESSION AND CONTROL OF MAGNETIC ISLANDS IN TOROIDAL PLASMAS

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Abstract
SUPPRESSION AND CONTROL OF MAGNETIC ISLANDS IN TOROIDAL PLASMAS.

Some recent theoretical developments on the formation and dynamics of magnetic islands in toroidal plasmas are reported. It is shown that an energetic ion population can have a significant effect on the non-linear stability of a single helicity tearing mode. A dynamical equation for the magnetic island width is derived from kinetic theory. It is shown that island growth can be suppressed in tokamak plasmas by injecting an energetic ion beam with a density profile which peaks just outside the rational surface. This technique can be used to suppress major disruptions caused by the non-linear growth of \( m = 2, n = 1 \) islands in a tokamak. The formation of magnetic islands as a result of plasma pressure in three-dimensional equilibria is also studied. The width of the equilibrium islands is shown to depend on the resistive interchange properties of the plasma. This analytical theory is applied to the Heliotron-E configuration. Equilibrium beta limits due to the criterion of island overlap are investigated. It is suggested that experimental observations of internal disruptions on Heliotron-E can be interpreted as a loss of equilibrium due to island overlap. Energetic ions can also be used to modify magnetic islands in three-dimensional equilibria. It is shown that energetic ions can be used to reduce the size of equilibrium islands in a stellarator due to vacuum field errors.

1. INTRODUCTION

The formation of magnetic islands in toroidal magnetic configurations generally has a detrimental effect on the confinement properties of plasmas. Islands caused by resistive tearing instabilities [1, 2] are believed to play a role in a variety of relaxation phenomena in toroidal discharges. In particular, the non-linear growth of the \( m = 2, n = 1 \) tearing mode can trigger major disruptions. For confinement systems without continuous symmetry, such as stellarators, magnetic islands exist in equilibrium. If the magnetic islands are large and overlap with each other, magnetic confinement is lost since stochastic magnetic fields cannot sustain a plasma pressure gradient.
In this paper, we examine two problems involving the formation of magnetic islands. In Section 2, we propose a method for drastically reducing the size of non-linear $m \geq 2$ tearing modes by introducing a population of energetic ions in the vicinity of the magnetic island [3, 4]. This technique can be used for disruption control in tokamaks by suppressing the growth of $m = 2$, $n = 1$ magnetic islands. In Section 3, we calculate the widths of magnetic islands induced by plasma pressure in stellarator equilibria [5] which extends earlier work on this subject [6]. When neighbouring magnetic islands overlap, magnetic confinement is lost. This island overlap condition can then set an equilibrium beta limit in stellarators. The theory is applied to the Heliotron-E device in Section 4. It is demonstrated that experimental observations of internal disruptions on Heliotron-E are consistent with the interpretation of an equilibrium beta limit [7]. Finally, we apply the method introduced in Section 2 to show that an injected beam of energetic ions can be used to control the size of islands in stellarator equilibria [4].

2. SUPPRESSION OF TEARING MODES BY ENERGETIC IONS

In this section we present a method which can be used to suppress the non-linear growth of $m \geq 2$ tearing modes. To understand the underlying physics of the calculation, we first give a simplified heuristic interpretation of the results in slab geometry [3, 4]. Consider the equilibrium magnetic field given by $B = B_0 z + B_y (x/L_s) y$, where $x = 0$ is the location of the rational surface, $L_s$ is the local shear length, and $B_0$ is a large constant magnetic field in the $z$-direction. If a coherent symmetry breaking perturbation $B_i = b_0 \sin(ky)x$ is imposed, magnetic islands of half-width $w = 2(b_0 L_s/kB_y)^{1/2}$ form at the rational surface. Fig. 1(a) shows the projection of the magnetic field in the $x$-$y$ plane.

The guiding centre motion of an energetic ion is given by $v = v_i b + v_d$, where $b = B/B$, $B = |B|$, and $v_d = [(v_i^2 + v_\text{cyc}^2)/\Omega_i] b \times \nabla \ln B$ is the magnetic drift. For $B = B(x)$ and $B > 0$, the drift velocity is predominantly in the $y$-direction. If the drift velocity is added to the field aligned velocity of the equilibrium field, the null line of the velocity of the equilibrium field is shifted by an amount of

$$x_s = - \frac{(v_i + v_\text{cyc}^2/2v_i)}{\Omega_i} \frac{L_s B_0}{L_i B_y}$$

from the null line of $B_y$, where $L_i = (d\ln B/dx)_1^{-1}$ and $\Omega_i$ is the hot-ion cyclotron frequency. (For toroidal geometry, $|x_s| \approx \epsilon q v_i / \Omega_i$, where $\epsilon$ is the inverse aspect ratio and $q$ the safety factor.) In the presence of the perturbation, the spatial contours of the hot-ion velocity field show islands similar to the magnetic islands of Fig. 1(a). This is shown in Fig. 1(b), where the assumption $|x_s| > w$ is used. Note that the sign of $v_d$ determines whether the islands form above or below $x = 0$. The contours of Fig. 1(b) represent constant density contours of the hot ions.
FIG. 1. (a) Projection of magnetic field on x-y plane; (b) projection of guiding centre velocity field for $v_1 = v \cdot b > 0$ and $v_1 < 0$; (c) effect of energetic ions with $|x| > w$. The solid lines are the magnetic field lines, with the X's representing constant density contours for ions with $v_1 > 0$ and O's the constant density contours for ions with $v_1 < 0$. 
If untrapped energetic ions have a net fluid velocity, they produce an electrical current. The plasma electrons tend to follow the ions in order to cancel this current; however, since the electrons scatter into the trapping loss cone faster than the ions, a net current in the direction of the ion flow results [8, 9].

In the vicinity of the island, as shown in Fig. 1(c), the effect of the perturbing field is to slightly deform the constant energetic ion contours from horizontal lines, as long as the inequality $|x_i| > w$ holds. We now allow for an energetic ion density gradient. For $n'_h > 0$, there are more ions at the top of Fig. 1(c) than at the bottom. For this density gradient, the energetic ions produce a current profile as one passes from the X-point of the island to the O-point with $j(X) > j(O)$. This spatial dependence of the current produces a magnetic field that is stabilizing. If the sign of the density gradient or the magnetic drift is reversed (given by $B'$), the energetic ions cause a field that enhances the perturbation. Note that the stabilizing effect is independent of the direction of injection of the energetic ions and depends only on the sign of $n'_h B'$.

The details of the calculation in toroidal geometry are carried out using a kinetic theory in the long mean free path regime [4], along with the usual Rutherford analysis [10]. Effects due to resistive interchanges [11], and bootstrap currents [12, 13] are included in the calculation. The injected energetic ions are assumed to be circulating, so that the effects due to magnetic trapping of the energetic ions can be ignored.

The dynamical equation for the magnetic island half-width $w$ of a non-linear tearing mode is given by

$$\frac{1}{\eta_n} \frac{dw}{dt} = k_0 \Delta' - Nw + \frac{Q}{w}$$

where $\eta_n$ is the neoclassical resistivity,

$$k_0 = c^2 |\Phi|^2 / 4\pi, \quad N = 1.46 \sqrt{\varepsilon} k_v \omega_{ph} d_4 \ln n_h / 4\pi d_4 \ln B$$

$$Q = 0.75 (E + F) c^2 |\Phi|^2 - 0.5 \sqrt{\varepsilon} p' R^2 c^2 q_0 / q_0'$$

where $\Phi$ is the toroidal flux function that labels magnetic surfaces, $k_v$ is a numerical coefficient that is approximately one, $\omega_{ph}$ is the hot ion plasma frequency, $R$ is the major radius, and $q_0$ and $q_0'$ the safety factor and its derivative evaluated at the rational surface. The first term on the right hand side of Eq. (1) describes the magnetic free energy (measured by $\Delta'$) available to the tearing mode [10]. The term $Q$ contains the effects due to resistive interchanges, described by the quantity $E + F$ [14], and bootstrap currents. The new result of this analysis is the term $Nw$ in Eq. (1). As mentioned above, this term is stabilizing if $n'_h B' > 0$. For discharges with $\Delta' > 0$, it is possible to control the island width by tailoring the hot-ion density profile. The saturated island width is given by

$$w_s = k_0 \Delta' / 2N + \sqrt{(k_0 \Delta' / 2N)^2 + Q/N}$$
Numerical estimates of $w_s$ consistent with the inequality $|x_s| > w_s$ indicate that, by using energetic ions, the saturated island width can be made much smaller than the usual quasi-linear saturated island width [15]. The energy requirements for this scheme appear to be quite modest and involve a small fraction of the energy expended on Ohmic or neutral beam heating. Thus, our analysis suggests that it is possible to suppress the $m = 2, n = 1$ island in tokamaks by having the energetic ion profile peak just outside the $q = 2$ surface.

3. MAGNETIC ISLAND FORMATION IN THREE-DIMENSIONAL PLASMA EQUILIBRIA

Well defined magnetic surfaces do not generally exist for three-dimensional equilibria [16]. However, if one postulates the existence of magnetic surfaces, it can be shown that the general solution of the equilibrium equations contains singularities in the plasma current. To see this, let us assume that the magnetic field can be written

$$B = \nabla \Phi \times \nabla (\theta - \iota \phi)$$

(3)

Here, $\Phi, \theta$, and $\phi$ are magnetic co-ordinates, where $\Phi$ is the toroidal flux function, $\theta$ and $\phi$ are the poloidal and toroidal angles, respectively, and $\iota$ is the rotational transform. The magnetic field lines lie on surfaces of constant $\Phi$. The Jacobian, $J = (\nabla \Phi \cdot \nabla \phi \times \nabla \phi)^{-1}$, and the parallel current profile, $Q = J \cdot B / B^2$, are represented by a Fourier series

$$J = \sum_{mn} J_{mn} e^{i n \theta - i m \phi}, \quad Q = \sum_{mn} Q_{mn} e^{i n \theta - i m \phi}$$

(4)

for equilibria with no symmetry. Using the force balance equation $J \times B = \nabla p$ and the quasi-neutrality condition $\nabla \cdot J = 0$, we find $p = p(\Phi)$ and the current amplitude $Q_{mn}$ has the general solution

$$Q_{mn} = -p'[J_{mn}/(\iota - n/m)] + \hat{Q}_{mn} \delta(\Phi - \Phi_r)$$

(5)

where $\hat{Q}_{mn}$ is an undetermined amplitude of a current sheet at the rational surface $\iota(\Phi_r) = n/m$.

The singularity is resolved by allowing for the formation of a magnetic island at the rational surface. The magnetostatic equations are then solved for self-consistency. The detailed calculation [5] leads to an equation for the magnetic island half-width. Written in terms of the extent of the magnetic island as measured by the rotational transform, $\delta \iota$, the half-width of the magnetic island is given by the expression
\[ \delta_t = \frac{\rho}{2} + \sqrt{\left(\frac{\rho}{2}\right)^2 + |C|} \] (6)

where

\[ \rho = \frac{(E + F)Z}{m_r} |\varepsilon(a) - \varepsilon(0)| \] (7)

\[ C = \frac{\beta}{m_t^2 \varepsilon_t^2} \left( \frac{J_{m_t}}{J_{\infty}} \right) \] (8)

$E + F$ is the resistive interchange instability criteria [14], $Z$ is a numerical constant $\approx 0.5$, $|\varepsilon(a) - \varepsilon(0)|$ is the total shear, and $m$ is the poloidal mode number of the island. If $E + F$ is negative (indicating resistive interchange stability), Eq. (6) predicts a small island since the term $|C|$ is usually made small by design. However, if $E + F$ is positive (resistive interchange instability), magnetic islands may be large. Furthermore, if $E + F > 0$, it can be demonstrated that island overlap is inevitable [6]. The island overlap criterion establishes an equilibrium beta limit that is generally more stringent than the ad hoc limit obtained by assuming that the flux surfaces are perfect, and then identifying the plasma beta at which the Shafranov shift exceeds one-half of the plasma radius.

The present calculation assumes that the vacuum magnetic field has well defined magnetic surfaces. If vacuum magnetic islands exist, Eq. (6) is modified by letting $|C| \to |C| + (\delta_t)^2$, where $\delta_t$ is the amplitude of the vacuum magnetic island. We notice that Eq. (6) predicts that devices with favourable resistive interchange properties can reduce the size of magnetic islands as plasma pressure is introduced [17].

To understand the physics of this result, it is useful to draw an analogy between 2-D axisymmetric systems with saturated 3-D instabilities, and intrinsically 3-D configurations [6]. In the 2-D case, the perturbations grow from the axisymmetric equilibrium until some non-linear process saturates the mode. A 3-D equilibrium can then be thought of as a 2-D equilibrium with intrinsic symmetry breaking perturbations. If the stellarator has unfavourable resistive interchange properties, the equilibrium has the same island structure as the axisymmetric device with saturated instabilities [11].

4. EQUILIBRIUM BETA LIMITS IN HELIOTRON-E

The theory of the previous section is now applied to the Heliotron-E experiment [7]. The island widths, given by Eq. (6) in the previous section, are evaluated numerically by using a modified version of the STEP code [18]. Since Heliotron-E has
unfavourable resistive interchange properties over a large part of the plasma, substantial magnetic island formation is predicted for Heliotron-E equilibria. Figure 2 plots the magnetic island half-width $\delta_{t_1}$ computed for plasmas with $\beta(0) = 2\%$ and mode numbers $m = 10-29$. Also plotted is the width computed from the Chirikov criterion (given by $\delta_{t_c} = |t_1 - t_2|/2$, where $t_1$ and $t_2$ are neighbouring resonant surfaces). We notice that, for $0.65 < t < 1.0$, island overlap is predicted since $\delta_{t_1} > \delta_{t_c}$. Although the analytical calculation breaks down when magnetic stochasticity occurs, this result is suggestive of an equilibrium beta limit due to the onset of island overlap.

These results motivate us to revisit experimental observations of internal disruptions in Heliotron-E [19]. It is observed that as beta increases for discharges with peaked pressure profiles, internal disruptions occur at $\beta \sim 2\%$ that lead to the flattening of the pressure profile. A possible explanation of these phenomena is that as the Heliotron-E plasma relaxes through a series of quasi-static, 3-D equilibria, magnetic islands form and increase in size with beta. When low order islands become sufficiently large that they overlap with each other, magnetic stochasticity occurs and the pressure relaxes to a flat profile. The computation suggests that this occurs at $\beta \geq 2\%$.

5. SUPPRESSION OF MAGNETIC ISLANDS IN STELLARATOR EQUILIBRIA WITH ENERGETIC IONS

An extension of the technique to suppress non-linear tearing modes with energetic ions can be used to control island sizes in stellarator equilibria as well [4]. For this problem, we consider island widths that satisfy the inequality $|x_1| < w < a$, where $a$ is the minor radius and the length $|x_1|$ is introduced in Section 2. (For the limit $|x_1| > w$, the physical picture given in Section 2 applies.)
When $w$ exceeds $|x_\phi|$, the energetic ions near the rational surface are trapped in the magnetic island at the rational surface. For this problem, Fig. 1(a) also represents constant density surfaces for the energetic ion population. In this limit, the direction of the net ion current and the sign of the shear determine whether the ions enhance or reduce the island size. If the islands are injected parallel to the magnetic field, with $\epsilon' > 0$, the ions produce a magnetic field with the same helicity as the perturbation and the island size increases; if, however, the ions are injected antiparallel to the magnetic field, the magnetic island is suppressed.

In the detailed calculation, we assume that the source of the magnetic island is a vacuum field error. In the presence of a plasma and the injected ions, the island half-width is computed self-consistently. The island equation is given by

$$\delta_t = \frac{S}{2} + \sqrt{\left(\frac{S}{2}\right)^2 + (\delta_t)^2} \quad (9)$$

where

$$S = \frac{8\pi}{c} \frac{k_s}{m_e} \frac{R f_\epsilon j_0}{B} \kappa \quad (10)$$

$\delta_t$ is the vacuum magnetic island half-width, $k_s$ is a numerical factor of order unity, $m$ is the poloidal number of the rational surface, $R$ is the major radius, $f_\epsilon$ is the electron trapping fraction, $\kappa = \text{sgn}(\epsilon')$ and $j_0$ is the injected energetic ion current. The sign of $S$ is determined by the sign of $j_0\kappa$, so that, if $j_0\kappa < 0$, the magnetic island width is smaller in the presence of the energetic ions than in the vacuum configuration.

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INVESTIGATION OF TRANSPORT PROCESSES NEAR THE DENSITY LIMIT

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Abstract

INVESTIGATION OF TRANSPORT PROCESSES NEAR THE DENSITY LIMIT.
The paper models the experimentally observed mechanisms of ECRH regimes near the density
limits, including such features as preferential electron cooling at the plasma column periphery, the
abrupt burst of an MHD m = 2 mode, a rise in the density limit with increasing power absorbed by
the plasma, an increase in neutral particle fluxes into the plasma, etc. No contradictions are found with
the experimental data.

1. BASIC MODELLING TASKS

Experimental studies were carried out at the T-10 facility on regimes where the
density limit \(\bar{n}_{cr}\) is attained, the main task being to study the causes limiting the
increase in density [1–3]. The experiments were conducted under both Ohmic heating
and auxiliary ECR heating conditions. The density limit was attained by switching
on additional gas puffing.

The aim of this paper is to model the experimentally observed mechanisms of
ECR heating regimes which occur when the density limit \(\bar{n}_{cr}\) is approached:

1. Preferential cooling of electrons at the periphery of the plasma column as \(\bar{n}_{cr}\)
is approached, leading to a transformation of the electron temperature and
current profiles \(T_e(r)\) and \(j(r)\);
2. The abrupt burst of the MHD \(m = 2\) mode, which in experiments always
preceded the disruption and was treated in the calculations as if the density limit
\(\bar{n}_{cr}\) had been attained;
3. The rise in the density limit as the actual power absorbed by the plasma
increases, in accordance with the expression \(\bar{n}_{cr} \sim P_{ex}^{1/2}\);
4. The rise in the fraction of radiation losses \(P_{rad}\) in the electron energy balance
as heating power increases;
5. The increase in neutral particle fluxes into the plasma as \(\bar{n}_{cr}\) is approached;
6. The weak dependence of \(\bar{n}_{cr}\) on the gas puffing rate \(\Gamma_c\) with a slight rise in \(\bar{n}_{cr}\)
as \(\Gamma_c\) increases;
7. Variation in the moment of the MHD \(m = 2\) mode burst with changes in the
conditions for achieving \(\bar{n}_{cr}\), the duration of the RF-pulse and the gas puffing
rate.
Free parameters were selected in the stationary stage so that the model could describe the discharge dynamics only by opening the puff valve (together with ECR heating) with a neutral flux similar to the experimental. The purpose of modelling the discharges with increasing density was as follows:

(1) To analyse the relative role of radiation and electron transport in limiting the growth of $n_e$;

(2) To discover whether satisfactory agreement with experiments can be obtained by using an electron transport model similar to the T-10 model which gives a good description of electron transport in the majority (both Ohmic and ECR heating at densities $n_e < \bar{n}_e$) of T-10 regimes [1, 4] (the question of an additional electron transport mechanism near $\bar{n}_e$);

(3) To explain the role of the region beyond the limiter, which sets the conditions at the boundary of the plasma column. These conditions are difficult to study experimentally.

2. MODEL DESCRIPTION

The proposed model, which allows self-consistent calculations of the development of the discharge across the entire cross-section of the plasma, including the boundary area (the scrape-off layer, SOL), consists of:

(1) A system of equations for the density $n_e$, the electron and ion temperatures $T_e$ and $T_i$ and the poloidal magnetic flux $\psi$, solved by standard ASTRA code methods [5];

(2) The SOL model, described in detail in Refs [6–8]. The model uses integral zero-dimensional relations which determine the balances of energy and particle fluxes in the plasma and neutral gas along and across the magnetic field and make it possible to take account of the basic effects essential for the peripheral zone, such as ionization of neutrals, radiation, longitudinal heat conduction and convective transport, and transverse Bohm diffusion;

(3) Analysis of the stability of the MHD $m = 2$ mode, which, using the calculated current profile, enables the moment of formation and the width of the magnetic island $m = 2$ to be determined [9]. This feature of the model reflects the attainment of the density limit $\bar{n}_e$, which is in accordance with the experimental results.

In the system of equations for the bulk plasma, the electron thermal flux $q_e$ was determined by the coefficient of heat conduction $\chi_e = \chi_e^n + \chi_e^l$, which is close, in terms of magnitude and parameter dependences, to the T-10 transport model obtained from analysing T-10 experimental data in Ohmic and ECR heating regimes. 
Under these conditions

\[ \chi^s = C^s \frac{c^2v_e}{\omega \rho qR} \sim \frac{T^{1/2}}{n} \]

is the transport due to stochastic diffusion in drift waves [10] and describes the central part of the column,

\[ \chi^r = 3 C^r \left[ \left( \frac{U_{\text{tor}} q}{2\pi B s L_T} \right)^{4/3} \left[ \frac{v_i m^2 s^2}{\nu_i R^2 q^2} \right]^{-1/3} \right] \approx \frac{10^{-2} q^2 n^{1/3}}{T^{5/6}} \sim 1 \]

is the transport due to the rippling mode [11] and, as shown by the calculations [5], is the most appropriate to describe the column periphery. In these expressions, \( v_e \) is the thermal velocity of the electrons, \( L_T = T/\nabla T \), \( s \) is the shear, and \( C^s \) and \( c^r \) are fitting coefficients of the order of unity. For agreement with transverse heat conduction in the SOL, which is considered Bohmian,

\[ D_{\text{Bohm}} = C^\text{Bohm} \frac{1}{16} \frac{cT}{eB} \]

the electron heat conduction was used in the form:

\[ \chi_e = \left( \frac{1}{\chi^s} + \frac{1}{\chi^r} + \frac{1}{D_{\text{Bohm}}} \right)^{-1} \]

The ion thermal flux was determined by the neoclassical coefficient of heat conduction \( \chi_i \) and was not very significant owing to the heat transfer taking place mainly via the electron channel. The particle flux \( \Gamma \) was determined by the diffusion coefficient \( D = \chi_e/2 \) and the pinch velocity \( V_p \):

\[ V_p = c \frac{E}{B_p a^2} \left( 1 - \frac{r^2}{a^2} \right) C^{vp} \]

which is approximately five to ten times greater than the neoclassical value.

The profile of RF power absorbed in the plasma, \( P_{\text{ex}} = 0.5-1.5 \, \text{MW} \), was taken from experiments, which made it possible to take into account the change in \( P_{\text{ex}}(r) \) with increasing \( \bar{n} \), owing to RF wave refraction.

The source of neutral particles entering the plasma was given by the equation:

\[ \Gamma_n = \xi \Gamma_s + \Gamma_c \]
where $\xi \Gamma_s$ is the flux of neutrals ejected from the limiter and $\Gamma_c$ is the flux of neutrals from the valve. The recycling coefficient $\xi = 0.99$ was selected in the stationary stage of the discharge.

The thermal and particle fluxes $Q_s$ and $\Gamma_s$ at the boundary between the bulk plasma and the SOL are determined by the equations for the bulk plasma. The boundary parameters $n_s$ and $T_s$, and also the parameters in the SOL (temperature $T_d$ and density $n_d$ close to the limiter, width of the SOL, $\Delta$) are calculated by using the SOL model and are functions of the fluxes $Q_s$ and $\Gamma_s$.

The radiation losses consisted of losses in the bulk plasma zone calculated in the coronal equilibrium approximation for carbon and iron and losses in the SOL, which were estimated as the 'ionization cost' $W_i$ multiplied by the ionization rate in the SOL. Impurities were taken into account by increasing the 'cost' of a single ionizing event, which was taken as $W_i \sim 50-100$ eV.

3. MODELLING RESULTS

It is clear from the above analysis that the development of the discharge is determined to a significant extent by the properties of the plasma in the SOL. To understand the dynamics of the process, it is therefore useful to consider the behaviour of the region beyond the limiter separately [7, 8]. Figure 1 shows the results of the numerical solution to the SOL model equations for T-10 in the form of the level lines of the boundary density $n_s$ and temperature $T_s$ in the plane ($\Gamma_s$, $Q_s$).

**FIG. 1.** Level lines of $N_s$ and $T_s$ and discharge trajectory in the ($\Gamma_s$, $Q_s$) plane for the limiter regime at T-10. $N_s (10^{19} \text{m}^{-2})$ is the plasma density at the boundary, and $T_s (\text{eV})$ is the electron temperature at the boundary.
FIG. 2. Evolution in time of the calculated parameters. $P_{\text{ex}}$ (MW): total power of auxiliary heating; $\langle n \rangle$ ($10^9$ m$^{-3}$): volume averaged plasma density, calculated (calc) and experimental (exp); $Q_s$ (MW): total thermal flux through the separatrix; $\Gamma_s$ ($10^{22}$ s$^{-1}$): particle flux through the separatrix; $T_s$ (eV): boundary temperature of the bulk plasma; $n_s$ ($10^{19}$ m$^{-3}$): boundary density of the bulk plasma; $P_{\text{rad}}$ (MW): radiation intensity in the SOL; MHD (a. u.): MHD signal, calculated (calc) and experimental (exp); $H_\alpha$ (a. u.): luminescence of $H_\alpha$ line; $T_e(0)$ (keV): central electron temperature, calculated (calc) and experimental (exp).

It is clear from the diagrams that, when some critical density is reached on the periphery $n_s^* \sim (2-3) \times 10^{19}$ m$^{-3}$, the dependence of $n_s$ on $\Gamma_s$ becomes very weak; under conditions of almost constant density at the boundary $n_s \sim n_s^*$, this may bring about a transition of the periphery to a large flux regime. As the neutral atoms entering the plasma are already ionized in the SOL, this increases particle circulation in the peripheral area and hence radiation losses and cooling of the plasma periphery.

Figure 2 shows the dynamics of the development of discharge No. 48255 with ECR heating power $P_{\text{ex}} \sim 1$ MW and with auxiliary gas puffing $dN/dt \sim 8 \times 10^{20}$ s$^{-1}$. The discharge trajectory in the plane ($\Gamma_s$, $Q_s$) is shown in Fig. 1.
FIG. 3. Evolution in time of the heat conduction profile, $He(r)$. $T = 0.5\ s\ (1), \ t = 0.57\ s\ (2), \ t = 0.64\ s\ (3)$.

The original (until its opening of the auxiliary puff valve and the switching on of RF pulse) values $Q_s \sim 100\ kW$, $\Gamma_s \sim 2.5 \times 10^{21}\ s^{-1}$, $n_s \sim 4 \times 10^{18}\ m^{-3}$, $T_s \sim 25\ eV$, are close to the experimental ones. When the RF pulse is switched on, the thermal flux $Q_s$ increases steadily for the first 70 ms up to $\sim 0.85\ MW$ and then stabilizes. The heat source $P_{ex}$ decreases during this time by 30%, owing to RF wave refraction. Opening the valve leads to an increase in $\bar{n}_e$, which agrees well with the experiment. Following the opening of the valve, the density at the periphery ($n_s$, Fig. 2) increases and then stabilizes at the value $n_s \sim 3 \times 10^{19}\ m^{-3}$, and the particle flux $\Gamma_s$ increases by a factor of approximately four. To make sure that this increase in the particle flux takes place, the plasma parameters must be altered so that the transport coefficients are increased at least at the periphery. Figure 3 shows the change in the heat conduction profile $\chi_e$ and, consequently, in the diffusion $D = \chi_e/2$ as the density increases. It is clear from this figure that the dependence of $\chi_e$ on $n_e$ and $T_e$ makes it possible, in accordance with the variation in density and temperature, to increase transport, initially by increasing thermal conduction and then by expanding the poor confinement zone.

The circulation of neutral atoms in the SOL increases with the increase in the flux $\Gamma_s$; this causes the intensity of radiation $H_n$ to rise by approximately one order of magnitude and radiation losses at the column periphery and in the SOL to rise up to $Q_{rad} \sim 0.8\ MW$ (before the disruption), which is close to the measured value; it then causes consequential cooling of the column periphery, a redistribution of the flux density $j(r)$ and, finally, the burst of the MHD $m = 2$ mode 105 ms after switching on the gas puff and RF pulse. The moment of the $m = 2$ mode burst is also close to that observed experimentally ($\sim 120\ ms$).

Figure 4 shows the variation in mean density and in the amplitude of the $m = 2$ mode for discharges where the gyrotrons are switched off at various moments.
FIG. 4. Evolution in time of the mean density \( \langle n \rangle \) \( (10^{19} \text{ m}^{-3}) \) and of the MHD signal in regimes where the gyrotrons are switched off at different moments in time: (1) 560 ms, (2) 590 ms, (3) without switching off ECR heating.

FIG. 5. Evolution in time of the mean density \( \langle n \rangle \) \( (10^{19} \text{ m}^{-3}) \) and of the MHD signal in regimes where varying ECR power is deposited: (1) 1 MW, (2) 0.75 MW, (3) 0.5 MW.

Since the temperature stops rising when the gyrotrons are disconnected, it peaks at an earlier stage. For the same reason the earlier the auxiliary heating is switched off, the earlier the \( m = 2 \) mode bursts take place, which agrees with the experiment (see Fig. 1 from Ref. [3]).

Figure 5 shows the variation in mean density and in the amplitude of the \( m = 2 \) mode for discharges with varying input power. The less power deposited, the smaller the increase in temperature and the quicker the cooling of the periphery, which leads to the earlier disruption of the discharge. An approximate root dependence on the power deposited of the density at which the disruption occurs has been obtained: \( n_{cr} \sim P_{ex}^{1/2} \). This corresponds approximately to the dependence on power of the critical boundary density, \( n_b^* \sim P_{ex}^{5/8} \) [7] obtained from the SOL model.
Figure 6 shows the variation in mean density and in the amplitude of the \( m = 2 \) mode for discharges with varying gas puffing rates \( \Gamma_e \), i.e. with varying rates of density increase. The accelerating rate of density increase causes the mean density \( n_5 \) to reach high values before the periphery cools. However, the greater density produces larger fluxes \( \Gamma_s \), and the disruption therefore occurs earlier than observed in the experiment \[3\].

4. CONCLUSIONS

(1) The proposed model yields a satisfactory description of the dynamics of the discharge under conditions where the density increases up to \( \bar{n}_e \) and describes the main mechanisms of discharge development observed experimentally.

(2) No contradictions were found with the experimental data when using an electron transport model which describes Ohmic and ECR heating regimes in the T-10 over the entire range of \( \bar{n}_e \).

(3) Modelling shows that variations in the conditions at the periphery leading to a lessening of the dependence of the boundary density \( n_s \) on the flux from the plasma \( \Gamma_s \) play a significant role in limiting the density.

(4) \( m = 2 \) mode bursts, which occur because of the changes in the current density profile \( j(r) \) as the critical density is approached, give a quantitatively correct description of the moment when \( \bar{n}_{cr} \) is reached.

REFERENCES


ANOMALOUS ELECTRON THERMAL DIFFUSIVITY, ANOMALOUS PARTICLE PINCH AND ISOTOPE EFFECT DUE TO SKIN SIZE ELECTROMAGNETIC DRIFT MODE

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Abstract

ANOMALOUS ELECTRON THERMAL DIFFUSIVITY, ANOMALOUS PARTICLE PINCH AND ISOTOPE EFFECT DUE TO SKIN SIZE ELECTROMAGNETIC DRIFT MODE.

The collisionless electromagnetic drift mode has its maximum growth rate of the order of the ion transit frequency at \( k_t \approx \omega_i/c \) (the inverse skin depth) over a wide range of plasma beta. A finite beta is destabilizing. The particle flux due to the instability is radially inward (anomalous pinch), and the predicted electron thermal diffusivity agrees well with those diffusivities experimentally inferred from JET (Joint European Torus) both in magnitude and in reproducing the radial profile.

Identifying the key instability causing the anomalous particle and thermal transport in tokamaks has been a challenging theoretical problem. Numerous models have been proposed in the past based on the well known instabilities, such as the dissipative trapped electron (DTE) mode, the ion temperature gradient (ITG) mode, and the electron temperature gradient (ETG) mode. Each model has advantages and disadvantages when compared with experimental data. In the lowest order, the success of a model may be assessed by whether it explains at least the two outstanding anomalies well established, namely, the anomalous electron thermal diffusivity approximately proportional to the inverse of the electron density (\( \chi \propto 1/n \)) and concurrent anomalous particle pinch (\( \Gamma < 0 \)). The DTE mode predicts \( \chi \propto 1/n \), but \( \Gamma > 0 \). [1] The ITG mode predicts \( \Gamma < 0 \) only when the density profile is relatively flat [2]. Furthermore, \( \chi \propto 1/n \) is difficult to explain in terms of the ITG mode. (Besides, the role played by the ITG mode in the tokamak transport is becoming increasingly questionable [3-6].) The toroidal ETG mode [7] is characterized by almost adiabatic ion response because of the short wavelength nature. Therefore, the ETG mode is unable to explain the anomalous pinch.

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Since Ohkawa [8] proposed the well-known empirical formula for the anomalous electron thermal diffusivity in tokamaks based on the assumption that electromagnetic suprathermal fluctuations in the region \( k \approx \omega /c \) (the inverse skin depth) are somehow excited, several attempts have been made to clarify the mechanism of fluctuation enhancement [7,9]. However, searches for linearly unstable electromagnetic modes in that particular wavelength regime have not been successful. Then, Horton et al. [7] have argued that the short wavelength ETG mode should nonlinearly down cascade to the region \( k \approx \omega /c \) to excite electromagnetic modes. Skin size electromagnetic turbulence is attractive, for the dependence \( \chi \approx 1/n \) naturally emerges through the crossfield random walk distance. However, the mechanism proposed in Ref. [7] may not be required if the particular wavelength region of interest is linearly unstable. Recently, it has been shown [10] that in a low beta tokamak the growth rate of the electromagnetic toroidal drift mode peaks in the region \( k \approx \omega /c \). The instability is further destabilized by trapped electrons and electron temperature gradient. Collisional damping is weak because of the high frequency nature. Therefore, the instability is expected to be robust under realistic discharge conditions.

In this paper, it will be shown that the anomalous electron thermal diffusivity predicted from the instability is capable of explaining several anomalous features experimentally observed. At the same time, the predicted particle flux is radially inward (anomalous pinch). The anomalous electron thermal diffusivity emerging from this mode is given by

\[
\chi_e = \left( \frac{c}{\omega_{pe}} \right)^2 \frac{1 + Aq^2}{qR} \frac{1 + \eta_e}{\eta_e} c_s
\]

where \( c/\omega_{pe} \) is the skin depth, \( c \) the ion acoustic speed, \( q \) the safety factor, \( R \) the major radius, \( A \) a constant of order unity, and \( \eta_e \) the electron temperature gradient factor. The appearance of the skin depth as the random walk distance is due to the fact the growth rate of the instability peaks at \( k \approx \omega /c \). The ion acoustic transit frequency, \( c_s /qR \), is approximately the maximum growth rate of the instability and gives rise to the ion mass dependence (isotope effect) of the thermal diffusivity. The factor \( (1 + \eta_e)/\eta_e \) is due to the strong disparity between the electron thermal diffusivity and particle diffusivity \( \chi_n \gg D \), as will be explained. When \( Aq^2 > 1 \), \( \chi_e \) becomes proportional to \( q \). The appearance of the safety factor in the numerator, rather than in the denominator [7,8], agrees with the general experimental trend that the global energy confinement time improves with the plasma current almost linearly. The anomalous particle flux caused by the instability is against the density gradient (anomalous particle pinch). Therefore, the instability explains
several outstanding transport anomalies in tokamaks. Further experimental support for the present theory is provided by the fact that the predicted high frequency, short wavelength electromagnetic fluctuations have been observed in several tokamaks [11,12].

The linear aspects of the subject instability can be qualitatively revealed from the following electromagnetic local dispersion relation [10]

\[
\left\{ \left[ \frac{c k_\perp}{\omega_{pe}} \right]^2 + 2 F_{e u 2} \right\} \left[ F_{e u 0} + F_{e T 0} - \tau (1 - F_{10}) \right] - 2 F_{e u 1} \phi = 0 \quad (1)
\]

where \( \tau = T_e / T_1 \),

\[
F_{10} = \left\{ \frac{\omega + \omega_{\perp 1}}{\omega + \omega_{D 1} - k_\perp v_\perp} \right\} J_0^2 \left( k_\perp v_\perp / \Omega_1 \right) \quad (2)
\]

\[
F_{e j n} = \left\{ \frac{\omega - \omega_{\perp e}}{\omega - \omega_{D e} - k_\perp v_\perp \delta v_j} \right\} J_0^2 \left( \Lambda_{e j} \right) \quad (3)
\]

with \( \langle \rangle \) indicating averaging over the velocity space with a Maxwellian weighting, \( J_n \) is the Bessel function, \( j = U (|v_\parallel| > v_{ce}) \) for untrapped electrons, \( j = T (|v_\parallel| < v_{ce}) \) for trapped electrons, \( \Lambda_{e T} = q k_\perp v_\perp / v_{ce} \) (which approximately takes into account the finite banana orbit of trapped electrons), \( \Lambda_{e u} = k_\perp v_\perp / \Omega_e \), and

\[
\bar{\omega}_{\perp j} = \omega_{\perp j} \left[ 1 + \eta \left( \frac{\varepsilon_j}{T} - \frac{3}{2} \right) \right] , \quad \varepsilon_j = \frac{1}{2} \frac{m_j v^2}{T}
\]

\[
\bar{\omega}_{D j} = \omega_{D j} \left[ 1 + \eta \left( \frac{c T}{2 T} + \frac{1}{2} \right) v^2 \right] , \quad \omega_{D j} = \frac{2 c T}{e B^2} \langle \vec{\nabla} B \times \vec{B} \rangle \cdot \vec{K}
\]

Eq. (1) has been derived under the following assumptions: \( \omega_{b 1} \ll \omega \ll \omega_{ce} \) (\( \omega \), the bounce frequencies), low \( \beta \) so that compressional magnetic field perturbation is ignorable, and negligible collisionality. The norms of operators for a strongly ballooning mode are [10]:

\[
\langle k_\perp^2 \rangle = k_\theta^2 \left[ 1 + \frac{\pi^2}{3} - 2.5 \right] s^2 - 10 \alpha s / 9 + 5 \alpha^2 / 12 \quad (4)
\]

\[
\langle \omega_{D j} \rangle = 2 c \frac{\omega_{\perp 1}}{2 \Omega_j} \left( \frac{5}{3} + \frac{5}{9} \alpha - \frac{5}{12} \alpha \right) \quad (5)
\]

\[
\langle k_\parallel^2 \rangle = \frac{1}{3 (q R)^2} \left[ 1 + \left( \pi^2 / 3 - 0.5 \right) s^2 - 8 \alpha s / 3 + 3 \alpha^2 / 4 \right] \quad (6)
\]

where \( s \) is the shear parameter and \( \alpha \) is the ballooning parameter.
Figure 1 shows the growth rates normalized by the ion transit frequency versus $k = c k_\theta / \omega_{pe}$ when $\beta = 0.2$ and $1\%$, $\tau = 1$, $\epsilon_e = 0.3$, $\epsilon = 0.25$, $s = 1$, $q = 2$, $\eta_e = \eta_i = 2$. The frequency for $k = c k_\theta / \omega_{pe} > 0.3$ is approximately constant and given by $\omega = \omega_{\parallel} \approx 2 \epsilon \omega_{pe}$. It can be seen that a finite $\beta$ is destabilizing, although rather weakly. The maximum growth rate occurs at $k_\theta = \omega_{\parallel} / c$ (the inverse skin depth) relatively independent of $\beta$. For typical tokamak discharge parameters, the maximum growth rate is approximately given by $\gamma = \omega_T$, the ion transit frequency, or the ion acoustic frequency in an isothermal discharge with $T_e = T_i$. It is noted that the low $\beta$ case ($\beta = 0.2\%$) is relevant to typical ohmic discharges and the higher $\beta$ case ($\beta = 1\%$) approaches the ballooning limit. (For the parameters chosen, $\alpha$, the ballooning parameter is 0.35 when $\beta = 1\%$.)

As shown in Ref. [10], the electron temperature gradient and trapped electrons are not essential to the instability, for the inverse Landau damping of untrapped electrons alone can excite the mode. However, since both have destabilizing influences, their effects should be retained to model realistic discharges.

The quasilinear particle flux ($\Gamma$), ion and electron thermal fluxes ($Q_i$, $Q_e$) are proportional, respectively, to the following moments,

\[ \Gamma \propto - \text{Im} F_{10} \]
\[ Q_i \propto - \text{Im} F_{1q} = - \text{Im} \left\langle \frac{M v^2}{2 T_i} \frac{\omega + \bar{\omega}}{\omega_{D1} - k_\parallel v_\parallel} J_0^2(\lambda_1) \right\rangle \]
FIG. 2. Imaginary parts of the moments $F_{i0}$, $F_{iq}$, and $F_{eq}$ for the mode shown in Fig. 1 ($\beta = 0.2\%$).

\[
Q_e \propto \text{Im} \, F_{eq} = \text{Im} \left\langle \frac{m v^2}{2T_e} \frac{\omega - \bar{\omega}_e}{\omega - \bar{\omega}_{De}} J_0^2 \left( A_{eT} \right) \right\rangle_T \\
+ \text{Im} \left\langle \frac{m v^2}{2T_e} \frac{\omega - \bar{\omega}_e}{\omega - \bar{\omega}_{De} - k v_{\|}} \right\rangle_T \left| 1 - \frac{D_{es}}{\text{Feu1}} \right|^2 
\]

(9)

where $D_{es} = F_{e\omega0} + F_{e\omega1} - 1 - \tau(1 - F_{i0})$ becomes the electrostatic dispersion relation when equated to zero.

Fig. 2 shows the imaginary parts of $F$'s as functions of $k$ for the mode shown in Fig. 1(a) ($\beta = 0.2\%$). Except in a limited region at long wavelengths, the particle flux is predominantly negative ($\text{Im} \, F_{eq} > 0$). The electron thermal flux is radially outward over the entire $k$ region. The ion thermal flux is outward for $k < 0.25$ and inward for $k > 0.25$. The inward ion thermal flux is negligibly small compared with the electron thermal flux. These general features remain unchanged when $\beta$ is increased to 1%. 
The unstable region in $k_\theta$ extends up to the inverse skin depth, $\omega /c$. The diamagnetic frequency with $k_\theta = \omega_p /c$ in a typical ohmic discharge [$\beta = 0(0.1-0.3\%)]$ is of order of $1\,MHz$. Recently, such high frequency (thus short wavelength) density and magnetic fluctuations have been identified in several tokamaks with ohmic heating alone [11,12]. In particular, close correlation between the density and magnetic fluctuation spectra reported in Ref.[11] clearly suggests that even in low $\beta$ ohmic discharges, high frequency drift type modes have significant magnetic components.

As shown in Fig. 1, the growth rate slowly increases with $\beta$. This is in contrast to the case of the ITG mode for which electromagnetic corrections are stabilizing. [13] (This conclusion remains unaltered when trapped electrons and kinetic effects are incorporated in the ITG mode. [14])

In Ref. [15], it has been shown that a simple function

$$\frac{1}{n_e} \frac{1 + \eta_e}{\eta_e}$$

can well reproduce the radial profiles of the electron thermal diffusivity in several tokamaks (JET, PLT, and TFR). The function has also been tested against the $\chi_e$ profiles in TEXT [16] with satisfactory reproduction. The $1/n$ dependence is consistent with the skin depth playing the role of cross-field random walk distance as previously noted [7-9]. The factor $(1 + \eta_e)/\eta_e$ has been given an explanation based on the nonlinear saturation of the drift mode. However, an alternative explanation may be given as follows. In fully developed density and magnetic turbulence, almost complete pressure mixing is expected, implying

$$|p_e| \sim \frac{1}{k_\perp} \left| \frac{dP_e}{dr} \right|$$

where $p_e$ is the electron pressure perturbation and $dP_e/dr$ is the equilibrium pressure gradient. The resultant anomalous total thermal flux is therefore

$$Q_e = \langle v_e p_e \rangle = - \frac{v_r}{k_\perp} \frac{dP_e}{dr} = - n_e \chi_e \frac{dT}{dr} - T_e \chi_e \frac{dn}{dr}$$

If $\chi_e >> D$, we obtain

$$\chi_e = \frac{v_r}{k_\perp} \frac{1 + \eta_e}{\eta_e} \sim \frac{\Omega}{k_\perp^2} \frac{1 + \eta_e}{\eta_e}$$

where $\Omega$ is the nonlinear decorrelation frequency. The factor $(1 + \eta_e)/\eta_e$ thus emerges independent of the amplitude and nature of turbulence provided $\chi_e >> D$. In JET, it has recently
been reported [17] that there exists a significant disparity between $\chi_e$ and $D$, $\chi_e \approx 7D$, together with clear evidence for the anomalous pinch.

Rigorous evaluation of the decorrelation frequency $\Omega$ is a difficult theoretical problem. However, as usually done, if the growth rate, which is of the order of the ion acoustic frequency, is taken as $\Omega$, we obtain the following estimate for the anomalous electron thermal diffusivity,

$$\chi_e \approx \left( \frac{c}{\omega_{pe}} \right)^2 \frac{(1 + Aq)^2 c_s}{qR} \frac{1 + \eta_e}{\eta_e}$$

(11)
where the factor $1 + A q^2$ is to take into account a possible enhancement in the diffusivity due to the toroidal geometry with $A$ a numerical factor of order unity. Figure 3 shows comparison between the electron thermal diffusivity experimentally deduced from three types of discharges in JET [18] and that predicted in Eq. (11) with $A = 2$. (It is noted that the parameter $A$ is the only unknown. Other discharge parameters are either given in Ref. [18] or can be deduced from the data.) The three types of discharges are: (a) ohmic discharge, (b) ohmic plus ion cyclotron heating, and (c) ohmic plus neutral beam heating [with a lower plasma density than in (a) and (b)]. Considering the large variation in the discharge parameters across the cross-section and between different types of discharges, the agreement, both in magnitudes and in radial profiles, is surprisingly good. The formula has also been tested by Hiroe [19] against the TFTR ohmic data [20] with a satisfactory agreement.

In summary, it has been shown that the high frequency electromagnetic drift mode recently revealed can explain several features of the anomalous thermal and particle transport in tokamaks. The particle flux is anomalous (against the density gradient) and the proposed electron thermal diffusivity can not only predict the magnitude but also reproduce the radial profiles of the experimental diffusivities in three different types of JET discharges and in TFTR ohmic discharges. The favorable dependence of the energy confinement times on the plasma current and ion mass (the isotope effect) may also be explained by the instability. Density and magnetic fluctuations with frequencies and wavelengths expected from the theory have recently been observed in several ohmic discharges.

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THE ISOTOPIC EFFECT ON PLASMA CONFINEMENT

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Abstract

THE ISOTOPIC EFFECT ON PLASMA CONFINEMENT.

Plasmas whose main component nuclei are deuterons have been observed to exhibit superior confinement properties to those of equivalent proton plasmas. The theoretical explanation given involves the excitation of modes driven by the density gradient of the impurity population in the peripheral region of the plasma column and the hypothesis that these modes control the energy transport from this region to the edge of the plasma column. Considering the ratio of the main ion thermal velocity to the mode phase velocity as a measure of the instability window, for the mode frequency, the proposed theoretical model indicates that this window is reduced considerably in a hydrogenic plasma when the mass of the main nuclei population is doubled. The mode properties are also consistent with the observation that no significant difference in the energy confinement time is noticed experimentally between deuterium and helium plasmas with similar characteristic parameters.

The possibility of obtaining hydrogenic plasmas with a high degree of purity ($Z_{eff}$ close to unity) has been demonstrated through a series of experiments [1] carried out by the Alcator high field machine since late 1974. A discovery that followed from these experiments is that the plasma energy confinement time increases significantly, by about a factor $\sqrt{2}$, when the component nuclei are changed to deuterons from protons. This observation has since been widely confirmed by a large variety of experiments and has in fact been well exploited in order to obtain record values of the confinement parameter $n_0T_E$.

In spite of the evident importance of this fact, no theoretical model explaining it has been advanced yet. An example of the difficulties encountered is the following: if the energy is assumed to be transported by ordinarily considered collective modes and an effective diffusion coefficient $D_{th}$ is evaluated following the procedure indicated in Ref. [2], the
relevant confinement time is found to scale as $1/\sqrt{A_i}$, $A_i$ being the mass number of the component nuclei. Specifically,

$$D_{th} \propto \frac{eT}{eB} \rho_i \lambda_\parallel$$

where $T$ is the plasma temperature, $B$ the confining magnetic field, $\rho_i$ the average gyroradius of the nuclei, $\lambda_\parallel \equiv |d\ln T/dr|^{-1}$ the temperature gradient scale distance, and $\lambda_\parallel$ the typical mode wavelength along the magnetic field.

By now, it is also commonly accepted that the observed temperature profiles comply [3] with the "principle of profile consistency" in that they are, under a large variety of conditions, well behaved, monotonically decreasing functions of $r/a$, $a$ being the radius of the plasma column and $r$ the distance from its center. This means that, for instance, if we write the thermal energy balance equation as

$$S = -\frac{1}{r} \frac{d}{dr} \left( r D_G n \frac{dT}{dr} \right)$$

and $T \approx T_0 \exp[-\alpha_T r^2 / a^2]$, where $\alpha_T > 1$ is a weak function of $\xi \equiv r^2 / a^2$, we have

$$D_G \approx a^2 \left[ \int_0^\xi S(\xi')d\xi' \right] / \left[ 4nT(\xi)\xi \frac{d}{d\xi}(\xi\alpha_T) \right]$$

Then it is easy to verify that $D_G$ is a monotonically increasing function of $\xi$. On the other hand, a diffusion coefficient of the type represented by Eq. (1) would be maximum within the radius where the temperature gradient is maximum.

To resolve this difficulty we assume that the class of inclusive diffusion coefficients $D_G$ that can reproduce the experimentally observed profiles is the result of at least two classes of process:
- one that can transport energy from the center of the plasma column toward its peripheral region,
- another that transports this energy, away from the peripheral region toward the plasma outer edge.

Therefore, we may assume that collective modes can be excited by the density and temperature distributions typical of the edge of the
plasma column and that these provide a thermal conduction in series to that resulting from the excitation of modes in the interior of the plasma column.

For this we consider that a second ion population is present at the edge of the plasma column as a consequence of its interaction with the first wall. We call this the "impurity species" and take it to be characterized by a mass number $A_j$ and a charge number $Z_j$. We refer for simplicity to a one-dimensional plane configuration with the magnetic field in the $e_z$ direction and consider electrostatic modes with a fluctuating field $\hat{E} = -\nabla \hat{\phi}$ and

$$\hat{\phi} = \phi(x) exp(-i\omega t + ik_\perp y + ik_\parallel z)$$

The densities of the electrons, the main nuclei, and the impurity species are indicated by $n_e(x)$, $n_i(x)$, and $n_j(x)$ respectively and we assume that all the relevant temperatures are equal, that is $T_e = T_i = T_j = T(x)$. Now we notice that "impurity driven modes" [4,5] can be found in the frequency range

$$k_\parallel^2 \frac{T}{m_p A_j} < \omega^2 \leq k_\parallel^2 \frac{T}{m_p A_i}$$

Our argument to explain the isotopic effect is that by changing the main nuclei species from protons to deuterons, the window given by the inequalities (4) shrinks by a factor two and the width of the relevant fluctuation spectrum is, therefore, significantly decreased. Here we consider the mode number $k_\parallel$ to be bound by a typical scale distance, in the direction of the magnetic field, of the considered confinement configuration. Thus for a toroidal configuration whose major radius is $R$ we take $k_\parallel \approx \rho/R$, where $\rho(r)$ is the helical parameter.

In order to give an idea of the characteristics of the mode of interest, we consider a weakly collisional regime where

$$\omega < \nu_i k_\parallel^2 \lambda_i^2 = \nu_i V_{thi} \times (k_\parallel \lambda_i)$$

$\lambda_i$ is the mean free path, and $k_\parallel \lambda_i < 1$. Thus the perturbed ion density is given by the momentum conservation equation

$$-ik_\parallel \left( n_i \dot{T}_i + n_i \dot{T}_i + \alpha_i n_i \dot{T}_i \right) - ik_\parallel n_i e\phi Z_i \simeq 0$$
where $\alpha_i \dot{T}_i$ accounts for the thermal force due to collisions between the main nuclei population and the impurity species. The main nuclei thermal energy balance equation is

$$\frac{3}{2} \frac{\dot{V}_{Ez}}{dx} n_i - T_i \left(-i \omega \dot{n}_i + \dot{V}_{Ez} \frac{dn_i}{dx} \right) = -k_i^2 V_{thi}^2 \chi_i^o \dot{T}_i \dot{n}_i$$

(7)

where $\chi_i^o$ is the finite numerical coefficient characterizing the ion thermal conductivity in the direction of the magnetic field, and $\dot{V}_{Ez} = -i k_y c \hat{\phi} / B$. This gives, for $|\omega| < k_y c T_i (dn_i / dx) / (eB)$,

$$\frac{\dot{T}_i}{T_i} = \nu_{ii} \frac{1}{k_i^2 V_{thi}^2 \chi_i^o} \times \dot{V}_{Ez} \left( \frac{1}{n_i} \frac{dn_i}{dx} - \frac{3}{2} \frac{dT_i}{dx} \right)$$

(8)

Since we consider the limit $\omega^2 > k_i^2 V_{thi}^2$, we may ignore the longitudinal motion of the impurity population and obtain, from the relevant mass conservation equation,

$$-i \omega \dot{n}_I + \dot{V}_{Ez} \frac{dn_I}{dx} = 0$$

(9)

The perturbed electron density is obtained from the equivalent of Eq. (6), where after considering Eq. (8) the contribution of $\dot{T}_e$ can be neglected. Thus

$$\dot{n}_e = \frac{e \hat{\phi}}{T_e} n_e$$

(10)

Since $\dot{n}_e$ and $\hat{\phi}$ are in phase no net electron transport is produced by these modes, while if hot impurity nuclei are transported out, cold main nuclei are transported in or vice versa. Thus, the thermal energy of the main nuclei population tends to be decreased by the onset of the considered instability. The dispersion relation resulting from the quasi-neutrality condition

$$Z_I \dot{n}_I + Z_i \dot{n}_i = \dot{n}_e$$

(11)

is then

$$\omega = \frac{-k_y D_B Z_I \frac{dn_I}{dx}}{1 + Z_i^* n_e}$$

$$\times \left\{ 1 + i \frac{1 + \alpha_i I}{1 + Z_i^*} \frac{T_i}{T_e} \frac{\nu_{ii}}{k_i^2 V_{thi}^2 \chi_i^o} k_y D_B \left( \frac{1}{n_i} \frac{dn_i}{dx} - \frac{3}{2} \frac{dT_i}{dx} \right) \right\}$$

(12)
where $Z_i^* \equiv Z_i \left(1 - Z_i n_I/n_e\right) T_e/T_i$, and $D_B \equiv c T_e/\left(e B\right)$. From this we see that the instability condition is

$$\left(3 \frac{1}{2} \frac{dT_i}{dx} - \frac{1}{ni} \frac{dn_i}{dx}\right) \frac{1}{n_e} \frac{dn_I}{dx} > 0$$

Therefore, if $\eta_i \equiv d ln T_i/d ln n_i > 2/3$, the considered mode can be excited in the region of the plasma column where the density of the hot impurity population is decreasing. In particular we assume that the local profile of the impurity density $n_I(x)$ is the result of the inflow process due to normal collisional transport and the outflow due to the instability under consideration.

When referring to non-hydrogenic plasmas with $Z_i > 1$, the width of the mode existence window (4) is to be assessed by considering that the frequency $\omega$ of the proposed modes is a decreasing function of $Z_i$. Therefore the transition from a deuterium to a helium plasma cannot be expected to be characterized by the same change in the confinement time as the transition from hydrogen to deuterium. The ratio $k/V_{thi}/\omega$ can in fact be written as $(k/\omega)V_{thi} \left(1 + Z_i^*\right)/V_{*I}$, where $V_{*I} \equiv D_B Z_I (dn_I/dx)/n_e$. For the modes with the longest wavelengths that are expected to give the prevalent contribution to transport, the ratio $k/\omega$ may be related to

![Graph](image)

**FIG. 1.** Growth rates and instability window for the collisionless impurity driven mode obtained by the numerical solution [6] of the relevant dispersion equation for $Z_i = A_i/2 = 8$, $n_i/n_e = 2.5\%$, $T_i = T_e = T_f$, $d ln n_i/d ln n_e = 10/3$, and $d ln T_i/d ln n_i = 3$. 
the geometrical characteristics of the considered confinement configuration and to the gradients of the temperature and the densities involved.

The existence of a finite instability window in the parameter \( b_i \equiv k_y^2 \rho_i^2 / 2 \) can be seen with particular clarity for the collisionless version of the impurity driven mode, through the numerical solution [6] of the relevant dispersion relation. An example is illustrated in Fig. 1, and the near disappearance of the instability window when deuterium is considered instead of hydrogen is evident. We notice that the collisionless instability is definitely weaker than the collisional one as the mode-particle resonance that involves only a small fraction of the main nuclei distribution in velocity space replaces the effect of finite longitudinal thermal conductivity that involves the entire particle population.

The "effective" diffusion coefficient of the nuclei thermal energy resulting from this mode may be estimated as

\[
D_I \sim \gamma k_y^2 \rho_i \frac{Z_I n_I}{n_e} \frac{c T_e}{k_y r_{nI} eB} \times F_I
\]

where we take \( \omega_{\xi I}^T \equiv k_y c (dT_z/dx)/(eB) \sim k_{\parallel} V_{thi} k_{\parallel} \lambda_i \), and \( 1/r_{nI} \equiv |dn_I/dx|/n_I \). The function

\[
F_I \equiv F_I \left[ \left( \frac{k_{\parallel}}{\omega} \right)^2 \left( k_{\parallel}^2 \lambda_i^2 V_{thi}^2 - V_{thI}^2 \right) \right]
\]

that is introduced to represent the width of the excited spectrum of modes, is a decreasing function of \( A_I \) for hydrogenic plasmas, and depends on other plasma parameters such as \( \eta_i \). The coefficient \( D_I \) is of sufficient magnitude to allow a rate of transport of plasma thermal energy that is compatible with those experimentally observed.

On the other hand, the global transport coefficient \( D_G \) to be used in order to reproduce the experimental results has to include also the features of the collective mode that can take the electron or the ion thermal energy from the central part of the plasma column toward the periphery. One microinstability that is consistent with the characteristic parameters of existing advanced confinement experiments, in regimes where injected heating is prevalent over Ohmic heating, is the trapped electron "ubiquitous" mode [7]. In order to construct a form for \( D_G \) we have to consider,
besides the characteristics of this mode the fact that $D_G$ has to take into account macroscopic constraints on the electron temperature profiles and comply with the so-called "principle of profile consistency". The coefficient of this kind that has been proposed in Ref. [8] can be rewritten as

$$D_G \propto D_G^2 \frac{\varphi(r)}{RV_{th_e}} \frac{Z_{eff}^{1/2}}{A_i}$$

in the case where $T_i \simeq T_e = T$. Here $Z_{eff}$ is the effective plasma charge number,

$$D_G^2 \equiv \left[ \frac{c^2}{4\pi ne} \frac{dp}{dI_i} \right]^2 (ar_e) \left( \frac{r_e}{R} \right)^{1/2}$$

$I_i = I_\parallel(r)$ is the plasma current flowing within the magnetic surface of radius $r$, $p$ is the plasma pressure, $a$ the minor radius of the plasma column, $r_e \equiv r + r_{tb}$ and $r_{tb}$ is a small fraction of the plasma radius such as $r_{tb} \simeq a/10$. The distance $r_{tb}$ is introduced in order to take into account that the effects of the considered modes extend to the center of the plasma column. It is easy to verify that $D_G$ that has to be supplemented by other (non-vanishing) transport coefficients in the center of the plasma column increases monotonically toward the periphery as required by Eq. (2). The $Z_{eff}$ dependence introduced in Eq. (16) is consistent with that found for the outer region of the plasma column produced by the JET machine [9].

This expression for $D_G$ is being incorporated in existing 1+1/2D transport codes after rewriting $D_G$ in general co-ordinates, an extension that is made particularly easy by the adopted form of $D_U$. Meanwhile, it is worth noticing that the scaling for the energy containment time that is obtained from $D_G$ is similar to one proposed by Perkins [10] on the basis of the assumed excitation of ordinary electron drift modes. On the other hand, electron drift modes are easily stabilized by the electron temperature gradient and most experiments have, characteristically, $\eta_e = d\ln T_e/d\ln n_e > 2$, which is sufficient to stabilize these modes. Therefore we can argue that these modes are not good ingredients in a realistic transport model. More recently, Lackner and Gottardi [11] have made a fit of a set of experimental data on a variation of this scaling based on the assumption that generic trapped particle effects rather than the
excitation of electron drift modes were involved. We note, however, that
the diffusion coefficient proposed in Ref. [10], as a basis of the proposed
scaling, does not appear consistent with any of those that can be derived
from the theory of trapped particle modes. On the other hand, the
relevant database can be utilized to derive, starting from the form (16)
for \( D_C \), the following scaling for the energy confinement time in regimes
where Ohmic heating is no longer prevalent:

\[
\tau_E \approx C_E \left( \frac{n_e \kappa}{W_h} \right)^{3/5} \left( \frac{I_p}{\alpha_T} \right)^{4/5} a R^6 \left( q \tilde{A}_i \right)^{2/5} / \left( Z_{eff} \right)^{2/5}
\]

\( (s, m^{-3}, MW, MA, m) \)  

(18)

When MKS units are used as indicated within parentheses, \( C_E \approx 1.75 \times 10^{-13} \). In addition, \( q = 1/ \kappa_0 \), \( \kappa_0 \) is the value of \( \kappa \) evaluated at the plasma
boundary, \( I_p \) is the total plasma current, \( W_h \) is the heating power, \( \kappa \) is the
elongation of the plasma column, \( \alpha_T \) is the temperature profile parameter
defined, for a circular plasma cross-section, before Eq. (2), and \( n_e \) is the
line average electron density.

The scaling (18) refers to the case of maximum degradation of con-
finement (frequently referred to as “L-mode” of confinement) that oc-
curs when vigorous injected heating is applied relative to the case where
Ohmic heating is prevalent. In the latter case we have argued [3] that the
electron thermal energy transport and the current density transport are
related. Thus the global scaling for \( \tau_E \) is different while the isotopic im-
provement of \( \tau_E \) persists when the form of heating changes. We consider
this fact to be consistent with the interpretation offered in this paper that
the isotopic effect is the result of a persistent anomalous transport pro-
cess in the peripheral region of the plasma column that “filters” different
forms of thermal energy transport toward this region.

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CONFINEMENT IMPROVEMENT IN ECH AND NBI HEATED HELIOTRON E PLASMAS

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Abstract

CONFINEMENT IMPROVEMENT IN ECH AND NBI HEATED HELIOTRON E PLASMAS.

Confinement optimization has been performed experimentally in the \( \alpha^*-\beta^* \) space (\( \alpha^* = B_\alpha/B_0; \beta^* = B_\beta/B_0; B_\alpha: \) helical field on axis, \( B_\beta: \) the additional toroidal field, \( B_0: \) the vertical field produced by both standard vertical coils and auxiliary vertical coils), where \( \alpha^* \) essentially changes the plasma radius and \( \beta^* \) shifts the magnetic axis. By magnetic surface measurement using the resistance method in the vacuum magnetic field and by a double probe and a Li beam probe in ECH plasmas, the change of the plasma periphery is confirmed to be consistent with that of the calculated magnetic field. For ECH and NBI plasmas, the optimum condition of confinement is found to be \( \alpha^* = 0.05 \) and \( \beta^* = -0.192; \Delta_\alpha = -2 \text{ cm} (\Delta_\alpha: \) the shift of the vacuum magnetic axis). Its confinement time is a factor of up to 1.5
better than the standard configuration ($\alpha^* = 0.0$, and $\beta^* = -0.185$; $\Delta_r = 0$ cm). For the same magnetic configuration as described above, density clamping of an ECH plasma was successfully suppressed mainly by the effect of $\alpha^* = 0.05$ (presumably, by an improvement of the particle confinement and/or change of recycling), and, moreover, an ECH plasma with a density higher than the cut-off density was produced in Heliotron E for the first time.

1. INTRODUCTION

Heliotron E is a unique $i = 2/m = 19$ helical system [1] with a high rotational transform ($\epsilon(0) \sim 0.5$, $\epsilon(a) \sim 2.5$), a strong magnetic shear ($\Theta = (r^2/R)\epsilon_r/dr \sim 0.15$ at $r = 2a/3$), and a deep helical field ripple ($\epsilon_h(a) \sim 0.3$). The major radius is 2.2 m and the average minor radius changes from 0.18 to 0.24 m, depending on $\alpha^*$ (it is 0.21 m for the standard configuration, see the definition of $\alpha^*$ in Section 2). So far, Heliotron E has demonstrated proof-of-principle of the basic heliotron concept, achieving the highest beta, $n_T/\rho$ and density values among the existing helical systems [2] with its standard magnetic configuration. The experimental data have contributed to the confinement physics and to the global confinement scaling of helical systems [3-5]. At present, one of the main Heliotron E activities is the optimization of plasma confinement in order to clarify the transport mechanism and to obtain a well established database for more accurate design of future larger machines. Thus, with expectation for

(i) the improvement of plasma confinement,
(ii) a better understanding of the edge plasma properties, and
(iii) the control of MHD activities,

the main emphasis in the Heliotron E experiments in f.y. 1989–1990 was placed on magnetic configuration variation studies [6].

2. EXPERIMENTAL SET-UP

An application of both the additional toroidal field $B_t$ and the vertical field $B_v$ produced by both standard and auxiliary vertical coils offers a good opportunity for magnetic configuration control, where some favourable effects have also been predicted theoretically [7-9].

A plan and a cross-sectional view of the coil arrangements, together with the vacuum vessel, are shown in Fig.1(a). Two pairs of auxiliary vertical field coils and 19 auxiliary toroidal field coils are used to change the magnetic field configuration. An example of magnetic surfaces varied by the toroidal field is shown in Fig. 1(b).

The variation of $\alpha^*$ ($= B_t/B_h$; $B_h$: helical field on axis) causes mainly a change in the mean plasma radius $a$ and in the rotational transform $\epsilon(r)$, while that of $\beta^*$ ($= B_v/B_h$) causes a shift of the vacuum magnetic axis ($\Delta_r$) and a variation of the
shear parameter ($\Theta$). In this connection, edge plasma parameters were also diagnosed with Langmuir probes, calorimeters, a thermal Li-beam probe and a laser Thomson scattering system. The bulk plasma properties in $\alpha^* - \beta^*$ space were also studied, and the optimum condition of $\alpha^* - \beta^*$ combination was sought for currentless ECH/NBI plasmas.

3. EDGE PLASMA STUDY

To study the magnetic surface properties in this series of experiments, the edge characteristics are studied. The vacuum magnetic surface structure was diagnosed in $\alpha^* - \beta^*$ space by measuring the resistance between a gun electrode and the chamber wall [10]. This was done without plasma, but with a certain amount of rarefied neutral gas. Figure 2(a) shows probe currents (inverse of resistance) for two different
FIG. 2(a). Radial profile of probe current intensity (conductivity between probe electrode and chamber wall) of vacuum magnetic field configuration for $\alpha^* = 0.1$ and $-0.1$ at fixed $\beta^* (= -0.185)$. The calculated $\epsilon$ value is also plotted.

FIG. 2(b). Periphery density profiles of ECH plasma. There are two points where the density gradient is discontinuous. The inner point is denoted by $R_L$ and the outer one by $R_n$. 
FIG. 2(c). $\alpha^*$ dependence of location of $R_n$ ($R_{n}^{DP}$: measured by a double probe, $R_{n}^{Li}$: measured by a Li beam probe.) $R_s$: position of calculated outer edge of separatrix region; $R_p$: radius of plasma limited by carbon protector installed inside vacuum chamber; $R_{p}^{'}$: the same as $R_p$, but without carbon protector.

$\alpha^*$ values ($-0.1$ and $+0.1$) at a fixed $\beta^*$ value of $-0.185$. The calculated value for $t$ is also shown in the figure. The change of the resistance is also similar to that in the other configuration, including the case of axis shift by changing $\beta^*$. Moreover, when the error field of $m = 1/n = 1$ is added with ‘MI’ coils as shown in Fig. 1(a), an island with the expected width was observed. Thus, these coils have led to qualitative agreement between the magnetic surface calculation and the measurement. The periphery density profiles of the ECH plasma were measured by a double probe and a thermal Li-beam probe [10]. The edge density profile shows two-staged exponential decay in the radial direction. The outer points in the profiles indicated as $R_n$ in Fig. 2(b), where the gradient of the curve is discontinuous, are plotted as a function of $\alpha^*$ in Fig. 2(c). The data obtained from the two independent methods agree rather well. The outermost surface seems to be formed almost as expected, at least for $\alpha^*$ less than 0.5, based on a consideration of these data and the above resistance data. For $\alpha^*$ larger than about 0.1, the carbon protector installed inside the vacuum chamber touches the outermost plasma surface. In this case, the average radius of the plasma limited by the protector is denoted by $R_{p}^{'}$ in the figure, while $R_p$ indicates the case without the effect of the carbon protector.

The consistent change of the minor radius of ECH plasmas was confirmed with a far infrared (FIR) interferometer. The expected temperature decrease at the fixed observing point for an NBI plasma with decreasing $\alpha^*$ value was also confirmed by a laser Thomson scattering method.
4. CONFINEMENT OF BULK PLASMAS

The parameter range for ECH/NBI plasmas in this series of experiments is as follows:

\[1 < n_e (10^{19} \text{ m}^{-3}) < 10, \quad 0.3 \leq T_e(0) \leq 1 \text{ keV}, \quad 0.2 \leq T_i(0) \leq 0.9 \text{ keV}\]

\[\langle \beta \rangle \leq 1\%, \quad P_{\text{NBI}} \leq 2.5 \text{ MW}, \quad P_{\text{ECH}} < 0.6 \text{ MW}, \quad 0.94 \leq B \leq 1.9 \text{ T}\]

\[-0.1 \leq \alpha^* \leq 0.15 \text{ and } -0.198 \leq \beta^* \leq -0.172 \quad (-6 \leq \Delta_r \leq 4 \text{ cm}).\]

Gas and/or pellet fuelling \cite{6} was used to control the temporal evolution of \(n_e\) evolution. Carbon coating on the wall was carried out during some shots. In this paper, when the special notification is not given, the data are without carbon coating.

With regard to \(\beta^*\) or \(\Delta_r\), at fixed \(\alpha^* (\approx 0.0)\), it is shown that the best confinement is in the region \(-4 \leq \Delta_r \leq -2 \text{ cm}\) at \(\alpha^* = 0\) for low collisionality, low \(\beta\) ECH/NBI plasmas, as is shown in Fig. 3(a). In this region, the decay time, \(\tau_d\), of the ECE central electron temperature of the 53.2 GHz ECH plasma after ECH turn-off reaches a peak at \(\Delta_r \approx -2 \text{ cm}\) \cite{11}. Axis shifts of \(\Delta_r > 2 \text{ cm}\) or \(\Delta_r < -5 \text{ cm}\) caused a rapid deterioration of confinement. The central electron temperatures of ECH plasmas are also plotted. For NBI plasmas, confinement times \(\tau_e^B\) and \(W_p\) are also shown. Theoretically, the inward shift permits the helical field ripple to be localized at the inner side of the torus. The drift optimization of the Heliotron E configuration was carried out \cite{9} to predict a minimum loss region condition with respect to the shift of the magnetic axis. Figure 3(b) shows the normalized area enclosed by the largest drift surface of confined \(v_B = 0\) particles (so-called \(B_{\text{min}}\) contour) and \(a^2 P_{\text{heat}}\) calculated by the orbit tracing Monte Carlo code, HELIOS. This suggests two significant effects: (i) improvement of bulk plasma confinement, and (ii) improvement of fast ion confinement. Item (ii) increases the available net heating power, \(P_{\text{heat}}\). Experimentally, there exists a good confinement window with regard to the axis shift, and the optimum position corresponds to the predicted minimum loss cone configuration, although the window width for the experiment is narrower than that in the theory. The improvement of particle confinement by magnetic axis shift is also confirmed by \(H_\alpha\) measurement calibrated by a laser fluorescence method \cite{12} as shown in Fig. 4. The improvement of particle confinement time for \(B = 1.76/1.9 \text{ T}\) is 20-30\%, although this is not clear for the low field case, \(B = 0.94 \text{ T}\). For studying the mechanism of particle confinement, density fluctuations are also being measured by a Fraunhofer diffraction method \cite{13}.

With regard to \(\alpha^*\), the best confinement is obtained in the region of \(0.05 \leq \alpha^* \leq 0.1\) for \(\Delta_r = -2 \text{ cm}\). One of the marked features of this region is that the density clamping, observed previously for high power ECH plasmas, was overcome as shown in Fig. 5, where the ECH power of about 450 kW and the gas puff rate were kept constant, and \(\alpha^*\) was changed from 0 to 0.1 at a fixed \(\beta^*\) value of \(-0.192\).
FIG. 3(a). Plasma parameter dependence on magnetic axis shift $\Delta r$.

For $\alpha^*$ of 0.0, the density clamping occurred at about 20 ms after the ECH pulse start. For $\alpha^*$ of 0.05 ~ 0.1, such clamping was suppressed. The density profile was rather flat. Moreover, the density could be increased more easily by increasing the gas puff rate. Thus, a positive $\alpha^*$ value makes it easy to control the density.
For ECH plasmas with $\alpha^* = 0.05$ and $\beta^* = -0.192$, the dependence of the electron and ion temperatures and central density on a line averaged density is plotted in Fig. 6. A remarkable point is that an ECH plasma with density higher than the cut-off density of $3.5 \times 10^{19}$ m$^{-3}$ for 53.2 GHz fundamental heating could be
produced, which was observed in Heliotron E for the first time, although the electron pressure becomes maximum at $\bar{n}_e = 2.5 \times 10^{19} \, m^{-3}$, not at the highest density, which may be due to a change in the ECH wave coupling efficiency.

With increasing $\alpha^*$, the density increased also for an NBI plasma under the same gas puff conditions, as shown in Fig. 7(a). This suggests (a) improvement of particle confinement and/or (b) a change of particle recycling due to the change of the area of plasma-wall interaction. From a spectroscopic measurement, the enhancement of plasma-wall interaction is clearly seen as shown in Fig. 7(b). The impurity line intensity normalized by the line averaged electron density remains at a low level when $\alpha^*$ is less than 0.05. The normalized intensity of heavy metals increases rapidly with $\alpha^*$, when $\alpha^*$ is larger than 0.05. This is presumably due to the larger size of the plasma minor radius with larger $\alpha^*$, which causes a larger plasma-surface interaction area. Bolometric power increases almost linearly from 20% to 40% of the NBI absorbed power, when $\alpha^*$ increases from -0.1 to 0.1.

Figure 8 shows the dependence of the plasma internal energy $W_p$ and the gross confinement time $\tau^8_{\text{E}}$ on $\alpha^*$ for NBI plasmas under the same NBI port-through power and the gas-puff conditions. Measurements of $\tau^8_{\text{E}}$ were made over the whole $\alpha^*-$-$\beta^*$ space by utilizing a 1-D profile analysis/transport/code [14] and the data from laser Thomson scattering $T_e(r)$, FIR $n_e(r)$, $T_i(0)$ (CXRS: O VIII) and other diagnostics. The increase in $\alpha^*$ causes an increase in the electron density as described above.
FIG. 7(a). Temporal evolution of line averaged density of NBI plasma for different $\alpha^*$ values at $\beta^* = -0.192$ under the constant gas puffing.

FIG. 7(b). Impurity line intensity normalized by line averaged density versus $\alpha^*$. $B = 1.9 \, T$ for $\alpha^* \geq 0$ and $B = 1.76 \, T$ for $\alpha^* < 0$. 
The maximum internal energy exists in the region $0.05 \leq \alpha^* \leq 0.1$. Compared with the plasma of the standard configuration ($\alpha^* = 0$, $\Delta_v = 0$), $W_P$ increases by a factor of $\sim 2.5$, and the NBI absorbed power also increases by a factor of $\sim 2$. Accompanied by these changes, the gross energy confinement time, $\tau_E^0$, in which definition the radiation power is not taken into account, reaches a maximum around $\alpha^* \sim 0.05$ (a factor of $\sim 1.3$ larger than that of the standard configuration). The electron temperature and density profiles for $\alpha^* = 0.05$ and $\beta^* = -0.192$ are apparently wider than those of the standard configuration. The electron temperature profile is almost parabolic, and the central electron temperature is 550 eV, while the central ion temperature measured by CXRS of the O VIII line is 600 eV, for this case.

To make an appropriate comparison of the confinement properties, the confinement time should be normalized in accordance with an appropriate scaling law since the heating power and density are different. As described in the previous paper [15], an empirical confinement scaling with respect to the density and heating power for the standard configuration is proposed, that is, $\tau_E^0$ is proportional to $P_{\text{abs}}^{0.64} \bar{n}_e^{-0.54}$. Here, it is assumed that the functional dependence on the density and the heating power is not changed with values of $\alpha^*$ and $\beta^*$. Thus, confinement enhancement factor $f_r(\alpha^*, \Delta_v)$ is defined as:

$$f_r(\alpha^*, \Delta_v) = \left(\frac{\tau_E^{0}(P_{\text{abs}}^{0.64} \bar{n}_e^{-0.54})_{(\alpha^* \Delta_v)}}{\tau_E^{0}(P_{\text{abs}}^{0.64} \bar{n}_e^{-0.54})_{(\alpha^* = 0, \Delta_v = 0)}}\right)$$

where the subscripts indicate the $\alpha^*$-$\Delta_v$ combination at which the quantities in the parentheses should be calculated. The dependence of $f_r$ on $\alpha^*$ for the case of $\Delta_v = -2$ cm is shown in Fig. 9. The case of the standard configuration is also shown.
Thus, the improvement of confinement by adjusting $\alpha^*$ and $\beta^*$ from the standard configuration is obvious. From the figure, the effect of $\beta^*$ ($\Delta_\nu$) contributes about 30% of the improvement, and $\alpha^*$ contributes about 20% of the improvement. The important point is that the two effects seem to be additive. The value of $\eta_a$, also shown in Fig. 9, is defined as $\eta_a = 1.3 \{a(a)/a(a = 0)\}^2$, where all values are taken at $\Delta_\nu = -2$ cm, and a constant factor of 1.3 has been used to fit the $f_\tau$ value at $\alpha^*$ of 0.0. The change of $f_\tau$ is very similar to that of $\eta_a$. This suggests that the dependence of $f_\tau$ on $\alpha^*$ is dominated by $\alpha^2$ scaling [4], although the finite effect of $\tau$ and shear on confinement still cannot be excluded.

In addition to confinement improvement, an MHD stabilization effect at the same $\alpha^*$ value has also been observed on the $m = 2/n = 1$ and $m = 1/n = 1$ pressure driven interchange modes, as described in a separate paper in these Proceedings [16].

5. CONCLUSION

Confinement optimization has been performed experimentally in the $\alpha^*-\beta^*$ space, where the former quantity essentially changes the plasma radius and the latter one shifts the magnetic axis, although the other parameters such as the rotational...
transform and the shear also have a changing effect. By magnetic surface measurement using the resistance method in the vacuum magnetic field and by a double and a Li beam probe in the ECH plasmas, the change of the periphery of the plasma is confirmed to be consistent with that of the calculated magnetic field configuration. This observation is also supported by the other measurements such as FIR and laser Thomson scattering. The optimum condition for confinement is found to be around $\alpha^* = 0.05$ and $\beta^* = -0.192$ ($\Delta_v = -2$ cm). The confinement time is a factor of up to 1.5 better than for the standard configuration ($\alpha^* = 0.0$ and $\beta^* = -0.185$; $\Delta_v = 0$ cm). The inward magnetic axis shift of 2 cm corresponds to the minimum loss region configuration. An MHD stabilization effect of $\alpha^*$ has also been discovered in the region around $\alpha^* \sim 0.05$.

For the above mentioned magnetic configuration, density clamping of an ECH plasma has been successfully suppressed, and, moreover, ECH plasmas with densities higher than the cut-off densities have been produced in Heliotron E for the first time.

A detailed study of the effects of the change of rotational transform and shear on the confinement remains to be done.

**REFERENCES**

DISCUSSION

P. SMEULDERS: You showed us \( t(r) \) profiles which increase with the minor radius. You also showed us that your discharges are characterized by sawteeth. Are the sawteeth due to the \( q = 1 \) surface and what effects do they have?

T. OBIKI: The sawteeth occur only under certain limited conditions and are considered to be a pressure driven interchange mode at the \( q = 1 \) resonant surface with the mode number \( (m, n) = (1, 1) \). The interchange mode at other \( q \)-values such as 1.5 or 2 may be unstable, depending on the pressure profile and the magnetic field configuration.

These sawteeth usually contribute only a small fraction to particle and heat transport.
CONFINEMENT PROPERTIES OF THE ‘ADVANCED STELLARATOR’ WENDELSTEIN W VII-AS


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Abstract

CONFINEMENT PROPERTIES OF THE ‘ADVANCED STELLARATOR’ WENDELSTEIN W VII-AS.

An extended parameter range \((T_e \leq 3 \text{ keV}, T_i \leq 0.7 \text{ keV}, n_e \leq 2.5 \times 10^{20} \text{ m}^{-3})\) was explored in the new modular stellarator Wendelstein W VII-AS using ECRH and NBI. At 2.5 T main field, central values of \(\beta\) up to 1.5% were achieved. A reduced Shafranov shift as consequence of the reduced Pfirsch-Schlüter currents for the advanced stellarator concept is verified. Depending on the plasma parameters, a net current is observed and identified as the bootstrap current. At the low shear stellarator W VII-AS, the magnetic configuration and the confinement depend sensitively on the plasma currents. Thus a control of the configuration by small currents (OH, ECCD, Ohkawa current) is applied to maintain optimum confinement. On the basis of profile measurements the electron heat conduction and the ion particle diffusion coefficient were evaluated for ECRH plasmas. As a result of statistical analysis, a phenomenological expression for the energy confinement time \(\tau_E\) is derived. After carbonization the high-Z impurities (Fe, Ti) and radiation losses were significantly reduced. In first experiments with NBI \((P_{\text{IN}} \leq 1.5 \text{ MW})\), discharges at densities of \(n_e \leq 2.5 \times 10^{20} \text{ m}^{-3}\) could be maintained and an energy replacement time of up to 30 ms was achieved.

1. INTRODUCTION

The new stellarator at Garching, the W VII-AS ‘Advanced Stellarator’ (major radius 2 m, plasma radius \(\leq 0.17 \text{ m}\) \([1]\)) has been operated since the summer of 1988. The experimental programme is devoted to the new possibilities offered by the concept of optimization and guides further theoretical models. The configuration of W VII-AS with \(m = 5\) periods, similar to five toroidally linked mirrors, is produced by a system of modular coils \([2]\). The innovative engineering of the magnetic system is testing a technique for the realization of favourable magnetic configurations, even for the dimensions of a reactor. The improved equipment for heating includes ECRF (electron cyclotron heating: four gyrotrons at 70 GHz with 200 kW each), NBI (neutral beam injection heating: tangential injection with 1.5 MW), ICRF (ion cyclotron heating: ‘experimental antenna’ with 1 MW) and allows more flexible operation of the device than the previous conventional stellarator W VII-A \([3]\). Especially, the sophisticated ECRF system enables a quasi-optical power launch by adjustable mirrors to vary the power deposition profiles and to be applied for local current drive by introducing \(k_{||}\) components \([4]\).

1.1. Parameter range

More than 10 000 plasma discharges have been recorded so far, nearly half of them at the full field of 2.5 T. Plasma buildup by ECF waves (70 GHz) works routinely for both fundamental and second harmonic launching. In the first experimental periods of the W VII-AS stellarator, investigations have mainly centred on ECR
heated discharges at $B_0 = 1.25$ T (second harmonic X-mode) and $B_0 = 2.5$ T (fundamental O-mode). In these quasi-stationary ECRH discharges, the maximum densities were limited by the cut-off condition to $<3 \times 10^{19}$ m$^{-3}$ for $B_0 = 1.25$ T and to $<6 \times 10^{19}$ m$^{-3}$ for $B_0 = 2.5$ T. Up to 800 kW ECF power was launched. Maximum electron temperatures of up to nearly 3 keV were measured. Typical energy exchange times of 5 to 20 ms were evaluated. For these ECRH discharges with a duration of up to 1.5 s, the L/R times were very long and up to 1 s was necessary to establish a stationary internal current density profile. The global confinement can sensitively depend on the internal rotational transform profile, and unstable plasma evolutions (bifurcations) were observed in phases with stored energy and constant line density [5]. NBI heated discharges in the first experimental phase without wall conditioning were dominated by radiative losses and, in most cases, terminated by radiative collapse. After carbonization, the radiation loss mainly due to heavy impurities (iron and titanium) was significantly reduced and operation at high density became possible.

2. MAGNETIC CONFIGURATION EQUILIBRIUM AND STABILITY

Excluding the direct vicinity of $\iota = 1/2$ and $\iota = 1/3$, very good agreement of all measured flux surfaces with code simulations was found in the plane where the cross-section has a triangular shape [6]. Flux surfaces were also measured in one elliptical plane, where the deviation of the measured from the predicted position of the magnetic axis was within the accuracy of the method. Electron temperature profile measurements (Thomson electron cyclotron emission (ECE) and soft X ray (SX)) at different toroidal positions indicate an average accuracy of the magnetic axis position of roughly 1 cm.

One of the aims of the partly optimized W VII-AS field configuration was the reduction of the Pfirsch–Schlüter (PS) currents. From the W VII-AS optimization, a PS current reduction by a factor of two was expected relative to a standard stellator. This leads to a reduced Shafranov shift. The outward shift of the magnetic axis with $\beta_0$ is confirmed by soft X ray observations. In ECRH discharges with highly localized central power deposition, the electron temperatures are peaked. Figure 1 shows the outward shift of the measured central X ray profile as a function of $\langle \beta \rangle$ for $\iota \approx 1/2$ and $\iota \approx 1/3$. Note that the shift of the magnetic axis, depending on the shape of the pressure profile, is about three times larger for $\iota \approx 1/3$ than for $\iota \approx 1/2$. The comparison with predictions based on the KW equilibrium code [7] verifies the PS current reduction by a factor of two. In addition, the $B_z$ field component originating from the PS currents, which is related to the volume averaged $\langle \beta \rangle$, is measured directly with magnetic loops and agrees with the relation $B_z \sim \langle \beta \rangle B_0/\iota$ as simulated by the VMEC equilibrium code [8]. Volume averaged $\beta$ values, $\langle \beta \rangle$, of up to 0.16% and 0.28% were found by using ECRH for $B_0 = 2.5$ T and $B_0 = 1.25$ T, respectively. The maximum peak $\beta_0$ values were
FIG. 1. Horizontal shift of magnetic axis versus volume averaged \langle \beta \rangle from X ray profile analysis for \( i = 1/2 \) (upper plot) and for \( i = 1/3 \) (lower plot). The straight lines result from equilibrium calculations (KW code).

0.52% and nearly 0.8%. Maximum values of \( \langle \beta \rangle = 0.62\% \) and \( \beta_0 = 1.5\% \) were found with NBI. So far, the maximum values of \( \beta \) seem to be limited by the available heating power only. The experimental \( \langle \beta \rangle \) values were much lower than the predicted \( \langle \beta \rangle \) limit of about 2%. However, localized fluctuations and mode activities were found by SX and ECE diagnostics, from reflectrometry and \( H_a \) measurements. These activities are mainly related to configurational effects for discharges with resonances \( i = 1/2 \) or \( i = 1/3 \) within the confinement region. Current profile analysis indicates that these fluctuations and mode activities not related to the stability limit are stabilized by sufficient internal shear as generated by the internal currents.

2.1. Bootstrap current

The magnitude of the bootstrap current in W VII-AS is comparable to the equivalent tokamak value, and the bootstrap current increases the rotational transform. In the ECRH discharges, bootstrap currents of several kA were measured which, in most cases, were fully balanced by applying a small loop voltage, \( U_l \), by
feedback control of the OH transformer. Without current control, the edge value of the rotational transform can be modified significantly, leading to confinement degradation due to configurational effects. In ECRH discharges with typically very broad \( n_e \) and peaked \( T_e \) profiles, the neoclassical bootstrap current is mainly driven by the electron temperature gradient, the ion contribution being rather small because \( T_i \ll T_e \) under these conditions.

For each magnetic configuration in W VII-AS, the bootstrap current is calculated by using the DKES code. On the basis of measured \( T_e \) and \( n_e \) profiles, the total bootstrap current as well as the plasma resistance are calculated. In Fig. 2, the neoclassical predictions for the dominant electron component, \( I_{b,e} \), are compared with the experimental values defined by \( I_p - U/R \) for ECRH discharges. The agreement with the neoclassical predictions is very good.

For higher heating power levels and lower densities, suprathermal electrons will contribute to both bootstrap current and plasma resistance. For ECRH discharges without electron cyclotron current drive (ECCD), the dependence of the experimental value of the bootstrap current, \( I_p - U/R \), on global plasma parameters is found by multiple linear regression. \( I_p - U/R \) scales linearly with the effective plasma radius, \( a \), and nearly quadratically with the volume averaged temperature defined by \( W_{\text{dia}}/n \). A slightly negative \( B \) and nearly no \( \iota \) dependence were found. These findings are consistent with the neoclassical predictions for which the temperature dependence of the bootstrap current coefficients is dominant. DKES calculations [9] show that the bootstrap current is rather insensitive to the different magnetic field configurations characterized by vacuum \( \iota, B, \) field and field ripple which can be modified.

FIG. 2. Electron component of bootstrap current, \( I_{b,e} \), calculated from DKES code versus experimental value, \( I_p - U/R \), for ECRH discharges (without ECCD) with central deposition.
by varying the currents in the modular coil system. This result of bootstrap current, being roughly the same for different W VII-AS configurations, is in contrast to the situation in the ATF experiment where the existence of the neoclassical bootstrap current was confirmed by changing the additional quadrupole field.

2.2. Configurational effects on confinement

As was the case for the W VII-A stellarator, the global confinement in W VII-AS depends strongly on the boundary value of the rotational transform, \( \iota(a) \) \cite{5}. A degradation of both energy and particle confinement is found for low order rational values of the rotational transform at the plasma edge, \( \iota(a) \), whereas optimum confinement can be established in narrow \( \iota \)-windows close to the resonances \( \iota(a) = 1/3 \) as well as \( \iota(a) = 1/2 \). The nearly shearless vacuum configurations are modified by plasma currents: the PS current and the neoclassical bootstrap current generated by plasma pressure, currents driven by the heating method (ECCD and Ohkawa currents during NBI) or externally by the OH transformer. As the contribution of the bootstrap current to the rotational transform is much larger than the narrow \( \iota \)-windows for optimum confinement, the total plasma current was controlled to avoid low order rational values of \( \iota \) or to keep \( \iota(a) \) within the range of optimum confinement. Current diffusion on the L/R time scale determines the time needed to reach stationary conditions. In most ECRH discharges, the bootstrap current was fully balanced by applying a small loop voltage, \( U_{\text{ls}} \), by feedback control of the OH transformer. In the stationary phase, the Ohmic current density profile is concentrated in the central plasma region for the peaked \( T_e \) profiles whereas the bootstrap current is driven more outside in the \( T_e \) gradient region. For \( \iota(a) \) being fixed, positive shear is generated in the confinement region. The situation for counter-ECCD with central power deposition is similar, the shear also being positive. For stronger co-ECCD, however, negative shear can be generated in the whole confinement region. For low internal shear operation, degraded confinement is found to be related to low order rational values of \( \iota(a) \), which indicates island formation and ergodization in the magnetic configuration. Even a local flattening of the \( T_e \) profile has been resolved for these conditions. With sufficient internal shear, nearly identical discharges could be established above and below both \( \iota(a) = 1/3 \) and \( \iota(a) = 1/2 \). Current profile estimates clearly show that the low order rational values are within the confinement region. By a scan of ECR heating power, a non-linear mechanism of 'self-stabilization' of the internal shear was demonstrated \cite{5}: both the bootstrap and the compensating Ohmic currents increase with the energy content, resulting in higher positive shear and in an improved confinement which leads to increasing energy content. The W VII-A experience, on the other hand, indicates a degradation of confinement with strong internal shear. As a consequence, the optimization of energy confinement depends on the optimization of the internal \( \iota \)-profile. First experiments using local ECCD for the \( \iota \)-profile shaping show promising results.
3. TRANSPORT ELECTRON HEAT CONDUCTION

For ECRH discharges, the electron densities were lower than \(5 \times 10^{19} \text{ m}^{-3}\), so the collisional electron–ion coupling and the radiative losses were rather small, and the electron heat conduction was the dominant loss channel. These discharges are the best candidates for electron energy balance analysis, which is based on density and temperature profiles measured by the Thomson scattering diagnostic. Furthermore, the \(n_e\) profiles were very broad and the \(T_e\) profiles highly peaked for central ECF power deposition. With increasing power density, the density profiles become hollow. The central ion temperature was estimated by passive CX neutral particle diagnostics with \(T_i\) values up to 450 eV. As \(T_i\) profile information was not available, the central \(T_i\) was assumed to be radially constant, and the edge values were fitted to the measured \(T_e\) profile. The total radiation loss was measured by bolometry. The radiation profiles were modelled by a simple corona model.

The stationary electron energy balance equation with radiative losses and electron–ion power transfer is solved with the diffusive ansatz for the radial electron energy flux, \(q_e = -n_e \chi_e T_e^\prime\), and an analytic ECF power deposition model which is highly peaked in agreement with ray tracing calculations. Solving the electron energy balance equation, the measured \(T_e\) profiles are fitted by a least squares technique using a power series of \(\log \chi_e\) in normalized radius, the power series coefficients being the fit parameters. This integration method of the electron energy balance leads to a smoothed representation of the electron heat conductivity, \(\chi_e\). For the \(n_e\) profiles, a standard fit function is used. With these \(T_e\) and \(n_e\) profiles, all neoclassical transport properties are estimated by using the DKES code [10]. In Fig. 3, the electron temperature, density, heat conductivity and rotational transform profiles are shown for typical high pressure \(<\beta> = 0.14\%\) ECRH discharge at \(B_0 = 2.5 \text{ T}\) with 640 kW ECF input power (four gyrotrons operating). The bootstrap current was compensated by an external loop voltage. For the highly peaked \(T_e\) profile due to central ECRH, the bulk part of the plasma is in the long mean free path regime (LMFP), neoclassical transport being dominated by ripple losses. The neoclassical \(\chi_e\) estimated by the DKES code (dot–dashed line in Fig. 3) significantly exceeds the axisymmetric contribution (dashed line). Only in the innermost part does the neoclassical \(\chi_e\) approach the experimental value. Close to the effective limiter radius of 15.5 cm, the \(T_e\) profile becomes flat and the experimental \(\chi_e\) increases strongly here. In the major part of the plasma, \(\chi_e\) is roughly constant (1 m²/s) and much larger than the neoclassical value. With an assumed value of \(Z_{\text{eff}} = 4\), the total electron bootstrap current was calculated to be 5.2 kA, nearly compensated by the Ohmic current. In the lower right plot, the resulting \(\iota\)-profile is shown. The resonances \(\iota = 1/2\) as well as \(\iota = 5/11\) (natural islands in the vacuum configuration) are located within the confinement region. At reduced power, \(\chi_e\) values much less than 1 m²/s were deduced which are of the order of the best \(\chi_e\) values found in optimum tokamak confinement. A minimum in the \(\chi_e\) profiles at about 2/3 plasma radius is typical for all discharges without degradation due to configurational effects. For most
FIG. 3. Transport analysis for case 7565-S7. Electron temperature, T_e, density, n_e, heat conductivity, \( \chi \), and rotational transform profiles for \( B_0 = 2 \cdot 3 \cdot 10^3 \) T and with \( P_{CET} = 640 \) kW input power. In the experiment, \( \chi \) (lower left hand plot) is also shown in the lower right hand plot, specifically, the rotational transform, \( \chi \), is shown in the lower right hand plot, specifically, the resulting (r) due to internal currents (solid line) and without internal currents (dashed line) are presented. Collimation: \( \chi \), is shown in the lower right hand plot, specifically, the resulting (r) due to internal currents (solid line) and without internal currents (dashed line) are presented.
discharges with optimum confinement, $n_e \chi_e$ is roughly constant (or even decreasing) over the whole plasma cross-section. For maximum limiter aperture, however, the increase in $n_e \chi_e$ indicates a degradation of the confinement due to the magnetic topology at the outermost radii.

3.1. Particle transport

The particle confinement in W VII-AS was investigated for ECRH discharges at 1.25 T and 2.5 T by coupling DEGAS code [10] simulations with $H_a$ emissions measured at relevant toroidal positions. Radially resolved ion fluxes were obtained from calculated neutral particle distributions, after calibrating them with the $H_a$ signals. Estimated $Z_{\text{eff}}$ were used to derive the electron particle fluxes and diffusivities. Central electron density profiles were found to become increasingly hollow as the ECR heating power with central deposition is increased. For an ECRH power scan at 2.5 T the electron fluxes in the central region have been compared with neoclassical predictions. As $T_e$ profile information is lacking, only the neoclassical electron fluxes were estimated by the DKES code. For the higher heating power levels, for which the central hollow profiles are more pronounced, the fluxes near the plasma centre are in fairly good agreement. This means that the observed hollow density profiles can be explained reasonably well by the neoclassical temperature gradient driven particle flux (thermodiffusion), without need for additional anomalous contributions. Reduction of the heating power down to the lowest level (1 gyrotron) yields density profiles that are flat and particle fluxes much larger than the neoclassical values. In the density gradient region, as compared to the central region, the particle fluxes increase strongly whereas the neoclassical fluxes are negligible because of the neoclassical $T_e$ dependence. The particle diffusivities, $D$, have been evaluated for discharges operated at half and full field and $\chi(a)$ values close to the major resonances 1/3 and 1/2, where optimum confinement has been found. Concerning the plasma refuelling sources, it should be mentioned that limiter recycling accounts for about 90% of the ion production in the small aperture, low-$\chi$ case (optimal wall screening from ion impact) and only for about 20% in the large aperture, high-$\chi$ case. A comparison between all analysed 1.25 T and 2.5 T ECRH discharges indicates an average improvement of the particle confinement with the magnetic field by a factor of $\sim 3$ throughout the density gradient region. A comparison of $D$ and $\chi_e$ for the analysed discharges yields $D/\chi_e$ values between 0.1 and 0.3 in the outer confinement region.

The impurity particle confinement in W VII-AS has been investigated by laser ablation of aluminium. Since hydrogen-like Al, which is peaked in the bulk plasma for ECRH discharges, can be assumed to evolve under ionization equilibrium conditions, the decay time of the Al XIII line radiation during stationary plasma conditions is a measure of the central confinement time for this impurity. For moderate and high ECRH power, the decay time is found to increase from $\approx 10$ ms at 1.25 T to $\approx 40$ ms at 2.5 T. Also, it decreases with ECRH power for a given limiter aperture.
The time evolution of Al XIII could be reproduced by transport simulations with the STRAHL code [11], which uses a constant diffusion coefficient and an inward convective term, \( v = -(2D/a) (r/a) \). The corresponding values of \( D \) decrease from 5000–8000 cm\(^2\)/s for 1.25 T to 1200–2000 cm\(^2\)/s for 2.5 T. On the other hand, if the velocity term is omitted, these numbers reduce by only \( \sim 20\% \). Discrepancies between the measured and the simulated decay times are, however, found for the lower ionization stages. More experimental data and more accurate simulations, including a spatial variation of \( D \), are needed to improve the description of the impurity transport behaviour under different experimental conditions.

### 3.2. First results with neutral injection (NBI)

During July 1990, the investigations at W VII-AS concentrated on experiments using neutral beam injection. After carbonization by glow discharges with a mixture of He and 30\% CD\(_4\) (or CH\(_4\)) the content of high-Z material (Fe and Ti) was significantly reduced. With reduced radiative losses, the parameter range of plasmas was extended to much higher densities than before and to pulse durations of up to 300 ms. The target plasma is produced by ECRH 70 GHz at a main field of 2.5 T or 1.25 T. The full power of NBI (\( P_N \leq 1.5 \text{ MW} \)) at W VII-AS injecting H\(^0\) with an accelerating voltage of 45 kV was used for further heating (Fig. 4).

With carbonized walls the recycling coefficient is larger than one and the evolution of the discharge is characterized by a steady increase in the density. Before saturation the rate of density increase is determined by gas released by plasma-wall interaction rather than by the flux associated with NBI. The maximum energy content of 28 kJ (\( \beta^0 \approx 1.5\% \)) at 1.4 MW absorbed power was achieved at a field of 2.5 T and a plasma radius of 0.176 m for the rotational transform \( \iota = 0.34 \). With balanced injection the observed bootstrap current of 1.2 kA agrees well with the calculated value. Unbalanced injection generates a variation in the net current by some kA, depending on the density. Unfortunately, no ion temperatures could be measured at these high densities since the diagnostic beam and the CX flux from the centre were completely absorbed. However, electron and ion temperatures should be equal at this very high density. The maximum obtainable density and, consequently, \( \beta \) seem to be related to the absorbed power as long as the electron temperature stays above 350 eV to minimize the radiative losses and prevent a radiative collapse.

Figure 5 presents the measured profiles. Compared to the typical situation for the ECRH discharges the ion heat losses should be significantly enlarged. More detailed information is necessary to evaluate local experimental data and deal with the influence of electric fields. At present, only a global description of the energy confinement is possible. Helium glow discharges and the reduction of the input power allow a stabilization of the density close to \( n_e = 1 \times 10^{20} \text{ m}^{-3} \). At these densities, with \( T_e \approx 0.75 \text{ keV} \) and \( T_i \approx 0.6 \text{ keV} \), a maximum replacement time of 30 ms was achieved.
FIG. 4. NBI injection after carbonization: shot 9901, 2.5 T, $\iota = 0.34$, balanced injection using four beams with $P_N = 7.5$ MW. Development of global parameters: energy content $W$(J); line density ($m^{-2}$) at the elliptical plane with $L = 0.63$ m by HCN interferometer; input power; ECF and NBI (W); radiative power by bolometer (W); time derivative of $W$; plasma current, $I_p$ (A); rotational transform ($\iota$).

Operating at low field, 1.25 T discharges at similar densities and slightly reduced temperatures could be maintained. By using only three injectors ($P_N \leq 1.1$ MW), plasmas with an averaged $\langle \beta \rangle = 0.65\%$ and a replacement time of, typically, 10 ms were produced.
3.3. Energy confinement scaling

The dominant parameter dependence of the global energy confinement time, $\tau_E$, is given by multiple linear regression analysis. This standard procedure is used to obtain the general trend in the confinement properties as well as the significant parameter dependences.

About 86 series of discharges with second and first harmonics, of ECRH as well as 14 series with NBI were selected from a database system for this regression. Discharges with confinement degradation due to configurational effects were excluded. The energy confinement time depends significantly on magnetic field strength, $B_0$, line averaged density, $n$, the absorbed power, $P_{abs}$, and the plasma radius, $a$. The edge value of rotational transform, $\tau(a)$, was found to have a smaller significance:

$$\tau_{E}^{W-VII-AS} = (1.68 \pm 0.02) \times 10^{-8} a^{1.28 \pm 0.16} B_0^{0.716 \pm 0.06} n^{0.33 \pm 0.03} \times P_{abs}^{-0.56 \pm 0.03} \tau(a)^{0.23 \pm 0.08}$$
The experimental $\tau_E$ versus the result of the regression, $\tau_{E}^{W \text{ VII-AS}}$, is shown in Fig. 6.

The regression coefficients for $n$ and $P_{\text{abs}}$ are very similar to those of the LHS or 'gyro-reduced Bohm' scalings. In spite of the differences in the parameter range for ECF ($v^*(0) \geq 10^{-3}$) and NBI discharges ($v^* \approx 0.1$), the different power deposition and the different power losses by electrons, ions and radiation, there is no significant deviation in such a global scaling. For pure ECRH discharges in W VII-AS, where the density profiles are flat and very broad, $n_e \chi_e$ was found to be roughly constant. The shape of the $T_e$ profiles does not change very much on variation of external plasma parameters. Within the important confinement region, the temperature gradient is only slightly affected by the heating power. A physical picture of this anomalous transport cannot be given by such a form of regression analysis. However, the fact that no local scaling of $\chi_e$ with $T_e$ was found hints at two possible explanations: first, the electron heat conduction cannot be treated by a regression ansatz as has been used so far for the accessible parameter range, or, second, the picture of transport being determined by only local plasma parameters is inadequate. ECRH related effects (distortion of the energy distribution, localized power deposition) must be eliminated by application of different heating methods and a more detailed local transport analysis.
4. CONCLUSIONS

The W VII-AS 'Advanced Stellarator' is being successfully operated. To date, there have been about 4000 plasma discharges at full field parameters with durations of up to 1.5 s. This experience allows the conclusion that also a large modular coil system can be realized, especially as the mechanical forces will not exceed those prevailing in W VII-AS. Together with the promising results of the first experimental period, a good basis for the further development of the 'Advanced Stellarator' concept to Wendelstein W VII-X [12], which shall include all theoretical improvements, is obtained.

The predicted reduction of the Pfirsch-Schlüter currents by a factor of two has been verified by measurements of the Shafranov shift. The pressure achieved was much below the predicted stability limit, and no significant mode activity related to the stability limit was observed. However, local mode activity and fluctuations which are related to low order rational values of the rotational transform were detected. The effect of these activities on transport has not yet been analysed.

The neoclassical bootstrap current is quantitatively confirmed in W VII-AS. The bootstrap current with several kA significantly affects the rotational transform profile and dominates the Pfirsch-Schlüter contribution. It depends sensitively on the average temperature. To avoid a strong modification of the \( \iota \)-profile, the bootstrap current has to be controlled. The control of rotational transform at the plasma edge has worked well for all scenarios under consideration. Internal shear is introduced by the bootstrap current depending on the energy content achieved. A future task of W VII-AS will be the analysis of confinement optimization by means of an appropriate shaping of the whole rotational transform profile using local ECCD.

For ECRH discharges, electron energy balance analysis based on measured \( T_e \) and \( n_e \) profiles yields the electron heat conductivity, \( \chi_e(r) \). The best \( \chi_e \) values which have been achieved are well below \( 10^4 \, \text{cm}^2/\text{s} \), which are in the range of the optimum \( \chi_e \) values found in tokamaks. For all discharges analysed, the neoclassical transport coefficients were calculated by using the DKES code and compared to the experimental values. Because of the strong \( T_e \) dependence, the neoclassical \( \chi_e \) values decrease rapidly with the radius and are typically about one order of magnitude smaller than the experimental values at half plasma radius. The neoclassical \( \chi_e \) comes up to the experimental value only for central high ECF power deposition with peaked temperatures where the neoclassical ripple losses dominate. Further investigations of transport in an extended parameter range with different heating methods are necessary to discriminate effects related to particular heating scenarios.

The regression analysis of the global energy confinement time agrees rather well with predictions based on the LHS- and the 'gyro-reduced Bohm' scalings.

From the DEGAS code, particle fluxes and diffusivities have been derived by using measured \( T_e \) and \( n_e \) profiles and absolute \( H_a \) intensities at relevant positions around the machine. The radial range of the diffusivities includes the density gradient region up to the limiter. Here, the value of \( D \) exceeds neoclassical predictions by
more than one order of magnitude. The ratio \(D/x_e\) was between 1/10 and 1/3. For the hollow profiles at high power level, the central particle fluxes from DEGAS simulations agree fairly well with the neoclassical fluxes which are dominated by thermodiffusion. Consequently, the ECRH density pump-out in the central region is consistent with neoclassical transport in W VII-AS.

Some preliminary results are obtained for NBI heated discharges. With carbonization, the very high wall recycling led to a strong density increase, and stationary conditions could not be obtained. The radiation losses were, however, significantly reduced, and the discharges were terminated by edge cooling at very high densities. Only limited by the available heating power, discharges at densities of up to \(2.5 \times 10^{20} \text{ m}^{-3}\) with favourable confinement were obtained. Acceptable discharge conditions with low radiation levels and lower recycling could be realized shortly after helium glow discharge cleaning. Boronization and the installation of pump limiters positioned in regions with high outward fluxes [13] may help to reduce the impurity production and to decrease recycling (perhaps owing to partial Ti gettering).

REFERENCES


DISCUSSION

T.N. TODD: In the past you have shown us plots of stored energy versus rotational transform resembling mountain ranges, and you have selected particular
medium transform irrational values as optima. Now you are indicating a scaling with $i$, such as $i^{0.23}$. You have also indicated a significant rise in stored energy (i.e. confinement time) with increased shear (e.g. by electron cyclotron resonance current drive cancellation of the net bootstrap current). Do these trends suggest a shift in the emphasis of future designs as compared to a few years ago, i.e. towards a large rotational transform and finite shear?

H. RENNER: W VII-X, as the next step, has a transform close to 1 and slightly positive shear. We have learned to accept modifications of the transform profile by beta effects and bootstrap current. Consequently, as a second choice, if resonances cannot be avoided, an improvement by small positive shear seems indicated. As long as magnetic surfaces persist, the confinement time increases experimentally with higher transform.
ENERGY CONFINEMENT AND BOOTSTRAP CURRENT STUDIES IN THE ADVANCED TOROIDAL FACILITY


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Abstract

ENERGY CONFINEMENT AND BOOTSTRAP CURRENT STUDIES IN THE ADVANCED TOROIDAL FACILITY.

Studies of energy confinement and bootstrap current are described, with emphasis on the role of magnetic configuration control in transport studies in the ATF torsatron. Long pulse plasma operation up to 12 s has been achieved with electron cyclotron heating (ECH). With neutral beam injection power of \( \geq 1 \) MW, global energy confinement times of 25 ms have been obtained with line average densities up to \( 1.2 \times 10^{20} \text{ m}^{-3} \). Energy confinement in ATF is roughly the same as that in tokamaks of similar size and field. Favourable density and field dependences offset an unfavourable power dependence. The empirical scaling observed is similar to gyro-reduced Bohm scaling. The toroidal current measured in ECH-only discharges is identified as bootstrap current. The observed currents agree well with predictions of neoclassical theory in magnitude and parametric dependence, as determined by systematic scans of quadrupole (shaping) and dipole (magnetic axis shift) moments of the poloidal magnetic field. Plasma transport responds to variations of the magnetic configuration and is compared with theoretical models.

1. INTRODUCTION

The objective of the Advanced Toroidial Facility (ATF) experiment is to investigate improvement of toroidal confinement concepts in a stellarator device. The stellarator geometry has many features in common with tokamaks, but the stellarator's ability to externally control the magnetic configuration can be exploited to improve understanding of toroidal physics issues. Here we address energy confinement, bootstrap current, and local transport and stability, emphasizing the role of configuration control in these studies. Companion papers on ATF discuss fluctuation studies [1], compare them with those in the TEXT tokamak [2], and describe impurity behavior and impurity transport [3].

ATF [4,5] has a major radius \( R_0 \) of 2.1 m, an average minor radius \( a \) of 0.27 m, and a maximum magnetic field on axis \( B_0 \) of 2 T. The heating power available for these studies is 0.4 MW of electron cyclotron heating (ECH) from two cw 53.2-GHz gyrotrons and 2 MW of neutral beam injection (NBI) from two opposing tangential 40 kV, 0.3 s injectors. The device is an \( \ell = 2, m=12 \) torsatron. In the standard configuration, the rotational transform \( \tau \) varies radially from 0.3 to 1.0, and a modest magnetic well extends out to about the \( \tau = 1/2 \) surface. External control of the magnetic configuration is provided by three independently driven pairs of axisymmetric poloidal field coils ('inner', 'mid', and 'trim' vertical field (VF) coils). These coils produce variations in the dipole and quadrupole fields from the standard ATF configuration, thereby changing the plasma shape, stability of various modes, and trapped particle confinement.

2. EXPERIMENTAL OPERATION

Recent work on ATF has emphasized two goals: (1) extending the discharge duration to long pulse/steady state and (2) enhancing the parameters of NBI discharges. Considerable progress has been made in both areas.
With ECH alone, quasi-stationary discharges lasting for 20 s have been obtained with second harmonic heating at $B_0 = 0.95$ T, as shown in Fig. 1. There is no sign of increasing impurity radiation in these helium discharges, as measured by spectroscopy (for example, FeXIV in Fig.1) and a bolometer array (on loan from PBX). Such long-pulse operation is useful in experimentation because it provides an opportunity to examine plasma response to different control parameters on a single-shot basis. Dynamic configuration scans were conducted during two 20-s ECH discharges, changing vertical elongation (in the $\phi = 0^\circ$ toroidal plane) from 2.0 to 2.4 in one discharge, and from 2.0 to 1.4
in the other. The plasma response to the changing vertical elongation is indicated by the ratio of the vertical chord integral of the electron density (from the central channel of the 7-channel far-infrared interferometer) to the horizontal value (from the 2 mm microwave interferometer). If $n_e$ were constant on a flux surface, this ratio would be equal to the ratio of chordal lengths, and this equality was observed for the duration of the discharges. The plasma current response to the configuration change will be discussed below. Transport studies with ECH discharges require knowledge of the ECH power deposition, which is provided by ECH turn-off experiments. Figure 2 shows the power deposition profile obtained through Thomson scattering measurements at and just after ECH was turned off in ECH. It accounts for a total absorption of 78% of the 'port-through' ECH power.

NBI into ECH target plasmas has produced discharges with parameters comparable to those in a tokamak of similar size (e.g. ISX-B). Figure 3 shows the time evolution of several plasma parameters for a typical discharge with NBI ($H^0$ injection into $D_4$, 1.3 MW total from co- and counter-injecting beams) at $B_0 = 1.9$ T. The strong gas puff at the beginning of the beam pulse raises the line-average electron density $n_e$ from $\lesssim 1 \times 10^{19} \text{ m}^{-3}$ to $1.1 \times 10^{20} \text{ m}^{-3}$, which is maintained until the end of the beam pulse. Improvements in wall conditioning (titanium gettering with $\approx 70\%$ coverage) and gas fueling have eliminated
thermal collapses [3] in the high-density regime. In the quasi-stationary state, the plasma stored energy \( W_{\text{dia}} = 25 \text{ kJ} \) from the diamagnetic measurement yields a global energy confinement time \( \tau_{E*} = W_{\text{dia}}/P_a \) of 20 ms, where \( P_a \) is the estimated absorbed power. This value agrees well with the (thermal) gross energy confinement time, \( \tau_E(a) = (W_e + W_i)/(P_{be} + P_{bi}) \), of 21 ms calculated from the profile analysis (with the PROCTR-Mod code [6]). The profile analysis is usually based on a 15-point Thomson scattering measurements along a single vertical chord, which is interpreted in the flux surface coordinate derived from a finite-\( \beta \) equilibrium (with VMEC [7]). Excellent consistency is observed between experiments and theory for MHD equilibria, as illustrated by the result of a full radial (‘two-dimensional’) Thomson scattering scan in a
FIG. 4. Electron temperature profile and comparison of the corresponding electron temperature contours and flux surfaces.
TABLE I. MAXIMUM PLASMA PARAMETERS ACHIEVED IN ATF

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Line average density $n_e$</td>
<td>$1.3 \times 10^{20}$ m$^{-3}$</td>
</tr>
<tr>
<td>Global confinement time $\tau_E^*$</td>
<td>25 ms</td>
</tr>
<tr>
<td>Plasma stored energy $W_p$</td>
<td>28 kJ</td>
</tr>
<tr>
<td>Electron temperature $T_e(0)$</td>
<td>1.5 keV</td>
</tr>
<tr>
<td>Ion temperature $T_i(0)$</td>
<td>1.0 keV</td>
</tr>
<tr>
<td>Central beta $\beta_0$</td>
<td>3%</td>
</tr>
<tr>
<td>Volume average beta $\langle \beta \rangle$</td>
<td>1.5%</td>
</tr>
<tr>
<td>Discharge duration $t_d$</td>
<td>20 s</td>
</tr>
</tbody>
</table>

**FIG. 5. Volume average beta and line average density as functions of magnetic field.**

different NBI-heated discharge (at $B_0 = 0.95$ T), Fig. 4. The measured $T_e$ contours are consistent with the flux surfaces derived from a self-consistent MHD equilibrium.

Other NBI operational regimes have contributed to the record ATF plasma parameters listed in Table I. At low density with NBI, the central ion temperature $T_i(0)$ has increased to 1.0 keV with $T_e(0) = 1.0$ keV at $n_c = 1.0 \times 10^{19}$ m$^{-3}$ and $B_0 = 1.9$ T. Target plasmas have been successfully created at lower fields, 0.63 T (third harmonic heating at 53.2 GHz) and 0.48 T (fourth harmonic heating). NBI heating of such target plasmas has led to the highest volume average beta $\langle \beta \rangle$ of 1.5%, as shown in Fig. 5. The maximum densities obtained during NBI are about equal to the density limits predicted by Sudo et al. [8]. Modest heating has been observed with application of ICRF power ($\leq 0.2$ MW at 18 MHz) to H-minority, $D^+$ plasmas heated by helium neutral beam injection (1 MW).
TABLE II. COEFFICIENTS AND STATISTICS FOR SCALING LAW FITS (UNITS ARE kJ, 10^19 m^-3, T, kW)

<table>
<thead>
<tr>
<th>W_{dia} fit</th>
<th>Constant</th>
<th>( \alpha_n )</th>
<th>( \alpha_p )</th>
<th>( \sigma_p )</th>
<th>R²</th>
<th>( \sigma^2 )</th>
<th>F_{\alpha}</th>
</tr>
</thead>
<tbody>
<tr>
<td>LHD</td>
<td>0.43</td>
<td>0.69</td>
<td>0.84</td>
<td>0.42</td>
<td>88.9%</td>
<td>6.0%</td>
<td>1.35</td>
</tr>
<tr>
<td>GRB</td>
<td>0.38</td>
<td>0.60</td>
<td>0.80</td>
<td>0.40</td>
<td>89.0%</td>
<td>5.9%</td>
<td>1.21</td>
</tr>
<tr>
<td>Regression 1</td>
<td>0.15</td>
<td>0.42 ± 0.05</td>
<td>0.68 ± 0.06</td>
<td>0.48 ± 0.08</td>
<td>89.4%</td>
<td>5.7%</td>
<td>1.00</td>
</tr>
<tr>
<td>Regression 2</td>
<td>0.19</td>
<td>0.44 ± 0.02</td>
<td>0.67 ± 0.06</td>
<td>0.44 ± 0.02</td>
<td>89.4%</td>
<td>5.7%</td>
<td>1.00</td>
</tr>
<tr>
<td>'ATF'</td>
<td>0.13</td>
<td>1/2</td>
<td>2/3</td>
<td>1/2</td>
<td>89.2%</td>
<td>5.8%</td>
<td>1.11</td>
</tr>
</tbody>
</table>

FIG. 6. Global energy confinement time versus gyro-reduced Bohm scaling.

3. SCALING OF GLOBAL ENERGY CONFINEMENT

Energy confinement in ATF is comparable to that in a tokamak (e.g. ISX-B) with the same minor radius and magnetic field. As described above, \( \tau_E^* \) in ATF improves with density and magnetic field, and this behavior tends to offset the deterioration with power [9]. Table II lists measures of goodness of fit (R² and \( \sigma^2 \)) for the scaling laws and for full regression. The regression is for \( W_{dia} = P_a \tau_E^* = \text{const} n_e^{\alpha_n} B_0^{\alpha_0} P_a^{\alpha_p} \) rather than for \( \tau_E^* \) because \( W_{dia} \) is the measured quantity. The ATF data match well the 'LHD scaling' [8] based on various stellarator results and (as shown in Fig. 6) the gyro-reduced Bohm (GRB) scaling based on drift turbulence models [10,11]; the exponents for either err by a common factor \( F_{\alpha} \) near unity. In regression 2, a constraint
\( \alpha_n = \alpha_p \) brings no significant statistical penalty. Dependence on ion species 
(A_i) is almost absent (\( \alpha_{A_i} = 0.05 \pm 0.07 \)) in ATF data (H, D, and He working 
gas; H and He neutral beams), in contrast to tokamak experience, but roughly as in the GRB scaling 
(\( \alpha_{A_i} = -0.2 \)).

The dependence of \( W_{\text{dia}} \) is roughly as \( (B_0^{4/3} n_e \rho_p)^{1/2} \) — here called 
ATF scaling. This is close to the regression result and suggests a thermal 
diffusivity behaving as \( (T/B_0^{4/3}) \), a scaling intermediate between Bohm \( (T/B_0) \) 
and GRB \( (T^{3/2}/B_0^{2}) \), taking \( T - W_{\text{dia}}/n_e \sim (T_e + T_i)/2 \).

The data are used to check constraints suggested by scaling invariance arguments [12]. Within regression estimates of coefficient error, the energy 
confinement scaling is consistent with the quasi-neutral high-\( \beta \) models, but 
does not support quasi-neutral low-\( \beta \) or single-fluid ideal or resistive MHD 
models. However, a two-fluid MHD model with \( \omega^* \) correction carries the 
single Kadomtsev constraint, which is consistent with the ATF data. A short-
wave length gyro-kinetic model with beta and collisionality effects (\( \alpha_{\nu^*} = -0.21 \) 
and \( \alpha_\beta = 0.35 \)) is a better fit than a long-wave length (Bohm-like) version 
(\( \alpha_{\nu^*} = -0.08 \) and \( \alpha_\beta = 0.16 \)). This is similar to the tokamak result [13]. Studies 
of local transport, as in Section 5, and fluctuation measurements [1,2] are 
needed to narrow the range of instabilities. Adjustability of the ATF magnetic 
configuration should provide an important control factor.

4. BOOTSTRAP CURRENT STUDIES

The bootstrap current studies have exploited the stellarator advantage of 
magnetic configuration control. The existence of bootstrap currents has been 
confirmed in several toroidal devices [14]. It is important to improve our 
understanding of the physics of the bootstrap current and to find ways of con-
trolling it in toroidal devices, both for reduction of the current drive needs in 
tokamaks and for current-free operation in stellarators.

The toroidal currents in ATF ECH discharges have been compared with 
the neoclassical theory [15]. In the low-collisionality limit, the predicted 
bootstrap current is \( j_b = -3(f_v/f_c) \) \( G_b \) \( B_p^{-1} \) \( \nabla \rho \). The comparison [16] primarily 
tested the dependence on the geometry factor \( G_b \) by changing the harmonic 
content of \( |B| \) through variation of the quadrupole (shaping) or dipole (vacuum 
axis) fields, in the absence of other current sources (negligible Ohmic and 
ECH-driven current without NBI).

In the most extensive set of tests, the quadrupole field scan, the current 
of the mid-VF coils was changed while the magnetic axis position was fixed at 
\( R_0 = 2.08 \) m. The variation of the parameter \( f_m \) (ratio of ampere-turns in the 
mid-VF coil to those in the helical coils) led to a change in the toroidally aver-
ged plasma shape from horizontally elongated (\( f_m < 0 \)) to vertically elongated 
(\( f_m > 0 \)). Throughout the scan, the line-average electron density was kept 
constant (\( n_e = 5.5 \times 10^{18} \) m\(^{-3} \)), as shown in Fig. 7(a). Stored energy peaks at
about $f_m = +0.10$. Both electron density and temperature profiles broaden in going from the $f_m < 0$ to the $f_m > 0$ configuration, Fig. 7(b). Figure 8 shows the measured toroidal current (solid points) as a function of the quadrupole field. The current decreases systematically with increasing vertical elongation and goes negative at about $f_m = +0.15$. The bootstrap current predictions (open points) are calculated based on the experimental temperature and density profiles. $G_B$ is calculated as in Ref. [15] for the low collisionality regime. The calculation uses an analytical expression for the finite collisionality correction that interpolates between different collisionality regimes. The agreement between experiment and theory is good; the uncertainty is probably caused by uncertainties in the profiles and in the value of $Z_{eff}$. 

FIG. 7. Plasma responses in a quadrupole scan with ECH only.
FIG. 8. Measured toroidal currents and neoclassical predictions for bootstrap current in the quadrupole scan (Fig. 7).

FIG. 9. Current as a function of the quadrupole moment. Experimental values are from stationary discharges (closed points) and configuration scans during long ECH discharges (continuous curves). Theoretical estimates (dashed curves) are based on two fixed pressure profiles which bound the measured profiles.

The agreement of the experimental and theoretical trends with the quadrupole moment is more clearly seen in Fig. 9, where measured toroidal current is plotted as a function of quadrupole moment ($\Delta Q_{20}$) for a number of quadrupole scans. Use of the moment (Fig. 9) rather than the field parameter $f_m$ (Fig. 8) permits including other combinations of currents in the three vertical fields [18]. Data from the dynamic configuration scans during long 20 s ECH
discharges are similar to those from quasi-stationary discharges. Also shown are theoretical estimates of bootstrap current with fixed pressure profiles (at two extreme cases that bracket the experimental profiles). The experimental and theoretical trends with the quadrupole moment variation agree well with each other. The agreement indicates that the basic dependence on the quadrupole moment is through the geometry factor $G_b$ rather than through changing plasma parameters.

Other experiments in ATF support the neoclassical description of toroidal current flow with ECH alone. Results of a dipole field scan are consistent with the neoclassical prediction that the current increases as the magnetic axis is shifted inward [16]. In operation at 1.9 T the observed current is lower by about a factor of two than that at 0.95 T for the same quadrupole moment, supporting the assertion that the bootstrap current is inversely proportional to $B_0$.

These results are also significant in that they verify the intended ability to control toroidal current in ATF through the quadrupole and dipole moments of the vertical field and to control the current to a zero net value. Currentless operation is expected to be important in future stellarator devices.

5. LOCAL TRANSPORT AND STABILITY

Although parallel (to field lines) transport (as in bootstrap current) is close to neoclassical, perpendicular transport (as in energy transport) is substantially anomalous. The local power balance for the ECH-only discharges in the quadrupole scan (Figs 7 and 8) is dominated by electron transport loss, in particular heat conduction. Figure 10 shows the electron neoclassical anomaly

![Figure 10: Neoclassical anomaly in electron transport loss at $r/a = 1/3$ and $2/3$ in the quadrupole scan.](image-url)
FIG. 11. Electron temperature and density profiles for low density NBI discharge ($P_{\text{enj}} = 1.5$ MW, $n_e = 1.0 \times 10^{19}$ m$^{-3}$, $B_0 = 1.9$ T).

(ratio of the experimental transport loss to the neoclassical transport loss [20]) at two different radii, $r/a = 1/3$ and $2/3$. Although the loss in the central region is only a few times the neoclassical electron transport loss, the anomaly in the confinement region ($r/a = 2/3$) is much larger and decreases with increasing $f_m$. Any possible anomaly in bootstrap current is substantially smaller than the anomaly in heat transport, as expected from theory [17].

Consistency of the global energy confinement scaling with the GRB scaling favors an anomalous transport mechanism based on drift wave
turbulence, and several theoretical considerations lead one to anticipate the emergence of dissipative trapped electron modes (DTEMs) \[19\] in ATF. The result of the stability analysis for these ECH discharges indicates that the DTEMs are close to marginal stability. They can be unstable (only) at the edge, but their level of saturation is well below that of resistive interchange modes. It is possible that the profiles evolve by staying close to marginal stability to DTEMs. The low-density NBI plasmas mentioned in Section 2 appear to be more deeply in the DTEM regime. Figure 11 shows the temperature and density profiles for a sequence of such discharges where the central ion temperature of 1 keV was achieved. Results of stability analysis for these NBI plasmas show that the confinement region is unstable to DTEMs and the plasma edge should be dominated by the resistive interchange instability. Figure 12 shows theoretical estimates of thermal diffusivities \(\chi_e\) due to these instabilities \[21,22\]. The magnitude and radial dependence of \(\chi_e\) are similar to those obtained experimentally from local power balance. (Incidentally, the existence of dual mechanisms is roughly consistent with the implication obtained through the comparison of global energy confinement scaling with theoretical models). The plasma loss in the confinement region could be driven by DTEMs. The levels of theoretically predicted fluctuations are close to 10%, which should be detectable with the fluctuation diagnostics being prepared \[1\].
6. FUTURE DIRECTIONS

The ATF studies of transport and stability will be expanded to lower collisionality and higher $\beta$ regimes by using higher input power. Enhanced diagnostic capability coupled with variation of the magnetic configuration should lead to a more definitive comparison of theory and experiment in the areas of anomalous transport (e.g. DTEM and resistive interchange) and confinement improvement (e.g. electric field effects and second stability regime). The program is also preparing for steady state operation (initially with ECH only). This will give us the opportunity to conduct experiments of higher quality than those with pulse operation in many areas, such as plasma optimization, modulated transport, and fluctuation measurements.

ACKNOWLEDGMENTS


REFERENCES


DISCUSSION

B. COPPI: I was pleased to see that your experiments confirm the evidence offered by tokamak experiments in support of the so-called gyro reduced Bohm scaling, given its relationship to the theories that we have been pursuing.

On the other hand, I do not see how you can reconcile this with the excitation of the dissipative trapped electron mode around the centre of the plasma column and the resistive interchange mode at the edge.

M. MURAKAMI: The gyro reduced Bohm scaling represents a wide range of theoretical models, including both dissipative trapped electron and resistive interchange modes. The resistive interchange modes come about through two fluid MHD with $\omega^*$ correction, rather than by the single fluid resistive model. In addition,
regression analysis indicates that a small but important correction is needed for beta and $v^*$. The true significance of global energy confinement scaling would be understood only if we had a correlation between local transport, fluctuations and theoretical modelling. We believe that ATF's unique external magnetic configuration control is an important feature which makes it possible to distinguish the various transport mechanisms.

K. ITOH: The two-dimensional $T_e$ profile measurement is impressive; careful measurement indicates that both $\nabla T_e$ and $T_e$ are low in the region $0.8a \leq r \leq a$ (Fig. 4 of your paper). The contribution of thermal insulation to global $\tau_e$ is large. What causes the deterioration in confinement in this region? This observation also contradicts the 'gyro reduced Bohm' picture of $\chi_{\text{eff}}$ which decreases in the low $T_e$ regime. Thus the conclusion regarding the gyro reduced Bohm scaling picture is misleading.

M. MURAKAMI: I should think that, in general, the global energy confinement represents transport in the radial range further in, say, $r = 2a/3$, than what you are suggesting. Loss mechanisms in the outer region ($0.8a \leq r \leq a$) may be a combination of resistive interchange and various atomic processes such as radiation, charge exchange, and so on. The gyro reduced Bohm relationship is only the leading term, though again a small but significant correction is needed for beta and $v^*$. Specific transport mechanisms may vary according to the radial range and operating regimes. Again, we need more studies of the correlation between local and global transport, and also a comparison with theoretical work.
CONFINEMENT CHARACTERISTICS OF HIGH POWER HEATED PLASMA IN CHS

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Abstract

CONFINEMENT CHARACTERISTICS OF HIGH POWER HEATED PLASMA IN CHS.

High power heated discharges in the Compact Helical System (CHS) are studied from the viewpoint of optimizing plasma performance in a low aspect ratio heliotron/torsatron (R/a = 100 cm/20 cm). With tangential neutral injection the stored energy, especially the electron temperature, strongly depends on the location of the magnetic axis. The optimal magnetic axis (R_{ax} = 92 cm), however, does not coincide with the location where theory predicts the most favourable drift surface (R_{ax} = 88 cm). A high beta plasma (\beta = 1.5\%) can also be obtained in inward shifted configurations at the toroidal field B_t = 0.47-0.6 T. The optimal location is also R_{ax} = 92 cm, although the observed plasma centre shifts outward by several centimetres owing to Shafranov shift. So far, the obtained stored energy stays in the range that the Large Helical Device scaling predicts. This fact means that the performance of CHS, i.e. a low aspect ratio helical system, is as good as that of other helical systems. The induced currents are observed in both ECH and NBH discharges. In NBH discharges, the currents consist of bootstrap and Ohkawa currents. In fact, current reversal is often observed when the density is high or B_t is low, which is consistent with the theory of bootstrap current in the plateau regime.

1. INTRODUCTION

Helical systems are attractive because of their potential for steady state, current disruption free operation. Recent technical developments in plasma production and heating have made it possible to realize a high performance plasma. Then the issue of designing the optimal helical system arises from its wide variety of configurations.

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The low aspect ratio heliotron/torsatron configuration has recently drawn attention because of its compact size and its potential for high MHD stability [1]. Although the size of the loss cone and the fragility of the magnetic surfaces are considered possible problems in this configuration, recent theoretical studies suggest that additional dipole and/or quadrupole magnetic field components improve the situation [2].

The Compact Helical System (CHS) is a medium sized heliotron/torsatron device \((R = 1 \text{ m}, a = 0.2 \text{ m})\) with \(\ell = 2\) and \(m = 8\). It is characterized by its low aspect ratio of 5 [3–5]. The major objectives of CHS experiments are therefore the study of transport phenomena and MHD characteristics to optimize the low aspect ratio helical system.

2. EXPERIMENTAL SET-UP

Plasma production and heating are performed by ECH with fundamental (28 GHz, 120 kW) and/or second harmonic (53.2 GHz, 150 kW) resonances. Plasma production by ICRF (7.5–18 MHz, 350 kW) is also available. Additional heating is effected by a neutral beam (40 keV, 1.1 MW, 1 s) with scannable injection angle. ICRF plasma production is a useful tool because it has a wide operating region on the toroidal field \(B_t\), as shown in Fig. 1. With titanium gettering a high power tangential neutral injection can sustain the discharges started by ICRF. Therefore the operating toroidal field strength of CHS is not limited by the ECR condition.

CHS has a poloidal coil system that can change dipole and quadrupole magnetic field components independently. The shape and the location of the plasma can be varied by these coils. With a scannable neutral beam injector, it is possible to optimize the power deposition profile for each configuration.

![Diagram](image)

*FIG. 1. Line averaged electron density produced with the Nagoya Type III antenna as a function of the magnetic field strength.*
The main diagnostics of CHS are: HCN laser interferometry for density measurement, a diamagnetic loop for stored energy, Thomson scattering for spatial profiles of electron temperature and density, charge exchange recombination spectroscopy (CXRS) for spatial profiles of ion temperature and rotation, and VUV spectroscopy for impurity behaviour. Visible spectroscopy, a Rogowsky coil and magnetic probes are also used.

3. HIGH POWER NEUTRAL BEAM HEATED DISCHARGE

CHS has a single neutral beam line which can deliver a 40 keV, 1.1 MW hydrogen beam through the port for 1 s. The injection angle can be varied from normal to tangential direction by rotating the whole beam line [6]. The experiments started with tangential injection because of the lack of armour plate and the expected good beam deposition and confinement.

The plasma was able to be sustained by NBI alone from the first trial. Both ECRH and ICRH are available for production of target plasma for NBI. However, the operation is sensitive to the wall condition. Titanium gettering is essential to
increase the operational range of density or pulse length [7]. By Ti gettering, which covers more than 85% of the wall, the line average density $n_e$ of $8 \times 10^{19} \text{m}^{-3}$ was achieved. The radiation power is 10–20% of absorbed power for $n_e < 5 \times 10^{19} \text{m}^{-3}$, while it increases up to 50% for $n_e = 8 \times 10^{19} \text{m}^{-3}$, where the radiation loss is mainly due to oxygen in high density plasma. Oxygen is also a dominant impurity at the radiation collapse, which limits the maximum density. The obtained density is still below the scaling [8]. The plasma can be sustained almost steadily up to 800 ms under the moderate density of $3 \times 10^{19} \text{m}^{-3}$.

The plasma stored energy $W_p$ as measured by the diamagnetic coil roughly agrees with the Large Helical Device (LHD) scaling [8]. Figure 2 shows the comparison of experimental data with the LHD scaling: $W_p(\text{kJ}) = \alpha P_{\text{abs}}(\text{MW})^{0.42}n_e (10^{20} \text{m}^{-3})^{0.69}B_0(\text{T})^{0.84}a(\text{m})^{2.0}R(\text{m})^{0.75}$, where $P_{\text{abs}}$ is the absorbed power and the coefficient $\alpha$ is 170. The experimental data show a weaker density dependence as $n_e^{0.6}$.

4. MAGNETIC AXIS SHIFT EXPERIMENTS

The magnetic field configuration varies according to the location of the magnetic axis, which is controlled by the magnetic dipole field. In CHS the magnetic axis (in vacuum) $R_{ax}$ can be varied from 88.8 to 101.6 cm. Generally speaking, an outward shift increases magnetic well and makes a divertor configuration, while an inward shift increases magnetic shear and improves high energy particle confinement. Indeed, NBH plasmas are affected strongly by the magnetic axis shift.

Figure 3 shows the dependence of $W_p$ on $R_{ax}$ for NBH plasma under fixed injection power. The maximum stored energy can be obtained at $R_{ax} = 95$ cm. Because the plasma volume also changes as the magnetic axis shifts, the stored energy is normalized by the volume to eliminate the volume effect on confinement. This is also shown in Fig. 3. Then the optimal position is $R_{ax} = 92$ cm. The finite $\beta$ plasma axis $R_0$ differs from $R_{ax}$ owing to Shafranov shift, which is large in the low aspect ratio machine. In the case of Fig. 3, the optimum position corresponds to $R_0 \approx 95$ cm, where the field ripple is almost zero on the axis. Figure 4 shows the behaviour of the central electron temperature $T_e(0)$ as a function of electron density. At the same density, $T_e(0)$ is largest when $R_{ax} = 92$ cm. So far we cannot measure $T_e(0)$ for $R_{ax} < 92$ cm. The CX and orbit losses of fast ions calculated by the HELIOS code vary only from 10% to 20% as the axis goes from 92.1 cm to 101.6 cm. Therefore these results show that the confinement, at least in the core, is improved as the magnetic axis shifts inward down to 92 cm. This improvement can also be seen in Fig. 2, where the data show that the coefficient of LHD scaling $\alpha$ depends on $R_{ax}$.

This tendency is not yet clearly understood. From the viewpoint of particle orbit, the deviation of the drift surface from the magnetic surface becomes minimized at $R_{ax} = 89$ cm. On the other hand, the interference between the plasma and the inner wall becomes stronger when the plasma moves inward ($R_{ax} < 98$ cm). In fact
FIG. 3. Dependence of stored energy (open circles) and volume normalized stored energy (full circles) on the location of the magnetic axis; $B_i = 1.05\ T$, $P_{inj} = 1\ MW$, co-injection.

FIG. 4. Dependence of the central electron temperature on the electron density at four different locations of the magnetic axis. Circles and squares correspond to the different injection angles of NBI, where $R_{tan}$ indicates the tangency radius of the beam.
the attainable density is limited by radiation collapse for $R_{ax} < 92$ cm. Therefore, the strong plasma-wall interaction might spoil the good confinement of fast ions. This kind of dependence is not clearly seen for ECH plasma ($88.8$ cm $< R_{ax} < 97.4$ cm), where it is difficult to estimate the change of the power deposition profile.

The local transport analysis has been done on the basis of the measured density and temperature profiles using the ORNL PROCTR-mod code [9]. The typical values of obtained electron thermal conductivity $\chi_e$ are 2–10 $m^2/s$ for both NBH and ECH plasmas, in spite of different plasma parameters and collisionality [5]. The anomaly compared with the neoclassical value appears to be higher when the electron density is high. In CHS the ion temperature profile can be obtained by CXRS for NBH plasmas. However, the estimate of ion thermal conductivity $\chi_i$ is more ambiguous because of the lack of $Z_{eff}$ measurement. Assuming that $Z_{eff} = 2$ and is uniform, $\chi_i$ is of the same order as $\chi_e$ and is also anomalous.

5. HIGH $\beta$ STUDY

One of the motivations for studying a low aspect ratio helical system is its good MHD stability for high $\beta$ plasma. The theoretical prediction based on the H-APOLLO and H-ERATO codes shows that the $\beta$ limit also depends on the location of the magnetic axis. $\beta$ is limited by stability for $R_{ax} < 92$ cm because of the magnetic hill configuration, and by equilibrium for $R_{ax} > 92$ cm because of the large shift of magnetic axis and the small plasma cross-section. The predicted maximum $\beta$ of 6% is thus attained at $R_{ax} = 92$ cm for a parabolic pressure profile.

FIG. 5. Equilibrium $\beta$ values obtained at different magnetic axes. Arrows show the Shafranov shift. Also shown are theoretical equilibrium and stability limits calculated under fixed boundary conditions by the H-APOLLO and H-ERATO codes, respectively.
The experiments have been carried out at $B_t = 0.45-0.6$ T with ECH (28 GHz, second harmonic) or ICRF (7.5 MHz) startup. From the diamagnetic measurement, volume averaged $\beta$ of 1.5% is achieved including the beam pressure. The attained $\beta$ values are almost the same for $B_t = 0.45-0.6$ T because the plasma performance degrades as $B_t$. The obtained density and temperature profiles show a large Shafranov shift. The finite $\beta$ equilibrium is consistently obtained by PROCTR-mod and VMEC using these profiles. Figure 5 shows the calculated equilibrium $\beta$ values using observed pressure profiles at different magnetic axes. An $R_{\alpha}$ dependence of $\beta$ similar to that in Fig. 3 can be seen in this low toroidal field operation. MHD activities of low $m/n$ coherent modes are observed during operation. The amplitude becomes large as the magnetic axis shifts inward, but it still remains on a low level.

6. INDUCED CURRENTS

Because CHS has a single beam line, and the beam is injected tangentially in most experiments, a large amount of beam driven current is expected. However, in CHS the behaviour of measured current is more complicated. Figure 6 shows typical time evolutions of plasma current in three cases. When the density is low, a large current of up to 15 kA is observed in the same direction as beam injection (a). When the density is high (b) or $B_t$ is low (c), the current flow is reversed, i.e. in the counter direction. (Note that the current does not reach the steady state, especially in the low density discharge, because the slowing down time is long and the magnetic diffusion time is also long owing to high temperature.) The reversed current also appears when the injection angle is varied from tangential to normal direction. This result suggests that the observed currents consist of two components, one of which is a beam driven Ohkawa current and the other the bootstrap current.

When the density is low, the parameter dependence of currents fits that of the beam driven current, $I_p \propto P_{abs}\langle T_e \rangle/\bar{n}_e R$. Subtracting the beam driven component using this formula, the amount of the bootstrap component is 2–4 kA. This value and the direction as well agree with the theoretical prediction of the bootstrap current in the plateau regime [10]. No reversed currents are observed in the counter-injection experiments because both the beam driven and the bootstrap currents flow in the same direction in this case.

The induced currents are also observed in ECH plasma. The direction of current is the co direction, and its amount depends on the magnetic quadrupole field. When the cross-section of the plasma is elongated vertically, the amount of current decreases and the direction is reversed in some cases. These results are consistent with the theory of the bootstrap current in the collisionless regime.

So far the induced current has not been found to affect the global confinement characteristics.
FIG. 6. Typical time evolutions of plasma current and line average density in three different cases: (a) low density, (b) high density and (c) low $B_t$. 
7. SUMMARY AND CONCLUSION

A survey of the characteristics of tangential NBH plasma has been performed in CHS. Although there had been concern that the confinement of fast ions was not good in the low aspect ratio machine, the experimental results show that the obtained plasma performance is as good as that of other helical systems. The location of the magnetic axis is a very important parameter to optimize the global confinement characteristic. A higher pressure plasma can be obtained as the magnetic axis shifts inward down to \( R_{ax} = 92 \) cm. A greater shift degrades the operating density limit, which suggests that plasma-wall interaction (at the inner wall) may occur. A similar tendency can also be found in low toroidal field operation. Indeed, the highest \( \beta \) value, 1.5\%, is also obtained in the inward shifted configuration. Although MHD activities are observed in high \( \beta \) operation, they do not seem to affect the confinement. So far the \( \beta \) value is limited by heating power.

The observed currents can be explained by the beam driven and bootstrap currents in the plateau regime for NBH plasma. The efficiency of the beam driven current seems to be of the same order as in tokamaks. The theoretically predicted bootstrap current can explain the experimental results. The behaviour of the currents observed in ECH collisionless plasma also agrees with the bootstrap theory.

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FIRST EXPERIMENTAL RESULTS ON THE URAGAN-3M TORSATRON


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Abstract

FIRST EXPERIMENTAL RESULTS ON THE URAGAN-3M TORSATRON.

Basic results of calculations and measurements of magnetic field characteristics for the modified \( \ell = 3 \) Uragan-3M torsatron are presented together with data on the behaviour of RF produced and heated plasma. It is shown that a twofold increase in the poloidal compensation magnetic field and a higher precision in making the helical winding resulted in an increase of the plasma volume bounded by the last closed magnetic surface and in essentially improved confined plasma parameters.

1. INTRODUCTION

The \( \ell = 3, m = 9 \) Uragan-3 (U-3) torsatron [1] was built to study currentless plasma confinement in a stellarator magnetic configuration with a natural divertor under a magnetic field of up to 2 T and with the mean radius of the last closed magnetic surface (LCMS) \( \bar{a} = 13 \) cm. However, insufficient rigidity of the helical winding support structure restricted the magnetic field value to a level of \( B_0 \leq 1 \) T, while inaccuracies in fabrication of the magnetic system together with a low multipolarity of the compensation winding (\( p = 2 \)) restricted the average radius of the LCMS to \( \bar{a} \leq 9 \) cm [2]. In 1989, to improve magnetic field structure, a new helical
winding bounded by nine strengthening rings and two additional inner compensation field coils were mounted on the modified Uragan-3M (U-3M) torsatron. The new magnetic system permits operation with a magnetic field of up to $B_0 = 2\, \text{T}$.

2. THE U-3M MAGNETIC FIELD CONFIGURATION

Two methods of magnetic field mapping, namely the emissive triode [3] and the luminescent probe [4], were used in the U-3M. These allowed us (i) to carry out magnetic configuration fine tuning and to demonstrate the existence of nested magnetic surface families in the range of vertical magnetic field variations.

![Diagram of U-3M magnetic field configurations](image)

**FIG. 1.** U-3M magnetic field configurations for several $B_1/B_0$ values; $B_1/B_0 = -1.0\%$ (a), $0.0\%$ (b), $+0.5\%$ (c) and $+1.0\%$ (d).

**TABLE I. BASIC PARAMETERS OF THE MAGNETIC FIELD CONFIGURATIONS SHOWN IN FIG. 1(a–d)**

<table>
<thead>
<tr>
<th>$B_1/B_0$ (%)</th>
<th>$\Delta$ (cm)</th>
<th>$\rho$ (a)</th>
<th>$\beta$ (%)</th>
<th>$D\left(10^3 , \text{cm}^{-3}\right)$</th>
<th>$(N), \Delta \rho/\Delta a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1.0</td>
<td>9.5</td>
<td>0.24</td>
<td>+6.0</td>
<td>0.03</td>
<td>(8) 0.04</td>
</tr>
<tr>
<td>0.0</td>
<td>11.4</td>
<td>0.29</td>
<td>0.0</td>
<td>0.14</td>
<td>(3) 0.1</td>
</tr>
<tr>
<td>+0.5</td>
<td>11.0</td>
<td>0.28</td>
<td>-6.0</td>
<td>0.09</td>
<td>(4) 0.08</td>
</tr>
<tr>
<td>+1.0</td>
<td>10.0</td>
<td>0.28</td>
<td>-6.0</td>
<td>0.13</td>
<td>(4) 0.13</td>
</tr>
</tbody>
</table>
−1.0% ≤ B_x/B_0 ≤ +1.5%; (ii) to demonstrate the positive effect of additional compensation field coils on the magnetic field configuration compared with the configuration in the U-3 torsatron; (iii) to demonstrate the existence of configurations with a magnetic 'well' and a magnetic 'hill', and also with different geometric factors for transport coefficients under the same boundary conditions (\(a, \ell(a)\)). Figure 1 and Table I present U-3M magnetic configurations for several \(B_x/B_0\) values and the basic parameters of these configurations (\(V''\): magnetic well/hill value; \(D\): geometric factor; \(N\): number of islands adjacent to the LCMS; \(\Delta \rho/\rho\): relative size of the island).

FIG. 2. Arrangement of plasma diagnostics in the U-3M torsatron.
FIG. 3. Time behaviour of plasma parameters.

FIG. 4. Gasdynamic plasma pressure $nT$, longitudinal current $I_{pl}$ and radiation losses $P_{rad}$ versus input RF power.
FIG. 5. Comparison of $T_e$ and $n_e$ profiles in U-3 and U-3M.

FIG. 6. (a) Time behaviour of input RF power, mean electron density and antenna loading resistance; (b) $\bar{n}_e$ versus input RF power.
3. RF DISCHARGE PLASMA BEHAVIOUR IN THE PLASMA COLUMN

As in the U-3 torsatron [1, 2], the plasma in the U-3M was produced and heated by RF waves in the \( \omega \leq \omega_{ci} \) range of frequencies. The waves were generated in the plasma column by two different helical frame-like antennas. The antenna similar to that in U-3 (the A1 antenna) was used to operate with a low density plasma \((n \leq 5 \times 10^{12} \text{ cm}^{-3})\). The other antenna (A2), its current conducting surface being formed by parts of two helical winding casings, was used to produce a higher density plasma \((n \leq 2 \times 10^{13} \text{ cm}^{-3})\). By means of these two antennas, the plasma can be produced in the U-3M in the whole range of vertical field variations \((-0.1% \leq B_i/B_0 \leq +1.5\%)\). After the surfaces were cleaned by an RF discharge with a low magnetic field \(B_0 = 0.02 \text{ T} \), discharges with a quasi-stationary behaviour of plasma parameters were obtained \((\vec{n}_e = (1-4) \times 10^{12} \text{ cm}^{-3}, T_i(0) = 200-300 \text{ eV}, T_e(0) \sim 100 \text{ eV}, \Delta t = 20-50 \text{ ms})\) for the input RF power injected by the A1 antenna \(P_{RF} \leq 250 \text{ kW}\). Initially these discharges were investigated with a magnetic configuration similar to that in the U-3 \((B_i/B_0 = 1.26\%)\) and the same value of \(B_0 (=0.45 \text{ T})\). The layout of all the diagnostics is shown in Fig. 2. Figure 3 depicts the time behaviour of plasma parameters in the discharges under study. According to diamagnetic measurements, the energy stored in the plasma column \(W = (3/2)n_e(T_i + T_e)V\), as well as the unidirected current generated in the plasma and radiation losses, are proportional to \(P_{RF}\) (Fig. 4). The global confinement time \(\tau^G_E = W/P_{RF}\) does not exceed 0.2 ms. It follows from energy decay time measurements, carried out 0.5 ms after RF power was switched off, that \(\tau^G_E = W/W\) varies from 2.6 to 3.5 ms with \(P_{RF}\) rising from 80 to 240 kW. The values of \(\tau^G_E\), \(n_e(T_i + T_e)\) obtained in this range of \(P_{RF}\) variation are 1.3–1.5 times as large as those observed on the U-3 torsatron. This result is attributed to the more flattened density and temperature profiles (Fig. 5).

To pave the way for experiments at higher magnetic fields \((B_0 \leq 2 \text{ T})\), RF production of the plasma by the A2 antenna was studied (Fig. 6(a)). The density value here depends on the input RF power and attains \(n_e = 1.4 \times 10^{13} \text{ cm}^{-3}\) at \(P_{RF} = 120 \text{ kW}\) (Fig. 6(b)). This dense plasma was produced in the 0.35–1.2 T range of magnetic field strength. In high density discharges at a quasi-stationary ion temperature \(T_i(0) \sim 100 \text{ eV}\), a fast degradation of the electron temperature and an increase in radiation losses were observed. In order to eliminate \(T_e\) degradation, special measures to reduce the impurity influx from the walls should be taken (carbonization, gettering, etc.).

In the case of low \(n_e\), only a small fraction of the power injected by the antenna is absorbed in the plasma column \((\sim 10\%)\). The rest of the RF power apparently goes to maintain the plasma in the regions of the divertor layer and the plasma mantle surrounding the helical winding in the active stage of the discharge [2].
4. PERIPHERAL PLASMA BEHAVIOUR

As probe measurements show, during the RF pulse a divertor layer of the plasma is formed with a quasi-stationary time behaviour of plasma parameters ($T_i \sim 100$ eV, $T_e \sim 20-40$ eV, $n_e \sim (6-10) \times 10^{10}$ cm$^{-3}$). The width of the divertor layer in the region near the separatrix is $\sim 1$ cm. Near the apex of the separatrix this layer widens and extends behind the helical windings.

In all modes of operation the peripheral plasma acquires a quasi-stationary positive potential with respect to the walls; its value can attain $\geq 200$ V near the separatrix surface. The highest value of the radial electric field associated with this potential is 200 V/cm. The spatial distributions of ion density, ion saturation current and current onto an earthed Langmuir probe are characterized by two peaks in the space between two helical windings (Fig. 7), thereby confirming the existence of diverted plasma flows [1]. A special feature of these distributions is the dependence of both absolute and relative values of the peaks and their positions not only on the $B_\parallel/B_0$ ratio but on the absolute value of the confining magnetic field and the input RF power. Apparently, this is caused by the effect of the radial electric field mentioned above and the resonance character of RF power absorption on the diverted plasma flows.

![FIG. 7. Charged particle current to grounded Langmuir probes.](image-url)
Figure 8 shows the time behaviour of the plasma density in the column, in the divertor layer, and in the mantle, as observed at \( n_e < 5 \times 10^{12} \, \text{cm}^{-3} \). A fast decay of the density (\( \tau_{\text{ed}} = 0.8 \, \text{ms} \)) and of the electron temperature (\( \tau_{\text{Te}} = 0.3 \, \text{ms} \)) takes place in the divertor layer after RF power is switched off. A similar situation is observed in the mantle. The density decay in the divertor layer and in the mantle results in the penetration of the neutral gas into the plasma column, the rise in the density and the cooling of the electron component with a characteristic time of \( \approx 0.8 \, \text{ms} \). Note that a decrease in the mantle plasma is observed as \( n_e \) increases in the plasma column.

5. CONCLUSIONS

In summary, our studies show that:

(a) In the modified U-3M torsatron the magnetic configurations with the families of nested magnetic surfaces of different well/hill values and D factors can be formed owing to vertical magnetic field variation at nearly the same values of \( \bar{a} \) and \( \ell(a) \).
(b) A plasma with a density of \((1-14) \times 10^{12} \text{ cm}^{-3}\) can be produced by the RF method in all magnetic configurations.

(c) The values of \(n_T\) and \(\tau_B^D\) in the U-3M are 1.3–1.5 times as high as in the U-3 torsatron with a similar magnetic configuration, as a result of an increase in \(\bar{a}\) and \(n_e\) and \(T_e\) profile flattening.

(d) At a density of \(n_e > 5 \times 10^{12} \text{ cm}^{-3}\) fast radiation cooling of the electron component of the plasma is observed.

(e) Clearly pronounced diverted plasma flows are observed at the periphery. The height and position of the peaks of these flows depend not only on the \(B_\perp/B_0\) ratio but also on the confining magnetic field and the input RF power, this apparently being due to the influence of a strong radial electric field and the resonance character of RF power absorption.

The transition to studies of plasma confinement and equilibrium at \(B = 1-2 \text{ T}\) and \(\bar{n}_e \geq 10^{13} \text{ cm}^{-3}\) will require additional efforts to reduce impurity influx (prolonged cleaning discharges, surface gettering, etc.).

REFERENCES


PLASMA RELAXATION HEATING AND TRANSPORT IN REVERSED FIELD PINCHES

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Abstract

PLASMA RELAXATION HEATING AND TRANSPORT IN REVERSED FIELD PINCHES.

The behaviour of a reversed field pinch (RFP) plasma is governed by relaxation phenomena which sustain the spatial distribution of the magnetic field against resistive diffusion. Relaxation phenomena are associated with the reorganization of the magnetic field profile and changes occurring in magnetic topology through reconnection processes. During relaxation a significant amount of energy can be transformed from magnetic into kinetic energy and/or can be dissipated through plasma resistivity and viscosity. Furthermore, these processes can determine a situation in which magnetic flux surfaces are poorly defined and stochastic fields are generated in large portions of the plasma volume. New measurements have been taken on ETA-BETA II to further analyse these problems. In particular, results on electron and ion temperatures in the bulk of the plasma and on heat flux in the edge region are reported.

1. ION TEMPERATURE AND CARBON EMISSION MEASUREMENTS

The ion temperature \( T_i \) has been evaluated from \( \text{C V} \) 2271 Å line Doppler broadening measurements performed by a 1 m Czerny–Turner spectrometer equipped with a high resolution 2400 grooves/mm grating [1]. The time evolution of the \( \text{C V} \) 2271 Å emission has been detected by a photomultiplier, and the Doppler broadening has been evaluated by scanning the line spectral profile on a shot by shot basis.

The ion temperature obtained from \( \text{C V} \) has been assumed to be representative also of the deuterium gas; in fact, the collisional equilibrium time in the density and temperature ranges typical of ETA-BETA II is very short, of the order of microseconds. Moreover, this assumption is consistent with the results of other RFP experiments showing that the impurity \( T_i \) measured from Doppler broadening is approximately equal to the deuterium \( T_i \) measured with neutral particle analysers [2, 3]. The time evolution of \( T_i(\text{CV}) \) and of the electron temperature \( T_e \) is shown in Fig. 1. The electron temperature, obtained from a Si(Li) detector, is representative of the hottest central region of the plasma. Spatially resolved measurements of \( \text{C V} \) emission indicate that also \( T_i(\text{CV}) \) can be considered representative of the plasma.
FIG. 1. Time evolution of central electron and ion temperatures obtained experimentally in ETA-BETA II.

centre if the ion temperature profile is not much more pronouncedly peaked than parabolic. $T_i$ and $T_e$ show rather different temporal behaviour: while $T_i$ at 0.4 ms after the current start has already reached 200–250 eV and subsequently, within the statistical errors, remains constant, $T_e$ increases continuously from about 120 eV at 0.4 ms to about 350 eV at 1.7 ms, in correlation with the density decay (Fig. 2).

An anomalous ion heating mechanism has to be invoked to account for the observed ion temperatures which are higher than those expected on the basis of classical electron–ion equipartition.

It has been found that the power necessary to sustain an RFP is almost twice that necessary to compensate for the diffusion of magnetic fields on a global scale length, $P_w = \int \eta J^2 \mathrm{d}V$, and is well described by the power required to maintain the total helicity content of the RFP distribution, $P_k = (1/\phi) \int \eta \mathbf{J} \cdot \mathbf{B} \, \mathrm{d}V$, at a constant value.

It has been suggested [4] that this extra power $P_k - P_w$, which is related to the internally generated dynamo electric field associated with fluctuations as $P_k - P_w = \int \mathbf{E}_d \cdot \mathbf{J} \, \mathrm{d}V$, is stored in the magnetic configuration and then dissipated during the relaxation. Magnetic reconnection, in fact, provides dissipation that is greater than that produced by Joule dissipation of the large scale field and allows the configuration to reach a lower energy state with a simpler topology.

The details of this magnetic energy transfer and the problem of whether ions or electrons or both are affected are still open questions. One way of energy dissipation is resistive diffusion. In fact, magnetic reconnection involves the formation and destruction of thin current sheets [5]. Current sheets can accumulate excess magnetic energy and, as a result of instability development, a rapid release of this energy may occur [4]. Moreover, there is also a flow of particles out of the reconnection region.
We suggest that viscous dissipation of this momentum flow could be an effective ion heating mechanisms.

To study the plasma global confinement properties, $\beta_\parallel$, $\langle \beta \rangle$ and $\tau_E$ have been evaluated from the measured $T_e$, $T_i$, and $n_e$, assuming steady state conditions and parabolic profiles. Values of $\beta_\parallel \geq 15\%$ and of $\langle \beta \rangle \geq 5\%$ are obtained at the beginning of the discharge as is shown in Fig. 3. The ratio, $Z_{\text{eff}}^*$, between the resistivity from Ohm's law on the axis and that from the Spitzer expression with $Z = 1$, is plotted as a function of time in Fig. 4, where we see that a plasma resistivity value close to the classical one is obtained at early times. From the evaluated $\beta_\parallel$ and $Z_{\text{eff}}^*$ the energy confinement time has been estimated, leading to values of $\tau_E \geq 100 \mu s$ at the start of the discharge. The decrease in $\beta_\parallel$, $\langle \beta \rangle$ and $\tau_E$ at late times and the increase in $Z_{\text{eff}}^*$ seem to be essentially related to the electron density decay.

The emission from carbon has been measured along seven plasma chords, in order to investigate the impurity transport processes. The data have been inverted by an asymmetric inversion technique based on a forward optimization and have been simulated by a one-dimensional collisional-radiative impurity diffusion model. In Figs 5 and 6, the line emission profiles obtained by inverting the experimental data and those obtained from the model are shown at three different times (the first time corresponding to the emission peak and the other times to the current flat-top phases).

The time behaviour of the C V 2271 Å line emission profile obtained from the simulation is consistent with the experimental observation: around the emission peak, in the early phase of the discharge, the emission is essentially peaked at the centre, while hollow profiles are obtained at later times. Figure 7 shows the carbon ion populations corresponding to the previous, simulated emission profiles. The simulations have been obtained by a classical transport model with the particle fluxes enhanced
by a space dependent coefficient, whose value is 10 in the central region (up to $r = 0.8a$) and increases up to 100 at the wall. The particle diffusion coefficient corresponding to these fluxes is about 10 m$^2$/s in the central region and 100 m$^2$/s at the wall and is consistent with the experimentally deduced global energy confinement time.

With the same values of particle fluxes the time behaviour of O IV, O V and O VI lines has been simulated and, by comparing the line intensities obtained from the model with the measured ones, the carbon and oxygen content may be evaluated, leading to $n_c/n_e = n_{ov}/n_e = 0.5$--1%. Typical values of the effective charge corresponding to this impurity content are about 1.2 at the start and 1.5 at the end of the discharge, and, hence, contribute only little to the previously discussed total anomaly factor, $Z_{eff}^*$. 
FIG. 5. Radial CV emission profiles obtained from inversion (a) and their chordal reconstruction (b); the squares represent the experimental points. The three times correspond to the emission peak (0.3 ms) and to the plasma current flat-top phase (0.7 ms, 1.1 ms).

2. EDGE PROPERTIES AND ANOMALOUS HEAT FLUX

The boundary region of ETA-BETA II has been investigated by movable instrumented limiters equipped with Langmuir probes, heat flux probes and surface collectors [6]. The plasma outward displacement (typically, < 1 cm), creates a cooler layer (whose width has been estimated to be about 1 cm) in the outer region where the magnetic field lines intercept the liner.

The probes are protected by graphite tiles which, when inserted into the plasma, act as poloidal limiters with a poloidal connection length of the order of half the minor circumference. At \( I_0 \sim 150 \text{ kA} \), average values of electron temperature and density
at the edge during flat-top are $T_e \sim 10-20$ eV and $n_e \sim (0.5-1) \times 10^{19}$ m$^{-3}$. These values decrease radially with e-folding lengths of $\lambda_T \sim 2-3$ cm and $\lambda_n \sim 1-2$ cm. From the ion saturation current collected by Langmuir probes, the charged particle flux parallel to the field lines is estimated to be of the order of $10^{23}$ m$^{-2}$·s$^{-1}$, with an e-folding length of $\lambda_T \sim 0.5-1$ cm. The measured power flux intercepted by these limiters is plotted in Fig. 8; it shows a strong directionality with the magnetic field [7]. The measured thermal load reaches the maximum value, of the order of 300 MW·m$^{-2}$, with an e-folding length of the order of 0.5–1 cm on the electron drift side [6]; it is almost an order of magnitude lower on the opposite side. The measured thermal load is found to be, at least, one order of magnitude higher than the value derived from the edge parameters $T_e$ and $n_e$, by applying a simple scrape-off layer model [7]. This large thermal load and the asymmetry in the energy deposition can be interpreted as due to current carrying electrons coming from the hotter region of the plasma through the stochastic region [8]. In fact, the RFP plasma is characterized by a substantial level of magnetic fluctuation ($b/B \approx 1\%$ for plasma currents of the order of hundreds of kA), with a wide toroidal periodicity spectrum. These fluctuations are interpreted in terms of ideal and resistive MHD instabilities, whose growth and non-linear interaction can result in a large plasma volume where the magnetic field lines wind ergodically. In particular, in ETA-BETA II, a wide stochastic region is predicted in the plasma outer region (near the toroidal field reversal surface), where the resonant surfaces of the instabilities accumulate. The effect is a radial diffusion of the magnetic field lines, and a magnetic diffusion coefficient can be defined as $D_m = \langle \Delta r^2 \rangle / S$ (where $\Delta r$ is the radial excursion for a longitudinal length $s$), which is of the order of $10^{-5}$ m in ETA-BETA II. Owing to the radial diffusion coefficient of the magnetic field lines, fast electrons can leave the inner region of the plasma travelling along the magnetic field lines. Computing the radial excursion relative to the electron mean free path $l_e$ to be $\Delta r = \sqrt{D_m l_e}$ at low densities ($n_e < 5 \times 10^{19}$ m$^{-3}$) and high temperatures ($T_e > 100$ eV), a $\Delta r$ of the order
FIG. 7. Radial distribution of carbon ion populations obtained from diffusion model.
of a few centimetres results, i.e. sufficient for the accelerated electrons to cross the boundary and hit the wall or the limiter before undergoing a collision. Furthermore, SiLi detector electron temperature measurements [9] do not indicate the presence of suprathermal electrons, suggesting that the incident particles essentially belong to the thermal tail of the energy distribution. Thus, a small fraction of energetic electrons (a few per cent of the average density) at a temperature comparable to or higher than that on the axis can account for the experimental high heat flux measured at the edge on the electron drift side.

3. CONCLUSIONS

The relaxation process is fundamental in RFP physics, and the measurements reported here support the important impact of relaxation on both plasma heating and transport. These phenomena depend crucially on the amplitude of the MHD fluctuations. In higher temperature experiments such as RFX (a = 0.5 m, R = 2 m, I = 2 MA) with reduced levels of fluctuation, there could still be a significant power for ion heating scaling as the Ohmic heating power since $P_i - P_w \approx P_i$. However, transport in stochastic magnetic fields could become less of a problem, reducing correspondingly also the wall loading problem. Whether these favourable scaling will be found experimentally remains to be proven and it is one of the main objectives of RFX.

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STABILITY AND ION DYNAMICS IN A REVERSED FIELD PINCH PLASMA


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Abstract

STABILITY AND ION DYNAMICS IN A REVERSED FIELD PINCH PLASMA.

Passive control of thin shell modes, by the addition of an extra shell in HBTX1C, has reduced their growth rate, while decreasing the loop voltage and ion heating and increasing the energy confinement time. Active control has also been demonstrated. Operation at ultra low q (0 < q < 1) is found to be restricted to values near major rational fractions. Models of plasma resistance, ion heating and transport are tested by the laser ablation of trace and large quantities of impurities and by comparing present results with those from HBTX1B which had a thick shell and a lower loop voltage. By treating the electron and ion dynamics separately, radiation is found to be an important power loss channel for the electrons, and a simple scaling of the ion temperature is derived which is corroborated by the measurements.

1. INTRODUCTION

A secondary shell has been added to the HBTX1C Reversed Field Pinch (RFP) device [1] to reduce the growth rate of the thin shell modes, to extend the plasma duration and to reduce the loop voltage. The new shell is sufficiently thin (wall time constant $\tau_w = 5.5$ ms) to retain the previous advantages of the original shell, allowing active (feedback) suppression of the thin shell kink mode ($m,n = 1,2$), control of the plasma position and modulation of the plasma current. Thin shell modes are also important in ultra low q plasmas and restrict the operating range of q values.

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The first application of laser ablation of impurities in RFPs is exploited to measure particle confinement times [2] and to examine how impurities affect the loop voltage, ion heating and power balance of the electrons [3]. These investigations will be continued in a new device under construction whose main features are a close fitting conducting shell and a smooth graphite wall.

2. MHD STABILITY WITH RESISTIVE SHELLS

Operation of HBTX with a resistive shell ($\tau_w = 0.5$ ms) located at 1.15 times the plasma minor radius, $a$, leads to the growth of low frequency magnetic 'thin shell' modes [1]. The dominant $m = 1$ instabilities can be categorized into three toroidal mode number groups: $n = (-5, -6, -7), (2, 3)$ and $(9, 10, 11)$. The existence of instabilities in the first two groups is in agreement with zero beta thin shell stability calculations [4] and the third group, which appears to be marginally non-resonant at the wall (i.e. with small $k \cdot B$), has been locally analysed and shown [5] to be ideally unstable because of finite pressure gradient effects.

With the addition of a secondary shell [6] ($\tau_w = 5.5$ ms), outside the windings (located at $1.6 \times a$), growth rates, $\gamma$, are typically halved, falling from $\gamma \sim 0.7$ ms$^{-1}$ to $\sim 0.4$ ms$^{-1}$, with discharges lasting about twice as long (up to 10 ms). The same groups of modes are present with similar growth rates, between the groups, as shown in Fig. 1, which implies some non-linear coupling between the modes. (Linear theory predicts preferential suppression of the low-n modes.) The relatively large radial fields, at current flat-top, are also reduced from $<B_r(a)> \approx 6.5$ mT to $\approx 4$ mT, with the addition of the secondary shell. Ion heating is decreased,

![FIG. 1. Temporal behaviour of the dominant $B_\theta$, $m = 1$ modes; open symbols relate to thin shell only and filled symbols refer to the secondary shell.](image_url)
with the ratio of ion to electron temperature $T_{i0}/T_{e0} = 1.2 \pm 0.2$ with the secondary shell compared with $T_{i0}/T_{e0} = 1.6 \pm 0.3$ with the thin shell alone, both at $I_0 = 170$ kA and $n_e = 2 \times 10^{19}$ m$^{-3}$. The growth rate of the external kink mode ($m,n = 1,2$) increases by a factor of $\sim 4$ as the pinch parameter $\Theta$ is increased from 1.6 to 2.2, which is in qualitative agreement with linear theory.

In ultra low-q (ULQ) operation ($0 < q < 1$), thin shell modes have been observed with growth rates comparable to RFP operation and their growth to large amplitude similarly limits the pulse duration. The resistive shell mode can be suppressed by maintaining $q$ to within 10% above its limiting value near major rational fractions ($1/3, 1/2, 2/3$, etc.). This can be achieved by controlling the toroidal field, by controlling the current or by ‘locking’ the toroidal and poloidal windings in a series connection. A fast growing ($\gamma \tau_w \sim 10$) resonant mode is also observed which leads to a minor plasma current collapse. The combined effect of these phenomena restricts ULQ operation to a narrow range of $q$-values near the major rational fractions [7].

3. ACTIVE PLASMA CONTROL

Three methods of active control of RFP plasmas have been applied to HBTX: (i) The horizontal position of the flux surface at the wall has been actively controlled to within $\pm 1$ mm by using high current switching circuitry [8]. (ii) This circuitry was subsequently adapted to drive currents in two helically wound conductors outside the secondary shell in order to control the $m,n = 1,2$ kink thin shell mode [9]. Mode detection was monitored from two arrays of pick-up coils located just outside the vacuum vessel. The resulting mode amplitude was reduced to $< 2$ mT throughout the discharge compared with $\sim 5$ mT at termination without feedback. (iii) The plasma current was modulated up to $\pm 30\%$ at $\sim 0.5$ kHz, by using a ringing circuit to extend the plasma duration and to investigate dynamic stabilization of the modes.

However, none of these techniques give rise to any measurable improvement in plasma confinement. Calculations from a 3-D code [10] indicate that the confinement might be more dependent on the dynamic modes ($m,n = 1, -4, -7$) which sustain the magnetic configuration.

4. POWER BALANCE, ION TEMPERATURE SCALING AND ION HEATING

In determining the sharing of input power, $I_0 V_\phi$, between the electron and ion channels, it is important first to account for the observed loop voltage, $V_\phi$. A general expression for $V_\phi$, appealing to global magnetic helicity, is given by $V_\phi = (1/\Phi) \int \eta_k \cdot \vec{B} dx^3 + \Delta V_\phi$, where $\eta_k$ is the resistivity appropriate to the helicity dissipation in the plasma. If the measured Spitzer resistivity, $\eta_S$, is assigned to
\section*{FIG. 2. Temporal behaviour of the total Spitzer power, (X), and radiated power, (□), as measured by a bolometer, preceding and during the laser ablation of a large quantity of impurities (maximum $n_{\text{carbon}}/n_e \sim 10\%$) into the plasma.}

$\eta_k$, then the change in $V_\phi$ with $I_\phi$ can be quantitatively explained \cite{11}, leaving a constant anomalous voltage term, $\Delta V_\phi$. More recent work has supported this finding by the application of laser ablation to inject large quantities of impurities. This has the required features of increasing the $Z_{\text{eff}}$ and decreasing $T_e$ and both enhancing $\eta_S$. The observed increase in $V_\phi$ can be accounted for using Spitzer effects alone \cite{3}.
The local electric field is given by $\mathbf{E} = \eta_S \mathbf{j} + \mathbf{E}_a$ where $\mathbf{E}_a = -\langle \nabla \times \mathbf{B} \rangle$ in the MHD dynamo field model. In this case, the local resistive heating of the electrons is pure Spitzer ($\eta_S j^2$) and the residual power, $\mathbf{E} \cdot \mathbf{j} - \eta_S j^2$, is available to heat the ions, and possibly the electrons. It is found that although radiation losses are generally only $\approx 10\%$ of the total input power, they can represent a large fraction of the Spitzer power. This can be seen in Fig. 2, where radiative losses are compared with the total Spitzer power, before and during the injection of impurities in HBTX1C. The radiated power, even before impurity injection, is seen to comprise a significant portion of the Spitzer power and, therefore, might be expected to play a more important role in electron confinement than previously thought.

Charge exchange losses have been shown to be relatively unimportant in HBTX1C and convective losses appear to dominate [3]. The latter fact is supported by recent results from laser ablation [2] in measuring the particle confinement time ($\tau_p \approx 1$ ms) and diffusion coefficient ($D \approx 15$ m$^2$ s$^{-1}$). The energy confinement time, $\tau_E$, for turbulent convection is described by [12] $\tau_E \propto \rho^{1/2} \bar{B}^{-1}$, where $\rho$ is the mass density. Here we explore the possibility that $\bar{B}$ is related to the MHD dynamo electric field (MDF). In the HBTX experiments the loop voltage $V_\phi \gg 2\pi \eta_S j_a$, so that, from the MDF description, it is reasonable to assume that $\langle \nabla \times \mathbf{B} \rangle \propto \Delta V_\phi$. This suggests that $\bar{B} \propto \Delta V_\phi^2$ and that the ion energy confinement time, $\tau_{Ei}$, should scale as $\tau_{Ei} \propto n_i^2 \Delta V_\phi^{-2}$. Assuming, as previously [13], that the power $I_0 \Delta V_\phi$ provides the ion heating, the ion power balance yields a scaling of $T_i$ given by: $T_i \propto I_0 \Delta V_\phi^2 n_i^{-1}$. A comparison of measured ion temperatures with that calculated from this scaling is shown in Fig. 3. Results from both HBTX1C and HBTX1B are included because of the different $\Delta V_\phi$ values ($\sim 50$ V and $\sim 10$ V, respectively). Reasonable agreement is seen between measured and scaled $T_i$ values for both devices.

A theory has been developed [14] which may account for the enhanced ion heating found in RFPs. The classical Braginskii coefficient of parallel viscosity is first modified to accommodate the low collisionality of the plasma. Appeal is then made to the omnipresent dynamo fluctuations of an RFP which can provide a distributed heat source that may furnish the required ion heating, via parallel ion viscosity.

5. NEW FIRST WALL DEVICE

Previous work [11] has revealed the importance of the edge conditions, in particular, perturbations at the edge in the guise of field errors and material obstructions. Unique features of a new HBTX device, under construction, are a close fitting copper shell ($\tau_w = 20$ ms at 1.02 x a) and a smooth, thin (3 mm), bakeable graphite wall [15]. This structure is predicted to reduce the radial field penetration of the wall in the frequency range $1 - 20$ kHz by a factor of six compared with the lowest values achieved on HBTX1B which had a thick shell. Calculations based on magnetic helicity balance indicate that $\Delta V_\phi$ will fall to $\sim 5$ V compared with the best value of
\(~10\) V with a thick shell. Energy confinement times exceeding 1 ms are also predicted.

6. CONCLUSIONS

Passive and active techniques have been successfully applied to reduce the growth of thin shell modes. Thin shell modes are also observed in ULQ operation but controlling the edge q value limits their growth. The first use of laser ablation of impurities in RFPs has enabled us to measure \(\tau_p\), to assess the importance of radiation losses and to test models of anomalous voltage and ion heating. A derived scaling of \(T_i\) is supported by the experimental data.

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CONFINEMENT STUDIES IN THE ZT-40M REVERSED FIELD PINCH*

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Abstract

CONFINEMENT STUDIES IN THE ZT-40M REVERSED FIELD PINCH.

Measurements of electrostatic and magnetic fluctuations in the ZT-40M Reversed Field Pinch (RFP) are used to estimate fluctuation driven transport. Edge electrostatic fluctuation driven transport is consistent with other estimates of the total edge particle transport, analogously to some tokamak and stellarator observations. However, in contrast to the case of tokamaks, electrostatic fluctuations do not explain the heat flux through the edge. Instead, transport of suprathermal electrons along fluctuating magnetic field lines constitutes the major electron heat loss. Ion losses in ZT-40M appear to be dominated by charge exchange.

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1. Introduction

The recent focus of ZT-40M research has been on elucidating the confinement properties of the Reversed Field Pinch (RFP), with an emphasis on the relationship between transport and the measured edge fluctuations\cite{1,2,3}. Profile data for plasma pressure are also becoming available. The work described here is part of a collaborative effort directed at understanding the mechanisms that drive fluctuational transport in the edge of toroidal devices, encompassing the RFP, tokamak and stellarator. The relevant parameter space under study includes magnetic shear and curvature, gradients in temperature and density, impurity effects, and the ratios $j_{\parallel}/B$, and $E_\phi/E_R$. ($j_{\parallel}$ is the current density parallel to the magnetic field, $B$; $E_\phi$ is the applied electric field and $E_R$ is the runaway field.) In examining data from a diversity of conditions one can, in principle, separate the physical mechanisms that determine fluctuation driven transport.

The RFP operates within a narrow range of relatively high beta values (set by power balance), even as the radiated power fraction is changed (by adding impurities) from 0.15 to 0.9 of the input Ohmic power. This is shown in Fig. 1, where the poloidal electron beta is computed from the line average electron density and the on-axis (Thomson scattering) electron temperature\cite{4}. These data have been simulated by a 1-D transport code\cite{5}; the simulation values are designated by triangles.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{beta_vs_radiation_fraction.png}
\caption{Electron poloidal beta as a function of the fraction of input power radiated\cite{4}.}
\end{figure}
2. Fluctuations and confinement

The emerging confinement picture may be conveniently viewed with reference to Fig. 2, which reflects our present view of power flow in ZT-40M. At the time of the 1988 IAEA meeting, we had identified that ~ 0.3 of the Ohmic input power was available to drive fluctuations, and hence heat the ions[1]. In terms of loss processes, kinetic electrons had been observed, and their importance in determining electron losses were indicated[1]. Significant new data have recently been obtained from a number of diagnostics, providing further insights into RFP confinement. Specifically, the magnitude of the various loss channels can now be identified as indicated in Fig. 2.

Detailed measurements have been made in the edge plasma using Langmuir probes, a tungsten calorimeter/ Langmuir probe and an electrostatic electron energy analyzer[2]. Suprathermal electrons with $T_{\parallel} \sim 2 - 3 \times T_e(0)$ are measured in the cold

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**GLOBAL RFP POWER FLOW (TYPICAL ZT-40M)**

![Diagram of global RFP power flow](image)

**FIG. 2.** Global power flow schematic diagram for ZT-40M. Approximate percentages are shown as a function of the Ohmic input power for typical 120 kA flat-top current discharges.
$T_e(a) \sim 10 - 30$ eV edge plasma at concentrations $n_{\text{supra}} \sim 0.05 \times n_e(a)$. These suprathermal electrons are generated inside the $B_\phi$ reversal surface[6], and we hypothesize that they transport to the edge by means of magnetic flutter. The suprathermal electrons can account for most of the electron power flux to the wall, and can, under all operating conditions examined to date, carry most of the edge ($r/a \geq 0.7$) current density, thus maintaining the local magnetic field distribution ("kinetic dynamo"[7]). These suprathermal electrons are responsible for 35 to 40 percent of the Ohmic input being lost under typical, 120 kA discharge conditions.

A 35 mm square array of four magnetic field probes, each measuring the field along three axes, has been used in the ZT-40M edge plasma[3]. The magnetic field fluctuation level for a 60 kA discharge as measured 10 mm inside the vacuum vessel wall is typically $\sqrt{\langle B_r^2 \rangle} / B = 0.015$, a level much higher than found in modern tokamaks. By measuring the phase shift between the probes as a function of frequency one determines the mode numbers $m$ and $n$. The mode spectrum is predominantly $m = 1$ and is peaked at $n$ values resonant with the magnetic field line pitch inside the $B_\phi$ reversal surface, which is typically located at $r/a \sim 0.85 - 0.90$. Utilizing the edge spectral measurements of $\dot{B}_r$ and extrapolating into the plasma assuming quasi-static modes[3,8], we calculate several quantities of interest: (1) the Chirikov

\[ Z' \equiv \eta_i / \eta_{\text{spitzer}} \]

\[ E_\phi / E_R \]

\[ FIG. 3. \text{ Resistive anomaly factor } \eta_i / \eta_{\text{spitzer}} \text{ as a function of the ratio of applied electric field } E_\phi \text{ to the critical field for runaway, } E_R. \]
(island overlap) parameter $S$; (2) the quasi-linear field line diffusivity $D_m$; and (3) the Kolmogorov entropy (field line divergence scale length), $L_K$. These data give $S > 1$ for $r/a < 0.7$, indicating stochasticity in the core plasma. The small population of suprathermal electrons present in ZT-40M obey the collisionless criterion $\lambda >> L_K$, where $\lambda$ is the mean free path.

The low frequency portion of the $\tilde{B}$ spectrum agrees qualitatively with 3-D MHD computations, which show that these low frequency modes are associated with MHD dynamo activity\[9\]. The experimental data also show higher frequency modes with higher mode numbers; these may reflect fluctuational activity resonant in the plasma exterior ($r/a > 0.6$). Experimentally typical low frequency global magnetic fluctuation amplitudes decrease with the Lundquist number as $S^{-0.4\pm0.1}$. Global confinement times improve with increasing $S$. Without detailed radial profile data on these fluctuations, it is difficult to relate this scaling to a particular model. However, as an example, the turbulence associated with electrostatic resistive $g$-modes, which are resonant in the plasma exterior, should give $\tilde{B}_r \propto S^{-0.5}$ with constant electron beta[10,11], in reasonable agreement with these data.

Preliminary modeling of the suprathermal electron transport by solving the long mean free path (collisionless) limit of the Boltzmann equation, assuming magnetic flutter driven transport, yields qualitative agreement between the calculated and measured electron distribution function, the magnitude of the magnetic field flutter, and the applied electric field $E_{\phi}$\[12\]. As an example, one can estimate the additional resistivity of the plasma due to momentum transfer to the wall as\[13\]:

$$\eta_K/\eta_{\text{Spitzer}} \simeq [1 + (\kappa P/\Theta)(E_{\phi}/E_R)]$$

where $\kappa \sim 2.5$, $\eta_K$ is the helicity balance resistivity, $P$ is the power deposition asymmetry factor on the wall, $\Theta \equiv B_{\phi}(a)/ < B_{\phi} >$ and $E_{\phi}$ is the applied electric field. A plot of ZT-40M data (Fig. 3) shows a similar dependence, although the ratio $\eta_K/\eta_{\text{Spitzer}}$ here also includes contributions from impurities.

Langmuir probe measurements are complicated by the presence of the suprathermal electrons and uncertainties in the secondary and thermionic electron emission from the probe tips. No correction is made for suprathermals in the present analysis, though a model for their effect is being developed. We estimate that the ion saturation current overestimates the mean density by perhaps 50 percent, while errors in the
mean temperature are relatively small. It is found experimentally by comparing the results to those obtained inside the limiter shadow, where suprathermal electrons and probe heating are much reduced, that no significant differences exist to suggest that the fluctuation measurements were dominated by these effects.

The k-spectrum is obtained from a two-point correlation of the floating potential and has a spectral width $\sigma_k \approx k_L$ and $< k_L > \rho_i \approx 0.05$. A linear fit to the phase spectra gives a propagation speed $v_{ph} = 4 \pm 1 \times 10^4 m/s$ in the electron diamagnetic drift direction for 60 kA discharges. This velocity is comparable to the estimated electron diamagnetic drift speed $v_d = T_e/(eBL_n) \sim 2 \times 10^4 m/s$, and is not dominated by the $E \times B$ drift in the ion diamagnetic direction. In contrast to TEXT observations[14], a velocity shear is not found.

The quasi-linear particle flux driven by these fluctuations is given by: $\Gamma_E = < \mathbf{n} \mathbf{u}_r > = - < \mathbf{n} \nabla \phi > / B$, where $\mathbf{u}_r$ is the radial fluctuating velocity driven by the fluctuating local electric field $\mathbf{E} = -\nabla \phi$, $\Gamma_E$ is approximately equal to the total particle flux estimated spectroscopically[15]. Thus, as in a tokamak[14], electrostatic turbulence is identified as an important edge particle transport mechanism.

The electrostatically driven heat flux, $Q_E$, may be described in terms of a conductive and a convective component: $Q_E = Q_{cond} + Q_{conv}$ where $Q_{cond} \equiv \frac{3}{2} n k < \mathbf{T} \mathbf{u}_r >$, and $Q_{conv} = \frac{3}{2} kT < \mathbf{n} \mathbf{u}_r >$. In ZT-40M $Q_{cond}$ and $Q_{conv}$ are both small (~ 5 percent) compared with the total heat flux. This is consistent with the hypothesis that $\mathbf{B}$ transport dominates energy confinement in present RFPs and contrasts with observations in tokamaks[14] and stellarators[16] where $Q_E$ contributes strongly to the total edge thermal flux. Another observation is that $\mathbf{T}$ and $\mathbf{n}$ are anti-correlated, which implies $Q_{cond}$ is directed inward.

One observes: $\mathbf{n}/n \sim 0.3 - 0.5 < \mathbf{\Phi}_{fl}/kT_e$, as also seen in a tokamak[14] and in models that involve the effects of radiation on edge turbulence. Further work is required in this area; data from highly radiating plasmas will be especially instructive in this regard.

Ions are heated more rapidly and to higher temperatures than expected from electron-ion equipartition. The ion heating is strongly correlated to the macroscopic level of fluctuational activity in ZT-40M[1,17]. Ion temperature profile data have been obtained for 120 kA and 180 kA discharges with flat-topped currents using Doppler
broadening of C V impurity ions. The present data indicate that the $T_i(r)$ is spatially flatter than $T_e(r)$ (based on Thomson scattering) for these conditions. Thus, significant $T_i$ gradients exist in the plasma between the last measurement location ($r/a = 0.8$) and the wall. (Different ion temperature profiles have been observed under other conditions, see, for example, Ref. 18.) Charge exchange losses have been estimated from the flux of neutrals observed by a time-of-flight neutral particle spectrometer; these losses amount to 10 to 20 percent of the Ohmic input power.

Gradients in electron temperatures and densities have also been measured[2]; the resultant pressure gradients provide a drive mechanism for fluctuational activity in the edge plasma.

3. Summary

We conclude that, in ZT-40M, suprathermal electron transport along weakly stochastic magnetic field lines is a major electron energy loss channel, accounting for about half of the electron energy input. Radiative losses account for most of the other electron energy losses as indicated on Fig. 2. The suprathermal electron current density $j_{\text{hot}}$ is comparable to the parallel current density $j_{\parallel}$ at least for $r/a \geq 0.7$. In the edge region, transport by electrostatic fluctuations is sufficient to explain the particle flux as in the TEXT tokamak and ATF stellarator. However, in contrast to TEXT and ATF results, we find $Q_E$ is small ($\sim 5$ percent).

Ion heating is well correlated with fluctuational activity[1,17] and ion losses in ZT-40M are dominated by charge exchange. Significant ion pressure gradients exist, which can provide a local drive mechanism for pressure driven fluctuations.

The specific underlying mechanisms that drive the electrostatic and magnetic fluctuations are presently unknown. There is some evidence that current driven dynamo activity, pressure gradient driven drift wave turbulence, and possibly radiation enhanced turbulence are important drive mechanisms. Further research in this area, in conjunction with tokamak and stellarator research, is clearly necessary.

Implications of this and other work are encouraging for the next generation of multi-mega-ampere RFP experiments (RFX and ZTH). These devices will have much lower intrinsic magnetic field errors and low-Z wall materials will be used to reduce metal
impurity radiation losses. These devices are expected to have a Lundquist number S some two orders of magnitude larger than ZT-40M, and a factor five lower ratio of $E_d/E_R$. Based on our present understanding of RFP transport, this will lead to much lower magnetic fluctuations, reduced suprathermal electron acceleration and associated kinetic electron losses.

REFERENCES


**DISCUSSION**

P.M. BELLAN: Could you tell us what the frequency of electrostatic fluctuations was in ZT-40M, and the $k_\perp$ in MST?

P.G. WEBER: Most of the ZT-40M electrostatic fluctuation power is at low frequencies (tens of kilohertz). There is as yet no reliable measurement of $k_\perp$ in MST.
CONFINEMENT AND FLUCTUATIONS IN THE MST REVERSED FIELD PINCH

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Abstract

CONFINEMENT AND FLUCTUATIONS IN THE MST REVERSED FIELD PINCH.

The Madison Symmetric Torus (MST) is a large (R_0/a = 1.5/0.52 m) RFP which to date has obtained 80 ms discharges at a peak plasma current of 0.6 MA. Low loop voltages (15 V) and modest temperatures (T_e/T_i = 350/250 eV) are routinely obtained, giving estimated unoptimized energy confinement times of about 1 ms. Loop voltage and ion temperature are anomalous. Magnetic fluctuations are typically 0.5%, with most of the power at frequencies below 30 kHz and mode numbers in agreement with MHD prediction for tearing modes. Electrostatic fluctuations are typically 20-50% with a spectrum that decreases with frequency.

1. THE MST DEVICE

The Madison Symmetric Torus (MST) is a large reversed field pinch device (R_0 = 1.5 m, a = 0.52 m) that began operation in June 1988. It was designed for an ultimate plasma current of 1 MA and a pulse length of 50 ms, limited by its 2 Wb iron core. It is thus intermediate between existing devices such as ZT-40 and HBTX and the next generation devices such as ZTH [1] and RFX [2]. It incorporates a number of unique design features [3] to reduce magnetic field errors, facilitate disassembly and access, and reduce the separation of the plasma from the surrounding conducting shell. Its large size offers the opportunity for a definitive assessment of RFP confinement scaling.

2. PLASMA CHARACTERISTICS

During its first year of operation the major design objectives of MST were met, despite the use of a temporary Ohmic heating winding that produced significant magnetic field errors at the poloidal gap in the shell. Plasma currents of 0.5 MA and pulse lengths of 35 ms were obtained [4]. The second phase of operation began in February.
1990 with a new Ohmic heating winding, upgraded capacitor banks, graphite limiters and improved diagnostics. The new winding produces field errors whose RMS value is about a sixth that of the previous winding. Under similar conditions (0.4 MA), the loop voltage has been halved (to about 15 V) and the discharge duration doubled to 80 ms (Fig. 1(a)).

For a density of $6 \times 10^{18} \text{ m}^{-3}$, the electron temperature (as determined by Thomson scattering) is 350 eV and the ion temperature (as determined by charge
exchange analysis) is 250 eV. These values imply an unoptimized energy confinement time of about 1 ms. Charge exchange and Doppler measurements of the ion temperature indicate a non-collisionally heated ion population, with $T_i > T_e$ in some cases. The neutral energy spectrum is Maxwellian. The ratio $T_i/T_e$ is a function of electron density.

Plasma confinement in discharges with reversal parameter $F \approx -0.15$ may be approximately described by two empirical relations: $T_{e0}$ (eV) $\approx 30I/N \times 10^{-14}$ A-m, where $N = \pi a^2 \langle n \rangle$ is calculated for a parabolic density profile; and $Z^* \approx 0.03T_{e0}$ (eV), where $Z^*$ is the enhancement of plasma resistance over that given by Spitzer's formula with $Z = 1$ and is in part a result of the helical current path. Similar relations have been observed on smaller RFP devices. Optimization of MST discharges is ongoing, and these relations are indicative of present operation, not optimal RFP confinement.

3. MAGNETIC FLUCTUATIONS

In addition to the gross confinement properties of MST, a detailed investigation of magnetic and electrostatic fluctuations is under way. Edge magnetic fluctuations have been studied from the point of view of distinguishing between tearing mode fluctuations, which are expected at low frequency, and resistive interchange fluctuations, expected at higher frequency. Within the vacuum vessel there are about 400 magnetic coils, attached to the wall, to provide detailed spatial properties of the turbulence. The relative amplitude of the magnetic fluctuations appears to decrease with plasma current and is about 0.5% of the equilibrium field, at a current of 0.5 MA. Most of the fluctuation power (90%) is contained at frequencies below 30 kHz. At low frequency ($f < 25$ kHz) the mean poloidal mode number $\langle m \rangle$ is unity, with a narrow spread of $\delta m < 1$. The mean toroidal mode number $\langle n \rangle$ is about 5–7, with a spread of $\delta n \approx 20$. The mean mode numbers at low frequency are in excellent agreement with the MHD prediction for tearing modes. At higher frequency $\langle m \rangle$ remains about 1–2, but $\delta m$ increases to about 3 at 100 kHz. The $n$ spectrum broadens and shifts, with $\delta n$ increasing to about 100 and $\langle n \rangle$ increasing to 20–40. Moreover, $\langle n \rangle$ appears to change sign from low to high frequency; this suggests that the modes are shifting from being resonant inside the reversal surface at low frequency to resonant outside the reversal surface at high frequency. The shorter wavelengths and resonant location at higher frequency might be compatible with resistive interchange turbulence. The lowest fluctuation levels coincide with the best plasma quality, suggesting the possibility of a causal relation.

Often a rotating localized disturbance is observed (Fig. 2(a)), similar to the 'slinky' mode observed in the resistive wall OHTE experiment [5]. The disturbance represents a phase locking of several toroidal modes, typically 5 through 7. Sometimes, particularly at the highest values of plasma current, the disturbance ceases rotation in the vicinity of the vertical gap in the shell (Fig. 2(b)). This locking to the
FIG. 2. Time dependence of low frequency magnetic fluctuations at 32 toroidal locations for (a) a 'long' shot and (b) a 'short' shot. The rotating mode locks to the gap (toroidal angle = 0°) during a 'short' shot at the time the m = 1 field error begins to grow.

gap usually occurs at the crash of a sawtooth oscillation. Following this locking, an m = 1 radial error magnetic field grows at the gap, causing early discharge termination (Fig. 1(b)). It is anticipated that active field error control will eliminate the early termination.

4. ELECTROSTATIC FLUCTUATIONS

Langmuir probes have been employed in the edge of MST to measure electrostatic fluctuations. The probes consist of four graphite tips, two of which measure
floating potential ($\phi_0$) and two of which operate as an ion saturation current ($I_{sat}$) double probe. Preliminary results indicate that the observed fluctuations have amplitudes given by $\delta I_{sat}/I_{sat} \approx \delta n_i/n_i = 50\%$ and $\delta T_e/T_e = 20\%$. The coherence amplitude of $\delta n_e$ and $\delta T_e$ is about a half, and the relative phase shift is about 45 degrees, yielding pressure fluctuations of the order of 55\%. If the toroidal electric field is approximated by the gradient of the floating potential, the particle transport due to electrostatic fluctuations can be found using $\langle \delta n_e \delta E_t \rangle / B$. The phase shift between $\delta n_e$ and $\delta E_t$ is such as to produce maximum outward radial transport. The particle confinement time deduced from electrostatic fluctuation induced transport is of the same order as the particle confinement time deduced from spectroscopic data. The energy transport due to electrostatic fluctuations is given by $\langle \delta p_e \delta E_t \rangle / B = T_e \langle \delta n_e \delta E_t \rangle / B + n_e \langle \delta T_e \delta E_t \rangle / B$. The conductive term has not been directly measured; however, given the observed amplitude of the temperature fluctuations, this term should not be large enough to cancel the convective term. Therefore, the energy confinement time is expected to be of the same order as the particle confinement time deduced from spectroscopic data. The electric field is given by the gradient of the plasma potential; therefore, the gradient of the temperature fluctuations must also be measured to obtain a reliable value of the resulting edge transport rates. Measurements also indicate that $e \delta \phi_p / k T_e \sim 1.3 > \delta p_e / p_e \approx 0.55$, which implies that the plasma behaviour is non-Boltzmann.

ACKNOWLEDGEMENT

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ANOMALY OF ION TEMPERATURE IN THE REVERSED FIELD PINCH AND ULTRALOW Q PLASMAS OF REPUTE-1 AND 1Q

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Abstract

ANOMALY OF ION TEMPERATURE IN THE REVERSED FIELD PINCH AND ULTRALOW Q PLASMAS OF REPUTE-1 AND 1Q.

The anomaly of ion temperature \( T_i \) in the reversed field pinch (RFP) is studied in REPUTE-1, and the anomaly of \( T_i \) in ultralow Q (ULQ) discharges is examined in REPUTE-1 and 1Q. The ion temperatures measured by CV Doppler broadening and neutral particle analyser and the observed plasma resistance \( R_p \) in RFP increase anomalously when the electron density \( n_e \) decreases while the plasma current is kept constant. Both \( T_i \) and \( R_p \) increase with an increase of the MHD fluctuation level of the \( m = 1 \) mode and with increasing peakedness of the soft X-ray radial distribution. Possible mechanisms of direct ion heating are discussed. In ULQ discharges anomalous ion heating is also observed during MHD active phases. A significant feature of anomalous heating is the difference in temperatures of the different species. A simple model is used to explain the charge and mass dependences of the heating power of the fluctuations.

The phenomena of MHD relaxation and anomalous ion heating have been investigated in RFP and ULQ plasmas in REPUTE-1 [1-4]. ULQ plasma in a higher toroidal field is studied in the new REPUTE-1Q device.

1. ANOMALY OF ION TEMPERATURE AND PLASMA RESISTANCE IN RFP PLASMA

It has been observed in several experiments that the ion temperature \( T_i \) is higher than the electron temperature \( T_e \), especially in the regimes...
of high-Θ operation or of low electron density, \( \bar{n}_e \). In REPUTE-1, the anomaly of \( T_i \) and its correlation with MHD activities are studied in detail.

Figure 1(a) shows the dependence of the ion temperature of CV Doppler broadening on the line averaged density \( \bar{n}_e \) measured by CO\(_2\) laser interferometry at \( t = 1.0 \) ms after the plasma current rise, which almost corresponds to the current flat-top in the case of \( I_p = 300 \) kA. The dependence of \( T_e \) at the centre measured by Thomson scattering and soft X-ray pulse height analysis as well as the dependence of plasma resistance \( R_p \) on \( \bar{n}_e \) at \( t = 1.0 \) ms are shown in Fig. 1(b).

The plasma resistance \( R_p \) of RFP can be decomposed into two parts: one part from Ohmic dissipation, \( R_{pe} \equiv \int \eta j^2 dV/I_p^2 = \eta_{sp}(0) \left( \frac{2\pi R_p}{\pi a^2} \right) \xi_w \) (\( \eta_{sp}(0) \) is the Spitzer resistivity at the centre; \( Z_{eff} = 1.5 \) is assumed; the space factor of the modified Bessel function model is given by \( \xi_w \approx 8 \)); and the other part is due to the fluctuation, \( \Delta R_p \equiv R_p - R_{pe} \). Figure 2 shows the relations of the estimated \( \Delta R_p \) and \( T_i \) versus \( \bar{n}_e \). Both \( T_i \) and \( R_p \) tend to increase with increase of the relative fluctuation levels of the \( m = 1 \) poloidal component of the toroidal field at the plasma boundary as well as of the soft X-ray signal from the centre chord of the plasma. It
is observed that the radial profile of the soft X-ray signal is more peaked in the lower electron density regime, where the MHD activity is stronger and $T_i$ is higher. Figure 3 shows the time variation of $T_{CV}$ of CV and $T_H$ measured by neutral particle analyser of the H-plasma. $T_{CV}$ is usually higher than $T_H$ in the low density regime ($n_e < 5 \times 10^{19} \text{ m}^{-3}$).

The equation of energy balance is $I_p V_L = \frac{\partial}{\partial t} \int \frac{B^2}{2 \mu_0} dV + \int \left[ \eta j^2 + (j \times B) \cdot \mathbf{v} \right] dV$. The volume integral of the acceleration term $\int (j \times \mathbf{B}) \cdot \mathbf{v} dV$
must be equal to $\Delta R_p I_p^2$. When the MHD equation of motion is used, the acceleration term is reduced to

$$(\mathbf{j} \times \mathbf{B}) \cdot \mathbf{v} = \frac{\partial}{\partial t} \left( \rho_m v^2 - \frac{p}{\gamma - 1} \right) + \nabla \cdot \left[ \left( \frac{\rho_m v^2}{2} - \frac{p}{\gamma - 1} + p + \Pi \right) \mathbf{v} \right] - \sum \frac{\partial u_i}{\partial x_j} \Pi_{ij}$$

and the volume integral of the time average of this term becomes

$$- \int \langle \sum (\partial u_i / \partial x_j) \Pi_{ij} \rangle \, dV \quad [3].$$

The Braginskii viscous term consists of compressional and shear flow components. The former term is the so-called gyrorelaxational heating [5], and its power density is

$$Q_{\text{vis}}^c \sim \int \left[ \frac{n_i T_i}{\nu_i} \frac{\left( \delta B \right)^2}{B} \frac{\omega^2 \nu_i^2}{\nu_i^2 + (4/7) \omega^2} \right] \, dV$$

where $\omega$ is the typical frequency of the magnetic fluctuation $\delta B$ and $\nu_i$ is ion-ion collisional frequency. The latter term is given by

$$Q_{\text{vis}}^s \sim \int \left[ 10 \frac{n_i T_j}{\Omega_i \nu_i^2} \frac{\partial u_k}{\partial x_j} \right] \, dV$$

where $\Omega_i$ is the ion cyclotron frequency [5]. However, the standard Braginskii expression of the viscous term, which is not necessarily valid for fast varying phenomena, is not sufficient to explain the observed ion heating power. According to the Petschek model [6] and a numerical simulation [7], the plasma near the reconnection separatrix is considerably accelerated at the driven magnetic reconnection (as high as $0.8v_A$; $v_A$ is the Alfvén velocity) at the expense of the magnetic energy. The accelerated ions heat the bulk ions by energy relaxation with a heating power density of

$$Q_{\text{acc}} \sim \varepsilon 0.6 \rho_m v^2 \gamma (\tau_{pi} / \tau_{si}) \sim 1.2 \varepsilon (B^2/2\mu_0) \gamma (\tau_{pi} / \tau_{si})$$

where $\varepsilon$ is the fraction of accelerated ions, $\gamma$ is the repetition frequency of the MHD relaxation, $\tau_{pi}$ and $\tau_{si}$ are the particle confinement and slowing down times due to collisions of accelerated ions with bulk ions, respectively ($\tau_{pi} < \tau_{si}$ is assumed). Generalized viscous heating following this mechanism is a possible candidate to explain the observed heating power.

2. ULQ DISCHARGE

In ULQ discharges, anomalous ion heating is observed during MHD active phases. The most significant feature of the anomalous heating is
FIG. 4. Temporal evolution of plasma parameters: the temperatures of $\text{O}^{4+}(T_i^{\text{OV}})$ and $\text{H}^+(T_i^{\text{TOF}})$ and the amplitude of magnetic fluctuations in ULQ discharge as well as safety factor $q$, plasma current $I_p$ and line averaged electron density $\bar{n}_e$.

the difference in the temperatures among the different species. Figure 4 compares the temperatures of $\text{H}^+$ ions measured by a time-of-flight method and of $\text{O}^{4+}$ ions measured by Doppler broadening. It is observed that $\text{O}^{4+}$ ions are strongly and rapidly heated up in phase with the excitation of the MHD fluctuations, while $\text{H}^+$ ions are not heated significantly. The tight correlation between the heating of $\text{O}^{4+}$ and the MHD fluctuations implies that the MHD relaxation dynamics plays an essential role in the anomalous heating. This heating mechanism can be explained by a simple model assuming acceleration of particles by the component of fluctuating electric field perpendicular to the magnetic field. When the relevant fluctuation has a frequency $\omega$ in the range $\omega \ll \Omega_{ci}$, where $\Omega_{ci}$ is the ion cyclotron frequency, the heating power for ions is proportional to the ion mass. The electron temperature is typically 80~100 eV, and is also lower than the $\text{H}^+$ temperature in the same experimental condition.
Another aspect of the ULQ discharge anomaly is its high resistivity, as is that of RFP. We have studied the dependence of the effective resistivity, \( \eta^* \equiv (V_L/I_p)(a^2/2R) \), on the toroidal field, especially in a higher field range than was studied before, using the new device REPUTE-1Q \((R/a = 0.55 \text{ m}/0.15 \text{ m}, B_t = 0.7 \text{ T})\). Figure 5 shows the effective resistivity as a function of the toroidal field. Data from other devices are included[8-10]. Despite a large variation in device sizes, operational regimes and electron temperatures, we observe that the effective resistivities in these experiments follow an universal scaling, implying that \( \eta^*B_t \approx 4 \ \mu\Omega \cdot \text{mT} \). This scaling suggests that the resistance anomaly is related to the helicity balance condition [11].

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ULQ OPERATION AND TRANSITION TO RFP IN EXTRAP T1 WITH A RESISTIVE SHELL

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Abstract

ULQ OPERATION AND TRANSITION TO RFP IN EXTRAP T1 WITH A RESISTIVE SHELL.

The modified Extrap T1 device has been run also as an RFP and as an ultra low q (ULQ) configuration. The device now has a stainless steel bellows vacuum vessel and a resistive shell located inside the Extrap octupole field conductor system. The equilibrium field is controlled by the distribution of the currents in the ohmic heating primary winding. The main objective of the present experiments was to study the ULQ regime. These discharges, with toroidal currents in the range 20-50 kA, show the typical stepwise decay of the plasma current. The current steps corresponded to transitions of the edge q-value across rational values. By applying the octupole magnetic field, discharges in a magnetic separatrix configuration were obtained with toroidal plasma current 15-20 kA, q-value in the range 0.3-0.5, conductivity temperature 10-20 eV and duration 1 ms. With a programmed toroidal field RFP discharges with $\Theta = 1.6$ and $F = -0.2$ were obtained. The pulse lengths were up to 0.5 ms, limited by the termination of the reversed current in the toroidal field circuit. The duration is five times longer than the vertical field penetration time of the shell.

1. INTRODUCTION

The Extrap T1 device [1] has been rebuilt and run also as an RFP and as an ultra low q (ULQ) configuration. In particular the behaviour in the region of ULQ [2,3] is of interest, since earlier Extrap T1 experiments were run with an on axis safety factor of $q(0)=0.1$ with a paramagnetism in agreement with the anisotropic resistivity. We have investigated toroidal equilibrium control, on time scales longer than the shell vertical field penetration time, using a programmed external vertical field. In order to compare with the Extrap mode we have studied ULQ plasmas in a magnetic separatrix configuration. We have applied a poloidal octupole field, which combined with the plasma poloidal field gives rise to a magnetic separatrix inside the vacuum vessel. We observe globally stable ULQ discharges in the magnetic separatrix configuration with about the same duration as the conventional discharges bounded by the liner.
2. DESCRIPTION OF THE EXPERIMENTAL DEVICE

The rebuilt Extrap T1 device has stainless steel bellows vacuum vessel with major radius 0.5 m and inside minor radius 5.7 cm located on the inside of the Extrap octupole field conductor system. This liner has a penetration time of 3 \( \mu \)s for the perpendicular magnetic field and 50 \( \mu \)s for the toroidal field. The liner is surrounded by a 2.5 mm stainless steel resistive shell, with 12 poloidal and 2 toroidal gaps, and a perpendicular magnetic field penetration time of about 100 \( \mu \)s. The equilibrium vertical field is controlled by the distribution of currents in the ohmic heating primary winding. A system of eight toroidal coils outside the shell produces the octupole magnetic field. The toroidal magnetic field coil is placed outside the octupole coil system.

3. TOROIDAL EQUILIBRIUM CONTROL

The displacement \( \Delta \) of the outermost flux surface relative to the geometrical center of the vessel was measured by combining the signals from toroidal loops and a cosine coil. The measured displacement \( \Delta \) was only a few mm. ULQ discharges were obtained with duration 1–2 ms, which is equivalent to about 10–20 times the shell time constant, maintaining a good equilibrium position throughout the discharge. The inboard and outboard primary turns were connected in parallel so that there was a possibility to adjust the equilibrium position by introducing a choke inductance in either of the windings. We could deliberately produce initial radial displacements of the plasma and subsequent radial drift motion in either the inward or outward direction. Zero displacement was thus found to correspond to minimal plasma loop voltage although the loop voltage was practically unchanged for displacements up to about 10 mm in either direction.

4. ULQ OPERATION

The ULQ discharges were run with a toroidal magnetic field of <0.25 T and toroidal currents in the range 10–50 kA, yielding an average current density reaching 5 MA/m\(^2\). The filling pressure (\( \text{H}_2 \) or \( \text{He} \)) was 0.2–1 Pa and the loop voltage 200–500 V, corresponding to a Spitzer resistivity temperature of 10–20 eV. The resistive diffusion time at 15 eV is 200 \( \mu \)s and the Alfvén transit time of the order of 1 \( \mu \)s. A monitor of the \( \text{H}_\alpha \) radiation showed that ionization generally occurred within the first 100 \( \mu \)s of the pulse when the current was rising. In the high current discharges, 30–50 kA, the total radiated power was measured with a bolometer. The ratio of radiated to input power usually decreased substantially after 300–400 \( \mu \)s of the discharge, indicating radiation burn-through. The plasma current decayed in a stepwise fashion with each step corresponding to a transition across a mode rational value. As an example, we show in Fig. 1 the time evolution of the plasma current and the cosine and sine signals. Here, the steps occur at edge q-values of 1/3, 1/2, and 2/3. The current in the lowest level was finally terminated due to the saturation of the transformer core. The current steps were correlated with an increased magnetic fluctuation amplitude, detected by the cosine and sine coils, as well
as a momentarily increased impurity line radiation. In some cases, we observed a coherent oscillation on the cosine and sine coil signals during the constant current periods, indicative of a rotating helical structure. The frequency of the oscillation changed after each transition suggesting a simultaneous change in the helical structure. We often observed current levels corresponding to edge q-values near, or slightly above, the mode rational values q=1/n, n=1, 2, 3, ... This behaviour could be explained by the observed plasma resistance. The resistance, measured during the first flat-top period of the current, showed a marked increase for edge q-values near the mode rational values. For a series of shots with ohmic heating bank voltages kept unchanged, the resistance increase was coincident with a gradually decreased plasma current as the rational q-value 1/4 was approached from above by lowering the toroidal field. For a given voltage, the higher resistance near the mode rational q-value limited the plasma current for a certain range of toroidal field strength. For a sufficiently low toroidal field, the limit was overcome and the next q-window below the rational value was entered. The plasma resistance was here again low and the plasma current higher.

5. MAGNETIC SEPARATRIX CONFIGURATION.

A flux plot corresponding to the experimental parameters is shown in Fig. 2. The octupole field, which was applied prior to the discharge initiation, clearly affected the plasma current buildup. The plasma current rise time
increased significantly compared to discharges without octupole field. The discharge presumably was initiated along the low field region of the octupole field near the minor axis. The separatrix radius then increased with the plasma current amplitude. After the startup phase, the plasma current and octupole coil current exhibited similar time dependences resulting in a stationary separatrix. The experiments have shown globally stable plasma discharges, with q-values in the range around 0.3–0.5, lasting longer than 1 ms, as in the case without octupole field.

6. TRANSITION TO RFP

For discharges where q approached 1/6, corresponding to a pinch parameter of $\Theta=0.6$, the fluctuation level increased and the generation of toroidal field was stronger than the paramagnetic effect due to anisotropic resistivity. The RFP dynamo was starting the field reversing process. By programming the toroidal field coil current we obtained sustained pinch discharges with $\Theta=1.6$ and $F=-0.2$. The pulse lengths were limited by the termination of the reversed current in the toroidal field circuit to 0.5 ms, about 5 times the vertical field penetration time of the shell.
ACKNOWLEDGEMENTS

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DEVELOPMENT OF TANDEM MIRROR EXPERIMENTS IN GAMMA 10

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Abstract

DEVELOPMENT OF TANDEM MIRROR EXPERIMENTS IN GAMMA 10.
Extensive investigations of potential formation and confinement in a tandem mirror plasma have led to a new understanding of mirror confinement systems. End losses of the mirror plasma are greatly reduced in association with the formation of plug/thermal barrier potentials. The highest confining potential of 1.7 kV is obtained under the barrier depth of 1.1 kV. The improvement of particle confinement is more than $10^1$ times over that of a single mirror. The energy confinement time estimated from rate analyses of the density and temperature of potential-trapped plasma is seen to be improved by more than $10^2$ times over that of a single mirror as the particle confinement time increases.

1. INTRODUCTION

This paper reports on the results of recent experiments in the tandem mirror GAMMA 10. GAMMA 10 has obtained very encouraging results in suppression of radial as well as axial losses by potential plugging [1]. Significant improvements have been attained in the confined plasma due to additional installation of heating sources, ICH and ECH, under improved vacuum conditions. The plasma parameters satisfy the scaling of confining potential versus thermal barrier potential and the Pastukhov scaling of particle confinement. The scaling of energy confinement time is also shown as a function of the particle confinement time during the period of potential plugging.

2. GAMMA 10 TANDEM MIRROR

GAMMA 10, shown in Fig. 1, is an axisymmetrized tandem mirror with a vacuum vessel 27 m in length and 150 m$^3$ in volume. Four 28 GHz, 100 kW gyrotrons are
used to generate mirror-confined hot electrons that produce a thermal barrier potential depression \(2\omega_{ce} \text{ ECH}\) and warm electrons for the positive potential peak \(\omega_{ce} \text{ ECH}\) that confines central cell ions. An RF oscillator with 200 kW used to be the only heating source for the confined plasmas. Since the 1988 IAEA Conference, two heating sources, a 28 GHz gyrotron and a second RF oscillator, have been newly installed to the central cell in order to attain high \(\beta\) plasma. Two 41 GHz gyrotrons are also installed to the anchor cells in order to produce hot electrons in addition to the NBI heated hot ions for MHD stabilization. A long-lasting good vacuum condition is maintained by a liquid helium cryogenic pumping system operating at a temperature of 3.5 K. The pumping speed of the system is \(3 \times 10^6 \text{ L} \cdot \text{s}^{-1}\). The base pressure is kept below \(1 \times 10^{-6} \text{ Pa}\), and the pressure rise is restricted to the range of \(\sim 10^{-5} \text{ Pa}\) during the plasma shot. At each end, radially and azimuthally segmented plates are installed which can be biased in order to control the potential distribution as well as be floated through a 1 MΩ.
resistor. In standard operation, the end plates are floated so that the net current to the ends is effectively zero.

Typical single-shot parameters are also shown in the lower part of Fig. 1. In different discharges, the maximum plugged density $n_c \gtrsim 1.1 \times 10^{19} \text{ m}^{-3}$ and ion temperature $T_i \gtrsim 5.6 \text{ keV}$ have been obtained by using the newly installed ICH system.

3. POTENTIAL FORMATION PHYSICS

We have confirmed the following basic and important properties of thermal barrier operation: i) the confining potential, $\phi_c = \phi_p - \phi_c$ increases effectively with increasing thermal barrier depth, $\phi_B = \phi_C - \phi_B$; ii) the plug electron temperature is greater than the central one.

Figure 2(a) shows the scaling of $\phi_C$ with $\phi_B$ obtained recently for a ratio of the plug to the central cell density $n_p/n_c = 0.4-0.5$. Strong plugging is achieved with the highest confining potential of 1.7 kV and a thermal barrier depth of 1.1 kV. The solid curve is predicted by Cohen’s strong ECH theory [2] for $n_p/n_c = 0.5$. The experimental results are in good agreement with the theoretical calculations. In the strong ECH model, the heating rate dominates over Coulomb collisions in the plug region, and the electron velocity distribution is distorted from the Maxwellian. In Fig. 2(b) is shown an example of an X-ray spectrum of the plug electrons measured with pulse height analyses (PHA) using a Si(Li) detector. The spectrum is well fitted to the one calculated from the $\phi_{pb} = \phi_c + \phi_B$ trapped plateau electrons, where $\phi_{pb} = 2.7 \text{ kV}$ is used. This plateau distribution is clearly different from the 60 keV Maxwellian in the thermal barrier plasma and also from the 0.2 keV Maxwellian in the central cell plasma [3]. These findings not only support the basic idea of the role of the thermal barrier but they also provide experimental bases to Cohen’s scaling, which is constructed on a model of velocity distribution shaped in a plateau under strong ECH. These results indicate a significant scaling basis for the future advancement of tandem mirrors.
FIG. 2. (a) Scaling law of $\phi_c$ versus $\phi_b$. Solid curve is predicted by Cohen's strong ECH theory. (b) X-ray spectrum observed for $\phi_{th} (= \phi_c + \phi_b)$ of 2.7 kV. The solid fitting curve representing the plateau distribution with cut-off energy at 2$\phi_{th}$ fits quite well with the observed spectrum, while comparison of the dashed curve derived for a Maxwellian distribution with observation shows a large discrepancy.
4. PARTICLE AND ENERGY CONFINEMENT

The large confining potential $\phi_C$ results in a drastic improvement of the axial confinement time following the Pastukhov scaling [4]. The Pastukhov scaling is confirmed up to $n_C T_p \sim 1 \times 10^{19}$ m$^{-3}$ s. The improvement of the confinement is more than $10^3$ times over that of a single mirror. Note that the radial transport consisting of ambipolar and nonambipolar losses has been identified to be smaller than the axial one under the conditions of strong plugging, partly due to elimination of the asymmetries of the magnetic field and heating systems and also due to a reduced level of fluctuations such as flute mode and drift wave mode [5].

The temporal evolution of plasma parameters shows that $\phi_C$ formation leads to an increase of the central cell density and of the temperature of the plugged ions. When $\phi_C$ disappears, the density and the temperature start to decay. These observations indicate that the energy confinement is improved in association with the increased particle confinement time. The ion energy confinement time $\tau_{Ei}$ can be estimated from the following energy balance equation,

$$\frac{d}{dt} W_i \cdot V_t = P_{hp} \cdot V_C - \frac{W_i \cdot V_t}{\tau_{Ei}} ,$$  

where $W_i$ is the energy density of the confined plasma, $P_{hp}$ is the slowing down power from the high energy ions produced by ICH, $V_t$ is the volume of the confined plasma between the two plugs, and $V_C$ is the volume of the central cell plasma. In Fig. 3, the values of $\tau_{Ei}$ obtained from the time evolution of the ion energy content are plotted as a function of particle confinement time $\tau_p$. The energy confinement time improved in association with improved particle confinement. The solid lines are calculated by the prediction formula of the Pastukhov theory:

$$\tau_{Eii} = \frac{3}{2} \cdot \frac{\tau_{pii}}{1 + \phi_C / T_{i1}} ,$$

for various values of $\phi_C / T_{i1}$. The values are in good agreement.
FIG. 3. Energy confinement time as a function of particle confinement time. Solid lines are predicted by the Pastukhov theory for various values of $\phi_c/T_i$.

FIG. 4. Radial profiles of plasma parameters with (○) and without (●) plugging. $n_H$: neutral density.
Figure 4 shows radial distribution of plasma parameters. The open circles represent parameters in the presence of $\Phi_C$ with $\Phi_B$, while the solid circles stand for those in the absence of the potentials. The profile of $\Phi_C$ is associated with the flattened electron distribution at the plug, and it is closely connected to the local absorption of ECH power. It is noted that the density of hydrogen neutral atoms as determined from calibrated Hα measurements reduces drastically in the core region ($r < 10$ cm) during the plugging. This indicates the presence of strong shielding mechanisms against penetration of neutrals into the hot core and the reduction of the charge exchange loss. An increase of electron temperature by 50% has been observed by adding the central cell ECH. This results in the reduction of neutral particles by ionization and the increase of the confining potential.

The fluctuation-induced transport due to the drift wave mode is also estimated for various conditions of potential formation. The fluctuation level measured by the far-forward scattering method is observed to depend sensitivity on radial profiles of the plasma potential. It has a maximum value when a slightly negative potential is formed, and decreases with an increase in the electric field $|E_r|$ [5]. In the presence of large $\Phi_C$, the normalized density fluctuation is suppressed in the low level of $n_C/n_C < 0.01$, and the associated radial diffusion evaluated from the quasilinear theory gives an energy confinement time of 0.6 s. This value is comparable to that evaluated from the energy balance equation.

5. SUMMARY

In summary, the physical mechanism of potential formation and the scaling law of plasma confinement have been studied in the tandem mirror GAMMA 10. Substantial progress has been made in expanding the plasma parameters during the past two years. The product of $n_C \cdot T_e \cdot T_{i\perp} = 1.3 \times 10^{19}$ m$^{-3}$ s keV is achieved in line with Pastukhov's scaling law. The energy confinement time increases with particle confinement time. These values are comparable to those evaluated from Pastukhov's relation and also from the experiment of fluctuation-induced transport ($n_e \sim 4 \times 10^{18}$ m$^{-3}$, $\tau_E \sim 0.6$ s, $T_{i\perp} \sim 5$ keV).
REFERENCES


DISCUSSION

H. IKEGAMI: You applied an additional microwave power to increase the tandem potential, and the energy confinement time was observed to increase drastically. As a result, plasma energy is also seen to increase.

What is the effective heating power to the plasma compared to the input microwave power?

S. MIYOSHI: The key heating system for potential formation is the plug microwave (ECH). The confinement time of the order of a millisecond is obtained with 50 kW of plug microwave power, the confinement time of approximately a second with 100 kW of power. Note that the confinement is improved by three orders of magnitude with only twice the input power, since GAMMA 10 confinement follows the Pastukhov scaling.

R. COHEN: You showed the X ray spectrum as an indication of the departure of the distribution function from Maxwellian. There was a curve passing more or less through the data points. Was that curve a fit to data or was it theoretical? If theoretical, which version of the strong ECRH theory was used — the earlier one described in Physics of Fluids Letters, or the later one by Cohen and Lodestro in the Varenna conference proceedings? They differed in the distribution of non-potential-trapped electrons.

T. CHO: The curve fitted to the plug X ray spectra was calculated from strong ECH theory. In addition to the plateau electrons (less than 5.4 keV = 2(\phi_h + \phi_t)), 60 keV Maxwellian electrons (2.5% of the total density) are observed. These are not potential trapped but plug/barrier mirror trapped electrons as reported in our barrier X ray paper\(^1\). In this high potential region, an extremely strong ECH effect may be expected, as shown in your Physics of Fluids Letters contribution.

STUDIES OF PLASMA HEATING BY RELATIVISTIC ELECTRON BEAMS IN LONG SOLENOIDS


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Abstract

STUDIES OF PLASMA HEATING BY RELATIVISTIC ELECTRON BEAMS IN LONG SOLENOIDS.

The interaction of relativistic electron beams (REBs) with plasma in a wide range of beam parameters is studied. On the GOL-M device it is shown that the efficiency of the REB-plasma interaction increases with magnetic field increase in accordance with the theoretical predictions. Direct evidence of strong Langmuir turbulence excitation is obtained. The turbulence level is in reasonable agreement with the modulations instability threshold. The experiments on the GOL-3 device are carried out with a new level of REB generation technique. The beam (\( E_b = 1 \text{ MeV}, I_b \leq 60 \text{ kA}, r_h < 5 \times 10^{-6} \text{s} \)), with a current density of 2 kA/cm\(^2\), is passed through a 7 m long chamber with \( 10^{14} \text{ cm}^{-3} \) plasma or neutral hydrogen density along a 4-6 T magnetic field. Macroscopically stable transport of the beam is observed. Maximum beam energy losses of about 25\% are achieved at a plasma density of \( 5 \times 10^{14} \text{ cm}^{-3} \). The average energy of the plasma electron-ion pair reaches 2 keV under these conditions. The dependence of the plasma energy content on the plasma density has been established. The experiments on 'two stage' plasma heating have demonstrated the possibility of energy transfer from a rare plasma (\( n_e = 5 \times 10^{14} \text{ cm}^{-3} \)) to a dense one. On the U-2 device the generation and transport of a beam with a ribbon cross-section are investigated with the purpose of carrying out the successive injection of several beams into a plasma. Recent experiments have been carried out with a ribbon REB with a 140 cm \( \times \) 4 cm cross-section and 150 kJ energy content.

Studies on the interaction of a high power relativistic electron beam (REB) with plasma have been in progress in Novosibirsk for several years. The aim of this programme is the heating of a dense plasma up to fusion temperatures in long solenoidal systems. Beams in a wide range of parameters are used in the experiments: from several kilojoules of energy content and several hundred nanoseconds' duration up to 100 kJ and several microseconds' pulse duration.
1. GOL-M EXPERIMENTS

Theory predicts that the main mechanism of REB-plasma interaction is an excitation of Langmuir turbulence [1]. Recent experiments [2] on the GOL-M device have given the first direct evidence confirming the theoretical predictions. This paper presents new experimental data concerning the frequency and wavenumber spectra of oscillations, the level of turbulence and the energy deposition behaviour in relation to magnetic field strength. The experiments have been carried out under the following conditions:

**GOL-M plasma parameters**
- Electron density \( n_e = 5 \times 10^{14} \) to \( 2.5 \times 10^{15} \) cm\(^{-3} \)
- Plasma column length \( L = 750 \) cm
- Homogeneous magnetic field strength \( 1 \) T < \( B_0 \) < 5 T
- Mirror ratio \( R = 1.75 \)

**REB parameters**
- Maximal energy of electrons \( E_b = 0.5-0.7 \) MeV
- Total beam current \( I_b \leq 70 \) kA
- Beam current density \( 1 \) kA/cm\(^2\) \( \leq j_b \leq 15 \) kA/cm\(^2\)
- Beam FWHM \( \tau_b = 100-120 \) ns

It follows from theoretical calculations (see, for instance, Ref. [3]) that a specific energy deposition of an REB into a plasma, \( q \), increases as \( q \sim B^{2/3} \) with the increase of the external magnetic field strength. The predicted dependence is valid for the case \( \theta_0^3 \ll z/l \). Here \( \theta_0 \) is the initial angular spread of the beam, \( z \) is the distance from the REB injection plane and \( l \) is a value determined by

\[
l = 0.1 \frac{c}{\omega_{pe}} \gamma^2 \frac{mc^2}{T_e} \left( \frac{\omega_{pe}}{\omega_{pe}} \right)^2
\]

The inequality is satisfied for distances less than a few tens of centimetres from the plane of injection. The experimental data on the specific energy content of REB heated plasma as a function of magnetic field strength in the vicinity of the beam injection plane are shown in Fig. 1. The solid curve corresponds to the diamagnetism growing as \( B^{2/3} \). In spite of the experimental data spread, there are no contradictions between the theoretical calculations and the experimental results presented in Fig. 1.

A laser scattering method was applied for a detailed plasma turbulence study. A pulse CO\(_2\) laser with a wavelength \( \lambda = 10.29 \) \( \mu \)m was used as a source of radiation. The energy of the pulse was 10 J and the pulse duration could be varied between 70 ns and 2 \( \times 10^{-6} \) s. The CO\(_2\) laser wavelength selected made it possible to reduce very effectively stray radiation at the laser frequency. The strong suppression of stray light became possible owing the coincidence of a single absorption
FIG. 1. REB heated plasma energy content (per unit of length) versus magnetic field strength $B$ in the vicinity of the anode foil. $j_b = 2.3$ kA/cm$^2$, $n_e = 6 \times 10^{14}$ cm$^{-3}$. The angular spread of the beam $\theta^2 = 20^\circ$ and the cross-section of the beam $S_b = 30$ cm$^2$. Solid curve corresponds to $nTS \sim B^{3/2}$.

line of ammonia with the wavelength of transition R14 of CO$_2$ ($\lambda = 10.29$ $\mu$m). An absorption cell with longitudinal dimension $\ell = 40$ cm and NH$_3$ pressure $p = 2.5 \times 10^4$ Pa provided the suppression of laser radiation down to the level $I/I_0 = 10^{-13}$. With the aid of the described technique the scattered radiation was observed in the angle interval $1 \times 10^{-3} < \alpha < 2 \times 10^{-1}$ rad to the laser beam direction. The most important data gathered from CO$_2$ scattering are the following.

(a) The frequency shift of a satellite observed in the scattered radiation coincides exactly with a value $\Delta \nu = \nu_pe/2\pi$. The scattered radiation power exceeds the equilibrium level by 8 orders of magnitude. These facts confirm the theoretical predictions on the excitation of strong Langmuir turbulence by high current beams.

(b) The duration of the observed scattered radiation corresponds to the duration of the REB current exciting oscillations, as it should be in the case of Langmuir turbulence excitation.

(c) The level of turbulence estimated both from scattering experiments and from measurements of the temporal behaviour of the main body of the plasma electron non-equilibrium distribution function [3] is in reasonable agreement with the modulational instability threshold ($W/n_eT_e = (\nu_p/\omega_p)^2 = 0.03-0.1$).

The angular dependence of the scattered radiation without spectral dispersion makes it possible to study the distribution of scattered radiation power, $W_\gamma$, over the wavenumbers. To extend the range of the studied wavenumbers, in some experiments the laser radiation was directed simultaneously at angles $\gamma = 90^\circ$ and $11^\circ$ to the direction of the electron beam propagation. Such a system provides a possibility to
detect the radiation scattered by plasma oscillations with wave vectors oriented at $0^\circ < \beta < 180^\circ$ to the direction of beam propagation. Simultaneous measurements of the scattered radiation power at five different angles have been carried out to study the wavenumber spectrum of oscillations $W_\kappa$ with the longitudinal component of the wave vector $\vec{k}$ close to the resonant one ($k_\parallel = \omega_{pe}/v_b = \omega_{pe}/c$) and with different transverse components. The data obtained in these experiments are shown in Fig. 2.

One can see that the distribution of the spectral power density of oscillations $W_\kappa$ is located between $W_\kappa \sim 1/\kappa^3$ and $W_\kappa \sim 1/\kappa^4$, where $\kappa = k_\perp/(\omega_{pe}/c)$. In other words, the spectral power density of the oscillations decreases with the increase of $\kappa$, the normalized wavenumber value, faster than the growth rate of a two stream instability ($\Gamma_b \sim 1/\kappa^2$).

One of the unexpected results is that enhanced scattered radiation due to the oscillations coming from the REB cross-section was observed. The group velocity of Langmuir waves in a magnetic field $v_{group} = c(\omega_{Be}/\omega_{pe})^2 \sim 10^8-10^9$ cm/s for $k \geq \omega_{pe}/c$ permits the turbulent oscillations to penetrate to a sufficiently large distance in the outer plasma with a favourable plasma profile during the REB injection.

2. EXPERIMENTS AT GOL-3

Until recently nanosecond beams were used in the experiments on plasma heating [4]. Now a technique of microsecond beam generation with an energy content of about 100 kJ [5] has been developed. It has provided a basis for the construction
of the GOL-3 device [6], intended for the study of dense plasma heating in a multimirror system. A 'two stage' scheme of plasma heating is assumed in these experiments. This means that the electron beam deposits a large portion of its energy in the central section of the long solenoid with a plasma density of about $10^{15}$ cm$^{-3}$. Then this energy is transferred by binary collisions to the adjacent regions with a dense ($n_e \sim 10^{17}$ cm$^{-3}$) plasma.

The first stage of the GOL-3 device has recently been put into operation (Fig. 3). It consists of the U-3 generator of the microsecond REB [7], a vacuum plasma chamber inside the solenoid, 10 MJ capacitor storage for magnetic field creation and control and diagnostic systems. The electron beam is generated in the diode with a cold emission cathode and then is injected into the plasma chamber after 20-fold magnetic compression. The beam diameter in the plasma is 6 cm (the current density is up to 2 kA/cm$^2$) and its total energy content is about 100 kJ. An initial hydrogen plasma with a density of $10^{14}$-$10^{15}$ cm$^{-3}$ is produced in the chamber by a high voltage discharge [8]. The electron beam can also be injected into neutral hydrogen at the same densities. No macroscopic instabilities have been observed during the passage of the microsecond beam through the 7 m long solenoid with a 4-6 T magnetic field either in the preliminary plasma or in the neutral gas. In the second case gas ionization by the electron beam has been produced during a time interval of less than 1 microsecond. Figure 4(a) shows the beam energy losses $\Delta Q_b$ measured by a calorimeter versus the initial plasma density $n_0$. One can see that the percentage of beam energy losses $\Delta Q_b/Q_b$ is small enough at the density $5 \times 10^{15}$ cm$^{-3}$ and reaches 15-25% in the vicinity of $n_0 = 5 \times 10^{14}$ cm$^{-3}$ (this value of the losses has been obtained by two methods: measurements by the calorimeter and an analysis of the electron energy of the beam). The most probable explanation for the experimentally observed effect of the beam electrons' collective deceleration is an excitation of the two stream instability. It should be noted that similar results were obtained in the experiments on the U-1 device [5]. In this case a 100 kJ electron beam was injected both into a $10^{12}$ to $3 \times 10^{16}$ cm$^{-3}$ neutral gas and into preformed He plasma in the 1 m long, 4 T magnetic field solenoid.

Plasma energy content at the GOL-3 device obtained by means of diamagnetic measurements is shown in Fig. 4(b) as a function of the plasma density. The most efficient energy transfer from the beam into the plasma occurred in the density range $n = (2-5) \times 10^{14}$ cm$^{-3}$. The specific energy content of the plasma with the density $5 \times 10^{14}$ cm$^{-3}$ is equal to $3 \times 10^{19}$ eV/cm. For these conditions the average energy of an electron-ion pair reaches 2 keV. The losses of the beam energy exceed significantly the final plasma energy content. The result can be explained by the existence of a considerable flux of hot plasma electrons leaving the trap during the heating process, because no attempts have been made to provide efficient axial plasma confinement. The time behaviour of the diamagnetism indicates the non-equilibrium nature of the distribution function of the plasma electrons.

To test the two stage scheme of dense plasma heating, special experiments were carried out. A thin organic film used as an absorbing target was placed across the
FIG. 3. Schematic of the first stage of the GOL-3 device.
chamber filled with hydrogen. The plasma diamagnetism in the vicinity of the film was 3–8 times higher than without the film. This result shows that the film material is ionized and heated by fast electrons produced during the plasma heating by REB.

3. EXPERIMENTS AT U-2

To obtain a dense plasma with a temperature close to thermonuclear it is necessary to increase the total energy of the injected beam. One means for doing this consists of using a successive injection of several microsecond beams. The most suitable beams in this respect are those with a ribbon cross-section. The U-2 generator was constructed for experimental studies on ribbon beam physics [5]. The generation of the ribbon beam, its transport and the transformation of the cross-section have been investigated in experiments at a beam cross-section of 20 cm × 2 cm and energy content of about 50 kJ. The diode unit intended for the generation of the full scale ribbon REB with a 140 cm × 4 cm cross-section was tested experimentally. A 1 MeV, 10 μs ribbon beam with a total energy content of about 150 kJ has been obtained.
4. CONCLUSIONS

(1) Direct experimental evidence of excitation of strong Langmuir turbulence has been obtained. The level of turbulence is in reasonable agreement with the modulational instability threshold.

(2) The first stage of the GOL-3 device, the purpose of which is to investigate plasma heating by a 100 kJ, microsecond REB, has been constructed and put into operation.

(3) The macroscopic stability of the microsecond beam passing through a 7 m long plasma in a strong magnetic field is shown experimentally.

(4) Beam energy losses up to 25% have been observed.

(5) Preliminary experiments have proved the feasibility of a two stage scheme of dense plasma heating.

(6) A ribbon beam with a total energy content of about 150 kJ has been obtained as the first step in the programme of successive beam injection.

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ELECTRON CYCLOTRON CURRENT DRIVE AND WAVE ABSORPTION EXPERIMENTS IN THE W VII-AS STELLARATOR

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Abstract

ELECTRON CYCLOTRON CURRENT DRIVE AND WAVE ABSORPTION EXPERIMENTS IN THE W VII-AS STELLARATOR.

Experiments on non-inductive current drive by electromagnetic waves in the vicinity of the first and second harmonic of the electron cyclotron frequency (ECCD) were performed at the W VII-AS stellarator with up to 1 MW RF power at 70 GHz in long pulse operation (< 1.5 s). The single pass absorption as an important input quantity for current drive investigations was directly measured for first harmonic O-mode and second harmonic O- and X-mode operation, respectively, in a wide plasma parameter range. A good agreement with a 3-D ray tracing model was found. — ECCD was investigated by (a) a toroidal launch angle variation of the microwave beams while the total net current was kept close to zero (I_p < 0.2 kA) by feedback control of the OH transformer. The change of the required loop voltage with respect to perpendicular launch (no ECCD) was measured as a function of the launch angle; (b) the adjustment of the launch angle of the microwave beams to balance the bootstrap current without making use of the OH transformer (counter current drive). Here, the EC wave driven current is measured in units of the bootstrap current; (c) a perturbation experiment at up/down shifted frequencies, where 0.2 MW were launched at a fixed toroidal angle and the EC resonance layer was shifted out of the confinement region (B_0 variation). The loop voltage change required for compensation of the

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EC driven current was measured as a function of the magnetic induction. — The parameter dependence on the launch angle, the electron temperature, the plasma density and the microwave power for all types of ECCD experiments is in good agreement with a linear theoretical model, which takes into account quasi-linear and trapped particle effects. The results are confirmed by Fokker–Planck calculations.

1. INTRODUCTION

Experiments on current drive by electromagnetic waves in the vicinity of the electron cyclotron frequency and the comparison with theory attract increasing interest for both tokamak as well as stellarator research to provide a reliable database for electron cyclotron current drive (ECCD) scenarios in next step devices such as NET and W VII-X. The high localization of the driven currents and the capability to penetrate the plasma centre even in large machines together with the technical advantage of a simple remote launching structure may overcompensate the disadvantage of a small ECCD efficiency (compared to lower hybrid current drive) for particular applications such as MHD mode control, current profile shaping or, especially for stellarators, bootstrap current compensation. Basic experiments were performed at the W VII-AS stellarator, where the small EC driven currents are not masked by large inductively driven currents as in tokamaks. The control of a pressure driven net current was experimentally demonstrated and is mandatory in low vacuum shear configurations such as W VII-AS to maintain good confinement properties [1].

The theoretical treatment of ECCD would require a Fokker–Planck solution in full phase space, which is out of scope. In a first approach, we compare the experimental results with a theoretical model, which in the simplest version neglects trapped particle effects [2]. In a second step, we have analysed the sensitivity of this model with respect to simplified assumptions on trapped particle and quasi-linear effects [3, 4]. The experimental investigation of the single pass absorption of a microwave beam in the electron cyclotron frequency range is of crucial importance as an input quantity for CD calculations. Measurements of the single pass absorption are compared to ray tracing calculations.

2. WAVE ABSORPTION

Plasma buildup and heating are achieved at W VII-AS with a 1 MW, 70 GHz ECRH system. The resonant magnetic field and the cut-off densities are 2.5 T and \( n_{e,\text{crit}} = 6.2 \times 10^{19} \text{ m}^{-3} \) for the first harmonic and 1.25 T and \( n_{e,\text{crit}} = 3.1 \times 10^{19} \text{ m}^{-3} \) for the second harmonic experiments, respectively. The single pass absorption was directly measured for perpendicular launch by a 35 channel pick-up wave guide array mounted opposite to the launching mirrors at the inner vacuum vessel wall. In the investigated electron temperature range of \( 0.6 \text{ keV} < T_{e0} < 1.8 \text{ keV} \) total absorption of the waves in a single transit through the plasma was
found in all cases with second harmonic X-mode launch, whereas for the second harmonic O-mode the plasma is optically thin (single pass absorption <7%) in agreement with theory. A deviation from total absorption is expected for second harmonic X-mode at a central electron temperature of $T_{e0} < 200$ eV. For the second harmonic O-mode, $T_{e0} > 4$ keV is required to obtain considerable absorption.

For first harmonic O-mode heating, however, the single pass absorption is sensitive to the electron temperature and density in the experimentally accessible parameter range of $0.6$ keV < $T_{e0}$ < $2.7$ keV and $1 \times 10^{19}$ m$^{-3}$ < $n_{e0}$ < $5 \times 10^{19}$ m$^{-3}$. The single pass absorption for a density scan at a fixed central electron temperature of 1.5 keV and a temperature scan at a fixed electron density of 2.5 $\times$ 10$^{19}$ m$^{-3}$ was measured ranging from 60% ± 3% up to 95% ± 5%. A comparison of the measurements with a 3-D ray tracing model based on measured spatial profiles of $T_e$ and $n_e$ (Thomson scattering, ECE) shows an excellent agreement within the experimental error bars.

3. ELECTRON CYCLOTRON CURRENT DRIVE EXPERIMENTS

The ECCD experiments were performed at the W VII-AS stellarator with up to 0.7 MW power in long pulse operation (< 1.5 s). Up to four linearly polarized RF beams were launched at both the first harmonic ordinary and the second harmonic extraordinary wave polarization, respectively, from the low field side. The RF beams were directed towards the plasma at arbitrary toroidal launch angles by a set of independently movable focusing mirrors mounted inside the vacuum chamber [5].

In a first experiment, the toroidal launch angle of the RF beams was varied while the total net plasma current was kept close to zero (I < 0.2 kA) by feedback control with the OH transformer. The measured change of the required loop voltage $\Delta U$ with respect to perpendicular launch (no ECCD) is plotted in Fig. 1(a) as a function of the launch angle at the resonance layer for constant input power of 0.35 MW in first harmonic O-mode. The dots refer to experiments with one beam (0.17 MW) at perpendicular launch (no ECCD) and one beam (0.17 MW) at oblique launch angles. The crosses refer to a variation of the launch angle of both beams. The right hand wing of the curve with positive loop voltage increment corresponds to a situation where the bootstrap current has the same direction as the EC driven current. The left hand wing of the curve corresponds to ECCD in counterdirection to the bootstrap current. Equivalent experiments at $B_0 = 1.25$ T show similar results [6]. The change of the internal current distribution results in a change of the total stored plasma energy of about 20%. The influence on the confinement is explained by magnetic field configuration and internal shear effects [7]. The ECCD efficiency is evaluated in a 3-D ray tracing code by means of the adjoint approach [8]. In the limit of low collisionality, the evaluation of the local efficiencies was generalized to include trapped particle effects in W VII-AS magnetic configurations [4]. A comparison of
FIG. 1(a). Loop voltage increment $\Delta U$ as a function of the launch angle for 0.17 MW (dots) and 0.35 MW (crosses) ECCD power. The EC driven current is balanced by inductive current drive (OH transformer feedback). The total input power for both cases is 0.35 MW.

FIG. 1(b). EC current drive modelling based on the measured $n_e$ and $T_e$ profiles for the same discharges at 0.17 MW (dashed curve) and 0.35 MW (solid curve).
the experimental data with this linear model [3] is shown in Fig. 1(b). The calculations are based on the measured $n_c$ profiles and the measured change of the $T_e$ profiles for each launch angle. The experimental findings are well described by the model. In particular, the launch angle for maximum current drive and the linear increase of ECCD with RF power agree well. The absorption layer is shifted radially outward with increasing oblique launch. The current drive efficiency decreases for large launch angles (far in the Doppler regime) because the absorption depends sensitively on $T_e$ and the absorption layer is then within the $T_e$ gradient region. Furthermore, the number of trapped particles increases and the mismatch of the incident linearly polarized waves is no longer negligible [6]. It should be noted that the measured current inversion at large launch angles is also found in the calculations and is related to the particular W VII-AS magnetic field topology. The absolute value of the EC driven current at maximum launch angle derived from the experiment is about 3–5 kA, which is in satisfactory agreement within the simplified assumptions of the theory and the experimental error bars.

In a second experiment, the launch angle of up to four microwave beams (0.7 MW) was adjusted to balance the bootstrap current without making use of the OH transformer (counter-current drive). The net plasma current was kept below ±0.3 kA under all plasma conditions. The EC wave driven current is then measured under steady state conditions in units of the bootstrap current, which is varied by a variation of the plasma parameters with increasing microwave power. The dominant electron component of the bootstrap current ($T_e \gg T_i$) was calculated by the DKES code for the measured spatial profiles of $n_e$ and $T_e$. Scanning the RF power from 0.17–0.7 MW, the electron temperature from 0.8 keV < $T_e$ < 1.9 keV and the electron density from $1.1 \times 10^{19} \text{ m}^{-3} < n_{e0} < 2.8 \times 10^{19} \text{ m}^{-3}$, the bootstrap current varies from 0.8 kA < $I_{\text{boot}}$ < 4.3 kA (second harmonic X-mode). The current drive model overestimates the EC driven current by typically a factor of 3 if trapped particles are neglected. If trapped particles are taken into account, this discrepancy almost vanishes within the experimental accuracy and the uncertainties to derive the bootstrap current from the measured plasma profiles.

In a third experiment, ECCD in the up/down shifted regime was investigated by making use of a peculiarity of the magnetic field configuration of W VII-AS, i.e. wave launching at a poloidal plane with an almost vanishing magnetic field gradient. Under such conditions, a sufficiently high single pass absorption is obtained even at moderate electron temperatures of about 2 keV. Here, one microwave beam (0.2 MW) is launched at a fixed angle of 11° with respect to perpendicular incidence. A magnetic field variation from 2.4 up to 2.6 T shifts the EC resonance completely out of the confinement region in this poloidal plane as seen in Fig. 2(a). The target plasma is maintained by 0.7 MW at perpendicular launch (no ECCD) in a poloidal plane with a strong tokamak-like magnetic field gradient, where the EC resonance layer remains well within the confinement region for the given magnetic field variation. The calculated spatial current density distribution is given in Fig. 2(b) for the three cases of $B_0$ = 2.4 T (up-shifted), 2.5 T (resonant), and 2.6 T (down-shifted).
FIG. 2(a). Position of the resonance layer (dotted line) in a poloidal cross-section with small magnetic field gradient at $B_0 = 2.4\,\text{T}$ (top), $B_0 = 2.5\,\text{T}$ (middle) and $B_0 = 2.6\,\text{T}$ (bottom). The solid lines indicate $|B| = \text{const}$ contours, the dashed lines give the nested flux surfaces.
FIG. 2(b). EC driven current density distribution from 3-D ray tracing calculations for 0.1 MW microwave power at the upshifted resonance (top), the resonant case (middle) and the downshift case (bottom).
For the resonant case (Fig. 2(b), middle) an almost zero net current is found because the vanishing magnetic field gradient and the corresponding broad power deposition profile create counterstreaming currents on both the low and the high field sides of the resonance layer. The current inverses the sign while varying the magnetic field from an upshifted scenario (Fig. 2(a) and (b), top) to a downshifted scenario (Fig. 2(a), and (b), bottom). The measured loop voltage increment required to balance the EC driven current is given in Fig. 3(a) for the full magnetic field scan. The change of the sign as well as the transition through zero loop voltage increment at the resonant magnetic field is in agreement with theory as shown in Fig. 3(b), where the EC driven current is normalized to the launched microwave power. We assume, however, that only the first transit absorption, which is given in Fig. 4(a) as a function of the magnetic induction, contributes to current drive. The relevant EC-current drive efficiency is then obtained by normalizing the total current to the power absorbed in a single pass, which is shown in Fig. 4(b). In all calculations, trapped particle effects are taken into account. An improved current drive efficiency as compared to the two experiments mentioned first is clearly deduced.

FIG. 3(a). Loop voltage increment ΔU_{loop} required to balance the EC driven current as a function of the magnetic induction on axis. The left hand part (B₀ = 2.4 T) corresponds to an upshifted, the right hand part (B₀ = 2.6 T) to a downshifted scenario.
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FIG. 3(b). EC current drive modelling of the experiment. The current drive efficiency \( \eta_{\text{lin}} \) is defined as the driven current \( I_{\text{CD}} \) normalized to the launched power \( P_{\text{in}} \).

![Graph](image)

2.4 2.5 2.6

FIG. 4(a). Single pass absorption \( P_{\text{obs}} \) normalized to launched power \( P_{\text{in}} \) from ray tracing calculation for the same discharges.
FIG. 4(b). EC current drive efficiency $\eta_{\text{abs}}$, with the driven current $I_{\text{CD}}$ normalized to the single pass absorption $P_{\text{abs}}$, for the same discharges.

4. CONCLUSIONS

Fundamental experiments on wave absorption and ECCD were performed in the W 7-AS stellarator. The single pass absorption of the microwaves was measured at perpendicular launch for first harmonic O-mode and second harmonic O- and X-mode launch, respectively. An excellent agreement with ray tracing calculations was found. The parameter dependence of ECCD on the launch angle of the incident waves, the electron temperature, the electron density and the microwave power was investigated for both first harmonic O-mode and second harmonic X-mode operation. Perturbation experiments at up-down shifted resonance for first harmonic ECCD were performed by shifting the resonance layer across the confining region of the plasma. The experiments are in good agreement with a linear theoretical model and Fokker-Planck calculations if trapped particles are taken into account.

REFERENCES

DISCUSSION

J.L. JOHNSON: The bootstrap and ECH driven currents, and the Ohmic currents for compensation, all show different radial dependences. Thus there will be a local current. How great an effect will this have on the rotational transform and shear, as this is an effect which could affect the MHD properties?

V. ERCKMANN: Numerical calculations of the radial distributions of the three current contributions show a modification of the internal rotational transform profile of up to $\Delta \approx 0.3$. Localized high shear is introduced because of the narrow EC driven current profile.

L.M. KOVRIZHNYKH: Did you assume that the distribution function was Maxwellian when calculating the absorption ratio?

V. ERCKMANN: The single pass absorption is determined theoretically by ray tracing calculations where a Maxwellian electron distribution function is assumed in the dielectric tensor.
STABILIZATION OF THE INTERCHANGE MODES BY A MAGNETIC AXIS SHIFT AND A TOROIDAL FIELD IN HELIOTRON E, AND A NEW LOW-n MODE STABILITY ANALYSIS

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Abstract

STABILIZATION OF THE INTERCHANGE MODES BY A MAGNETIC AXIS SHIFT AND A TOROIDAL FIELD IN HELIOTRON E, AND A NEW LOW-n MODE STABILITY ANALYSIS.

Pressure driven MHD instabilities in Heliotron E were studied by shifting the vacuum magnetic axis position outward ($\Delta_v > 0$) or inward ($\Delta_v < 0$), and/or applying an additional toroidal field additively or subtractively. The global behaviour of the experimental results is consistent with theoretical studies using the ideal and the resistive MHD model based on the stellarator expansion approximation. The pressure profile was also changed systematically by the above mentioned control of the vertical magnetic field and/or the toroidal field. Stability improvement was obtained for the $\beta(0) \lesssim 1\%$ regime in the case where the additive toroidal field was ($3-8\%$ of the toroidal component from the helical coils for $-2\ cm \leq \Delta_v \leq 0\ cm$. It is believed that the pressure profile was unintentionally adjusted to improve stability; however, this improvement was not clear for $\beta(0) \gtrsim (2-3)\%$. A new type of ideal low-n stability code

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was developed which relies on an averaging procedure in the toroidal direction of a three-dimensional finite beta MHD equilibrium. This approach seems to include realistic finite beta configuration in the stability calculation better than the usual stellarator expansion method. For the Heliotron E the difference between the two results is fairly small.

1. INTRODUCTION

Previous confinement studies on Heliotron E have been extended by changing the vacuum magnetic configuration to stabilize the ideal and resistive interchange modes. Two methods have been proposed to improve the beta limit of Heliotron E. One is to produce a sufficiently deep magnetic well by shifting the magnetic axis outward[1]. The other is to apply an additional toroidal magnetic field for shifting the $i = 1$ surface resonant with the $(m,n) = (1,1)$ mode into an outside region where shear stabilization is expected[2]. High beta experiments were tried again to study these theoretical predictions and to improve the beta limit considerably by using both additional vertical field coils and nineteen toroidal field coils. Finite beta plasmas were produced by injecting NBI (maximum power = 2.5 MW) into the target ECRH plasmas at $B_{tot} = B_{ho} + B_{to} = 1.9 \, T$ or $B_{tot} = 0.94 \, T$, where $B_{ho}$ and $B_{to}$ are toroidal fields at the centre of the vacuum chamber produced by the helical coils and the toroidal coils, respectively.

The main purpose of this paper is to compare the results of the new high beta experiment with the recent theoretical stability analyses including the realistic vacuum magnetic configuration of Heliotron E. For the ideal MHD stability we mainly use the STEP code modified to include the toroidal corrections correctly[3,4]. We have already compared the STEP code results for low-n mode stability with the FAR code[5] and for the Mercier criterion with the VMEC code[6]. For the resistive modes an initial value solver (RESORM code) was developed for the equilibrium given by the STEP code[7]. It is well known that the beta limit determined by the low-n ideal interchange mode is somewhat higher than that given by the Mercier criterion[8,9]. The resistive interchange mode is always unstable in the magnetic hill region, which is inevitable in the heliotron/torsatron configuration. On the basis of a comparison between the data and the theoretical studies, we find that the experimental beta value exceeds the Mercier criterion and is close to the low-n ideal interchange limit when the Mercier limit is low or $\beta_n(0) \lesssim 1\%$. However, when $\beta_n(0) > 1\%$, the sawtooth oscillations correlate well with the low-n resistive interchange mode growth rate, $\gamma_g$. Roughly, $\gamma_g \approx 0.01$ at $S = 10^6$, normalized with respect to the poloidal Alfvén transit time, provides a good measure for
the appearance of sawtooth oscillations. Here, $S$ is the magnetic Reynolds number. It is noted that $\gamma_g \approx 0.01$ is obtained always above the Mercier limit in Heliotron E. This conjecture is also consistent with the Heliotron DR data[7].

In Section 2 we discuss results of the magnetic axis control experiment and compare them with the theoretical stability analysis. In Section 3 results of the additional toroidal field experiment are shown and compared with the theoretical predictions based on ideal and resistive MHD. Here, the parameter $\alpha^* \equiv B_{to}/B_{ho}$ is changed from $\alpha^* = 0.1$ to $\alpha^* = -0.1$.

The recent development of three-dimensional MHD codes has made MHD equilibrium calculations for heliotron/torsatron efficient. Two-dimensional numerical codes, such as the STEP and FAR codes, are efficient in studying the linear MHD stability properties of heliotron/torsatron. It is natural to combine the two types of code to study the global stability in three-dimensional configurations[4]. We have coupled the VMEC code for MHD equilibrium with the STEP code for low-n ideal mode stability, introducing numerical averaging procedures to transfer the equilibrium data to the stability code. In Section 4 results from this new VMEC-STEP code are presented and compared with the STEP code for Heliotron E.

2. MAGNETIC AXIS CONTROL EXPERIMENT

In Heliotron E the magnetic axis position can be controlled by changing the vertical field. MHD stability theory predicts that the magnetic well is deepened by an outward shift, which improves the stability beta limit. On the other hand, an inward shift of the magnetic axis degrades the stability. Figure 1 shows the MHD stability diagram in the $\beta(0) - \alpha^*$ plane for $n = 1$ and $n = 3$ global modes and the Mercier mode at the flux surfaces resonant with these modes for $\Delta_v = -2 \text{ cm}$, $\Delta_v = 0 \text{ cm}$ and $\Delta_v = 2 \text{ cm}$. A pressure profile $P \propto (1 - \psi)^2$ is assumed, where $\psi$ is a poloidal flux function. The points for the low-n mode stability show $\gamma_I = 0.01$ and $\gamma_g = 0.01$ at $S = 10^6$, where $\gamma_I$ and $\gamma_g$ are growth rates of the ideal and resistive modes normalized by the poloidal Alfvén time, respectively. If we consider $\gamma_I = 0$ for the low-n ideal mode, it should coincide with the Mercier limit.

Figure 2 (a) shows the oscillations on the soft X-ray signal, $I_{sx}$, for several cases with an inward magnetic axis shift. Here, discharges with $\beta(0) < 1\%$ or $B_{tot} = 1.9 \ T$ were selected. For the standard case with $\Delta_v = 0$, the sawtooth-like oscillations were never observed for $\beta(0) < 1\%$. At $\Delta_v = -2 \text{ cm}$ the sawtooth amplitude is $\Delta I_{sx}/I_{sx} \sim 60\%$, and at $\Delta_v = -4 \text{ cm}$ $\Delta I_{sx}/I_{sx} \lesssim 10\%$; however, the sawtooth repetition time reduces to $(2 \sim 3) \text{ ms}$ compared to $\sim 10 \text{ ms}$ at $\Delta_v = -2 \text{ cm}$. For the
outward shift case of $0 < \Delta_y \lesssim 4$ cm, there were no detectable oscillations on the soft X-ray for $\beta(0) < 1\%$. These results are consistent with the stability diagram in Fig.1. The large sawtooth at $\Delta_y = -2$ cm occurred near the $\iota = 1/2$ surface (see Fig.2(b)). This is a new phenomenon observed with an inward axis shift, and is expected from Fig.1. Post-cursor oscillations with $(m,n) = (2,1)$ were observed on the soft X-ray signal, which means that the $m = 2$ magnetic islands survive after the sawtooth crash. This shows the role of the resistive interchange mode since resistive reconnection is required to produce the magnetic islands. Figure 2(b) shows...
FIG. 2. (a) Soft X-ray traces for various $\Delta_0$ cases at $\alpha^* = B_0/B_{m0} = 0$. Both central and edge chords are shown. (b) Phase inversion radius of soft X-ray signal at the sawtooth crash as a function of $\Delta_v$ at $\alpha^* = 0$. Circles and triangles show low ($\beta(0) < 1\%$) and high ($1\% < \beta(0) < 3\%$) beta case, respectively. Resonant surfaces and separation lines between magnetic well and magnetic hill are shown.

the phase inversion radius of the sawtooth as a function of $\Delta_v$. Circles correspond to gas puffed discharges with $\beta(0) < 1\%$, and triangles belong to $1\% < \beta(0) < 3\%$ obtained by pellet injection. Resonant surfaces with $\iota = 1/2$, 2/3 and 1 (or $q=2$, 3/2 and 1) at $\beta(0) = 0\%$ and separation lines between the magnetic well and the magnetic hill at $\beta(0) = 0\%$, 2% and 3% are shown. Figure 2(b) clearly shows that the phase inversion radius of the sawtooth for the higher beta plasmas appears in the outer region where the $\iota = 2/3$ and $\iota = 1$ surfaces exist. One explanation is that the resistive interchange modes at the $\iota = 1/2$ surface, which trigger the sawtooth for $\beta(0) < 1\%$ and $\Delta_v \approx -2\, \text{cm}$, are stabilized by the expansion of the magnetic well region and the pressure profile broadening associated with the increase of $\beta(0)$.

3. STABILITY PROPERTIES WITH ADDITIONAL TOROIDAL FIELD

When a toroidal field is added to decrease the rotational transform or $\alpha^* > 0$, the outermost flux surface expands to increase the average plasma radius, $\bar{a}$. For $\alpha^* > 0$, if $\bar{a}$ is held fixed for a given peaked pressure profile,
FIG. 3. Time evolution of line averaged density (left) and soft X-rays (right) along the central chord for various $\alpha^* = R_{o0}B_{ho}$ at $\Delta_v = -2$ cm. One division corresponds to $15$ ms. Here, all cases belong to $\beta(0) < 1\%$.

FIG. 4. (a) Peaking factors given by $n_e(0)/\langle n_e \rangle$ shown as functions of $\alpha^*$ for $\beta(0) < 1\%$ plasmas. In the dotted regions, sawtooth type oscillations were observed. (b) Magnetic fluctuations showing coherent $(m, n) = (3, 2)$ mode as functions of $\alpha^*$ for plasmas with $1\% < \beta(0) < 3\%$. 
the dangerous resonant surface with \( \iota = 1 \) moves outwards or the pressure gradient at the \( \iota = 1 \) surface decreases. In this situation the ideal MHD stability for the \( n = 1 \) mode improves significantly [2]. In the Heliotron E experiment no material limiter was used to fix \( \bar{a} \). The \( \gamma_I = 0.01 \) and \( \gamma_B = 0.01 \) points shown in the \( \beta(0) - \alpha^* \) plane in Fig. 1 are with the variation of \( \bar{a} \). It is noted that the toroidal field with \( \alpha^* < 0 \) increases the rotational transform and decreases \( \bar{a} \). For \( \alpha^* < 0 \) the stability degrades compared to \( \alpha^* = 0 \) (standard case) and for \( \alpha^* > 0 \) the stabilizing effect is milder than the results in Ref. [2]. Figure 3 shows experimental results for various \( \alpha^* \) for \( \Delta \nu = -2 \text{ cm} \) and \( \beta(0) < 1\% \). As discussed in Section 2, the sawtooth oscillations appear even for \( \beta(0) < 1\% \) under \( \alpha^* = 0 \) and \( \Delta \nu < 0 \). The sawtooth amplitude is enhanced by \( \alpha^* < 0 \). We note that, when \( 0.03 \leq \alpha^* \leq 0.08 \), the sawtooth oscillations were suppressed. For \( \alpha^* > 0.1 \), however, they appeared again. An other important characteristic of the \( \alpha^* \) effect is that the peaking factor of the density profile is changed as shown in Fig. 4(a). This factor \( n_e(0)/< n_e > \) can be controlled by about a factor of two in the range of \( 0 \leq \alpha^* \leq 0.1 \). However, the electron temperature profile was almost the same for \( -0.1 \leq \alpha^* \leq 0.1 \). From the STEP and RESORM code results, when the pressure profile becomes broad, the stability beta limit increases. For \( \alpha^* \gtrsim 0.1 \), the plasma radius expands excessively and the plasma-wall interaction becomes strong, which was shown by spectroscopic measurements of impurity lines. This may suggest that the temperature profile at \( \alpha^* = 0.1 \) becomes relatively sharp compared to that in \( 0.03 \leq \alpha^* \leq 0.08 \). This means that the pressure profile becomes peaked again, since \( n_e(0)/< n_e > \) is almost constant for \( \alpha^* \gtrsim 0.08 \). From the stability diagrams shown in Fig. 1, \( \alpha^* > 0 \) and \( \Delta \nu > 0 \) is sufficient to obtain \( \beta(0) \gtrsim 3\% \) if \( \gamma_I = 0.01 \). However, for \( \Delta \nu = -2 \text{ cm} \), \( \beta(0) \sim 3\% \) may not be expected. An experiment to improve the beta limit at \( \Delta \nu = 2 \text{ cm} \) was tried but degradation of confinement limited the beta value to \( \beta(0) \lesssim (1 \sim 2)\% \).

Magnetic fluctuations were measured by using magnetic probes inside the vacuum chamber. Usually, \( B_\theta \) with \( (m,n) = (1,1) \) was observed at \( f \approx 11kHZ \) for almost all high beta discharges. This coherent magnetic fluctuation was usually dominant even though the \( (m,n) = (2,1) \) or \( (m,n) = (3,2) \) mode was observed clearly on the soft X-ray fluctuation and the line density fluctuations (see Fig. 3). Figure 4(b) shows \( B_\theta \) as a function of \( \alpha^* \) at \( \Delta \nu = -2 \text{ cm} \) for \( 1\% \leq \beta(0) \leq 3\% \). In this case the \( (m,n) = (2,3) \) mode is dominant and stronger than the \( (m,n) = (1,1) \) mode. Since the pressure profile becomes broad because of the carbonization of the wall for \( 0.03 \leq \alpha^* \leq 0.08 \), the pressure gradient becomes large at the \( \iota = 3/2 \) resonant surface which excites the \( (m,n) = (2,3) \) resistive interchange mode. We
note that this magnetic fluctuation is large where the stability improvement is observed for $\beta(0) < 1\%$ plasmas (see Fig. 3).

4. A NEW LOW-$n$ MODE STABILITY ANALYSIS

We have developed a new code to study low-$n$ ideal mode stability by coupling the VMEC code to the STEP code. In the VMEC code, the magnetic field is described by $\mathbf{B} = \nabla s \times \nabla (\psi' \phi - \chi' \zeta - \lambda)$, where $\zeta$ means the angle in the toroidal direction, and $\psi$ and $\chi$ are the toroidal and the poloidal fluxes, respectively. The prime denotes the derivative with respect to $s$. Since space co-ordinates $(R, Z)$ and $\lambda$ are related to $(s, \theta, \zeta)$, the magnetic field components are given by $B_R = B^\theta \frac{\partial R}{\partial \theta} + B^\zeta \frac{\partial R}{\partial \zeta}$, $B_Z = B^\theta \frac{\partial Z}{\partial \theta} + B^\zeta \frac{\partial Z}{\partial \zeta}$, and $B_\phi = B^\zeta R$, where $B^\theta = (\chi' - \frac{\partial \lambda}{\partial \zeta})/\sqrt{g}$ and $B^\zeta = (\psi' - \frac{\partial \lambda}{\partial \theta})/\sqrt{g}$. On the other hand, the magnetic field in the STEP code is given by $\mathbf{B} = \nabla s \times \nabla (\psi' \hat{\theta} - \chi' \phi)$, where $\hat{\theta} = \theta - \lambda(s, \theta, \zeta)/\psi'$, which gives straight magnetic field lines in the $(\hat{\theta}, \phi)$ plane. By numerical calculations we find the correspondence between $\theta$ and $\hat{\theta}$. Using $\hat{\theta}$, we obtain $\{R(s, \hat{\theta}, \phi), Z(s, \hat{\theta}, \phi)\}$ and $\mathbf{B} = \{B_R(s, \hat{\theta}, \phi), B_Z(s, \hat{\theta}, \phi), B_\phi(s, \hat{\theta}, \phi)\}$.

FIG. 5. Growth rate of $n = 1$ mode versus $\beta(0)$ by VMEC-STEP code ($S = \infty$), STEP code ($S = \infty$) and RESORM code ($S = 10^6$) for Heliotron E standard configuration ($a^* = 0$ and $\Delta_e = 0$).
We apply the averaging procedure over the $\phi$ co-ordinate, and obtain \{\(\hat{R}(s, \hat{\theta}), \hat{Z}(s, \hat{\theta})\)\} and the magnetic curvature term:

$$\Omega(s, \hat{\theta}) \equiv \frac{\overline{R^2}}{\overline{R_0^2}} - \frac{\overline{B_\phi^2}}{\overline{B_0^2}} - 1$$

where bars denote the averaged quantities. Here \(B_\delta\) is the non-axisymmetric stellarator magnetic field, and \(R_0\) and \(B_0\) are taken at the centre between the two helical coils of Heliotron E. By constructing the \{\(\psi, \theta\)\} co-ordinate system from \{\(\vec{R}, \vec{Z}\)\} for the STEP code, we have combined the VMEC equilibrium code with the STEP low-n stability analysis. Figure 5 shows a comparison between the STEP code and the new VMEC-STEP code for the ideal n=1 mode in Heliotron E. Both the growth rates and the threshold beta values show good agreement. We think that the VMEC-STEP code is useful for studying the stability of torsatrons with low aspect ratio or helical pitch number, since the VMEC code gives realistic three-dimensional finite beta equilibrium data, such as the position of resonant surface, magnetic shear and local average curvature, to the STEP stability solver.

ACKNOWLEDGEMENTS

Theoretical work was supported by the US-Japan Co-operative Stellarator/Heliotron Program and the JIFT Program.

REFERENCES

COMPARISON OF TOROIDAL/POLOIDAL
ROTATION IN CHS HELIOTRON/TORSATRON
AND JIPPT-IIU TOKAMAK

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Abstract
COMPARISON OF TOROIDAL/POLOIDAL ROTATION IN CHS HELIOTRON/TORSATRON
AND JIPPT-IIU TOKAMAK.

Toroidal and poloidal rotation profiles have been measured on the JIPPT-IIU tokamak and CHS
heliotron/torsatron devices with charge exchange recombination spectroscopy. A comparison is made
of the toroidal/poloidal rotation profiles between CHS and JIPPT-IIU for L-mode plasmas. Parallel
viscosity damping and anomalous radial momentum transport are studied. The electric field profiles
inferred from plasma rotation and ion pressure are also compared. The plasma in CHS rotates poloidally
near the plasma periphery and toroidally at the centre. The profile of the toroidal rotation velocity is
localized at r < 0.3a, which is consistent with the estimate of neoclassical parallel viscosity due to
helical ripple. The plasma in JIPPT-IIU rotates toroidally. The poloidal rotation is damped to less than
a few km/s, which also supports neoclassical parallel viscous damping. The radial transport of the
toroidal rotation velocity on both devices is found to be anomalous, i.e. 2-4 m²/s. This value is
comparable to the thermal diffusivity in JIPPT-IIU. The radial electric field in CHS has a negative
gradient (∂E_r/∂r < 0), while the electric field in JIPPT-IIU has a positive gradient (∂E_r/∂r > 0) near
the plasma periphery. The relation of the space potential to the ion temperature is also discussed.

1. INTRODUCTION

Plasma rotations play an important role in plasma
confinement. The peaked density profile mode is often
associated with peaked toroidal rotation[1]. Recently, a jump
in the radial electric field and the poloidal rotation velocity
has theoretically and experimentally been found to be
important in the transition from the L- to the H-mode [2-5].

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In the H-mode model, the neoclassical parallel viscosity is used to obtain solutions for the poloidal rotation velocity. Hence, it is a crucial issue to study the parallel viscosity (the poloidal viscosity in tokamaks) experimentally. It is, however, difficult to estimate the poloidal viscosity in tokamaks since the source term of the poloidal momentum contains bipolar flux due to ion orbit loss, which is still undetectable experimentally. On the other hand, in a helical device, we can estimate the parallel viscosity (the toroidal viscosity in heliotron/torsatron) from the measurements of the toroidal rotation velocity profile, in the presence of momentum input associated with tangential neutral beam injection (NBI).

The radial electric field, which affects energy and particle confinement in the plasma, can be measured indirectly from the plasma rotation velocity. The $V \times B$ radial force, by poloidal and toroidal rotations, balances the electric field and pressure gradients on an MHD equilibrium time-scale. In this paper, we describe the radial electric field measured in JIPPT-IIU tokamak and CHS heliotron/torsatron and compare the results with estimates of the neoclassical values. Parallel viscosity and momentum in the transport in radial direction in JIPPT-IIU and CHS are also discussed.

2 EXPERIMENTAL SET-UP

2.1. Charge exchange spectroscopy

A multichannel space resolved visible spectrometer system using a CCD detector coupled with an image intensifier has been developed to measure the profiles of ion temperature and toroidal and poloidal rotation velocity simultaneously, with a time resolution of 16.7 ms, using charge exchange spectroscopy. Charge exchange transfer between fully ionized carbon and the fast neutrals of NBI
heating results in excited ions with one more electron per ion. The emission of hydrogen-like carbon (C VI \( n = 7-6 \) and \( n = 8-7 \)) is localized at the cross-section of the neutral beam line and the line of sight of the viewing array. Two sets of viewing optical fibre arrays, one viewing a fast neutral beam and the other viewing off the neutral beam line, have been installed on CHS and JIPPT-IIU to subtract the background radiation [6]. The background radiation (cold component) is mostly due to the charge exchange reactions between fully ionized impurities and background thermal neutrals at the plasma periphery.

Table I. PLASMA PARAMETERS FOR COMPARISON

<table>
<thead>
<tr>
<th>parameter</th>
<th>JIPPT-IIU</th>
<th>CHS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Magnetic configuration</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Toroidal rotation, ( i(0) )</td>
<td>0.22</td>
<td>1.2</td>
</tr>
<tr>
<td>Ohmic heating power, ( P_{OH}(\text{MW}) )</td>
<td>0.3</td>
<td>0.2</td>
</tr>
<tr>
<td>NBI power, ( P_{NBI}(\text{MW}) )</td>
<td>0.3</td>
<td>0.9</td>
</tr>
<tr>
<td>NBI direction</td>
<td>counter</td>
<td></td>
</tr>
<tr>
<td>NBI tangency radius, ( R_{\text{nbl}}(\text{cm}) )</td>
<td>84</td>
<td>87</td>
</tr>
<tr>
<td>ICRF Power, ( P_{RF}(\text{MW}) )</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Electron density, ( n_e(0) (10^{13}/\text{cm}^3) )</td>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>Electron temperature, ( T_e(0)(\text{keV}) )</td>
<td>1.6</td>
<td>0.2</td>
</tr>
<tr>
<td>Ion temperature, ( T_i(0)(\text{keV}) )</td>
<td>0.5</td>
<td>0.3</td>
</tr>
<tr>
<td>Collisionality, ( v_s(i/2) )</td>
<td>0.6</td>
<td>2.4</td>
</tr>
<tr>
<td>Toroidal rotation, ( V_\theta(0)(\text{km/s}) )</td>
<td>+34</td>
<td>-75</td>
</tr>
<tr>
<td>Poloidal rotation, ( V_\theta(a)(\text{km/s}) )</td>
<td>&lt;2.3</td>
<td>6</td>
</tr>
<tr>
<td>Plasma potential, ( \Phi(0)(\text{keV}) )</td>
<td>0</td>
<td>-2.6</td>
</tr>
</tbody>
</table>
2.2. Plasma parameters in JIPPT-IIU and CHS

The plasma parameters of the JIPPT-IIU tokamak [7] and the Compact Helical System CHS heliotron/torsatron ($l = 2$, $m = 8$) [8] investigated in this study are summarized in Table I. These devices have similar plasma sizes but quite different magnetic field structures and characteristics of the parallel viscosity associated with transit time magnetic pumping (TTMP) [9]. A 28 GHz gyrotron with a power of 0.1 MW produces ECH plasmas of low density below $1 \times 10^{13} \text{ cm}^{-3}$, while tangential NBI (0.9 MW) can sustain plasmas with high density up to $1 \times 10^{14} \text{ cm}^{-3}$. The plasma in JIPPT-IIU is heated by Ohmic input, perpendicular NBI (0.7 MW) or tangential NBI (0.4 MW) and ICRF (1-1.5 MW). Radial profiles of electron density and electron and ion temperatures in these devices are shown in Fig. 1(a) and (b). The electron density profile in CHS is hollow, while the electron density in JIPPT-IIU is slightly peaked at the centre.
3. TOROIDAL ROTATION VELOCITY AND VISCOSITY

A comparison of the toroidal rotation profiles in the presence of parallel NBI is made between CHS and JIPPT-IIU for L-mode plasmas to study parallel viscosity damping due to helical ripple in CHS, and the radial momentum transport in JIPPT-IIU. For this study, the neutral beam is injected tangentially (tangency radius of beam line: \( R_{\text{nbi}} = 84\text{cm} \) in JIPPT-IIU and 87 cm in CHS). Figure 2 shows the toroidal rotation profiles measured in JIPPT-IIU and CHS for plasmas with tangentially injected neutral beams.

The toroidal rotation velocity in CHS has a very narrow profile since the toroidal rotation velocity is strongly damped at \( r > 0.3a \), where the neoclassical parallel viscosity [9,10] due to helical ripple become large (\( c_h = 0.25\rho^2 \)). The solid line indicates the limit set by neoclassical (NC) parallel viscosity. Although the error bar is large, the experimental data at \( r > 0.25a \) agree with the parallel viscosity prediction.

FIG. 2. Radial profile of toroidal rotation velocity normalized by ion thermal velocity for tangential \( (R_{\text{nbi}} = 84\text{ cm}) \) co-NBI in CHS and tangential \( (R_{\text{nbi}} = 84\text{ cm}) \) co- and counter-NBI in JIPPT-IIU. Solid lines are calculated toroidal rotation velocity with neoclassical parallel viscosity damping.
Since there is no helical ripple at the plasma centre, the toroidal rotation velocity is not limited by the neoclassical parallel viscosity, but by anomalous radial momentum transport. The radial momentum transport coefficient estimated from the gradient of the toroidal rotation velocity is $2 \text{ m}^2/\text{s}$, which is comparable to the electron thermal diffusivity ($2-10 \text{ m}^2/\text{s}$) [11].

A broader toroidal rotation profile in JIPPT-IIU is mainly determined by radial transport. The absolute value of the toroidal rotation velocity in counter-injection case (NBI is injected in the direction opposite to the plasma current) is larger than that in the co-injection case (NBI is injected parallel to the plasma current). There is a rotation in counter direction in the Ohmic plasma without an apparent momentum source. The change in the toroidal rotation velocity from an Ohmic plasma is comparable for co- and counter-injection cases. The toroidal momentum diffusivity in JIPPT-IIU is found to be anomalous and is $3-4 \text{ m}^2/\text{s}$ at $r/a < 1/2$, which is comparable to the ion and electron thermal diffusivity, showing a qualitative agreement with the theoretical estimate [12].

4. RADIAL ELECTRIC FIELD

The radial electric field profiles are obtained from the profiles of the ion pressure gradient and the toroidal/poloidal rotation by using the momentum balance equation:

$$E_r = \frac{\partial p_i}{e Z_i n \hat{r} r} - (B_\theta V_\phi - B_\phi V_\theta)$$  \hspace{1cm} (1)

where $i$ stands for the measured impurity species. Here, the radial frictional force between different species is small enough to be neglected.
4.1. CHS

The toroidal rotation contribution is very small in CHS. The radial electric field in CHS as in Fig. 3(a) shows a strong shear at the plasma periphery associated with the poloidal rotation velocity. This edge electric field increases as the electron density is increased. This negative electric field is -80 V/cm for low density and -120 V/cm for high density plasmas. The ion collisionality $v_i$ at $r = a/2$ is 2.4 for low density ($n_e = 2 \times 10^{13} \text{cm}^{-3}$) and 22 for high density ($n_e = 6 \times 10^{13} \text{cm}^{-3}$) discharges. The radial electric field shows a different behaviour in the collisionless regime ($v_e = 0.2$) produced by ECH in CHS. $E_r$ at $r = 0.7a$ is 16 V/cm (positive) in a plasma with low density ($n_e = 4 \times 10^{12} \text{cm}^{-3}$).

The radial electric field profiles measured in CHS are compared with neoclassical estimates [13,14] in Fig. 3(a).

![FIG. 3. Radial electric field profiles measured in (a) CHS and (b) JIPPT-IIU with neoclassical estimates. (a) Solid lines are calculated radial electric field for high density plasma; dashed lines refer to low density plasma. (b) Solid lines are calculated radial electric field for perpendicular ($R_{nbi} = 20 \text{ cm}$) counter-NBI; dashed lines refer to co-NBI.](image)
These calculated electric fields are smaller than those measured from the poloidal rotations. Theory does not explain the electric field shear near the plasma edge \((r/a > 0.6)\). The large electric field measured near the plasma edge may need other explanation such as, e.g. orbit losses.

4.2 JIPPT-IIU

The pressure gradient and the toroidal rotation are the dominant terms in Eq. (1), since the poloidal rotation is small. Poloidal rotation velocities of 3-5 km/s inside the plasma are measured from intrinsic C VI line radiation without neutral beams. They are found to be below the detection limit (2-3 km/s). This does not contradict the neoclassical estimate [15]. The electric field \(E_r\) can be derived from the measured toroidal rotation velocity \(v_\phi\) and the ion pressure gradient and the neoclassical estimate for \(v_\theta\). To eliminate the influence of the toroidal momentum input on the electric field, a perpendicular neutral beam has been injected. The beam line of NBI is still tilted by \(\pm 9^\circ\) \((R_{nbi} = 20 \text{ cm})\). The direction of the plasma current was reversed to check the effect of the momentum input of NBI. The plasma rotates in the counter-direction during NBI, even when NBI is tilted to co-direction \((v_\phi(0)\) is -30 km/s for co- and -60 km/s for counter-injection). The radial electric field profile is peaked at a radius of \(r = (2/3)a\) as shown in Fig. 3(b). The peak values are -190 V/cm in co- and -250 V/cm in counter-injection for NBI plus ICRF heated plasmas. The plasma space potential depends mostly on the ion temperature, and these values were found to be \(\Phi(a) - \Phi(0) = 1.5T_i(0)\), where \(\partial \Phi/\partial r = -E_r\), for both NBI and ICRF heated plasmas[16]. Bipolar flux should exist in the plasma since negative plasma potentials are observed. According to the neoclassical ripple transport theory [17,18], there is a finite
residual toroidal rotation velocity due to toroidal field ripple in the banana regime, and the electric field can be predicted to balance this toroidal rotation velocity. The measured values as shown in Fig.3(b) are smaller than those predicted.

5. DISCUSSION

The radial profile of the toroidal rotation velocity in CHS demonstrates the neoclassical parallel viscosity damping process. Poloidal rotation measurements in JIPPT-IIU support the existence of this damping. The radial momentum transport in both devices was found to be anomalous (2 m²/s in CHS and 3-4 m²/s in JIPPT-IIU for the case of this study). The radial electric field in CHS has a negative gradient ($\partial E_r/\partial r < 0$), while the electric field in JIPPT-IIU has a positive gradient ($\partial E_r/\partial r > 0$) near the plasma edge. The effect of the electric field on helical ripple will be studied in the future. The mechanism which causes the difference in density profiles between CHS and JIPPT-IIU is still an open question; however, the difference in toroidal rotations is one of the candidates for this mechanism since the rotation velocity shear produces a particle pinch [19].

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The authors would like to thank Dr.K.C.Shaing (ORNL) for useful discussions. The calculation with the neoclassical estimate of the CHS electric field has been carried out by Dr.K.Yamazaki.

REFERENCES

DISCUSSION

X.Z. LI: Your comments about the radial electric field and poloidal velocity due to parallel damping of the neutral beam are interesting. My question is — what would happen if there were no neutral beam injection, but just the ICRH?

K. IDA: We have no measurements of toroidal or poloidal rotation without neutral beam injection since rotation measurements by charge exchange spectroscopy require a neutral beam. However, rotation measurements have been done for ICRH plus NB heated plasma in JIPP TII-U. The radial electric field becomes more negative as the ICRH power is increased with fixed NB power. This increasingly negative radial electric field is mainly due to the increase in the ion temperature by ICRH.

G. GRIEGER: Let me point out that poloidal rotation was carefully measured in Wendelstein VII-A some years ago. The corresponding increase in confinement has been demonstrated. The momentum input can be estimated from the lost orbits of nearly perpendicular neutral beam injection.
COMPARATIVE STUDIES OF EDGE TURBULENCE IN THE TEXT TOKAMAK AND THE ATF STELLARATOR

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Abstract

COMPARATIVE STUDIES OF EDGE TURBULENCE IN THE TEXT TOKAMAK AND THE ATF STELLARATOR.

The paper presents experimental results on edge turbulence and transport from the tokamak TEXT and the stellarator ATF. A naturally occurring velocity shear layer determines a characteristic edge radius. The measured electrostatic fluctuations can explain the transport of particles and perhaps energy in the far edge close to the outermost closed flux surface. Certain drives (radiation, ionization, Kelvin–Helmholtz, as well as density and temperature gradients) and stabilizing terms (velocity shear) are suggested by the results. The experimental fluctuation levels can be reproduced by considering the nonlinear evolution of the reduced Braginskii equations, incorporating thermal and ionization drives.

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1. INTRODUCTION

The motivation for studying the fluctuations and transport in the plasma edge is manifold: (1) experimental data exist which show the electrostatic fluctuations are large enough to explain the total transport; (2) theories are better developed than for the core; and (3) modeling can be performed mainly using fluid equations. In an attempt to understand the edge turbulence a comparative study of the ATF currentless stellarator and TEXT tokamak has been initiated. We begin by summarizing and comparing measurements of edge turbulence studies on TEXT and ATF—the similarities in the results are quite striking. We then describe theoretical studies aimed at developing a physical picture that is compatible with the experimental results from the two devices.

<table>
<thead>
<tr>
<th>Table I. Edge plasma characteristics</th>
</tr>
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<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>$B_\theta$ (T)</td>
</tr>
<tr>
<td>$n$ (cm$^{-3}$)</td>
</tr>
<tr>
<td>$T_e$ (eV)</td>
</tr>
<tr>
<td>$q(a)$</td>
</tr>
</tbody>
</table>

2. EXPERIMENTAL STUDIES

Device configurations and parameters. Table 1 shows the device and edge plasma parameters for the TEXT tokamak [1–3] and the ATF stellarator [4–6]. The edge plasmas in the two machines are generally comparable, although the density is lower in ATF as a consequence of the use of 53.2 GHz, second-harmonic ECH for plasma production and heating. The principal differences lie in the magnetic configurations of the two devices. In TEXT, plasma current is used to produce a configuration in which the safety factor is $q(0) \sim 1$ in the center and $q(a) \approx 3$ at the edge. In ATF, external fields produce a configuration without plasma current in which the shear is opposite in sign and $q(0) \approx 3$, $q(a) \approx 1$. 
Profiles, shear layer. Fig. 1a shows a typical edge density profile $n_e(r)$ in ATF and TEXT as measured by a fast reciprocating Langmuir probe (FRLP) and Fig. 1b shows the electron temperature profile $T_e(r)$. In this figure the radius is normalized to the shear layer radius $a_s$. Density and fluctuation measurements with the FRLP show that, in both devices, the steepest gradient in the profile corresponds to the location of a shear layer where the poloidal phase velocity $v_{ph}$ and radial electric field $E_r$ change sign; for $r < a_s$, the fluctuations propagate in the electron diamagnetic direction and $E_r < 0$, and for $r > a_s$, the fluctuations propagate in the ion diamagnetic direction and $E_r > 0$. When the radial profiles of the fluctuating quantities (e.g., $\tilde{n}/n$, shown in Fig. 2) are plotted as a function of normalized radius $r/a_s$, the results for TEXT and ATF are similar.

In TEXT, which is limited by a fixed poloidal belt limiter, the radial location of the shear layer changes as the plasma column is shifted in major...
radius, effectively varying the minor radius of the plasma. The shear layer position can be controlled by biasing limiters, and multiple shear layers can be generated by stochastic magnetic regions due to externally produced resonant fields.

In ATF, which can be operated either with a magnetic limiter or with a movable toroidal rail limiter (toroidal extent $\approx 5^\circ$), the location of the shear layer shifts with the vacuum magnetic configuration when the externally imposed vertical field is varied. The shear layer moves radially inward when...
the limiter is moved inside the nominal last closed magnetic surface where $q \approx 1$. The shear layer is also affected by the main plasma parameters: it moves radially outward by $\approx 2$ cm as the plasma stored energy increases in both ECH and neutral beam injection heated plasmas; the location of maximum density gradient similarly shifts outward. Elucidation of the physical mechanisms involved in the formation of the shear layer is essential to understanding edge turbulence and transport.

*Characteristics of the edge turbulence.* In both TEXT and ATF, probe measurements show that the frequency spectra of density ($\bar{n}$) and floating potential ($\tilde{\phi}_{fl}$) fluctuations (Figs. 2a, 2c) are broad, and dominated by components at $f < 200$ kHz. The poloidal wave number ($k_{\theta}$) spectra are also broad, with $\bar{k}_{\theta} \sim 1$-5 cm$^{-1}$ and $\sigma_{k_{\theta}}/\bar{k}_{\theta} \sim 0.5 - 1$; here $\sigma_{k_{\theta}}$ and $\bar{k}_{\theta}$ are, respectively, the variance and mean of the $k_{\theta}$ spectra. Figure 3 shows $\sigma_{k_{\theta}}$ as function of local poloidal fluctuation velocity $v_{\theta}$ in the laboratory frame of reference for positions excluding the shear layer region. In both devices $\sigma_{k_{\theta}} (\propto 1/\ell_{\theta}$, where $\ell_{\theta}$ is the poloidal correlation length) varies systematically with $v_{\theta}$. Typically, $\sigma_{k_{\theta}}$(ATF)/$\sigma_{k_{\theta}}$(TEXT) $\approx q_a$(ATF)/$q_a$(TEXT) $\approx 1/3$, as would be expected for fluctuations resonant with the pitch of the magnetic field near the plasma edge. Measurements of radial and poloidal correlation

![Graph showing phase velocity dependence of the width $\sigma_{k_{\theta}}$ of the fluctuation $k_{\theta}$ spectrum excluding the velocity shear region. Results for ATF (open circles) and TEXT (dots) are shown.](image)
lengths in TEXT show that typically $\ell_\theta \approx 1$ cm, $\ell_r \approx 0.5$ cm. In both devices, the minimum correlation length and time are measured in the shear layer where $v_{ph} \approx 0$.

Triple probe and swept double probe measurements [7] have been used to determine tentative electron temperature fluctuations $T_e$. Substantial systematic errors may occur due to phase shifts between probe tips and/or shielding effects. Radial profiles of $T_e/T_e$ and $\bar{n}/n$ for ATF and TEXT are compared in Fig. 2b. In both machines only limited measurements have been made inside the shear layer, but the existing data suggests that the character of the fluctuations may change at the shear layer, with $T_e/T_e \geq \bar{n}/n$ for $r/a_s \leq 1$. More accurate measurements are required to better establish the level of temperature fluctuations inside the shear layer. Even more stringent precision is necessary to determine the plasma potential fluctuations $\langle \phi_i \rangle$ from the floating potential fluctuations, since $\phi_i(t) = \phi_f(t) + \beta kT_e(t)$ with $\beta \sim 3$. Large $T_e$ may also affect the density fluctuation measurement. Such studies are in progress.

**Fluctuation-induced transport.** We can estimate the fluctuation-induced particle flux $\Gamma'$ from probe measurements of $\langle \bar{n}\bar{\phi} \rangle$, where the floating potential is used for $\bar{\phi}$. We find that $\Gamma'$ increases with increasing $n$ and is comparable to the edge particle transport determined from global particle balance if the fluxes are estimated to be poloidally and toroidally uniform. In TEXT, regression analysis of the electrostatic fluctuation driven flux $\Gamma'$ for $r \approx a$ shows that the results can be fit by the expression $\Gamma' \propto nB_{\phi}^{-2\pm 1}$.

In TEXT, the existing data on $T_e$ outside the shear layer (where $\bar{n}/n$ is $> T_e/T_e$) [7] have been used to determine that the fluctuation-induced energy flux ($\langle \bar{T}_e\bar{\phi} \rangle$ and $\langle \bar{n}\bar{\phi} \rangle$) is large enough to account for the total energy flux [3]. A more complete picture of energy transport requires a better knowledge of $T_e$ in both devices, as was noted earlier.

**Driving terms.** The overall similarity of the results for zero-current plasmas in ATF and ohmically heated plasmas in TEXT suggest that plasma current and applied toroidal electric fields $E_{\phi}$ are not the dominant drive for edge turbulence. This conclusion is further supported by experiments in
TEXT in which the applied toroidal electric field was significantly reduced with no accompanying change in properties of the edge turbulence [8]. This rules out the simple resistivity-gradient driven turbulence mechanism as the dominant mechanism for the edge plasma turbulence.

Injection of an impurity ($N_2$) into TEXT discharges cools the plasma and modifies the potential fluctuations $\tilde{\phi}_i/kT_e$ [9]. Such an effect could be evidence for an impurity line radiation drive.

The strong variation of the turbulence characteristics in the vicinity of the shear layer in both devices (e.g., Figs. 2 and 3) suggests that shear flow effects may be important in driving and/or stabilizing the edge fluctuations.

3. THEORETICAL STUDIES

The analysis of TEXT and ATF experimental results has identified three distinct regions at the edge of the plasma: the outboard side of the velocity shear layer (VSL), $r/a_s > 1$, the VSL itself acting as the edge of the confinement region, and the inboard side of the VSL, $r/a_s < 1$.

Thermally driven turbulence. On the main plasma side of the VSL ($r/a_s < 1$) impurity radiation is operative. With $E_\phi \neq 0$, as is the case for Ohmic TEXT discharges, our early thermally driven convective cell turbulence model [10] included, in addition to the resistivity and impurity gradient drives [11], both the condensation component $I_z/T$ of the impurity radiation [12], which drives density fluctuations, and its thermal counterpart $-dI_z/dT$, which drives temperature fluctuations. Here, $I_z$ is the cooling rate. We have since measured [7] and shown theoretically [13,4] that condensation is not effective because pressure balance does not hold (i.e. $\tilde{T}_e$ and $\tilde{n}$ are only partially correlated) given that $\gamma > k_{\|}c_s$, with the net result that the thermal drive $\gamma_T = -2/3n_zdI_z/dT$ (with $n_z$ the impurity density) and only the thermal component, which acts as a source for temperature fluctuations, remains.

When the loop voltage drops and diamagnetic effects take over, i.e. in the case of edge current free TEXT discharges and ATF where $E_\phi = 0$,
thermally driven convective cells evolve into thermally driven drift waves [14]. These dissipative drift waves, whose available free energy is drastically reduced or eliminated by $\nabla T_e$, are excited by the thermal component of the radiation drive. For both thermally driven convective cells and thermally driven drift waves, analytical theory shows that the turbulence saturation condition is created by balancing the parallel diffusion dissipation (enhanced by the radial transport) with the resistivity gradient and/or thermal instability drives. In the radiation dominated limit, the saturation condition yields

$$\frac{\tilde{e}\tilde{\Phi}}{T_e} = 2.95 \frac{\gamma_R^{3/2}}{\chi || k'^2} \bar{k}_g \rho_s c_s$$ \hspace{1cm} (1)

Here $\chi ||$ denotes the parallel thermal conductivity, $k' = k_g/L_s$ and $c_s$ the sound speed. Since the radiation drive preferentially excites temperature fluctuations, it is expected that $\tilde{T}_e/T_e > n/n$ inside the plasma.

Computations have been performed with our 3D nonlinear MHD code KITE [15]. The model radiation function $I_z$ is based on a shifted coronal equilibrium. $I_z$ is normalized to give a radiation power density peak in the range of 0.08 to 0.2 W/cm$^3$. The measurements in TEXT are in the

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**FIG. 4.** Radial dependence of normalized potential fluctuation levels measured and predicted for TEXT: experiment (dots), resistivity gradient model with radiation (open squares) and thermally driven drift wave model (solid triangles).
upper end of this range. The computations show that the fluctuation levels are in good agreement with theoretical predictions (Eq. 1). As shown in Fig. 4, there is reasonable agreement between normalized fluctuation levels from the experiment and computations for both thermally driven convective cells and thermally driven drift waves. The models can explain both the measured \( e\bar{v}/T_e > 40\% \), and the measured \( e\bar{v}/T_e > \bar{n}/n \). The calculated \( \sigma_{k\theta} \) values range from 1 to 2 cm\(^{-1}\) for convective cells and from 0.75 to 1.5 cm\(^{-1}\) for drift waves as compared to experimental values of 2 cm\(^{-1}\). The computations yield \( \ell_\theta \approx 2.5 \text{ cm} \) and \( \ell_r \approx 0.27 \text{ cm} \), whereas \( \ell_\theta \approx 1 \text{ cm} \) and \( \ell_r \approx 0.5 \text{ cm} \) are measured in TEXT. The computations also show there is a strong dependence of \( \sigma_{k\theta} \approx \bar{k}_\theta \) on \( q \), with \( \sigma_{k\theta} \) scaling as \( q \) in a radially averaged sense. Since \( q_a \sim 3 \) in TEXT and \( q_a \sim 1 \) in ATF, we predict \( \sigma_{k\theta} \) for ATF to be about a factor of 3 lower than for TEXT [16]. This was confirmed in Figure 3. Parameter scans in TEXT show both \( \bar{k}_\theta \) and \( \sigma_{k\theta} \) to be an increasing function of \( q \).

The computations also yield \( \bar{T}_e/T_e \geq \bar{n}/n \). This result has been confirmed analytically [13] and agrees with preliminary experimental measurements.

Velocity shear layer. The TEXT and ATF results presented in Sec. 2 show that the VSL has a profound influence on the characteristics of the turbulence. Analytical calculations [17] indicate that the turbulence correlation length shrinks when the shearing frequency \( k_\theta v'_\theta \Delta \) exceeds the decorrelation rate of the ambient turbulence. Here, \( \Delta \) is the characteristic radial scale size of the turbulence. This feature is observed experimentally as an increase of \( \sigma_{k\theta} \) at the VSL [Fig. 3]. The calculations also show that sheared flow induces a coupling between turbulent radial diffusion and poloidal shearing, which results in enhanced decorrelation and a concomitant reduction in the size of the turbulent convection cells. These effects suppress turbulence in the presence of strongly sheared poloidal flows, and improve confinement. Evidence for this has been shown on TEXT [18].

Nonlinear computations show that the VSL can be generated in a self-consistent way by thermally driven drift waves. This result agrees with recent calculations [19,20] which demonstrate quasi-linear flow generation for radially propagating modes. Thus the VSL can form regardless of whether
a limiter is present (as in TEXT) or not (as in ATF). As shown in Fig. 5, the computations reproduce well the VSL measured in TEXT, both in terms of the velocity jump (5 km/s computed as compared to 10 km/s measured) and the radial extent of the VSL.

**Ionization driven turbulence.** Outside the VSL, which includes the scrape-off-layer, no detailed modeling of the magnetic configuration has yet been done. Here, dissipative drift waves, which are otherwise rendered stable by $\Delta T_e$, can be excited by particle sources introduced by ionization. A four-field model for $\tilde{n}$, $\tilde{\phi}$, $\tilde{T}$ and $\tilde{v}_\|$, evolution has been derived [14]. The equations include ionization induced growth and charge exchange induced damping effects. In the quasi-local limit, the ionization driven drift wave model yields $\omega = \omega_m + i(\gamma_l - 1.71n_{\text{neut}}^2/\chi_hk^2)$ so that long wavelength modes are destabilized by ionization. Here $\omega_m$ is the electron diamagnetic drift frequency and the ionization drive $\gamma_l = n_{\text{neut}}(\sigma v)_I$. Note that $\nabla T_e$ stabilization constitutes a short-wavelength energy sink. The $E \times B$ nonlinearity induced by the non-adiabatic electron response dominates over the nonlinear polarization drift so mode coupling to small scales (note the waves are dispersion free) can be expected. As $\nabla T_e$ stabilizes short wavelength
modes, a saturated state results. Detailed calculations in the quasi-local limit indicate

\[ \frac{\tilde{n}}{n} \approx \left( \frac{\chi k^2}{c} \right) \ \left( \frac{\gamma_l}{1.71 k e c_T} \right) \ \frac{1}{(k e \rho_s)^2} \]  

(2)

Numerical calculations with a constant rate \( \gamma_l \) have been performed. The value for \( \gamma_l \) taken in these calculations has been 3 to 5 times the value of \( \langle \sigma v \rangle_I \) for hydrogen at \( T_e = 20 \) eV times a neutral density of \( 10^{11} \) cm\(^{-3} \). At saturation, density fluctuation levels of 50 to 60\% have been obtained. A more accurate modeling of the edge plasma is needed for a detailed comparison with experimental results.

Preliminary nonlinear computations indicate that the density fluctuations saturate at a finite amplitude, that \( \tilde{n}/n \geq e \phi / T_e \) and that the spectral characteristics of the turbulence are comparable to those for thermally driven drift waves. Sensitivity studies to the neutral density profile still remain to be done.

An interesting feature of this model is that the particle flux is purely inward, i.e.,

\[ \Gamma = \langle \tilde{n} \tilde{v} \rangle = - \sum_k \frac{1.71 n (k e \rho_s c_T)^2}{\chi k^2 c_T} \left| \frac{e \phi_k}{T_e} \right|^2 \]  

(3)

This seemingly implausible result is entirely consistent with the fact that \( \nabla T_e \) effects are stabilizing, here. Thus, particles pay an energy penalty to climb the temperature gradient, so that \( \gamma_l > 1.71 \eta_{\omega c_T} k^2 \) is required for instability (i.e. the “drive” is not related to the gradients here).

The ionization driven drift waves model suggests an interesting scenario for explaining particle uptake during a gas puff. Normally, the \( \gamma_l \) drive is weak. However, injection of neutrals strongly perturbs the turbulence, destabilizing the ionization mode. The resulting purely inward flux results in rapid penetration of ionized gas, thus depleting the reservoir of neutrals and returning the system to quiescence. Of course, it is clear that some other mechanism is needed to explain the background, steady-state, outward flux of particles.
4. SUMMARY

TEXT and ATF both exhibit a shear layer which, when used to normalize minor radius, results in both machines exhibiting similar fluctuation characteristics. This shear layer profoundly influences the turbulence characteristics themselves. The electrostatic fluctuations explain particle transport and perhaps energy transport at the edge. At the edge, several driving terms are relevant, impurity radiation, ionization and velocity shear may be important. Many features of the turbulence can be explained by using the reduced Braginskii equations with those drives. These nonlinear calculations also show that the shear layer can be self-consistently generated by the turbulent fluctuations.

Several outstanding issues remain:

1) Measurement of the drives inside the plasma and outside the velocity shear layer. While the model $I_z(T)$ is in agreement with TEXT bolometric data, recent TEXT and ATF measurements indicate that the radiated power for a single line is peaked at smaller radii than we have used. This discrepancy suggests not only that the choice of the functions $I_z(T), n_z(r), Z^r$ are critical to the results, but also that other drives such as ionization (an electron thermal energy sink and density drive) and charge exchange (momentum and ion thermal sink) are important.

2) Accurate measurements of temperature fluctuations inside the plasma.

3) Assessment of the reliability of poloidal and radial correlation length measurements in order to resolve the discrepancy in the ratios between experiment ($\ell_\theta/\ell_r \approx 2$) and computations ($\ell_\theta/\ell_r \approx 5 - 10$).

4) Calculation of the frequency spectra in the models and comparison with experimental results, including their respective phase relations and cross-correlations.

The dual observations that fluctuations induce sheared flows and that such flows in turn help regulate turbulence levels naturally suggest that fur-
ther theoretical work on self-consistent models of turbulence in differentially rotating plasmas and further experimental work on momentum transport and shear effects receive high priority. Moreover, the manifestation of shear suppression of turbulence in an OH plasma strongly suggests that the H-mode, characterized by strong, extended shear, may be a natural parametric continuation of the shear layer edge, rather than a fundamentally new or different regime.

ACKNOWLEDGMENTS

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REFERENCES

DISCUSSION

R.J. TAYLOR: You do not seem to have an inward pinch term for the mass transport in your theoretical picture. What is your justification for this?

C.P. RITZ: Experimentally we observe a fluctuation induced flux sufficient to describe the outward particle transport. Poloidal asymmetries in the potential profiles may have additional effects. However, no pinch term is necessary in the edge region of the discharge, as source effects are still substantial. On TEXT, charge exchange neutrals may be sufficient to fuel the plasma further towards the interior as well. Theoretical predictions suggest that ionization driven drift wave turbulence will also produce a local inward flux.

P.R. THOMAS: We talk all the time as if the plasma boundary were the only place where the action is. The H mode is seen to improve confinement across the entire cross-section of the plasma. What is the connection between this and the boundary shear layer?

C.P. RITZ: What I reported can only explain the confinement improvement in the shear layer region. Because of the short radial correlation length, it cannot have a direct effect on the interior. However, local profile changes in the shear layer (density, temperature, source, $E_r$, etc.) can have an indirect effect on the profiles in the adjacent segment further towards the centre.

G. FUSSMANN: It seems to me that the impurity simulations would have to be performed on a 3-D basis for low ionization stages of importance in the edge region. In other words, I suspect that the effects which you observe depend strongly on the exact position at which the impurities are blown into the discharge.

C.P. RITZ: Impurity puffing causes a temperature collapse in the edge immediately after the puff, which is possibly characterized by substantial asymmetries. Over successive shots with nitrogen puffing, the impurity content increases. The increase in the floating potential fluctuation level which we report is due to an increased overall impurity content. Asymmetries are thus probably not much more important than in normal discharges.
OPTIMIZATION, MHD MODE AND ALPHA PARTICLE CONFINEMENT BEHAVIOUR OF HELIAS EQUILIBRIA

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Abstract

OPTIMIZATION, MHD MODE AND ALPHA PARTICLE CONFINEMENT BEHAVIOUR OF HELIAS EQUILIBRIA.

Result and interpretation of the optimization of Helias equilibria with respect to seven stellarator specific goodness criteria are described. The MHD stability investigation of such optimized configurations in terms of global internal (i.e. fixed boundary) modes leads to the conclusions that Mercier mode stability has to be satisfied to avoid global modes and that low poloidal node number ballooning modes then do not exist. The collisionless confinement of alpha particles in the optimized configuration is found to be sufficiently good at finite beta.

1. INTRODUCTION

Stellarators need optimization because classical physics issues seriously limit their viability as fusion devices. Such issues are for example their magnetic surfaces, their MHD and neoclassical properties, and their realization by coils. On the other hand, stellarators can really be optimized. An example for their optimization potential are quasi-helically symmetric toroidal magnetic fields [1] which show that the magnetic geometry of a stellarator can be decoupled from its real space geometry. Thus, stellarator optimization does not only mean an improvement of some given basic concept but also primarily the selection of basic physics properties.

Stellarator optimization is naturally divided into a set of general guidelines and a set of specific physics objectives.

The plasma behaviour in the confinement region can be optimized by noting that the geometry of the confinement boundary within the last closed flux surface completely determines the properties of the confinement region. Thus, boundary value problems may be solved during optimization, the parameters of the boundary being the optimization variables. Boundary value problems are the basic steps of the optimization procedure employed here, which allows large steps to be taken in the stellarator...
configurational space. The boundary representation used for Helias [2] equilibria appears to provide a suitable configuration space.

In this paper, the objectives of the optimization and its result as well as aspects of the MHD stability and the collisionless alpha particle confinement behaviour of the optimized configuration are described. An overview of the physics and engineering aspects of such a stellarator is given in paper G-I-6 of this Conference.

2. OPTIMIZATION OF HELIAS CONFIGURATIONS

For the optimization of Helias configurations for W VII-X the following set of criteria has been used [3]:

1. high quality of vacuum field magnetic surfaces (sufficiently small thickness $\Delta_{ix}$ of islands),
2. good finite beta equilibrium properties (sufficiently high $\beta_{eq}$),
3. good MHD stability properties (sufficiently high $\beta_{stab}$),
4. small neoclassical transport in the $1/\nu$-regime (small equivalent ripple $\delta_e$),
5. small bootstrap current in the lmfp-regime ($J_{BS,stell}/J_{BS,tok}$ sufficiently small),
6. good collisionless alpha particle containment (fraction of prompt loss $f_{a}$ sufficiently small),
7. good modular coil feasibility (sufficiently large distances $\Delta_c$ and radii of curvature $R_c$ of the coils).

The constructiveness of this set of classical physics goals in connection with an optimization procedure results from the dimensionless goodness parameters indicated above.

Criteria 1 and 7 are taken into account by solving Helias boundary value problems with side conditions on the shaping parameters. Criteria 2 and 3 are satisfied by maintaining resistive interchange and ballooning stability at ($\beta$) $\approx 0.05$ for configurations with 5 periods and aspect ratio of approximately 10. While maintaining resistive interchange stability is directly incorporated into the optimization, ballooning stability is taken into account through its driving terms [4]. Criteria 4, 5, and 6 are taken into account by optimizing the structure of $B(\theta, \phi)$ in magnetic co-ordinates. This optimization procedure constitutes an inner optimization loop.

The evaluation of ballooning stability and of the three neoclassical properties 4, 5, and 6 leads to an iteration of this inner loop until sat-
isfactory properties are found. Goodness parameters $\Delta_{is}$, $\beta_{eq}$, $\beta_{stab}$, $\delta_e$, $J_{BS,stell}/J_{BS,tok}$, $I_0$ which can be achieved simultaneously appear to be $0.1, 0.05, 0.05, 0.01, 0.1, 0.1$ [5].

An explanation for the compatibility of the seven criteria listed above can be obtained in terms of a unified optimization procedure. Key ingredients are:

(i) it suffices to consider the structure of $B(\theta, \phi)$; the real space geometry is a result of the optimization;

(ii) the spectrum of $B(\theta, \phi)$ can be optimized;

(iii) a unified optimization procedure simply consists in keeping the spectrum pure and in minimizing the helical and toroidal curvature terms under the constraints of small bootstrap current and location of trapped particles in the weak curvature region.

Indeed, optimization in a large (approximately 20-dimensional) space of boundary variables [6], in which only an ellipticity and a triangularity parameter are kept fixed, with the above prescription yields the configuration in Fig. 1, which is nearly identical with the one obtained by optimization according to the seven criteria listed above. Ingredient (iii) of the optimization procedure described above elucidates the degree of unambiguity of the optimization result. The major type of a qualitatively different result would be obtained by requiring the principal toroidal curvature term to be zero; this would result in quasi-helically symmetric equilibria with a finite bootstrap current. The solution chosen here essentially eliminates the bootstrap current as an alien element of stellarators proper and, on
FIG. 2. Flux surfaces of an unstable Helias configuration with 5 periods, rotational transform $\approx \beta_4$, and approximately marginal magnetic well in the vacuum field.

the other hand, achieves the other neoclassical physics requirements (see Section 2, principles 4 and 6); here it has to be noted that the good collisionless alpha particle confinement requires a non-vanishing $\beta$-value, see Section 4.

3. MHD MODE BEHAVIOUR

Global mode calculations made with the Finite-Element Fourier code CAS3D [7] (Code for the Analysis of the MHD Stability of 3D Equilibria) concentrated on applications to Helias configurations with five equilibrium periods, aspect ratio 10, $(j^2_\parallel/j^2_\perp) \approx 0.7$ and a small shear $\iota$-profile including the rational $\iota = \frac{3}{4}$ (see Fig. 2).

Parameter studies in a set of configurations of this type show that, though the low poloidal node number non-local modes are less restrictive than the Mercier criterion, the two stability limits are so close that, in practice, low shear stellarators have to be Mercier stable. The results obtained from CAS3D demonstrate various critical aspects of the 3-D linear stability analysis. For optimal convergence the calculation has to be based on a large number $L$ (up to $L = 60$ has been considered) of Fourier components in the perturbation functions. For this purpose CAS3D provides an automatic selection process. Discussion of the various contributions to the energy functional shows that energy minimization occurs simultaneously with the annihilation of the field compression term. Near the Mercier stability limit the mode localizes radially (see Fig. 3).

These and other aspects were successfully embedded into the code development (CAS3D1, which only employs the perturbation component $\xi^s$ in the eigenvalue problem). Furthermore, the dominating poloidal mode number $M$ of the perturbations enters the code as an input parameter.
FIG. 3. Various contributions to the MHD energy functional for a high-M-number mode \((M = 24)\) in the equilibrium of Fig. 2 versus the normalized flux label \(s\): 1 field line bending term, 2 "Mercier" term, 3 field line compression term, 4 field line curvature term, and 5 energy per flux shell. The rational value of \( \epsilon = 2 = \frac{18}{24} \) occurs at \(s \approx 0.63\).

which is not connected with the spatial resolution of the stability calculation so that high-\(M\) unstable modes could be obtained. With a further modification concerning the numerical stability of the discretized eigenvalue problem [8], the code has also been used to investigate modes with ballooning character in tokamaks and stellarators. Ballooning modes in Mercier stable stellarators with significant negative shear (i.e. a rotational transform decreasing towards the plasma edge) have been obtained. Modes of the type occurring in these configurations are however absent in configurations as shown in Fig. 2.

As results of these investigations the following statements can be made: In Helias stellarators Mercier mode stability has to be satisfied to avoid global modes and low poloidal node number ballooning modes then do not exist.

4. COLLISIONLESS ALPHA PARTICLE CONFINEMENT

The collisionless alpha particle confinement is assessed by guiding centre orbits of a sample of alpha particles started at aspect ratio \(A = 40\) (which corresponds to \(1/4\) of the plasma radius \(a\)) with random values in the angular-like magnetic co-ordinates \(\theta\) and \(\phi\) and the pitch angle
FIG. 4. Alpha particle losses in Helias50B with $\langle \beta \rangle = 0(\circ), 0.024 (\times),$ and $0.049 (\Delta);$ as a function of collisionless time of flight. A random sample of $\alpha$-particles (ratio of plasma to gyroradius 30) is started at aspect ratio 40. Shown is the fraction of reflected particles which is lost. Number of reflected particles is 100 in all cases. Altogether there are 260 particles, i.e. 160 passing particles. Each symbol indicates the loss of one particle.

$\eta = v_{||}/v$. In W VII-AS all alpha particles that undergo reflections are quickly lost, but no passing particle is lost. In a quasi-helically symmetric stellarator all particles are completely confined. A particular characteristic of the optimized configuration of Section 1 is that the beta effect at $\langle \beta \rangle \approx 5\%$ is sufficient to improve the alpha particle confinement in such a way that the fraction of prompt losses is reduced to approximately 0.1 [9](see Fig. 4).

A significant improvement of the fast particle losses already occurs at the modest value of $\langle \beta \rangle = 0.024$, which may also be of importance for NBI and ICR heating.

The favourable collisionless particle confinement result can be understood in terms of the creation of a maximum-$J$ configuration, with $J$ the second adiabatic invariant. A direct verification of this concept is seen in Fig. 5, which shows a poloidally closed drift orbit of a reflected alpha particle. Figure 6 shows the formation of poloidally closed $J$-contours as $\beta$ is increased.
FIG. 5. Left: Shown is a $\sqrt{s}$, $\theta$-plane with $s$ the flux label and $\theta$ the poloidal magnetic coordinate. Poloidally closed drift orbit of a reflected alpha particle starting on the magnetic surface indicated by the dashed circle in the direction of the magnetic field (+ positive $v_\parallel$; o negative $v_\parallel$). In the limit of small gyroradius this Poincaré plot converges to the contour of the constant-$J$ surface. Right: Typical drift of a localized alpha particle in W VII-AS magnetic geometry.

FIG. 6. Constant $J$-contours in Helias50B with $(\beta) = 0.0$ and $(\beta) = 0.049$. Dashed lines indicate regions close to maxima, dotted lines those close to minima. The reflection value of $B$ is a constant and defines the reflected particles considered as moderate-deeply trapped.

5. DISCUSSION

While the optimization only takes into consideration classical physics goals, it results in interesting perspectives as far as anomalous transport is concerned: several candidate mechanisms (stochasticity in vacuum and finite beta fields, instabilities such as ballooning, tearing, trapped particle drift modes) are influenced in a beneficial way. In particular, with respect
to trapped-particle modes, in the maximum-$J$ situation prevailing here
the diamagnetic drift fluid velocities and the poloidal drifts of localized
particles are of opposite sign (see Fig. 5). Another aspect which was not
an explicit goal of the optimization is the adaptability of the optimized
configuration to some form of divertor operation. Again, the perspectives
are favourable because a reasonably insensitive separatrix region with ade-
quately sized islands exists in "helical stripes" which connect the bottom
and the top of indented cross-sections one period apart. These stripes –
although they lie in outward parts (with respect to the toroidal radius)
of the configuration – are situated in favourable curvature regions because
the helical curvature dominates the toroidal one.

The MHD equilibrium and stability behaviour has here been described
with the underlying assumption of nested surfaces throughout the confine-
ment region. For example, the VMEC [10] code is used for the equilibrium
calculations. A particular MHD equilibrium aspect is the behaviour of
the equilibrium profiles near a medium order resonance of the rotational
transform per period. With the above assumption this question can be
approached by calculating equilibria with pressure profile flattening near
the resonance and non-singular parallel current density, for which the de-
tailed profile description is obtained by requiring marginal stability with
respect to resistive interchange stability as a particular convenient means
to obtain such a prescription [3]. With this procedure it is found that
the $\beta$-value achievable in optimized Helias configurations is only slightly
reduced. Dropping the assumption of nested surfaces, the behaviour of
magnetic islands at finite beta is investigated in paper D-III-1 (C) of this
Conference.

The collisionless alpha particle confinement behaviour is further being
investigated with respect to the critical number of modular coils which has
to be used in order not to destroy the favourable confinement and with
respect to the angular location of the loss fraction.

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IDEAL STABILITY STUDIES FOR THREE AND FOUR PERIOD FLEXIBLE HELIACS

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Abstract

IDEAL STABILITY STUDIES FOR THREE AND FOUR PERIOD FLEXIBLE HELIACS.

The stability properties of two realizations of the flexible Heliac concept presently under construction — HI in Australia and TJ-II in Spain — are described. HI and TJ-II equilibria, calculated by using the VMEC code, are analysed for their stability with respect to Mercier modes and, in the TJ-II case, to low-n modes. In the helically symmetric limit, heliacs have been found to have very favorable properties as fusion machines. It is shown in the paper that, thanks to the built-in flexibility of the machines, a thorough and complementary programme can be pursued that will allow a full characterization of the stability of Heliac plasmas.

0. INTRODUCTION

The two medium-sized flexible heliacs under construction (HI in Canberra and TJ-II in Madrid — see Table I and Refs. [1] and [2]) complement each other both in design and in scientific aims. Both machines will have access to a great variety of plasma configurations encompassing a wide range of important stellarator parameters (such as the rotational transform ($\alpha$), magnetic well and shear). They differ, however, in their number of field periods and characteristic aspect ratios ($A= 5$-$10$ in HI as against $A= 7$-$14$ in TJ-II ). With its smaller number of periods and lower heating power, HI has been designed for the exploration of plasma instabilities and fluctuations at modest $\langle \beta \rangle \leq 1\%$ whereas TJ-II, with its planned inclusion of 4 MW of neutral beam heating, will extend the study of pressure-driven phenomena to much higher $\langle \beta \rangle \geq 4\%$. Where the two machines overlap in $\langle \beta \rangle$, it is expected that their results should provide a starting point for the derivation of heliac scaling laws. We note in this respect that recent work on the scaling laws from the tokamak database [3] has shown the importance of having machines well separated in aspect ratio but of similar cross-sectional shape to avoid colinearity between these variables.
### TABLE I. MAIN PARAMETERS OF HI AND TJ-II

<table>
<thead>
<tr>
<th>Parameter</th>
<th>HI</th>
<th>TJ-II</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of field periods</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>Number of toroidal coils</td>
<td>36</td>
<td>32</td>
</tr>
<tr>
<td>Swing radius of toroidal coils (m)</td>
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<td>0.28</td>
</tr>
<tr>
<td>Major radius (m)</td>
<td>1</td>
<td>1.5</td>
</tr>
<tr>
<td>Typical mean minor plasma radius (m)</td>
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</tr>
<tr>
<td>Plasma volume</td>
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</tr>
<tr>
<td>Maximum field (T)</td>
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<td>1</td>
</tr>
<tr>
<td>Pulse duration (s)</td>
<td>∞/1</td>
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</tr>
<tr>
<td>Maximum ring current (kA)</td>
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<td>480</td>
</tr>
<tr>
<td>Mean radius of control helix (mm)</td>
<td>96</td>
<td>70</td>
</tr>
<tr>
<td>Heating power (kW)</td>
<td>200</td>
<td>600/4000</td>
</tr>
<tr>
<td>$t_0$ (“standard case”)</td>
<td>1.1</td>
<td>1.44</td>
</tr>
<tr>
<td>$t$ range</td>
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<td>0.96 to 2.5</td>
</tr>
<tr>
<td>Well range</td>
<td>-0.02 to 0.07</td>
<td>-0.01 to 0.06</td>
</tr>
</tbody>
</table>

A valid way of estimating the MHD stability of low-shear, net-current-free stellarators is to evaluate the Mercier criterion for stability against ideal interchange modes [4]. The 3D Mercier criterion for closed magnetic flux surfaces can be written as

\[ D_M = D_S + D_I + D_W + D_G > 0 \]

where $D_S$, the main contribution due to the shear, and $D_I$, which includes the contribution of a net current, are small for low-shear, net-current-free stellarators. Thus, instability comes about as a result of the pressure-induced currents parallel to the magnetic field (Pfirsch-Schlüter currents) causing the negative term $D_G$ to dominate the stabilizing well term $D_W$. In this paper we evaluate the Mercier criterion for MHD equilibria calculated by the fixed boundary version of the VMEC 3D-equilibrium code [5] for both HI and TJ-II.

#### 1. HI STUDIES

The results presented in Table II represent a scan of some of the configurations accessible to HI by varying the currents in the control helix and the vertical field coils [1]. The Mercier criterion has been evaluated using the JMC code of Nührenberg [6].
TABLE II. HI MAGNETIC CONFIGURATIONS

| Case | $I_{ho}/I_{ce}$ (%) | Aspect ratio | Well (%) | $\varepsilon(0)/\beta$ | $|\Delta \varepsilon|/\varepsilon_0$ (%) | $<\beta>_c$ (%) | X(2,1) | X(1,0) | B(0,1) | B(1,0) |
|------|----------------------|--------------|----------|-----------------|-----------------|----------------|--------|--------|--------|--------|
| (a)  | 0                    | 8.1          | 1.3      | 0.37            | 3               | 0.95           | 0.74   | 0.5    | 0.4    |
| (b)  | 0                    | 7.1          | 0.3      | 0.37            | 4               | 0.42           | 0.25   | 0.5    | 0.2    |
| (c)  | 0                    | 7.7          | 1.6      | 0.37            | 3               | 0.95           | 0.91   | 0.5    | 0.6    |
| (d)  | 8                    | 6.6          | 1.2      | 0.43            | 1               | 0.50           | 0.36   | 1.1    | 0.4    |
| (e)  | 8                    | 6.6          | 3.2      | 0.43            | 1               | 0.60           | 0.73   | 1.3    | 0.5    |
| (f)  | 8                    | 9.0          | 3.2      | 0.44            | 1               | 0.82           | 0.56   | 1.4    | 1.2    |
| (g)  | -8                   | 7.0          | -0.3     | 0.29            | 14              | 0.08           | 0.05   | 0.2    | 0.03   |
| (h)  | -8                   | 8.1          | 0.8      | 0.29            | 9               | 0.45           | 0.60   | 0.2    | 0.4    |
| (i)  | 9                    | 4.8          | 0.2      | 0.45            | 3               | 0.07           | 0.07   | 2.0    | 0.04   |
| (j)  | 9                    | 9.9          | 3.8      | 0.45            | 1               | 0.45           | 0.41   | 1.9    | 1.5    |
| (k)  | 24                   | 6.6          | 2.8      | 0.54            | -5              | 0.16           | 0.16   | 2.7    | 0.4    |
| (l)  | 24                   | 8.8          | -0.5     | 0.54            | -4              | 0.05           | 0.05   | 1.5    | 0.6    |
| (m)  | 30                   | 7.3          | 1.9      | 0.58            | -5              | 0.40           | 0.60   | 1.2    | 0.5    |
| (n)  | 30                   | 8.4          | 0.3      | 0.58            | -4              | 0.16           | 0.27   | 1.0    | 0.7    |

We emphasize that there has been no attempt to optimize $<\beta>$ and that good equilibria can be obtained with the VMEC code at $<\beta>$ values well above $<\beta>_c$. The survey considered two pressure profiles — a broad profile which was linear in the flux $\Phi$ ($p \propto 1-\Phi$) and had its largest pressure gradients at the edge of the configuration, and a more peaked profile ($p \propto (1-\Phi)^2$) in which the largest pressure gradients were closer to the magnetic axis. Whereas in many cases the broader profile was stable at higher $<\beta>$, there are also many examples where an unfavorable magnetic structure near the edge of a set of flux surfaces gives rise to a region of instability there at $<\beta>_c$ lower than that for the more peaked profile.

Two main conclusions can be drawn from Table II. Firstly, the H1 plasma should show the effects of local instability at readily accessible $<\beta>$ values ($\leq 1\%$),
TABLE III. TJ-II MAGNETIC CONFIGURATIONS

<table>
<thead>
<tr>
<th>Case</th>
<th>$t/4$ (0)</th>
<th>$I_{C}$ (kA)</th>
<th>$I_{HC}$ (kA)</th>
<th>Well (%)</th>
<th>av. minor radius (m)</th>
<th>$\langle \beta \rangle_c$ (%) broad</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>0.365</td>
<td>290</td>
<td>92</td>
<td>1.5%</td>
<td>0.16</td>
<td>0.4</td>
</tr>
<tr>
<td>A2</td>
<td>0.365</td>
<td>219</td>
<td>95</td>
<td>2.2%</td>
<td>0.15</td>
<td>2.5</td>
</tr>
<tr>
<td>A3</td>
<td>0.365</td>
<td>105</td>
<td>82</td>
<td>3.5%</td>
<td>0.14</td>
<td>3.8</td>
</tr>
<tr>
<td>A4</td>
<td>0.370</td>
<td>65</td>
<td>55</td>
<td>4.5%</td>
<td>0.13</td>
<td>6.7</td>
</tr>
<tr>
<td>B1</td>
<td>0.285</td>
<td>163</td>
<td>39</td>
<td>1.5%</td>
<td>0.12</td>
<td>Self-Stable</td>
</tr>
<tr>
<td>B2</td>
<td>0.300</td>
<td>82</td>
<td>42</td>
<td>1.7%</td>
<td>0.08</td>
<td>Self-Stable</td>
</tr>
</tbody>
</table>

with the "standard" configuration (with no current in the control helix) being the most stable. Secondly, the global magnetic well depth alone is not a good parameter for ideal stability. In H1 this is due to three factors: the shape of the magnetic well profile across the flux surfaces (particularly interesting for case (i) where a 2% well across the inner flux surfaces changes to a hill at the edge, causing a localized region of instability there for both pressure profiles); the stabilizing effects of shear (important for cases (g) and (h)); and the dependence of the destabilizing Pfirsch-Schlüter currents on the rotational transform. We have found that the normalized parallel current density, $X = J.B/B^2$, can be strongly dominated by its $m=2$, $n=1$ component (where $m$ is the poloidal and $n$ is the toroidal mode number per period) even when the $t = 1/2$ surface is outside the plasma. This is the case for the high transform configurations shown in the table (where representative values of $X$ are given at the surface corresponding to one half of the normalized flux). Thus the high transform, high-well cases studied do not yield a dramatic improvement in stability. We note that the resistive interchange criterion [7] yields a lower critical $\langle \beta \rangle$ for the high-shear configurations (g) and (h) and that the negative-shear configurations may be susceptible to ballooning modes [8].

Another feature of Table II is the variation in the shape of the magnetic field strength, B, across the configurations (as shown by the ratio of its $m=0$, $n=1$ component to its axisymmetric component, $m=1$, $n=0$, at the outermost flux surface). This suggests a significant variation in neoclassical transport over the parameter space of H1.

2. TJ-II STUDIES

We have studied two families of configurations accessible to TJ-II, briefly described in Table III.
2.1 Mercier studies

Figure 1 summarizes the results obtained when the 3-D Mercier criterion is applied to configurations of family A, all having the same iota/period (0) = 0.36 and different values of the magnetic well. We have found that instabilities in TJ-II onset at the boundary of the configuration and as β increases the region of instability broadens towards the center of the configuration [9]. For configurations having the same iota, but different values of the vacuum magnetic well, the magnetic well acts as a clear stabilizing mechanism and as is shown in figure 1, the onset of the instabilities is at higher values of <β> as the well deepens. Configuration A4 (magnetic well in vacuum = 4%) is still stable at <β> = 6%. Nevertheless as found in H-1, the stability of the configuration cannot be foreseen from the knowledge of the vacuum magnetic well depth alone and the existence of a global magnetic well is no guarantee of stability. The pressure profile used in this study was the broad profile described in section 1, for a comparison between broad and peaked profiles in TJ-II equilibria, please see Ref. [9].

Family B of configurations having iota/period = 0.3 shows a qualitatively different behavior when studied with the Mercier criterion. Figure 2 shows the Mercier criterion in arbitrary units for several values of the normalized radius of the
configuration as a function of the central pressure. The Mercier criterion becomes more positive over the whole configuration as $\beta$ increases exhibiting a self-stabilizing behavior characteristic of a second stability regime. It should be emphasized that the vacuum magnetic well has the value of 1.7% for this configuration. The different behavior of the two families is understood when one analyzes further the $D_w$ term in the Mercier criterion, for family A, $D_w$ grows linearly with $\beta$, but for family B, $D_w$ goes like $\beta^2$ therefore offsetting the negative dependence with $\beta$ of the Pfirsch-Schlüter currents.

### 2.2 Low-n studies

The flexibility of the TJ-II device allows to control the rotational transform profile and the shear. The configurations of family A scan, $\epsilon/4 = 0.36$, were chosen in order to avoid the presence of the most dangerous low-order resonances in the plasma. However, for the $\epsilon/4 = 0.30$ case, resonances corresponding to $\epsilon = 6/5, 7/6$ appear at some values of $\beta$. To study their low-$n$ linear stability properties, we use a set of reduced MHD equations which have been derived by applying the averaging method to

![Diagram showing the Mercier criterion in arbitrary units vs. $\beta(0)$ for TJ-II configuration B2 at several normalized radii.](image)
an equilibrium represented in a straight field coordinate system [10]. We have found that these configurations are stable to low-n modes. The results are thus in agreement with the Mercier criterion.

ACKNOWLEDGEMENTS

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EQUILIBRIUM PROPERTIES AND APPROACH TO HIGH MAGNETIC FIELD IN A SPHEROMAK

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Abstract
EQUILIBRIUM PROPERTIES AND APPROACH TO HIGH MAGNETIC FIELD IN A SPHEROMAK.

The paper presents the results of spheromak research done by four separate groups in Japan: (1) The best fitting curve for the observed magnetic field in an equilibrium configuration in CTCC-II is obtained by using an anisotropic pressure. (2) A theoretical study on the configuration and stability of a flux core spheromak (FCS) with finite pressure is presented. (3) Additional helicity injection into a spheromak is investigated in FACT. The results obtained demonstrate three to four times flux amplification of the seed spheromak. (4) Experimental results on the flux amplification and the sustainment of FCS are also presented by two types of experiment in TS-3. The sustainment time is observed to be four times the resistive decay time.

1. Introduction

Research on a spheromak configuration is carried out in detail to improve the confinement properties of the CTCC-II device at Osaka University. The effect of anisotropic pressure is taken into consideration since the spheromak configuration shows a large difference in magnetic field strengths between an inner and an outer part with respect to the magnetic surface (high mirror ratio) [1].
Additional helicity injection into a spheromak plasma is expected to achieve the production and sustainment of a high field spheromak, which is the next milestone. For this purpose, three groups in Japan have made research on the helicity injection of spheromak. Kaneko's group of the University of Tokyo theoretically demonstrates the equilibrium configuration and the stability of the flux core spheromak (FCS) with finite pressure for the CTCC-II device. The FACT experiment at the Himeji Institute of Technology shows an amplification of the fluxes of the seed spheromak by both inductive and electrostatic methods [2]. The TS-3 experiment at the University of Tokyo demonstrates the production of free boundary FCS and the amplification of FCS flux by two methods [3].

2. Equilibrium properties in the CTCC-II device

In the CTCC-II experiment, the initial plasma is produced by a magnetized gun and ejected into an aluminum flux conserver (FC) with a thickness of 15 mm. The spheromak is formed in the FC with a lifetime of 1.5 ms. Measurements show that the average electron density is $2-4 \times 10^{19} \text{ m}^{-3}$ and the electron temperature is 20-60 eV, the beta value on the magnetic axis is estimated to be about 7-12%, from measurements by Thomson scattering. The field profile is measured by an array of magnetic probes and is compared in detail with the theoretical profile which is analysed by the anisotropic Grad-Shafranov equation [1].

The result is depicted in Fig. 1. Profile (a) is roughly fitted by a theoretical profile assuming a low beta limit. However, there is a systematic deviation between measured and theoretical profiles. Profile (b) is fitted by assuming an isotropic plasma pressure. In this case, when the parameters including the beta value are optimized, the beta value obtained is above 35%, which is much larger than the experimental result. Profile (c) is fitted by assuming anisotropic pressure. Then, the best fitting profile is obtained, and the estimated beta value is 10%. The result is consistent with the experimental findings. The degree of anisotropy defined as 2
3. Stability of FCS with finite pressure

The work dealt with in the foregoing section concerns the equilibrium configuration. Another investigation on the stability of the CTCC-II spheromak was performed by using isotropic pressure obtained by optimization according to the Mercier criterion [5]. The maximum value, $\beta_{\text{max}}$, of the beta
ratio on the magnetic axis is shown to be 6% at most. This is a reasonable value in comparison with the experimental values stated in the previous section.

The equilibrium and the stability of the FCS in CTCC-II are investigated in this section. As is well known, FCS is designed for the helicity injection. Let us choose the FC hole as the entrance of the flux core and open another hole at the bottom of the FC to exhaust the flux core. Let us assume two electrodes placed at the entrance and the exit. The electric current flowing along the flux core is supposed to be force free.

The Grad-Shafranov equation is solved by the grid generation method. A model for the functions of pressure and toroidal magnetic field is adopted. An example for an equilibrium configuration is shown in Fig. 2, where a choking coil current of $I_c = 0.0$ is assumed. The parameters $\lambda$ and $\lambda_0$ are the ratio between total current and total flux in the flux core and the lowest eigenvalue when all plasma boundaries are conductive, respectively.

The stability of the equilibrium is investigated by the Mercier criterion (Fig. 3). We have two regions satisfying the Mercier criterion. One is a spheromak-like region ($\lambda \approx \lambda_0$).

![Figure 2. Equilibrium configuration with $I_c = 0.0$.](image)
where the safety factor $q(\phi)$ assumes a maximum value on the magnetic axis and decreases monotonically towards the separatrix. The other one is a tokamak-like region ($\lambda < \lambda_0$), where $q(\phi)$ increases monotonically. There is an unstable region between these two regions. The effect of $I_c$ is also investigated: it has a stabilizing effect in the spheromak-like region if the proper value of $I_c$ is used. In the tokamak-like region, there is a destabilizing effect.

4. Flux amplification and sustainment of the spheromak in the FACT experiment

In the FACT experiment, additional helicity injection into a seed spheromak is carried out to amplify magnetic fluxes by both inductive and electrostatic methods [2]. The seed spheromak is produced by a magnetized coaxial gun.

In the inductive method, a toroidal voltage, $V_t$, is applied to the seed spheromak in the flux amplifying (FA) region. $V_t$ is generated by an oscillating current in the FA coil, which is embedded in the extended internal gun electrode.

In the electrostatic method, a seed spheromak is translated into the FC, and a FCS is formed with external
fluxes encircling the plasma. To inject magnetic helicity, a DC voltage is supplied between the two electrodes which the external fluxes intersect. Then, a DC current flows along these external fluxes.

We have two manners of applying the DC voltage to the FCS in the FC. In the first manner, the DC voltage is applied between the extended internal gun electrode and the FC wall. The FA coil is used to apply the external fluxes. In the second manner, a pair of columnar and hollow electrodes are placed near the exit hole of the FC. The voltage between the two electrodes is applied through pulse forming networks (PFNs) for long pulsed discharges. In this case, a divertor field is formed by using an external coil, as shown in Fig. 4. The DC current flows along the lines of force of the divertor field.

The effect of flux amplification due to helicity injection is investigated by measurements of the magnetic-field profiles and the magnetic-fluxes of the formed spheromak in the FC. Both inductive and electrostatic methods show a three to four times amplification of the seed spheromak. The maximum poloidal flux is about 16 mWb. In the first manner of the electrostatic method, the poloidal magnetic field is measured at 100 mesh points. The magnetic flux contours obtained are shown in Fig. 5. A spheromak configuration with external flux is initially formed in the FC and subsequently the flux.
increases during helicity injection. In addition, the flux conversion is shown to be related to the $m = 1$ helical kink distortion of the external flux (the flux core) and of the current flowing in it. In the second manner of the electrostatic method, the spheromak configuration is observed to be maintained as long as the voltage between the two electrodes is maintained.

5. Formation and sustainment of free boundary FCS in the TS-3 device

The TS-3 experiments with an axial $z$-discharge electrode [3] successfully demonstrate the production and the sustainment of a novel type of spheromak configuration, i.e. the FCS [6]. Two types of experiment have been carried out. In the first type, the $z$-$\theta$ discharge is initiated in a reversed bias magnetic field and a conventional spheromak (seed spheromak) is produced. This spheromak has a currentless flux
hole in the central, axial part. For an additional helicity injection, a couple of central electrodes with an end separation of 52 cm are used and an axial z-discharge current $I_c$ is applied. Results show that both the toroidal flux $\Phi_t$ and the toroidal current $I_t$ increase with increasing $I_c$ as shown in Fig. 6.

In the course of these experiments, it is found that, with linked mode operation (the outer part of the poloidal fluxes of FCS is linked with the internal PF coils), global instabilities are suppressed, and the FCS configuration is sustained for 120 $\mu$s for $I_c$ with a pulse width of 160 $\mu$s. This
value is about four times as much as the resistive decay time without helicity injection.

In the second type of experiment, the FCS is only produced by the central \( z \)-discharge in the reversed bias field (Fig. 7, \( \text{time} = 0 \)). In this case, a stable FCS is obtained when the separation of the two center electrodes is reduced to less than 32 cm. The axial field is measured at 30 mesh points (6 x 5)
in a poloidal plane simultaneously at each shot to calculate the poloidal flux. An example for the poloidal flux contours obtained is shown in Fig. 7, where the poloidal flux surrounding the magnetic axis is 2.5 mWb. Sometimes a doublet type configuration appears. An axial compression is performed by increasing the PF coil current (rise-time about 30 μs). Then, the doublet type FCS can be shaped into a single-magnetic-axis type one and the closed poloidal flux of the FCS increases to about 6 mWb. This value is 2.4 times the maximum value of the non-compression case.

6. Summary and conclusions

(1) The observed magnetic field profile is consistent with the theoretically estimated profile when the effect of anisotropic pressure is taken into consideration.
(2) The stability of an FCS equilibrium is investigated by the Mercier criterion. Two regions (a spheromak- and a tokamak-like region) are shown to satisfy this criterion.
(3) Helicity re-injection results in three or four times flux amplification of the seed spheromak by both inductive and electrostatic methods.
(4) Production and sustainment of FCS is demonstrated by two types of experiment. The sustainment time is 120 μs, that is, about four times the resistive decay time.

References

EXPERIMENTAL INVESTIGATION OF THE EFFECTS OF HELICITY ON MAGNETIC RECONNECTION

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Abstract

EXPERIMENTAL INVESTIGATION OF THE EFFECTS OF HELICITY ON MAGNETIC RECONNECTION.

The effects of helicity on magnetic reconnection are investigated by taking into account all three vector components of the magnetic field in a laboratory experiment. Two toroidal, magnetized plasmas of spheromak configuration carrying identical toroidal currents and poloidal field configurations are made to collide, thereby inducing magnetic reconnection. It is found that plasmas of antisymmetric helicity merge appreciably faster to create a high beta field reversed configuration (FRC) than those of equal helicity.

1. INTRODUCTION

In current carrying plasmas for magnetic fusion research such as tokamaks, reversed field pinches (RFPs) and spheromaks, magnetic reconnection plays an important role in determining their magnetic configurations and transport properties [1, 2]. In these plasma devices, magnetic reconnection has often been investigated as a global phenomenon by monitoring the total magnetic flux, helicity and energy of the magnetically confined plasmas. However, the local features have not been documented because of the difficulty of directly measuring the internal field line structure. To understand magnetic relaxation mechanisms such as internal disruptions in the toroidal plasmas [2], a clear local picture of the magnetic reconnection processes in toroidal geometry is desired.

This paper addresses two important issues: (a) how the third-dimensional component of the magnetic field line affects the reconnection, and (b) how the global plasma characteristics influence the local features of the reconnection.

The direction of the toroidal field plays an important role in the reconnection process. For example, we expect that two plasmas of antiparallel toroidal field merge
Formation of Spheromak

 Formation of FRC

(a)

(a')

(b)

(b')

(c)

(c')

FIG. 1. Three-dimensional effects of magnetic reconnection. (a), (a'): 2-D poloidal picture of evolution of magnetic field line before and after merging; (b), (b'): 3-D description of evolution for merging two toroidal plasmas with equal helicity to form a spheromak, before and after reconnection; (c), (c'): 3-D description of evolution for two plasmas with opposite helicity to form an FRC, before and after reconnection.

differently from those of parallel toroidal field, as shown in Fig. 1. The most commonly used description of magnetic field line reconnection is shown in Fig. 1(a'), based on two-dimensional (2-D) analyses of magnetic field evolution as made by Sweet, Parker and Petschek [3, 4]. In actual reconnection phenomena however, the magnetic field lines have significant components in all three dimensions, as is observed in solar flares and in most laboratory experiments. The same 2-D picture of the field line shown in Figs 1(a) and (a'), describing the merging of two plasma toroids carrying equal currents, appears quite differently in the 3-D sketches shown in Figs 1(b) and 1(b') as well as 1(c) and 1(c'). Even though their 2-D representations are identical, the three-dimensional pictures of the merging of two otherwise identical toroidal plasmas differ strongly, depending on whether their initial helicities were parallel or anti-parallel. In the former case, the field lines merge at various angles, while in the latter case the field lines merge exactly with anti-parallel symmetry. In addition, the internal toroidal field is necessarily accompanied by a poloidal plasma current and the additional $\vec{j} \times \vec{B}$ force changes the character of the magnetic reconnection. In general, in the case of merging counter-helicities, there is a parallel poloidal local current on both sides of the reconnection region, while the currents flow with an angle to each other for co-helicity merging.

There is another important difference in the reconnection patterns shown in Figs 1(b) and (c). Conserving helicity, the transition from the configuration of 1(b) to (b') should be globally smooth. But in the case of counter-helicity merging, 1(c) and (c'),
the pitch of the field lines changes abruptly at the reconnection point. One expects violent plasma acceleration as the field lines contract after reconnection (a slingshot effect), and a substantial heating can occur during the formation of an FRC with the total helicity being reduced to zero.

To describe the effects of the toroidal field on the magnetic reconnection quantitatively, one can introduce a notion of 'magnetic helicity' for the topological structure of plasma configuration. Taylor [5] defined the magnetic helicity to measure the linkage of magnetic field lines as

\[ K = \int A \cdot B d^3x \]  

where \( A \) is the vector potential for the magnetic field vector \( B \). Using the Stokes' theorem, we express the magnetic helicities of the two spheromaks as

\[ K = \pm c \psi_s \phi_s \]  

in which \( \psi_s \) and \( \phi_s \) are the poloidal and toroidal fluxes contained in the spheromak plasmas, and \( c \) is a profile factor [6].

2. EXPERIMENTAL SET-UP

Experiments are carried out in the TS-3 spheromak device at the University of Tokyo [7], in which two toroidal shape spheromaks (with a currentless regime in the symmetric axis) are generated to merge together. In the earlier experiments global features of merging spheromaks were studied [8].

The polarity of \( K \) for the two spheromaks is determined independently by the direction of the initial discharge currents to create toroidal fields. The average plasma density is about \( 3 \times 10^{14} \) cm\(^{-3} \) (for H and He discharges), the electron temperature \( T_e \approx 5-15 \) eV, the peak toroidal field \( B_t \leq 1 \) kG, the average beta \( \langle \beta \rangle \leq 20\% \), the magnetic Reynolds number \( S \approx 300 \), and the toroidal plasma current \( I_{pt} \approx 30-50 \) kA. To investigate magnetic field line reconnection in the neighbourhood of the midplane, \( z = 0 \), the plasmas of \( R_p \approx 15 \) cm and \( r_p \approx 8-10 \) cm are made to collide. The ion gyroradii are much smaller (2-5 mm) than the plasma sizes. To document the internal magnetic structure of the reconnection on a single shot, a two-dimensional magnetic probe array is placed on an \( r-z \) plane of the vessel. This \( 5 \times 7 \) array (grid spacing 5 cm \( \times \) 5 cm) is composed of 35 small pick-up coils inside five glass tubes of 5 mm diameter. Signals from additional monitoring probes showed this array did not disturb the plasma magnetics by a large amount (\( \delta B/B \leq 5\% \)).
3. EXPERIMENTAL RESULTS

As a first step in the three-dimensional analysis of the magnetic reconnection laboratory experiment, the present TS-3 experiment focuses on the effects of the third (toroidal) vector component of the magnetic field, i.e. on magnetic helicity effects.

To investigate the effects of toroidal field on reconnection, the merging of two toroidal plasmas of the same helicity (K + K) is compared with the merging of opposite helicities [K + (−K)]. Figure 2 presents a time evolution of experimentally measured poloidal flux (assuming axisymmetry) taken for two cases of merging plasmas, for co- and counter-helicity merging. The other plasma parameters were kept identical for each discharge.

As is shown in the figures, spheromaks of opposite helicity merge more efficiently than those with the same helicity. Generally speaking, the former merge violently, in agreement with the expectations mentioned above. Merging is often
accompanied by a magnetic fluctuation of 100 kHz whose dominant toroidal mode number is measured to be \( n = 1 \) and/or \( n = 2 \). The phase velocity of the mode is \((1-2) \times 10^7 \text{ cm/s}\), roughly equal to \( v_{\text{Alfvén}} \). The merging of two spheromaks with the same helicity occurs rather smoothly, and the total helicity of the spheromaks tends to conserve, as was reported previously.

In the case of co-helicity merging, the reconnection rate is observed to slow down significantly as the merging progresses while, in the case of counter-helicity merging, reconnection continues until the spheromaks merge completely. To describe the reconnection quantitatively, we introduce a coefficient, \( \alpha_c \) (termed the common flux ratio) to represent how much poloidal flux is common to the two plasmas:

\[
\alpha_c = \psi_c/\psi_p
\]

where \( \psi_c \) and \( \psi_p \) are the values of highest common flux and peak flux of each plasma, respectively. If the peak values of the two plasmas do not agree (generally, \( \Delta \psi_c/\psi_p < 0.1 \)), the smaller value is used. By monitoring \( \alpha_c \) versus time, one can quantify the rate of magnetic field connection by \( d\alpha_c/dt \) in the regime occupied by the two plasmas. In the present experiment, the reconnection is analysed as a local phenomenon between the two plasmas — we count \( \psi \) only inside the separatrix region shown in Fig. 2. Figure 3 depicts \( \alpha_c \) versus time for various colliding velocities for co- and counter-helicity merging. It is observed that \( \alpha_c \) increases initially with almost the same speed for co- and counter-helicity merging, but the reconnection rate slows down significantly after \( \alpha_c \) has reached 50% in co-helicity merging, while it progresses with more or less the same speed in counter-helicity merging until it reaches 100%.

Here, we should note that the angle of merging field lines changes gradually from 180° to 0° for co-helicity merging as reconnection progresses because the rotational transform of the flux hole spheromak [5] varies drastically with respect to radius (\( q = 0 \) at the edge and \( q = 0.6 \) at the magnetic axis, where \( q \) is the inverse of the rotational transform). For counter-helicity merging, the angle is always 180°. In a recent computer simulation [9], it was shown that the reconnection occurred most efficiently for the merging angle of 180° and least efficiently for 0°. The observed inefficiency of co-helicity merging in the later phase is consistent with the simulation.

As was mentioned earlier, a formation of FRC (field reversed configuration or \( \theta \)-pinch configuration [10]) is expected as a result of the counter-helicity merging. The toroidal field components have been measured by the magnetic probe arrays. It is found that the peak toroidal field diminishes quickly in the case of counter-helicity merging, although it should be noted that oppositely directed weak toroidal fields sometimes co-exist during and after the merging. This result is in clear contrast to that of co-helicity merging in which the toroidal field profile in the closed poloidal flux regime indicates the normal additive nature of the co-helicity spheromak mergings.
FIG. 3. Common flux ratio $\alpha_c$ versus time for reconnection of two counter-helicity plasmas (a) and of two co-helicity plasmas (b). For each case, the relative velocity of the plasma merging is varied from $2.5 \times 10^5$ cm/s ($\sim 0.02 V_A$) to $7.5 \times 10^5$ cm/s.

Furthermore, the beta values of the final plasma configurations are evaluated from the measurements of the average density by CO$_2$ laser interferometer, the magnetic configuration and Langmuir probes. The average density of the final plasma after counter-helicity merging is appreciably larger than that after co-helicity merging. It was estimated that $0.5 \leq \langle \beta \rangle \leq 1$ for the counter-helicity merged plasmas, which suggests a formation of the FRC configuration, while $\langle \beta \rangle < 0.3$ for the co-helicity merged spheromaks. To date, no significant change in the electron temperature has been detected during the mergings, and it is suspected that most of the dissipated magnetic energy from the reconnection is lost through the ionization of residual neutral atoms and the impurity radiation channels.

Another significant result of the present preliminary experiment is the observation of a strong dependence of the reconnection rate on the mutual approaching speed of the two plasmas as seen in Fig. 3. The approaching speed of two spheromaks, which is much smaller than $v_{Alfv\text{\textae}}$, can be controlled by adjusting the poloidal bias field. This trend suggests the importance of an external driving force, supporting a certain aspect of a driven reconnection model. In many recent tokamak experiments, a rather fast magnetic reconnection has been observed during the sawtooth
crashes, and present results can support the notion that a fast plasma motion inside the $q = 1$ flux surface triggers the fast reconnection [2, 11].

In the future, we plan to examine the role of helicity in energy conversion to directed flows, the role of turbulence in thermalizing directed flow energies, and the role of magnetic fluctuation in changing the reconnection region from the x-point-line of 2-D pictures to 3-D volume filling processes. In the case of antisymmetric helicity, the resulting FRC configuration [10] is expected to have an $s$-value ($s$ is the ratio of separatrix radius to average ion gyroradius) exceeding ten, offering the chance to investigate the MHD stability of large-$S$ FRCs.

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EXPERIMENTS ON THE MM-4U, MM-2 AND HER MIRRORS

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Abstract

EXPERIMENTS ON THE MM-4U, MM-2 AND HER MIRRORS.

Three lines of experiments on open confinement systems have been followed which are concentrated on the key issue of experimental investigation of various MHD stabilization methods in axisymmetric end cells. (1) MM-4U is a small tandem mirror with cusp end cells. By use of electron beam injection from the ends to build the plasma, a negative potential plasma has been formed and preliminary measurements of the axial potential profile have been carried out. (2) After formation of a hot electron ring in the MM-2 simple mirror, the spatial distribution of hot electrons and ECRH trapping of an electron beam have been investigated. (3) The stabilizing effect of line tying of the conducting wall on the interchange mode is studied in the HER mirror.

1. INTRODUCTION

To develop the tandem mirror into a fusion reactor, the radial transport, driven by the asymmetric magnetic field component of the end cell, has to be overcome. So in recent years there has been an interest among mirror researchers in seeking various new methods to stabilize the MHD modes in a full axisymmetric configuration or with an axisymmetric mirror as an end cell. As is well known, these methods include the cusp end cell, the RF stabilization effect, an octupole plus a hot electron ring,

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conducting wall stabilization for high $\beta$ plasma, sloshing ion distribution, or a combination of these. In China, three mirrors are now in operation. They are MM-4U and MM-2 in Leshan and HER in Hefei. The experimental results obtained from these mirrors in the past two years are presented in Sections 1, 2 and 3, respectively.

2. NEGATIVE PLASMA POTENTIAL PROFILE IN THE DOUBLE CUSP TANDEM MIRROR MM-4U

Obviously the plasma confined in a cusp is MHD stabilized. The MM-4 is an electrostatic plugging cusp device. By use of electron beam injection from the end to build the plasma and supplying an inner plugging positive electrode, a negative potential plasma has been built in MM-4 [1].

On the basis of the experimental results of MM-4, the negative potential tandem mirror is very attractive, so we modified the single cusp MM-4 to a small double cusp tandem mirror, MM-4U. Schematic diagrams of the MM-4 device, its magnetic field and diagnostic arrangements are shown in Fig. 1. The main parameters of MM-4U are: device length 3.5 m, maximum external diameter 0.8 m, distance between edges of outer coils 1.28 m, mirror space of the central cell 0.6 m, magnetic field at point cusp 0.5 T and at ring cusp 0.36 T, mirror ratio of central cell 11.3, background pressure in the end cells $3 \times 10^{-4}$ Pa and in the central cell $1 \times 10^{-3}$ Pa; a maximum plugging voltage of 8 kV and an electron beam current of 25 to 500 mA are to be used. By use of a rank of electric probes, capacitive probes, microwave interferometer and diamagnetic probes, the electron density and temperature, the floating potential and the plasma potential at different points along the axis, and the $\beta$ value of the plasma can be measured. The first results obtained are: electron density $n_E = 1.7 \times 10^{11}$ cm$^{-3}$ (E: east end cell), $n_W = 4.7 \times 10^{10}$ cm$^{-3}$ (W: west end cell) and $n_C = 7.5 \times 10^7$ cm$^{-3}$ (C: central cell); electron temperature $T_E = T_W = 20$ eV; plasma potential $V_{PE} = -180$ V, $V_{PW} = -160$ V and $V_{PC} = -1.8$ V. The axial potential profile measured during electron beam injection is shown in Fig. 2. The density profile obtained is similar to that of the potential. In the central cell a 7–9 kHz oscillation has been measured under the above experimental conditions.

From the results mentioned above, we can find that the plasma potential in the whole system is negative. A more interesting result is that there are two potential wells between the cusp and the central cell. Their action seems to be like that of a thermal barrier. The difference between $V_{PE}$ and $V_{PW}$ may be caused by the different intensity of electron beam injection between east and west end cells. In the central cell the absolute values of the plasma density and potential are fairly low and a low frequency oscillation has been observed. These facts indicate that in the central cell there exists an MHD instability which resists the raising of the plasma parameters (density, potential, etc.). But in the end cusps the stabilizing effect of minimum B on the MHD mode is so strong that MHD oscillation cannot be monitored as the density approaches $10^{12}$ cm$^{-3}$. 
FIG. 1. (a) Schematic diagram of the MM-4U device. 1: electron gun; 2: end electrode; 3: ring electrode; 4: end cell cusp; 5: central cell; 6: magnetic force line; 7: adjustable electrode. (b) Magnetic field strength profile along the device axis. (c) Arrangement of diagnostics. 1: plugging positive electrode; 2: diamagnetic probes; 3: magnetic force line; 4: microwave interferometer; 5: Langmuir probes.
As our next steps, we plan to improve the magnetic field configuration between the cusp end cell and the central cell, to enhance the field strength of the central cell, i.e. to decrease the mirror ratio, for suppressing MHD instability in the central cell, to alter the electron beam injection (raise the energy and enhance the intensity), for studying the potential confinement configuration for the tandem mirror, to use RF plasma heating in the central cell and to improve particle transport in the thermal barrier-like region, etc.

3. SPATIAL DISTRIBUTION OF HOT ELECTRONS AND ECRH TRAPPING OF ELECTRON BEAMS ON THE MM-2 MIRROR

The MM-2 is a simple mirror adapted from a previous small minimum B set-up. On the basis of the hot electron ring experiment in MM-2 [2], we plan to further investigate the MHD stabilization methods, RF stabilization, an octupole plus a hot electron ring and a sloshing ion distribution. So the MM-2 is being developed to the larger MM-2U mirror, with a mirror ratio of 2.2 to 1, a mirror space of 0.8 m and a central field of 0.3 T. The Ioffe bar of the previous MM-2 will be modified to octupole coils in MM-2U. MM-2U has an RF power supply (30 kW, 5 MHz) to run the RF heating and RF stabilization experiments. An 80 kW neutral beam injector will be used on MM-2U to build a sloshing ion distribution.

While the MM-2U mirror is being assembled, ECRH experiments are continuing in the MM-2 device. Spatial distribution of the hot electron emission has been obtained using a hard X ray pinhole camera. The camera, with a microchannel plate and screen intensifier, is set along the device axis. Clear visual pictures of the electron annulus can be taken under suitable discharge conditions to deduce a two dimen-
FIG. 3. Two dimensional contour of hard X ray radiation intensity from MM-2 (arbitrary units).

sional contour of radiation intensity. A typical result is shown in Fig. 3. This direct display method, without disturbing the plasma, is useful for acquiring information on the structure and movement of the annulus. Usually azimuthal asymmetric electron annuli were observed, some with a gap at the bottom of the ring. As described in Ref. [2], the microwave was injected from the top window of the device. The wave energy deposition is azimuthally asymmetric. If we increase the power of microwave injection, the hot electron population at the centre will be increased and the azimuthal asymmetricality of the annulus can be changed.

Meanwhile, by injecting the electron beam (1–5 keV) into the mirror at an angle $\phi$ to the magnetic force line, where $\phi$ is approximately equal to the loss cone angle, experiments on electron beam trapping by ECRH have been carried out. The main results obtained are as follows: (1) Owing to ECRH trapping of the injected electrons, the preionization time of the plasma can be shortened greatly. (2) The $\beta$ value of the hot electron ring increases by about 62%. (3) The trapping efficiency of the electron beam is about 30–40%. These results are shown in Fig. 4.
FIG. 4. Variation of the preionization time, X ray counts and diamagnetism with neutral pressure (open symbols: with EBl; closed symbols: without EBl).
4. **LINE TYING EFFECT ON PLASMA MHD INSTABILITY IN THE HER MIRROR**

In microwave produced hot electron plasma, the line tying stabilizing effect should be eliminated in order to determine the hot electron component stabilizing effect on cold plasma.

The experiment on the line tying effect was performed in the HER simple mirror. Plasma was produced by a Ti gas loaded, washed stack plasma gun. The plasma temperature was 5–10 eV and the density was approximately $2 \times 10^{13}$ cm$^{-3}$. The line tying effect could be controlled by changing end plates and side wall conductance. The four conditions used for experiments were (see Fig. 5): (1) The whole side wall and end plates were conductors. (2) Only part A was covered by a glass plate.

![FIG. 5. Experimental set-up of the HER mirror.](image)

![FIG. 6. Critical $\beta$ for the four types of line tying conditions as a function of plasma radius.](image)
(3) Both part B and part C were covered by a glass tube. Part A was the same as for case (2). (4) Parts A, B, C and D were all covered by glass. The marginal $\beta$ values at the onset of disruption under the four line tying conditions are shown in Fig. 6. It is clear that a strong line tying effect favours the increase of the $\beta$ value. The relation between $\beta$ value and plasma radius $r_p$ indicates that for larger $r_p$, i.e. weaker density gradient, relatively lower fluctuation levels are excited, leading to a relatively larger $\beta$ value (because the probe measurement shows that the peak density fluctuation of 20 kHz, $m = 1$ is at the radius $r_p = 6$ cm, where the density gradient is maximized).

Besides the experimental investigation, we have carried out some theoretical analyses. The line tying effect can be introduced into the dispersion relation by considering leakage of net charge density due to the response of electrons to plasma potential fluctuations. Using the linearized MHD equation and the dispersion relation of the interchange mode, a quadratic equation has been derived. From the dispersion relation, it was found that in the case of there being no line tying effect, the marginal stability criterion is

$$\beta > \frac{4\xi}{(1 + 2\xi)}$$

where $\xi = r_p/R_c$, and $R_c$ is the magnetic curvature radius. The growth rate of the interchange mode in the case of a strong line tying effect has also been derived.

REFERENCES

TILT STABILITY AND COMPRESSION HEATING STUDIES OF FIELD-REVERSED CONFIGURATIONS

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Abstract
TILT STABILITY AND COMPRESSION HEATING STUDIES OF FIELD-REVERSED CONFIGURATIONS.

The first comprehensive observations of internal tilt instabilities in field-reversed configurations (FRCs) are reported. Comparisons with 3-D simulations establish that data from an array of external magnetic probes are signatures of these destructive plasma instabilities. This work is a major step towards reconciling theory and experiments, and it suggests that grossly stable FRCs are restricted to very kinetic and elongated plasmas. Self-consistent 3-D numerical simulations demonstrate tilt stabilization by the addition of a beam ion component. High-power compression heating experiments with stable equilibrium FRCs are also reported. Plasmas formed in a tapered theta pinch coil have been translated along a guide magnetic field into a new single-turn compression coil where the external field is increased up to 7 times the initial value in 55 µs. Substantial heating is observed accompanied by a decrease in confinement times.

1. TILT STABILITY STUDIES

1.1 In-situ confinement

For the last five years, the world's largest field-reversed configurations (FRCs) have been studied in the FRX-C/LSM experiment[1]. The experimental apparatus (Fig. 1), diagnostics, and FRC formation method are described elsewhere.[1] FRCs are formed with reverse bias fields, 0.03 ≤ Bb ≤ 0.11 T, and fill pressures, 2 ≤ po ≤ 12.5 mtorr. However, FRCs with good confinement have

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been restricted to $B_\phi < 0.08 - 0.09$ T and $p_0 < 5$ mtorr for unclear reasons.\[1\] Detailed confinement studies\[2,3\] have recently explored the transition from good to bad FRC confinement. Inferred field null resistivities and electron thermal diffusivities are higher than classical for the well-confined FRCs by factors $5 - 20$ and $35 - 140$, respectively. These factors rapidly increase to much higher values during the transition to bad confinement, suggesting non-local, non-diffusive transport mechanisms such as formation dynamics or instabilities. Many studies have attempted to explain the above limitations in terms of some formation inadequacy, but they have proved inconclusive.\[4\] Other studies, reported below, have then attempted to identify FRC gross instabilities.

1.2 Observations of tilt instabilities

A Mirnov loop array consisting of 64 external $B_\phi$ pick-up loops (8 axial\times 8 toroidal) has been recently installed on FRX-C/LSM.\[5\] This diagnostic, shown partially in Fig. 1, allows separation of axially-even and odd components and toroidal Fourier analysis for the $n \leq 3$ components. Several instabilities have been detected, including rotational modes\[6\], transient flutes\[1\], and tilt instabilities\[7\]. Only the latter (axially-odd, $n = 1$ modes) are strongly correlated with poor FRC confinement. The tilt data have been compared with calculated tilt asymmetries in 3-D resistive-MHD simulations\[8\]. The 2-D FRC equilibrium in Fig. 1 was given a 1% tilt perturbation and the $n = 1$ tilt instability developed as shown in Fig. 2(a). Such comparisons have identified many features (time histories, axial dependence) of the data with signatures of internal tilt instabilities. Calculations show a rapid transition from closed to open field lines around peak tilt amplitude. The $B_\phi$ data track the global features of this transition, while soft x-ray imaging, excluded flux and interferometer arrays all display sudden changes consistent with field line opening.

Finite Larmor Radius (FLR) theory\[9\] applied qualitatively to the FRC tilt mode suggests stability for $s/e \ll 1/4$, where $s$ is the ratio of FRC minor radius to average ion gyroradius and $e$ is the FRC separatrix elongation. Recent calculations which isolate Hall-fluid\[10\] and FLR\[11\] effects support this trend. In FRX-C/LSM, values of $s = 1 - 5$, $e = 3 - 8$, and $s/e = 0.1 - 1.5$ have been achieved. Analysis of the entire data base reveals a gradual increase in tilt amplitudes (the best correlation found so far) and a degradation in FRC confinement as $s/e$ increases.\[7\] Consistently large tilt amplitudes and poor confinement are observed for $s/e > 0.5$. 
1.3 Theoretical FRC tilt stabilization with ion beams

If tilt stability requires $s/e < 0.5$ while ignition may require $s = 20 - 40$, FRC fusion reactors would be unattractively long. This illustrates the need for additional stabilizing techniques. Tilt stabilization is difficult\([8]\) but it has been recently demonstrated\([12]\) by adding an ion beam component as a collisionless Vlasov species in the 3-D simulation of Fig. 2(a). Tilt stability of the resulting self-consistent equilibria has been studied for several values of $f_B$ (beam/total particle inventory ratio). The tilt growth rate is reduced by about a factor of two for $f_B = 3 \times 10^{-3}$ and no growth is observed (Fig. 2b) for $f_B = 6 \times 10^{-3}$. The latter corresponds to fractions (beam/total) of 0.5 for energy and 0.25 for current. Further calculations will explore ways of reducing these fractions by optimizing the beam ion energy and its spatial distribution.
The above results suggest the feasibility of FRC tilt stabilization studies in FRX-C/LSM. Available technologies are now being considered to assess whether an ion beam can be introduced during FRC formation. In particular, the required ion beam could perhaps be obtained by plasma capture of a neutral hydrogen pulse from an intense ion diode.[13] Studies are also underway to find practical techniques in future large-size FRC devices. Partial loss of fusion products may naturally produce some beam ion component in ignited FRCs.[14]

2. COMPRESSION HEATING STUDIES

2.1 FRC translation

Preparatory translation experiments were performed on FRX-C/LSM prior to the installation of the magnetic compression hardware. FRCs were moved axially through a sharp transition and trapped inside a flux-conserving chamber (6.7-m length, 0.40-m i.d.). The confinement properties of translated FRCs are not very different from their in situ, non-translated, counterparts. An applied helical quadrupole field of 3% of the external field B stabilizes the destructive n=2 mode and FRC lifetimes of up to 400 μs have been observed.[15] Internal magnetic field measurements in these translating plasmas reveal a largely force-free structure (J || B) with a significant toroidal field component.[16]

2.2 High-power compression heating

The FRX-C/LSM device has recently been modified (Fig. 3) to allow high-power magnetic compression heating experiments. Kinetically-stabilized FRCs (s = 1.2, e = 5) formed in a deuterium puff inside a tapered θ-pinch coil are translated along a guide field into a new single-turn magnetic compression coil. The plasma enters the compressor approximately 30 μs after formation, reflects from a downstream mirror, and becomes trapped near the center of the compression coil at 100 μs. At that time B is raised from 0.2 T up to 1.8 T in 55 μs. Significant heating, consistent with the B^{4/5} adiabatic scaling, is measured (see Table I) even though a sizeable fraction of the particle inventory is lost. These observations are consistent with an energy confinement that is dominated by particle diffusion, as found in in-situ experiments[3]. The significant electron heating suggests that there is no fundamental thermal conductivity limit, although it may simply be a consequence of open field line confinement. The
FIG. 3. Schematic diagram of the FRX-C/LSM device modified for high-power FRC compression heating experiments.

TABLE I. FRX-C/LSM DATA

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Units</th>
<th>0-pinching source†</th>
<th>Before compression</th>
<th>Near peak compression</th>
</tr>
</thead>
<tbody>
<tr>
<td>time</td>
<td>µs</td>
<td>30</td>
<td>100</td>
<td>150</td>
</tr>
<tr>
<td>B</td>
<td>T</td>
<td>0.41 ± 0.02</td>
<td>0.23 ± 0.01</td>
<td>1.56 ± 0.04</td>
</tr>
<tr>
<td>r_s</td>
<td>mm</td>
<td>152 ± 11</td>
<td>145 ± 5</td>
<td>54 ± 6</td>
</tr>
<tr>
<td>&lt;n&gt;</td>
<td>10²¹m⁻³</td>
<td>0.7 ± 0.0</td>
<td>0.6 ± 0.1</td>
<td>6.4 ± 1.5</td>
</tr>
<tr>
<td>T_e + T_i</td>
<td>eV</td>
<td>516 ± 34</td>
<td>189 ± 49</td>
<td>970 ± 23</td>
</tr>
<tr>
<td>T_e(r = 0)</td>
<td>eV</td>
<td>132 ± 26</td>
<td>80 ± 15</td>
<td>340 ± 40</td>
</tr>
<tr>
<td>poloidal flux</td>
<td>mWb</td>
<td>3.4 ± 0.8</td>
<td>2.7 ± 0.3</td>
<td>0.7 ± 0.3</td>
</tr>
<tr>
<td>s</td>
<td></td>
<td>1.2 ± 0.2</td>
<td>1.9 ± 0.3</td>
<td>0.7 ± 0.2</td>
</tr>
<tr>
<td>c</td>
<td></td>
<td>4.7 ± 0.4</td>
<td>5.7 ± 0.5</td>
<td>4.3 ± 0.8</td>
</tr>
<tr>
<td>T_φ</td>
<td>µs</td>
<td>210 ± 67</td>
<td>147 ± 46</td>
<td>33 ± 16</td>
</tr>
<tr>
<td>T_N</td>
<td>µs</td>
<td>175 ± 48</td>
<td>129 ± 66</td>
<td>33 ± 21</td>
</tr>
<tr>
<td>T_E</td>
<td>µs</td>
<td>79 ± 15</td>
<td>51 ± 20</td>
<td>21 ± 7</td>
</tr>
</tbody>
</table>

† Data from source are for non-translated FRCs from in-situ experiments.[3]

inferred poloidal flux and particle confinement times, T_φ and T_N, remain approximately equal during compression, scaling roughly with the square of the separatrix radius r_s. The confinement times for the compressed FRCs are similar to those obtained on smaller devices with comparable plasma parameters.[17]
FIG. 4. (a) Ion and (b) electron temperatures versus compression field. The $T_i$ data are averages from pressure balance, and from the neutron flux, while the $T_e$ data represent single-point Thomson scattering measurements at $r = 0$ for 64 separate discharges.

Higher energy densities are achieved with hotter initial conditions in the θ-pinch and earlier compression during the first FRC transit through the compression coil. For this case, the total FRC plasma energy is tripled ($5 - 15$ kJ) with total neutron yields up to $1 \times 10^9$, $T_e = 0.4$ keV, $T_i = 1.5 - 2.0$ keV, $n_e = 3 \times 10^{21} \text{m}^{-3}$, $E_p/V = 1 \times 10^6 \text{ Jm}^{-3}$, and $n_\tau_E \leq 10^{17} \text{m}^{-3} \text{s}$. However, in contrast to the colder initial conditions, the quadrupole stabilization field (of up to 4% B) is insufficient to control the $n=2$ rotational mode which often terminates the FRC before peak compression. Volume-averaged measurements of neutron emission with absolutely calibrated instruments support the peak compression ion temperatures determined from pressure balance (Fig. 4a), while Thomson scattering measurements near the geometric axis reveal substantial electron heating (Fig. 4b).

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EXPERIMENTAL MHD STABILITY LIMIT IN THE GAS DYNAMIC TRAP

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Abstract

EXPERIMENTAL MHD STABILITY LIMIT IN THE GAS DYNAMIC TRAP.

The paper summarizes the results of extensive experimental research into plasma MHD stability in gas dynamic traps (GDTs), which has been carried out over the past three years at Novosibirsk. The objective of the experiments reported was to give experimental evidence of plasma stability in a GDT and to compare the conditions in which it would be achieved with the theoretical predictions. The theoretical predictions generally agree with the measurements, with only one notable exception, i.e. the limit in the mirror ratio for the plasma stability, which is discussed in detail in the paper.

1. INTRODUCTION

The gas dynamic trap (GDT) is an axisymmetric mirror device with a large mirror ratio R and a length ℓ, which considerably exceeds the effective ion mean free path for scattering into the loss cone [1].

The main advantage of the trap is that it can provide plasma MHD stability even for axisymmetric geometry. MHD stability is possible even in axisymmetric geometry because plasma density and momentum flux just beyond the mirror throats in the expander region, with a favourable curvature of the field lines, are large enough to

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1 Deceased.
balance the destabilizing contribution between the mirrors of the trap. This is in contrast to the conventional mirror machines with nearly collisionless plasma, for which the pressure as well as the momentum flux in this region are obviously negligible so that the MHD stability is obtained for axisymmetric geometry.

The objective of the reported experiments was to give experimental evidence of plasma stability in a GDT and to compare the conditions in which it can be achieved with the theoretical predictions. In particular, the MHD stability limit of warm plasma confinement in GDT is also a critical point for the startup physics of prospective upgraded devices with denser and hotter plasmas.

Figure 1 shows the GDT layout schematically. The magnetic field at the mid-plane was varied from 1.5 to 2 kG. The stainless steel vacuum chamber of 11 m length and 0.5 m radius was evacuated to a base pressure of $10^{-6}$ torr.

The curvature of the field lines in the region outside the mirrors, where the plasma stream leaking out of the trap expands, is controlled by switching on the coils with currents opposite to those in the central cell coils (Fig. 1). These coils are energized by independent supplies.

Both mirror coils are composed of two parts that are inserted into one another. The outer part is energized in series with the central cell coils, whereas the inner coils are powered independently and produce a field of up to 100 kG. These coils provide a variable mirror ratio ranging from 12.5 to 100.

The warm plasma under investigation is produced in the trap by a plasma source — a hydrogen fed washer stack gun located in one of the end tanks. In Fig. 1 the magnetic lines of force are also shown for various regimes of operation.

2. SUMMARY OF THEORY

A stability analysis has been carried out by the Rosenbluth-Longmire pressure weighted curvature criterion [2]. For a collisional tandem mirror open trap with five
to ten ion–ion mean free paths between the mirrors and good curvature within the
mirror cell, this stability agreed quantitatively with the experiments [3]. A modification
of the criterion applicable to the gas dynamic trap was considered in Ref. [4].
According to Refs [2, 4], the condition for stability against curvature driven flute
modes can be represented as follows:

\[ W = \int \frac{\kappa (P_\perp + P_\parallel)}{rH^2} \, dl \geq 0 \]  

(1)

where \( \kappa \) is the normal curvature, and the components of the pressure tensor are given
by \( P_\perp \) and \( P_\parallel \). The integration is performed along a line with radius \( r(\ell) \) and a mag-
netic field \( H \). Inside the trap, \( P_\parallel \) and \( P_\perp \) should be taken equal to one another. In the
expanders, \( P_\parallel \) is substituted by \( P + \rho \nu^2 \).

It was found convenient to split the integral (1) into two different parts: the first,
\( W_1 \), extends along a line inside the trap and the other one, \( W_2 \), in the expanders.
For the central cell filled with an isotropic plasma, the integral \( W_1 \) is obviously
negative. It is also convenient to express the stability condition, which is equivalent
to expression (1), in terms of the 'safety factor' \( Q \), defined as the absolute value of
the ratio of contribution made by expanders \( (W_2) \) to that from the central \( (W_1) \) cell
(we assume \( W_2 \) to be positive). For a plasma to be stable against flute modes, the
safety factor \( Q \) should exceed unity. Furthermore, we will use the fact that, for a
GDT, \( Q \) is obviously inversely proportional to the mirror ratio (it changes as the
plasma pressure in the expanders). For large scale flute modes, criterion (1) should
be somewhat modified to take into account the spatial structure of the perturbations.

As obviously follows from expression (1), regions with small magnetic field
make the greatest contributions to the stability integral. Therefore, the estimate of the
safety factor is highly sensitive to the choice of the upper limit to this integral, which
corresponds to spatial points in the expanders. In practice, the integration is per-
formed until one of the two imposed limitations is violated. The first condition was
that the ion Larmor radius should be small compared to the local curvature radius
\( \langle \rho_i \kappa \rangle \leq \langle \rho_i \kappa \rangle_{cr} \). The second limitation imposed is that the plasma beta should be
smaller than unity. Both limitations are imposed by the theory and, if violated, make
the theoretical predictions rather unreliable. The part of a flux line where these limi-
tations are violated should be considered as a region that does not make any contribu-
tion to the stability criterion.

3. STABLE AND UNSTABLE REGIMES OF CONFINEMENT IN THE GDT

A warm plasma was injected into the trap during 3 ms. The plasma parameters
inside the trap depended upon the discharge current and on the gas feed rate. The den-
sity was varied from \( 1.5 \times 10^{13} \) cm\(^{-3} \) to \( 7 \times 10^{13} \) cm\(^{-3} \), the temperature varied
from 5 to 15 eV, and the plasma radius was 6.5 cm [5]. For all regimes of operation, the ion mean free path was substantially smaller than either the total machine length or the distance between the mirrors.

In all cases, during gun operation neither MHD instability nor substantial transverse plasma losses occurred, probably because of line tying. After the plasma gun had been turned off, the plasma behaviour became sensitive to the curvature of the field lines in the expanders.

As has been reported previously [5–7], when the curvature of the field lines in the expanders was favourable and the safety factor for the large scale modes with $m = 1$ was calculated to be about 5, a stable plasma confinement regime was observed.

The plasma lifetime (~2.5 ms) was in good agreement with the calculated flow rate through the mirrors. Running the machine without energizing the coils in the expanders made the curvature of the field lines there negligible. In these conditions, during plasma decay flute growth was observed, which resulted in precipitous plasma losses within about five time-scales, for large scale modes.

The theoretical predictions are generally consistent with the experimental observations, with only one notable exception, i.e. the Q-factor, whose measurements will be discussed in greater detail in the next section.

4. EXPERIMENTS ON SAFETY FACTOR MEASUREMENTS IN GDT

Additional efforts were made to quantitatively verify the agreement of the calculated Q-value with the experimental value. Experimentally, two different approaches were used to measure this value. Both used a controllable change of the contribution to the stability criterion (1), coming from a certain region occupied by plasma. During these changes the MHD stability limit, corresponding to $Q = 1$, was reached, and then, after rather simple calculations, it was possible to evaluate $Q$ for the initial, undisturbed conditions.

As was mentioned before, the contribution made by the expanders is proportional to $R^{-1}$. One can then change this contribution by varying the current in the inner mirror coils. Varying the mirror field in this way caused negligible field changes in the central cell and the expanders, of the order of $\delta H \approx 10^{-3}$ H.

Another approach to measuring $Q$ was to change the unfavourable contribution of the central cell in an easily controllable manner. We changed the magnitude of the field locally in the central cell, using an additional coil placed near the midplane. Simulation runs were carried out to obtain integral (1) over the central part of the trap for a certain current in this coil. If energized, the additional coil disturbs this optimal configuration and thus reduces the safety factor right up to the stability limit when $Q = 1$. In Fig. 1 we see a schematic representation of the field lines for this case.
We discuss the experimental results obtained by making use of both approaches. Plasma lifetime versus mirror ratio ranging from $R = 12.5$ to 75 is plotted in Fig. 2. We observed the expected linear relationship, slightly corrected for the mirror ratio dependence of the lifetime, within the interval $R \leq 35$, as is indicated in the figure.
Whenever we exceeded a mirror ratio of 35, the plasma lifetime was found to decrease because of the instability drive and enhanced transverse losses. For such mirror ratios, an MHD activity of a plasma was observed, which can be interpreted as a passage through an interchange instability threshold to be attributed to reducing the expander’s favourable contribution to the stability criterion (1). An analysis of the data represented in Fig. 2 indicates that in our operating regime with \( R = 25 \), the value of the Q-factor can be estimated to be 1.5–2. Approximately the same value of the safety factor was inferred from the data obtained at the shots when we changed the configuration of the field lines inside the trap by an additional coil (see also Fig. 3).

5. DISCUSSION

The major success of GDT experiments in terms of the physical basis has been the absence of flute-like modes in the trap as was predicted by theory [1]. At the same time, throughout the measurement of the stability limit with respect to the mirror ratio, we have observed Q to be approximately 20% of the theoretically predicted value. The data taken for the unstable regimes show that the driven perturbations have the form of flutes extending through the trap (an estimation of \( k_B \) shows it to be less than \( 10^{-3} \)). The azimuthal spectrum of unstable oscillations was dominated by modes with \( m \approx 1 \).

The exact reason for the Q-reduction is unknown, so far. One of the possible reasons may be an uncertainty in the parameter \( k_B \), which defines the upper limit of integration in expression (1), especially, since the value of the safety factor is very sensitive to this limit. On the other hand, this may be a result of some processes in the expanders, which alter the plasma flow and have not been taken into account by the theory, as yet. While we observed the stability limit, insufficient measurements were made to determine the plasma flow parameters inside the expanders, which determine the MHD stability in the entire trap. This led to additional experimental efforts to determine the characteristics of the plasma flow. The evaluation of integral (1) over the expanders requires the data on the components of the pressure tensor at all points along the field lines, which constitutes an extremely complex problem. Therefore, using simple diagnostics, we have tried to find out the differences between the plasma flow regime and the theoretically calculated values.

Measurements with Faraday cups and probes have shown that, at least up to a magnetic field of \( H \geq 10^{-2} H_{\text{max}} \), the density of the plasma flow changes in accordance with the flux conservation law, \( j/H = \text{const} \). This means that neither significant ion losses nor significant deviations of the ions from the field lines have occurred. Measurements of the electron temperature just beyond the mirror throat have shown it to be much lower than that inside the trap. Hence, the regime of the plasma flow should be very close to an adiabatic regime, when longitudinal heat transport can be neglected. According to our simulation runs the regime with
FIG. 4. Potential profile in the expander. $U_m$, $Z_m$ — electric potential and co-ordinate of mirror, respectively; $T_0$ is the plasma temperature inside the central cell.

$T_e \approx 5$ eV should also be nearly adiabatic. To identify the flow regime in the expanders, we have tried to use data on the potential distribution along the field lines, which are rather sensitive to the plasma parameters. In addition to the distribution, a knowledge of the drop in the potential between mirror throat and end wall is also interesting. This enables us to evaluate the ion momentum flux in the near-end-wall regions where the magnetic field is low.

We have measured the potential by making use of emissive probes [8] moved along a radius inside the trap and along the axis in the expander. The potential profile in the expander is shown in Fig. 4. Also shown are potential profiles for two opposite limits with an infinite and a negligible longitudinal plasma thermal conductivity. We have carried out calculations taking into account the plasma viscosity and the finite thermal conductivity and found the calculated potential curve for the experimental conditions to be lying in between these limiting curves. Apparently, neither model for the potential distribution uniquely fits the experimental observations. It has, however, been observed that an increase in the electron temperature up to 10 eV provides a potential profile approaching that calculated for the adiabatic regime. This is the reason why we can hope that a further increase in temperature will produce a plasma flow with parameters within the domain where the results of our simulation runs prove to be applicable. The safety factor has been obtained by use of isothermal as well as adiabatic approximations. Its calculated value is found to be only slightly different (by about 25\%) for these regimes. A considerable decrease in $Q$ should be expected from the fact that the experimental curves of the potential lie below the adiabatic curve.
An exact calculation of $Q$ for our experimental conditions ($T_e \approx 5$ eV) requires the data on the density profile along a field line in the expander which are not available at present.

Additional effects have been predicted theoretically [9, 10] to modify the stability condition from that determined by the magnetic field structure only. They are connected with a finite resistivity of the end plates. The electron temperatures along a field line in the expander are unknown at present. Nevertheless, the shape of the experimental potential curves indicates them to be substantially lower than those given by the adiabatic model of flow for the same spatial points. If this is the case, theory [10] predicts a value of the safety factor which is much smaller than that calculated without taking the end wall conductivity into account.

The stability criterion formulated in terms of the pressure weighted curvature obviously needs to be additionally verified in experiments. Apparently, the problem of quantitatively fitting the safety factor obtained from formula (1) is not only inherent in GDTs. Thus, although this stabilization criterion has been checked qualitatively in ambipolar trap experiments [3, 11], in Ref. [11] a substantial difference between the MHD stability limit and the theoretically predicted value has been noted.

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PHYSICS STUDIES FOR THE URAGAN-2M TORSATRON

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Abstract

PHYSICS STUDIES FOR THE URAGAN-2M TORSATRON.

Results of theoretical studies on the URAGAN-2M torsatron at Khar’kov are presented. The magnetic field configuration, plasma equilibrium and stability, equilibrium currents, neoclassical transport in the low collisionality regime and impurity flux reversal at RF heating are studied.

1. INTRODUCTION

The torsatron with an additional toroidal field, URAGAN-2M (U-2M) [1], which is in the final phase of construction in the Khar’kov Institute of Physics and Technology, was designed to study beta limits and transport at low collisionality. So far, as in the classical stellarator, the $v_l$ transport scales as $m^{9/2}$ (where $m$ is the helical winding field period number), and the helical winding ponderomotive forces decrease with decreasing helical pitch angle; the $m$ value for U-2M was chosen to be $m = 4$. In a torsatron with such a small field period number, closed flux surfaces exist only when an additional toroidal magnetic field is superimposed. The helical winding law for U-2M was chosen to maximize the rotational transform $\iota$ and to ensure the existence of a magnetic well. This law is closely similar to the law governing a toroidal helix with constant pitch angle. The number of toroidal field coils $T (T = 16)$ was chosen to minimize the toroidal field ripple, and the poloidal field coils $P (P = 8)$ provide a good compensation for the transverse helical magnetic field.

This paper presents results of theoretical studies of the U-2M magnetic configuration and its effect on plasma equilibrium, stability and transport.

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FIG. 1. Cross-section of URAGAN-2M showing relative locations of its main features (in centimetres).

FIG. 2. Main magnetic configuration parameters versus $K_\phi$ for URAGAN-2M ($B_z/B_0 = 1.5\%$).
2. MAGNETIC FIELD CONFIGURATION IN U-2M

The magnetic field configuration in the U-2M torsatron is the superposition of magnetic fields produced by

— an $\ell = 2$ helix wound on a torus of major radius $R = 1.7$ m and minor radius $a_h = 0.445$ m with the following winding law:

$$2\varphi = -\theta + 0.2168\sin \theta - 0.0171\sin 2\theta$$ (1)

— eight poloidal field windings, and

— 16 toroidal field coils (Fig. 1).

The parameters of the magnetic configuration in U-2M — the radial profile of the rotational transform, $\iota(r)$, the average radius of the closed flux surface, $a$, and the well depth — can be changed over a wide range (Fig. 2) by varying the parameter $K_\varphi = B_h/(B_h + B_t)$. Here, $B_h$ is the toroidal magnetic field produced by the helical winding ($B_{h,\text{max}} = 0.75$ T), and $B_t$ is the additional toroidal magnetic field ($B_{t,\text{max}} = 1.8$ T). We note that, for $K_\varphi < 0.375$, the radius of the last flux surface is defined by the limiter ($a_L = 0.32$ m), but for $K_\varphi > 0.375$ the last closed surface lies inside the limiter.

The magnetic configuration parameters can also be changed by varying the correcting transverse field $B_L(0)$.

In the operational range of $K_\varphi$, a variety of configurations can be studied: from the largest plasma radius, $a = 0.27$ m, a small rotational transform, $\iota = 0.18$, and zero shear at $K_\varphi = 0.2$ to a smaller plasma ($a = 0.17$ m) with moderate $\iota$ and shear at $K_\varphi = 0.4$. The choice of the specific values of $K_\varphi$ depends strongly on a property of the magnetic configurations that is affected by ‘natural’ island formation at rational values of $\iota$. By ‘natural’ islands we understand islands that result from the real geometry of the magnetic coils and the current ratios prevailing in them. An example of natural islands at $\iota = 4/6$ is shown in Fig. 3(a) for the U-2M standard case ($K_\varphi = 0.375$, $B_L/B_0 = 2.8\%$). It is evident that such large islands are unacceptable so that much effort was aimed at eliminating the islands.

For the standard configuration, substantial $\iota = 4/6$ island suppression was achieved by a proper choice of the second modulation coefficient in the helical winding law (1) [2]. Thus, the change of this coefficient from $-0.0171$ to 0.007 results in a strong decrease of the $\iota = 4/6$ island width (from $\Delta r = 3$ cm to $\Delta r = 0.3$ cm). However, for a torsatron with variable configuration, this approach looks unpractical.

It is evident that ‘natural’ islands are a result of resonance perturbations inherent in a specific magnetic coil configuration in the torsatron. Spectral analysis of the scalar potential $H$ of the helical coil field and the vector potential $A$ of the poloidal coil field for the standard case configuration (Fig. 3(a)) and the configuration with diminished islands (Fig. 3(b)) has allowed the harmonics of $H_{mn}$ and $A_{on}$ affecting
FIG. 3. Flux surfaces in URAGAN-2M at the beginning of the field period (volume helical pole model): (a) standard configuration ($K = 0.375, B_L/B_0 = 2.8\%$); (b) standard configuration with changed winding law; (c) standard configuration with changed PF coil currents.

the size of the $\ell = 4/6$ islands to be singled out. It was shown [3] that by an appropriate choice of the vector potential harmonic amplitudes it is possible to diminish the $\ell = 4/6$ and $\ell = 4/7$ island widths considerably (Fig. 3(c)). The necessary spectrum of harmonics can be obtained by choosing the poloidal field coil positions or, for given coil positions, by suitably choosing the currents in these coils. The latter approach is most practical since it allows an optimization of the magnetic surfaces for a wide range of configurations realized by varying $K_\phi$ and $B_L/B_0$, presumably with finite beta plasmas.
FIG. 4. Rotational transform profile and radial variation of specific volume $V'$. 

The above mentioned calculations did not take into account magnetic field perturbations connected with the specific design of the U-2M magnetic coil. Effects of HF coil current leads and bolted joints on the magnetic configuration were also studied. It was shown that, for the U-2M standard case, these perturbations result in an ergodization of the outer magnetic surfaces and in an increase of the $\iota = 4/6$ island width. Methods of designing magnetic field error compensation for U-2M are being studied.

3. PLASMA EQUILIBRIUM AND STABILITY

The MHD equilibrium for U-2M plasmas has been studied by means of the ORNL 3-D numerical code VMEC [4]. Zero current equilibrium with a pressure profile of $p = p_0(1 - \Psi)^2$ ($\Psi$ is the normalized poloidal flux) was considered.
Since the VMEC code can calculate the equilibrium for vacuum configurations with closed magnetic surfaces we have studied finite beta effects for the two configurations shown in Fig. 4. The magnetic axis shift, $\Delta r/a$, for both configurations is plotted in Fig. 5. Taking $\Delta r/a \approx 50\%$ as a measure of the equilibrium beta limit, we find that both configurations have a peak beta limit, $\beta_0 \approx 9\%$.

The ideal MHD stability studies consisted of testing the Mercier criterion [5] for the two U-2M configurations. The Mercier criterion can be written as

$$D_m = D_s + D_l + D_w + D_k > 0$$  \hspace{1cm} (2)
The condition $D_m > 0$ implies stability. Here, the terms $D_s$, $D_w$, $D_g$ and $D_I$ are related to shear, magnetic well, geodesic curvature and plasma current, respectively.

For the standard configuration, the dominant terms determining the Mercier stability are the magnetic well and the geodesic curvature contributions. For the range of beta values analysed the magnetic well dominates over the geodesic curvature, and the beta value is limited by equilibrium convergence rather than by stability.

The configuration with larger minor radius has stability properties different from those of the standard configuration (Fig. 6). The contribution of the magnetic well is not positive over the whole radial range; near the boundary it is negative because of the magnetic hill. Moreover, the shear contribution compensates for the magnetic well at the edge. The main stability problem comes from the geodesic curvature at the $\iota = 1/2$ resonance, which makes $D_m$ negative around the radius $r = 0.4 \, a$. As the unstable region is very narrow, a local flattening of the pressure can probably overcome this problem. At higher beta values, $\beta_0 = 6\%$, this region broadens, and at the same time the shear contribution is not large enough to compensate for the magnetic hill so that there is another unstable region near the edge. Therefore, the stability beta limit is $\beta_0 < 5.7\%$.

The distortion of the rotational transform profile with beta is also a very important effect in this configuration (Fig. 7). The use of a quadrupole field in controlling the transform could help in extending the range of stable operation for this configuration.
FIG. 8. $(|j||j_\perp|)$ for URAGAN-2M and other stellarators.

FIG. 9. Radial profile of $G_d$ for URAGAN-2M.
4. EQUILIBRIUM CURRENTS

A poloidal variation of the integral $\int \frac{df}{B}$ allows the magnitude of $\langle |j_r/j_\perp| \rangle$, the ratio of the parallel and perpendicular equilibrium current densities averaged over the magnetic surface, to be estimated. For the U-2M standard case, this quantity is shown in Fig. 8, together with data for other stellarators.

The bootstrap current in the plateau regime for the U-2M standard case is a factor of 0.8 the value for the axisymmetrical configuration with the same values of aspect ratio and rotational transform [2].

The geometrical factor of the bootstrap current $G_b$ in the low collisionality regime for the two U-2M configurations was calculated according to Ref. [6] and is shown in Fig. 9.

5. NEOCLASSICAL TRANSPORT IN THE LOW COLLISIONALITY REGIME

5.1. Particle orbits and velocity space loss regions

The numerical integration of the guiding centre equations shows the existence of all kinds of particle trajectory for U-2M (passing and trapped particles, passing and trapped helical bananas and transitional orbits) [7]. Charged particle trajectories display large deviations from the magnetic surface which are characteristic for the motion of helically trapped particles with unfavourable magnetic field modulation (Fig. 10(a)). However, the radial drift of the helically trapped particles may be

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FIG. 10. Helically trapped particle orbits (without and with electric field).
substantially compensated for by a radial electric field of the correct sign in order to increase the poloidal velocity of the helically trapped particles (Fig. 10(b)).

As far as computations confirmed the adiabatic conservation of the longitudinal invariant \( J = \oint v_t d\ell \), the loss regions in phase space were studied by using the guiding centre approximation with an idealized magnetic field model, which takes into account the basic toroidal and helical harmonics and the nearest sidebands of the basic helical harmonics [8]. It was shown that the dimensions of the loss regions are larger that those of the conventional \( \ell = 2 \) stellarator because of satellite harmonics of the same sign as that of the main helical harmonic, but an electric field of a sign that yields the physical effect mentioned above, decreases the loss regions.

5.2. Diffusion in the 1/\( \nu \) regime

A major problem in stellarator confinement is the level of losses in the 1/\( \nu \) regime, which is not affected by the electric field effects. The particle flux in the 1/\( \nu \) regime has been calculated in Refs [9, 10] for multiple helicity stellarators. When only the toroidal ripple, \( \epsilon_t \), and the helical ripple, \( \epsilon_h \), are considered, the particle flux in the 1/\( \nu \) region scales as \( \epsilon_h^{3/2} \epsilon_t^2 / r^2 \). The presence of other harmonics introduced by the magnetic field geometry can be accounted for by the geometrical factor \( D_0 \) [9, 10]:

\[
D_0 = \frac{\epsilon_h^{3/2} \epsilon_t^2}{r^2} \left[ \gamma_1 - \left( \frac{\epsilon_h}{\epsilon_t} \right) \gamma_2 + \left( \frac{\epsilon_h}{\epsilon_t} \right)^2 \gamma_3 \right]
\]

where \( \gamma_1, \gamma_2, \gamma_3 \) are functions of the ratios \( \epsilon_{t \pm 1}/\epsilon_h \), and \( \epsilon_{t \pm 2}/\epsilon_h \), where \( \epsilon_{t \pm 1}, \epsilon_{t \pm 2} \) are the amplitudes of the poloidal satellite harmonics. The value of \( D_0 \) is particularly sensitive to the sign of the nearest satellites.

The Fourier analysis of the magnitude of the magnetic field for the U-2M standard configuration showed the unfavourable harmonic spectrum with the nearest positive satellites (for example, \( \epsilon_h = 0.045 \), \( \epsilon_{h+1} = 0.033 \), \( \epsilon_{h-1} = 0.012 \) at \( r = 12 \) cm). The effect of satellite harmonics is shown in Fig. 11, where the radial profiles of \( \epsilon_h \) and \( \left< \epsilon_h \right> \) are plotted. Here, \( \left< \epsilon_h \right> \) is effective helical ripple defined by the relation

\[
\left< \epsilon_h \right> = \epsilon_h \left[ \gamma_1 - \left( \frac{\epsilon_h}{\epsilon_t} \right) \gamma_2 + \left( \frac{\epsilon_h}{\epsilon_t} \right)^2 \gamma_3 \right]^{2/3}
\]

The rather large effective ripple on axis is the result of a modulation of the toroidal magnetic field component \( B_h \) (m = 4) produced by the helical winding.

This result can be greatly improved by using differing currents in the TF coils. In the standard case of U-2M, each of the 16 TF coils carries a current of 666.7 kA. If this is changed so that the inner two TF coils in each field period carry currents of 764.7 kA and the outer two TF coils carry currents of 626.7 kA (the U2-M power
supply and circuit allow such a mode of TF coil operation), then the ripple on the magnetic axis can be essentially eliminated (Fig. 11), in which case the neoclassical transport properties of the device are greatly improved. This is also done without significantly altering the rotational transform and shear and actually leads to a slight increase in the depth of the magnetic well.

In the presence of plasma ($\beta(0) = 2.7\%$), the factor $D_0$ is increased by a factor 1.5 at half-radius.

6. IMPURITY FLUX REVERSAL AT RF PLASMA HEATING

Currentless plasma production and heating for U-2M will be brought about by RF waves in the ion cyclotron and Alfvén frequency ranges [1]. It is well known that properly phased RF power absorption can be used for impurity flux reversal in a tokamak [11].

We have studied the RF impurity flux reversal in a torsatron, taking into account inhomogeneous heating of impurity ions and the thermal force due to the longitudinal gradient of the impurity ion temperature. An analysis of RF power absorption by ions in the torsatron magnetic field geometry showed that a local, helically inhomogeneous heat source can be created and used for ion impurity flux
reversal. Impurity ion cyclotron absorption of fast magnetoacoustic waves (FMAW) was considered as a candidate mechanism for helical heat source creation. A simple antenna system consisting of two pairs of loop antennas placed diametrically opposite along the major circumference of the torus and excited by $\omega \sim 0.1 \omega_c$ can be used for this purpose. The RF power which is necessary for metal and carbon ion flow reversal in U-2M is in the range of 20 kW [12].

7. CONCLUSIONS

The evaluation of the magnetic configuration, equilibrium, Mercier stability and neoclassical transport properties of the URAGAN-2M torsatron leads to the following results:

1. The vacuum magnetic field configuration in the torsatron with an additional toroidal field can be changed in a wide range by varying the toroidal and transverse magnetic fields. Natural islands at rational $i = 4/6, 4/7$ can be avoided by a proper choice of the transverse magnetic field configuration.

2. For the U2-M configuration with a plasma minor radius of 17 cm we have found that peak beta values of up to about 6% are stable to Mercier modes. Plasma equilibrium currents change the rotational transform profiles in such a way that they may cross rational values that were avoided in the vacuum magnetic field configuration. External control of the $\iota$ profile may be very important.

3. The neoclassical transport properties of the configuration with a plasma radius of 17 cm are as follows:

(a) The ratio of the equilibrium currents $\langle |j_x|/|j_y| \rangle$ is near $\sqrt{2}/i$.

(b) In the plateau regime, the diffusion is a factor of 1.5 and the bootstrap current a factor of 0.8 the values valid for the equivalent tokamak.

(c) In the long mean free path regime the bootstrap current is a factor of 0.5 the value valid for the equivalent tokamak, and in the $1/\nu$ regime the diffusion is a factor of 2 the value for the classical stellarator.

The last result is valid for a vacuum magnetic configuration as well as for finite beta equilibrium.

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FLUCTUATIONS AND STABILITY IN THE ATF TORSATRON*

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Abstract

FLUCTUATIONS AND STABILITY IN THE ATF TORSATRON.

The results of experimental and theoretical studies of fluctuations and instabilities in the ATF torsatron, a type of stellarator are presented. Measurements of globally coherent magnetic fluctuations in high beta plasmas with narrow pressure profiles show evidence of self-stabilization ('second stability'); the trends are compatible with theoretical analyses of self-stabilization of resistive curvature driven instabilities, but there are discrepancies between the absolute experimental and theoretical fluctuation amplitudes. Fluctuation measurements in plasmas with broad pressure profiles reveal new phenomena — specifically, toroidally localized magnetic fluctuations, whose amplitudes increase with plasma pressure, and coherent density fluctuations with significant radial width.


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1. INTRODUCTION

The Advanced Toroidal Facility (ATF) [1] is an $l = 2$, 12 field period torsatron with major radius $R_0 = 2.1$ m, average plasma radius $\bar{a} = 0.27$ m, and mean field on axis $B_0 \leq 2$ T. While the rotational transform profile can be adjusted externally, typically the central transform $\iota_0 \approx 0.3$ and the edge transform $\iota_s = 1$. The configuration was designed to study access to the MHD second stability regime. Currentless plasmas are produced by up to 400 kW of 53.2 GHz electron cyclotron heating (ECH) at the fundamental, second or third harmonic and up to 1.5 MW of tangential neutral beam injection (NBI) from two injectors.

2. EXPERIMENTAL STUDIES OF SECOND STABILITY IN PLASMAS WITH NARROW PROFILES

Operation with a field error [2] that reduced the effective plasma radius to 60% of its design value and $\iota_s$ to $\sim 0.5$ resulted in peaked pressure profiles ($\beta_0/\langle \beta \rangle \approx 6$) and increased Shafranov shifts that lowered the theoretical transition to ideal MHD second stability to $\beta_0 \approx 1.3$%; the experimental $\beta$ values ($\beta_0 \leq 3\%$) are well above this transition. No evidence of instabilities inside the plasma is seen on soft X ray diodes. The magnetic fluctuations measured outside the plasma ($B_e$) have a toroidal mode number $n = 1$ with principal poloidal harmonics $m = 2$ and 3 and thus appear to be connected with pressure driven instabilities at the $\iota = 1/2$ and $1/3$ rational surfaces. The fluctuation amplitudes first increase with beta, then saturate (maximum values of $B_e(n = 1)/B_0 \sim 10^{-7}$ at the $B_e$ coils, about 30 cm outside the reduced plasma radius) and then decrease somewhat with increasing beta above $\beta_0 \approx 1.3\%$. The pressure profile also broadens with increasing beta. These results [3] are suggestive of self-stabilization.

3. THEORETICAL STUDIES OF SECOND STABILITY IN SHEARED STELLARATORS

We have theoretically studied the resistive stability of toroidally averaged [4] stellarator equilibria having $\beta_0$ values from 0.26% to 2.3% and peaked pressure profiles. For these calculations we used the computer code FAR [5], which uses reduced MHD equations [6]. These equilibria are ideally stable. A linear, resistive study revealed, however, two different instabilities: (1) a resistive interchange mode driven by the normal curvature and (2) a resistive ballooning mode driven by the geodesic curvature.

The large-$n$ limit of the equations (ballooning equations) in the FAR code is a useful diagnostic of the nature of the instability. The local growth rate obtained with these equations agrees well with the low-$n$ stability results. This agreement between
low- and high-n stability results is better for stellarators than for tokamaks, probably because current driven modes are absent in stellarators. The driving term responsible for resistive interchange modes and the driving term responsible for resistive ballooning modes were selectively used to identify the modes in the plasma interior as resistive ballooning and the modes toward the edge as resistive interchange. A comparison of the full toroidal and the cylindrical calculations confirms this identification. As beta increases beyond $\beta_0 = 0.8\%$, the plasma stabilizes because (1) resistive interchange growth decreases owing to the deepening of the magnetic well caused by the Shafranov shift, and (2) resistive ballooning growth decreases because of the combined effect of the well and reduction of the shear. This shear reduction is caused by a change in the $t$ profile induced by the zero net current requirement on the equilibria. The reduction in shear in the plasma core also reduces the number of rational surfaces that can support the instability non-linearly. With these effects, the saturation and slight decrease in $\bar{B}$ with increasing beta that is seen in the experiment can be qualitatively reproduced [7]. In Fig. 1, calculated contours of constant pressure from the non-linear evolutions (at twice the saturation time) are shown for $\beta_0 = 0.8\%$ and $\beta_0 = 2.3\%$. The pressure contours at the lower beta indicate quite turbulent behaviour while those at higher beta are more like those of a quiescent equilibrium, particularly in the plasma interior. The reduction in the strength of the resistive instabilities, signaling entry into the second stability regime, is clear. Two additional non-linear effects have been found at finite beta. The first is a bulk plasma displacement caused by the non-linear interaction of modes with the same value of $n$ and values of $m$ differing by 1. These instabilities drive the $(m = 1, n = 0)$ mode non-linearly. This could be an experimental signature of the presence of the ballooning instability and can be seen in the pressure contours of Fig. 1. The second non-linear effect is a flattening of the central pressure profile caused primarily by the $(m = 1, n = 0)$ mode interacting with itself.

Considerable further work is required to develop an accurate model for the fluctuation behaviour seen in the ATF experiment. The magnitudes of the magnetic fluctuations found in the calculations so far are $\sim 100$ times larger than those

FIG. 1. Perturbed pressure contours in ATF (calculation).
measured in the experiment. The discrepancy can be reduced somewhat (by a factor of \(\leq 10\)) if one takes into account possible uncertainties in the pressure profile measurements and the large difference in the radial position of the locations where the fluctuation is calculated (at the plasma edge) and measured (at the vacuum chamber wall), but a significant quantitative disagreement remains. Additional reductions in the fluctuation level caused by diamagnetic rotation, compressibility, and ion viscosity have been estimated. Each mechanism separately reduces the fluctuation level by a factor of \(\sim 2\). Thus, while no single effect could reduce the fluctuation level by the order of magnitude or more that is needed, the inclusion of several effects acting collectively could provide the reduction needed.

4. FLUCTUATIONS IN ATF PLASMAS WITH BROAD PRESSURE PROFILES

Results from recent experiments in ATF with the field error repaired [8] show that the pressure profile is now broader, with \(\beta_0/\langle \beta \rangle < 4\). The highest values of \(\langle \beta \rangle\) (up to 1.4%) have been achieved by operating at high density, \(\bar{n}_e \geq 5 \times 10^{13} \text{ cm}^{-3}\), where the contributions of fast ion pressure are minimal; this can further flatten the pressure profile.

The characteristics of the magnetic fluctuations are now quite different. The root mean square fluctuation level of coherent fluctuations increases with \(\langle \beta \rangle\) to \(\langle B_0 \rangle/B_0 \approx 10^{-5}\) at the largest values of \(\langle \beta \rangle = 1.4\%\) studied so far (Fig. 2) and is an order of magnitude larger than the fluctuations observed with the narrow \(p(r)\) at comparable values of beta. (This could reasonably be attributed to the reduction of the radial separation between the effective plasma edge and the magnetic pick-up coils.) With narrow \(p(r)\), \(\tilde{B}_0\) contained globally coherent components; in contrast, with broad \(p(r)\) the fluctuations have short coherence lengths \(\sim 1\) field period (1/12 of the toroidal circumference). The geometrical structure of the fluctuations is also different. Figure 3 shows the apparent spectrally averaged toroidal and poloidal mode numbers as a function of \(\langle \beta \rangle\). The toroidal mode numbers \(\langle n \rangle\) generally increase from \(\sim 1\) at low \(\langle \beta \rangle\) to \(\sim 3\) at high \(\langle \beta \rangle\). Detailed studies of the harmonic spectra show that, as \(\langle \beta \rangle\) increases, components with progressively higher toroidal mode numbers \(n = 2-6\) increase in amplitude, while the \(n = 1\) contribution actually decreases. The apparent 'poloidal mode numbers' decrease with increasing \(\langle \beta \rangle\) to values between 0 and 1. These results suggest that for broad \(p(r)\) at high \(\langle \beta \rangle\) the simple picture of global magnetic fluctuations that resonate geometrically with rational values of \(\ell\) inside the plasma no longer applies. Instead, the \(\tilde{B}_0\) signals may be dominated by more toroidally localized disturbances at the edge of the plasma, where the local magnetic field curvature in the helical ripple wells is strongly destabilizing.

The measured \(\tilde{B}_0\) are still too small (\(\tilde{B}_0/B_0 \sim 10^{-5}-10^{-6}\)) to produce significant transport so that it is important to study the role of other internal plasma fluctuations.
FIG. 2. Magnetic fluctuation amplitude as a function of $\langle \beta \rangle$ for ATF (after repair of field error).

FIG. 3. Apparent average toroidal $\langle n \rangle$ and poloidal $\langle m \rangle$ mode numbers for magnetic fluctuations in ATF.

Langmuir probe studies [8] of the edge region ($T_e \leq 40$ eV) of ECH and some NBI plasmas show broadband fluctuations in density, $n/n \sim 5\%$, and floating potential, $\Phi/kT_e \sim 10\%$, just inside the last closed magnetic surface ($r/a \sim 0.95$). The outward particle transport induced by these fluctuations is comparable to that estimated from the global particle balance if the flux is assumed to be poloidally and toroidally uniform. Co-ordinated experimental and theoretical modelling studies of these edge
fluctuations are described in a companion paper [8]; these studies suggest that a possible mechanism for the observed edge fluctuation transport is a termally driven instability.

A two channel, 30–40 GHz tunable microwave reflectometer with quadrature phase detection [9] is now being used to study density fluctuations somewhat deeper inside \((r/a \approx 0.8–0.9)\) the plasma. For ECH discharges \((n_e \approx 5 \times 10^{12} \text{ cm}^{-3})\), the reflectometer spectra show peaks in the frequency range 35–80 kHz that are thought to be associated with rational magnetic surfaces in the gradient region of the plasma at \(\iota \simeq 1/2\). Figure 4 shows density fluctuation spectra obtained using two frequency correlation reflectometry with a numerical fringe counting technique to calculate the density fluctuations from the fluctuations in microwave phase delay. For the edge gradient lengths \(L_n \sim 5 \text{ cm} \) measured in these plasmas, the small reflectometer frequency difference \((1.5 \text{ GHz})\) used for Fig. 4 corresponds to a radial separation

---

**FIG. 4.** Fast Fourier transform (FFT) histograms of cross-power and coherence spectra for two frequency correlation reflectometer on ATF. The reflectometer frequency separation is 1.5 GHz. The integrated amplitude of the coherent peak in the range 35–80 kHz is \(\langle n \rangle/n = 5.7\%\); for the frequency range 20–250 kHz, \(\langle n \rangle/n = 10.7\%\). The second coherence peak at \(f = 100 \text{ kHz}\) may be a harmonic introduced by the signal detection process.
\( \sim 3 \) mm between the two reflecting layers. Although the fluctuation amplitudes at low frequency \((f \leq \text{kHz})\) are uncertain because of noise effects in the fringe counting, \( \langle n \rangle / n \) at higher frequencies can be determined; the amplitude of the 35–80 kHz feature in Fig. 4 is \( \langle n \rangle / n \sim 6\% \). Measurements of the radial coherence spectrum obtained by varying the frequency difference between the two reflectometer channels show that the fluctuation peak has a large radial correlation length \( L_c \sim L_n \) and near zero radial propagation phase. The coherent peak is also seen in some of the edge Langmuir probe fluctuation spectra; however, it contributes negligibly (< 10%) to the fluctuation induced \( \vec{E} \times \vec{B} \) particle flux measured by the probes at the plasma edge. In ECH plasmas, the edge \( \vec{B}_e \) signals are quite small and do not show coherence with the density fluctuation feature; in some high pressure NBI heated discharges, however, there is some evidence of correlation between a similar coherent density fluctuation at the plasma surface and \( \vec{B}_e \) [10, 11]. These observations suggest the presence of large (radial width \( \sim 1-5 \text{ cm} \)), coherent, rotating density structures, possibly with a magnetic component, in the gradient region of ATF.

5. DISCUSSION

The importance of curvature driven modes in stellarator configurations such as ATF is demonstrated by studies of self-stabilization in plasmas with peaked pressure profiles. The localized magnetic fluctuations seen in ATF plasmas with broad pressure profiles are likely to be connected to similar instabilities, although a detailed theoretical analysis remains to be done. Initial results from reflectometer measurements of density fluctuations in the gradient region inside the plasma show significant coherent structures. Studies to determine the relationship of these fluctuations to core plasma confinement using systematic variation of the ATF magnetic configuration [12] and improved fluctuation diagnostics (heavy ion beam probing and microwave scattering) are in preparation.

ACKNOWLEDGEMENTS

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IMPURITY EFFECTS AND IMPURITY TRANSPORT IN ATF*

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Abstract

IMPURITY EFFECTS AND IMPURITY TRANSPORT IN ATF.

Impurity radiation and its effects on plasma performance have been investigated spectroscopically for various types of wall conditioning in the Advanced Toroidal Facility (ATF). The total radiated power is estimated by using an impurity transport code in which parameters are adjusted so that the calculated spectral line intensities match measured intensities of the major contaminants. The correct magnitudes of the transport coefficients used in the modelling are ascertained from laser ablation experiments and from intrinsic impurity behaviour. Results indicate that a progression of wall conditioning techniques beginning with glow discharge cleaning and baking through gettering with chromium and titanium have made significant reductions in the radiative losses. In addition, gettering with titanium has led to improved control over the electron density. As a result, it has been possible to improve plasma parameters steadily and to operate with neutral beam injection under conditions where plasma collapses can be avoided.

1. INTRODUCTION

A succession of increasingly more effective wall conditioning techniques has allowed plasma parameters to be improved steadily in the Advanced Toroidal Facility (ATF). Initially, only glow discharge cleaning and baking were used to condition the vessel, but gettering, first with chromium and subsequently with titanium, has become a standard procedure. The motivation for improving the wall conditioning was driven primarily by the fact that neutral beam heated
plasmas in the early stages of ATF operation always collapsed to low temperature, low density afterglows within about 70 ms after the start of injection [1]. It was believed that radiation might be a contributing factor to this phenomenon, and gettering was introduced in an effort to reduce the concentrations of low-Z impurities, which dominated the radiation losses. Throughout the different stages of wall conditioning a great deal of emphasis was placed on analysing the total radiation from spectroscopic analysis in order to assess the influence of individual impurities on the plasma behaviour. To support the radiation modelling and to characterize impurity transport, the diffusion and convection coefficients were determined both from studies of intrinsic impurities and from elements introduced by laser ablation [2].

2. IMPURITY TRANSPORT COEFFICIENTS

Laser ablation of both aluminium and scandium has been employed to measure diffusion and convection coefficients \((D,V)\) of impurities in plasmas heated by electron cyclotron heating (ECH)

![Graph](image)
alone and in plasmas heated by neutral beam injection (NBI). Figure 1 compares the measured time history of the 333 Å line of Al X with the results of modelling for quasisteady NBI plasmas. The e-folding decay time for this interior ion is approximately 100 ms, at least five times longer than for a tokamak of comparable minor radius such as ISX-B [3]. The transport is found to be mainly diffusive with little or no convective component. It is necessary to use a radially dependent diffusion coefficient to reproduce the observed spectral line intensities; the best fits yield $D = 1000 + 4000\rho^2$ cm$^2$/s for ECH plasmas and $D = 5000 - 4500\rho^2$ cm$^2$/s for NBI plasmas, where $\rho$ is the normalized plasma radius. Global impurity confinement times are 65 ms for the ECH plasmas and 40 ms for the NBI plasmas; they are two to three times longer than the energy confinement times. These results are substantiated by analysis of the intrinsic metallic impurity radiation following the startup of a discharge. As metallic impurities released from the walls drift inward, the time lag between the radiation from edge ions and central ions after initiation of a plasma provides a measure of the transport coefficients, since, unlike in a tokamak, the flux surface configuration is established before the plasma is produced.

3. WALL CONDITIONING AND IMPURITY RADIATION

In the initial stages of ATF operation, NBI plasmas always suffered a collapse of stored energy within 70 ms of the beginning of injection, even though wall conditioning and baking alone proved adequate for operating quasi-steady ECH plasmas. Gettering was initiated to reduce the low-Z impurity content which was thought to be largely responsible for the collapse. Chromium gettering using two sources that provided approximately 30% wall coverage was employed first. Subsequently, four sources that gave 50% wall coverage were used. These were replaced by four titanium sources after several months of operation, and ultimately the number of titanium getters was increased to a maximum of seven that provided about 70% wall coverage.
By comparing discharges having very similar values of $\bar{n}_e$ it was observed that the initial chromium gettering did not reduce the total radiative power appreciably from the non-gettered case, although the peripheral radiation from low-Z elements decreased by factors of two to three. Radiation from the He-like and H-like stages of carbon, nitrogen, and oxygen remained about the same in both types of discharges, and the metallic concentrations increased. Toward the end of the period in which chromium gettering was used, it appeared that a reduction of the total radiated power by a factor of 1.4 compared to the non-gettered discharges could be achieved by the regular use of four sources. Gettering with titanium rather than chromium proved to be much more effective in reducing the total radiated power. Spectroscopic analysis indicated that after titanium was employed, the edge radiation from low-Z impurities was reduced by as much as a factor of 50 from non-gettered cases, and total radiated power was lowered by a factor of three or more to approximately 20% of the input power.

Table I shows the spectroscopic estimates for radiated power from the major impurities, other than nickel, in NBI discharges. The results shown in the first four columns refer to plasmas that collapsed between 70 and 100 ms after injection; the analyses of the radiative losses were performed near the time of the peak in the stored energy. The plasmas characterized in the last column were maintained in a quasisteady state. The uncertainty in the total radiated power is estimated to be 25%. Radiation losses approximately equal to the beam power were observed in the non-gettered cases. (The density over most of the plasma cross section is above the cutoff limit so that ECH is not an efficient heating source during NBI.) Chromium gettering lowered the losses to about 63% of the NBI power, but titanium gettering caused a much more dramatic reduction to only about 21% of the beam power at the peak of the stored energy. The fact that low density NBI plasmas still collapsed at such a low level of impurity radiation was a strong indication that radiation by itself did not initiate the collapse.
TABLE I. SPECTROSCOPIC ANALYSIS OF RADIATED POWER DURING DISCHARGES WITH NBI

<table>
<thead>
<tr>
<th>Getters</th>
<th>None</th>
<th>None</th>
<th>4 Cr</th>
<th>4 Ti</th>
<th>6 Ti</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam power $P^*_n$ (kW)</td>
<td>675$^a$</td>
<td>650$^a$</td>
<td>850</td>
<td>750$^a$</td>
<td>750</td>
</tr>
<tr>
<td>$\bar{n}_e (\times 10^{12} \text{cm}^{-3})$</td>
<td>10.9</td>
<td>17.6</td>
<td>16.7</td>
<td>8.3</td>
<td>53.0</td>
</tr>
<tr>
<td>Emission rate for 1032 Å line of O VI (GR$^b$)</td>
<td>150</td>
<td>400</td>
<td>150</td>
<td>12</td>
<td>50</td>
</tr>
<tr>
<td>Radiated power (kW)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Oxygen</td>
<td>263</td>
<td>364</td>
<td>216</td>
<td>18</td>
<td>92</td>
</tr>
<tr>
<td>Carbon</td>
<td>298</td>
<td>219</td>
<td>95</td>
<td>7</td>
<td>11</td>
</tr>
<tr>
<td>Nitrogen</td>
<td>26</td>
<td>93</td>
<td>8</td>
<td>1</td>
<td>11</td>
</tr>
<tr>
<td>Iron</td>
<td>56</td>
<td>72</td>
<td>100</td>
<td>41</td>
<td>22</td>
</tr>
<tr>
<td>Chromium</td>
<td>17</td>
<td>20</td>
<td>77</td>
<td>22</td>
<td>9</td>
</tr>
<tr>
<td>Titanium</td>
<td>0</td>
<td>0</td>
<td>55</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>Total spectroscopically measured power (kW)</td>
<td>681</td>
<td>798</td>
<td>536</td>
<td>160</td>
<td>183</td>
</tr>
</tbody>
</table>

$^a$200 kW of ECH power applied during the entire neutral beam pulse.
$^b$1 gigarayleigh = $10^{15}$ photons·cm$^{-2}$·s$^{-1}$.

This conclusion has been reinforced by several other studies including operation of high density discharges similar to that represented by the results shown in column 5 of Table I. As described in Section 4, titanium gettering has allowed the density limits in ATF to be increased substantially, and at these higher densities it is possible to operate without inducing a collapse. However, it can be seen by comparing columns 4 and 5 in Table I, that the estimates of radiated power are slightly greater for the high density discharges than for the low-density discharges, so that the global radiation does not seem to be the primary driving mechanism for the collapse.

4. PLASMA IMPROVEMENT WITH GETTERING

Gettering has led to significant improvements in plasma parameters, although reducing the concentration of low-Z impurities has not by itself eliminated the problem of collapse. A major contribution of
the gettering is the reduction of hydrogen released or recycled from the walls during a discharge. This reduction allows more control over the evolution of the electron density by permitting tailored gas fuelling, a procedure that has produced improved stored energy and confinement times. Density control has led to quasisteady operation without collapse either at high densities ($n_e > 5 \times 10^{13}/\text{cm}^3$) or at densities so low that the ECH is not cut off during NBI. However, the highest values of stored energy (28 kJ), electron density ($n_e = 9 \times 10^{13}/\text{cm}^3$), and energy confinement times (20 ms) have been obtained in transient plasmas. Figure 2 shows the stored energy as a function of $n_e$ for both 1 T and 2 T discharges. Clearly the stored energy improves with density, and the ability to increase the density steadily over the operating history of ATF has been directly connected with the reduction of impurity levels and recycling.

Figure 3 illustrates the typical evolution of several parameters in a plasma where quasisteady operation is achieved by means of a strong increase in the gas fueling rate at the same time the NBI is initiated. The line averaged electron density ($n_e \ell/65 \text{ cm}$) rises
from $5.0 \times 10^{12} \text{cm}^{-3}$ to $5.6 \times 10^{13} \text{cm}^{-3}$. Concurrently, the electron temperature drops from 800 eV to less than 100 eV, but then reheats to 275 eV; this behaviour is reflected in the soft X-ray signal. Following the reheat, a quasi-steady state is maintained until injection is terminated. It is not possible to control the density in this manner without gettering, nor at the other extreme, is it possible to keep the density below the ECH cutoff without gettering.

5. SUMMARY

Absolute spectral line measurements and measured transport coefficients have been used to evaluate the total radiated power from ATF for various wall conditioning techniques. The radiation level is approximately 20% of the input power when using titanium
gettering with about 70% wall coverage. Improvement of cleanliness and hydrogen pumping by titanium have led to significant increases of stored energy and to modes of operation without collapses of stored energy during NBI.

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CURRENT FREE PLASMA HEATING IN THE L-2 STELLARATOR AT THE SECOND HARMONIC OF THE ELECTRON CYCLOTRON FREQUENCY


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Abstract

CURRENT FREE PLASMA HEATING IN THE L-2 STELLARATOR AT THE SECOND HARMONIC OF THE ELECTRON CYCLOTRON FREQUENCY.

The paper presents results of experiments carried out on the L-2 stellarator on current free plasma production and heating at the second harmonic of the electron cyclotron frequency with the use of an extraordinary wave.

1. INTRODUCTION

In this paper we present the results of current free plasma production and heating at the second harmonic of the electron cyclotron frequency with the use of an extraordinary wave. The experiments were carried out in the L-2 stellarator (R = 1 m, a = 11.5 cm, \( l = 2 \), \( \iota_0 = 0.19 \), \( \iota_a = 0.78 \)).

A gyrotron with frequency \( f_0 = 75 \) GHz, power \( P_0 = 300 \) kW and heating pulse duration up to 10 ms was employed as a heating source. If the cyclotron resonance condition \((\omega_0 = 2\omega_{Be})\) is satisfied on the stellarator axis, a plasma with an average density up to \( 2 \times 10^{19} \) m\(^{-3}\) and an electron temperature up to 1 keV is produced. In this experiment the volume averaged plasma heating power reached a value of the order of 1 MW/m\(^3\).

In contrast to the previous L-2 experiments on ECRH of a current free plasma with the use of an ordinary wave at the first harmonic of the cyclotron frequency \((f_0 = 37 \) GHz) [1], in the experiments considered here we succeeded in more than doubling the average plasma density and in substantially increasing the specific heating power in the cyclotron resonance region.
2. EXPERIMENTAL CONDITIONS

Microwave power was introduced into the vacuum chamber of the stellarator through a quasi-optical system consisting of four copper mirrors [2]. The cylindrical surfaces of the mirrors caused the focusing of the wave during its motion from the gyrotron towards the stellarator. The microwave power losses in the quasi-optical system did not exceed 10% of the gyrotron output power.

The computation of the wave power absorption along the ray traces shows that, in the case considered of an extraordinary wave (E \perp B_0) and central resonance (B_0 = 1.34 T), the single pass absorption coefficient \( \eta \) is practically equal to unity for the plasma parameters of the L-2. Figure 1 presents the results of the calculations; together with the absorption coefficient, the radial dependence of the absorption power is given. The specific power, plotted on the ordinate, is the ratio of the power of the wave absorbed between the magnetic surfaces separated along the radius by the distance \( \Delta r \) to the total plasma volume between these surfaces. In accordance with the calculations, the specific absorption power in the L-2 experiments reached a value of the order of some tens of watts per cubic centimetre, which is, apparently, essentially higher than in the other experiments on ECRH in toroidal systems (e.g. [3]).

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**FIG. 1.** Radial profiles of (a) the local coefficient of extraordinary wave absorption \( (\eta_0 = 0.96) \) and (b) the absorbed power density \( (P_0 = 150 \text{ kW}) \).
3. PLASMA PROPERTIES DURING CENTRAL HEATING

Figure 2 shows the time evolution of plasma formation, heating and decay after the end of the microwave pulse. The formation of the plasma column is completed during 1.5 ms, the main density rise time taking about 0.5 ms. It should be noted that we have not practically studied the mechanism of the gas breakdown and the initial stage of heating, which give rise to some questions. In this paper we discuss mainly the next stage of the ECR discharge when the average plasma density was maintained constant during the heating with an accuracy to not less than 10%. The time evolution of the plasma heating is seen in Fig. 2(c), which shows the diamagnetic signal. In the case considered the plasma energy reaches a stationary value in the middle of the heating pulse. For ECRH in the L-2 a lower (compared with the OH regime) level of radiation losses is typical; in all regimes investigated these losses were less than 10% of the total input power.

After the end of the microwave pulse the plasma energy falls with a time constant of about 3 ms. At the same time, no peculiarities can be observed in the signal of the average plasma density at the moment of termination, and the density remains at the same level during 4 ms, after which plasma decay with a time constant of about 8 ms begins.

**FIG. 2.** Time dependence of the main discharge parameters: (a) ECRH power pulse, (b) average plasma density, (c) plasma energy and (d) radiative loss power.
The measurements of the radial plasma density and temperature distributions indicate the flattening of the density distribution with plasma heating. Figure 3 presents examples of such distributions for three cases which differ in their values of heating power and mean plasma density. Despite the fact that the mean plasma density is constant in time, the radial density distribution is changed with time. Figure 3 shows that at the centre of the plasma column the distributions become more and more flat with the increase in specific energy input due to either an increase in heating power or a decrease in plasma density. Accordingly, the gradient at the plasma boundary grows. For example, for the case of the greatest energy input per particle (Fig. 3(c)) the distribution, described initially by an almost quadratic parabola $n_e(r) = n_0[1 - (r/a)^2]$, by the end of the heating pulse turns out to be close to $n_0[1 - (r/a)^9]$. The radial electron temperature distribution (Fig. 4) noticeably differs from the corresponding density distributions and is characterized by a gradient slightly varying with radius.

The values of the plasma pressure obtained in this experiment are not yet high ($\beta(0) \approx 0.5\%$) and perturb only slightly the initial structure of the magnetic surfaces. Figure 5 presents the radial distributions in the horizontal plane of the electron temperature at the centre of the plasma column measured by means of X ray diagnostics. The rise in the plasma pressure leads to the shift of the axis of the magnetic surfaces and the absolute value of this shift does not exceed approximately one tenth of the plasma radius.

Figure 5 shows one more regularity in ECRH for the L-2 stellarator. For the given heating power the value of the plasma electron temperature weakly depends on the plasma density values. The dependence of the total plasma energy on the value
FIG. 4. Electron temperature profile (open squares: soft X rays; open circles: spectroscopy; full circles: Thomson scattering; triangles: electron cyclotron emission; $P_0 = 280$ kW, $n_e = 1.4 \times 10^{13}$ cm$^{-3}$).

FIG. 5. Displacement of electron temperature distribution with increase in the average plasma density (1: $0.9 \times 10^{13}$ cm$^{-3}$; 2: $1.4 \times 10^{13}$ cm$^{-3}$; 3: $1.8 \times 10^{13}$ cm$^{-3}$; $P_0 = 280$ kW).

of the mean density at a fixed level of input power is given in Fig. 6. It is clearly seen that the energy increases with the increase in plasma density. The experimental data may be quantitatively represented by the dependence $W \approx (n_e)^{\alpha}$, where the index (taking account of the spread of experimental points) lies in the range $\alpha = 0.7-1$, i.e. the heating efficiency noticeably improves with the plasma density.
4. EFFICIENCY OF MICROWAVE POWER ABSORPTION

The measurements by microwave detectors located in the cross-section of the wave input window on the inner side of the torus indicate, in the central resonance case, a practically complete absence of the passing power, beginning with the moment of the plasma column formation. Simultaneously it was found that the microwave power in the other chamber sections ceased to be recorded, starting at a distance of 20 cm from the input window. Meanwhile, no enhanced wave reflection on the input port of the stellarator chamber was observed, i.e. according to theoretical predictions all the introduced power is absorbed in the plasma near the cross-section of the microwave beam injection.

However, the measurements made by a diamagnetic technique of the microwave power absorbed by the plasma yield a value noticeably less than that of the introduced power. This method of determining the absorbed power is based on the plasma energy balance equation:

$$\frac{dW}{dT} = P_{ab} - \Gamma - P_{rad} - P_{cx}$$  \hspace{1cm} (1)

where $W$ is the total plasma energy, $\Gamma$ is the heat flux transferred by diffusion and thermal conductivity, $P_{rad}$ is the radiative loss power, $P_{cx}$ are the charge exchange...
losses and $P_{ab}$ is the microwave power absorbed by the plasma. At the moment of heating pulse termination the jump of the derivative of the diamagnetic signal shows the value of $P_{ab}$. This is valid only if all the other terms of Eq. (1) have no discontinuity. The constancy of $P_{rad}$ and $P_{cx}$ follows from the direct measurements of these quantities. As for the flux $\Gamma$, generally speaking it may be supposed that the presence of high power HF fields in the plasma volume could deteriorate its confinement in the stellarator magnetic field. However, this hypothesis does not find any experimental support. The time evolution of the plasma density curve does not indicate a noticeable change in particle confinement at the moment of heating termination (Fig. 2). Also, no changes are observed in the behaviour of plasma fluctuations measured by a method of 2 mm beam scattering. The boundary plasma parameters also do not vary, which indicates the absence of a gap in the thermal conductivity flux by which the boundary plasma is heated.

To measure correctly the jump of the derivative of the magnetic flux inside the plasma in the conducting closed vacuum chamber, it is necessary to take account of the time constant of the chamber. In our case this constant, $\tau_c = L_c/R_c$, amounted to 110 $\mu$s according to both calculations and measurements. Hence it is clear that the chamber may cause essential distortions in the measurement of the fast processes, i.e. if the plasma parameters change over a time comparable with $\tau_c$. In this case the true value of the jump of the derivative of the diamagnetic signal may be obtained from the following simple calculations.

The change in the magnetic flux inside the plasma due to the change of its energy, $\Phi_0$, results in the change of the flux inside the chamber, $\Phi_c$, which produces the current flowing through the chamber, $I_c$. The change of the flux inside the longitudinal field’s solenoid results in a current change in the solenoid coils by the value $I_s$, which causes a change in the magnetic flux inside the chamber, $\Phi_{cs}$. Hence the diamagnetic loops located at the surfaces of the chamber record the following changes in the magnetic flux inside the chamber:

$$\frac{d\Phi_c}{dt} = -(d\Phi_0/dt + d\Phi_{cs}/dt + I_c R_c)$$ (2)

and the diamagnetic loop voltage

$$u = -d\Phi_c/dt + d\Phi_{cs}/dt + d\Phi_0/dt$$ (3)

Taking into account (2), (3), relations $d\Phi_s/dt = 0$ and $d\Phi_c/dt = \tau_c (du/dt)$, one can obtain:

$$u = -[1 - (r_c/r_s)^2](d\Phi_c/dt + d\Phi_0/dt)$$ (4)

$$\Phi_0 = \frac{u + \tau_c (du/dt)[1 - (r_c/r_s)^2]}{1 - (r_c/r_s)^2}$$ (5)
Usually the diamagnetic signal \( u \) gives the value of the derivative of the magnetic flux change in the plasma. In the case of the closed chamber, in order to determine this value it is necessary also to incorporate the derivative of the signal from the diamagnetic loops.

The analysis of experimental data from the ECRH measurements shows the importance of incorporating the additional term \( \tau_c (du/dt) \) for determining the value of the jump of the derivative of the plasma energy, i.e. the value of the absorbed power. The processing of the diamagnetic signal with account taken of the chamber effect shows that the real value of the derivative jump \( (W_1 - W_2) \) exceeds that observed, in some cases, by more than 40%. Such a picture may emerge in the case when there exists in the plasma some fraction of energy with a short lifetime \( \tau_l \ll \tau_E \) (the plasma energy lifetime \( \tau_E = 3 \) ms). The fast phase of decay, which is not recorded owing to the integrating effect of the chamber, corresponds just to this short lived component. The energy removed from the plasma during the fast phase of decay does not, as a rule, exceed 5% of the plasma heat energy. However, the absorbed power increment \( \eta \), incorporating this decay phase is in some cases fairly noticeable. Figure 7 shows the change of the values \( P_{ab} \) and \( P_f \) during the heating pulse at \( P_0 = 200 \) kW. It is seen that the microwave power \( P_f \) absorbed by the short lived plasma component by the end of the heating pulse amounts to 30 kW. The higher the level of the input power and the lower the plasma density, the larger the fraction of the component \( P_f \) in the total energy balance. Figure 8(a) shows the dependence of \( P_{ab} \) and \( P_f \) on the input power at \( n_e = 9 \times 10^{12} \) cm\(^{-3}\) and Fig. 8(b) their dependence on the density at \( P_0 = 200 \) kW. The absorption coefficient dependence is weak and the absolute value \( \eta = P_f / P_0 \) does not exceed 60%. Taking the power absorbed by the short lived plasma component into account does not yield a substantial correction to the absorption coefficient.

The analysis of the evolution of the diamagnetic signal is based on the assumption of longitudinal isotropy of both longitudinal and transverse plasma pressures. We may suppose that the short lived component is localized in the region of HF power.

![FIG. 7. Evolution of \( P_{ab} \) and \( P_f \) during ECRH.](image-url)
injection. The presence of the longitudinal inhomogeneity of the pressure leads to difficulties in determining the absolute values of longitudinal and transverse fields and their relation to plasma energy.

It is not clear now how such a channel of absorption and fast loss of microwave energy can exist in the plasma, especially in the case of second harmonic ECRH, when the tails of fast particles and HF plasma oscillations were not recorded. Further experiments will have to study this phenomenon.

The modelling of the heat transport in the L-2 stellarator, carried out under the assumption of HF beam absorption in the centre of the plasma column and using $P_{ab}$ and neoclassical transport coefficients, showed fairly good agreement with the experimentally measured value of the electron temperature, its radial profiles and the plasma confinement time.

REFERENCES

TWO-DIMENSIONAL DENSITY STRUCTURES AND THE EFFECTS OF EXTERNAL BIASING ON POTENTIAL AND DENSITY FLUCTUATIONS IN THE INTERCHANGEABLE MODULE STELLARATOR

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Abstract

TWO-DIMENSIONAL DENSITY STRUCTURES AND THE EFFECTS OF EXTERNAL BIASING ON POTENTIAL AND DENSITY FLUCTUATIONS IN THE INTERCHANGEABLE MODULE STELLARATOR.

Two-dimensional density measurements in the Interchangeable Module Stellarator during ECRH show a hollow profile and island structures for hydrogen and xenon plasmas. Vacuum magnetic surface measurements demonstrate an island at the \( i = 3 \) surface. Divertor strike plate biasing changes the radial electric field profile and the fluctuation induced transport outside the separatrix.

1. INTRODUCTION

The Interchangeable Module Stellarator (IMS) is a seven field period, \( l = 3 \) modular stellarator with a major radius of 40 cm and an average plasma minor radius of 4 cm capable of operating at magnetic fields of up to 7 kG. Plasmas were produced with a 10 ms, 0.5–2.0 kW pulse of electron cyclotron resonance heating (ECRH) at 7.26 GHz, corresponding to a central magnetic field strength of 2.6 kG. The observed plasma densities were approximately \( 1.0 \times 10^{11} \text{ cm}^{-3} \) with a flat electron temperature profile of the order of 5 eV.

2. TWO-DIMENSIONAL DENSITY STRUCTURES

Previous measurements of hydrogen plasmas in the Interchangeable Module Stellarator (IMS) during ECRH showed increasingly hollow density profiles as the cyclotron layer was moved from the inboard to the outboard side of the torus [1]. The two-dimensional structure of the plasma density has been investigated at two separate toroidal locations and was measured on a shot-to-shot basis by scanning a
FIG. 1. Contours of constant density for a hydrogen plasma during ECRH (contour intervals are marked in units of $1 \times 10^{10} \text{ cm}^{-3}$).

FIG. 2. Contours of constant density for a xenon plasma during ECRH.
single Langmuir probe in 5 mm increments across the plasma cross-section. Figure 1 shows a contour plot of the plasma density for hydrogen discharges in which the data has been smoothed over 1 cm $\times$ 1 cm grid. In addition to the hollow profile, structures resembling cells, or 'islands', can be seen in the data. Fourier analysis of the density variation on ideal magnetic surfaces shows strong $m = 2$ and $m = 3$ components.

Measurements of density and floating potential in the JIPP stellarator demonstrated that cell structures could be reduced when a heavier gas such as xenon was used [2]. Similar experiments were undertaken in IMS to observe whether the density structures seen in a hydrogen plasma could be eliminated. Figure 2 shows the contour plot obtained in xenon discharges under the same conditions as Fig. 1 for hydrogen. While the xenon line average density is approximately four times that of the hydrogen data, the locations of the relative maxima and minima, and the basic structures, agree rather well.

A comparison of the density structures observed for the two gases to the confining magnetic field configuration has been performed by mapping the magnetic surfaces at the same field strength at which the plasma measurements were made. Using a video camera and recorder, the toroidal transits of a 70 eV electron beam were monitored by capturing the image of a highly transparent fluorescent screen, which fluoresced upon electron impact [3]. An $m = 3$ magnetic island with a width of $1.2 \pm 0.2$ cm was found at the $\iota = 1/3$ surface. The location and the phase of the island are in reasonable agreement with the $m = 3$ plasma density structure.

3. EDGE FLUCTUATIONS WITH DIVERTOR STRIKE PLATE BIASING

Edge studies in IMS have shown the possibility of altering diverted particle flows by applying an electrostatic bias to remote divertor strike plates [4]. Investigations have been performed to examine the effects of this strike plate biasing on fluctuations of the density and potential in the edge plasma region. An array of Langmuir and emitting probes was used to measure the fluctuations under the same conditions, which resulted in significant poloidal redistribution of the diverted plasma flows.

The divertor strike plates are directly connected to specific edge regions of the separatrix through the diverted magnetic field lines, and a fraction of the potential applied to these plates propagates to these locations [4]. Fluctuation measurements were made along a minor radial chord at two different toroidal angles; one location corresponds to a region magnetically connected to the biased strike plate (called the 'S' port), while the other (the 'S' port) is not magnetically connected to the biased plate. Figure 3 shows the normalized density fluctuation profile at the 'S' port under the conditions of $+50$ V, $-50$ V, and no applied bias. The figure clearly shows a decrease of edge density fluctuation levels with either sign of applied bias, with a reduction factor as large as 2. The 'Z' port density fluctuation levels, as seen in Fig. 4, increased with either sign of applied bias. The edge potential fluctuations were only marginally affected by the biasing.
Estimates of the edge fluctuation induced transport indicate a reduction in particle transport for the 'S' port when the divertor strike plate is biased, and an enhancement of the 'Z' port under the same conditions. Direct measurements of the (DC) radial electric field with emissive probes show an increase in shear at the 'S' port and a flattening of the field at the 'Z' port region. The reduced transport in the 'S' port region is consistent with an increase in the electric field shear decorrelating the fluctuations [5].
4. CONCLUSIONS

Two dimensional measurements of the density in IMS during ECRH indicate that the profile is hollow and that island structures are evident for both hydrogen and xenon plasmas. Measurements of the vacuum magnetic surfaces with an electron beam and a fluorescent screen indicate that an island exists at the $t = 1/3$ surface with a width of $1.2 \pm 0.2$ cm. Biasing of the divertor strike plates has been shown to change both the radial electric field profile and the fluctuation induced transport in the region outside the separatrix.

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PHYSICS STUDIES ON
HELICAL CONFINEMENT CONFIGURATIONS
WITH $\ell = 2$ CONTINUOUS COIL SYSTEMS

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Abstract

PHYSICS STUDIES ON HELICAL CONFINEMENT CONFIGURATIONS WITH $\ell = 2$ CONTINUOUS COIL SYSTEMS.

Physics studies have been carried out to optimize $\ell = 2$ heliotron/torsatron configurations having continuous coil systems for the Large Helical Device (LHD), focusing on the beta orbit–divertor compatibility requirement, neoclassical and anomalous transports, bootstrap and Ohkawa currents, and divertor layer analysis. The optimal m number is found to be $\sim 10$ for the LHD system on the basis of three physics criteria: MHD stability ($\beta \geq 5\%$), particle orbit confinement (loss cone free radius $r_L \geq \frac{1}{2} a_p$ (plasma minor radius)) and a clean divertor layer (divertor–wall clearance $\Delta_{dw} \geq 3$ cm). The pitch parameter of the helical coil $\gamma_c$ is determined mainly from the beta–divertor condition. For the detailed prediction of LHD plasma parameters, 2-3-D equilibrium–transport simulations including empirical or drift wave turbulence models are carried out, which reveal that the global confinement time is sensitive to the edge electron anomalous transport, although ripple loss through the ion channel is dominant in the plasma core. When ripple transport optimized configurations are adopted, the bootstrap current is increased; however, the Ohkawa current may be utilized to cancel this current. The divertor layer study clarifies the peculiar magnetic properties of thin curved divertor layers and suggests the effectiveness of helical divertors. From these physics considerations within the related engineering constraints, a final standard LHD configuration ($m = 10, \gamma_c = 1.25, \alpha^* = 0.1$, where $m$, $\gamma_c$ and $\alpha^*$ are toroidal mode, pitch and pitch modulation parameters of the helical coil, respectively) is determined.

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1. INTRODUCTION

Helical systems have distinct advantages in achieving steady state operation without plasma current disruptions. To demonstrate these advantages using a built-in helical divertor configuration, the Large Helical Device (LHD, major radius $R \sim 4$ m, magnetic field $B \sim 4$ T, plasma minor radius $a_p \sim 0.6$ m) [1] is being built. The $\ell = 2$ continuous coil system is adopted for the LHD because of its clean divertor configuration and adequate experimental database. Related to this project, physics studies extended from previous work [2] have been carried out to optimize heliotron/torsatron configurations within the engineering constraints [3, 4] with respect to physics requirements such as high beta ($\beta \geq 5\%$), good high energy particle confinement (loss cone free radius $r_L \geq \frac{1}{3}a_p$ for 70% ICRF heating efficiency), sufficient divertor–wall clearance ($\Delta_{dw} \geq 3$ cm) and optimization of transport properties.

2. MHD BETA AND PARTICLE ORBIT CONFINEMENT

The low $m$ (toroidal period number) system has good MHD properties but particle orbit confinement is a concern. This orbit problem is significantly mitigated by the inward shift of the plasma column. Extensive studies on high beta stability in the LHD have been done using H-APOLLO and H-ERATO codes [2], and orbit confinement conditions for high heating efficiency have been analysed by using parallel adiabatic invariants [5]. The beta–divertor condition mentioned in Section 1 dictates an $m$ number below $\sim 10$ for a medium $\gamma_c$ configuration ($\gamma_c = m a_c / l R \sim 1.2-1.3$, $a_c$ is the minor radius of the helical coil), as shown in Fig. 1(a). A higher $\gamma_c$ configuration ($\gamma_c \geq 1.3$) is not appropriate with respect to the divertor–wall clearance for 4 m/4 T LHD designs. Figure 1(b) shows beta orbit conditions in terms of $m$ number and percentage of inward shift of the plasma axis ($\delta_{ax} = \Delta_{ax} / R$), which suggests that a configuration with $m = 10$ ($\gamma_c = 1.20$) and a slight inward shift ($\delta_{ax} = -3\%$) is optimum.

In the LHD a slight positive pitch modulation of the helical windings ($\alpha^* = 0.1, \theta = (m/l)\phi + \alpha^* \sin(m/l)\phi$) is found to improve the physics and engineering compatibility. The positive pitch modulation leads to more improvement in orbit confinement for a large inward shift ($\delta_{ax} \leq -3\%$) than the negative pitch modulation.

The detailed engineering design makes high coil current density available (up to 53.3 A/mm$^2$ at $R = 4.0$ m and $B = 4.0$ T), which permits a higher $\gamma_c$ configuration (up to 1.25) and gives rise to higher beta achievement without deterioration of the orbit confinement compared with the $\gamma_c = 1.20$ configuration.
FIG. 1. Schematic m–g, and m–δax plots for LHD design optimization. Relevant physics requirements are fulfilled in the shaded area. (a) Beta and divertor constraints. The divertor condition is shown for a 4 m/4 T machine with a coil current density of 33.3 A/mm². Other helical devices are plotted in the same figure. In the LHD the helical pitch parameter γ is adjustable by the multilayer operation of the helical coils. (b) Beta and loss cone free conditions. An appropriate coil pitch parameter γ is chosen for each m number (γ = 1.15, 1.20, 1.23, 1.25, and 1.30 for m = 8, 10, 12, 14, and 19, respectively).

3. NEOCLASSICAL AND ANOMALOUS TRANSPORTS

For the analysis of the LHD transport, 2\(\frac{1}{4}\)-D equilibrium–transport simulations (a combination of 3-D equilibrium VMEC [6] and 1-D transport HTRANS codes) with NBI deposition and its slowing down calculations have been done which take into account several anomalous transports as well as neoclassical transport. The neoclassical ripple transport model adopted here includes radial electric field and multihelicity effects of magnetic configurations, and its validity is checked by the DKES code [7]. As for anomalous transports, modifying the global LHD scaling [8],
we adopted (1) an empirical semilocal transport coefficient with an improvement factor $f^{\text{imp}}$,

$$\chi_{\text{emp}}(r) = 1.47 f^{\text{imp}} R^{0.58} B^{-0.84} R^{-0.75} n(r)^{-0.69}$$

or (2) a drift wave turbulence (DWT) model [9] with a helical ripple ($\epsilon_h$) contribution on the collisionless and dissipative trapped electron modes (CTE and DTE):

$$\chi_{\text{te}}(r) = \frac{5}{2} \min \left( \sqrt{\epsilon_h} \frac{\omega_{pe}}{k^2_\perp}, \sqrt{\epsilon_h} \frac{\omega_p}{k^2_\perp}, \frac{\omega_{pe}}{\nu_{ei}/\epsilon_h} \right)$$

These models do not contradict the CHS experimental data of Ref. [10], as shown in Fig. 2.

Typical simulation results of 20 MW NBI heated LHD plasmas are shown in Fig. 3 for (a) empirical scaling with $f^{\text{imp}} = 1.0$ and (b) the electrostatic DWT model. Neglecting anomalous particle inward flows, flat or hollow density profiles are obtained, as seen in the existing experiments. The empirical electron thermal conductivity is larger than the neoclassical values; on the other hand, the ripple
contribution is not neglected for ion energy transport (Fig. 3(a)). From the prediction of the DWT model with neoclassical transport, the ion temperature gradient (ITG) mode due to the flat density profile is dominant in the ion transport process of the plasma core, instead of CTE, DTE, collisionless circulating electron (CCE) and collisional circulating electron (XCE) modes (Fig. 3(b)). This prediction from ITG modes should be checked by experiments.

Related to the effect of the magnetic configuration, a moderate positive pitch modulation of the helical coil slightly reduces the central temperature owing to high ripple ion loss, but leads to a slight increase in the plasma radius and hence in the global confinement. Moreover, the positive $\alpha^*$ configuration with higher effective helical ripple provides easier access to the hot ion regime with positive electric field at low density operation.
4. BOOTSTRAP AND OHKAWA CURRENTS

Currents parallel to magnetic field lines, consisting of the bootstrap current, Ohkawa current and Spitzer current in the LHD, are analysed in the banana regime using the flux-friction relation. The bootstrap current depends strongly on the magnetic geometry because of the strong dependence of the radial diffusion on the magnetic field structure. Vertical elongation of the toroidally averaged magnetic surface, outward shift of the magnetic axis, and positive pitch modulation of the helical coils make the bootstrap current small. Vertical elongation is especially effective [11, 12]. However, its dependence on the geometry is opposite to that of the particle orbit confinement. In the optimal configuration discussed in Section 2, the bootstrap current amounts to 150–300 kA, depending on the density profile. The neoclassical conductivity and the ratio of the net Ohkawa current to the fast ion current have the same expressions as the ones in the case of a tokamak, because the source term is insensitive to the detailed magnetic field structure. Influences of the magnetic field structure appear through the fraction of trapped particles. The Ohkawa current is obtained using the electron drift kinetic equation [13] as well as the moment approach, which suggests that ion rotation does not contribute to the Ohkawa current. A unidirectional parallel injection of hydrogen atoms with 10 MW and 120 kV generates a current of roughly 100 kA. One method to reduce the total plasma current is to balance the bootstrap current by the Ohkawa current, if needed.

![Diagram of periphery field lines for LHD divertor.](image)
5. HELICAL DIVERTOR STUDY

The helical divertor is a key ingredient of the LHD. Like a good tokamak divertor, the LHD configuration has a good capability of recycling control and high edge shear, two possible key conditions for achieving a good H mode. The major difference between the helical and tokamak divertors is the magnetic configuration.

The LHD configuration can be divided into four regions: (1) the stochastic region just outside the last closed surface, which is due to overlapping of the natural islands with a toroidal mode number of 10; (2) the divertor thin curved layers, as shown in Fig. 4, which are caused by the existence of high ‘local’ magnetic shear at the periphery on the large major radius side of the torus and the radial movement of the ‘X point’ [14]; (3) the region between the thin curved layers, in which field lines directly connect two different points on the divertor plates within several helical pitches; and (4) the divertor channel with very short field line length (~πR/m) beyond the X point.

The heat flux from the core plasma region first passes through the stochastic region and flows along the divertor thin curved layers with a connection length of \( \geq 3 \times 2\pi R \). Then it flows into the divertor channel (region 4) and reaches the divertor plates. The most important design requirement for the divertor is that these thin curved layers do not touch the first wall, which is satisfied in the present LHD design.

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MHD AND KINETIC DYNAMO ACTIVITIES AND CONFINEMENT IN THE REVERSED FIELD PINCH EXPERIMENT ON TPE-1RM15

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Abstract

MHD AND KINETIC DYNAMO ACTIVITIES AND CONFINEMENT IN THE REVERSED FIELD PINCH EXPERIMENT ON TPE-1RM15.

In the reversed field pinch experiment on TPE-1RM15, three kinds of limiter material (stainless steel, graphite and molybdenum) are studied. In the present operating region, molybdenum gives the best results. The central electron temperature ($T_e(0)$) and the density increase linearly with the plasma current ($I_p$) up to 200 kA. The highest electron temperature of about 1 keV and a density of about $3 \times 10^{19}$ m$^{-3}$ are obtained. The heat flux of the high energy electron flow (HEEF) in the boundary region is estimated to be about 500 MW/m$^2$ from the increase in the bulk temperature and the variation of the surface temperature of the movable graphite limiter. A high energy component of about 10 keV is observed in HEEF. The power loss fraction associated with HEEF is about 50-70%. The detailed mechanism of the MHD dynamo becomes clear where toroidally localized distortions of $m = 1$ and $m = 0$ modes are observed. It is found that $V_{loop}$ depends weakly on $I_p$ and $T_e(0)$. The $V_{loop}$ anomaly related to the dynamo activity is estimated to be about 10-15 V.

1. INTRODUCTION

In the reversed field pinch (RFP) experiment on TPE-1RM15 ($R/a = 0.7/0.135$ m) [1], the error magnetic field has been substantially reduced [2, 3], and the limiter material has been changed to improve plasma confinement in the high plasma current ($I_p$) region.

The mechanism relevant to the configuration sustainment (dynamo action) of RFP is studied. In several RFP experiments, the high energy electron flow (HEEF) is observed [4], which causes the damage of the limiter surface and is thought to be one of the phenomena supplying evidence in favour of the kinetic dynamo theory (KDT) [5]. On the TPE-1RM15 experiment with a graphite limiter, intensity, direction and energy distribution of the heat flux associated with the HEEF are estimated

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FIG. 1. Dependence of central electron temperature on $I_p$. Results with three limiter materials are compared during the flat-top phase of the plasma current. ○: SUS, △: graphite, ●: molybdenum.

in the boundary region. The detailed mechanism of the MHD dynamo is also studied by extensive measurements of the magnetic fluctuation. The effect of the dynamo activity on the loop voltage ($V_{\text{loop}}$) is estimated by using the $I_p$ ramp-up/down technique.

2. IMPROVEMENT OF CONFINEMENT WITH MOLYBDENUM LIMITER

To improve plasma performance in the high $I_p$ region, the limiter material is changed from graphite to molybdenum. Scaling of the central electron temperature ($T_e(0)$) with $I_p$ for a molybdenum limiter is compared in Fig. 1 with the cases of stainless steel (SUS) and graphite. The dependence of $V_{\text{loop}}$ on $I_p$ is also compared in Fig. 2. As is shown in these figures, molybdenum gives the best results. With molybdenum, $T_e(0)$ reaches the highest value of about 1 keV at $I_p = 195$ kA, with an electron density ($n_e$) of about $3 \times 10^{19}$ m$^{-3}$. The electron density increases with $I_p$, and the poloidal beta ($\beta_p$) remains approximately constant ($\equiv 0.01$). The loop voltage does not show an abrupt increase in the high $I_p$ region (about 30 V at $I_p = 195$ kA). The energy confinement time ($\tau_E$) increases almost linearly with $I_p$ ($\tau_E \equiv 0.2$–0.3 ms at $I_p = 195$ kA).

The deterioration in the SUS limiter case can be consistently interpreted as a radiation collapse due to enhanced metal impurities from SUS. A two-dimensional calculation shows that the surface temperature of the SUS limiter can reach 2800 K,
which is well above the melting point with a heat flux of 500 MW/m². A rapid increase in radiation power (P\text{rad}) is observed in the high I\text{p} region.

In the graphite limiter case, the calculation shows that the surface temperature can reach about 2800 K, which agrees with the value measured by the far infrared temperature monitor. Radiation enhanced sublimation could produce a large influx of carbon impurities. However, P\text{rad} and the intensities of the carbon and oxygen lines do not show any abrupt increase in the high I\text{p} region. The deterioration is not caused by the radiation loss. A possible explanation is the destabilizing effect of the enhanced resistivity near the boundary, which concentrates the current profile since a large amount of neutral influx from the highly heated graphite limiter surface is observed in the high I\text{p} region, strongly localized in the toroidal direction.

In the case of molybdenum, the characteristics of the discharge are essentially the same as in the case of SUS, but molybdenum can withstand much higher I\text{p}. The calculated limiter surface temperature is about 2000 K, which is well below the melting point. The improvement of scaling with the molybdenum limiter indicates that the degradation of the RFP confinement in the high I\text{p} region was not caused by the confinement physics of RFP (transport, stability, etc.) but by the plasma–wall interaction in the present operating region.

3. CHARACTERISTICS OF HIGH ENERGY ELECTRON FLOW

In the graphite limiter experiment, intensity and direction of the heat flux are measured by inserting a movable graphite limiter into the boundary region [6]. The
increases in the bulk temperature on the electron and ion drift sides are separately measured by thermocouples. The temporal and spatial variations of the surface temperature are measured by an infrared temperature monitor. In a discharge with $I_p = 135-140$ kA, a Joule input power of $P_{\text{OH}} = 4.5-5$ MW, $V_{\text{loop}} = 30-35$ V, $F = -0.11$ and $\theta = 1.55$ at $t = 4.5$ ms, the energy deposited on the electron drift side at fixed limiter position ($r = 135$ mm) is, typically, 2.2-2.3 MJ/m$^2$. The heat flux is estimated to be 400-600 MW/m$^2$. A large temperature rise is observed on the surface facing the direction of the electron drift. The temperature of the hottest spot reaches 2800 K. On the other hand, the ion drift side remains below 1200 K, which is the lower measurable limit of the temperature monitor. By comparing the measured temperature with the calculation, the intensity of heat flux is estimated to be about 500 MW/m$^2$. This value shows fairly good agreement with the estimation from increase in the bulk temperature.

The energy distribution of the HEEF is estimated by measuring the spectrum of soft X ray emission from metal targets (Ti, Cr, Fe and Ni) at the plasma boundary [7]. As is shown in Fig. 3, the energy is distributed over a wide range (several keV). The peak at the Ni K$_\alpha$ line is clearly observed, indicating that electrons with energies of more than 8.3 keV (ionization energy of the Ni K shell) exist. This value is about two or three orders higher than the bulk electron temperature at the edge (10-20 eV) and about one order higher than $T_e(0)$ (about 400 eV). The power loss fraction to the fixed limiters by HEEF can be about 50-70%, on the assumption that about 30 limiters (60 mm width) are exposed to the electron flow with a depth of 5 mm.

The statistical treatment of the electron velocity distribution measured by Thomson scattering at off-axis ports (±7 cm apart from the centre) indicates the pos-
4. DETAILED STRUCTURE OF MHD DYNAMO MECHANISM

To study the detailed mechanism of the HEEF dynamo the fluctuations of magnetic field are measured extensively [8]. Figure 4 shows the time variation of the $m = 1$ fluctuation as a function of the toroidal angle ($m$ is the poloidal mode number), and Figure 5 shows the variation of the n-spectrum ($n$ is the toroidal mode number) during one dynamo event in a high $\theta$ discharge (1.82). Large $m = 0$ distortion is always observed at places where the $m = 1$ mode is large. Toroidal mode numbers of the $m = 0$ mode are typically 1, 2 and 3. These observations agree qualitatively with the results of recent simulations [9, 10]. The most interesting feature reported here is that the largest distortions of $m = 1$ and $m = 0$ modes are toroidally localized, where the mode conversion seems to take place actively, and they move around the
torus along the helix of the dominant \( m = 1 \) mode (typically, \( n \) is about 10). The localized mode structure, which is similar to the slinky mode found in OHTE with resistive shell [11], seems to be a common feature, even with a conductive shell. Movement of the \( m = 0 \) mode around the torus has been observed in ZT-40M [12]. These observations suggest that mode conversion (reconnection of magnetic field lines) is a toroidally localized phenomenon although the place where it happens moves around the torus and an almost axisymmetric configuration is generated again after one dynamo event.

5. DYNAMO EFFECT AND \( V_{\text{loop}} \) ANOMALY

The energy confinement time for ohmically heated plasmas is generally expressed as \( \tau_E(s) = 0.47 \times 10^{-6} R_0 \beta_p L_p / V_{\text{loop}} \), where \( r_0 \) is the major radius. \( \tau_E \) in present RFP experiments is restricted by the limitation of \( \beta_p \) and by the high, anomalous \( V_{\text{loop}} \) value, which is associated with the dynamo activity. The limitation
of $\beta_p$ is probably caused by a pressure driven instability. The effect of finite pressure on the transition to a helical symmetry state might be a candidate for this phenomenon [13].

In the experiment with graphite limiter the effect of the dynamo activity on $V_{\text{loop}}$ is estimated by controlling the fluctuation level which is known to be a function of the current ramp-up/down rate. By this method, the comparison of $V_{\text{loop}}$ with the same configuration parameters such as $\theta$, $F$ and $I_p$ is possible. Figure 6 shows the dependence of $V_{\text{loop}}$ on the squared fluctuation level for $I_p = 145 \text{ kA}$ with $\theta = 1.58$, where the inductive part of $V_{\text{loop}}$ is corrected by a polynomial function model. Extrapolation to the zero fluctuation limit gives about 20 V, which can be interpreted as the Joule heating part. The anomaly part is 10–15 for the flat-top $I_p$ case. This value is close to that estimated by a different method in HBTX [14].

6. SUMMARY

By using the molybdenum limiter, scalings with $I_p$ are improved in the high $I_p$ region. $T_e(0)$, $n_e$ and $\tau_E$ increase almost linearly with $I_p$ and become about 1 keV, $3 \times 10^{19} \text{ m}^{-3}$ and 0.2–0.3 ms at $I_p = 195 \text{ kA}$, respectively. $\beta_p$ stays nearly constant. The heat flux of the high energy electron flow (HEEF) is about 500 MW/m$^2$, by which the surface of the graphite limiter is heated up to 2800 K. The energy spectrum of HEEF is distributed over a wide range. A very high energy component of about 10 keV is observed. The detailed mechanism of the non-linear phenomenon in the MHD dynamo becomes clear, where the toroidally localized distortion of the
m = 1 and m = 0 modes are observed which move around the torus along the dominant m = 1 mode helix. The $V_{\text{loop}}$ anomaly which is associated with the dynamo activity is about 10–15 V, which is 30–50% of $V_{\text{loop}}$.

REFERENCES

HEATING, POTENTIAL FORMATION AND BARRIER PUMPING USING MODE CONTROLLED ICRF IN THE HIEI TANDEM MIRROR

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Abstract

Plasma heating and potential formation by mode controlled ICRF are investigated in the HIEI tandem mirror. Central cell plasma heating is provided by a shear Alfvén wave which is mode converted from the helicon/fast wave launched at the midplane. The ion confining potential is created in the plug cells by a combination of sloshing ion formation and electron heating due to the central cell ICRF propagating to the plug cells. It is shown that additional application of the plug cell ICRF produces a thermal barrier resulting in enhancement of the ion confining potential. The mechanism of the thermal barrier pumping is attributed to the ICRF ponderomotive force.

1. INTRODUCTION

The formation and sustainment of thermal barrier potential are necessary for efficient and long time ion plugging in tandem mirrors. The pumping of cold ions trapped in the thermal barrier has so far been provided by NBI with a shallow injection angle or by the $\mu \times B$ force of applied ICRF [1]. In this paper we study the ICRF ponderomotive pumping together with the formation of confining potential.

The HIEI tandem mirror [2] is a purely axisymmetric three cell device with a length of 4.6 m, a magnetic field strength in standard operation of 0.1–0.7–0.25–0.7 T (from the central cell to the outer plug mirror point) and a central cell plasma radius of 14 cm. Two dual half-turn antennas [3] with an axial separation of 48 cm are installed in the central cell midplane and are driven at a frequency of 8 MHz for the excitation of a helicon/fast wave (H/FW). In each plug cell a dual half-turn antenna with a frequency of 1.5 MHz is located 8 cm inboard from the midplane for the excitation of a shear Alfvén wave (SAW) or ion Bernstein wave (IBW). Typical plasma parameters in the central cell for plugged operation are: peak density $n = 3 \times 10^{12}$ cm$^{-3}$, $T_{i\perp} = 100$–200 eV, $T_{i\parallel} = 50$–110 eV and $T_e = 20$–50 eV.
2. CENTRAL CELL PLASMA PRODUCTION AND HEATING

With the application of an RF pulse to two dual half-turn antennas, a helium plasma with a line density \( n_1 \sim 3.1 \times 10^{14} \text{ cm}^{-2} \) \( (n \sim 1.7 \times 10^{13} \text{ cm}^{-3}) \) was created for a net RF power of 200 kW, \( \omega/\Omega_i \sim 57 \) and \( \Delta \psi \sim \pi/2 \), which corresponds to right rotating excitation. Here \( \Omega_i \) is the local majority ion cyclotron frequency at the antenna location and \( \Delta \psi \) is the phase difference of RF currents in the two antennas. The \( \beta \) value is about 10\%. We have found that the right rotating excitation of the antennas efficiently generates the \( m = -1 \) H/FW, resulting in the highest plasma production rate. The central cell plasma is MHD stable, owing to the RF stabilization effect of the H/FW. The low frequency (~10 kHz) fluctuation level near \( r = 9 \text{ cm} \) was significantly reduced by increasing the RF antenna current.

By adding minority hydrogen (\( \leq 30\% \)) to the helium plasma, perpendicular ion temperature \( T_{i\perp} \) as high as 200 eV was obtained for \( \omega/\Omega_i \sim 23 \) and \( n \sim 2.2 \times 10^{12} \text{ cm}^{-3} \) (\( \beta \sim 3\% \)). Figure 1 shows (a) \( T_{i\perp} \) and (b) \( nT_{i\perp} \) divided by the net input RF power \( P_{\text{net}} \) versus \( \Delta \psi \) for a fixed antenna current. The maximum value of \( T_{i\perp} \) (130 eV) as well as the maximum heating efficiency are obtained for \( \Delta \psi \sim \pi/2 \). For this phasing, the H/FW is excited efficiently, as mentioned earlier. According to the mode conversion theory [4, 5], the H/FW, which is excited at the midplane, encounters cut-off near the throats as it propagates towards the higher magnetic field in two-ion-species plasmas. If the energy of the incident wave is normalized to 1, the energy of \( 1 - T^2 \) is reflected, where \( T \) is the transmission coefficient. The reflected wave propagates towards the midplane to approach the two-ion

\[ \text{FIG. 1. (a) } T_{i\perp} \text{ and (b) } nT_{i\perp} \text{ divided by the net input RF power } P_{\text{net}} \text{ versus the phase of the antenna current, } \Delta \psi, \text{ for a fixed antenna current amplitude.} \]
hybrid resonance. Then, the energy fraction of $T_2(1 - T_2)$ of the H/FW converts into a SAW, which damps at the minority ion cyclotron resonance. The maximum mode conversion rate, i.e. the maximum value of $T_2(1 - T_2)$, is 0.25. We measured the RF magnetic field near the mode conversion region. The left rotating component of the RF magnetic field, $B_+$, is significantly enhanced over the right rotating component $B_-$ when the two-ion hybrid resonance layer is located in the central cell. The value of $B_+/B_-$ at $r = 0-2$ cm is 0.8 for the case without the hybrid resonance, while it is 2.1 with the resonance. We estimated the mode conversion rate from the measured magnetic field amplitude. The value of the mode conversion rate is 0.17, which is close to its theoretical maximum. Therefore, the observed heating is due to the mode converted SAW.

3. POTENTIAL FORMATION AND BARRIER PUMPING

We measured the axial potential profile by using probes and multigrided end loss analysers (ELAs). The potential is almost flat ($\pm 15$ V) from the central cell midplane to the outer mirror point of the plug cell for a plasma without the two-ion hybrid resonance. With the resonance, the potential in the plug cell increased for plug cell density smaller than the central cell density by a factor of 5. For $n_1 \sim 9.5 \times 10^{13}$ cm$^{-2}$ and a minority concentration of $\leq 5\%$, the plug cell inboard potential was higher than the central cell potential by ~70 V. The potential profile in the plug cell shows a small dip at the midplane. It was also observed that the plug cell electron temperature is 3-4 times higher than in the case without the two-ion hybrid resonance. The potential of the floating end plates decreased significantly, showing the existence of high energy electrons. The incident H/FW from the central cell mode converts into a SAW near the two-ion hybrid resonance point at the inboard side of the plug cell to heat minority ions and electrons. Since the ion heating occurs far from the midplane, sloshing ion distribution is formed with a potential dip at the midplane. The dip was larger for higher minority concentration. The strong electron heating by the SAW enhances the plug cell potential.

We observed a reduction of the ion end loss flux $\Gamma_{\text{end}}$ in the presence of the ion confining potential $\phi_\text{C}$. The value of $\phi_\text{C}$ is obtained as the difference of the plug cell potential measured by ELA and the central cell potential measured by the probe. The value of $n/\Gamma_{\text{end}}$ is plotted in Fig. 2 versus $e\phi_\text{C}/T_{ij}$ for $n_1 \sim 5.1 \times 10^{13}$ cm$^{-2}$ and a minority concentration of ~20%. The horizontal bars indicate an error involved mainly in determining the central cell potential from the probe data. It is seen that the parallel confinement is improved by increased $\phi_\text{C}$. By using the Pastukhov formula, $\tau_{\text{Pl}}$ is estimated to be about 7 ms for $e\phi_\text{C}/T_{ij} \sim 1$.

We then applied the plug cell RF (1.5 MHz) of the $m = \pm 1$ mode using the dual half-turn antenna with $\omega/Q_i = 1.5$. The major effect of the plug cell RF is that the potential near the plug cell midplane decreases significantly. The value of the potential in the plug cell relative to that in the central cell ($z = 60$ cm) is plotted in
FIG. 2. $n/T_{\text{end}}$ versus $e\phi_C/T_i$ for $n_i = 5.1 \times 10^{13} \text{ cm}^{-2}$ and a minority concentration of $\sim 20\%$.

FIG. 3. Value of potential in the plug cell relative to that in the central cell ($z = 60 \text{ cm}$) as a function of axial distance.
Fig. 3 versus the distance from the central cell midplane for \( n_i \sim 4.0 \times 10^{13} \text{ cm}^{-2} \), a minority concentration of \( \sim 30\% \) and \( T_{i\perp} \sim 110 \text{ eV} \). It is clearly seen that the potential dip is formed near the midplane with the potential hill on the outboard side. The thermal barrier depth is \( \phi_B \sim 100 \text{ V} \), and the ion confining potential is \( \phi_C \sim 100 \text{ V} \). The radial profile of the potential shows that the barrier depth is largest at \( r = 0 \) and gradually decreases towards the outside. The value of \( \phi_B \) increases with the plug cell RF power, as shown in Fig. 4(a). The maximum value of \( \phi_B \) obtained so far is 170 V. When the thermal barrier is created, as shown in Fig. 3, the density at \( z = 103 \text{ cm} \) increases significantly. The ratio of the density at this point to that at \( z = 124 \text{ cm} \) is greater than 4. The large amplitude local RF field of the \( m = \pm 1 \) mode at \( z = 122 \text{ cm} \) produces axial ponderomotive force primarily on ions. The effective pseudo-potential pumps ions out of the local potential dip near \( z = 124 \text{ cm} \) and increases the barrier depth. This ponderomotive force also acts as a pumping force for cold ions. Since \( \nabla B \) is weak near the potential dip, the pumping mechanism is not attributed to the \( \mu \nabla B \) force.

Another effect of the plug cell RF is the enhancement of the potential at the outboard side of the plug cell. Figure 4(b) shows the difference in potentials with and without the plug cell RF, \( \Delta \phi_P \), measured by ELA as a function of the magnetic field strength at the throat, \( B_m \). The large increase in \( \Delta \phi_P \) is notable for \( B_m \) greater than 0.6 T. This condition corresponds to the fact that the majority ion cyclotron...
resonance for the plug cell RF locates at \( z = 145 \text{ cm} \) or closer to the midplane. A factor of 2 increase in \( T_e \) at \( z = 142 \text{ cm} \) was observed, as well as a moderate increase in \( T_{i\perp} \) for \( B_m \geq 0.6 \text{ T} \). The electron heating is considered to be caused by the IBW propagating towards the outboard side through the midplane. As the wave approaches the cyclotron resonance, the axial phase velocity decreases and electron Landau damping takes place. Plug electrons trapped in the potential well near \( z = 140 \text{ cm} \) are then pumped out to enhance the potential hill.

In conclusion, it has been demonstrated that thermal barrier formation and plasma confinement can be achieved in the purely axisymmetric tandem mirror by ICRF only.

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SUPPRESSION OF ROTATIONAL INSTABILITY IN FRC BY AN AXIAL CURRENT

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Abstract

SUPPRESSION OF ROTATIONAL INSTABILITY IN FRC BY AN AXIAL CURRENT.

The theoretical analysis based on an ideal MHD model shows that a weak axial current is effective in suppressing the n = 2 rotational instability when the radial distribution of the current density is bell shaped so that the greater part of the axial current flows through a narrow region around the axis of symmetry of the FRC. In the case where the axial current flowing inside the null point flows backward outside, the stabilizing effect of the axial current becomes stronger because the circulating axial current produces no azimuthal field in the vacuum region which could destabilize the FRC. The theory also shows that the alternating axial current whose angular frequency is much greater than the growth rate of the n = 2 rotational instability is effective in suppressing the instability even if the axial current flows at the surface of the plasma column. The stabilizing effect of the current is confirmed by the NUCTE-III experiment, where an alternating current is applied to the FRC. The n = 2 rotational instability is suppressed when the current amplitude and frequency increase up to 60 kA and 90 kHz, respectively. The effective strength of the field produced by the oscillating current corresponds to about 25% of the confinement field at the plasma surface. The angular frequency of $5.7 \times 10^5$ $s^{-1}$ of the current is much higher than the maximum growth rate of $3 \times 10^4$ $s^{-1}$ of the n = 2 rotational instability. An n = 1 kink instability is excited by the current in every discharge. In the high current case where the current exceeds the Kruskal–Shafranov limit at the plasma surface, the FRC is moved over a large distance towards the wall.

1. INTRODUCTION

Some years ago, the PIACE experiment showed that weak multipole fields could suppress the n = 2 rotational instability, which was the only gross instability from which FRCs suffered [1].
The suppression effect of the multipole fields was confirmed by many other FRC experiments. At present, almost all FRC devices are equipped with multipole field systems to obtain FRC plasmas that are free of the \( n = 2 \) rotational instability. This method has, however, some disadvantages. For instance, if the multipole field penetrates into the edge layer of the FRC, the particle loss to the wall along the lines of force shortens the FRC lifetime. Therefore, it is desirable to develop an alternative method of suppressing the \( n = 2 \) rotational instability. In this paper, the results of theoretical and experimental investigations are reported on the effect of axial current suppressing the \( n = 2 \) rotational instability.

2. THEORETICAL ANALYSIS OF THE STABILIZING EFFECT OF AN AXIAL CURRENT

We consider an infinitely long cylindrical plasma column rotating around its axis of symmetry. The plasma is assumed to be an incompressible, conductive fluid. The axial field \( B_z \) in the plasma column is given by

\[
B_z = B_W \tanh \left( K \left( \frac{2r^2}{r_s^2} - 1 \right) \right) \tag{1}
\]

where \( B_W \) is the axial field in the vacuum region, \( r_s \) the radius of the plasma column, and \( K \) the constant from which the average beta \( \beta_{AV} \) is determined as \( \tanh(K)/K \). The axial current is assumed to be so weak that the azimuthal field \( B_\theta \) produced by the current scarcely affects the FRC equilibrium. The linearized MHD equations for perturbations of the form \( \exp \{ i (n \theta + kz - \omega t) \} \) are solved to obtain the following dispersion relation:

\[
\rho \Omega - (n-1) \Omega r^2 = \frac{1}{\mu} \left[ n(n-1) \left\langle \frac{B_\theta^2}{r^2} \right\rangle_n + 2(n-1)K \left\langle \frac{B_\theta B_z}{r} \right\rangle_n \right]
+ K^2 \left\langle B_z^2 \right\rangle_n + \frac{1 + X_s^{2n}}{1 - X_s^{2n}} \left( \frac{nB_\theta}{r_s} + KB_W \right)^2 - \frac{nB_\theta^2}{r_s^2} \right] - (n-1) \rho \Omega^2 \tag{2}
\]
where $\rho$ and $\Omega$ are the mass density and the rotational angular velocity of the plasma column, respectively, $x_s$ the ratio of the plasma radius $r_s$ to the wall radius $r_w$, $B_a$ the vacuum azimuthal field at the plasma edge, $\mu$ the vacuum permeability, and $\langle f \rangle_n$ the average value of $f(r)$, defined by

$$\langle f \rangle_n = \frac{2n}{r_s^{2n}} \int_0^{r_s} f(r) r^{2n-1} dr \tag{3}$$

To obtain the stability criterion for the FRC of $l_s$ in length, we assume that the both ends of the plasma column are fixed so as to limit the perturbations to the standing waves whose wavelengths are $2l_s$, $2l_s/2$, $2l_s/3$... Then, we obtain the Kruskal-Shafranov stability criterion against the $n = 1$ kink mode from Eq. (2). Against the $n = 2$ rotational mode, the stability criterion becomes

$$\alpha_2 \equiv \delta_2 b_a^2 + b_c^2 \tag{4}$$

where

$$\alpha_2 = \frac{1 + x_s^4}{1 - x_s^4} + \frac{x_s^2}{2} \tag{5}$$

$$b_a = \frac{l_s B_a}{\pi r_s B_w} \tag{6}$$

$$b_c = \frac{l_s B_c}{\pi r_s B_w} \tag{7}$$

$$B_c = r_s |\Omega| \sqrt{\mu \rho} \tag{8}$$

and the coefficient $\delta_2$ is a dimensionless quantity which depends on $x_s$ and the radial distribution of the axial current density. The lefthand side of Eq. (4) represents the wall stabilizing effect,
the first term on the righthand side the effect of the axial current, and the second term the destabilizing effect of the plasma rotation. When the axial current flows on the boundary surface of the plasma column, the coefficient $82$ is positive so that the axial current destabilizes the plasma column. However, when the axial current is concentrated around the axis of symmetry of the FRC, the stabilizing effect of the magnetic shear produced by the axial current is superior to the destabilizing effect of the vacuum azimuthal field so that the axial current stabilizes the plasma column. For example, when the current density is uniform for $0 < r < r_s/2$ and zero for $r > r_s/2$, the coefficient $82$ is $-5.4--5.5$ for $0 < x_s < 0.5$.

In the case where the axial current flowing inside the null point ($r = R = r_s/\sqrt{2}$) flows back outside, the vacuum azimuthal field which destabilizes the plasma column vanishes. Then the dispersion relation given by Eq. (2) reduces to

$$
\rho \{ \omega - (n-1)\Omega \}^2 = \frac{1}{\mu} \left[ n(n-1) \left\langle \frac{B_r^2}{r^2} \right\rangle_n + 2(n-1)k \left\langle \frac{B_r B_z}{r} \right\rangle_n \\
+ k^2 \left\langle B_z^2 \right\rangle_n + \frac{1 + x_s^{2n}}{1 - x_s^{2n}} k^2 \left\langle B_w^2 \right\rangle_n \right] - (n-1) \rho \Omega^2
$$

(9)

Since the righthand side of Eq. (9) is positive for $n = 1$, the plasma column is stable against the $n = 1$ kink mode. Against the $n = 2$ rotational mode, the stability criterion becomes

$$
\alpha_2 \equiv \delta_2' b_t^2 + b_e^2
$$

(10)

$$
b_t = \frac{l_s B_t}{\pi r_s B_w}
$$

(11)

where $B_t$ is the azimuthal field at the null point and the coefficient $\delta_2'$ is a functional of the axial current density distribution.

A straightforward calculation shows that, when each component of the axial current inside the null point flows back outside along the same line of force, $\delta_2'$ is negative so that the axial current has a stabilizing effect on the $n = 2$ rotational
mode. For example, when the current density is uniform inside the null point and outside, $\delta_2' = -1.55$. When the axial current is concentrated around its axis of symmetry and flows back around the plasma edge, $\delta_2'$ becomes so large that the stabilizing effect of the axial current increases.

Finally, we consider the case where the axial current is an alternating current and its angular frequency is much higher than the growth rate of the $n = 2$ rotational instability. In this case, we can show that, even if the axial current flows on the surface of the plasma column, the plasma column can be stabilized by the axial current. The dispersion relation for the surface current becomes

$$\rho \{ \omega^2 - (n-1) \Omega \}^2 = \frac{1}{\mu} \left[ \frac{n}{2} \left( \frac{1+X_s^{2n}}{1-X_s^{2n}} - 1 \right) \overline{B_a}^2 + \frac{1+X_s^{2n}}{1-X_s^{2n}} K^2 B_W^2 \right]$$

$$+ K^2 <B_z^2> \right\} - (n-1) \rho \Omega^2 \quad (12)$$

where $\overline{B_a}$ is the amplitude of the vacuum azimuthal field at the plasma edge. Using Eq. (12), we obtain the stability criterion against the $n = 2$ rotational mode:

$$\alpha_2 \equiv \delta_2'' \overline{B_a}^2 + \sigma c^2. \quad (13)$$

$$\delta_2'' = -\frac{1}{\alpha_2} \left( 2 \frac{1+X_s^4}{1-X_s^4} - 1 \right) \quad (14)$$

$$\overline{B_a} = \frac{l_s \overline{B_a}}{\pi l_s B_W} \quad (15)$$

Since $\delta_2'' < 0$, the axial current has a stabilizing effect.

3. STABILIZING EXPERIMENT OF ROTATIONAL INSTABILITY BY ALTERNATING CURRENT

The FRC plasma is confined in an axial field of 0.7 T for 70-80 $\mu$s on the NUCTE-III device [2]. The rotational instability starts to grow at about 30 $\mu$s after FRC formation and is observed up to the end of the discharge as seen in Fig.1-(a). The angular
FIG. 1. Time evolution of FRC plasma line integrated density measured at plasma column midplane: (a) $I = 0$, (b) $I = 50$ kA, (c) $I = 57$ kA, (d) $I = 63$ kA.

The experimental range of the current is 0-63 kA in amplitude and 0-90 kHz in frequency. The stabilizing effect of the current on the rotational instability appears clearly above 50 kA and 70 kHz. The characteristic oscillation arising from the instability seen in the line integrated density signal becomes unimportant with increasing current. The current effectively stabilizes the rotational instability at $I_Z = 57$ kA with $f = 90$ kHz in the present experimental range as shown in Fig. 1(c). Since the...
The small amplitude fluctuation in the signal sometimes remains between 30 and 60 $\mu$s even if the rotational instability is stabilized. The fluctuation becomes remarkable at high current operation, for instance, at $I_z = 63$ kA as shown in Fig. 1(d). The dip on the signal is caused by the radial shift of the plasma column towards the tube wall. It can be understood from our previous studies that the FRC with $I_z = 63$ kA is in the marginal state of a kink instability [3]. The kink mode is also observed by a magnetic probe array which detects the azimuthal field generated by the axial current. It is found from a Fourier analysis of the probe signals that the mode appears even at low
current as the line density measurement does not perceive the kink instability. The mode clearly develops with increasing current as shown in Fig. 2.

4. CONCLUSIONS

A theoretical analysis has been done on the effect of an axial current on the $n = 2$ rotational instability. When the axial current concentrates around the axis of symmetry of the FRC, it has a stabilizing effect on the $n = 2$ rotational mode. Furthermore, when the axial current flowing around the axis of symmetry flows back around the plasma edge, the stabilizing effect increases. When the axial current is an alternating current, a dynamic stabilization of the $n = 2$ rotational mode is possible.

The NUCTE-III experiment shows that the $n = 2$ rotational instability is suppressed by an alternating axial current of an angular frequency of $5.7 \times 10^6 \text{s}^{-1}$, which is much higher than the maximum growth rate $3 \times 10^4 \text{s}^{-1}$ of the instability. With increasing amplitude of the axial current, the deformation of the plasma column decreases and finally disappears. However, a further increase causes an $n = 1$ kink instability, because the Kruskal-Shafranov stability criterion is violated.

REFERENCES

SOME PHYSICAL CONSEQUENCES OF MAGNETIC INSULATION IN A COMPACT TORUS

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Abstract

SOME PHYSICAL CONSEQUENCES OF MAGNETIC INSULATION IN A COMPACT TORUS.
Magnetic insulation during reversal based on the azimuthal field $B_\phi$ of an axial current pulse $I_z$ is investigated. It is shown that the plasma relative pressure $\beta_z$ (diamagnetism) in the channelled Z-discharge is one of the most important parameters determining the effectiveness of magnetic insulation. In turn, the main discharge parameters including the electron temperature are strongly dependent on a number of experimental parameters such as the temporal delay between bias field $B_z$ and $Z$-current, the relative position of electrodes and separatrix between the trigger and mirror fields, etc.

1. INTRODUCTION

For a long time, experiments on Compact Torus (CT) formation had an unpleasant defect: plasma-wall contact during $B_z$ reversal, which resulted in a break of the magnetic insulation and a strong limitation of many CT parameters (such as RFTP of the sixties). By now, this difficulty has been overcome and consecutive magnetic insulation has been realized [1]. In this context, two questions are to be answered:

(1) Which of the previously determined CT characteristics are retained in the new conditions and what corrections must be made to them?
(2) Which new, substantial phenomena are associated with the magnetically insulated formation and what new control means can be used in this case?

The present paper intends to clarify these problems.

2. EXPERIMENTAL RESULTS

Experiments were made on the TOR device, which is described elsewhere [1], at $B_z \sim 5-10$ kG, $n \sim 10^{14}-10^{15}$ cm$^{-3}$.

Figure 1 shows the general situation in a CT.
1. As has been shown in present experiments, radial plasma confinement and prevention of plasma-wall interaction have a significant effect on the radial structure and other plasma characteristics. Subsequent system relaxation to equilibrium is, however, still governed by axial shock compression. It is important that this process can be characterized in terms of 'hard' (H) and 'soft' (S) modes (Fig. 2(A)), which were determined in the early experiments.

This result is of fundamental importance because the classification soft and hard modes is inherently related to the wide spectrum of relaxation and equilibrium characteristics observed in experiments [2, 3].

FIG. 2. (A) Two types of Compact Torus axial relaxation: 1,2: soft mode with diamagnetic plasma in channelled discharge; 3,4: hard mode. The plasma during the first stage is paramagnetic, 5: $\Phi$, (t). (B) Behaviour of plasma density at three radii in a quasi-equilibrium phase. (C) Radial distribution of optical radiation plasma in soft (1) and hard (2,3) relaxation modes. (D) $Z_{\text{eff}}$ obtained from absolute X ray measurements. (E) Evolution of end region in a hard compression process: 1: separatrix radius $r_s(z)$; 2: radial intensity distribution of $I_{CIV}$; 3: axial 'piston' velocity; 4: $n(t)$ at $r = 1$ and $r = 12$ cm. (F) CT annihilation as a result of magnetic barrier break-up: 1 separatrix profile, $r_s(z)$, 2: radial distribution of plasma density $n(r)$ and C IV line width. (G) Magnetic flux lifetime versus ion temperature.
2. The magnetically insulated (channelled) preionization process (for the first stage of formation, see Fig. 1(A)) is accompanied by a substantial level of plasma dia-(or para-)magnetism; this new feature to a great extent determines the generation of a hard or a soft compression mode. Plasma diamagnetism can be varied by means of programming the axial current $I_z(t \sim 50 \text{ kA}$, which is introduced into the plasma through two circular electrodes at the ends of the chamber. These electrodes are placed near the separatrix inside a cusp-like magnetic structure which is created during the first stage of the $\theta$-pinch discharge [1].

More precise programming was achieved by means of varying the following experimental parameters: (a) electrode position relative to the separatrix between mirror, $\Phi_m(t)$, and trigger, $\Phi_t(t)$, magnetic fluxes; (b) the relative values of these fluxes, and, finally, (c) the temporal delay between $I_z(t)$ and $B_z(t)$. An additional positive effect of this optimization procedure was the reduction of the lower density limit down to $10^{14} \text{ cm}^{-3}$, where the compact torus could be formed.

Simultaneous measurements of $I_z(t)$, $B_z(t)$, $\Phi_z(t)$, $D(z, t)$, and $n(r,t)$, as well as end and side spectral diagnostics provide a basis for clarifying specific properties of the dia- and paramagnetic modes (Fig. 3(A), (B)) and determine the marginal conditions when the channelled discharge converts to an electrical break along the inner surface of the quartz tube (Fig. 3(C), (D)). The most important condition which must be fulfilled for the required process development to take place is the formation of an initial axial plasma core (current channel), which is brought about by both a Z-pinch effect during the initial phase ($\tau \sim 0\text{-}1 \mu\text{s}$) and an appropriate junction of the circular electrodes with the near-axis magnetic lines $B_2$. The subsequent monotonical (diffusive) evolution of the $n(r,t)$ profile preserves an arch-type distribution with sharp effective boundary, $r_p$. (This enables arbitrary adjustment of the $r_p/r_w$ value at the reversal time).

The transition from a dia- to a paramagnetic mode is performed by increasing the $I_z(t)$ delay relative to $B_z(t)$ and a correspondent rise of the initial trapped flux $\Phi_z$. Weakening mirrors also contribute to this transition. The following MHD equilibrium relation demonstrates the dia- and paramagnetic parts:

$$\frac{B_{2z}(r)}{B_{ze}} = \left(1 + 2(B_{pe}^2 - B_{p}^2)/B_{ze}^2 - \frac{8\pi nT/B_{ze}^2}{1}\right)^{1/2}$$

$$\begin{align*}
1 &\leftarrow \text{para} \rightarrow 1 \\
1 &\leftarrow \text{dia} \rightarrow 1
\end{align*}$$

Fig. 3. (A) Time dependence of channelled discharge parameters (diamagnetic mode). $I$: axial current; $\Phi$: bias flux; $D$: plasma diamagnetism; $n(r = 0, 6, 12 \text{ cm})$: optical plasma length; $N$: particle inventory. (B) The same for the paramagnetic mode. (C,D) Non-channelled z-discharge with quartz wall arcs and impurity illumination (Si II, Si III). Streak pictures are taken side-on. (E) Slow reversal with magnetic (barrier) insulation and $X_s \sim 1$. 
The simplest estimation of the channelled discharge parameters can be made for the marginal case of equal dia- and paramagnetic parts (discharges with very small diamagnetic signals $D(t) \sim 0$ are observed experimentally). Assuming $B_{zi} \sim B_{ze}$, we obtain $\beta_s (r = 0) \sim 8 \pi n T B_{pe}^2 \sim 2$ as long as $B_{pe} \sim B_{ze}$. For $J_z (r) = \text{const}$, $\beta_z = 2 (B_{pe}^2 / B_{ze}^2) (1 - r^2 / r_p^2)$ remains greater than 1 up to $r = 0.7 r_p$. Thus, for typical experimental conditions, $B_{ze} = 1 \text{ kG}$, $B_{pe} = (0.8 - 1.2) B_{ze}$ and $n_e = 1 - 3 \times 10^{14} \text{ cm}^{-3}$, the electron temperature is about 100 eV. This result is in accordance with other well known characteristics of a channelled discharge: high level of trapped flux, $\Phi_{tr} \sim \pi r_p^2 B_e$, and small anomalous annihilation losses during reversal, due to a comparatively high electron temperature and a diffusive $J_z (r)$ distribution. For example, in a favourable case, $V_{dr} / C_s < 1$ at any radius, so an ion sound instability can be excluded. Sharp $I(z)$ current excursions (Fig. 3(B)) are not allowed because this would lead to violent radial dynamics, a steeply skinned plasma boundary, and wall breakdown with Si II and Si III illumination.

3. Figure 3(E) presents some results of a model experiment on slow formation CT with a diamagnetic plasma in the first stage. The process of full magnetic closing, which takes $\Delta t \sim 10 \mu$s, and the subsequent closed structure with $X_s = 1$ are shown. In a ‘standard’ case with a reversed-to-bias-field ratio, $B_n / B_1 \sim 5$, an interesting phenomenon is observed. If no measures are taken to delay the axial contraction (such as the ‘ballooning’ configuration), a longitudinal plasma motion manifests itself in some effects on the plasma surface as soon as $B_z$ crosses zero well before the reversal is completed and even before the beginning (!) of the radial compression. The C IV line width ($\lambda = 465.8$ nm, $\Delta \lambda \sim 3.5$ A) corresponds to 1000 eV carbon energy — a rather non-trivial result for the early stage of reversal.

Estimations of $Z_{eff}$ (Fig. 2(D)) were made with the help of collimated X ray detectors through thin Al foils (3.5 and 7 $\mu$m) absolutely calibrated on Al target.

4. Another observation is related to the late phase of the relaxation. Even in a most favourable ‘soft’ case, full equilibration of outer plasma boundary (separatrix) does not mean completion of the axial relaxation because intense plasma flows (Fig. 2(B)) are preserved (likely along the magnetic lines of force) for $10^2 \mu$s, in contrast with transit time $\tau_{tr} \sim \ell / 2 V_A \sim 6 - 8 \mu$s.

5. Magnetic insulation complicates and, to some extent, changes the physics of losses and global degradation of the compact torus because it has a potential ability to stimulate transport processes and MHD instabilities.

The first problem to be solved is enhanced magnetic annihilation in the neutral layer, due to a possible current (ion sound) microinstability. A current layer is formed in the region of a high density gradient, $(1 - 5) \times 10^{14} \text{ cm}^{-3} / (1 - 3) \text{ cm}$, so that the conditions for an ion sound instability, $V_{dr} / C_s > 1$, can be met at some radius with sufficiently low electron density. Experiments (Fig. 2(F)) show that, for certain initial conditions, rapid magnetic field annihilation can result in a substantial
(2–3 times) drop of the plasma cross-section in 1–3 μs. This process gives rise to a noticeable heat release that prevents any axial contraction of the antiparallel configuration.

6. Another example deals with MHD plasma behaviour in the hard mode of axial relaxation. The increased Alfvén velocity in a relatively low density neutral layer is a potential reason for much more vigorous axial dynamics and a resulting total energy loss. This aspect of the problem is due to the paramagnetic preionization mode because the viscosity in the neutral layer and the damping mechanisms are weakened in this regime.

An interesting effect of anomalous losses from the end parts of the separatrix (Fig. 2(E)1,2,3) during axial contraction is also observed in the present experiments. A picture of periodic loss bursts can be reconstructed from the separatrix evolution at the ends, the radial distribution of highly ionized impurity lines (C IV), and the axial velocity modulation of the X points.

As is shown in Fig. 2(C)2,3, strong plasma contamination is characteristic for the ‘H’-mode where a dense impurity spectrum including low ionization states is registered up to the wall. This fact points out that the ‘H’-mode leads to an intense transport through the separatrix and plasma interaction with the wall. A hard formation case with a relatively long lifetime of $\tau_\phi \sim 80 \mu$s (Fig. 2(A)3) cannot be treated as being in order because the rising experimental energy is not consistent with the ‘H’-regimes.

Finally, it should be noted that a high plasma temperature is the most interesting feature in further Compact Torus investigations. As is shown by the TOR (USSR) experiments (Fig. 2(G)), even the 1–3 keV region gives a noticeable rise in $\tau_\phi$ compared to the Hoffman scaling [2] although these experiments suffered from a wall-dominated reversal.

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OGRA-4K EXPERIMENTAL RESULTS

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Abstract

OGRA-4K EXPERIMENTAL RESULTS.

Confinement of a hot electron plasma has been investigated on the OGRA-4K linear cusp device. The plasma was generated by ECRH and formed as two rings. The plasma parameters depending on gas puffing are: density \((2-6) \times 10^{12} \text{ cm}^{-3}\), average electron energy \(2-20 \text{ keV}\), \(\beta\) up to 3\%. It has been shown that plasma losses are approximately classical.

A linear cusp with a hot electron plasma as an MHD anchor for stabilization of hot ion plasma in open magnetic systems was suggested in Ref. [1]. The possibility of obtaining \(\beta\) of a few per cent in cusps, MHD stability with a pressure profile decreasing towards the cusp centre and the effect of the non-adiabaticity zone on plasma energy losses from the cusp were the main problems of experimental studies at the first stage.

In Ref. [2] the first experimental results obtained on the OGRA-4K cusp device were reported. The results of further investigation, which have provided positive answers to the above mentioned problems, are discussed below.

1. OGRA-4K FACILITIES

A cusp configuration was produced by two superconducting coils with counter-directed magnetic fields (Fig. 1). A hot electron plasma was produced in a hydrogen superhigh frequency discharge. Linearly polarized radiation from a gyrotron was launched along the magnetic field (z axis) through an axial mirror.

The field lines, the magnetic surfaces where the ECR condition \(\omega_g = n\omega_e\) for the fundamental frequency \(n = 1\) and second harmonic \(n = 2\) is satisfied and the gyrotron power distribution measured in the absence of plasma are shown in Fig. 2 (\(\omega_g\) is the gyrotron radiation frequency, \(\omega_e\) is the electron cyclotron frequency). The distance \(l\) across the magnetic field lines is measured along lines passing through...

FIG. 2. Magnetic field geometry and gyrotron power $P_z$ distribution. Shaded areas denote zones of hot electron production.
magnetic field minima points at field lines [3]. The main cusp and gyrotron parameters are:

- Magnetic field in axis mirror, $B_a$ = 2.46 T
- Magnetic field in slit mirror, $B_s$ = 1.57 T
- Field rise gradient along $\ell$, $G$ = 13 T/m
- Gyrotron emission wavelength, $\lambda$ = 8.2 mm
- Launched gyrotron power, $P_g$ = 40 kW
- Gyrotron pulse duration, $T_g$ = 30 ms

The mirror ratio respective to the field in the slit is well represented by the expression $R = 12/\ell$ (cm) and the magnetic field strength at the minima points $B_{\text{min}} = G \ell$ (T/m). The whole plasma flux exiting through a slit was collected with the circular plate placed within a weak magnetic field zone (Fig. 1). The plate was insulated from the ground and the plate potential $\Phi$ could be changed within the range $\pm 300$ V. A grid for suppression of the secondary electron emission was installed in front of the plate. The total current flowing out through the slit mirror was measured. A multigrid analyser, which made it possible to measure the ion and electron fluxes on the plate, had the same potential as the plate. The diagnostic apparatus employed was described in Ref. [2].

2. EXPERIMENTAL RESULTS

2.1. Measurements of plasma fluxes beyond the mirrors along the magnetic field lines

An analysis of plasma fluxes has been carried out with the multigrid analysers using retarding potential methods. Some typical curves showing the dependence of ion and electron currents on retarding potential $V_r$ are shown in Fig. 3. The ion retarding curve is interpreted as an ion beam with an ion temperature of 15 eV exiting along field lines, displaced by a positive plasma potential $\Phi$ ($\Phi = 80$ V in the given case). The measurements show that $T_i$ has a weak dependence on $\ell$, plasma heating and production conditions and its value is within the range 10–15 eV. Thus from the ion curves (Fig. 3(a)) the ion current density $j_i(\ell)$ and the potential $\Phi(\ell)$ are found.

The electron current retarding curves are more complicated (Fig. 3(b)). One can distinguish three groups of electrons: cold electrons, with a temperature $T_{e}^{c}$ of about a few tens of electronvolts; warm electrons, with a temperature $T_{e}^{w}$ of about 0.5 keV; and hot electrons. An analysis of electrons has been done up to $V_r = 1.5$ kV. The diamagnetic measurements show that the average energy of hot electrons, $E_{e}^{h}$, is about 10 keV. From experimental data it follows that $T_{e}^{w}$ and $E_{e}^{h}$ have a weak dependence on the transverse co-ordinate $\ell$.

The results of measuring the plasma fluxes through the left axial mirror at small $\ell$, where the manifestation of non-adiabaticity [3] and an increase in the losses are
possible, are given in Fig. 4. One can see that there are no peculiarities in $j_i(\ell)$. However, electron current density $j_e$ is a few times greater than $j_i$ at $\ell < 2$ cm.

The relationships $j_i(\ell)$, $T_e^*(\ell)$ and $\Phi(\ell)$ obtained from measurements through an axial mirror are shown in Fig. 5. Figure 6 shows similar relationships for fluxes through a slit. The measurements show that $j_i = j_e$ everywhere except for a small region near the field zero. In Figs 5 and 6 it can be seen that practically the whole

FIG. 3. Normalized ion current $I_i$ (a) and electron current $I_e$ (b) versus retarding potential $V_r$. 
current along the field lines is transferred at $\ell > 5$ cm. Figure 7 shows the dependence of the ratios $j^w_e/j_e$ and $j^h_e/j^w_e$ on $\ell$, where $j^w_e$ is the current density of warm electrons and $j^h_e$ is the current density of hot electrons. One should note the absence of $j^h_e$ at $\ell < 7$ cm.

2.2. Experiments on plasma potential control

As pointed out in Section 1, it was possible to vary the potential $\Phi_p$ of the circular plate (Fig. 1), receiving the whole plasma flux through the annular slit.
Plasma flux through the axial mirrors arrives at the ground of the facility. With $\Phi_p$ varied within the range $\pm 300$ V no essential change in the main plasma parameters was obtained.

Total current on the plate, $I_{pl}$; cold electron current to the slit mirror, $I_s$, and to the axial mirror, $I_a$; plasma potential respective to the ground, $\Phi$; and plasma potential respective to the plate, $\Phi_{pl}$, essentially depend on $\Phi_p$ (Fig. 8). The distri-
Fig. 8. Normalized $I_{pl}$, $I_{pl}$, $I_s$, $\Phi$ and $\Phi_{pl}$ versus circular plate potential $\Phi_p$.

Fig. 9. Profiles $j_e(\ell)$. Squares: $\Phi_p = 250$ V; triangles: $\Phi_p = -120$ V.

Distribution of electron current density $j_e(\ell)$ through the left axial mirror for two values of $\Phi_p$, normalized to the current density at $\Phi_p = 0$, is shown in Fig. 9. It can be seen that at $\ell > 5$ cm it is possible to cut off an electron current or to nearly double it by a change in $\Phi_p$. The effect of $\Phi_p$ on the central zone of the cusp is considerably smaller.
3. DISCUSSION

3.1. Plasma production zones and parameters

It is known that a hot electron plasma is produced when the ECR condition is satisfied. The resonance surface $n = 1$ intersects all the field lines in the zone of confinement (Fig. 2) and resonance conditions are realized at the magnetic field minimum for lines with $\ell = 9.5$ cm. From experimental data it follows that a hot electron plasma is produced at $\ell > 5$ cm and thus the strongest electron heating occurs not too far from the magnetic field minimum. With a reduction in the magnetic field strength the diamagnetic signal drops abruptly. These data confirm the insignificant role of ECR $n = 2$ under the given experimental conditions.

The measurements of $j_\| (\ell)$ together with diamagnetic and interferometric data have allowed us to determine the production zone for a hot electron plasma and its average density. Such a plasma is produced as two rings (Fig. 2) weakly bound to each other. The insertion of a metallic rod 10 mm in diameter between the rings along the separatrix does not change the plasma parameters in the rings.

Depending on the puffed-in gas flow, the average plasma density in the rings is in the range $(2-6) \times 10^{12}$ cm$^{-3}$ at the average electron energy 2-20 keV and the highest average energy is observed at a lower density. The volume of the hot electron plasma rings is about $10^3$ cm$^3$. A value of $\beta$ up to 3% was obtained in a stable regime. Restricted plasma pressure instability is discussed in Ref. [4].

3.2. Non-adiabatic effects

From theory [3] it follows that adiabaticity of the motion can be violated at $\ell_e < 2$ cm for 1 keV electrons. From the measurements (Figs 3, 4) it follows that $j_\perp$, $\Phi$, and $T_e$ have no rough peculiarities at low $\ell$. The total particle and energy flux from the non-adiabaticity zone is a small fraction of the flux from the whole cusp. The considerable excess of $j_\parallel$ over $j_\perp$ observed at $\ell < 2$ cm (Fig. 4) is probably related to the microwave power launch along the $z$ axis from one side of the cusp, and the driven electron currents freely pass through the non-adiabaticity zone. As seen in Fig. 2, the microwave power maximum is in the non-adiabaticity zone. Hot electrons are not observed in this zone (Fig. 7).

The external pressure slope (toward the wall) is MHD stable. For internal pressure slope stability it is necessary to have a not very steep pressure reduction to the cusp centre [5]. Steep transverse profiles of hot electron fluxes (Fig. 7) are experimentally observed with no indication of MHD instabilities. The reasons for such contradictions require additional theoretical and experimental studies.
3.3. Plasma potential control

In Figs 8, 9 it can be seen that a change in $\Phi_p$ mainly affects the plasma potential and results in the redistribution of electron fluxes streaming out through axial and slit mirrors.

Hot electrons are magnetically confined in the trap. Cold ions can be confined in a trap for a long time only in a potential well at least a few ion temperatures deep. It is clear that a change in $\Phi_p$ within the range $\pm 300$ V cannot essentially affect hot electrons and cold ions. It has been experimentally observed that the plasma parameters and ion fluxes through the mirrors do not depend on $\Phi_p$. The electron fluxes are essentially redistributed. If the plate potential is positive (Fig. 9), the potential barrier for cold electrons in the slit will be lower, and they will escape from the trap mainly through the slit. The energy of ions escaping along the z axis increases proportionally to the potential growth. If $\Phi_p < 0$, the potential barrier, on the contrary, is lower from the axial mirror side, and cold electrons escape from the trap mainly along the z axis. In this case a current of about 1 A passes through the plasma. This current changes its sign with a change in $\Phi_p$.

Such a classical behaviour of fluxes confirms the absence of transverse cold electron transport.

3.4. Particle and power balance and plasma losses

The measured plasma fluxes from a cusp and their energy spectra, diamagnetic signals, puffed gas flux and launched microwave power allow an approximate balance between the entering and exiting fluxes of particles and power.

The total ion flux $I_i$ leaving a cusp along magnetic field lines is approximately equal to the puffed-in gas flux $Q$: $I_i \approx Q \approx (2 \pm 0.5)$ A. Measurements of the electron current along field lines give the following distribution of components: hot electrons 0.2 A, warm electrons 0.8 A, cold electrons 1 A. The outstreaming power loss is mainly through the flux of hot electrons. The total power of plasma fluxes is 3–4 kW. Independent diamagnetic measurements and measurements of plasma decay time give a similar value.

As pointed out above, the gyrotron power is 40 kW. However, two factors should be taken into account to estimate the absorbed power. First, only 0.3$P_g$ enters the zone of plasma production (two rings), owing to the microwave radiation power distribution (Fig. 2). Second, the coefficient of absorption in the ring measured by a bolometer was about 0.3. Taking into account these coefficients, the absorbed microwave power, within the accuracy of the measurements, is equal to the power of particle fluxes streaming along field lines from the cusp.

The measured diamagnetic signal decay times, when the gyrotron is switched off in a stable regime, are close to the Coulomb time.
REFERENCES

INTERPRETATION OF EXPERIMENTS WITH Z PINCHES

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Abstract

INTERPRETATION OF EXPERIMENTS WITH Z PINCHES.

The approach taken for the neck ignition experiments is to compress the neck as strongly as possible. From this point of view, experiments with a radiation collapse are interpreted as having a 'Joule' collapse. The dependence of stability on current rise is considered. An estimate for the resistance of Z pinches is presented.

In this paper experiments are considered from the point of view of obtaining maximum energy density in Z pinches. This is necessary for both the well known projects on the realization of nuclear fusion in linear Z pinches with a condensed D-T mixture [1-3] and for a project based on high neck compression in such a mixture and detonation [4]. To obtain detonation (Lawson criterion for α particles) it is necessary to achieve a neck diameter of $10^{-4}$ cm with a current of 10 MA, starting from a diameter of 1 cm and including an energy of about $10^5$ J with a characteristic time of $10^{-7}$ s.

1. MODELS FOR Z PINCH NECK COLLAPSE

The most natural model for hydrogen pinches with high linear density is the Kolb-Vikhrev model [5, 6]. It is based on two assumptions:

(i) The current in the pinch is conserved and the Bennet evaluation is valid (the magnetic pressure is of the order of the thermal pressure):

$$I^2 = 3.3 \times 10^{-22} \pi r^2 (n_e + n_i)T$$

where $I$ is the current in MA, $r$ is the radius in cm, $n$ is the density in cm$^{-3}$ and $T$ is the temperature in eV.

(ii) The temperature grows adiabatically:

$$T \sim n^{2/9}$$
The characteristic time of the process development is determined by the acoustic velocity:

$$\tau \approx 10^{-6} r T^{-1/2}$$

This model is most favourable for ignition in the neck because of the possibility of achieving a radius of $10^{-4}$ cm with a suitable linear density.

From the point of view of achieving high densities the most impressive results are those on micropinch compression up to a radius of the order of $10^{-4}$ cm [7]. Unfortunately these results were obtained not in hydrogen but in heavy element plasmas. In Ref. [8] they were interpreted as a radiation collapse.

We have another viewpoint on these experiments. The neck collapse model described in previous work [5, 6] has a limited scope of application for heavy element plasma. The reason for this is the small value of the ionization adiabatic exponent, as a result of which the heavy element plasma temperature increases too slowly with growing density and the growth of temperature does not compensate the increase in the resistance due to the radius decrease. Once the Joule heating is equal to the gas dynamic power, a new collapse regime will come into play which can be called the Joule regime. In this model

$$\frac{R}{c_s} = \frac{4\pi \sigma R^3}{c^2}$$

(2)

For elements with $Z > 50$ the following simple approximations are possible:

$$c_s = 10^5 T^{3/4}, \quad \sigma = 10^{13} T$$

The evaluation (2) yields

$$T = \left( \frac{100}{R} \right)^{4/7}$$

(3)

which at $R = 10^{-4}$ cm leads to $T = 3$ keV, in accordance with experimental results [7] ($r = 10^{-4}$ cm, $T \approx 3-5$ keV).

If (3) is supplemented with the Bennett condition, one can obtain the Joule polytope exponent

$$T \sim n^{0.4}$$

(4)

In vacuum spark experiments this power is close to 0.5, which is in agreement with the Joule regime and points directly to the increase in specific entropy, in contrast to the purely radiation collapse regime [6].
This model does not work for hydrogen. The significance of experiments with micropinches lies in the demonstration of the possibility of collapse up to \( r = 10^{-4} \) cm, which is prevented neither by the pressure of the surrounding plasma [9] nor by the violation of symmetry.

2. MECHANISMS STOPPING NECK DEVELOPMENT

The most serious limitation on compression was proposed by Sasorov in Ref. [9], which related to a low pressure rare plasma surrounding the pinch. The problem is that only the plasma inside the skin layer can flow out from the neck, while the external plasma is frozen into the magnetic field and thus should be compressed with the rise of density \( n \sim r^{-2} \), which with allowance for heating through the compression results in \( n \sim r^{-10^{19}} \), i.e. the thermal pressure increases more rapidly than the magnetic pressure. This leads to a current redistribution and the formation of a stable (according to Kadomtsev) configuration. This means that if we want to compress the plasma by \( 10^4 \) times over the radius the thermal pressure of the surrounding plasma should be low:

\[
\frac{8\pi nT}{H^2} < \left( \frac{r}{R} \right)^{49} \approx 10^{-5}
\]

For a kinetic limit this criterion was generalized, in unpublished work, by Filippov and Yan’kov.

Another mechanism preventing the neck instability is the magnetic field convection effect in the framework of the electron magnetohydrodynamics model, which is applicable once the linear number of particles \( N = \pi nr^2 \) decreases so that the parameter \( \Pi_i \) ceases to exceed unity:

\[
\Pi_i = \frac{ZNe^2}{AMc^2} < 1
\]

and the applicability of the usual one fluid hydrodynamics is violated. The current velocity and thermal drift velocities of particles begin to exceed the Alfvén velocity and anomalous resistance occurs. A normal evolution of the neck is impossible in this case, which can easily be seen under the assumption of an ideal freezing in.

At \( \Pi_i < 1 \) the current velocity exceeds the Alfvén velocity and to a first approximation one can consider a stationary flow of electrons through motionless ions distributed with a prescribed concentration \( n(r, z) \). The magnetic field has the sole component \( H_r = H(r, z) \).
An exact solution of the electron MHD equations gives a formula for the current $I$

$$I = I(nr^2)$$

which flows through the circle of radius $r$. The neck evolution is associated with the decrease in the linear density $nr^2$, but from (5) it is seen that the current cannot flow into the region with reduced linear density, and therefore the magnetic field is carried away from the neck by the electron current and the neck should no longer be developed.

It is possible that just this mechanism led to the stabilization found in numerical simulations [10]. It would be of great interest to study this effect with the code ANTHEM.

While interpreting all the experiments with Z pinches, one should bear in mind that the neck never tears up to a violation of quasi-neutrality and its resistance (even at an ideal conductivity) is estimated by the formula (6), where $a$ is the radius and $R$ is in ohms

$$R = 30 \frac{u}{c}, \quad u = \frac{1}{2\pi nea^2}$$

In this state the neck exists for 10 to 100 MHD times, whereupon the number of particles increases and the resistance (6) is decreased.

The experiment on explosion of frozen deuterium fibre [11] has attracted much attention. In this work an anomalously long lifetime of the pinch before the development of the neck was observed. The beginning of instability was connected with a change of current derivative sign. In our opinion this dependence is inverse. A specific feature of the dependence $I(t)$ is that the neck's development caused a sharp increase in resistance and a subsequent decrease in current.

Nevertheless, a problem of the anomalous stability of fibre pinches exists. The arguments of Ref. [1] explaining high stability by finite resistance [12, 13] or other deviations from ideal MHD are realistic. Note that taking account of the finite Larmor radius only decreases instability growth rates [14, 15].

The most difficult experiments to interpret are those with strongly radiative pinches at a current of several megamperes, for example at the Saturn facility [16]. It is difficult to combine observation of untrapped Kr K lines with a current of several megamperes and a small pinch radius (or to explain why only high current pinches are stabilized at large radii). It should be recalled that the model of radiative collapse [8] considers lines to be trapped already at a current of 0.5 MA.
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PRODUCTION AND IDENTIFICATION OF THE ION TEMPERATURE GRADIENT INSTABILITY

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Abstract

PRODUCTION AND IDENTIFICATION OF THE ION TEMPERATURE GRADIENT INSTABILITY.

The production and identification of the ion temperature gradient ($\eta_i$) instability has been performed in the Columbia Linear Machine (CLM). CLM is a steady state linear machine with a quiescent, cylindrical, collisionless plasma confined by a solenoidal magnetic field. The creation of plasma parameters appropriate for the instability is described, followed by its observation and identification.

To produce and study the ion temperature gradient instability, the Columbia Linear Machine has been modified to yield a peaked ion temperature and flattish density profiles. Under these conditions the parameter $\eta_i (= d \ln T_i / d \ln N)$ exceeded the critical value and a strong instability has been observed. Further identification has been based on observation of the azimuthal and axial wavelengths, and the real frequency, appropriate to the mode.

Anomalous ion thermal conductivity remains an open physics issue for the present generation of high temperature tokamaks and future reactor type tokamaks. It is now widely believed to be due to the ion temperature gradient instability ($\eta_i$ mode) [1-4]. However, it has been difficult, if not impossible, to identify this instability directly in tokamaks. Therefore, the production and identification of this mode is, at present, being pursued in the simpler and experimentally convenient configuration of the Columbia Linear Machine (CLM) [5]. In the CLM a hydrogen plasma, with coincident radial temperature and density gradients, is produced by an E||B discharge source. The machine is approximately 3 m in overall length and has a uniform axial magnetic field. The typical parameters of the CLM before modifications are: $N \sim 6 \times 10^8$ to $2 \times 10^9$ m$^{-3}$, $T_e \sim 5$ to 9 eV, $T_i \sim 5$ eV, $P_n \sim 7 \times 10^{-7}$ torr, $r_p \sim 2.7$ cm, $1/(r_p L_n) \sim 0.27$ cm$^{-2}$, $L_e \sim 50$ to 150 cm, $B_0 = 1$ kG, $\nu_e/\omega^* \sim 0.0015$ to 0.2, $\nu_e/\omega_e \sim 0.15$ to 0.01; $\nu_e/\omega_n$, $\nu_e/\omega_e \sim \nu_i/\omega_i$, $\nu_i/\omega_i \sim 3 \times 10^{-5}$ to $10^{-3}$; $\eta_e \sim 1.0$ to 2; $\eta_i \sim 0$, $\rho_i/r_p \sim 0.2$ to 0.1. It is clear that for our present purpose the only inadequate parameter is $\eta_i$. This parameter has to be increased beyond the critical value $\eta_{ic}$ for the instability, which is generally estimated to be of the order of one. The methods and modifications used to accomplish this are described below.
The parameter $\eta_\parallel$ can be increased either by flattening the density gradient and/or by increasing the ion temperature gradient (from zero). It must be emphasized that, even for zero density gradient, one must have an appropriate ion temperature gradient to excite this instability, which is basically an ion pressure gradient mode. A ‘feathered’ screen installed at the entrance of the experimental region can significantly lower the density gradient. A theory of ion temperature gradient instability with anisotropic $\eta_\parallel$ has been developed [6], which indicates that $\eta_\parallel$ alone cannot drive the instability; however, $\eta_\perp$ alone can do so. This has motivated the development of a scheme for parallel heating of ions.

The basic idea of the scheme is to accelerate the ions from the plasma source before they enter the experimental cell as shown in Fig. 1. The acceleration parallel to the magnetic field is achieved via a 70% transparent tungsten mesh at both ends of the plasma column, both biased to $-30$ to $-50$ V. Lastly, the terminating end plate has a bias of $+4$ to $+8$ V to contain the heated ions, while the typical plasma potential is $-6$ to $-8$ V. Thermalization in the high neutral pressure transition region, subsequent to acceleration, produces parallel ion heating roughly in the region covered by the accelerating meshes (of radii 1 and 0.6 cm). This leads to a peaked $T_{i\parallel}$ profile. Furthermore, the mesh in the transition region reduces the density in the central core and helps to reduce the density gradient. Therefore, the biased mesh can act both as a density flattening and parallel temperature peaking device and can produce high values of $T_{i\parallel}$.

Profiles of plasma parameters in the absence of parallel ion heating (with no accelerating bias on the meshes) are shown in Fig. 2(a). It is seen that $\nabla T_{i\parallel} \sim 0$, $\nabla T_{i\perp} \sim 0$ over the entire plasma core. As the parallel ion heating is turned on (with accelerating bias on the meshes), profiles change as shown in Fig. 2(b). It is clearly
FIG. 2. Radial profiles of plasma parameters. $N$, $T_{\parallel}$, $T_{\perp}$, $T_e$ and $\Phi$ are equilibrium plasma density, parallel ion temperature, transverse ion temperature, electron temperature and potential, respectively. (a) Profiles without parallel heating of the ions (i.e. 0 bias on the accelerating meshes); (b) profiles with parallel heating of the ions (i.e. -50 V bias on both meshes).

seen that the $T_{\perp}$ profile remains essentially flat, but the $T_{\parallel}$ profile has developed strong gradients in the vicinity of the accelerator mesh location. This is a clear consequence of the fact that the biased meshes can accelerate ions only in the parallel direction and yield an enhanced parallel temperature, while leaving the transverse ion temperature unaltered. We can now examine the fluctuation spectra in the plasma with and without ion parallel heating. In the absence of parallel heating, we have the profiles of Fig. 2(a), where $\nabla T_{\perp} \sim 0$, $\nabla T_{\parallel} \sim 0$. The corresponding density
fluctuation spectrum (from the fluctuating ion saturation current drawn by a Langmuir probe) is shown in Fig. 3(a). Under these circumstances, we really do not expect any $\eta_1$ mode. However, there is one spectral feature at 27 kHz, which is an $E_0 \times B_0$ rotationally driven mode, always present in our machine with a radial equilibrium electric field produced by the potential ($\phi$) profile in Fig. 2(a). As the parallel heating is turned on (via the biases on the meshes), the profiles change as shown in Fig. 2(b); now $\nabla T_{\perp} \sim 0$, and $\nabla T_{\parallel}$ is quite large. The corresponding fluctuation spectrum is shown in Fig. 3(b). The usual $E_0 \times B_0$, rotationally driven mode is still there at a slightly higher frequency of 35 kHz, but a new mode is clearly visible at 62 kHz. Under a variety of plasma conditions, this new mode correlates with the existence of a strong $\nabla T_{\parallel}$ and is, therefore, considered as a strong candidate for the $\eta_1$ mode.

The azimuthal mode number and axial wavelength of both modes are determined via cross-correlation of two appropriately placed Langmuir probes. The results show that the first mode at 27–35 kHz is $m = 1$ and $\lambda_1 \gg L$, which conforms with the notion that it is a $E_0 \times B_0$ rotationally driven flute. The second mode at 62 kHz is $m = 2$, with $\lambda_1 \sim 330$ cm $\sim 2L$; it is necessarily a drift-like mode. A detailed shooting code analysis using the actual profiles shows that the $m = 1$ rotationally driven flute mode is unstable both in the absence and presence of parallel heating. However, the $m = 2$ rotationally driven flute is stable in both cases. Therefore, the $m = 2$ second mode at 62 kHz is not a rotationally driven mode and very likely a $\eta_1$ mode.

We now discuss the $\eta_1$ stability character of this mode by examining the marginal stability criteria. For our experimental parameters with $\nabla N \approx 0$, $\nabla T_{\perp} \sim 0$, we calculate the critical temperature gradient scale length and compare with the experimental value as:
Theoretical: \((L_{T_{\parallel}})_{\text{crit}} = (\delta_{T_{\parallel}})_{\text{crit}}^{-1} = 2.2\) (m = 1), 4.0 (m = 2) cm.

Experimental: \((L_{T_{\parallel}})_{\exp} \sim 2.4\) cm.

The experimental measurement of \(L_{T_{\parallel}}\) has been averaged over the mode width to yield \((L_{T_{\parallel}})_{\exp}\). From the above numbers for theoretical and experimental ion temperature gradient scale lengths, it is clear that m = 1 mode is stable, but the m = 2 mode will be strongly unstable. In fact, the solution of the local dispersion relation for our experimental parameters yields \(\gamma/k_{\parallel}v_{\text{th}} \sim 0.4\) and \(\omega_{*}/k_{\parallel}v_{\text{th}} \sim -0.8\) for m = 2. This is in agreement with the experimental evidence of the emergence of a m = 2 mode as seen in Fig. 3(b). Furthermore, the observed real frequency of the mode in the laboratory frame is dominated by the Doppler shift due to \(E_{0} \times B_{0}\) rotation with a small downshift in frequency, roughly consistent with the above estimate of the real frequency of the mode in the ion diamagnetic direction.

In conclusion, we have succeeded in producing and identifying the \(\eta_{i}\) instability. Its identification has been based on its appearance in conjunction with high \(\nabla T_{\parallel}\) and low \(\nabla N\), and the observed mode with wavenumbers \(m = 2\), \(k_{\parallel} = \pi/L\) is predicted to be strongly \(\eta_{i}\) unstable. Furthermore, as predicted, the real frequency of the mode is observed to be in the vicinity of the \(E_{0} \times B_{0}\) Doppler shifted frequency, with a slight downshift due to the plasma frame frequency of the mode in the ion diamagnetic direction.

REFERENCES

THE FUSION RELATED DENSE Z-PINCH PROGRAMME AT IMPERIAL COLLEGE


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Abstract

THE FUSION RELATED DENSE Z-PINCH PROGRAMME AT IMPERIAL COLLEGE.

A universal stability diagram for the Z-pinch has been found in which ln (I/a) is the ordinate and ln N is the abscissa, where I is the current, a is the radius and N is the line density. Straight lines represent the critical Lundquist number for stability; the onset of viscous damping and pressure anisotropy; values of the ratio a/a (the measure of finite ion Larmor radius effects); and \( \Omega \tau_1 = 1 \). The experimental results are interpreted in terms of their loci in this diagram. The compression pinch shows a weak \( m = 0 \) structure for \( a/a = 0.18 \) and \( \Omega \tau_1 = 7 \). A carbon fibre Z-pinch goes violently unstable after 20 ns when \( \Omega \tau_1 \) is less than one. The theory for both \( m = 0 \) and \( m = 1 \) resistive stability of an ohmically heated equilibrium shows a critical Lundquist number of about \( 10^2 \) below which the pinch is stable. Pressure equilibria can evolve through self-similar current profiles that can be centrally peaked and in the outer region possess a profile increasing with radius. Theoretical stability results in the high \( I^4a \) regime have been developed for low N (Vlasov model with skin currents) and high N (CGL anisotropic regime). The DZP project is now being designed and the generator assembled to yield up to 2 MA at 2.4 MV for 200 ns. Not only fusion conditions will be studied, but also radiative collapse at a current I above the Pease-Braginskii limiting current. Collapse to over \( 10^4 \) times the solid density is predicted in a 1-D simulation that includes degeneracy and radiation transport effects.

1. INTRODUCTION

There is a renaissance in Z-pinch research following both the introduction of pulsed-power generators to drive the discharge with very high rates of rise of current (\( 10^{12} \) to \( 10^{13} \) As\(^{-1}\)), and a greater understanding of the theoretical conditions required for both fusion and stability. Typically a current of \( 10^6 \)A with a voltage driver greater than 1MV for a pulse exceeding 100 ns is required in a narrow filamentary pinch of radius of 10 microns and line density \( 5 \times 10^{18} \) m\(^{-1}\). In Section 2 we will develop the theoretical principles that lead to the universal diagram that identifies stability regimes. With this it is possible to identify what theoretical model applies to each experiment. Ideal MHD theory which predicts the worst unstable case is shown to be confined to a

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narrow region of parameter space. In Section 3 results from a novel side-on laser holographic diagnostic of a compression Z-pinch will be presented. In Section 4 results from fine carbon fibres carrying currents up to 100 kA indicate the later growth of an m=0 instability. Dense Z-pinches tend to evolve under ohmic heating with conditions close to pressure balance. Several self-similar current profiles are predicted and compared to 1-D simulations in Section 5. Resistive stability, a Vlasov skin current model, and a fluid model with anisotropic pressure are presented in Sections 6, 7 and 8. A one-dimensional simulation of the pinch under conditions in which the current exceeds the Pease-Braginskii limit is shown to follow quite closely the earlier semi-analytic model except that the final state of collapse is limited by degeneracy and opacity effects. This is presented in Section 9. The last section, Section 10, shows some design features of the large dense Z-pinch experiment being built. The generator, MAGPIE, will develop 2MA at 2.4MV with a pulse length of 200 ns.

2. REGIMES FOR STABILITY MODELS

Ideal MHD theory represents the plasma as a single fluid, neglecting the Hall effect, electron inertia, electron pressure, resistivity, viscosity, thermal conduction, and finite ion Larmor radius effects, and usually assumes equal electron and ion temperatures. Recently Coppins [2] extended the earlier analytic work of Kruskal and Schwarzschild [3] and Taylor [4] in ideal MHD stability theory. When resistivity (\(\sigma^{-1}\)) is introduced to Ohm's law we find that, in contrast to tokamaks and reversed field pinches, its effect is not localised because there are no singular surfaces. For this reason it is not a conventional tearing mode, but a mode that requires also the inclusion of Joule heating in the energy equation. Therefore we perturb an evolving equilibrium in which the Joule heating continually raises the temperature. The current in turn is continually raised to maintain pressure balance at constant radius, and it follows the Haines-Hammel curve [5], i.e. \(I \propto (\text{time})^{1/3}\). Culverwell and Coppins [18] have shown that then there is a critical Lundquist number \(S\),

\[
S = \mu_0 \sigma v_A a
\]

(1)
of about 100, below which the plasma is stable to \(m=0\) instabilities. Using the Bennett relation

\[
16\pi NkT = \mu_0 I^2
\]

(2)

\(S\) can be written as

\[
S = 3.86 \times 10^{23} \frac{I^4 a}{N^2}
\]

(3)
where $N$ is the line density $\int 2\pi r n(r) r \, dr$. Figure 1 shows in a plot of $\ln (I^4a)$ versus $\ln N$ the line corresponding to $S=10^2$.

The effect of viscosity is described by the Reynolds' number $R$ defined by

$$R = \rho v_A a / \mu_{||}$$

(4)

where a parallel viscosity $\mu_{||}$ is equal to $n_i k T_i / \tau_i$, $\tau_i$ being the ion–ion collision time. $R$ is the inverse of $\gamma \tau_i$ where $\gamma$ is $v_A / a$ i.e. the inverse of the radial Alfvén wave transit time which characterises the growth rate for ideal MHD modes. Therefore $\gamma \tau_i < 1$ is also the condition for the perturbed pressure to be isotropic. Using Eq.(1) this can also be written in terms of $I^4a$ and $N$,

$$\frac{1}{R} = \gamma \tau_i = 2.07 \times 10^{39} \frac{I^4a}{N^2}$$

(5)

for $Z=1$. The line $R=1$ is shown in Fig.1.
Recently Cox [6] and Spies [7] studied the combined effect of resistivity and parallel viscosity but in absence of Joule heating of the equilibrium of perturbed state. Cox also studied anisotropic viscosity; for \( \Omega_i \tau_i > 1 \) the effect is greatly reduced.

When the perturbed pressure is anisotropic, the model of Chew, Goldberger and Low [8] must be employed. Coppins [9] and Coppins and Scheffel [10] have treated the case of \( m=0 \) in the limit of \( \gamma \tau_i >> 1 \), showing a marked reduction in growth rates. Fig. 1 also shows when electron anisotropic pressure applies.

When in pressure balance the ratio of ion Larmor radius \( a_i \) to plasma radius \( a \) depends only on line density; for deuterium

\[
\frac{a_i}{a} = \frac{8.08 \times 10^8}{N_i^2} \tag{6}
\]

Vertical lines for \( a_i/a = 0.1 \) and 1 are shown in Fig.1, the former being where finite ion Larmor radius effects and the latter where large Larmor radius effects apply. This can only apply if the ions are magnetised, i.e. \( \Omega_i \tau_i > 1 \) where \( \Omega_i \) is the mean ion cyclotron frequency. This condition is also shown in Fig.1, because of

\[
\Omega_i \tau_i = 2.20 \times 10^{32} \frac{I_{14} a}{N_i^{5/2}} \tag{7}
\]

Interestingly \( \Omega_i \tau_i = 1 \) passes through the middle of the wedge-shaped region where ideal MHD theory applies.

3. THE COMPRESSIONAL PINCH

Here we report side-on holographic interferograms of the compressional pinch. We have previously [11] reported end-on observations but these are not ideal because interferometric measurements integrate over the optical path. Thus with end-on observations it is difficult to detect both \( m=0 \) and higher \( m \) instabilities if the the wavelength is longer than the optical path. Furthermore, end-on measurements of density include a contribution from the plasma which has flowed out of the pinch through the hollow electrodes and thus can give an erroneous confinement time.

The pinch apparatus used in the observations reported here was similar to that described in [11] but a longer (75 mm x 20 mm dia.) pinch tube was used. The filling gas was hydrogen at 1 and 2 torr and it was preionised by a 5 kA current discharge which began 200 ns before the main current. As previously reported [12] there was a well developed shock implosion
which formed a plasma on axis of diameter about 3 mm. There was no subsequent radial bounce of the plasma column, only a slow expansion. This is shown in the optical streak photograph in Fig. 2. The corresponding current waveform is also shown in the figure. The photograph shows that a wall plasma was generated soon after pinch formation presumably due to UV irradiation. A current crowbar through this wall plasma is presumed to be responsible for the observed behaviour of the pinch after pinch formation [13,14].

Side-on interferometric measurements are difficult due to refraction in the wall of the quartz tube and poor optical quality of the tube. These difficulties were overcome by using an image-plane holographic, double-exposure interferometric system in which one-half of the outside of the quartz tube was sandblasted to make a diffuse screen. The rest of the holographic arrangement was conventional. A 300 mJ, 12 ns fwhm ruby laser pulse was split into two beams which were expanded to 40 mm diameter. The light from the diffuse screen was collected by a 200 mm compound lens after passing through the plasma and imaged on the holographic plate.

Some typical side-on interferograms are shown in Figs. 3 and 4. Fig. 3: the fringe patterns with their corresponding radial electron density profiles obtained by Abel inversion. Figure 3a is at 100 ns after current rise, which is 10 ns after pinch formation, and Fig. 3b at 130 ns. These interferograms show an axially uniform, radially confined pinch with a radius of approximately 3 mm and wings out to 6 mm. The line density inferred from the density profiles is $4 \times 10^{19}$ m$^{-1}$. This is consistent with all the gas being swept-up and ionised by the imploding sheath. Using this value of the line density and the observed current at these times a temperature of $85 \pm 15$ eV is calculated from the Bennett relation assuming
equi-partition between the ions and electrons. The wall plasma has a strong effect on the fringe pattern near the wall as can be seen in Fig. 4b. An electron density of about $10^{24} \text{ m}^{-3}$ peaked on the inside surface and width of about 1.5 mm can be inferred.

At late times the interferograms show some axial non-uniformity as shown in Fig. 4. It is not possible to Abel invert the data at these times due to the axial density variation. However, the line density can be estimated from the average fringe shift and the radius. These estimates show that the line density decreases in time, halving in about 100 ns. This

FIG. 3. Interferograms and radial density profiles in the compressional pinch (2 torr of hydrogen).
FIG. 4. Observed and computed interferograms showing $m = 0$ instability at late times in the compressional pinch (1 torr hydrogen).

is in contrast to the end-on observations previously reported which showed a decrease of the line density over a much longer time. This is presumably due to the contribution to the line-integral in the end-on observations of the plasma which had escaped by flowing through the hollow ring electrodes. The observed time for this loss obtained from the side-on observations is consistent with the calculated Bennett temperature.

The axial variation evident at late times (Fig. 4a) is consistent with a periodic axial perturbation which may be an $m=0$ instability. Figure 4b shows a computed fringe pattern where the $m=0$ instability is modelled as a perturbation of the radius, $\Delta a \sin (kz)$, of the plasma column radius.
The line density $N$ is taken to be constant at all axial points. In the computation $ka = 16$, $N = 2 \times 10^{19} \text{ m}^{-1}$, and $\Delta a/a = 0.3$, $\Delta a$ being the perturbation in $a$.

The instability was first detected at 140 ns when it has a perturbation $\Delta a/a$ of 5%. From the measured radius and assuming that the plasma is at the Bennett temperature we calculate that the ideal MHD growth time for the instability is 20 ns. The pinch was observed from its formation at 90 ns until 280 ns. This duration of 190 ns is approximately 9 MHD growth times. If the instability had grown from 5% at 140 ns it would have disrupted in only 4 MHD growth times.

For a pinch in Bennett equilibrium the ratio of ion Larmor radius to the pinch radius, $a_i/a$, depends only on the line density (see Section 2, Eq. 6). For our hydrogen pinch with our measured line density $a_i/a = 0.18$. Further, the ions are magnetised ($\Omega_{iT_i} = 7$). The observed enhanced stability in this hydrogen pinch could be due to ion Larmor radius effects.

4. THE FIBRE PINCH

Experiments have been performed on 7$\mu$m diameter carbon fibres. The low line density ($N_P = 4.2 \times 10^{18} \text{ m}^{-1}$) means that for a fully ionised pinch ($Z = 6$) in Bennett equilibrium the ratio of the ion Larmor radius to the pinch radius is about 0.3; but for this effect to apply it is necessary for $\Omega_{iT_i}$ to be greater than unity. The enhanced stability observed in [16] could be ascribed to such an effect if there was incomplete ionisation of the fibre.

The $Z$-pinch was driven by a 3$\Omega$ transmission line with a maximum current of 100kA and a rise time of 45ns. The fibre was suspended vertically between the electrodes and was 20mm long. A variety of diagnostics was used to study the plasma. These included holographic interferometry, visible streak and framing photography, dual X-ray time integrated pinhole photography using various filters in the region below 2keV and PIN diodes to monitor the time resolved total X-ray output in the same energy region. Harder X-rays in the range 5–50keV emitted from the anode were observed using appropriately filtered PIN diodes. Pocket dosimeters and a scintillator–photomultiplier combination were used for hard X-rays above 50keV. The current through the fibre and the voltage on the transfer section was also monitored.

Figure 5 shows four optical framing photographs of 1ns exposure taken at 5, 20, 40 and 50ns in different shots. The discharge at 5ns in the first exposure (a) appears quite uniform and with a radius of about 100$\mu$m. In the second frame (b) at 20ns instabilities are seen as a series of bright spots distributed along the axis. These instabilities are characteristic of the $m=0$ instability. From the photographs a typical value for $ka = 2.5$ for the instability can be deduced. In the next frame (c) at 40ns the spots are at their greatest size. The last frame (d) at 50ns shows only the faint
emission from a single spot along with light from the cathode. These spots were thus formed at early times during the current rise and have a lifetime of 20—40ns, some being extinguished before the current has risen to its peak value. Optical streaks of the axis taken simultaneously show that these instabilities were formed within the first 10ns and that there was little evidence of axial movement of these spots. This is contrary to previously reported work on glass fibres [11]. From the photographs an average
expansion velocity of about $2 \times 10^6 \text{cm/s}$ can be deduced, which increases to $10^7 \text{cm/s}$ when the instabilities set in. Holograms taken after this time, when no plasma light was evident, showed a diffuse plasma column without any localised density maxima. At this time there was no detectable X-ray emission from either the plasma or the anode. Figure 6 shows (a) a time integrated X-ray pinhole photograph, (b) a holographic interferogram and (c) an optical frame. These observations were made simultaneously, roughly the middle 7mm of the pinch being sampled. They show quite clearly that there is an exact spatial correspondence of the regions of most intense time-integrated X-ray emission, the radially expanded regions of plasma density and the optical bright spots. The hologram shows that the plasma line density in some of the interconnecting regions was below the resolving limit of the diagnostic. At this time the instabilities were well developed. This would indicate that material was ejected axially from the neck of the instability as it developed. It should be noted that, since the bright spots associated with the instability occurred at random sites along the axis and existed for arbitrary times, optical radial streaks that sample one point of the axis are unreliable for determining the equilibrium and stability of the plasma.

The ratio technique on the differentially filtered time integrated dual pinhole X-ray camera images has been used to estimate the temperature. Sampling the bremsstrahlung emission between the copper L-edge and aluminium K-edge yields a temperature of about $125 \pm 30 \text{eV}$ for the spots and about $175 \pm 50 \text{eV}$ for the regions in between. X-ray photographs with a resolution of $15 \mu\text{m}$ did not show limb brightening as was reported in the glass fibre work [11]. This indicates that in our experiments the whole of the carbon fibre was fully ionised. The time resolved soft X-ray signals from the PIN diodes generally showed a few peaks, the origin of which is not fully understood.

Electron beams generated by the plasma were observed using PIN diodes and X-ray film. These occurred while the current was rising and their interaction with the anode gave rise to copious amounts of radiation in the $5-30 \text{ keV}$ regime. Using appropriate filters it was found that the majority of these X-rays were emitted by the K-shell transitions of the constituent elements of the steel anode. Preliminary estimates from the data indicate a beam of $10-15 \text{keV}$ and about $10-20 \text{kA}$. Dosimeters placed around the chamber showed that there was also present an X-ray flux sufficiently energetic to penetrate the chamber wall (2 cm of dural) and 15mm of steel. This would indicate an energy greater than $100 \text{keV}$. These X-rays were emitted roughly isotropically and obeyed the inverse square law. Simultaneous measurements taken with a scintillator/photomultiplier arrangement showed that these X-rays occurred in a 35ns burst that began with the onset of voltage across the electrodes. From this it is clear that these hard X-rays were generated by electron acceleration from the cathode onto the anode.

In this experiment assuming the carbon fibre was fully ionised, $a/a = 0.3$, $\Omega r_f = 2 \times 10^{-4}$ and the Lundquist number $S = 8$. It should also be noted that the instability was observed to grow during the time of current rise.
5. EQUILIBRIUM EVOLUTION

Previously reported self-similar equilibria [11,17] assume that ion cross field thermal conductivity dominates the thermal transport in the pinch, i.e. they assume high $\Omega_i \tau_i$. In the Z-pinch the average value of $\Omega_i \tau_i$ is proportional to $I^4$ for fixed line density and pinch radius (Eq. 7). Since a current rising as $I = \phi^{1/3}$ is used to maintain pressure balance [5], the assumption of high $\Omega_i \tau_i$ is satisfied at late times.

At low currents electron parallel thermal conductivity dominates, and then the solutions are not valid. However, as $\Omega_i \tau_i \to 0$ the ratio of (electron parallel) thermal conduction to ohmic heating tends to zero. Thus in the low $\Omega_i \tau_i$ regime we are justified in neglecting thermal conduction in

![Graphs showing number density, temperature, and current density as functions of radius.

FIG. 7. Low current self-similar equilibrium (number density, temperature and current density) for $N = 10^{20}$ m$^{-1}$, $\phi = 10^8$ A·s$^{-1}$, $I = 100$ kA, at the maximum attainable radius.]}
the energy equation. Making this assumption we find a new class of self-similar equilibria which have the same power law dependences on time as in the high $\Omega_I$ case (i.e. $j$ and $B \propto t^{1/3}$, $T$ and $P \propto t^{2/3}$). However, the spatial parts of the solutions, i.e. the profile shapes, are strikingly different from the high $\Omega_I$ solutions. They are "gas-embedded" (the plasma edge density is finite) and the current density is peaked on axis. In contrast the high $\Omega_I$ equilibria possess density profiles which fall to zero at the edge, and edge peaked current densities. There is a maximum possible radius for the low $\Omega_I$ solutions given by

$$r_{0\text{max}} = 1.12 \times 10^{-7} \frac{N^{3/4}}{\varphi^{3/2}} \text{ metres}$$

where $N$ is the line density and $\varphi$ is the current time dependence parameter. Figure 7 shows the low $\Omega_I$ self-similar density, temperature and current density profiles corresponding to the maximum attainable radius.

A single fluid 1-D Lagrangian code [14] has been used to model Z-pinch equilibrium evolution and dynamics, in particular the spontaneous generation of self-similar solutions. Simulations show that at early times in the current rise two regions of the pinch can be distinguished — an inner region in which the profiles are the 'low $\Omega_I$' type, and an outer 'high $\Omega_I$' region. This characteristic is illustrated in Fig. 8 which shows the resulting profiles at the time at which the average value of $\Omega_I$ is 0.01. As the current rises the width of the inner region diminishes but does not vanish completely because of the field null on axis. This behaviour prevents perfect convergence to the high $\Omega_I$ self-similar solutions.
6. RESISTIVE STABILITY THEORY

In the resistive phase the equilibrium pinch is evolving on a timescale comparable to the MHD growth time. Thus exponential growth will not occur and there is no obvious way to formulate the problem as an eigenvalue equation. We have avoided this difficulty by using a linearized initial value code. An arbitrary initial perturbation is imposed and the linearized equations together with the equilibrium variables are advanced in time. For the equilibrium we use our self—similar profiles [17] in which exact pressure balance is maintained while the current is rising as \( I \sim t^{1/3} \).

We have used this approach to study both \( m = 0 \) [18] and \( m = 1 \) linear free boundary modes. The results are best understood with reference to \( S \), the Lundquist number (Eq. 3). Since for our rising current \( S \sim t^{d/3} \) the plasma will eventually become 'ideal' and the time dependence of the equilibrium can be ignored. In this phase exponential growth of the instability occurs. For both \( m = 0 \) and \( m = 1 \) it is found that the pinch is stable during the early part of the current rise when \( S \) is below a certain critical value, denoted \( S^* \). Once the pinch current is sufficiently high that \( S > S^* \) an exponentially growing instability, with a growth rate agreeing with ideal MHD, is found. Thus \( S^* \) represents the threshold of ideal MHD validity.

The value of \( S^* \) depends both on the value of \( ka \) (\( k \) is the axial wavenumber of the instability, \( a \) is the pinch radius) and on the form of the initial perturbation. However, for any given value of \( ka \) it is found that upper and lower bounds on the value of \( S^* \) can be obtained by using for the initial perturbation either the fastest growing ('parallel') ideal MHD mode, which gives the lowest \( S^* \), and the second fastest ('orthogonal'), which gives the highest \( S^* \). In particular initialising with random noise gives a value of \( S^* \) between the two limits. This feature of the solution is probably related to the orthogonality of the ideal MHD eigenfunctions. Whatever the initial perturbation of course the mode which eventually emerges in the ideal phase is always the fastest growing one.

For \( m = 0 \) impressive agreement is found with results from 2—D non—linear simulations at NRL, Washington [19]. The values of \( S^* \) obtained using this fundamentally different approach lie between the bounds described here.

Figures 9 and 10 show \( S^* \) as a function of \( ka \) for \( m = 0 \) and \( m = 1 \) respectively. Taking a mean value, the scaling with \( ka \) is found to be

\[
<S^*> \sim 50 \ (ka)^{-0.86} \quad \text{for} \quad m = 0
\]

\[
<S^*> \sim 40 \ (ka)^{-0.75} \quad \text{for} \quad m = 1
\]

This work indicates that in the time dependent, resistive phase of the current rise the pinch is strongly stabilized to both \( m = 0 \) and \( m = 1 \) instabilities. This may explain the anomalous stability observed in deuterium fibre experiments [16].
FIG. 9. Critical Lundquist number $S^*$ against $ka$ for $m = 0$ 'parallel' and 'orthogonal' initial perturbations. Cochran and Robson's NRL results [19] are also shown.

FIG. 10. Critical Lundquist number $S^*$ against $ka$ for $m = 1$ 'parallel' and 'orthogonal' initial perturbations.
7. CGL STABILITY THEORY

The theoretical study of $m = 0$ linear stability in the small ion Larmor radius regime [9,11] using the Chew–Goldberger–Low equations [8] has been extended to include equilibria with anisotropic pressures [10] and equilibria partly confined by a skin current. This work is directly relevant at the high currents employed in the new fibre pinch experimental facilities, presently coming on-line or under construction (Sec. 10). The problem has been approached by examining stability thresholds (derived from the energy principle) as well as by direct solution of the eigenvalue equation.

For isotropic equilibria the CGL growth rate is always lower than ideal MHD. A further reduction in the growth rate of short wavelength (fast growing) modes can be obtained if $P_{\perp} > P_{\parallel}$. However the degree of anisotropy which is accessible is limited by both the requirement of pressure balance and by the onset of the mirror instability at large $P_{\perp}/P_{\parallel}$. For a typical case we find, for the maximum allowable $P_{\perp}/P_{\parallel}$, a reduction in the $ka = 10$ growth rate by approximately 30% compared to the CGL result for the isotropic case.

The previously obtained CGL $m = 0$ stability criterion for isotropic equilibria without a skin current [9,11] is always satisfied in the inner region of the pinch, for any isotropic equilibrium. In the outer part of the pinch the pressure profile must lie above the marginal profile given by

$$P(r) = \frac{C_1 r^{10}}{(1 + C_2 r^{3/2})^8}$$

(10)

where $C_1$ and $C_2$ are constants defined by the pinch parameters. This indicates that any isotropic equilibrium with zero pressure at the edge is CGL unstable. We have therefore considered equilibria partly confined by a skin current, for which the edge pressure is finite. Numerical solution of the CGL eigenvalue equation for any arbitrary equilibrium is straightforward. In addition, for a pure skin current Z-pinch an analytic solution is also found. This shows enhanced stability over ideal MHD but a higher growth rate than that obtained using the Vlasov fluid model (Sec. 8). This indicates that ion kinetic effects strongly stabilize the Z-pinch. Varying the fraction of the total current flowing in the skin changes the qualitative nature of the eigenvalue spectrum, but has little effect on the growth rate of the fastest growing mode.

8. VLASOV FLUID STABILITY THEORY

The collisionless linear stability of a Z-pinch confined by a skin current [20] has been studied using the Vlasov fluid model [21]. This model treats the ions by the Vlasov equation and the electrons as a cold quasi-neutralising background fluid. In contrast to previous applications of
this model we find that for our simple equilibrium the full eigenvalue equation does not need to be solved. Instead we derive a dispersion relation from the plasma edge boundary condition.

A significant reduction in growth rate compared to ideal MHD [4] is found. The Vlasov fluid growth rate saturates at \( ka \approx 5 \), and in the short wavelength limit the equations can be solved in closed form to give the following exact value for the asymptote, for all \( m \) numbers,

\[
\lim_{ka\to\infty} \gamma^* = 0.5\pi^{1/2}
\]

where \( \gamma^* \) is the growth rate normalized with respect to the radial ion thermal transit time. This result should be compared with the large \( ka \) ideal MHD growth rate which increases without limit as \( \lambda k \) [4]. Figure 11 shows \( \gamma^* \) as a function of \( ka \) for the \( m = 0 \) mode. Both the Vlasov fluid and ideal MHD results are shown.

This work has been generalized to include finite electron temperature. The effect is to increase growth rates from the cold electron value by a factor \( \delta \) which depends on \( T_e/T_i \) and \( ka \). For \( T_e = T_i \delta \) lies in the range \( 1.8 < \delta < 2.0 \) for all \( m \) and \( ka \), i.e. the predominant effect is to approximately double growth rates.
9. RADIATIVE COLLAPSE

At currents in excess of the Pease—Braginskii current $I_{PB}$ (roughly 1.2 MA in hydrogen) the approximate radial pressure balance in Z-pinches formed from fibres is upset by radiative cooling. Since the pinch must contract to restore pressure equilibrium the density is increased and thus the
radiation rate is also increased thereby accelerating the rate of compression and giving rise to a non-linear collapse. In a realistic system driven by an external circuit, the dramatically increasing pinch impedance during collapse causes the current to fall. However the falling Coulomb logarithm and rapidly changing internal structure of the pinch allow $I_{PB}$ to fall ahead of the current [Chittenden et al, [22] and thus collapse is not terminated until enormous densities have been achieved. Figure 12 shows the dynamics of a 3 cm long pinch of line density $2 \times 10^{19}$ ions per metre obtained using a 1-D radial Lagrangian resistive MHD code coupled to a 3 MV external circuit (the initial phase of formation of a fully ionized pinch from a cryogenic fibre has been by-passed here). Collapse is terminated at $2 \times 10^5$ times the density of solid hydrogen by the effects discussed below.

To ascertain what phenomena are responsible for collapse termination we compare the expected trajectory of the experiment in density–temperature space to lines representing the onset of various phenomena. In Fig. 13, line 1 represents when the volume integral of bremsstrahlung emission equals the surface integral of pinch emission, i.e. when the pinch becomes a black-body emitter. Below line 2 the electron chemical potential becomes positive and free electron degeneracy effects start to become important. Line 3 is given by 99% ionization from the Saha equation (including continuum lowering) and represents the onset of recombination. Finally, line 4 is given by the number of particles in a Debye sphere being unity and represents the onset of strong coupling effects. If the experiment encounters line 1 the radiation emission loses its density dependence and collapse is rapidly terminated. Figure 13 suggests
FIG. 14. Scaling diagram for $N = 10^{30}$.

FIG. 15. Radiative cooling rate/emission (arbitrary units) against frequency group and normalized radius at peak compression.
that free electron degeneracy is encountered before this occurs and that both effects participate in collapse termination at this line density. Figure 14 shows that for a line density of $10^{20}$ ions per metre opacity is expected to be dominant over degeneracy. Strong coupling and recombination effects are not expected to be of relevance to collapse termination over the range of line densities of experimental interest.
During radiative collapse the pinch emission undergoes a continuous transition from being almost completely optically thin to being optically thick to low frequencies in high density regions. The strong radial density gradient during collapse implies a high degree of radial anisotropy in the radiative flux whereas axial and azimuthal symmetry are assumed. A radiation transport package which integrates emission and absorption of radiative flux both radially inwards and radially outwards in order to find the flux divergence for 32 frequency groups, was coupled to the 1-D code. Figure 15 shows the ratio of the effective radiative cooling rate to the emission rate against frequency and radius taken from a simulation including radiation transport at peak compression ($2 \times 10^5$ times solid density). The increasing opacity at low radii and low frequency causes a fall-off in the rate of radiative cooling in these areas and gives rise to collapse termination. Figure 16 shows the evolution of the total pinch emission spectrum with increasing density from an optically thin bremsstrahlung spectrum to a spectrum which is becoming Planckian at low frequency but is still optically thin at high frequency.

Computational analogues for the effects of free electron degeneracy upon the transport coefficients, the equation of state and the opacity were implemented within the framework of the 1-D code. It is unlikely that the modifications made to the transport coefficients constitute a collapse termination process since the effect of free electron degeneracy is to increase the electron relaxation time and thus decrease the resistivity with respect to the classical Spitzer value. Electron degeneracy terminates radiative collapse through the equation of state by supplementing the radial pressure gradient with degenerate electron pressure until the electromagnetic pinch force is balanced. Figure 17 shows the ratio of the electron pressure to the perfect gas value throughout peak compression taken from a simulation including the modified equation of state but in the absence of radiation transport.

Results from simulations including radiation transport and all the effects of free electron degeneracy for various line densities are summarised in Table 1.

### TABLE I. KEY PARAMETERS FROM SIMULATIONS INCLUDING ALL EFFECTS

<table>
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<tr>
<th>Line density ($m^{-1}$)</th>
<th>$5 \times 10^{18}$</th>
<th>$2 \times 10^{19}$</th>
<th>$10^{20}$</th>
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<tr>
<td>$a_{\text{min}}$ (nm)</td>
<td>7.71</td>
<td>33.6</td>
<td>528</td>
</tr>
<tr>
<td>$n_{\text{mean}}/n_{\text{solid}}$</td>
<td>$4.3 \times 10^3$</td>
<td>$8.9 \times 10^4$</td>
<td>$1.8 \times 10^5$</td>
</tr>
<tr>
<td>$n_{\text{max}}/n_{\text{mean}}$</td>
<td>3.405</td>
<td>3.924</td>
<td>4.096</td>
</tr>
<tr>
<td>$\xi_{\text{max}}$</td>
<td>5.107</td>
<td>4.713</td>
<td>1.013</td>
</tr>
<tr>
<td>$P_e/P_{\text{pe}}$</td>
<td>2.074</td>
<td>1.991</td>
<td>1.303</td>
</tr>
</tbody>
</table>
In the table $a_{\text{min}}$ is the minimum pinch radius, $n_{\text{mean}}$ is the mean density at peak compression, $n_{\text{max}}$ is the maximum density at peak compression and $\xi_{\text{max}}$ is the maximum value of the degeneracy parameter ($\xi = \frac{\hbar \mu}{kT}$, $\mu$ is the chemical potential). With lower line densities the mean (Bennett) temperature, is higher and thus the onset of the collapse termination processes is delayed until higher density. As expected the level of degeneracy and its effect on the electron pressure $P_e$ (compared to the perfect gas value $P_{\text{pfg}}$) is greater for lower line densities. However the rapidly steepening radial density profile during collapse produces a highly centre peaked profile at maximum compression thus allowing degeneracy to be present even in larger line density simulations contrary to the expectations of Fig. 14.

10. THE DENSE Z-PINCH PROJECT

A new experimental facility, MAGPIE, is under construction to explore the physics of dense plasma pinches. MAGPIE, which is due to be completed by the end of 1992, is a 2 MA, 2.4 MV generator designed to provide a pulse of 200 ns into an inductive load. An artist's impression of MAGPIE is shown in Fig. 18. Four 86 kJ, 2.4 MV Marx generators
feeding four water dielectric pulse forming lines which are switched simultaneously into a single matched 1.25 ohm water transfer line. The final part of the generators is designed to be flexible and will be capable of being adapted for variety of Z-pinch loads, from high inductance fibre pinches to low inductance gas-puffs.

The first experimental load is scheduled to be a cryogenic hydrogen fibre of 10 to 100 μm diameter. The aim of these experiments will be to study the physics of radiative collapse and the $10^4$ times solid density predicted. The experiment would allow us to gain insight into the details of the mechanism contributing to both radiative collapse and the termination processes. By varying the initial fibre diameter from 10 μm through 100 μm and therefore the line density, it would be possible to investigate the different regimes of stability discussed above. The results will of direct relevance to the high density Z-pinch under the fusion conditions.

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FREE BOUNDARY IDEAL MAGNETOHYDRODYNAMIC STABILITY OF THREE-DIMENSIONAL STELLARATORS

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Abstract

FREE BOUNDARY IDEAL MAGNETOHYDRODYNAMIC STABILITY OF THREE-DIMENSIONAL STELLARATORS.

The 3-D ideal MHD stability code TERPSICHORE is used to investigate the global external stability properties of the ATF device and the Wendelstein 7-X device. The stability of 3-D ATF equilibria with zero net toroidal plasma current and bell shaped pressure profiles shows that (a) external modes alter the internal stability properties, (b) the coupling between the main toroidal Fourier components of the mode and their sidebands is destabilizing, (c) the stabilizing outward shift of the magnetic axis is confirmed, (d) the quadrupole field deteriorates stability by annihilating the vacuum magnetic well and (d) global stability is insensitive to pressure profile effects. The ideal MHD stability of a sequence of configurations with zero net toroidal current that varies from a conventional L = 2 stellator to the proposed Wendelstein 7-X device shows that the mode structures are essentially internal and that this proposed configuration is stable to global modes at $\beta = 4.5\%$ with a safe margin.
1. INTRODUCTION

The global external ideal magnetohydrodynamic (MHD) stability properties of three-dimensional (3-D) plasma confinement configurations are investigated with a new free boundary extension of the TERPSICHORE code [1-3]. The magnetic structure is constrained to have nested flux surfaces and 3-D equilibria are generated with the VMEC code [4]. The calculations in TERPSICHORE are performed in Boozer magnetic coordinates [5] and the incompressibility constraint is imposed so that only stability indices are computed. The components of the displacement vector are Fourier decomposed and a finite hybrid element radial discretization scheme is invoked to reduce the variational energy principle in its weak form to an eigenvalue problem in which the matrices have a special block pentadiagonal form. The vacuum region is treated as a pressureless, massless and shearless pseudoplasma, and the resulting matrix structure is identical to that in the plasma.

We investigate the global external MHD stability properties of the Advanced Toroidal Facility (ATF) [6] and of a sequence of configurations encompassing the proposed Wendelstein 7-X stellarator in this paper (see Paper G-I-6 of this Conference).

2. FREE BOUNDARY STABILITY STUDY OF THE ATF DEVICE

The pressure profile $p(r)$ in this study is bell shaped as a result of imposing $p'(0) = p'(1) = p(1) = 0$ and prescribing the location in the radial variable $0 < r < 1$ at which $p'(r)$ acquires its minimum value in order to vary the width of the profile. We vary $\beta$ with $p(0)$, $r$ is proportional to the volume of the plasma enclosed and in all the calculations zero net toroidal plasma current within each flux surface is imposed.

2.1. Effects of the vacuum and toroidal mode coupling

In the ATF there are six families of modes that decouple one from another depending on their toroidal mode number $n$ [3]. We concentrate in this subsection on the stability of the standard ATF configuration [6] to the family of $n = 2$ modes. The pressure gradient profile has its minimum at $r = 1/2$. The results are summarized in Fig. 1 which shows the converged eigenvalue as a function of the volume average $\beta$ for four cases:

(a) a fixed boundary sequence with only $n = 2$ components (limiting $\beta = 4.7\%$)

FIG. 1. Converged eigenvalue versus beta.

FIG. 2. Beta versus vacuum magnetic axis position.
(b) a fixed boundary sequence with \( n = 2 \) components and their toroidal sidebands \( (n = -2, 10, 14) \) (limiting \( \beta = 4.2\% \))
(c) a free boundary sequence with only \( n = 2 \) components (limiting \( \beta = 3.7\% \))
(d) a free boundary sequence with \( n = 2 \) components and their toroidal sidebands \( (n = -2, 10, 14) \) (limiting \( \beta = 3.1\% \)).

Although in all cases the mode structure is essentially internal, a conducting wall placed at the plasma-vacuum interface exerts a strong stabilizing influence. Furthermore, the mode structure though dominated by the \( n = 2 \) components is destabilized by coupling to the \( n \neq 2 \) members of the family.

### 2.2. Shift of the magnetic axis

The ATF device is designed with a vertical field coil system that can shift the position of the magnetic axis in the vacuum state [6]. An inward shift annihilates the magnetic well while an outward shift deepens it and widens its radial extent. In Fig. 2, the critical \( \beta \) value imposed by the \( n = 1 \) and the \( n = 2 \) families of global external modes with a conducting wall placed far from the plasma-vacuum interface plotted as a function of the vacuum magnetic axis position shows an enhancement of the stability conditions when the shift of the magnetic axis is outward (corresponding to the conditions of deepest magnetic well).

### 2.3. The quadrupole magnetic field

The vertical field system contains a set of coils that induce a quadrupole magnetic field that is used to control the rotational transform profile \( \iota \) in the ATF [6]. The current in the quadrupole field coils normalized to the current in the main helical field coils is denoted by \( I_{mid} \). As in Subsection 2.2, the pressure profile is bell shaped with its gradient troughing at \( r = 1/3 \). Monotonicity in the \( \iota \) profile is maintained at finite \( \beta \) by increasing \( I_{mid} \). The global external MHD stability properties imposed both by the \( n = 1 \) and the \( n = 2 \) families of modes yield curves of very similar values of critical \( \beta \) that deteriorate as \( I_{mid} \) is increased, as is shown in Fig. 3. The reason for this is that as the quadrupole field is incremented, the vacuum magnetic well becomes shallower and its radial extent contracts reverting virtually to a magnetic hill at \( I_{mid} = 0.19 \). In fact, an improvement in global stability is realized by reversing the current in the quadrupole field coils. Local Mercier stability, however, is optimal for the standard configuration \( (I_{mid} = 0) \) and exceeds the \( \beta \) limits imposed by the global modes except at the extreme ends of both curves. The mode structure of the dominant components of the \( n=1 \) family in this sequence (as well as those in the sequences described in the previous subsections)
FIG. 3. Beta versus Imid.

FIG. 4. Dominant Fourier components for the $n = 1$ mode in ATF with $\text{Imid} = 0.07$ and peaked $p(r)$. 

$$\text{nsd} = 64$$
$$P(0) = 6.0063$$
$$q(0) = 2.1961$$
$$q(1) = 1.0959$$
FIG. 5. Perturbed pressure distribution in ATF with Imid = 0.07; global n = 1 instability.
**FIG. 6.** Beta versus pressure profile width.

**FIG. 7.** Dominant Fourier components for the $n = 1$ mode in ATF with $l_{mid} = 0.07$ and broad $p(r)$. 

- $nsd = 64$
- $P(0) = 4.0001$
- $q(0) = 2.5101$
- $q(1) = 1.1399$
concentrate in the region where the magnetic shear is negative (where the Mercier criterion is strongly stabilized). This instability structure is shown in Fig. 4 for the configuration with $Imid = 0.07$. A very important destabilizing feature to the structure that is observed is that the amplitudes of the dominant components at the plasma-vacuum interface exceed 10% of their peak values. The mode structure of the $n = 2$ family displays similar characteristics to the $n = 1$ family, except that it is more local and the external effects are less significant. The perturbed pressure distribution due to the $n = 1$ family of modes at five toroidal planes that span one field period is shown in Fig. 5 for the configuration with $Imid = 0.07$.

2.4. Pressure profile effects

We examine a sequence of equilibria in which we vary the width of the bell shaped pressure profile by making its gradient trough at $r = 1/3$, $r = 1/2$ and $r = 2/3$ for the ATF configuration for which $Imid = 0.07$. The result is shown in Fig. 6, where we note the $\beta$ value imposed by the $n = 1$ family of modes is insensitive to the width of the profile. A calculation with a more peaked parabolic profile with $p(r) = p(0)(1 - r)^2$ provides further confirmation of the scaling. The large pressure gradients in the magnetic hill region for the broad profile cases cause the Mercier criterion to become destabilized at very low values of $\beta$. For the peaked profile in the sequence, the bulk of the pressure gradient lies in the magnetic well region and the localized modes are stabilized. For this case, the global $n = 1$ mode is more restrictive and would only exceed the $\beta$ limit predicted by the Mercier criterion if the coupling between the $n = 1$ and the $n \neq 1$ components were ignored and a conducting wall were placed very close to the plasma-vacuum interface. The dominant Fourier component reveals an $n = 1$ external mode as is shown in Fig. 7 and the corresponding perturbed pressure distribution at a surface near the plasma-vacuum interface appears in Fig. 8. As in Fig. 5, the colour blue denotes regions where the perturbed pressure is minimum and the colour red where it is maximum. A local toroidal current driven near the plasma surface can enhance stability [7], if it increases the value of $\iota$ at the edge [3].

3. FREE BOUNDARY STABILITY FOR AN OPTIMIZED HELIAS CONFIGURATION

A second application concerns free boundary modes in a Helias configuration optimized for the proposed Wendelstein 7-X stellarator (see Paper G-I-6 of this Conference). Results on internal modes are described in Paper C-V-3 of this Conference.
FIG. 8. Periurbed pressure distribution of ATF with Imid = 0.07; global n = 1 instability.
FIG. 9(a). Pressure and flux contours of Wendelstein 7-X configuration ($T = 1$).
FIG. 9(b). Pressure and flux contours of a conventional $L = 2$ stellarator ($T = 0$).
FIG. 10. Perturbed pressure distribution for the configuration with $T = 0.6$ in the sequence.
Free boundary modes are studied in a sequence of equilibria of which one end-point is the optimized Helias configuration. The other end-point was not chosen in such a way as to span a typical sequence of unstable and stable Helias equilibria [8] but rather to connect the results with those already obtained for geometrically more simple stellarators [2]. A typical situation with respect to free boundary modes is studied, namely \( \iota \) at the plasma boundary just below unity, and the volume average \( \beta \) value is kept fixed at 4.5\%. For a dominantly \( m = 4, n = 4 \) mode a Fourier mode pattern for optimal convergence is obtained with the fixed boundary code CAS3D [9]. This pattern is expanded further to include the \( m = 10, n = 9 \) component that is resonant and its relevant poloidal and toroidal sidebands. Modes with higher dominating node numbers satisfying the resonance condition more closely yield lower eigenvalues. Up to 60 Fourier components have been used. Figure 9 shows the two end points of the equilibrium sequence. Figure 10 shows a mode structure by way of a perturbed pressure distribution (red indicates maxima, blue minima) which reveals it to be dominantly internal. Figure 11 shows the eigenvalue as a function of the equilibrium sequence parameter and indicates marginal stability close to the Mercier stability boundary.
4. CONCLUSIONS

The TERPSICHORE code has been applied to survey the ideal MHD stability properties to global external modes of 3-D equilibria that model the ATF device and to investigate a sequence that leads to the proposed Wendelstein 7-X configuration. For the ATF, we find that
(a) the mode structures although basically internal in nature have small but physically significant external destabilizing components [10]. ("The tail wags the dog");
(b) the effects of coupling between the dominant global components of the mode and the toroidal sideband mode numbers that belong to the same family have a noticeably destabilizing effect. The coupling is critical for the accurate determination of the stability properties;
(c) the stabilizing effect of an outward shift of the magnetic axis is confirmed;
(d) the quadrupole field deteriorates stability conditions because it annihilates the magnetic well;
(e) pressure profile effects do not alter the global stability, but local stability imposes peaked profiles.

The maximum critical $\beta$ value imposed by global modes that we calculated reached 3.7% by reversing the current in the quadrupole field coils. However, modest toroidal currents driven near the plasma surface such that $\iota$ at the edge increases can improve the $\beta$ value [3].

For the investigation of the sequence of configurations that starts with a conventional $L = 2$ stellarator and ends with the optimized Wendelstein 7-X device, we find that at $\beta = 4.5\%$ the marginal point of stability due to the global mode structures agrees with the Mercier boundary at the configuration that is 20% a conventional $L = 2$ device and 80% the proposed Wendelstein 7-X device. Therefore, Wendelstein 7-X is predicted to be stable to global ideal MHD instabilities with a safe margin at this value of $\beta$.

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We are greatly indebted to Dr. S. P. Hirshman for providing us the 3-D VMEC equilibrium code to perform part of this work. This work was supported by the Fonds National Suisse pour la Recherche Scientifique, by Euratom and by the U. S. Department of Energy.

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