The cover picture shows an inside view of the coils of Wendelstein 7-AS. By courtesy of the Max-Planck-Institut für Plasmaphysik.
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FOREWORD

The International Atomic Energy Agency Conferences on Plasma Physics and Controlled Nuclear Fusion Research are the largest and most significant conferences in the field. The 1992 conference in Würzburg was the 14th in a series of meetings which began in 1961 and which, since 1974, have been held on a biennial basis. The conference was highlighted by reports of recent results from all of the major fusion facilities around the world, including the milestone experiment at JET in which tritium was introduced for the first time into a tokamak fuel mixture.

The conference was organized in co-operation with the Max-Planck-Institut für Plasmaphysik, Garching, to which the IAEA wishes to express its appreciation and deep gratitude. The conference was attended by around five hundred participants representing some thirty countries and two international organizations.

The opening session of the conference was highlighted by a round table discussion on ITER and its Relationships to Ongoing Fusion Programmes and by the traditional Artsimovich Memorial Lecture, which was given by Professor P.K. Kaw. During the technical sessions, over two hundred papers were presented. Contributions were made on tokamak experiments, inertial confinement, non-tokamak confinement systems, magnetic confinement theory and modelling, plasma heating and current drive, ITER, and technology and reactor concepts.

These proceedings include all the technical papers and five conference summaries. For the first time, the summary talks are being published as a separate volume before the rest of the proceedings.

The IAEA contributes to international collaboration and exchange of information in the field of plasma physics and controlled nuclear fusion research not only by organizing these biennial conferences but in a number of other ways as well. The International Fusion Research Council is sponsored by the IAEA and provides advice to the Agency on all matters related to fusion. The Nuclear Fusion journal has been published continuously by the IAEA for over 32 years. The IAEA organizes and maintains databases of nuclear, atomic, molecular and plasma-material interaction data for applications in fusion research and engineering. It also regularly organizes co-ordinated research projects, technical committee meetings, workshops, consultants meetings and advisory group meetings on relevant topics. Through all these activities, the IAEA hopes to contribute towards achievement of the long range goal of controlled fusion as a future energy resource.
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(Sessions D-1 to D-3 and Poster Session D-4)

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Paper IAEA-CN-56/D-1-5-2 was presented by A.B. Kukushkin as Rapporteur
STUDY AND CONTROL OF ERROR FIELD INDUCED LOCKED MODES IN TOKAMAKS

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Abstract

STUDY AND CONTROL OF ERROR FIELD INDUCED LOCKED MODES IN TOKAMAKS.

There is growing evidence that error fields may have a much greater effect on the stability of large tokamaks than has previously been supposed. Small resonant magnetic perturbations (especially \( m = 2, n = 1 \) perturbations) can lead to the production of unexpectedly large phase-locked magnetic islands which degrade the plasma confinement, and in some cases trigger major disruptions. These effects have been studied experimentally, and have also been investigated using both analytic theory and numerical codes.

1. Error Field Experiments

Experiments on the COMPASS-C tokamak [1] using externally applied resonant magnetic perturbations [principally, \((2,1)\) and \((3,2)\)] have revealed the existence of a threshold perturbation strength above which a large locked magnetic island is formed, and the rotation of the tearing-mode frame is arrested. Typical experimental wave-forms are shown in Fig. 1. No rotating magnetic precursor to the locked-mode formation is observed. Below the threshold strength there is some dragging of the tearing-mode frame, leading to modification of the bulk plasma rotation, but very little reconnection is induced in the

* Work jointly funded by the UK Department of Trade and Industry and Euratom.
FIG. 1. A typical ohmic COMPASS-C discharge, in which application of a (2,1) magnetic perturbation leads to the formation of a locked mode. The feedback-controlled plasma current and toroidal field are held constant to within a few percent. The diagnostic data shown are the helical coil current ($I_c$), a central SXR chord ($I_{SXR}$), the $n = 1$ locked mode signal relative to that obtained in the absence of plasma ($B_r(n = 1)$), the radial $m = 2$ Mirnov signal ($B_{θr}$), and the boron IV impurity ion toroidal velocity ($V_{φ}$). The locked mode occurs at the time marked (a), and unlocking at the time marked (b).

plasma. The production of a (2,1) locked island is associated with a significant degradation of energy and particle confinement and suppression of the sawtooth oscillation in the plasma core. However, the production of a (3,2) locked island gives rise to a far weaker confinement degradation and has little effect on sawteeth. Locked-mode production generally leads to a reversal in the impurity ion toroidal rotation velocity from the electron to the ion drift direction. The threshold strength at the rational surface is found to reduce as the plasma density decreases. Large (1,1) perturbations can be applied without any obvious effect until the toroidally coupled $m = 2$ sideband in the plasma reaches sufficient amplitude to induce a locked mode at the $q = 2$ surface.

Similar effects are seen on DIII-D [2]. It is found that edge perturbations of order 1 gauss can lead to disruptions on DIII-D, even though the predicted island size (from vacuum fields) is less than 5% of the minor radius. This effect is partially explicable
as a manifestation of the well-known 'amplification' of the vacuum island by the plasma response [3], but this is incapable of accounting for the large amplification sometimes observed.

Experiments on COMPASS-C, DIII-D, and JET [4-5] have revealed that the threshold perturbation strength for locked-mode production is a strongly decreasing function of increasing machine size. This effect is thought to be primarily related to the slower rotation of the tearing frame in larger devices and explains why error fields have seldom been a problem on small tokamaks. The size scaling suggests that, in the absence of externally driven rotation, large devices such as ITER are likely to be significantly more sensitive to error fields than any existing device.

A series of experiments using high power neutral beam injection have been performed on DIII-D in order to study the effect of forced rotation and high-β on the threshold perturbation strength for locked-mode production [6]. This reveals that increased rotation leads to a higher threshold. For 3 MW of injected beam power, the rotation of the tearing frame is raised by a factor of about 6 over the ohmic value, and the threshold is approximately doubled, indicating a scaling $\propto f_{\text{tor}}^4$. However, as the plasma β is further increased the threshold falls, despite a continuing increase in the rotation. The existence of a β dependence which is distinct from the dependence on rotation has been confirmed by repeating the experiment with different injection geometries. It is conjectured that this β dependence is related either to the degraded plasma stability at high β (these plasmas being close to the normal β limit), or to more effective coupling at high β between the plasma at the rational surface and the external perturbation.

2 Analytic Theory of Tearing Mode Interaction with Error Fields

The tearing stability index of a mode interacting with a static external magnetic perturbation can be written $\Delta' = \Delta'_{\text{mode}} + \Delta'_{\text{coil}}$, where $\Delta'_{\text{mode}}$ is the standard stability index for a free-boundary tearing mode, and

$$\Delta'_{\text{coil}} \propto r_s \simeq 2m \times \left( \frac{W_{\text{vac}}}{W} \right)^2 \cos \Delta \varphi$$

parameterises the effect of the external perturbation on mode stability [7]. Here, $r_s$ is the radius of the rational surface, $W$ the plasma island width, $W_{\text{vac}}$ the 'vacuum' island width obtained by superimposing the vacuum perturbation onto the equilibrium field, and $\Delta \varphi$ the helical phase shift between the plasma and vacuum islands. Note that $d\Delta \varphi/dt \equiv \omega(t)$, where $\omega$ is the (angular) rotation frequency of the plasma island.
The total flux-surface-integrated toroidal electromagnetic torque acting across the island region is given by

\[
T_{\phi EM} \simeq 4\pi^2 R_0 \times \frac{m n}{\mu_0} \times \frac{(B_{\phi})^2}{28} \left( \frac{r_z^2}{R_0 q_a} \right)^2 \times \left( \frac{W_{\text{lim}}}{r_s} \right)^2 \times \sin \Delta \varphi.
\]

Here, \( R_0 \) is the major radius, \( B_{\phi} \) the toroidal field strength, \( q_a = m/n \), and \( s \) is the magnetic shear at the rational surface. This torque gives rise to a modification of the bulk plasma rotation, leading to the development of a viscous restoring torque in the vicinity of the island. In steady state, the relationship between the local plasma toroidal (angular) velocity shift (relative to the unperturbed plasma) \( \Delta \Omega_{\phi} \) at the rational surface and the flux-surface integrated viscous torque is

\[
T_{\phi VS} = -4\pi^2 R_0 \times \Delta \Omega_{\phi} \times R_0^2 \int_{r_s}^a \frac{1}{\mu_{\perp}(r)} \frac{dr}{r},
\]

where \( a \) is the minor radius, and \( \mu_{\perp} \) is the (anomalous) perpendicular plasma viscosity.

Provided that the magnetic island width is much larger than that of the linear layer (i.e., the island is 'non-linear') there is no net plasma flow across the separatrix (i.e., there is no 'slip'). The velocity shift at the rational surface is related to the shift in island rotation frequency via the 'no slip' constraint \( \omega - \omega_0 = -n \Delta \Omega_{\phi} \), where \( \omega_0 \) is the mode rotation frequency in the unperturbed plasma (the so-called 'natural' frequency). If the plasma island is unable to satisfy the 'no slip' constraint it will remain in a 'suppressed' state with a similar width to the linear layer and the plasma flow 'slipping' through it to some extent. The suppressed state can be approximated as a linear layer.

Two asymptotic limits can be identified in the interaction of a locked (i.e., \( \omega = 0 \)) non-linear island with a static external perturbation. In the first, the tearing mode is intrinsically highly unstable, so that its saturated width \( W_0 \) is essentially unaffected by the external perturbation. In the second, the tearing mode is intrinsically highly stable, but is driven by the external perturbation.

The island phase shift \( \Delta \varphi \) is obtained by balancing the electromagnetic and viscous torques in the vicinity of the rational surface. The stable phase shifts are found to be such that \( \Delta \varphi_{\text{col}} \geq 0 \). There is a maximum stable phase shift which can be maintained in the plasma (\( \Delta \varphi = \pi/2 \) in the saturated limit, \( \Delta \varphi = \pi/4 \) in the driven limit). Beyond this critical value, the viscous torque can no longer be balanced by the electromagnetic torque, and the locked non-linear island makes a 'mode unlocking' transition to a rotating non-linear state, if the tearing mode is intrinsically unstable, or a locked suppressed state, otherwise [see point (b) in Fig. 1]. Thus, there is a minimum perturbation amplitude...
(conveniently parameterised by a minimum vacuum island width) needed to maintain a locked non-linear island in the plasma. This is given by

\[ (W_{\text{vac}})_{\text{unlock}} = \frac{W^{2}_{\text{unlock}}}{W_{0}} \]

in the saturated island limit, and

\[ (W_{\text{vac}})_{\text{unlock}} = 2^{1/4} \left( \frac{\Delta^{\text{mode}} r_{s}}{2m} \right)^{1/2} W_{\text{unlock}} \]

in the driven island limit, where

\[ \frac{W_{\text{unlock}}}{r_{s}} = \left[ \frac{2^{8}}{m^{2}} q_{s} R_{0}^{2} \left( \omega_{0} \right)^{2} \frac{r_{s}^{2} \mu_{s}^{/}}{\mu_{s} (r_{s})} \int_{r_{s}}^{a} \mu_{s} (r_{s}) \, dr' \right]^{1/4} . \]

Here, \( \tau_{H} = R_{0} \sqrt{\mu_{s} (r_{s})} / (B_{s} r_{ns}) \) is the local hydromagnetic time-scale, \( \tau_{V} = r_{s}^{2} \mu_{s} (r_{s}) / \mu_{s} (r_{s}) \) the local viscous time-scale, and \( \rho \) the plasma density.

For a locked suppressed island interacting with a static magnetic perturbation in an ohmic plasma, the rotation frequency of the tearing frame \( (\omega'_{0}) \) is obtained by balancing the electromagnetic and viscous torques in the vicinity of the rational surface and obeys

\[ \frac{\omega'_{0}}{\omega_{0}} \approx \frac{1}{2} + \frac{1}{2} \left[ 1 - \left( \frac{W_{\text{vac}}}{W_{\text{pen}}} \right)^{4} \right]^{1/2} , \]

where

\[ \frac{W_{\text{pen}}}{r_{s}} = \left[ 67.31 \frac{m H^{2}}{r_{s}^{2}} q_{s} R_{0}^{2} \left( \omega_{0} \right)^{2} \frac{r_{s}^{2} \mu_{s}^{/}}{\mu_{s} (r_{s})} \int_{r_{s}}^{a} \mu_{s} (r_{s}) \, dr' \right]^{1/4} . \]

Here, \( \tau_{R} = \mu_{s} r_{s}^{2} / \eta_{||} (r_{s}) \) is the local resistive time-scale. When the tearing frequency \( \omega'_{0} \) is reduced to half of its unperturbed value \( \omega_{0} \) (i.e., when \( W_{\text{vac}} \geq W_{\text{pen}} \)) the system makes a sudden transition to a non-linear locked island state with \( \omega'_{0} = 0 \) [see point (a) in Fig. 1]. This transition is distinguished from conventional mode locking by the absence of any rotating precursor and is generally known as 'mode penetration'.

3 Numerical Simulation of Tearing Mode Interaction with Error Fields

Numerical simulations with a reduced set of MHD equations (cylindrical, low \( \beta \), single helicity) have been carried out to study the mechanism of mode locking. The chosen safety factor profile is \( q(r) = 1.2 [1 + (r/0.61)^{4}]^{1/2} \), so that the (2,1) rational surface lies at \( r_{s} = 0.7 \). (All lengths are expressed as fractions of the minor radius).

The time evolution of the width of a magnetic island is shown in Fig. 2(a). Here, time is normalised to the poloidal Alfvén transit time \( (\tau_{A}) \). Initially, the plasma column rotates
FIG. 2. (a) Time evolution of the width of a (2,1) magnetic island interacting with an external helical magnetic perturbation. (b) Time evolution of the poloidal velocity profile during the period when the helical perturbation is present.

as a solid body in the poloidal direction with a period of $50 \tau_A$. The magnetic Reynolds number at the rational surface is $S = 2 \times 10^4$ and the normalised viscosity (i.e., the ratio of $\tau_A$ to the typical viscous diffusion time-scale) is $\nu = 10^{-5}$. Just prior to $t = 3000 \tau_A$ the magnetic island is in a saturated steady state.

The (2,1) static external helical field is switched on at $t = 3000 \tau_A$ and exerts a drag torque on the rotating plasma in the vicinity of the rational surface. The time evolution
of the poloidal rotation velocity profile is shown in Fig. 2(b). Mode locking occurs at \( t = 3800 \tau_A \). The strong velocity shear induced in the vicinity of rational surface leads to a transient decrease in the island width. This is confirmed by linear stability analysis, including the effects of plasma flow, [8] which has been used to demonstrate that the velocity shear at \( t = 4500 \tau_A \) has a stabilising effect on the tearing mode. For \( t > 5000 \tau_A \) the island settles down to a locked state with relatively small velocity shear in the vicinity of the rational surface and with a large island width due to direct driving by the external helical field.

One possible technique for avoiding mode locking is to modulate the phase of the external field in time in order to induce quasi-steady stabilising velocity shear near the mode rational surface. This technique has been successfully simulated using rotating and oscillating external helical fields. A feedback control system with appropriate control parameters could also be used to suppress the locked mode.

REFERENCES


DISCUSSION

J.H. HARRIS: What modifications to the theoretical picture for mode locking do you expect to see at finite pressure? Could they be related to the phenomena seen in experiments at high \( \beta \)?

R. FITZPATRICK: Preliminary calculations have shown that the finite \( \beta \) correction to the zero \( \beta \) expression for the electromagnetic torque acting at a given rational surface tends to increase the torque, given similar ideal MHD eigenfunctions in the outer region. These results suggest that increasing \( \beta \) may facilitate mode locking.

K.M. McGUIRE: In your modelling of the effects of locked modes, you show an evolution of the poloidal velocity profile with no effect on the central rotation.
However, in the experimental data there was sawtooth stabilization and no $m = 1$ mode rotation. How do you explain this experimental result?

T.N. TODD: The profile referred to is based on a cylindrical calculation and therefore includes no toroidal mode coupling. It is thought that toroidal mode coupling between the $m = 2$ and $m = 1$ modes plays a key role in the experimentally observed suppression of sawteeth, although the precise mechanism remains to be elucidated.
SIMULATIONS FOR CONFINEMENT IN NEAR FUSION EXPERIMENTS

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Abstract

SIMULATIONS FOR CONFINEMENT IN NEAR FUSION EXPERIMENTS.

Numerical simulation techniques are used to examine several critical areas in present near fusion experiments: (1) L-mode and some hot ion mode tokamak profiles are found to be marginally stable against ion temperature gradient driven instabilities, contrary to previous results. Important stabilizing effects from impurities, beam and electron collisions are found which are usually neglected. (2) Non-linear analytical arguments reveal that strongly ballooning instabilities can be less stabilized by non-linear effects than is commonly assumed, possibly leading to the appearance of structures with a long radial correlation length. (3) Non-linear toroidal kinetic simulations also show that they can sometimes produce surprisingly large transport, even near marginal stability. (4) Instability stabilization due to sheared flows is important near the edge. Non-linear kinetic simulations show a strong correlation with linear stabilization results. (5) Simulations of sawteeth for low collisionality conditions reveal that they can saturate at very low amplitude.

1. INTRODUCTION

The goal of the numerical tokamak programme is to develop computational tools to simulate actual experimental conditions. We examine some of the successes of this development in the following sections.

2. MARGINAL STABILITY OF EXPERIMENTAL PROFILES

A very complete and comprehensive linear initial value code has been developed for kinetic microinstabilities such as ion temperature gradient driven (ITGD) modes. The code advances full gyrokinetic equations in ballooning co-ordinates for several species: ions, an impurity species, electrons, and a beam species. Full trapped particle effects and toroidal curvature are included, and a momentum conserving Lorentz pitch angle scattering operator is used. The code solves for the distribution function on a grid in all co-ordinates and is fully implicit in the parallel direction, thus allowing large time steps. Electromagnetic effects ($A_1$) are included. The code
has been run for parameters of several TFTR shots, using experimentally determined values for all profile parameters, $Z_{\text{eff}}$ (from carbon) and beam density. The code will be made non-linear shortly, but has already given important linear results.

Several remarkably strongly stabilizing effects have been identified, and some or all of these have been ignored in previous treatments. Stabilizing effects result from low-Z impurities (e.g. carbon), high beam fractions, and finite $\beta$ electromagnetic effects. In addition, collisionless trapped electrons can significantly destabilize the ITGD mode; however, a Lorentz collision operator strongly reduces this destabilization. We have found that some Krook collision operators [1] underestimate collisions for $\omega \sim \nu_{\text{eff}}$, which can result in a strong destabilization, particularly for hot ion modes and supershots.

For beam heated L-mode TFTR shots, the dominant instability in almost all cases is identified as a strongly ballooning ITGD mode with $\omega \sim \omega_{\text{ci}}$. The experimental data set consisted of the shots used to examine whether the plasma scales as Bohm or gyroBohm. In the inner one-third of the plasma, the experimental ion temperature gradient scale length $L_{\text{exp}}$ is almost always within about ±20% of marginal stability, which is within experimental errors. Around $r/a = 0.5$, the plasma is within 20% of marginal stability in about half of the cases, and within ±40% in almost all the remaining cases. Most points in the outer third of the plasma have ion temperature gradients above threshold by 50% or more.

Recently, Zarnstorff et al. [2] have reported a pellet injection experiment to test the marginal stability hypothesis and have concluded that the plasma could be induced to be above several theoretical thresholds. In the analysis of the experiment it was asserted that the ITGD mode must be unstable after the pellet. However, the impurity density and gradient can very strongly affect the stability. Data available from data analysis codes (TRANSP) have unrealistic assumptions about the impurity profile. We have found that more reasonable assumptions about the impurity profile can give $L_{\text{exp}}$ within 20–30% of $L_{\text{crit}}$. Therefore, we do not believe that this experiment can be used as a disproof of the marginal stability hypothesis.

The stability of a hot ion mode shot at $r/a = 0.73$ is shown in Fig. 1. Without carbon, beams or $\beta$ effects, the shot is far above marginal stability for almost all $k_{p}\rho_{i}$ values. However, with all effects included, we can see that the shot is within 20% of the marginal stability for modes with $k_{p}\rho_{i} \sim 0.1$.

The value of $\chi$ has been estimated by (1) mixing length estimates from the linear eigenfunction, $\chi \sim \gamma \Delta x^2$, and (2) inserting a spatial diffusion operator from turbulence into the kinetic equation and iteratively determining the value of the diffusion coefficient $D$ which stabilizes the mode in the linear code. We then take $\chi = C_1 \chi_{\text{mix}}$ or $\chi = C_2 D$, and find $C_1$ and $C_2$ by benchmarking against the non-linear codes for cases well above marginal stability (see below). $\chi$ is much smaller than the experimental $\chi$ (often by an order of magnitude, even if the temperature gradient is increased by 20–40%). Also, $\chi$ decreases with minor radius, in contradiction to experiments. Thus the strong turbulence mixing length paradigm is not consistent with the data.
FIG. 1. Ratios of critical temperature scale length to experimental value versus $k_\phi \rho_i$ for a TFTR hot ion mode for the full kinetic calculation, neglecting beam effects only, and neglecting impurity effects.

3. NON-LINEAR ANALYSIS

A possible alternative paradigm for transport near marginal stability has emerged from simulations and theory. First, recall that a ballooning mode has a structure $\phi \sim \Phi(r, \theta)e^{i\kappa}$, where the distribution function behaves similarly. Thus, $\nabla \phi \sim \Phi \nabla s$ and $\nabla f \sim f \nabla s$. Thus the non-linear terms in the gyrokinetic equation, $v_B \cdot \nabla f \sim n^2 \Phi f \nabla s \times B \cdot \nabla s = 0$, vanish to lowest order since the lines of constant $f$ are parallel to the lines of constant $\phi$.

The preceding discussion is slightly oversimplified since ballooning coordinates are not periodic, so that a transformation must be used to obtain the eigenfunction in real space. If the mode is strongly localized at the outside of the torus, the non-linear term is not exactly zero, but it is small, and it can be shown to be reduced by a factor $v_B \cdot \nabla f / |v_B| |\nabla f| \sim \phi(\pi)/\phi(0)$. For typical experimental parameters, the linear code finds that the instability eigenfunctions have $\phi(\pi)/\phi(0) \sim 0.05-0.15$. The previous arguments suggest that such instabilities can grow to larger amplitude than a mixing length estimate would suggest and may produce the transport needed to force the profiles close to marginality. Such instabilities would have an easily identifiable structure: they would have a radial correlation length much longer than $k_\phi \rho_i$, and neighbouring poloidal harmonics would remain locked together even in the non-linear phase.
Such structures have been found in non-linear simulations, where they produce large transport. We also note that they are indicated in recent beam emission spectroscopy measurements of fluctuations on TFTR.

4. COMPARISONS OF NON-LINEAR SIMULATIONS WITH EXPERIMENTAL TRANSPORT

Two non-linear gyrokinetic particle codes based on the $\delta f$ algorithm [3] (one based on particles, the other on spectral and grid methods) which were previously developed to simulate ITGD modes in slab geometry have been modified to include full toroidal effects, including trapped particles and toroidal drifts. A co-ordinate system has been found which combines features of sheared slab geometry and ballooning co-ordinates, which allows non-linear slab codes to be easily modified to accurately describe small scale instabilities in a narrow annular region of a torus. These codes evolve the ion dynamics fully kinetically and can simulate non-linear saturation and transport for actual experimental parameters. They can run with up to 40–120 modes for several hours CPU time on a CRAY 2.

However, they do not yet include the effects of impurities, beams, trapped electrons and electromagnetic effects. Thus, they are not appropriate to detailed comparisons with experiments. When run for experimental parameters, they are far from marginal stability, as expected from the linear analysis above. At saturation, the turbulent $\chi$ found by these codes is within a factor of 2–3 of experimental values. However, they give a $\chi$ which decreases with minor radius, which is the opposite of the experimental observations.

The codes are now being used to examine the dynamics of instabilities close to marginal stability. For such cases, they find evidence for coherent fluctuations of the type described above at non-linear saturation. For many cases, the heat flux has an extremely rapid increase as marginal stability is crossed. Some simulations with the particle code have found transport levels in the experimental range when the temperature gradient is within 10–20% of marginal stability.

Non-linear simulations from a full torus particle code (TPC) have also found coherent structures with long radial correlation lengths. Simulations have been performed, keeping one to three Fourier modes, to reduce noise to very small values. The unstable eigenmodes are observed to grow exponentially and then saturate at the same time that the temperature profile relaxes to a smaller value. To check if this profile is linearly stable, the simulation was stopped and recontinued with $\phi = 0$ to let the distribution become uniform over a flux surface. When restarted with $\phi$ included, no instabilities grew, showing that saturation was by quasi-linear flattening. The simulations must be run for values of $\rho/a$ about an order of magnitude larger than experimental values. However, when the relaxation time is scaled to TFTR using gyroBohm scaling, it is roughly 5–10 ms, which is an order of magnitude shorter than typical L-mode confinement times. Scaling by Bohm gives even shorter times.
These scalings imply that TFTR profiles would quickly relax to marginal stability and remain close to marginal. Most ITGD microinstability based theories of transport produce a \( \chi \) which decreases with radius, because the gyroBohm scaling gives a \( T^{3/2} \) factor in \( \chi \). However, if the instability forces the profile close to marginal stability it will almost always automatically produce a \( \chi \) which increases with the minor radius for a steady state driven tokamak. The small deviation from \( m \) adjusts to drive the transport level to the value needed to maintain a nearly stable profile. In steady state, the heat flux balances the integrated power sources \( S, \int r \, S_r \, dr = r_X \, dT/dr \); since the source term usually increases with radius and \( dT/dr \) decreases with the minor radius for most marginally stable profiles, the small deviation from marginal stability will increase with minor radius to produce a \( \chi \) which increases with radius. The full torus simulations above show a \( \chi \) which increases with radius. This is consistent with the linear code results, which show that the modes are further from marginal stability as the radius increases.

5. SHEAR FLOW STABILIZATION IN THE EDGE

Data from several current tokamak experiments indicate that the equilibrium poloidal velocity field can become strongly sheared accompanying the transition from L-mode to H-mode confinement and that fluctuation levels are reduced. Linear theory suggests that velocity shear can stabilize ion temperature gradient (ITG) modes when the frequency shift experienced by the mode due to the radial dependence of the Doppler shift is comparable to the growth rate [4]. To confirm linear theory and to determine saturation amplitudes and energy transport levels, two and three dimensional gyrokinetic simulations of ITG modes were performed. The simulations were done with and without magnetic shear in a slab configuration using the partially linearized (\( \delta f \)) algorithm to reduce statistical noise. The simulations confirmed theoretical analyses [1] of the effects of applied \( E \times B \) velocity fields with linear and parabolic spatial profiles in the following respects. For \( V_0 L_s / c_s \geq 3 \), there was significant stabilization of the ITG modes for \( L_n / L_T = \infty \) and \( L_T / L_s = 0.033 \), where \( L_n \) and \( L_T \) are the density and temperature gradient lengths; and the stabilization was independent of the sign of \( V_0 \). Here \( V_0 \) is the spatial derivative of the applied \( E \times B \) velocity, \( L_s \) is the magnetic shear length and \( c_s \) is the ion sound speed. The velocity shear had a destabilizing effect for \( V_0 L_s / c_s \sim 1 \). The second spatial derivative \( V_0'' \) had a minuscule stabilizing effect if it was negative and a destabilizing effect if it was positive for \( V_0'' \rho_s L_s c_s < 2 \). In both gyrokinetic and gyrofluid simulations [5], the stabilizing effect of large \( V_0 \) on ITG modes was not defeated by non-linearly induced, self-consistent ion flows. The saturated states were turbulent, and the ion energy transport levels at saturation followed \( \gamma / k^2 \) (Fig. 2). The gyrokinetic simulations agreed qualitatively with gyrofluid simulations [5] and exhibited similar peak thermal transport levels at saturation. These transport levels are lower than those typically reported in the laboratory experiments; much of this
A discrepancy may be attributable to the omission of toroidal effects in these simulations. Inclusion of toroidal drive terms in our simulations enhanced the transport levels by about an order of magnitude, bringing them into closer agreement with experimental observations.

6. PRINCIPAL COMPONENT ANALYSIS

We have performed principal component analysis (PCA) on data sets from two dimensional gyrofluid simulations containing finite Larmor radius terms. In spite of the turbulence evident in the data, we find that 20 principal components capture more than 90% of the variance in the data. PCA can therefore serve as a useful diagnostic procedure to extract dominant time series and spatial patterns from turbulent data sets. We have obtained encouraging preliminary results in simulations that use those spatial patterns as basis functions. The number of time evolution equations in such a simulation can be two orders of magnitude smaller than in a conventional simulation.

7. ELECTROMAGNETIC SAWTOOTH SIMULATIONS

We have implemented the QIP (quiet implicit PIC) plasma simulation method [6] in a three dimensional cylindrical geometry, representing a straight tokamak. To date, we have carried out a number of two-fluid calculations of the internal
m = 1 sawtooth instability in a straight tokamak. These calculations include the electron physics relevant to a high temperature discharge, in which the mode is weakly affected by collisions. It has been argued that the sawtooth mode would grow rapidly to observable amplitude when ideal MHD unstable, and that the lack of observable m = 1 activity under such conditions is one of the many data presenting difficulties with the present theory of the sawtooth. Calculations have been carried out for parameters representative of a large tokamak and have been followed far into the non-linear regime. For one such case the simulation parameters represented a straight tokamak with a = 1 m, R = 3 m, n = 1 x 10^{20} m^{-3}, B = 5 T, zero pressure and a q on axis of 0.9. Pure deuterium plasma with real electron mass was used. For these parameters, the linear growth time is about 25 μs, and the theoretical saturation level [7] is B_{r (r=0)} = 8 x 10^{-3} T. In contrast to the easily observable shift associated with this saturation level, the non-linear calculation indicates quite a different situation. After a short linear growth phase, the growth of the shift (equivalently B_{r}) slows down markedly. There then appears a power-like approach of the amplitude toward the saturation level, perhaps indicating some sort of similarity solution. Such behaviour has been discussed in related magnetic reconnection problems, where it is observed that the presence of an Alfvén resonance at the mode rational surface leads to a power law growth phase [8]. For our parameters, the growth slows down to an insignificant rate at a B_{r} of around 2 x 10^{-4} T. Subsequent growth to 20% of the saturation level requires nearly 5 ms. Thus, it may be that an ideally unstable profile does not lead to measurable m = 1 activity on the ideal time-scale, and such effective saturation at much reduced amplitudes could explain the lack of observed activity.

REFERENCES

B. COPPI: The results you presented on the $m = 1$ mode are quite surprising, as they refer to the case of ideal MHD instability. How do you compare them with those obtained by Sydora, Naitou and Dawson, who consider the weaker modes found under ideal MHD marginal stability?

M. KOTSCHENREUTHER: The results you refer to were obtained by the Los Alamos group, and I am afraid that I am not familiar enough with them to offer a better explanation.

R.J. GOLDSTON: Perhaps you could comment on the TFTR pellet injection experiments, which seemed to contradict the marginal stability hypothesis?

M. KOTSCHENREUTHER: The theoretical results are very sensitive to the impurity profile used. Some of the data we have received from TFTR indicate that the impurity profile decreases immediately after the deuterium pellet has been injected. I do not think this is reasonable, but if it is true, then the mode is indeed well above marginal stability. If, instead, we assume, as seems most reasonable, that the impurity profile is unchanged by the pellet, then the plasma is in fact close to marginal stability. We have to work with the TFTR group to obtain a more precise idea of impurity behaviour.
PHYSICS AND MODELLING OF SCRAPE-OFF LAYER TRANSPORT

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Abstract

PHYSICS AND MODELLING OF SCRAPE-OFF LAYER TRANSPORT.
Studies of three schemes for reducing the peak heat flux on divertor plates — impurity injection ('radiative divertor'), neutral gas injection ('gas target divertor'), and divertor biasing — are presented. A theoretical analysis of a likely source of turbulent transport in the scrape-off layer and incorporation of the resultant transport coefficients into self-consistent models are reported.

The high heat flux incident on divertor plates is one of the crucial problems for ITER and other large, long pulse tokamaks. It is important to have models that describe the behavior of the scrape-off layer (SOL) region of present-day diverted tokamaks such as DIII-D, both for scaling to larger devices and for obtaining quantitative evaluation of schemes that manipulate the SOL to reduce the peak heat flux.

Equilibrium Studies

We model the SOL equilibrium with 1-D and 2-D transport codes using fully implicit algorithms [1] that solve for particle continuity, parallel momentum, electron energy, ion energy, electrostatic potential, and neutral gas density.

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Parallel currents and cross-field drifts are included so that divertor biasing can be investigated. Such drifts can also be important in preventing impurity accumulation. Transport parallel to the magnetic field line is assumed classical while perpendicular transport is represented by anomalous diffusion coefficients.

For the radiative divertor configuration, we study the ability to maintain low $Z_{\text{eff}}$ near the core plasma while reducing the divertor plate electron temperature, $T_{\text{ed}}$, via radiation. Deuterium (D) injected upstream of the plate results in increased ion flow and friction force on the impurities toward the plate, which opposes the thermal force from the axial temperature gradients near the plate. We consider parameters for the TPX/SSAT U.S. design study. Two cases are considered here: distributed argon injection between the x-point and divertor plate, and localized injection near the divertor plate. Using a multispecies, non-coronal one-dimensional fluid code (NEWT-1D), we found $T_{\text{ed}}$ was reduced from $\sim 100$ eV to below 5 eV, and the peak heat flux was reduced by a factor of 12, while radiating about 80% of the incoming SOL power. The plate erosion from the argon is estimated not to be serious. For entrainment, distributed injection resulted in argon profiles which gave a high midplane $Z_{\text{eff}} > 3.5$ over a range of upstream D sources. Much better entrainment was achieved for localized injection; using an upstream source which gave a midplane separatrix density of $3 \times 10^{19}$ m$^{-3}$, an acceptable midplane $Z_{\text{eff}} < 1.6$ was obtained, although the density is somewhat high.

The $E \times B$ drift can also influence impurity entrainment. Because the power to the outer divertor plate is typically larger than that to the inner plate, the ion temperature gradient will be larger near the inner plate. Should the ion thermal force cause impurities to flow out of the low heat flux plate, the $E \times B$ ion flow is predicted to sweep them all the way to the high-heat-flux divertor, preventing an impurity accumulation outside of the divertor region. We have also used NEWT-1D to simulate DIII-D with and without nitrogen puffing. Without puffing, less than 1 MW of power was radiated by the carbon impurity from the plate out of a total of $\sim 3$ MW from the core. With puffing, the power radiated rose to 1-2 MW, consistent with measurements. Nitrogen puffing is also modeled using the output of LEDGE to give plasma profiles and neutral density for use with the MIST impurity radiation code [2]. Although the calculation is not yet self-consistent, it gives an estimate of about 5% nitrogen to account for the 1-3 MW of radiated power observed.

For the gas-target divertor, we model a region near the divertor plate with the 2-D LEDGE code, assuming close-fitting side walls through which one can simultaneously inject gas and collect plasma and gas. We use a two-species gas fluid description for the Franck-Condon and charge-exchange neutrals. Results are shown in Fig. 1 as $T_{\text{ed}}$ at the plate for various gas currents. The side walls are 25 cm long, and there are three cases for the divertor plate recycling coefficient $R_p$. The side wall recycling coefficient is set to 0.5 to simulate pumping of gas and plasma. The upstream plasma parameters are adjusted to maintain either 25 MW or, for one case, 100 MW flowing into the region. A grey scale plot of the electron temperature in the injection region is shown in Fig. 2 for the 100 kA, 25 MW case with $R_p = 0.9$. The light regions indicate high $T_e$ with a peak of 45 eV at the left entrance and a divertor plate maximum of 5 eV; in the darkest regions, $T_e \approx 1$ eV. The results in Fig. 1 can be understood from current balance where $I_{\text{gas}} = I_{\text{plate}}(1 - R_p) + I_{\text{pump}}$ and power balance where $P_{\text{in}} = I_{\text{plate}}2T_{\text{ed}} + P_{\text{rad}} + P_{\text{wall}}$.
\( \delta \approx 6.5 \) is the energy transmission factor. We find that the power to the wall \( P_{\text{wall}} \) is smaller than the radiation or plate powers, which yields \( T_{\text{ed}} \approx (P_{\text{in}} - P_{\text{rad}})(1 - R_p + \epsilon)/(I_{\text{gas}} \delta) \). We have assumed \( I_{\text{pump}} = \epsilon I_{\text{gas}} \) where \( \epsilon \approx 0.03 \) models the results. At the lowest \( T_{\text{ed}} \), \( P_{\text{rad}} \) is about 50% of \( P_{\text{in}} \). The main effect of injecting the gas is to raise the plasma density at the plate which then carries more power through the sheath to the plate; i.e., the high-recycling divertor action. The gas flow upstream remains moderate (\( \sim 800 \text{ A} \)) even for the short 25 cm length and is reduced by increasing the length (\( \sim 170 \text{ A} \) for an 89 cm length). However, the plasma flow at the inlet boundary opposite the divertor plate reverses direction as \( I_{\text{gas}} \) is increased for each \( R_p \) and reaches several kAmps for the cases shown; such currents would fuel the core plasma excessively unless they can be controlled by a pumping scheme.
We obtained results for LEDGE simulations of divertor plate biasing with classical cross-field drift effects. The electrostatic potential is obtained from the $\nabla \cdot \mathbf{J} = 0$ current continuity equation with the diamagnetic and $\mathbf{E} \times \mathbf{B}$ drifts, as well as parallel currents, included self-consistently. Parameters are chosen for single-null DIII-D discharges in the Ohmic regime ($D = \chi_i = \chi_e = 1 \text{ m}^2/\text{s}$, and plasma core drift boundary conditions $n = 2.0 \times 10^{19} \text{ m}^{-3}$ and $T_e = T_i = 100 \text{ eV}$). The ion $\mathbf{V}\mathbf{B}$ drift is towards the x-point. For no bias, the outer/inner plate power ratio is $0.63/0.07$ (powers in MW), and the poloidal current into the outer plate is $I_{pol} = -1.6 \text{ kA}$. For -27.5 V bias on the outer plate, the power ratio is $0.39/0.38$, and $I_{pol} = 2.8 \text{ kA}$. The dominant mechanism changing the power distribution is the convection of electron energy by the increased electron parallel flow toward the inner plate for negative bias. This convected power can be written as $\Delta P \approx -(5/2 + 0.71) \times (T_e I_{pol})/e$ which accounts for the shift in power obtained from LEDGE. Generally, the values of $\Delta P$ and $I_{pol}$ calculated for a given voltage are much larger than observed experimentally [3], which suggests an anomalous resistivity.

The cross-field drifts shift the mixture of ion heat flux to electron heat flux at a given plate because the ion and electron diamagnetic drifts have opposite sign, but the sum is nearly unchanged for $T_e \approx T_i$. The $\mathbf{E} \times \mathbf{B}$ drift gives an incremental shift in the total power from the inner plate to the outer plate for the ion $\mathbf{V}\mathbf{B}$ drift toward the x-point [4] resulting in an enhancement of the power asymmetry. The plasma density at the inner plate is increased by a factor of 2 and is shifted radially outward by the radial $\mathbf{E} \times \mathbf{B}$ drift. On the outer plate, the opposite shift in density occurs. These shifts can have important implications for pumping neutrals.

We have modelled several single-null DIII-D discharges between ELMs in the H-mode regime. The shots chosen represent data with the plasma current and the neutral beam heating power varied. The radial electron temperature and density profiles at the outer midplane are obtained from mapping Thomson scattering data along flux surfaces. The radial density diffusion coefficient, $D$, and the electron thermal diffusivity, $\chi_e$, were adjusted in LEDGE to fit the measured midplane profiles of electron density and temperature. Previously we assumed $\chi_i = \chi_e$. Recent ion temperature data in the boundary plasma indicates that this assumption is reasonable. For neutral beam powers between 5 and 10 MW, and plasma currents between 1 and 1.75 MA, we find transport coefficients in the range $0.05 \text{ m}^2/\text{s} < D < 0.2 \text{ m}^2/\text{s}$, and $0.2 \text{ m}^2/\text{s} < \chi_e < 0.5 \text{ m}^2/\text{s}$. The calculated width of the power profile on the divertor plate is then also consistent with that measured. However, the calculated peak power is higher than that measured by a factor of two to five. Experimentally, we find the power on the divertor plates is significantly less than the difference between the input power and the radiated power; hence we can not account for all the power. The relatively low $\chi$'s are associated with the relatively narrow $T_e$ profile width $LT_e \sim 0.5 \text{ cm}$ for these shots; however, particularly in this case a diffusive model is questionable as indicated below.

SOL Turbulence

The predictive ability of codes such as LEDGE has been limited by the use of ad-hoc radial transport coefficients. Knowledge of the radial transport is crucial as it directly impacts the SOL width and hence its power-handling
capability. We address this deficiency by exploring a suggestion made in Ref. [5] that an instability driven by end loss, electron temperature gradients and ion polarization drift could be responsible for SOL turbulence.

In the relevant electromagnetic regime, the origin of the instability is in fact in the axial variation of the equilibrium electric field. If we follow the field-line displacement $\xi$ resulting from a (Eulerian) potential perturbation $\phi$, then to leading order $n$, $T_e$ and $T_i$ are unperturbed. Hence the perturbation in the Debye sheath charges up the field line at the rate $-2(\partial J_\parallel/\partial \phi)\phi_L$, where $J_\parallel$ is the equilibrium end-loss current and $\phi_L$ is the Lagrangian potential perturbation. This is balanced by charging from the polarization current, at the rate $(\kappa \omega/c)\int ds n\Omega\phi$, where $\Omega$ is the wave frequency $\omega$ Doppler-shifted by the $E \times B$ frequency $\omega_E$. The balance yields instability because $\phi_L$ and $\phi$ differ; their difference is proportional to $\omega$ of the plasma relative to the conducting end wall (which in turn is proportional to $\nabla T_e$).

We analyze the mode using (electromagnetic) electron and multi-species ion fluid equations. The mode is electromagnetic (electrostatic) for $\kappa_\perp^2/\kappa_\parallel^2 > 1$. Equilibrium quantities are assumed to vary along $B$ in a "pre-sheath" region of length $L$ near the ends of the field line. The Bohm sheath boundary condition can be written as $\partial \xi/\partial s|_d = \pm i\omega\xi/v_w$ where $\xi = \xi(x, x_e)$ ($x = $ radial direction), $v_w^{-1} \equiv c_s n_d/v_A^2\kappa_\parallel^2\rho_{sd} n_0$, $v_A$ is the Alfvén speed, $c_s = 1 + \gamma_{si}$ times the sound speed, $\gamma_{si}$ is the coefficient for secondary electron emission, and the subscript $d$ denotes evaluation at the sheath, $s = s_d = \pm L/2$ where $L$ is the field line length, while the subscript 0 denotes a mid-field-line value. For $k_\parallel L \ll 1$ and $\omega_\perp^2/\kappa_\parallel^2 > 1$, this boundary condition and the standard ideal-MHD eigenmode equation imply the following dispersion relation: $f_\parallel o' + \omega = (\omega^2 - \omega_E^2 - \omega_\perp^2)/\nu_A^2$, $\omega_\perp^2$ is the ion diamagnetic frequency, $f_\parallel = k_\parallel L/2$, $h = \tan \theta$, $\omega_1 = \langle \Delta(2\omega_E + \omega_\perp^2) \rangle$, $\omega_2 = \langle \Delta(\omega_E(\omega_E + \omega_\perp^2)) \rangle$, $\Delta \alpha(s) \equiv \alpha_0 - \alpha(s)$, and $\langle \rangle$ denotes an average over the pre-sheath region. The $k_\parallel L \rightarrow 0$ limit was analyzed in Ref. 6; maximizing the growth rate over $k_\perp$ yields [6] a frequency $\omega_m = (c_s/L T_e)\Lambda^{1/3}\Omega$ where $\Lambda = d\Phi_0/dT_{ed} \sim 4$, $\Phi$ is the equilibrium electrostatic potential, $\nu = 2Zn_d c_s/L T_e/n_0 \Lambda L c_{se}$, $c_{se}$ is the electron sound speed, and $\Omega$ is a complex function of $\delta \equiv \omega_\perp^2/\omega_E$ and $K_T \equiv (1+\gamma_{si})Z(\beta n_d/2n_0)^{1/2}v^{-2/3}$ with $\beta$ = plasma pressure/magnetic pressure at the sheath. The maximum occurs at wavenumber $k_\parallel = k_\parallel(\omega_\perp^2/\omega_E)$ and $k_z = 0$; for finite $k_z/k_\parallel$, the maximum growth rate is reduced by $(k_z/k_\parallel)^{1/3}$. The parameter $K_T$ governs the importance of finite-$k_\parallel L$ effects; in particular, for $K_T \gg 3$, there are appreciable growth rates for multiple branches of tan$^{-1}$. Such is the case for reactor-class machines (e.g. ITER) but not for machines the size of DIII-D. Finite $K_T$ also destabilizes antisymmetric modes ($h = -\cot \theta$). For $K_T \rightarrow 0$ (flute limit) and $\delta \rightarrow 0$, $\Omega \approx 0.55 + 0.38i$ and $k \approx 1.83$. We estimate thermal diffusivity $\chi_m$ from the mixing-length estimate $\chi_m = \text{Im} \omega_\perp k_m^2$. Note that $\chi_m$ is sensitive to $T_e$ at the sheath and is thus increased by making the SOL less collisional.

Important modifications can emerge from the finiteness of $k_\parallel L$. The right-hand side of the above dispersion relation includes the additional drive [7] resulting from axial shear in the equilibrium drifts. For small but finite $k_\parallel L$ it is straightforward to demonstrate that the growth rate is increased. Axial shear can dominate the overall growth rate for some modes at large $K_T$ (long, hot
SOL's). It also dominates for some versions of the gas-target divertor. In particular, it survives if plasma contact with the end wall is broken, but is sub-dominant if contact is retained but $T_{ed}/T_0 \to 0$ and either $n_d/n_0 \sim \text{const}$ (pressure balanced by neutrals) or plasma pressure balance is approximately maintained. An instability persists even for a field line with a single sheath (absorbing boundary conditions at the other end), in which case $\omega(k_z = 0)$ is purely real. The above dispersion relation is valid for this case if we define $h = i$. We then find that increasing either $n$ or $T_i/T_e$ is stabilizing, which may be related to observations that SOL's are narrower in H mode than L mode. (Increasing $T_i/T_e$ is also stabilizing for at least the flute limit of the end-wall-driven mode when the energy end loss perturbation is included.)

The radial structure of the mode can be analyzed by restoring $k_x \to -i\partial/\partial x$ in the MHD eigenmode equation. In particular, we can examine the penetration of the SOL mode into the closed-flux-surface "edge" region. If we impose periodic boundary conditions in the edge region but neglect shear as well as radial variations of $\omega_E$, $c_s$ and $\omega_i^*$, it follows that a mode with wavenumber $k_y$ in the SOL decays exponentially with a skin depth $k_y^{-1}$ in the edge. The skin depth is somewhat modified in the electrostatic regime and is reduced (with an altered functional form) if shear is retained, but the qualitative conclusion is the same.

We have also examined the role of an axially constant equilibrium parallel current $J_{\parallel 0}$, such as could be produced by biasing divertor plates. Parallel current corrections enter in two places: through a first derivative term in the eigenmode equation, and through an additional term proportional to $\xi \cdot \nabla J_{\parallel}$ in the boundary condition. In the flute MHD limit, the former correction is destabilizing, the latter stabilizing, and the two terms cancel in the dispersion relation. An effect persists at finite $k_yL$, but for $K_T \sim 1$ (as in DIII-D) and $J \equiv J_{\parallel}/n_{ec} \lesssim 1$, it is very small for wavelengths which fit in the SOL (stabilizing at the peak growth rate and destabilizing at lower $k_y$). The largest effect is then through modification of equilibrium parameters; e.g., for fixed temperature and density profiles and fixed pre-sheath potential drop, introduction of a bias voltage $V$ changes $\Lambda$ by $\Delta \Lambda = \ln w(1 + w)^{-1} + \ln[(w + 1)/2w]$, where $w = \exp(eV/T_e)$, which moderately changes $\text{Im}(\omega)$.

We have performed nonlinear analyses at several levels, including mixing-length estimates, scaling analyses of the nonlinear equations, one-point renormalization calculations, and 2-D fluid simulations. These approaches all yield the same scaling in the limit where the basic equation set is justified [$k_x L_T e > 1$].

We have developed a nonlinear fluid simulation model for the instability, which includes closed ("edge") and open (SOL) field-line regions and nonlinearities from the $E \times B$ motion and the sheath boundary conditions. First results for a 2-D model with radially constant $\omega_E$, $\omega_i^*$, $n$, $T_e$ and $T_i$ show saturated turbulence, clear evidence of the expected mode penetration into the edge, indications of an inverse cascade of wave energy (toward long wavelengths), and saturated amplitudes somewhat above and flux levels somewhat below those of the simplest mixing-length estimates.

We combine the mixing-length diffusivity with perpendicular and parallel transport equations to obtain a self-consistent picture of the SOL. We have done this for a point model (calculating radially averaged quantities at mid-field line and at the sheath, and radial scale lengths), a 1-D (radial) model, and the 2-D LEDGE code. The point model yields the scalings $L_T e \propto (AL_p)^{1/3} \lambda^{-1/4} \times \ldots$
(P/nR)^{1/9} B_p^{-1/2} \text{ and } T_{e0} \propto A^{1/9} B^{2/3} B_p^{-1/3} \lambda^{1/2} L_p^{-2/9} (P/nR)^{5/9}, \text{ where } L_p \text{ is the poloidally projected field line length and } P \text{ is the power reaching the SOL. In these solutions } \lambda \equiv T_{e0}/T_{ed} \text{ is determined from the heat conduction equation. Profiles from LEDGE for DIII-D parameters are shown in Fig. 3. Note that } T_i \text{ is broader than } T_e; \text{ this results from the lower parallel thermal conductivity and lower energy end loss per particle for ions than electrons, and their partial thermal decoupling. The disparity would be increased in a hotter SOL (better decoupling). The disparity is also seen in DIII-D experimental results. The calculated broad density profile is expected for the high recycling coefficient used (0.98).}

Mixing-length estimates and simulation results indicate that, for typical SOL parameters, $\xi \sim L_T e$. Furthermore, the 2-D nonlinear equations imply, and the simulations suggest, an inverse cascade of wave energy. Hence the validity of a diffusive radial-transport description is questionable (no separation of scales). An alternative entails minimizing equilibrium enstrophy for fixed energy. For a simple SOL model of a limited plasma with poloidal and toroidal symmetry (except for the limiter), the minimization yields $\Phi \propto T_e \propto \exp[-(r-a)/L_T e]$, where $L_T e$ entered as a Lagrange multiplier and $r = a$ is the limiter tip. Thus thermal transport is solved up to two constants, $\Phi(a)$ and $L_T e$. Energy balance provides one relationship between them, much as in the conventional transport picture. A second relationship can be obtained from dimensional analysis, mixing-length arguments, a similarity solution, or renormalization group techniques.

We have compared our predictions with equilibrium and fluctuation data from DIII-D and found a number of points of contact. The observed relative and absolute fluctuation levels of $\phi$, $T_e$, and $n$ are at least roughly in accord with the theory, and the spectral shape is qualitatively consistent with an inverse cascade from the peak-growth-rate wavenumber. The self-consistent transport models predict approximately the observed SOL parameters (the point model gives $L_T e \approx 0.8 \text{ cm}, T_{e0} \approx 84 \text{ eV}, T_{ed} \approx 13 \text{ eV}$ for a nominal parameter set).

Extrapolation to ITER is complicated by its relatively large value of $K_T$, implying contribution of multiple $k||$ harmonics and small Im$(\Omega)/Re(\Omega) (~0.1)$. In the absence of a detailed nonlinear theory for this case, we have constructed a range of nonlinear estimates, implying, for ITER Conceptual Design parameters,
$L_{Te} \approx 1 - 2$ cm, which is broader than that inferred on the basis of the commonly used constant-$\chi$ extrapolation. The more favorable result is attributable primarily to the relatively high equilibrium electric fields one would expect in the ITER SOL.

REFERENCES


DISCUSSION

D. HILDEBRANDT: You said that the motion of the impurity ions is significantly influenced by $\mathbf{E} \times \mathbf{B}$ drift in the vicinity of the target plate. What is the strength of the electric field in this case and by what is it caused?

R.H. COHEN: The strength of the radial electric field is $\Delta T_{e}/eL_{Te}$, where $L_{Te}$ is the electron temperature gradient radial scale-length and $\Delta$ $\approx$ 3-4. This is set by the requirement of nearly ambipolar end loss and the Bohm sheath condition. The poloidal electric field is set by parallel Ohm’s law plus toroidal symmetry. The $\mathbf{E} \times \mathbf{B}$ flow can compete with the parallel flow, particularly for heavy impurities, because only the poloidal projection of $v_{t}$ is relevant.
3-D MHD STUDY OF HELIAS AND HELIOTRON

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Abstract

3-D MHD STUDY OF HELIAS AND HELIOTRON.

The properties of magnetic islands induced by the finite pressure effect are numerically analysed for three-dimensional magnetohydrostatic equilibria of the Helias and $\ell = 2$ heliotron/torsatron types. For Helias, it is found that an island chain is generated on the $5/6$ rational surface, when such a surface appears in the plasma region of a finite beta equilibrium. The island chain is not, however, so dangerous as to destroy the plasma confinement even if it appears in a vanishingly small shear region. Moreover, it is definitely confirmed that the finite pressure effect sometimes exhibits a surprisingly good property, namely, that the vacuum islands are removed as beta increases, which can be called 'self-healing' of islands. This feature can be explained by the numerically discovered fact that the phases of islands induced by the finite pressure effect are always locked, i.e. do not change — regardless of beta. The way islands appear at finite beta of heliotron/torsatron is significantly different from that of Helias. However, the self-healing of islands is found to occur also for the heliotron configuration. This phenomenon can partly explain why the 'fragility' of surfaces decreases when they are shifted inward by controlling the external vertical field $B_v$, though the configuration develops a magnetic hill. Furthermore, the diversion properties of the magnetic field outside the last closed magnetic surface for finite beta equilibria are analysed, and it is found that divertor concepts which have been developed from the diversion properties of the corresponding vacuum fields can be maintained for finite beta equilibria of both configurations.

1. INTRODUCTION

Whether or not a three-dimensional (3-D) magnetohydrostatic finite beta equilibrium can keep clearly nested magnetic surfaces is a long pending question whose physics has not fully been understood, as yet. Properties of magnetic islands induced by a finite pressure effect in typical helical systems, such as Helias proposed for W7-X and $\ell = 2$ heliotron/torsatron proposed for LHD, are quantitatively studied by a numerical method, the 3-D equilibrium code HINT [1, 2], which does not a priori demand the existence of regularly nested magnetic surfaces. The equilibrium
calculation is carried out for the half-pitch period of the configuration, with the stellarator symmetry being kept.

In the process of equilibrium calculations, we found a remarkable property of the magnetic islands induced by the finite pressure effect: the islands, in some cases, show the property of 'self-healing', i.e. the islands tend to shrink as beta increases.

2. FORMATION OF MAGNETIC ISLANDS AT FINITE BETA IN HELIAS

The magnetic surfaces of the W7-X configuration are designed in such a way that the rational surfaces, \( \ell = 1, \frac{5}{6} \), can be avoided. It is, however, of interest to study the finite beta behaviour of the \( n = 5/m = 6 \) island chain to assess whether this would be dangerous or not. For this study a vacuum field situation was selected in which this island chain occurs at finite beta. The rotational transform at the magnetic axis, \( \psi_0 \), for the vacuum field is set slightly greater than \( \frac{5}{6} \). One characteristic in the finite beta behaviour of the \( \ell \) profile for Helias is that \( \psi_0 \) decreases as beta increases; in the case of the \( \ell = 2 \) heliotron/torsatron configuration, \( \psi_0 \) usually increases as a function of beta. Because of this behaviour, the \( \frac{5}{6} \) rational surface can get into the plasma region; first, it appears near the magnetic axis and then moves outward as beta increases. Thus, the \( \frac{5}{6} \) island chain can appear when the \( \frac{5}{6} \) surface resides at about half the plasma radius, which is the position of vanishingly small shear. In this case the plasma beta at the magnetic axis is \( \beta_0 = 7.2\% \).

As is shown in Fig. 1, our results indicate that even this situation is not so dangerous as to destroy the plasma confinement. Nearly the entire plasma region is covered by clearly nested surfaces. This is, of course, a consequence of the optimization of the configuration.[3]. Corresponding to the formation of islands in the magnetic field structure, as is shown in Fig. 1(a), the island structure is formed also in the pressure

Helias

![Image of magnetic field and pressure contours](image_url)

**FIG. 1.** Magnetic surfaces (a) and pressure contours (b) of a finite beta Helias equilibrium selected for code validation. We note the coincidence of magnetic and pressure islands, which is demonstrated here in computational stellarator equilibria for the first time.
profile, as is shown in Fig. 1(b), where the pressure is almost flattened inside the $\frac{5}{6}$ island. We note that in the HINT code the pressure is automatically computed inside the islands as a result of the incorporated calculation step which makes the pressure uniform along each field line. The coincidence of magnetic and pressure islands, which is for the first time demonstrated explicitly here in computational stellarator equilibria, proves the validity of the code. Interestingly, local instabilities (Mercier and resistive interchange) are stabilized around the rational surface because of this flattening, the size of the islands being just large enough to bring about this stabilization.

When beta is further increased, the $\frac{5}{6}$ island chain approaches the boundary region. Although beta is increased, the size of the island does not change so much, presumably because of the increased shear in the boundary region.

3. 'SELF-HEALING' OF MAGNETIC ISLANDS IN HELIAS

So far we have analysed finite beta effects on equilibria from the point of view of formation of islands. Computational experience indicates that the phase induced by finite beta is independent of the value of beta. Taking advantage of this property, we can have situations in which the finite beta effect tends rather to suppress islands which have existed in a vacuum field if the phase of the vacuum field island is opposite to that of the beta induced ones. This process is confirmed in equilibrium calculations. A vacuum field is shown in Fig. 2(a), where a chain of magnetic islands is formed externally by controlling external currents, on the $i = \frac{5}{6}$ rational surface existing in the region of closed surfaces. Shown in Fig. 2(b) is the corresponding finite beta equilibrium with $\beta_0 = 9\%$. It is clearly observed that the $\frac{5}{6}$ island chain, which existed in the vacuum field, disappears almost completely, and a nice magnetic surface recovers by the effect of 'self-healing', though the $i = \frac{5}{6}$ surface still exists in the plasma region. Thus a high beta equilibrium keeping beautiful surfaces can be realized by making use of this feature.

When beta is further increased, as is shown in Fig. 2(c) for $\beta_0 = 12\%$, the position of the $i = \frac{5}{6}$ surface moves further towards the plasma boundary, and the $\frac{5}{6}$ island chain appears again. Note, however, that the phase of the re-appearing $\frac{5}{6}$ island chain is opposite to that of the vacuum one.

As islands may exhibit the property of self-healing described here, the physical equilibrium mechanism of formation of islands at rational surfaces in finite pressure equilibria appears to be such that they are governed by resonant magnetic fields generated by the global effects of the plasma current density, mainly of the Pfirsch–Schlüter current, integrated over the whole plasma volume. We note that part of the reduction of the island size as beta increases, shown in Fig. 2(b), may be attributed to the increase in the shear in the boundary region. However, the shear effect alone cannot explain the whole process shown here since it does not explain the inversion in the phase of islands shown in Fig. 2(c). It appears natural to consider that the
global plasma current effect, which determines the magnitude and the phase of the resonant field, governs the island behaviour in a dominant way.

4. 'SELF-HEALING' OF MAGNETIC ISLANDS IN \( \ell = 2 \) HELIOTRON/TORSATRON

We observe the nature of self-healing of islands not only in Helias but also in the \( \ell = 2 \) heliotron/torsatron configuration, which is characterized by medium to high shear. As is shown in Fig. 3(a) for typical vacuum surfaces of an \( M = 10 \) heliotron, the boundary region of the vacuum field is ergodic because of the loss of symmetry; in this case, two island chains, \( n = 10/m = 8 \) and \( n = 10/m = 7 \), are clearly visible near the outermost closed surface. It is a general property of vacuum fields of a wide range of \( \ell = 2 \) heliotron/torsatron configurations that the appearing islands are in phase at the outside of the torus when they are induced on several rational surfaces simultaneously [4]; here, both island chains have the x-point outside the \( \phi = \pi/10 \) (horizontally elongated) poloidal cross-section.
Because of the high shear the way the islands appear at finite beta for heliotron/torsatron is significantly different from that for Helias; for heliotron, in general, the boundary region is ergodized by overlapping of multiple island chains with smaller sizes, and the boundary ergodized region gradually expands as beta increases. However, we find that the self-healing of islands can occur also for the heliotron configuration. Shown in Fig. 3(b) are magnetic surfaces for finite beta, $\beta_0 = 4.5\%$. When we observe the fine structure of boundary islands, we see that the $n = 10/m = 8$ island chain shrinks significantly, and the phase of $n = 10/m = 7$ becomes inverted. When the $n = 10/m = 7$ island chain disappears at slightly lower beta, the minor radius of the outermost closed surface expands compared with that of the vacuum field. These types of island behaviour indicate that the finite pressure effect acts to generate resonant fields with a global structure with respect to the phase. In fact, when beta increases further, island chains, all having the O-point outside $\phi = \pi/10$, grow at several rational surfaces simultaneously, and eventually the boundary region is ergodized. In this way, the expansion of the boundary ergodic region is delayed as beta increases, owing to the opposite nature of the island chain phases for vacuum and finite beta fields in this case. This opposite nature
of the phase typically appears when the surfaces are shifted inwards by controlling the external vertical field $B_v$. This can partly explain why the ‘fragility’ of surfaces at finite beta decreases as the surfaces are shifted inwards although the configuration develops a magnetic hill [1].

5. DIVERTOR FIELD STRUCTURE IN FINITE BETA EQUILIBRIA

One property of magnetic fields of helical systems is that divertor structures can be realized without installing any additional coils. We analyse the diversion properties of the magnetic field outside the last closed magnetic surface for finite beta equilibria and investigate whether or not divertor concepts which have been developed from the diversion properties of the corresponding vacuum fields can be maintained for finite beta equilibria of heliotron and Helias, which has not been analysed so far. For the $\ell = 2$ heliotron/torsatron, the structure of magnetic surfaces near the plasma boundary for finite beta equilibria may be considerably different from that of the vacuum field as far as the value of the rotational transform or the extent of the ergodic region are concerned. However, the position of the divertor field remains almost the same, which suggests that the same divertor plate can be used even in equilibria with fairly high beta. It should be noted that the ‘width’ of the divertor structure broadens when beta increases. Therefore, we have to be careful in designing the divertor geometry such as the baffle plate. For Helias, it is found that the finite beta equilibrium shows the same diversion properties as the vacuum field. Since the positions of the helical edges [5] are almost the same, the helical ‘troughs’ defined for the vacuum magnetic field work just as well as in the case of the finite beta equilibrium.

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DISCUSSION

M. KOTSCHENREUTHER: John Cary and I have written a paper on healing of magnetic islands by finite $\beta$ in stellarators. Do you see any indication of local resonant currents contributing to the island healing?

T. HAYASHI: No. We have found that the global effect, which has not been considered in local theory, can play a significant, possibly dominant, role in determining the behaviour of islands induced in 3-D equilibria.
HEAT TRANSPORT BY CYCLOTRON WAVES IN PLASMAS WITH STRONG MAGNETIC FIELD AND HIGHLY REFLECTING WALLS

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Abstract

HEAT TRANSPORT BY CYCLOTRON WAVES IN PLASMAS WITH STRONG MAGNETIC FIELD AND HIGHLY REFLECTING WALLS.

Heat transport by transverse and longitudinal cyclotron waves in inhomogeneous thermonuclear plasmas and its non-local (non-diffusive) character are analysed both analytically and numerically within the framework of a self-consistent kinetic description of plasma electron and plasma wave transfer in a strong magnetic field. Universal formulas for total power losses on wave emission are obtained, which, in particular, generalize Trubnikov's formula for synchrotron losses to the case of arbitrary emission/absorption processes and inhomogeneous non-stationary plasmas. The results suggest a qualitative explanation of the interrelation between non-local and local features of global heat transport in a tokamak.

1. INTRODUCTION

The increasing interest in the advanced thermonuclear fuel D–^3^He suggests a more detailed analysis of heat transport by electron cyclotron waves, because of the dominant role of electron cyclotron radiation (ECR) losses (see, e.g., Ref. [1]) for both the total and the local energy balances of plasmas confined by strong magnetic fields. Another, possibly even more important reason for a thorough investigation of this problem is the fact that heat transport by both transverse (i.e. photons) and longitudinal (i.e. plasmons) electromagnetic waves across a magnetic field is characterized by a non-local correlation of plasma temperatures and, correspondingly, a non-diffusive law of heat propagation in thermonuclear and astrophysical plasmas. Therefore, such a mechanism may be of importance to the problem of global heat transport in a tokamak, especially with a strong magnetic field (including D–^3^He based and ITER-like reactors).

A detailed description of the total ECR losses of homogeneous Maxwellian electron plasmas has for the first time been developed by Trubnikov in Refs [2, 3] (homogeneous slab) and has been modified in Ref. [4] (homogeneous plasma, the scaling laws allowing for inhomogeneous toroidal magnetic field and reflecting walls). Further treatments within the same framework (including the papers by Drummond and Rosenbluth [5]) are fully reflected in a survey [6]. However, the
descriptions of the spatial profile of ECR losses, based on regarding the result for total loss divided by the plasma volume as a local quantity for energy balance, leads, in particular, to incorrect results for the energy balance in peripheral plasma, even as far as the sign is concerned (Table II). A consistent treatment of spatial profiles of the ECR energy balance in inhomogeneous plasmas, which includes a calculation of the local ECR intensity, has been achieved by Tamor [7-9]. These (and all other) calculations assumed a Maxwellian distribution of the plasma electrons.

The present paper analyses the effect of deviations from the Maxwellian distribution, produced by the propagation of intense electromagnetic (EM) waves spontaneously emitted by the plasma, within the framework of a self-consistent treatment of radiative transfer and quasi-linear (QL) electron diffusion (Section 2). A simple description of the combined action of two effects, (a) the 'enlightenment' of the medium due to the QL reduction of absorption and (b) the depletion of the electron velocity distribution 'tail' due to radiation emission [10], is obtained. The results are applied to the ECR transfer in an inhomogeneous plasma in a system with a strong magnetic field and highly reflecting walls (Section 4).

The non-local character of heat transport by EM waves allows us to obtain a universal, analytic description of energy losses on the emission of EM waves by an inhomogeneous plasma (Section 3) by exploiting the concept of non-local transport in the theory of radiative transfer in resonance atomic lines (RTRAL) (see, e.g., the survey [11]).

It should be noted that within the global problem of heat transport and energy losses the problem of radiative transfer is to be treated 'from top to bottom', whereas in another important problem, auxiliary heating and current drive by ECR waves, one may neglect spontaneous emission and reduce the radiative transfer problem to calculating the absorption of an injected ECR wave in a narrow spectral range.

2. EFFECTS OF QL DIFFUSION AND TAIL DEPLETION IN RADIATIVE TRANSFER BY INTENSE WAVES

The space–time evolution of radiation intensity $J(\phi, \vec{r}, t)$, differential with respect to the EM wave (photon or plasmon) parameters $\phi$, $\phi \equiv \{\omega, \vec{n}, \zeta\}$

\[
\begin{align*}
\left( \frac{1}{v_g} \frac{\partial}{\partial t} + \vec{n}_g \cdot \frac{\partial}{\partial \vec{r}} \right) \left( \frac{J(\phi, \vec{r}, t)}{(N_e)^2} \right) = -\kappa \left( \frac{J(\phi, \vec{r}, t)}{(N_e)^2} \right) + \frac{Q}{(N_e)^2}
\end{align*}
\] (2)
where $\kappa(\phi, \vec{r}, t)$ is the absorption coefficient, $Q(\phi, \vec{r}, t)$ the source function, $N$, the ray refractive index, and $v_g$ and $\vec{n}_g$ are the group velocity and its direction. In a dispersive medium the substitution of the dependence $k = k(\omega, \vec{n}, \gamma)$ from the dispersion relation is implied. Equation (2) should be solved with boundary conditions for the intensity including possible reflection of the waves at the plasma boundary.

Equation (2) has an analytic solution in the form of an integral over the ray path. However, in the case of multiple reflection at the medium boundary (e.g. the tokamak wall), direct integration may be too cumbersome. Alternatively, the process described by Eq. (2) can be simulated by Monte Carlo modelling of the photon (plasmon) history.

In the case of radiative transfer by emission and absorption of EM waves by plasma electrons, the quantities $Q$ and $\kappa$ are expressed in terms of the electron distribution function (EDF), $f(\vec{r}, \vec{p}, t)$. Thus,

$$Q(\phi, \vec{r}, t) = \int q_i(\vec{p}, \phi, \vec{r}, t) f(\vec{r}, \vec{p}, t) \, d\vec{p} \quad (3)$$

where the function $q_i$ is the rate of emission by an individual electron. The absorption coefficient, which allows for 'true' absorption and stimulated emission, can generally be expressed in the form

$$\kappa(\phi, \vec{r}, t) = \int \kappa_i(\vec{p}, \phi, \vec{r}, t) f(\vec{r}, \vec{p}, t) \, d\vec{p} \quad (4)$$

where the quantity $\kappa_i$ can, in the long wave limit, be reduced to a differential operator in momentum $\vec{p}$.

The source function $Q(\phi, \vec{r}, t)$ may implicitly depend on the intensity $J(\phi, \vec{r}, t)$ via corresponding distortions of the EDF, caused by radiative transfer (see Eq. (10) below). If these distortions are small, then $Q$ does not depend on $J$, and a direct solution of Eq. (2) practically settles the problem of radiative transfer. It is this framework within which the problem of radiative transfer has been considered in Refs [2-9].

The evolution of the EDF, $f(\vec{r}, \vec{p}, t)$, can be described by the Fokker–Planck equation which allows for distant Coulomb collisions (operator $\hat{C}$ in the non-relativistic, Landau, or fully relativistic, Belyaev–Budker, form [13]), spontaneous emission of waves (term $E_j$) and their absorption (term $A_j$, which allows for QL diffusion). In the long wave limit, emission and absorption terms can be expressed in a divergence form. Thus, the equation for the EDF has the form

$$\left( \frac{\partial}{\partial t} + \vec{v} \frac{\partial}{\partial \vec{r}} + \vec{F} \frac{\partial}{\partial \vec{p}} \right) f(\vec{r}, \vec{p}, t) = \frac{\partial}{\partial \vec{p}_j} (E_j f - \hat{A}_j f) + \hat{C}[f] \quad (5)$$

where $\vec{F}$ is the external force (e.g. macroscopic magnetic field and electrostatic potential), and

$$\hat{C}[f] = - \frac{\partial}{\partial \vec{p}_j} (\hat{A}_j f - E_j f) \quad (6)$$
where the operators $\hat{A}_j$ and $\hat{E}_j$ describe 'absorption' and 'emission' of the plasma electric microfield by an individual electron at distant collisions (treated as either purely Coulomb or with appropriate account of screening).

According to the general laws of conservation, the following relations hold:

$$\int d\Phi J(\Phi, \Pi, t) \hat{\kappa}_j(\Pi, \Phi, \Pi, t) = v_j \hat{A}_j = \hat{Q}_{\text{abs}}$$

(7)

$$\int d\Phi q_j(\Pi, \Phi, \Pi, t) = v_j \hat{E}_j = Q_{\text{em}} d\Phi = d\omega d\Omega_n \sum_j$$

(8)

where the operators $Q_{\text{abs}}$ and $Q_{\text{em}}$ yield the values of energy absorbed and emitted by an individual electron of momentum $\Pi$.

We also assume that the Coulomb (electron–ion) collisions ensure an isotropic (in pitch angles) form of the EDF, which therefore depends on the electron energy $\epsilon$ only, $f = f(\epsilon)$ (this is true, to a large extent, even for auxiliary heating by intense ECR waves). The isotropic form may additionally be maintained by the isotropic form of radiation intensity $J$. In this case, we have

$$\hat{Q}_{\text{abs}} f \propto - \int d\Phi \frac{J(\Phi, \Pi, t)}{N^2} \sigma_{\text{abs}}(\Pi, \Phi, \Pi, t) \hbar \omega \frac{\partial f}{\partial \epsilon} = Q_{\text{abs}}^{(1)} T_e \frac{\partial f}{\partial \epsilon}$$

(9)

where $\sigma_{\text{abs}}(\Pi, \Phi, \Pi, t)$ is the cross-section of the absorption of a wave quantum $\Phi$ of energy $\hbar \omega$, the quantity $Q_{\text{abs}}$ represents the energy (averaged over $\Phi$) absorbed by an individual electron and is precisely equal to this mean energy in the case of a Maxwellian plasma.

For magnetized plasmas the action of the term with the Lorentz force on the left-hand side of Eq. (5) is incorporated into the quantity $\sigma_{\text{abs}}$ (which is the cross-section of absorption by an elementary Larmor circle), so that the operator $\partial / \partial \Pi$ in Eq. (5) and, consequently, Eqs (7) and (8) should be properly modified. However, for an isotropic EDF the implications of this modification may sometimes be non-observable, e.g., the quantities $\delta(\omega - k v) \frac{\partial f}{\partial \Pi} \frac{\partial f}{\partial \Pi}$ and $(k_1 \partial f / \partial p_1 + (\omega - k_1 v_1) / u_x \times \partial f / \partial p_1)$ both reduce to $\omega \partial f / \partial \epsilon$.

Neglecting the effects of inhomogeneity described by the operator $\vec{V} \partial / \partial \vec{r}$ and solving Eq. (5) in the stationary case, we obtain

$$f(\epsilon) = f_0(\epsilon) A \exp \left[ \int_0^\epsilon \left( 1 - \frac{Q_{\text{em}}^c + Q_{\text{em}}^{(1)} / Q_{\text{abs}}^{(1)}}{Q_{\text{em}} + Q_{\text{abs}}} \right) \frac{d\epsilon}{T_e} \right]$$

(10)

where all quantities $Q_{\text{em}}$ and $Q_{\text{abs}}$ are averaged over the pitch angles, the superscript 'c' distinguishes contributions of distant Coulomb collisions, the operators $Q_{\text{abs}}$ and $Q_{\text{em}}$ being defined in terms of the operators $\hat{A}_j$ and $\hat{E}_j$ similarly to the definitions of $Q_{\text{abs}}$ and $Q_{\text{em}}$, respectively, $f_0(\epsilon)$ is the Maxwellian distribution function and $A$ is a normalization constant. Allowing for the above mentioned effects of inhomogeneity, which requires proper averaging procedures (bounce averaging, transition to another set of canonical variables, etc.), still preserves the qualitative picture described by Eq. (10).
Equation (10) gives a simple description of the combined action of two effects: (a) 'enlightenment' of the medium due to QL reduction of absorption, the term $Q_{bs}^{(1)}$, and (b) depletion of the electron velocity distribution 'tail' due to radiation emission, the term $Q_{em}^{[10]}$.

Substitution of Eq. (10) into Eqs (2)-(4) and allowing for Eq. (9) leads to a complex multidimensional integro-differential equation (in the variables $T, \omega, \bar{n}, \bar{f}$ and $\bar{p}$) to be solved numerically.

3. NON-LOCAL (NON-DIFFUSIVE) NATURE OF HEAT TRANSPORT BY EM WAVES.
UNIVERSAL FORMULAS FOR TOTAL ENERGY LOSSES

The heat transport by EM waves, both transverse (i.e. photons) and longitudinal (i.e. plasmons) waves, in a wide range of parameters (which includes, in particular, magnetically confined thermonuclear plasmas) is characterized by its non-local (non-diffusive) nature which manifests itself in a non-local correlation of the plasma temperatures and, correspondingly, a non-diffusive law of heat propagation. Mathematically, the non-diffusive nature is expressed by the fact that the original equation for the heat transfer (specifically, the equation for the plasma temperature or the mean energy), which is an integral equation in spatial variables, cannot always be reduced to a differential diffusion-type equation in those variables. The main reason is the crucial role of the long mean free path energy carriers (photons or plasmons) whose long flights are of medium length $L$ (or greater, $L/(1 - R)$, in the case of reflection at the boundaries, $R$ being the reflection coefficient), which leads to an infinite diffusion coefficient for unbounded media or to an $L$-dependent diffusion coefficient for bounded media so that the very concept of diffusion coefficient becomes meaningless.

It should be noted that the increasingly growing trend towards the recognition of a non-local (non-diffusive) character of the anomalous heat transport in a tokamak still peacefully coexists with the permanent endeavour of treating the problem in terms of an anomalous electron heat diffusivity. This may be explained by the fact that non-diffusive transport by plasma particles (e.g. by fast electrons in a stochastic, braided magnetic field) exhibits a subdiffusive character of heat propagation ($t \propto r^a$, $a < 2$) rather than a superdiffusive one ($a > 2$) (whereas non-diffusive transport by plasma waves always leads to a superdiffusive law). This fact gives a certain, appreciable advantage to plasma waves over particles as main energy carriers for anomalous heat transport in a tokamak, the particles invariably being the main accumulators of plasma energy.

The phenomenon of the non-local, non-diffusive nature of the transport mechanism has been revealed and thoroughly investigated in the theory of RTRAL in gases and plasmas in the case of the so-called complete redistribution in photon frequencies (in the individual act of its being scattered by the medium's atom), in the late forties.
and early fifties in the pioneer works by Biberman, Holstein and Sobolev (see, e.g.,
the survey [11] and references therein). First, the crucial role of the ‘wings’ of a spec­
tral line, within the framework of the Biberman–Holstein equation, basic for the
whole problem of radiative transfer, was demonstrated, and, secondly, an approxi­
mate analytic formula for the density of excited atoms, which allows a calculation
of the total losses on line radiation, was obtained (this approach and its further
advances are known in the literature as the ‘escape probability method’; see, e.g.,
Ref. [14]).

Meanwhile, in the kinetic theory of transport by plasma waves, for many years
the ‘public opinion’ was evidently to a large extent determined by Davydov’s work
[15], in which establishing a simple relation between the diffusion coefficient in
momentum space and that in co-ordinate space corresponds to an actual elimination
of the line wings and, consequently, to a compulsory reduction to a diffusion-type
transport equation with a small diffusion coefficient for transport by wave–particle
interaction compared to one for particle–particle interaction (i.e. Coulomb pair colli­
sion, the distant one in the sense of small momentum transfer, but the close one in
the sense that collective effects are exhibited only in the dynamic screening of
individual colliding particles).

An important step towards realizing the possible role of transport by plasma
waves (specifically, longitudinal electron and ion waves) has been achieved by
Rosenbluth and Liu [16], who revealed the significant role of plasmons with a free
path of the order of the plasma size for energy losses on wave emission by a station­
ary system with a fixed temperature profile. For transport by transverse waves (i.e.
conventional electron cyclotron radiation) in an inhomogeneous plasma, the concept
of non-diffusive transport within the same framework (fixed density and temperature
profiles) has been fruitfully exploited by Tamor [8, 9] in numerical modelling and
analysing its results.

The straightforward analogy between heat transport by plasma waves and radia­
tive transfer in resonance lines has been traced in Ref. [17] for the problem of propa­
gation of heat waves produced by an instantaneous point source in a homogenous
plasma (non-stationary Green function), where a non-linear integral equation was
obtained which was reduced, by linearizing, to the above mentioned Biberman–
Holstein equation. The resulting law of propagation of small perturbations of electron
temperature for the specific case of transport by Bernstein modes appears to be non­
diffusive (approximately $t \propto r$, in contrast to the standard diffusion law $t \propto r^2$).

The most straightforward analogy revealed between radiative transfer in
resonance lines and heat transport by plasma waves in continuous spectra in bounded
media, with fixed density and temperature profiles and possible reflection at the
boundaries, enables us to derive, within the framework of a non-diffusive concept,
a simple and universal analytic description [18] of an arbitrary emission–absorption
mechanism in the individual act of wave–particle interaction. The existing analytic
descriptions are, rather, a fit of numerical results and pertain, first, to a specific
mechanism of emission, i.e. cyclotron radiation, and, secondly, to specific profiles
of temperature and density: a uniform profile, described by the well known
Trubnikov formula [4], or a tokamak-like profile, described by the formula of
Ref. [19]. The result [18] in the specific case of a tokamak geometry in the usually
satisfied conditions of not too large aspect ratio, a non-circular (in particular, elon­
gated) cross-section and multiple reflection at the plasma boundaries (for transverse
waves the latter is ensured by highly reflecting tokamak walls, \((1 - R) \leq 0.1\), and
for longitudinal waves it is determined by edge plasma parameters) reduces to the
form (Eq. (2)):

\[
\frac{dE}{dt} = \sum_f \int d\omega \int dV \int d\Omega_n \frac{Q(\phi, \vec{r})}{1 + \tau_{eff}}
\]

\[
\tau_{eff} = \frac{\int dV \int d\Omega_n Q(\phi, \vec{r})}{\int d\Omega_n \int (\vec{n}, d\vec{S}_S) (1 - R(\phi, S_S))}
\]

where \(\tau_{eff}(\omega, \vec{r})\) is the effective (dimensionless) optical length which describes the
trapping (or, equivalently, the imprisonment, as it is called in the language of
RTRAL theory) of plasmons (photons). Here, \(V\) and \(S_S\) are plasma volume and sur­
face, respectively, and \(R(\phi, S_S)\) describes the dependence of the reflection coeffi­
cient on the wave parameters (first of all, the frequency). A comparison of formula
(11) with the results of numerical calculations, in particular, for the homogeneous [4]
and inhomogeneous [9] cases, and with the formula of Ref. [19] (and the formula of
Ref. [4] in the region of a successful approximation of numerical results [4, 6]) shows
good agreement in the regions where these results are applicable. The anti-diffusion
method also provides a universal analytic description of the spatial profile of the wave
energy balance, also for an arbitrary degree of mixing of different waves at the
plasma boundaries, both for stationary and non-stationary cases. Thus, the non-
stationary counterpart of Eq. (11) has the form:

\[
\frac{dE(t)}{dt} = \sum_f \int d\omega \int_0^t \frac{dt'}{\tau_{eff}} \int dV \int d\Omega_n Q(\phi, \vec{r}, t')
\times \exp \left[ - \int_{t'}^t \frac{dt''}{\tau_{eff}} (1 + \tau_{eff}(t'')) \right]
\]

where \(\tau_{eff}(t)\) is the characteristic time of wave trapping exclusively due to the reflection
by the boundaries:

\[
\tau_{eff}(t) = \frac{\int dV \int d\Omega_n v_k^{-1}(\phi, \vec{r}, t)}{\int d\Omega_n \int (\vec{n}, d\vec{S}_S) (1 - R(\phi, S_S))}
\]

(Eq. (12) assumes that the retardation effects are small).

Formulas (11) and (12) generalize Trubnikov's formula [4] for synchrotron
losses to the cases of (a) an arbitrary elementary process of radiation emission by a
high temperature plasma with highly reflecting walls and fixed (e.g. Maxwellian)
velocity distribution, and (b) inhomogeneous and non-stationary plasma parameters
(density, temperature, magnetic field).
4. HEAT TRANSPORT BY CYCLOTRON WAVES IN THERMONUCLEAR PLASMAS

Numerical calculations allowing for the combined action of two effects, (a) the 'enlightenment' of a medium due to the QL reduction of absorption and (b) the depletion of the electron velocity distribution 'tail' due to radiation losses (Section 2), with ECR local and total energy balance in inhomogeneous plasmas in systems with strong magnetic field and highly reflecting walls, are carried out for the parameters of interest in future reactors with a sufficiently strong magnetic field (both D–3He based, APOLLO-L3, and D–T based tokamaks, ITER and IGNITOR, Tables I and II). The radiative transfer problem is treated within a model which is in good agreement with Monte Carlo modelling [8, 9]. In our approach, the radiative transfer problem is coupled with the solution (10) of the Fokker–Planck equation for the electron velocity distribution.

Tables I and II illustrate the characteristic features of emitted ECR frequency distribution and energy balance profiles which are peculiar to heat transport in large tokamaks with high temperature plasma, i.e. (1) the concentration of radiation intensity at high cyclotron frequency harmonics and (2) the peripheral plasma attenuating the radiation emitted by the hot central plasma.

### TABLE I. CHARACTERISTICS OF THE ESCAPING ECR FREQUENCY DISTRIBUTION FOR A CYLINDER WITH STANDARD TOKAMAK DENSITY AND TEMPERATURE PROFILES AND FOR MAIN PLASMA PARAMETERS TYPICAL OF THE FOLLOWING PROJECTS:

<table>
<thead>
<tr>
<th></th>
<th>R</th>
<th>$\langle \omega \rangle$</th>
<th>$\Delta \omega$</th>
<th>Loss, MW/m</th>
</tr>
</thead>
<tbody>
<tr>
<td>APOLLO-like</td>
<td>0.98</td>
<td>15.5</td>
<td>5.0</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>0.9</td>
<td>13.2</td>
<td>4.6</td>
<td>24</td>
</tr>
<tr>
<td>ITER-like</td>
<td>0.9</td>
<td>5.6</td>
<td>1.7</td>
<td>0.029</td>
</tr>
<tr>
<td></td>
<td>0.7</td>
<td>5.2</td>
<td>1.5</td>
<td>0.053</td>
</tr>
<tr>
<td>IGNITOR-like</td>
<td>0.9</td>
<td>5.5</td>
<td>1.7</td>
<td>0.093</td>
</tr>
<tr>
<td></td>
<td>0.7</td>
<td>5.0</td>
<td>1.5</td>
<td>0.17</td>
</tr>
</tbody>
</table>

Here, $\langle \omega \rangle$ is the average frequency, $\Delta \omega$ is the dispersion of the frequency distribution, both in units of the cyclotron frequency, 'Loss' is the total power loss per unit length, in MW/m, and R is the wall reflection coefficient.
TABLE II. PROFILES OF THE ECR POWER DENSITY LOSS FOR THE
CONDITIONS OF TABLE I, IN W/m$^3$; $b(c) = b \times 10^c$

<table>
<thead>
<tr>
<th></th>
<th>r/a</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.05</td>
</tr>
<tr>
<td>APOLLO-like</td>
<td>0.98</td>
</tr>
<tr>
<td></td>
<td>0.9</td>
</tr>
<tr>
<td>ITER-like</td>
<td>0.9</td>
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<tr>
<td></td>
<td>0.7</td>
</tr>
<tr>
<td>IGNITOR-like</td>
<td>0.9</td>
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<tr>
<td></td>
<td>0.7</td>
</tr>
</tbody>
</table>

The results show that for the parameters of interest the non-Maxwellian effects may appreciably influence the spatial profiles of the ECR energy balance in the edge plasma whereas the total losses, in the conditions of interest, are weakly affected and may both decrease and increase as a function of the plasma parameters. The deviation from the Maxwellian result increases with decreasing plasma density and increasing reflection coefficient. The resulting losses depend strongly on the mutual cancellation of QL flattening of the electron velocity distribution and the depletion of its 'tail' due to losses on wave emission, so that allowing for only one of these factors leads to a drastic distortion of the whole picture. For example, neglecting $Q_{abs} = 0$ in Eq. (10) reduces the total losses for APOLLO-like parameters by a factor of about two.

An application of the results to the energy transfer by longitudinal waves across a strong magnetic field shows that thermoconductivity through plasmons in tokamak plasmas strongly depends on the character of the wave dispersion law because the dispersion, together with multiple reflection of the wave at the plasma boundary, 'transfers' the process of heat transport to the far-away wings of the emitted spectrum where the latter may easily be influenced by various factors.

The application of the anti-diffusion concept [18] to the heat transport in a tokamak suggests a qualitative explanation for the interrelation of non-local (general property of $T_e$ profile invariance, only partially disturbed by L–H transition; unexpectedly short duration of L–H transition within the diffusion time-scale) and local (crucial role of edge plasma conditions) features of global heat transport. This explanation is based on the fact that both non-local and local features are naturally incorporated into Eqs (11) and (12), which include spatially averaged emission–absorption
characteristics of the plasma and its essentially local characteristic, namely the reflection of EM energy carriers (plasmons and photons) at the plasma boundary. Such an interrelation suggests the possibility of effective control of plasma global (non-local) parameters via a proper control of the reflection of plasma waves in the edge plasma.

5. CONCLUSIONS

On the whole, we arrive at the conclusion that in the case of a strong magnetic field the energy balance studies require a description of the global heat transport within the framework of combined heat transfer by transverse and longitudinal waves and a kinetic description of the plasma electrons.

ACKNOWLEDGEMENTS

The author gratefully acknowledges that his interest in the radiative transfer problem was stimulated by V.I. Kogan. He is also indebted to V.S. Lisitsa for having aroused his interest in heat transport by longitudinal waves and for having cooperated fruitfully with him on this subject [17].

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SYNCHROTRON LOSS IN INHOMOGENEOUS TOKAMAK PLASMAS

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Presented by A.B. Kukushkin

Abstract

SYNCHROTRON LOSS IN INHOMOGENEOUS TOKAMAK PLASMAS.

Synchrotron emission in a tokamak configuration with inhomogeneous plasma parameters is studied as a means of investigating the effects of temperature profile and vertical elongation on radiation loss. By using the numerical solution of the transfer equation for ITER-like plasma parameters, several new results on the radiated energy in a Maxwellian plasma have been derived, in particular: (i) the synchrotron loss is profile dependent, i.e. at constant average thermal energy, the emitted radiation increases with the peak temperature; (ii) an analytical formula for the global loss in inhomogeneous tokamak plasmas with arbitrary vertical elongation is established; (iii) the maximum of the frequency emission spectrum is a linear function of the volume average temperature; and (iv) a passive diagnostic of the electron temperature is proposed.

1. INTRODUCTION

The synchrotron loss will play a role in the ignition capability of next generation devices such as ITER characterized by high central temperature ($T_e = 30–40$ keV). Synchrotron radiation may also be of interest to the problem of burn equilibrium and control of thermal runaway [1].

The basic theory of synchrotron radiation in a fusion device was initiated by Trubnikov [2] and extensively developed by him [3] and by Drummond and Rosenbluth [4]. The theory was applied to a homogeneous plasma cylinder. The case of an inhomogeneous cylinder was discussed by Tamor [5]. In this paper we generalize the earlier works to an inhomogeneous plasma confined in a tokamak with elliptical cross-section.

The paper is organized as follows: Section 2 briefly gives the general formulation for the global loss of a torus with elliptical cross-section in terms of the local absorption coefficient. In Section 3, we present some properties of the radiation loss for ITER-like parameters. Section 4 is devoted to the conclusions.

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2. GENERAL FORMULATION OF THE RADIATION LOSS

For simplicity, we assume the tokamak cross-section to be elliptic with elongation $\kappa = b/a$, where $a$ is the horizontal minor radius. Letting $n_e = n_e(\tau), T_e = T_{e0} \times g(\tau), g(0) = 1$, $B = B(\tau)$, assuming a Maxwellian distribution and neglecting reflections on the boundaries, we write for the power radiated by the plasma surface (see Ref. [3] for the solid angle):

$$P_0 = \frac{T_{e0} \omega_{c0}^3}{8\pi^3 c^2} \sum_{x,0} \int_0^\infty d\omega \int_0^\infty d\theta \int_{-\pi/2}^{+\pi/2} \sin^2 \theta \cos \beta \, d\beta$$

$$\times \int_{\ell_0}^{\ell_f} d\ell' \, g(\ell') \alpha_x(\ell') \exp[-\tau(\ell_x, \ell')] \tag{1}$$

where $\sigma$ is the surface, $\omega_{c0} = eB(0)/mc$ is the central electron cyclotron frequency, $\ell'$ is the abscissa along the ray, $\ell_0$ and $\ell_f$ are the initial and final abscissas along the ray, respectively, $\alpha_x$ is the absorption coefficient, $\tau$ is the optical thickness and $(x, o)$ denote the $x$- and $o$-modes, respectively.

The wave trajectory is obtained from the familiar ray equations and from the cold dispersion relation [6]. For a Maxwellian relativistic distribution the anti-Hermitian part of the dielectric tensor is given in Ref. [7].

3. NUMERICAL RESULTS

We show some properties of the synchrotron radiation loss using ITER-like parameters [8, 9] and the following temperature profile:

$$T_e(\psi) = T_{e0}(1 - \psi)^b$$

where $\psi$ is the normalized magnetic flux.

The ray trajectory in a plasma torus differs from the corresponding trajectory in a cylinder. In a torus $N_t$ varies in space, and multiple crossing of the plasma axis may occur [10]. These two toroidal effects will affect wave attenuation.

We first study the frequency dependence of the radiation spectrum $Q(\omega/\omega_{c0})$ defined as

$$P_0 = \int_0^\infty Q(\omega/\omega_{c0}) \, d\omega$$

for several values of the volume average electron temperature $\langle T_e \rangle$ and $(T_{e0}, h)$. The quantity $Q(\omega/\omega_{c0})$ differs from the $\omega^2$ distribution, and it appears that the frequency width of $Q(\omega/\omega_{c0})$ and the value of $Q_{\text{max}}$ increase with $T_{e0}$. This will enhance the radiation level for constant $\langle T_e \rangle$ [9, 11].
Within the accuracy of a few per cent the spatial dependence of the factor \((1 + 18 \, a/R_0 \, T_0^{1/2})^{1/2}\) defined in Ref. [3] can be neglected and an approximate expression for the power emitted by the plasma has been derived [9]:

\[
P_{\text{he}} \approx (2\pi E(e)/\pi) \, 41 \, R_0 \, a^{3/2} \, \beta_{e0}^{1/2} \, B_0^{7/2} \, T_0^{4/3} \times (1 + 18 \, a/R_0 \, T_0^{1/2})^{1/2} \, 0.4/(1/2 + h) \quad (2)
\]

where \(E(e) = E(e, \pi/2)\) is the complete elliptic integral of the second kind and \(\beta_{e0}\) is the electron beta at the plasma centre.

The emission from a torus with a non-circular cross-section varies with the position of the emitting surface element. We have calculated \(dQ/d\sigma\) versus \(\omega/\omega_{e0}\) for several different positions of the emitting surface. The emitted spectra exhibit a strong poloidal dependence due to the different values of wave damping. Specifically, we consider external side emission in the horizontal plane. An extensive series of computations show that \((dQ/d\sigma)_{\text{max}}\) is remarkably well described by the linear relation

\[
(dQ/d\sigma)_{\text{max}} = A(\kappa, T_{e0}) \, T_{e0} \left[ \langle T_e \rangle + k \, T_{e0} \, B(\kappa, T_{e0}) \right] \quad (3)
\]

where \((dQ/d\sigma)_{\text{max}}\) is in arbitrary units and \(A(\kappa, T_{e0})\) and \(B(\kappa, T_{e0})\) are weakly dependent functions of \(\kappa\) and \(T_{e0}\). This can be used to determine the electron temperature profile and \(T_{e0}\) from the best fitting of the theory to the measured spectrum [9].

The accuracy of the method will be enhanced if a single ray is used for which the wall reflection coefficient \(R_w\) opposite to the viewing direction can be accurately determined. We consider wave emission in the equatorial plane towards the low magnetic field side perpendicular to the magnetic field (\(N_1 = 0\)). The maximum of the emitted intensity \(I_w\) versus \(\omega/\omega_{e0}\) is found to be a linear function of \(\langle T_e \rangle\), i.e.

\[
(I_w)_{\text{max}} = A \, T_{e0} \left[ \langle T_e \rangle + B \, T_{e0} \right] \quad (4)
\]

where \((I_w)_{\text{max}}\) is in arbitrary units. An independent and similar relation is obtained for the o-mode. Combining these two relations we obtain

\[
\langle T_e \rangle = T_{e0} \frac{A_0 \, B_0 \, (I_{w0})_{\text{max}} - A_x \, B_x \, (I_{\omega0})_{\text{max}}}{A_x \, (I_{\omega0})_{\text{max}} - A_0 \, (I_{w0})_{\text{max}}} \quad (5)
\]

From Eq. (4) for one of the two modes and from Eq. (5), \(\langle T_e \rangle\) and \(T_{e0}\) are obtained from the measured \((I_{\omega0})_{\text{max}}\) and \((I_{w0})_{\text{max}}\) [9].

4. CONCLUSIONS

We have presented some results concerning the synchrotron radiation of an inhomogeneous, elongated tokamak plasma. The exact calculation of the synchrotron
loss is useful for the reactor design. The synchrotron emission can be used as a passive diagnostic tool giving the electron temperature. The work can be extended to non-Maxwellian plasmas. The important but very difficult problem of wall reflections is under consideration.

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ALPHA EFFECTS ON TAE MODES AND ALPHA TRANSPORT

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Abstract
ALPHA EFFECTS ON TAE MODES AND ALPHA TRANSPORT.

The properties of toroidicity-induced Alfvén eigenmode (TAE, also called the toroidal Alfvén eigenmode) and corresponding Alfvén continuum gap structures are investigated with the profile effects of plasma density, safety factor q, and plasma beta (pressure). Alpha particle driven TAE instabilities are investigated for the proposed TFTR D-T and ITER experiments by using the NOVA-K code. TAE instabilities with multiple toroidal mode numbers can be observed in TFTR D-T supershot runs for beam power less than 20 MW. For higher beam power operations, TAE instabilities can be excited in the transient phase lasting several hundred msec after the beam power is turned off. For ITER, the critical volume averaged alpha beta for the TAE instabilities is on the order of $10^{-3}$, which is smaller than the expected beta values of 0.5 – 1%. A general theory of stochastic particle transport due to MHD perturbations is presented in terms of overlapping of particle drift orbit islands. The ORBIT code calculations show that the stochastic amplitude threshold for TAE perturbation with a single toroidal mode ($n = 1$ or 2) is roughly $\delta B/B = O(10^{-4})$ for alpha particles in TFTR D-T supershot experiments, but is one order of magnitude smaller for TAE perturbations with multiple toroidal mode numbers (both $n = 1$ and $n = 2$). In the presence of both the $n = 1$ and $n = 2$ TAE modes with equal maximum amplitude at $\delta B/B = 10^{-3}$, there is a large fraction of diffusive alpha loss ($\approx 25\%$) in 500 transit times ($\approx 10^{-3}$ slowing down time) in addition to the transient loss of 25%.

In a fusion reactor any unanticipated loss of alpha power could result in serious wall damage, impurity influx, major operational control problems, and even a failure to sustain ignition. Neutral beam injection (NBI) experiments in large tokamaks have indicated that TAE (toroidicity-induced Alfvén eigenmode
and also called toroidal Alfvén eigenmode) [1,2] can be strongly unstable and cause the loss of about half of the fast beam ions [3,4]. This level of loss would be unacceptable in fusion reactors. Recent experiments in TFTR indicate that TAE modes can also be excited by energetic particles heated by ICRF. Stability of TAE modes in the presence of alpha particles had previously been studied [5,6] by employing a global kinetic-MHD stability code (NOVA-K) [7]. The alpha transport due to a single toroidal mode number TAE mode had also been studied [8,9] by using a Hamiltonian guiding center orbit code, ORBIT [10]. In Section 1 we present the properties of the TAE mode, the effects of alpha particles on TAE instabilities for TFTR D-T and ITER experiments. The alpha transport due to multiple toroidal mode number TAE modes and a stochastic transport theory are presented in Section 2. A conclusion is given in Section 3.

1. Alpha Effects on TAE Mode

TAE modes with frequency $\omega = v_A/2qR$, where $v_A$ is the Alfvén speed, can be driven unstable by energetic/alpha particles through wave-particle resonances by tapping the free energy associated with the alpha particle pressure gradient. This mechanism can be understood by considering the linear power transfer from the particles to the TAE mode which is given by

$$ P_\alpha = \pi \int \frac{d^3v}{d^3x} q_e^2 (\omega*F/\omega + T \partial F/\partial \epsilon) (v_\parallel E_\parallel + v_d E_\perp)^2 \delta(\omega - k_\parallel v_\parallel), $$

where $q_e$ is the charge, $\omega*$ is the diamagnetic drift frequency, $F$ is the particle distribution, $T$ is the particle temperature, $\epsilon$ is the particle energy, $v_d$ is the particle magnetic drift velocity, $k_\parallel$ is the parallel wave vector, and $E_\parallel$ and $E_\perp$ are the parallel and perpendicular components of perturbed electric field, respectively. Physically, the $\partial F/\partial \epsilon$ term gives the velocity space Landau damping, and the $\omega*$ term is associated with the destabilizing pressure gradient. To destabilize the TAE mode, the instability drive associated with the energetic particle pressure gradient must overcome damping effects due to the continuum damping (if it exists), trapped electron collisional damping, and the velocity space Landau damping from all particle species. The TAE mode growth rate is
typically of the order of $10^{-2} \omega_A$. In the following we will discuss the stability of low-n TAE modes for TFTR D-T experiments and ITER tokamaks. The stability of the TAE mode has been studied with the NOVA-K global stability code which calculates the destabilizing effects of fusion alphas as well as the stabilizing effects of thermal particles. The collisional damping effect due to trapped electrons is obtained by solving the bounce-averaged drift kinetic equation including the pitch angle scattering collision operator, and no assumption on the ordering of $v_e/\omega$ is made, where $v_e$ is the electron collisional frequency. Numerical results are given in terms of the volume averaged alpha particle beta $<\beta_\alpha>$ and $(\nu_A/\nu_A)$ parameters. If the TAE mode does not suffer continuum damping, the $<\beta_\alpha>$ threshold for TAE instability is on the order of $10^{-3} - 10^{-4}$.

1.1 Properties of TAE mode

The TAE modes [1,2] have discrete frequencies located inside the shear Alfvén continuum gaps created due to toroidal coupling of different poloidal harmonics. If the toroidicity is neglected, the shear Alfvén continuous spectrum can be described in its cylindrical form $\omega^2 = [(m-n)\Omega_A/qR]$, where $n$ and $m$ are the toroidal and poloidal mode numbers respectively. The spectrum is continuous because the safety factor $q$ and the Alfvén velocity are functions of minor radius. For example, the continuous spectra of the $(n,m)$ and $(n,m+1)$ modes cross at radial location $r_0$, where $q(r_0) = (m + 1/2)/n$. The degeneracy is broken by the toroidal coupling effect to form a gap at $r_0$. The continuum gap structure can be computed by a variational principle with a Lagrangian functional [1]. For a large aspect ratio, low-$\beta$ equilibrium with a circular outermost flux surface, the continuum gap boundary at $r_0$ is given to first order in inverse aspect ratio $\varepsilon_0$ by

$$\omega_\pm^2 = \omega_0^2 \pm 2 \omega_0^2 \left( r_0/R + \Delta'(r_0) \right),$$

and the center of the continuum gap at $r = r_0$ is given to $O(\varepsilon_0^2)$ by

$$\omega_0^2 = (\nu_A/2qR)^2 \left[ 1 + 3(r_0/R+\Delta')^2/2 + 4\Delta/R + (\nu/R)^2 - 2\nu\Delta'/R + 2g^{(2)} \right].$$
where \( \Delta(r) > 0 \) is the Shafranov shift of the non-concentric flux surfaces with \( \Delta' = d\Delta/dr > 0 \), and \( g^{(2)} = -P/B^2 + \int_r^a dr r(2-rq'/q)/q^2 R^2 \) is the plasma beta and magnetic shear correction of the toroidal magnetic field. Therefore, \( \omega_0^2 \) will shift upward more than the widening of the gap as plasma beta increases.

For TFTR supershots the Shafranov shift can be large with \( \Delta/a = 0.1 - 0.2 \), and \( \Delta' \) can be as large as 0.3. Thus, the finite pressure and aspect ratio effects on the continuum gap structure can be quite significant.

The TAE modes had been shown \[1,2\] to exist with discrete frequencies inside the continuum gap. For small (large) magnetic shear the TAE mode frequency is near the lower (upper) continuum gap boundary. Typically, for the \( n = 1 \) TAE mode the \( m = 1 \) and 2 poloidal harmonics are dominant components and peak near the \( q=1.5 \) surface. For the \( n = 2 \) TAE mode the \( m = 2 \) and 3 poloidal harmonics are dominant components and peak near the \( q=1.25 \) surface. The existence of TAE modes depends on plasma shaping, wall boundary condition, as well as the values and profiles of safety factor \( q \), plasma beta \( \beta \), and mass density \( \rho \). For some equilibria, more than one TAE mode with different dominant poloidal components and radial structure can exist. As the plasma beta increases or the magnetic shear decreases, the TAE frequency will move downward into the lower continuum \[11\] and the TAE mode will experience continuum damping. This beta limit above which TAE modes experience continuum damping will be lower for higher \( n \) modes. For the high-\( n \) mode, the beta threshold is roughly given by \( 2q^2 R(dP/dr)/B^2 = s^2/(1+s) \) in the limit of small magnetic shear \( s = rq'/q \) \[11\]. The ellipticity and triangularity of the plasma shaping will in general increase the critical beta as is the case for the beta limit of high-\( n \) ballooning modes. The vacuum energy increases the TAE frequency and the presence of wall reduces the critical beta.

One way to reduce or even stabilize the TAE instability is to control the profiles of plasma density and \( q \) so that the radial gap structure does not line up across the minor radius, and the TAE mode will experience continuum damping. The continuum damping effect on the TAE modes has been studied numerically with the NOVA-R resistive MHD code \[12\]. The continuum
damping rate is typically much less than 1% of the real frequency for the resonance surface near the plasma edge. For TAE modes that move down into the lower continuum due to higher plasma beta, the resistive damping rate of an $n = 3$ TAE mode for a TFTR NBI experiment is roughly given by $\gamma / \omega_A \approx 10^{2.37} \eta^{0.7}$, where $\eta$ is the normalized resistivity (inverse of magnetic Reynolds number). For $\eta = 10^{-7}$, the damping rate is $\gamma / \omega_A = 4.68 \times 10^{-3}$.

The continuum damping effect on the TAE modes has also been studied by a gyrofluid code (TAE/FL) [13]. The results have been compared against analytical calculations of continuum damping rates [14,15] and demonstrated similar scaling in $m_0 \epsilon$, where $m_0$ is the dominant poloidal mode number and $\epsilon = \epsilon_0[\partial \ln(q/\Lambda)/\partial \ln q]^{-1}$ with $\epsilon_0 = a / \Lambda$. The numerically obtained continuum damping rates show similar scaling between the various calculations and are weakest for high toroidal mode numbers and for density profiles for which the continuum gaps line up in radius and $\epsilon \to \infty$. The continuum damping is particularly important as the plasma beta approaches the Troyon limit, and the TAE frequency shifts downward into the lower continuum and enhances damping.

1.2 TAE instabilities in TFTR D-T experiments

To study the TAE instabilities for TFTR D-T experiments we will consider the supershot operational scenario. The core plasma parameters are scaled according to the TFTR supershot beam power scalings taken from the supershot operations in D-D experiments [16]. The plasma volume averaged beta in percent is scaled as $\beta = 0.13 + 0.025 P_B$, where $P_B$ is the neutral beam power in MW, the central electron temperature in keV is scaled as $T_e(0) = 5.7 + 0.11 P_B$, the central electron density in $10^{13}$ cm$^{-3}$ is scaled as $n_e(0) = 2.2 + 0.16 P_B$, and the central ion temperature in keV is scaled as $T_i(0) = 7.2 + 0.75 P_B$. Thus, both $T_i$ and $\beta$ are sensitive functions of beam power. The plasma profiles are chosen to model typical TFTR supershot profiles obtained from TRANSP simulations [17]. The pressure is chosen as $P = P(0) (1-\psi)^2$, where $\psi$ is the normalized poloidal flux and is zero at the magnetic axis and unity at the
plasma surface. The q-profile is chosen as \( q(\psi) = q(0) + \psi (q(1) - q(0)) + [q'(1) - q(1) + q(0)] (1 - \psi_s) (\psi - 1) / (\psi - \psi_s) \), where \( \psi_s = \left( \frac{q'(0) - q'(1) - 2[q(1) - q(0)]}{q'(0) + q'(1) - 2[q(1) - q(0)]} \right) \), \( q(0) = 1.1 \), \( q(1) = 5.5 \), \( q'(0) = 1 \), and \( q'(1) = 14 \). The electron density profile is given by \( n_e = n_e(0) (1 - 0.8 \psi) \). The central thermal ion density is taken to be \( n_i(0) = 0.7 n_e(0) \) \([17]\). The alpha particle pressure profile is chosen to have a Gaussian shape with \( P_\alpha = P_\alpha(0) \exp\left[ -(\tau / L_\alpha)^2 \right] \), where \( (\tau / a)^2 = \psi \). The fixed parameters of TFTR D-T experiments are taken to be a circular cross section with the major radius \( R = 245 \) cm, the minor radius \( a = 80 \) cm, the toroidal field \( B = 5T \), and the alpha particle pressure scale length \( L_\alpha / a = 0.3 \).

For the series of TFTR D-T equilibria there is only one marginally stable TAE mode for each toroidal mode number in the ideal MHD limit. For the equilibrium with the volume averaged plasma beta \( \beta = 0.59\% \), the \( n = 1 \) TAE mode frequency is given by \( (\omega / \omega_A)^2 = 2.215 \), where \( \omega_A = V_A(0)/q(a)R \), and the \( n = 2 \) TAE mode frequency is \( (\omega / \omega_A)^2 = 3.095 \). The radial eigenfunction of the \( n = 3 \) TAE mode becomes more singular and will not be discussed here due to the limitation of the theory. Figure 1 shows the critical volume averaged alpha particle beta, \( \langle \beta_\alpha \rangle \), versus \( \beta \) and \( P_B \) for the \( n = 1 \) & 2 TAE instabilities.
obtained from the NOVA-K code [7]. Also shown is a shaded region representing steady state values of volume averaged alpha particle beta expected in the D-T supershot experiments [16]. We see that the \( n = 1 \) TAE mode is stable except for a small range of core plasma beta values around \( \beta = 0.4\% \), where the \( n = 1 \) TAE mode is weakly unstable. On the other hand, the \( n = 2 \) TAE mode is unstable for \( \beta < 0.7\% \). Note that the critical alpha particle beta for TAE instabilities decreases rapidly as \( \beta \) decreases. This is because the ion Landau damping is the dominant damping mechanism. Both the electron Landau damping rate and the collisional damping rate are very small, on the order of \( 10^{-2} \) of the ion Landau damping rate. Since the ion Landau damping rate is roughly proportional to \( \beta_i^{3/2} \exp(-1/9\beta_i) \) and the thermal ion beta decreases as beam power (or core plasma \( \beta \)) decreases, the critical alpha particle beta \( \langle \beta_\alpha \rangle \) decreases rapidly with the plasma beta.

An instability scenario for the \( n = 1 \) TAE mode can be achieved for higher beam power operations by turning off the neutral beam power. After the beam power is turned off, the core plasma beta, electron density, and ion temperature will decrease rapidly, but the alpha particle beta will decrease at a much slower rate and can remain above the threshold of the \( n = 1 \) TAE instability for several hundred msec. Figure 2 shows a typical temporal trajectory of \( \langle \beta_\alpha \rangle \)
production and \((\nu_\alpha/\nu_A)\) obtained from a TRANSP simulation of a high beam power TFTR D-T experiment based on a supershot run (#55848) [17]. The arrow indicates the increase of time. During the neutral beam heating phase both \(<\beta_\alpha>\) and \((\nu_\alpha/\nu_A)\) increase, and the n = 1 TAE mode is stable. But after the neutral beam is turned off, \(<\beta_\alpha>\) remains fairly constant as \((\nu_\alpha/\nu_A)\) decreases due to decrease of plasma density, and the plasma enters into the unstable domain for the n = 1 TAE mode. One caution is that during the start of the beam heating phase the n = 2 TAE mode can be unstable and prevent the alpha particle beta from rising.

1.3 TAE instabilities in ITER

To study the TAE instabilities for ITER experiments we will consider a series of equilibria corresponding to the Conceptual Design parameters with varying plasma beta. The fixed parameters of the plasma geometry are taken to be the major radius \(R = 600\) cm, the minor radius \(a = 215\) cm, the ellipticity \(\kappa = 2\), the triangularity \(\delta = 0.4\), the toroidal field \(B = 4.85\) T. The pressure profile is chosen as \(P = P(0) (1-\psi^{1.05})^2\), where \(\psi\) is the normalized poloidal flux. The q-profile is chosen with \(q(0) = 1.1, q(1) = 3.2\), \(q(0)' = 0.9, q(1)' = 13\). The electron density profile is given by \(n_e = n_e(0) (1 - 0.8 \psi^2)\). The bulk plasma is assumed to have an equal mix of D-T, and the central thermal ion density equals the central electron density. The central electron density is taken to be \(n_e(0) = 10^{14}\) cm\(^{-3}\). The central electron and ion temperature are assumed to be equal and scaled as \(T_e(0) = T_i(0) = 9.1\ \beta\) with temperatures in keV and plasma beta in %. The alpha pressure profile is chosen to have a Gaussian shape with \(P_\alpha = P_\alpha(0) \exp\left[-(r/L_\alpha)^2\right]\), where \(L_\alpha\) is the alpha pressure scale length and \((r/a)^2 = \psi\).

For the series of ITER plasma equilibria there are two TAE modes for each toroidal mode number. Figure 3 shows the critical volume averaged alpha particle beta of \(<\beta_\alpha>\) versus the total volume averaged plasma beta \(\beta\) (and temperature) for the \(n = 2 \& 3\) TAE instabilities with \(L_\alpha/a = 0.3\). The \(n = 1\) TAE mode is stable for these parameters. The critical alpha particle beta decreases for higher \(n\) TAE modes. We see that the critical alpha particle beta
for the $n = 3$ TAE mode is on the order of $10^{-3}$ which is smaller than the expected volume averaged alpha beta values of 0.5 - 1%. The critical alpha particle beta increases as $\beta$ increases due to the higher ion Landau damping associated with the increase of ion beta. The ion Landau damping is dominant and other damping effects are very small, on the order of $10^{-2}$ of the ion Landau damping rate.

Figure 4 shows the dependence of the critical volume averaged alpha beta $\langle \beta_\alpha \rangle$ on the alpha pressure scale length $(L_\alpha/a)$ for an ITER equilibrium with $\beta = 2.67\%$. For the $n = 3$ TAE mode the critical $\langle \beta_\alpha \rangle$ seems to have an exponential dependence on $(L_\alpha/a)$. Note that the $(L_\alpha/a)$ dependence is extremely sensitive to the mode structure. For the other $n = 3$ TAE mode with a different mode structure, the critical $\langle \beta_\alpha \rangle$ is higher and its dependence on $(L_\alpha/a)$ is completely different. Note that for $(L_\alpha/a) > 0.35$ the critical volume averaged alpha betas for the $n = 2$ and the $n = 3$ modes are large ($>1\%$) and these two TAE modes are practically stable in ITER.

TAE modes with $n > 3$ also exist, and their critical volume averaged alpha betas are about the same as the $n = 3$ TAE mode shown in Figure 4. For example, the critical volume averaged alpha beta for the $n = 6$ TAE mode is about a factor of 2 below that of the $n = 3$ TAE mode. We have also found
FIG. 4. Dependence of the critical volume averaged alpha beta of $\langle \beta_\alpha \rangle$ on the alpha pressure scale length $(L_\alpha/a)$ for an ITER equilibrium with $\beta = 2.67\%$.

many ellipticity-induced Alfvén eigenmodes (EAE) for the ITER equilibria. Their frequencies are about a factor of 2 higher than the TAE frequencies, and they are all stable for $(L_\alpha/a) = 0.3$ due to enhanced alpha velocity space damping relative to the pressure gradient drive.

1.4 High-n theory of TAE instabilities in TFTR NBI experiments

TAE modes have been excited by energetic neutral beam ions in TFTR at low magnetic field ($B \approx 1$ T) [3]. The frequencies and mode structures are in reasonable agreement with theoretical solutions obtained from NOVA-K code. However, due to the narrow radial localization of the $n = 2, 3$ modes, the finite Larmor radius (FLR) and finite drift orbit width (FDW) effects of fast neutral beam particles are important in determining the instability drive. In addition, due to low plasma temperature of the experiment, the effects of electron collisional damping, and finite parallel perturbed electric field are also important in determining the damping mechanism. These effects are yet to be included in the global NOVA-K stability code. Instead, we present a theoretical stability calculation based on a high-n WKB-ballooning formalism to include these effects [18]. The FLR effect is stabilizing and dictates the maximum instability drive around $k\rho_\alpha = 1$. For trapped fast particles, the FDW effect is stabilizing, but for circulating fast particles the FDW effect is stabilizing for
large \((v_0/v_A)\) and destabilizing for small \((v_0/v_A)\). Using the TFTR NBI experimental parameters [3] of \(L_b/a = 0.44\), \(\varepsilon_b = 100\) keV, \(T_e = T_i = 1.1\) keV, and \(n_e = 2.7 \times 10^{13}\) cm\(^{-3}\) at \(r = 30\) cm, the critical beam betas for exciting the \(n = 2\) TAE mode at \(B = 1\)T and the \(n = 3\) TAE mode at \(B = 1.2\)T are both at \(\beta_b = 0.9\%\). The theoretical thresholds agree with the experimental observations within a factor of 2.

2. Alpha Transport

2.1 Stochastic transport theory

An analytical theory of stochastic particle transport mechanisms due to MHD perturbations has been developed [19]. Consider an MHD perturbation with a single helical component in a \((\psi, \theta, \zeta)\) flux coordinate system. The perturbed radial magnetic field is given by \(\delta B_r/B = - b(r) \cos \eta\), with amplitude \(b(r) = m \alpha(r)/(r/R)\), where \(\eta = n \zeta - m \theta - \omega t + \phi_{nm}\), \((r/a)^2 = \psi\), and \(\phi_{nm}\) is an arbitrary mode phase. The perturbation produces a magnetic island at the \(q = q_m = m/n\) resonant surface of half-width \(\delta r_{nm} = \left(4 q b/n R q^2\right)^{1/2}\). However, the same perturbation produces a series of drift orbit islands labelled by bounce harmonic \(p\), satisfying the magnetic drift-bounce resonance condition \(\omega = \omega_\delta + p \omega_b\) for trapped particles and the transit resonance condition \(\omega = k_i v_{||} + p \omega_t\) for passing particles, where \(p = 0, \pm 1, \ldots\). In the limit of zero energy particles with pitch \(\lambda = v_{||}/v = 1\) (which simply follow field lines), the widths \(\delta r_p\) of all these islands vanish, except \(\delta r_{p=0}\), which recovers \(\delta r_{nm}\). For larger energies, these island widths can become appreciable. Stochasticity ensues when the islands are large enough to overlap, \(\delta r_{p+1} + \delta r_p \geq r_{p+1} - r_p\), where \(r_p = r(q_p)\) is the resonant surface of the \(p\)-th island, and \(q_p \equiv (p+m)/n\).

The \(p \neq 0\) islands arise as follows. The radial drift of the particle's bounce/transit averaged radius \(\langle r \rangle\) is given by \(\langle v_r \rangle = - v_{||} b(r) \cos \eta\). For particles not too near the trapped/passing boundary, the deviations of the particle coordinates from their averaged values may be approximated as purely sinusoidal in the bounce/transit phase \(\theta_b\). Then, we have \(\cos \eta = \sum_p J_p(\eta_1) \cos\)
$\eta_p$, where $J_p$ are Bessel functions, phase $\eta_p = \langle \eta \rangle + p\theta_b$, and amplitude $\eta_1 = n \zeta_1 - m\phi_1$ is one half the change in mode phase during a bounce/transit time due to the oscillatory portion of the motion. $\langle v_r \rangle$ can then be expressed in a general form $\langle v_r \rangle = \sum_p v_{rp} \cos \eta_p$, where the amplitudes $v_{rp}$, whose particular form varies with particular mode model, are approximately proportional to $J_p$ for the present case. The time-development of $\eta_p$ is given by $d\eta_p/dt = \omega_d + p\omega_b - \omega$. Expanding $d\eta_p/dt$ about the resonant surface where $d\eta_p/dt = 0$, one obtains the island half-width for the $p$-th resonant surface given by $\delta_{rp}/R = \left( 4q_{vp}/(d\eta_p/dt) \right)^{1/2} = (4q_{vp}/\omega dq')^{1/2}$. Recent ORBIT code calculations indicate that for a single toroidal mode number the island overlap condition is satisfied with $\delta B_r/B = O(10^{-3})$ for both low and high frequency modes.

The above theory has been extended to multiple harmonics, to include the magnetic drift $\langle v_{RB} \rangle$ induced by the perturbations, and to apply to arbitrary $\omega$. The $\langle v_{RB} \rangle$ contribution can be competitive with that from $\langle v_r \rangle$ for MeV ions near the mode rational surface. The extension of the theory for high frequency perturbations is straightforward. As $\omega$ increases, the principal change is a shift in the positions $q_p = (\omega/\omega_{b,1} - p)/n$ of the resonant surfaces from their zero frequency values, with the island widths and resultant overlap condition varying only slowly with equilibrium parameters. For larger frequency shifts, since the island widths $\delta_{rp}$ fall off as $|p|$ increases, the wider primary islands can be shifted out of the range of $q$ in the plasma, and the stochasticity mechanism can be eliminated.

The presence of multiple harmonic perturbations, along with the Bessel functions $J_p$ from each harmonic, produces a stochastic transport theory with a structured, nonmonotonic dependence on energy. This opens the possibility of using the stochastic transport mechanism to remove helium ash at intermediate energies while remaining below the stochastic threshold for the bulk as well as for energetic alphas. For this, one needs to make use of the oscillatory character of the Bessel functions, and/or of the added structure provided by multiple harmonics.

The stochastic amplitude threshold $\delta B_r/B = O(10^{-3})$ cited above is toward the upper end of amplitudes observed experimentally. As the amplitude
FIG. 5. Prompt and total loss fraction $f_L$ of alphas versus maximum perturbation amplitude $\alpha(n=1, m=1)$ (note that $\delta B_t(r)/B = m\alpha(r)/(r/R)$) of an $n=1$ internal kink mode in a supershot TFTR equilibrium.

descends below the threshold, the range of the pitch angle values which remain stochastic decreases, and correspondingly so does the lost fraction of particles. This is illustrated in Figure 5 showing the prompt (curve with dark circles) and total loss (curve with dark rectangles) fraction $f_L$ versus maximum perturbation amplitude $\alpha(n=1,m=1)$ (note that $\delta B_t(r)/B = m\alpha(r)/(r/R)$) calculated from the ORBIT code, for an $n=1$ internal kink mode in a supershot TFTR equilibrium with the pressure chosen as $P = 0.6 (1-\psi)^2.5$, and the q-profile chosen with the parameters: $q(0) = 0.9$, $q(1) = 5.5$, $q'(0) = 0.65$, and $q'(1) = 14$. The fixed parameters of TFTR D-T experiments are $R = 245$ cm, $a = 80$ cm, the toroidal field $B = 5T$. From the NOVA-K code calculation, the dominant poloidal harmonics of plasma displacement eigenfunction $\xi_\psi (= \vec{\xi} \cdot \nabla \psi)$ and the perturbed magnetic field $\delta B_t$ of the internal kink mode are $m = 1, 2, 3$ which have large amplitudes within $r/a = 0.8$. In the ORBIT code calculation we launch 200 alpha particles with the birth density distribution $[1-(r/a)^2]^8$. Initially, they are randomly distributed to have a uniform distribution in pitch $\lambda$, $\theta$, and $\phi$. Each case was run for 800 transit times $\tau_t (= 2\pi R/V_\Omega)$. One notes from Figure 5 that the prompt loss fraction is almost independent of mode amplitude, while the remaining nonprompt fraction, due to stochastic loss, grows roughly linearly until saturating around $f_L = 0.75$. Most particles are lost just below the midplane on the outboard side, and most of these lost particles are co-going
particles \((\lambda > 0)\) in a range about the trapped/passing boundary. A smaller but significant number of particles launched with \(\lambda < 0\) also escape, striking below the midplane near \(\theta = \pi\), or by making a transition from counter-going to trapped, striking in the intermediate range \(-\pi < \theta < 0\), where the fixed alpha detectors on TFTR lie.

### 2.2 Alpha transport due to TAE modes

Alpha transport caused by a TAE mode with a single toroidal mode number \(n\) (but several poloidal harmonics) has previously been studied by employing a Hamiltonian guiding center code ORBIT for ITER like equilibrium \([9]\). In the phase space of particle energy \(E\), pitch angle \(\Lambda = \mu B / E\), and toroidal angular momentum \(P_\phi\), one can construct the prompt loss boundaries. Alphas born near these loss boundaries can drift resonantly with the TAE perturbation into the loss region even at \(\delta B / B\) below the stochastic threshold. The dominant alpha loss mechanism is stochastic diffusion to the loss boundaries. In the initial transient phase (lasting for \(\sim 50\) transit times) of near boundary loss, the total number of lost alphas is proportional to \(\delta B / B\). After the transient state the toroidal angular momentum diffusion \(D_{P_\phi P_\phi}(\equiv \frac{d(\langle\delta P_\phi \rangle^2 - \langle\delta P_\phi \rangle^2)}{\langle\delta P_\phi \rangle^2}) / dt\) and the energy diffusion \(D_{ee}\), and hence the number of lost alphas via diffusion scale with \((\delta B / B)^2\) and inversely with the square of the plasma current inside the continuum gap radius. The statistically most likely loss process is the conversion of resonant passing alphas near the passing/trapped boundary to large prompt loss trapped orbits. The alpha orbit stochasticity threshold depends on the TAE mode structure. If the TAE mode is localized in the plasma core, the particle drift orbits in the core can be stochastic and in the plasma edge still possess good drift surfaces. For \((\delta B / B) \geq 10^{-3}\) alpha orbit stochasticity sets in, and depending on the radial width of the fast alpha density profile, more than one half of alphas can be lost in a slowing down time. For a squared Gaussian initial \(n_\alpha(r)\) profile an amplitude of \(\delta B / B = 2 \times 10^{-3}\) yields a loss fraction of 50% in one slowing down time. For a broader trapezoidal \(n_\alpha(r)\) profile the loss fraction can be even larger.

Presently, the ORBIT code calculation has been extended to cover TAE perturbations with multiple toroidal modes, and a lower orbit stochasticity
threshold is obtained. We consider a supershot TFTR equilibrium with the volume averaged plasma beta $\beta = 0.43\%$. The plasma profiles are chosen to model typical TFTR supershot profiles with the pressure chosen as $P = 0.6 (1 - \psi)^{2.5}$, and the $q$-profile chosen with the parameters: $q(0) = 1.1$, $q(1) = 5.5$, $q'(0) = 0.65$, and $q'(1) = 14$. The plasma density profile is given by $\rho = \rho(0) [\exp(-4\psi) + 0.2\psi (2 - \psi)]$. The fixed parameters of these TFTR D-T experiments are taken to be a circular cross section with $R = 245$ cm, $a = 80$ cm, the toroidal field $B = 5$ T, and the plasma current is about 1.5 MA. From the NOVA-K code calculation, the $n = 1$ TAE mode frequency is given by $(\omega/\omega_A)^2 = 4.237$, and the $n = 2$ TAE mode frequency is given by $(\omega/\omega_A)^2 = 4.759$. For the $n = 1$ TAE mode the dominant poloidal harmonics of the plasma displacement eigenfunction $\xi_\psi (= \xi\cdot\nabla\psi)$ are $m = 1, 2, 3$ which are localized around $r = 0.5 - 0.7$. For the $n = 2$ mode the dominant poloidal harmonics of $\xi_\psi$ are $m = 3, 4, 5, 6, 7, 8, 9, 10$ which have large amplitudes from $r = 0.5$ to the edge.

In the ORBIT code calculation we launch 512 alpha particles with birth energy $(3.52$ MeV) at $\psi = 0.025$. Initially, they are randomly distributed and have a uniform distribution in pitch angle, $\theta$, and $\phi$. Figure 6 shows the transport behavior of alphas due to both the $n = 1$ and the $n = 2$ TAE modes with equal maximum amplitude $8B_r/B = 10^{-3}$. As in the single toroidal mode case, the dominant alpha loss process shown in Figure 6(a) is that particles resonating with the TAE modes are transported in phase space and converted into fat-orbit transition particles at the trapped/passing boundary (near pitch $\lambda = v_\perp/\nu \approx 0.5$) and lost to the wall. The lost alphas are mostly concentrated just below $\theta = 0$ as shown in Figure 6(b), and this will cause a very localized alpha heat load at the midplane on the outboard side of the wall. Figure 6(c) shows the number of alphas lost versus time in units of transit times, and a large alpha fraction (≈ 25%) escape due to near boundary transient loss and prompt loss occurring in 30 transit times. This is because the loss cone is greatly widened by finite amplitude perturbations of the TAE modes near the plasma edge due to large orbit width ($\Delta_B/a \approx 1/2$) and large Shafranov shift ($\Delta/a = a/R$) for the TFTR equilibrium. After the initial transient loss, the alpha loss is dominated by the diffusive process as shown in Figure 6(d), which plots $(<\delta P_\phi>^2)$ -
FIG. 6. Transport behavior of alphas due to both \( n = 1 \) and \( n = 2 \) TAE modes with equal maximum amplitude \( \delta B_r/B = 10^{-3} \) in a supershot TFTR equilibrium. (a) Number of particles lost \( N_L \) versus particle pitch \( \lambda = v_{\perp}/v \); (b) \( N_L \) versus poloidal angle \( \theta \); (c) total \( N_L \) versus time \( t/\tau_i \) (\( \tau_i = 2\pi R/V_{\phi} \)); (d) \( \langle \delta P_{\phi}^2 \rangle - \langle \delta P_{\phi} \rangle^2 \) versus \( t/\tau_i \).

\( \langle \delta P_{\phi}^2 \rangle - \langle \delta P_{\phi} \rangle^2 \) versus time, in units of transit time. There is a large fraction of diffusive loss (\( \approx 25\% \)) in 500 transit times, which roughly corresponds to \( 10^{-3} \) slowing down time. Since the q-profile is sharply increasing near the plasma edge, there are many closely packed resonant surfaces near the plasma edge, and overlap of resonant surfaces is much easier. Therefore, the stochastic threshold is greatly reduced compared with previous studies. The stochastic threshold, which is determined at the onset of diffusive loss, is \( \delta B_r/B \approx 2 \times 10^{-4} \).
**Diffusive Loss**

\[ N_{\text{Loss}} = D_{\text{WL}} \]

**Diffusion in Phase Space**

\[ D_{\varphi} = D\langle \Delta P_\varphi^2 \rangle / Dt \]

**FIG. 7.** (a) Total number of diffusively lost particles \( N_{\text{diff}} \) versus maximum TAE mode amplitude \( \delta B_r/B \) for \( n = 1 \) TAE alone, for \( n = 2 \) TAE alone, and for both \( n = 1 \) and \( n = 2 \) TAE modes. The corresponding toroidal angular momentum diffusion \( D_{\varphi,\varphi} = d\langle \Delta P_\varphi^2 \rangle / d\langle \delta P_\varphi^2 \rangle \) is shown in (b).

for the \( n = 1 \) TAE mode and is \( \delta B_r/B = 10^{-4} \) for the \( n = 2 \) TAE mode. The stochastic threshold is further reduced for multiple-\( n \) perturbation. Admitting both the \( n = 1 \) and \( 2 \) TAE modes with equal maximum amplitudes, the stochastic threshold is \( \delta B_r/B = 2 \times 10^{-5} \). The total number of diffusively lost particles versus maximum TAE mode amplitude \( \delta B_r/B \) is shown in Figure 7(a) for the \( n = 1 \) TAE alone, the \( n = 2 \) TAE alone, and for simultaneous \( n = 1 \) and \( 2 \) TAE modes. The corresponding toroidal angular momentum diffusion \( D_{\varphi,\varphi} \) versus maximum TAE mode amplitude \( \delta B_r/B \) is shown in Figure 7(b). The results clearly show the enhancement of alpha loss in the presence of multiple-\( n \) TAE modes.

Finally, the alpha transport due to TAE modes has also been simulated by the TAE/FL gyrofluid code [13] by including the \( \vec{E} \times \vec{B} \) convective
The code is based upon the technique of choosing appropriate closure relations for truncating moment equations in order to introduce Landau damping/growth effects. The equations for the energetic species are then coupled to a reduced MHD description for background plasma and are solved in an initial value time evolution code. The results demonstrate the rise and crash phases of TAE activity similar to experimental observations. The saturation is caused by generation of $m=0$, $n=0$ component through nonlinear beating of $n \geq 1$ modes; these couplings cause modifications of the original equilibrium profiles in such a direction as to decrease the instability drive. For example, in Figure 8 we show typical nonlinear modifications of the fast particle density profile. The peak magnetic fluctuation level increases with increasing energetic species beta, resulting in nonresonant stochastization of magnetic field lines (and profile flattening) above a certain beta. Comparison of the edge poloidal magnetic field fluctuations from the nonlinear simulation with the Mirnov
signals from the TFTR TAE experiments shows similarities in the peak frequency and width of the spectrum.

3. Conclusion

In the paper we have investigated alpha particle driven TAE instabilities for the proposed TFTR D-T and ITER experiments by using the NOVA-K code. Based on the TFTR supershot operational scenario, TAE instabilities can be observed for beam power less than 23 MW which corresponds roughly to 0.7% volume averaged plasma beta. For higher beam power operations, TAE instabilities can be excited by turning off the beam power, after which the core plasma beta, electron density, and ion temperature decrease rapidly, but the alpha particle beta decreases with a much slower rate and remains above the threshold of the TAE instabilities for several hundred msec. For ITER, the critical alpha particle beta for the TAE instabilities is on the order of $10^{-3}$ which is smaller than the expected volume averaged alpha beta values of 0.5 - 1%. The ORBIT code calculation shows that the stochastic amplitude threshold for TAE perturbation with a single toroidal mode ($n=1$ or 2) is roughly $\delta B_r/B = O(10^{-4})$ for alpha particles in TFTR D-T supershot experiments, but is one order of magnitude smaller for TAE perturbations with multiple toroidal mode numbers (both $n = 1$ and 2). In the presence of both the $n = 1$ and 2 TAE modes with equal maximum amplitude at $\delta B_r/B = 10^{-3}$, there is a large fraction of diffusive alpha loss ($\approx 25\%$) in 500 transit times ($\approx 10^{-3}$ slowing down time) in addition to the transient loss of 25%. These results may be applicable to ITER experiments, and such an alpha loss level is unacceptable for a fusion reactor.

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REFERENCES


DISCUSSION

H.L. BERK: Do you take into account the continuum damping, the $E_0$ collisional effects and the beam-like character of the energetic particles in your estimates of stability thresholds?

C.Z. CHENG: The continuum damping and electron collisional damping effects are included in our NOVA-K and NOVA-R models. These two damping effects are not important in our studies of low n TAE modes for TFTR D–T and ITER experiments. The non-thermal beam ions (which account for about one quarter to one third of ion beta) are not included in the present calculations. However, the non-thermal beam ions are probably destabilizing to TAE modes. Therefore our prediction of the $\langle \beta_n \rangle$ threshold is conservatively higher than it should be.
THEORY OF TOROIDICITY-INDUCED ALFVÉN EIGENMODES IN A FINITE-\(\beta\) ARBITRARY CROSS-SECTION TOKAMAK, AND ANALYSIS OF NONIDEAL EFFECTS, STABILITY THRESHOLDS, AND NONLINEAR BEHAVIOR

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Abstract

THEORY OF TOROIDICITY-INDUCED ALFVÉN EIGENMODES IN A FINITE-\(\beta\) ARBITRARY CROSS-SECTION TOKAMAK, AND ANALYSIS OF NONIDEAL EFFECTS, STABILITY THRESHOLDS, AND NONLINEAR BEHAVIOR.

The toroidicity-induced Alfvén eigenmode (TAE) has been predicted to be driven unstable by the \(\alpha\)-particles in a fusion plasma. Significant results in two different areas relevant for this instability are presented. The first is a detailed numerical study of the magnetohydrodynamic properties of these modes in a general cross-section, finite-\(\beta\) tokamak. The second is recent analytical advances in describing the mode structure, evaluating thresholds (with stability predicted for ITER parameters), and assessing nonlinear saturation of the TAE, as well as the discovery and analysis of a concomitant mode, the kinetic TAE.

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1. MHD PROPERTIES OF TAE MODES IN A NONCIRCULAR FINITE-$\beta$ TOKAMAK

A. Alfvén Gap Structure

Toroidal Alfvén Eigenmodes (TAEs) are global magnetohydrodynamic (MHD) modes with their real frequency inside the gaps of the frequency continuum of the Alfvén waves. TAEs have been predicted to be driven unstable by the $\alpha$-particles in a fusion plasma [1] and in turn can cause deleterious loss of confined $\alpha$-particles. For a low-$\beta$, large aspect ratio, circular tokamak, the gap in the Alfvén continuum is formed by the nonuniformity of the magnetic field strength on a flux surface. For a general cross-section tokamak, other plasma equilibrium properties such as elongation, triangularity, plasma $\beta$, and coupling to the sound waves also affect the Alfvén wave spectrum.

The general equations that describe the Alfvén wave-sound wave system on a tokamak flux surface have been given elsewhere [2]. It results in a complete (yet very complicated) frequency diagram. It is simplified, first, by invoking the Floquet theorem to locate the upper and lower bounds of the Alfvén and sound wave continua and, second, by selecting only waves with the Alfvén wave polarization. The following pair of equations that describe the gap locations of the Alfvén continuum are thus obtained [2,3]:

$$\frac{\omega^2 \rho |\nabla \psi|^2}{B^2} Y_1 + \frac{1}{J} \frac{\partial}{\partial \theta} \left( \frac{|\nabla \psi|^2}{B^2 J} \frac{\partial}{\partial \theta} Y_1 \right) + \gamma_\alpha \kappa \rho Y_2 = 0$$

$$\kappa Y_1 + \left( \frac{\gamma_\alpha \rho + B^2}{B^2} \right) Y_2 + \frac{\gamma_\alpha \rho}{\omega^2 \rho J} \frac{\partial}{\partial \theta} \left( \frac{1}{B^2 J} \frac{\partial}{\partial \theta} Y_2 \right) = 0$$

with $Y_1 (\theta + 2\pi) = Q_1 Y_1(\theta)$, $Q_1 = \pm 1$, $Y_2 (\theta + 2\pi) = Q_2 Y_2(\theta)$, and $Q_2 = \pm 1$. In here $Y_1$ is the plasma displacement in the flux surface perpendicular to the magnetic field and $Y_2$ is the divergence of the displacement vector.

These equations are independent of the toroidal mode number $n$. A typical diagram of the gap structure and its relation to the Alfvén spectrum is given in Fig. 1 for a high-$\beta$ equilibrium in DIII-D geometry. The dotted curves are the odd-odd ($Q_1 = Q_2 = -1$) and even-even ($Q_1 = Q_2 = 1$) envelopes. The solid lines are the Alfvén wave spectrum for $n = 3$ modes, and the vertical lines are the location of the toroidal gaps. Systematic studies of these diagrams showed the following features: for highly noncircular plasma, the first (toroidicity-induced) gap is widened by effects associated with the finite $\beta$-induced Shafranov shift. This gap results from the coupling of the $m$-th and $m+1$-th poloidal harmonic components. Coupling of the sound wave and the Alfvén wave creates a zeroth gap below the usual gap. The size of this gap is proportional to the local $\beta$ value of the plasma surface. Ellipticity and higher order toroidal effects create a wide second gap by coupling the $m$-th and $m+2$-th components. Plasma triangularity creates a third gap through
the coupling of $m$-th and $m+3$-th components, etc. These lower (zeroth) and higher (2nd and 3rd) order gaps allow the possibility of existence of other global MHD modes in the appropriate frequency ranges of the gaps. For up-down symmetric configurations, the gaps are always bounded by odd-odd or even-even envelopes.

B. Frequency and Mode Structure of Low-$n$ TAE Modes

The GATO ideal MHD stability code, which minimizes the potential energy according to a variational formulation, has been used to isolate and calculate the low-$n$ stable global eigenmodes. The existence of the TAE mode and its associated gap has been verified for a general cross-section finite-$\beta$ tokamak. The eigenfrequencies and eigenmodes obtained from this variational calculation are found to be in good quantitative agreement with those obtained from the nonvariational NOVA code [2]. In noncircular cross-sections, coupling of poloidal mode numbers $m$ and $m+2$ produces global ellipticity-induced Alfvén eigenmodes (EAE) in the second continuum gap. Higher order toroidal effects are found to couple poloidal numbers $m$ with $m+2$ to produce higher order TAE modes in the second gap. Toroidal coupling between the TAE modes associated with different $q$ surfaces or between the TAE and the Alfvén continuum further induces splitting of the TAE modes into two or more global modes at slightly different frequencies [4,5]. A systematic survey of the stable continuum has further revealed a surprising diversity in the structure of the
continuum Alfvén modes. Modes with toroidal mode number \( n = 1, 2, 3 \) are readily obtained. A typical mode structure for an \( n = 3 \) Alfvén eigenmode is shown in Fig. 2. The rich harmonic content of the eigenfunction attests to the width and extent of the gap structure which covers a large range of the poloidal mode numbers. The computed frequency and mode characteristics are in agreement with the DIII–D experimental observations reported at this conference [6].

C. High-\( n \) TAE Modes

The equations for high-\( n \) MHD modes at finite frequency have been used to study the high-\( n \) TAE modes. They are a pair of second order differential equations over the extended poloidal angle \( \theta \) on a flux surface. In the limit \( \theta \rightarrow \infty \), these equations reduce to Eq. (1), which determines the gap structure. The condition for the existence of undamped high-\( n \) TAE modes is thus that its frequency falls inside the envelopes of the gaps.

High-\( n \) TAE modes have been found to exist in the Alfvén continuum gap [3]. At low shear, the modes are located at the bottom of the gap. At high shear, they are located near the top. Finite \( \beta \) moves the mode toward

Fig. 2. Global TAE mode structure computed by the ideal MHD code GATO for the equilibrium shown in Fig. 1. (a) 2D mode structure, (b) Fourier amplitudes of displacement across (\( \xi_\varphi \)) and in (\( \xi_\chi \)) the flux surfaces.
the bottom of the gap, until its frequency is moved outside the gap and the mode disappears. Even at high \( \beta_N = \beta/(1/aB) = 3.5 \), the mode is still present in DIII-D geometry. In general, it is located outside the ballooning unstable region. In DIII-D discharge geometry near the \( \beta \) limit, \( \beta_N = 3.5 \), the high-\( n \) TAE modes remain present although ideal ballooning modes are calculated to be stable. The mode in the first gap typically decays to 0 at \( \theta \to \pm \infty \) with a period of \( 4\pi \). High-\( n \) TAE modes have also been found in the second gap. Typically, these have a period of \( 2\pi \) in \( \theta \). These should be contrasted with the ideal MHD ballooning modes with their frequency situated below the zeroth gap and their eigenfunctions decaying relatively nonperiodically to zero as \( \theta \to \pm \infty \).

D. Alfvén Continuum Damping

The global TAE mode can resonantly excite a localized Alfvén wave in the continuum when their frequencies coincide. This resonance results in phase mixing of the Alfvén waves near the resonance, channels energy away from the TAE, and contributes to its damping. The detailed damping mechanism depends on the specific plasma model adopted for the Alfvén resonance layer (see next section). However, the amount of damping depends only on the energy flux impinging on the resonance layer and is independent of the detailed dissipation process. This phase mixing, occurring in real space, can be formulated in direct analogy to the Landau damping process in velocity space. For this purpose, the MHD equations are first cast into a Hamiltonian form with the radial variable \( r \) taking the role of time. The equations are generalized to the complex plane in both \( r \) and the frequency \( \omega \) [7].

For a given \( \omega \), the complex Alfvén wave singularities \( r_i \) are treated through analytic continuation of the profiles into the complex plane, with the integration path chosen to go around the Alfvén singularities so as to satisfy causality. When the continuum dampings are weak, i.e., \( \text{Im}(\omega) \ll \text{Re}(\omega) \), analytic integration can provide jump conditions in the TAE mode amplitudes across these Alfvén singularities. Numerical tests on model equilibria reveal that the jump conditions are excellent approximations to the causal paths with the \( \{r_i\} \)'s. Thus, with the inclusion of these jump conditions, integration can again be performed on the real \( r \) axis.

An analytic model given in Ref. 8 was studied for the effect of continuum damping on TAE modes. It is found that, for a large aspect ratio circle, the damping is proportional to the inverse aspect ratio. For Alfvén wave speed and \( q \) profiles that give rise to multiple gaps, it is found that the number of independent TAE modes is equal to the number of gaps. A TAE mode that resonates with the Alfvén continuum both at the center and at the plasma edge is usually more heavily damped than a mode that resonates either only at the plasma center or the plasma edge. Application of this method to more general equilibria is in progress.
2. ANALYTICAL STUDIES OF TAE AND KTAE

A. Kinetic and Nonideal Effects

The ideal-MHD description of the TAE does not treat nonideal-MHD or kinetic effects, e.g., due to α-particle destabilization or core plasma dissipation mechanisms, which are essential for determining marginal stability conditions and, depending on parameters, also the mode structure.

A theory [9] has been developed that takes into account the interaction of the TAE with the kinetic Alfvén wave (KAW) and treats the electron parallel dynamics nonperturbatively. This analysis indicates that, for an extended range of parameters of relevance both to current-day fast beam experiments and proposed burning plasmas, the mode structure may take on a strongly kinetic character, leading to significant damping. The magnitude of the effect depends on the effective parallel conductivity $\sigma_e$ (which gauges the electron parallel dynamics) and other parameters. The analysis was carried out for the important special case of a single gap (i.e., toroidal coupling between only two poloidal harmonics) to illustrate the salient physical features of the overall mode that arises from the interaction of the ideal-MHD TAE and the KAW and to show when the nonperturbative effects may dominate.

The single-gap analysis also reveals a new mode [9], created by the toroidicity-induced coupling of two KAWs. This so-called kinetic toroidal Alfvén eigenmode (KTAE) is the lowest eigenstate of the continuum discretized by electron parallel dynamics (since higher-order radial eigenmodes are heavily damped). Its mode structure is similar to that of the TAE, although the two coupled poloidal harmonics are opposite in phase. Its damping rate scales as $\sigma_e^{-1/2}$ (same as for a KAW) and can be less than that for the TAE even for moderate mode numbers. The extension to multiple gaps is underway.

Numerical calculations are needed to obtain proper quantitative results with this theory, especially for low-$n$ modes (which may be the least damped) for which the details of the plasma profiles are essential. A fully-kinetic (including finite Larmor radius effects) toroidal numerical code was developed to study antenna-driven Alfvén waves. An earlier version [10] was extended to include coupling to an arbitrary number of poloidal harmonics. The code solves the full set of kinetic equations globally (to order $r/R$) without asymptotic matching. Discrete modes are identified as resonances in the antenna impedance as the frequency is varied, and the resonance half-widths yield the damping rates (without α-particle drive). The code also provides information about the mode structure and energy-deposition profiles useful to plan heating experiments and transport studies, as well as to analyze stability in the presence of α-particles. For a two-gap case ($n = 2$, $m = 2, 3, 4$), with TFR parameters, two eigenmodes were found whose radial energy deposition, shown in Fig. 3, illustrates the interaction with the kinetic Alfvén branch. The frequency of the higher mode is near that of the $(2,3)$ gap and lies inside both ideal gaps,
FIG. 3. Energy deposition versus radius for two $n = 2$ TAE modes from kinetic code. Nominal gap positions are $r/a = 0.35$ ($m = 2, 3$) and 0.64 ($m = 3, 4$), denoted by light and heavy dashed markers, respectively. (a) Light line: $\omega = 5.12 \times 10^5$ sec$^{-1}$. The only ideal continuum point is $r/a = 0.84$ ($m = 4$). (b) Heavy line: $\omega = 3.45 \times 10^5$ sec$^{-1}$. The ideal continuum points are $r/a = 0.26$ ($m = 2$), 0.40 (3), 0.61 (3), 0.61 (3), and 0.81 (4).

with large nonideal damping near the (3,4) gap and KAW deposition near the $m = 4$ Alfvén resonance. The frequency of the lower mode is near that of the (3,4) gap; it exhibits two deposition lobes from highly damped $m = 2$ and $m = 3$ KAWs at Alfvén resonances near the (2,3) gap. The damping rate for each eigenmode is $\gamma/\omega = 5 \times 10^{-2}$, close to the single-gap analytic prediction for either gap.

B. Eigenmode Structure

An asymptotic boundary-layer theory [8] was developed to describe the linear, radially global eigenmode structure for a large-aspect-ratio, low-$\beta$ tokamak plasma with multiple TAE gaps. This theory is valid for arbitrary toroidal ($n$) and poloidal ($m$) mode numbers; in the large-$n$ limit, a global ballooning-like mode profile is recovered. Also, the ideal MHD eigenmode equation of Ref. 8 was generalized to include nonideal effects.
The mode structure is derived by matching the solutions from two regions. In the outer-layer region, the eigenmode is described by the reduced ideal MHD equations with no toroidal coupling. For real frequencies \( \omega \), these solutions are logarithmically singular at points where the shear Alfvén resonance condition \( \omega^2 = k_{s,m,n}^2 v_A^2 \) is satisfied, with parallel wave number \( k_{s,m,n} = (n - m/q)/R \).

An inner-layer region arises when the frequency at such a singular point is degenerate, i.e., \( k_{s,m,n}^2 (r_m) = k_{s,m+1,n} (r_m) \), where \( q(r_m) = q_m = (m + 1/2)/n \) and \( \omega = \omega_m = v_A(r_m)/2 q(r_m) R \). Finite toroidicity effects resolve the degeneracy at this point, leading to a narrow, peaked eigenfunction, so that nonideal MHD effects (due to parallel electron dynamics and finite Larmor radius) also can be important within the inner layer.

Near each Alfvén singularity, the individual poloidal harmonics \( \Phi_m(r) \) of the perturbed electric field potential experience a rapid change in amplitude \( \Delta \Phi_m \), whereas the “flux” \( C_m(r) \approx r^3 (\omega^2/v_A^2 - k_{s,m}^2) (\partial/\partial r) (\Phi_m/r) \) varies slowly. Thus, if the \( C_m \) values are known at the degenerate singular points (call them \( C_{m+1}^{-} \) and \( C_m^+ \) at \( r = r_m \)), the eigenfunction is determined within the inner layer. For example, in low-\( \beta \) ideal theory, the functional form is essentially determined by

\[
\frac{d^2 \psi_m^{+}(k)}{dk^2} - \left[ 1 - (g_m - \tau_m k^2)^2 - 2 i \tau_m k \right] \psi_m^{+}(k) = -i C_m^{+} \delta^+(k)
\]

Here, \( \tau_m = (8 n^2 S_m^2 q_m^2 / \varepsilon^3 r_m^2) [3 \rho_i^2 / 4 + \rho_e^2 (1 - \lambda)] \) represents the nonideal effects, with \( \rho_i \) the ion Larmor radius, \( \rho_e^2 = \rho_i^2 (T_e/T_i) \), \( S_m \) the shear, and \( \lambda \) due to electron collisions (typically \( \lambda \ll 1 \)). If \( \text{Im} \tau_m / (1 - g_m^2) \ll 1 \), the coupling coefficients are \( \alpha_m = -g_m + i \exp [-2 f(g_m) / (\tau_m^2)] / (1 - g_m^2)^{1/2} + \tau_m / 2 (1 - g_m^2)^{5/2} \), etc., with the tunneling factor \( f(g_m) = \int_0^{(1 + g_m)^{1/2}} dk [1 - (k^2 - g_m)^2]^{1/2} \).

Matching the inner and the outer solutions leads to a generalization of the previously-derived three-point recursion relation for the \( C_m \)'s, but now with \( \alpha_m \) and \( \beta_m \) involving nonideal effects.

There also arise additional modes, the KTAE [9]. For example, if \( (2 \ell + 1)/r_m < 1 \), with \( \ell \) a non-negative integer, the KTAE roots (which are relatively insensitive to coupling to the outer-layer region) are approximately given by \( (\omega - \omega_m)/\omega_m \approx (\varepsilon/4) [1 + (r_m/2)^{1/2} (2 \ell + 1)] \). The different frequency scalings
for the TAE and KTAE modes can be illustrated for the single-gap case with a constant density profile, for $n = 1$, for which the three-point recursion relation of the asymptotic matching theory reduces exactly to the dispersion relation $\alpha_m + \Delta = 0$, with $\Delta$ a number whose exact value depends on the $q(r)$ profile. The solution for the complex eigenfrequency as a function of $|\tau_m|$ for $\tau_m = |\tau_m| e^{-0.1i}$ and $\Delta = -0.5$ is shown in Fig. 4. The TAE branch exhibits linear scaling, viz., $2\gamma/\dot{\epsilon} \omega_m = \text{Im}(\tau_m)/2(1 - g_m^2)$, for $|\tau_m| < 0.02$; exponential scaling for $0.02 < |\tau_m| < 0.4$, viz.,

$$\frac{\gamma}{\omega_m} = -\frac{\dot{\epsilon}(1 - g_m^2)^{3/2}}{2} \exp \left[ -\frac{2f(g_m)\tau_m^{-1/2}}{2} \right]$$

and $2\gamma/\dot{\epsilon} \omega_m = (1/2)\text{Im}(\tau_m e^{-i\alpha})^{1/3}$ for $|\tau_m| \gtrsim 3$. The KTAE branch scales as $\tau_m^{1/2}$ for $|\tau_m| < 0.1$, with nonlinear scaling (due to tunneling) for $|\tau_m| > 0.5$.

![Figure 4](image-url)

**Fig. 4.** Damping rate for the TAE and lowest KTAE as a function of $|\tau_m|$, the nonideal parameter, for the $n = 1$ single gap case.
At small $|r_m|$, the TAE branch is less damped. For $|r_m| > 1$, the TAE is more strongly damped than the KTAE branch. Note that $r_m \propto (q_a - q_0)^{5/2}$ for given temperature, mode number, and size, when $q(r) = q_0 + (q_a - q_0)/(r/a)^2$; hence the value of $r_m$ for DIII-D ($q_a \simeq 6$) is typically an order of magnitude larger than in TFTR ($q_a \simeq 3$). The more general problem of the TAE-KAW interaction for multiple gaps is being studied.

C. Stability Threshold

Estimates of energetic particle destabilization must take into account that their orbit size can be comparable to or larger than the eigenfunction scale length. This was done for the TAE in the $|r_m| \ll 1$ limit, where the solution of the ideal MHD three-term recursion relation for the eigenfunction and associated $C_m$'s of Ref. 8 were used. The solutions experience continuum mode damping in lowest order. These solutions can be employed perturbatively to obtain the total growth rate that results from the $\alpha$-particle instability drive and the background plasma dissipation mechanisms. In general, the growth rate is given by

$$\gamma = -\gamma_c + \int \frac{d^2 r}{d^2 r |B|^2/4\pi} = -\gamma_c + \sum_{m,j} \gamma_m^{(j)} \lambda_m \sum_m \lambda_m$$

where $-\gamma_c$ is the zeroth-order damping rate, $\lambda_m$ is a quadratic form in the $C_m$'s with $\sum_m \lambda_m$ the wave energy, and $\gamma_m^{(j)}$ is the growth rate due to the $j$-th instability source if only a single resonance pair of poloidal harmonics is excited.

Various contributions $\gamma_m^{(j)}$ are as follows:

1. Destabilization, $\gamma_m^{(\alpha^+)}$, from the $\alpha$-particle profile spatial gradient:
   For the important case when $\varepsilon r_m/4 m S(r_m) < \Delta_b < r_m/m S(r_m)$, where $\Delta_b = q_m v_i (1 + v_i^2/v_a^2)/\omega_{a0}$ is the $\alpha$-particle orbit excursion from a flux surface, this is given approximately by
   $$\gamma_m^{(\alpha^+)} = 5 q_m^2 (1 - g_m^2)^{1/2} r_m \left( \frac{d\theta_\alpha}{dr} \right) \left( \frac{v_A}{v_{\alpha0}} \right) \left( 1 - \frac{v_A^2}{v_{\alpha0}^2} \right) H \left( 1 - \frac{v_A}{v_{\alpha0}} \right)$$

2. $\alpha$-particle damping, $\gamma_m^{(\alpha^-)}$, from the energy gradient of a slowing-down distribution: Roughly, this is given by $\gamma_m^{(\alpha^-)} = -\gamma_m^{(\alpha^+)}(\omega/\omega_{\alpha}).$

3. Ion Landau damping [11], $\gamma_m^{(i)}$, from ion interaction with the magnetic curvature at the wave-particle sideband resonance $v_i = v_A/3$. 

Note that $\gamma_m^{(\alpha^+)}$ is independent of the mode number $n$. If $v_A < v_{\alpha0}$ (alpha birth velocity), there are additional, smaller contributions. [For calculations, a more general formula is used, which is valid when $\Delta_b < r_m/m S(r_m)$].
4. Collisional electron damping due to magnetic curvature [12], $\gamma_m^{(e,c)}$, and parallel electric field [13], $\gamma_m^{(e,n)}$, effects (the latter is here newly calculated for the TAE mode structure):

$$\frac{\gamma_m^{(e,c)} + \gamma_m^{(e,n)}}{\omega_m} = -0.52 \left\{ \frac{6.7 \beta_e q_m^2}{\varepsilon^2 (1 - g_m^2) \nu_m^2} + \frac{16 m^2 g_m^2 \rho_m^2}{1 + \left( \nu_e / \omega_m \right)^{1/2}} \right\} \times \frac{1}{\left[ 1 + 0.25 \ln \left( 1 + \frac{\omega_m \nu_m}{\nu_e R} \right) \right]^{3/2}}$$

with $\nu_e = 4 \pi n_e e^4 \ell n \Lambda / m_e^{1/2} (2 T_e)^{3/2}$. Since $\gamma_m^{(e,n)} \propto -m^2$, this contribution can cut off high-mode-number instabilities.

5. Ideal continuum mode damping [7,8]: this is part of $\gamma_e$.

6. Radiative damping due to coupling with kinetic Alfvén waves, $\gamma_m^{(n)}$: In the limit $|r_m| \ll 1$, this is given by Eq. (4). Since this scales as $m^2$, it, along with $\gamma_m^{(e,c)}$, is the principal dissipation mechanism at high $m$.

Equation (5) was evaluated numerically, with self-consistent eigenvalues and eigenfunctions. A profile for the slowing-down $\alpha$-particles based on fusion cross-sections with $T_e = T_i$ was used for ITER-like parameters: $R = 6$ m, $a = 1.9$ m, $B = 4.85$ T, with $n(r)/n_e(0) = 1 - (r/a)^4$, $T_e(r)/T_e(0) = \exp(-4 r^2/a^2)$, and $q(r) = 1 + 2 (r/a)^2$. Background dissipation mechanisms were found to be competitive with the $\alpha$-particle drive. Even though a local estimate may indicate instability, the global eigenfunction calculations do not, at least for $T_e(0) \approx 25$ keV. Figure 5 shows the ratio of the dissipation rate to the $\alpha$-particle destabilization rate, as a function of peak $\beta$ value, for an $n = 3$ TAE that tends to be mainly localized at the $m = 4/m = 5$ gap. Radiative damping due to coupling to KAWs is especially influential in achieving stability for all parameters. At higher $\beta$ values, ion Landau damping is also significant for stabilization; and at lower $\beta$ values, $\nu_A$ becomes larger than $\nu_{e0}$, which greatly diminishes the $\alpha$-particle instability drive.

There are indications that elements of the damping and growth rates given in Eq. (5) correspond to the general trends of the experimental observations in DIII-D [6].

D. Nonlinear TAE Behavior

The nonlinear evolution of the $\alpha$-particle distribution has been analyzed [14] as a special case of the problem of a distribution function with a weak beam-like source. The latter is generic to many plasma situations and can be considered in the context of the paradigm of the bump-on-tail instability, as well as for the more complicated $\alpha$-particle–TAE wave interaction. Whether unstable waves lead to spatial diffusion of $\alpha$-particles is determined by whether stochasticity due to the perturbed fields develops in the particle orbits. Without stochasticity, the wave amplitude saturates at the natural level determined
Stabilized by Ion Landau Damping

Fig. 5. Ratio of the power transfer from all bulk plasma dissipation mechanisms to the $\alpha$-particle destabilizing power transfer as a function of the plasma central $\beta$ value, with various plasma temperatures (10 to 25 keV), for an $n = 3$ TAE. Stability occurs because this ratio exceeds unity.

by when the resonant bounce frequency of a particle in the wave field becomes comparable to the linear growth rate. The amplitude of the radial bounce motion, $\delta q$, of a particle in the wave field has the dependence $\delta q \propto \delta B_r^{1/2}$, where $\delta B_r$ is the magnetic field perturbation and $q(r)$, the safety factor, is used as the radial variable; in the limit $\Delta r_m/4mS_m < \Delta_\theta < \Delta r_m/mS_m$, the explicit result is $\delta q/q_m \approx 4(2S_m R\delta B_r/nr_m B)^{1/2}$. For a single TAE mode, the neighboring gap resonances are separated by $\Delta q \approx 1/n$. Hence, stochasticity is achieved if $\delta q > 1/n$. If there is a spectrum of modes, however, the overlap condition can be considerably less; roughly, $\delta q > 1/np$, with $p$ the number of unstable modes that interact with the particle.

Recent experiments [15,16] have observed pulsation behavior in a steady-state, weakly beam-driven plasma. Such pulsations lead to benign oscillations if the wave saturation occurs below the stochasticity threshold, since then
global $\alpha$-particle diffusion does not occur. However, if the saturation level is above the stochasticity threshold, a phase-space "explosion" is predicted, in which the diffusion process itself allows the release of free energy that pumps the wave amplitude. Ultimately this causes rapid diffusion and either flattening or loss of the $\alpha$-particle distribution function, consistent with rapid particle losses observed in the TAE experiments with fast ions simulating $\alpha$-particles. These bursts occur periodically, with a relatively long quiescent phase during which a flattened $\alpha$-particle distribution builds up with benign pulsations, approximately according to classical theory, to the point where a phase space explosion occurs and the distribution again flattens.

Mapping methods [17] were developed to provide a quantitative analysis of the nonlinear pulsations. The map to describe wave-particle interaction for one particle transit is based on linear theory, whereas the orbit nonlinearity is described by the use of a map over many transits. The map is now being generalized to incorporate both particle and wave dynamics.

3. CONCLUDING REMARKS

A detailed numerical description of the ideal-MHD eigenfrequencies and associated eigenfunctions of the TAEs is now possible with the use of two-dimensional, finite-$\beta$, arbitrary-aspect-ratio numerical codes, which include elliptical shaping effects as in the DIII-D tokamak. Effects of other relevant details of the plasma equilibrium, such as plasma flow and separatrices, and MHD modes in the zeroth gap or higher gaps should be explored. Continuum damping is being incorporated.

Nonideal and kinetic effects on toroidal Alfvén waves are very significant, leading to enhanced damping of the TAE, as well as the excitation of the KTAE, which may be less damped. The relatively strong dependence of the nonideal effects on global shear is likely to favor excitation of the KTAE and stabilization of the TAE in the higher-shear DIII-D experiments, but excitation of the TAE in the lower-shear experiments on TFTR. Also, our analysis for an ITER-like burning plasma, even with a low-shear profile, predicts the TAE to be stable, albeit marginally so. Definitive conclusions about present-day experiments and their implications for reactor plasmas, however, will require a clear identification of the relevant modes. A refined evaluation of the linear instability thresholds that includes geometrical, finite-$\beta$, nonideal, and beam anisotropy and orbit size effects is essential.

Nonlinear TAE studies have estimated the saturation amplitude and the stochastic particle diffusion threshold. Still needed is a joint, self-consistent treatment of the nonlinear evolution of the waves and the $\alpha$-particle distribution, for experimental parameters. Mapping methods may be utilized for this generalization.
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REFERENCES

DISCUSSION

F.B. MARCUS: Using your extended theory, how well do theoretical and experimental instability thresholds agree for DIII-D?

M.S. CHU: This question is answered in great detail in paper D-3-3, "Stability of TAE modes in DIII-D", by E.J. Strait et al. I might say, however, that the elements of the excitation and damping mechanisms described in the present paper adequately explain the experimental results obtained on DIII-D.

F. ENGELMANN: You find that a temperature limit of 25 keV must be reached in ITER to ensure stability. Is this the peak temperature or an average temperature?

M.S. CHU: This is the peak of the electron and ion temperatures assumed for the ITER like plasma used in the present study.

R.J. GOLDSTON: Naively one might expect a high aspect ratio tokamak to be more stable to TAE modes than a low aspect ratio device, since for fixed $\beta_n$, $T$ and $q$, the absolute $\beta_a$ is lower. Can you draw any — even tentative — conclusions?

M.S. CHU: This is a complicated question, and it is difficult to give a definite answer. There are two damping aspects of the TAE modes: one scales favourably for large aspect ratio and the other inversely. At large aspect ratio, the gap size is narrower, leading to stronger radiative damping. On the other hand, the coupling of the different poloidal harmonics will be weaker, implying weaker continuum damping. Therefore it would appear that at larger aspect ratio higher $n$ ($\geq 3$) modes would be more stable and lower $n$ ($\leq 3$) modes would be more unstable. Other factors to consider are the gap width and the density and temperature profiles.

Ya.I. KOLESNICHENKO: The region of TAE mode localization depends essentially on the $q(r)$ profile. The instability threshold depends strongly on the bulk plasma radial profile and the fast ion distribution function. Non-linear theory gives rough estimates for the diffusion coefficient. In view of this, it would appear that your theory helps interpret existing results but that its predictions for ITER and other machines are not reliable. Do you agree?

M.S. CHU: Yes, I do. A systematic study covering a range of plasma density and temperature profiles would enable us to obtain a reasonable estimate of the impact of TAE modes on the ITER device, but we are not yet at a stage where we can do this.
ANALYSIS OF THE LINEAR AND NONLINEAR DYNAMICS OF ENERGETIC ION DRIVEN MHD INSTABILITIES AND ELECTROSTATIC MICROTURBULENCE

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Abstract

ANALYSIS OF THE LINEAR AND NONLINEAR DYNAMICS OF ENERGETIC ION DRIVEN MHD INSTABILITIES AND ELECTROSTATIC MICROTURBULENCE.

In the gaps of the continuum Alfvén spectrum, discrete, global modes induced by toroidicity (the so-called toroidal Alfvén eigenmodes (TAEs) have been shown to exist. These modes could be strongly destabilized by the resonant interaction with the parallel motion of the energetic ions such as alpha particles produced in fusion reactions. The TAE stability is determined by the competition between such driving mechanisms and the coupling with the Alfvén continuum, which provides damping. In the paper the linear and nonlinear TAE dynamics are studied. The dependence of the damping rate on resistivity and on the interaction with the Alfvén continuum is discussed. A simple nonlinear model for the nonlinear evolution is also given. The nonlinear dynamics of the ion temperature gradient driven mode (ηi mode) at short wavelengths are discussed using gyrokinetic particle simulations. A preliminary analysis of the effect of noncircular equilibria is also presented.

I. THE HYBRID CODE

The numerical study of the effect of high energy particles on MHD modes requires the solution of the fluid equations for the bulk plasma and of the kinetic equations for the energetic particles. The reduced resistive magnetohydrodynamic (MHD) equations expanded to the third order in the inverse aspect ratio e=a/R and for a low-β [O(e^2)] bulk plasma [1] have been numerically solved. This is in fact the lowest order at which the toroidal corrections enter in the equations. The driving term is obtained by numerically solving the gyrokinetic equations for the high energy particles using the electric potential and the parallel component of the vector potential as obtained by the MHD part of the code.

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A cylindrical coordinate system \((R, \phi, Z)\) is used. Following the so-called low-\(\beta\) tokamak ordering, the magnetic field \(B\) and the component of the fluid velocity \(v_\perp\) perpendicular to \(\phi\) can be written as:

\[
B = R_0 \nabla \psi \times \nabla \phi + (I_0 + I) \nabla \phi + O(\varepsilon^2) B_3, \quad v_\perp = (R^2/R_0) \nabla U \times \nabla \phi + O(\varepsilon^3) v_A,\]

where \(\psi\) and \(U\) are stream functions. \(R_0\) is the major radius of the vacuum chamber, \(I_0 = R_0 B_0\), \(B_0\) is the vacuum magnetic field at \(R = R_0\), \(I = \varepsilon^2 I_0\), and \(v_A\) is the Alfvén velocity. Using the above definitions, the following equation for the evolution of the magnetic stream function is obtained:

\[
\frac{\partial \psi}{\partial t} = \frac{R}{R_0} \left( (\nabla \psi \times \nabla) U + \frac{B_0}{R} \frac{\partial U}{\partial \phi} - \eta J_\phi \right) - O(\varepsilon^4) v_A B_\phi
\]

where \(\eta\) is the resistivity and \(J_\phi = - (R/R_0) \Delta^* \psi\) with \(\Delta^* = \{R (\partial/\partial R) [(1/R) (\partial/\partial R)] + \partial^2/\partial Z^2\}\). The vorticity equation can be written as:

\[
\hat{\rho} \left( \frac{D}{dt} - \frac{2}{R_0} \frac{\partial U}{\partial Z} \right) \nabla^2 U + \nabla \cdot \left( \frac{D}{dt} - \frac{2}{R_0} \frac{\partial U}{\partial Z} \right) \nabla U = \nabla \Delta^* \psi + \frac{1}{R_0} \nabla \cdot (R^2 \nabla \cdot \nabla H) \times \nabla \phi + O(\varepsilon^4) \rho \frac{\partial}{\partial a^2}
\]

where \(\hat{\rho} = \{R^2/R_0^2\} \rho\), \(D/dt = [(\partial/\partial t) + \nabla \cdot V]\), and \(\nabla H\) is the energetic particle stress tensor. As a boundary condition we have assumed a rigid conducting wall at the plasma edge.

A previously existing code [2] which solves the \(O(\varepsilon^2)\) reduced-MHD equations has been modified in order to solve the \(O(\varepsilon^3)\) equations. The code is written using the toroidal coordinate system \((r, \theta, \phi)\), with \(r\) being the radial coordinate \((r=0\) correspond to the geometrical centre of the vacuum chamber\), \(\theta\) and \(\phi\) being respectively the poloidal and the toroidal angles. The code uses finite difference in the radial direction and Fourier expansion in the poloidal and toroidal directions. It uses a semi-implicit algorithm where all the linear terms that couple with the cylindrical part of the equilibrium (that is with poloidal and toroidal mode number \(m=0,n=0\)) are done implicitly.

The term which depends on \(\nabla H\) in Eq.(2) is calculated by solving the gyrokinetic equations at each time step using the fields (the magnetic flux function \(\psi\) and the stream function \(U\)) obtained from the evolution of the reduced-MHD equations. The optimal ordering for the energetic particle component is obtained by imposing the resonant condition between the mode frequency and the energetic particle transit frequency which yields \(v_H \sim v_A\). In addition the magnetic drift frequency must be smaller than the ion transit frequency in order to avoid detuning. Therefore the hot ion Larmor radius is ordered as \(\rho_H \sim \varepsilon a\), which yields \(\omega_{dH}/\omega \sim \varepsilon\). On the basis of such an ordering the quantity \(\varepsilon H\phi/\Omega H \sim 1\). The gyro-averaged equations are then obtained in the form:

\[
\frac{dR}{dt} = \left( p - \frac{a_{||}}{m_H} \right) b + \frac{e_H}{m_H \Omega_H} b \times \nabla \left( \frac{\phi}{\varepsilon_H} \right) + \frac{a_{\perp}^2}{2m_H e_H} \left( \frac{M}{m_H} + \frac{p^2}{\Omega_H^2} \right) b \times \nabla \ln B,
\]
\[\frac{dp}{dt} = -\frac{eH}{m_H} \left( b + \frac{p}{\Omega_H} b \times (b \cdot \nabla) b \right) \cdot \nabla \left( \phi - \frac{p a_\parallel}{e_H^2} + \frac{a_\parallel^2}{2 m_H e_H} \right) - \nabla \cdot \nabla \ln B\]

Here \( R \) is the gyro-centre position, \( M \) is the exactly conserved magnetic momentum and \( p \) corresponds to the canonical parallel momentum; the fluctuating fields \( \phi \) and \( a_\parallel \) are calculated at the position of the gyro-centre and are related to the stream functions. The hot-particle stress tensor components can be written, in terms of gyro-centre coordinates as

\[\Pi_{Hkj}(x) = \frac{1}{m_H^2} \int d^6Z \left[ \frac{M \Omega_H}{m_H} \left( \delta_{kj} b_k b_j \right) + \left( \frac{a_\parallel}{m_H} \right)^2 b_k b_j \right] F_H(t,Z) \delta(x - R)\]

where \( d^6Z \) includes the Jacobian of the transformation from canonical to gyro-centre coordinates. The phase-space distribution function \( F_H \) is determined, at each time step, by its \( \delta F \) particle representation, \( F_H = F_{EQ} + \sum_{i=1}^{N_{\text{particles}}} W_i(t) \delta(R - R_i(t)) \delta(M - M_i(t)) \delta(p - p_i(t)) \), where \( F_{EQ} \) is the equilibrium distribution function, independent of time, and the simulation-particle coordinates and weights are evolved according to the equations \( \frac{dZ_i}{dt} = \frac{\partial F}{\partial Z_i} \right|_{Z=Z_i(t)} \) and \( \frac{dW_i}{dt} = W_i(t) \left( \frac{\partial}{\partial t} \frac{dZ_i}{dt} \right) \right|_{Z=Z_i(t)} \) with the initial conditions \( W_i(0) = 0 \) and \( \Delta_i(0) = (V/2\pi N_{\text{particles}}) (\langle n_H(R_i(0)) \rangle / F_M(Z_i(0))) \). \( V \) being the simulation volume and \( n_H \) the equilibrium hot particle density.

Random initial particle distribution is chosen in the meridian plane, with inversion of the cumulative function corresponding to \( F_{EQ} \) for the velocity-space coordinates \( p \) and \( M \). This distribution is then exactly reproduced for each toroidal slice of the simulation domain, in order to preserve the equilibrium toroidal symmetry and avoid unphysical linear coupling between different-\( n \) modes. Volume-weighting interpolation function is adopted to assign the pressure contribution of each particle to the adjacent grid points, as well as to interpolate the fluctuating fields at the particle positions.

II. NUMERICAL SIMULATIONS OF THE LINEAR TAE DYNAMICS

In the following the kinetic term in Eq.(2) will be neglected, and only the linear evolution of the perturbations with Fourier components \( m/n=1/1, m/n=2/1 \) will be considered. The equilibrium used is described by the safety factor \( q(r) = q(0)(1.0 + r^{2\lambda} [q(a)/q(0)]^{2 - 1.0})^{1/\lambda} \) with \( q(0)=1.1, q(a)=1.9, \lambda=1 \). The global nature of the mode is revealed by a Fourier analysis in time of \( U_{m,n}(r,t) \) for several values of the radial coordinate \( r \), as discussed in Ref. [3]. In Figure 1 the \( m/n=1/1 \) and \( m/n=2/1 \) components of the TAE eigenfunction are plotted for a case with \( \varepsilon=0.1 \) and \( \eta=1 \times 10^{-4} \).
The effect of finite resistivity on the evolution of the TAE mode has been studied. It is found that the damping rate scales as $\gamma \sim \eta^{0.6}$ for values of $\eta$ lying between $3 \times 10^{-5}$ and $3 \times 10^{-4}$ and $\varepsilon = 0.1$. A simple analytical calculation shows that for $\eta/\varepsilon^2 << 1$, the damping rate scales linearly with $\eta$, as $\gamma \sim (\eta/\varepsilon^2)$, whereas for $\eta/\varepsilon^2 \gg 1$ the damping rate scales as $\gamma \sim \eta^{1/3}$. For the parameters of our numerical results the transition between the two regimes occurs for $\eta \leq 10^{-6}$; above this value of $\eta$, the scaling which is obtained numerically for the damping rate as a function of $\eta$, i.e. $\gamma \sim \eta^{0.6}$, agrees fairly well with the analytical prediction.
The interaction of the TAE with the Alfvén continuum can occur at the edge of the plasma column if a density profile which decreases toward the edge is chosen. Analytical expressions for the damping rate of the TAE induced by the interaction with the Alfvén continuum are known only in the high-n limit [4]. In the low-n limit, the damping rate is expected to scale approximately as $\gamma \sim |L_n|^{-\alpha}$, with $\alpha \sim 1 + 1.5$ and $|L_n|$ the scale length of the density gradient where the interaction with the Alfvén continuum occurs. In the following results a radial dependence of the mass density of the form $\rho = (1-(r/a)^4)^{1/3}$ has been assumed. In Figure 2 the damping rate of the TAE versus the local value of $1/|L_n|$, evaluated at the radial position where the interaction occurs, is shown for an equilibrium with with $q(0)=1.1$, $q(a)=1.7$, $\lambda=1$ and $\epsilon=0.1$.

III. THE NONLINEAR TAE DYNAMICS

In order to analyze the effects of nonlinearities, we take $\beta=\rho_0$ in Eqs.(1) and (2), and assume that each field $\psi, U$ is written as a superposition of Fourier modes $m/n=2/1, 1/1$ and $1/0$. Typically the nonlinear effects are important when the $1/0$ component of the perturbations, significantly modifies the toroidal $O(\epsilon)$ corrections to the cylindrical equilibrium. Hence the nonlinear effects are expected to be important in the 'gap region', along with the effects of the cylindrical equilibrium. The $2/1$ and $1/1$ components exhibit a variation over two time scales, a fast variation on the scale $\omega_A^{-1}$ and a slow variation over the scale $(\epsilon \omega_A)^{-1}$. The $1/0$ component, which is generated by the beating of the $2/1$ and $1/1$ components varies only over the slow scale. Thus, it is convenient to define $\frac{\partial}{\partial t} = -i\omega_0 - i\epsilon_0 \omega_0 \delta$, with $\epsilon_0=3r_0/2R_0$, $q(r_0)=3/2$, $\omega_0=\nu_0/3R_0$ and $\lambda=O(1)$. Defining $U=(2/\epsilon_0)^{3/2}U(1,1)/(\omega_0 r_0^2)$, $V=(2/\epsilon_0)^{3/2}U(2,1)/(\omega_0 r_0^2)$, the full nonlinear equations can be written as (let $s=r_0 q(r_0)/q(r_0)$)

$$8|V|^2U'' = (\lambda-2sx) U'' + V' - 2V(2V* U'' + V* U') - 4V(3V* U'' + V* U')$$

$$2|U|^2V''' = -C_V + (\lambda+4sx) V' + U' + 2U(U* V'' + 2U* V') - 2U(3U* V'' + 2U* V')$$

(5)

where $C_V$ is a constant determined by the integration of the linear, cylindrical equation of the $2/1$ component outside the gap, the prime denotes the derivative respect to $x=2(r-r_0)/|\epsilon_0 r_0|$. The dispersion relation of TAE modes can be written as

$$\delta U + \frac{2}{\epsilon_0} A_1 r_0 = 0 \quad , \quad V(a) + K_1 V + K_2 \delta \dot{\theta} = 0$$

(6)

The cylindrical solutions have been taken in the form $U=(2/\epsilon_0)^{3/2}A_1(r/a)$, $V=(2/\epsilon_0)^{3/2}A_2(r/a)^2$ for $r/a \rightarrow 0$ ; $V(a)$ is the solution at the plasma wall,
obtained by continuously jumping all singularities in the cylindrical equations, and $K_1$ and $K_2$ are constants numerically determined from the cylindrical equations, $\delta U$ and $\delta V$ are the jumps of $U$ and $V$ solutions across the gap region, $\delta \Phi$ contains the linear contribution due to the resonance of the $2/1$ component with the Alfvén continuum close to the plasma edge.

In the linear limit, $\delta U = (\pi s) C_v (8 - 9 \lambda^2)^{-1/2}$, $\delta V = (-3 \lambda \pi /4 s) C_v (8 - 9 \lambda^2)^{-1/2}$, with the linear continuum at $\lambda_c^2 = 8/9$. In the nonlinear case, the eigenvalue $\lambda$ is shifted down towards the lower linear continuum, and stays real until $|C_v|$ reaches a certain critical value $C_{vc}$. At $C_{vc}(\lambda)$ the TAE mode coalesces with the "nonlinear Alfvén continuum", and above this value the mode is expected to undergo a nonlinear continuum damping, in analogy with what happens in the linear case. In the small shear limit, the analytical estimate for $C_{vc}$ is $C_{vc} = [3 \lambda + (10 + 9 \lambda^2)^{1/2}] [-9 \lambda + 2(10 + 9 \lambda^2)^{1/2}]^{3/2} / (2x5^{5/2})$. The behaviour of the eigenvalue $\lambda$ vs. $C_v$ is shown in Fig.3.

IV. ION TRANSIT AND BOUNCE RESONANCE EFFECTS ON MHD MODES

In the presence of a population of energetic ions the internal kink mode can be destabilized by the resonance with the bounce averaged magnetic drift frequency of the hot ions $\omega_{dh}$. The dynamics of such a mode (fishbone mode) can be affected by the resonance between mode frequency and both the transit frequency ($\omega_{ti} = \nu_{ti} / qR$) and the bounce frequency ($\omega_{bi} = (e/2)^{1/2} \nu_{bi} / qR$) of the thermal ions, since, for typical ignition tokamak parameters, $\omega_{ti} = \omega_{dh} \geq \omega_{bi}$. Previous papers have considered the effect of the transit resonance only in the inertial layer region. In the present analysis the kinetic effects have been fully accounted for both in the
layer and in the external region by solving the drift kinetic equation for the thermal ions. The inclusion of the transit resonance in the region outside the inertial layer results in a further stabilization of the mode. Such a mechanism is particularly effective in the frequency range $\omega_H > \omega_{bh} > \omega_{bh}$. The resulting threshold value for the hot particle beta $\beta_h$ in order to destabilize the fishbone mode turns out to be $\beta_{hc} = 3^{1/2} \varepsilon s \omega_{bh} / \pi \omega_A$ 
\[ \{1 + \left[ \pi^{1/2} \omega_H / (4 \omega_{bh}) \right]^{1/2} + 2 \pi^{3/2} \beta_i^{1/2} \left(3s\right)\} \] 
In addition, the nonresonant response of trapped ions in the layer region enhances the inertia term by a factor $e^{-1/2}$ for $\omega_{dh} < \omega_{bh}$, yielding $\beta_{hc} = 3^{1/2} \varepsilon s \omega_{bh} / \pi \omega_A \{1 + \alpha/(2e)^{1/2}\}$, with $\alpha$ being a numerical factor of order unity.

V. GYROKINETIC SIMULATIONS OF THE $\eta_\ell$-MODE

Previous fluid-code simulations of $\eta_\ell$-mode turbulence have shown that strong reduction of the ion thermal flux can occur in correspondence to the formation of large scale coherent structures in the turbulent electrostatic potential. In order to investigate whether the inclusion of kinetic effects related to wave-particle resonances can modify such a phenomenology, particle simulations based on the gyrokinetic formalism have been performed with a 2-D electrostatic code on a cluster of 8 workstations IBM RISC SYSTEM/6000. The code computes the time evolution of the distribution function $F(t, R, M, U)$ in the gyrocentre-coordinate space, using the $\delta F$ algorithm: $F$ is represented in a partially discretized form, $F = F_M(R, M, U) + \sum_{i=1}^{N_{\text{particles}}} W_i(t) \delta(R - R_i(t)) \delta(M - M_i(t)) \delta(U - U_i(t))$, where $F_M$ is the maxwellian time-independent distribution function and the coordinates and weight-factors of the particles are evolved according to $\dot{Z}_i = (dZ_i/dt)$ and $W_i = -\Delta_i \left( (dZ_i/dt) \right) \quad Z_i(t)$ and $W_i = -\Delta_i \left( (dZ_i/dt) \right) \quad Z_i(t)$, with $dM/dt = 0$, $dU_i/dt = -(elm) \cdot V^0 \cdot \nabla \cdot F$ and

\[
\frac{dR}{dt} = U \cdot b + \frac{e}{m} \cdot b \times \nabla \cdot \Psi + \left( \frac{M}{m} + \frac{U^2}{\Omega} \right) \cdot b \times (\nabla \ln B) \quad \tag{7}
\]

Here $\Delta_i = (V/2n^2)_{\text{particles}} (n_0(R_i(0))/F_M(Z_i(0)))$, $b = B/B$, $V$ is the simulation volume, $n_0$ is the equilibrium density and $\Psi$ is the gyro-averaged electrostatic potential, given by

\[
\Psi = \Phi - \frac{e}{m \Omega^2} b \cdot \nabla \cdot \Phi = \nabla \cdot \nabla \cdot \Phi - \frac{e}{2m \Omega^2} b \cdot \nabla \cdot \Phi \times \Phi - \frac{e}{2 \Omega} \frac{\partial}{\partial M} \frac{\Phi^2}{2} \quad \tag{8}
\]

with $\Phi = \int_0^{2\pi} d\theta / (2\pi) \Phi$, $\phi = \int_0^{2\pi} d\theta / (2\pi) \Phi$, $\Phi = \langle \phi - \langle \phi \rangle \rangle$. The last term in the expression for $dR/dt$ accounts for a constant magnetic field curvature and is responsible for the destabilizing magnetic drift resonance.

The $\delta F$ algorithm yields two main advantages: it reduces the noise level in the simulation and allows for proper linear simulations, obtained by retaining the perturbed electrostatic field terms only in the weight-factor evolution. As a benchmark for the code, linear simulations have
been performed. A comparison with the result of linear-theory [5] shows a fairly good agreement in the mode frequency and \( \eta_i \)-threshold.

Figure 4 shows the time behaviour of the ion thermal flux for a non-linear simulation. After a linear stage, during which the ion flux reaches very high values, large-size coherent structures appear. Correspondingly the ion flux suddenly reduces to very low values.

VI. LONG WAVELENGTH \( \eta_i \)-MODE STABILITY OF SHAPED EQUILIBRIA

In the long wavelength limit \( (b=(k_x \rho)_D^2 q \tau (\tau e_T)^{1/2} < 1) \), two branches exist, a toroidal and a slablike branch [6]. Here we focus only on the toroidal branch. A model equilibrium with noncircular cross section given in Ref. [7] is employed, where the elongation \( \kappa_0 \), the variation of \( \kappa_0 \) with respect to the minor radius, \( \kappa_0' \), the Shafranov shift \( \sigma \) and the triangularity \( \delta \) are all taken into account. The eigenfunctions belonging to the toroidal branch exhibit a fast variation along the equilibrium field over the connection length scale \( (X=\chi_0=O(1)) \) with a superimposed slow variation over a secular scale \( (X=\chi_1=O(b^{-1/3})) \). In order to solve analytically the eigen-mode equation in the fluid limit, it is convenient to conjecture the following form for the eigenfunctions: \( \phi=\Sigma_n C_n(\chi) \cos(n \chi_0/2) + S_n(\chi) \sin(n \chi_0/2) \).

A set of coupled differential equations for \( C_n \) and \( S_n \) up to \( n=4 \) have been solved under the ordering \( \Omega=\omega/\tau(\omega+\tau \omega D_i)^{1/2}=O(b^{-1/3}) \) and \( \sigma^2=\kappa_0'^2-\delta^2=O(b^{-1/3}) \). Toroidicity induces an \( n=1 \) unstable toroidal branch, propagating in the ion magnetic direction with \( \Omega=\exp(i 2 \pi/3)/(8 bq^2 \tau)^{1/3} \), and a marginally stable mode with \( \Omega=(8 bq^2 \tau)^{-1/3} \), propagating in the electron diamagnetic direction[6]. The noncircular effects give only higher order
corrections to the $n=1$ eigenvalue. The case with constant elongation ($\kappa_0'=0$) can not induce any new branch. However, if $\kappa_0'
eq0$, an $n=3$ branch is induced with the modes propagating in both the ion and electron diamagnetic directions. The existence of new branches due to the contribution of the Shafranov shift $\sigma$ are also found for $n=2$ and $n=4$ with unstable ion modes and for $n=3$ with a marginally stable electron mode. Furthermore, all of the three harmonic ($n=2,3,4$) branches may appear when triangularity $\delta$ is considered. The kinetic analysis of the above branches has been carried out yielding, for $\eta_1$-threshold, the value $$\varepsilon_{\tau_c}=(kx_{\eta q}(\eta)/(1-2/\eta_1)/(1+1/\tau))^{1/2}.$$ 

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REFERENCES


DISCUSSION

H.L. BERK: For the comparison with alpha particle non-linearity one needs to look at phase space flattening. Have you made the comparison with this saturation mechanism?

F. ROMANELLI: The effect of the alpha particle non-linearity could be important, even though, using the present estimate for the saturation amplitude, it turns out to be smaller by a factor of $e^{1/2}$. 
SELF-REGULATED SHEAR FLOW TURBULENCE IN CONFINED PLASMAS: BASIC CONCEPTS AND POTENTIAL APPLICATIONS TO THE L–H TRANSITION

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Abstract

SELF-REGULATED SHEAR FLOW TURBULENCE IN CONFINED PLASMAS: BASIC CONCEPTS AND POTENTIAL APPLICATIONS TO THE L–H TRANSITION.

The paper describes developments in the theory of edge plasma turbulence in a differentially rotating plasma. The thesis that such systems are dynamically self-regulating is presented. Results indicate that relevant fluctuations will generate a predominantly curved flow. Similarly, curvature is shown to be the predominant flow profile effect on fluctuations. A system fixed point is identified, the eigenfrequencies for small oscillations around it are calculated, and an overall stability criterion is determined.

1. INTRODUCTION

Despite significant recent progress, the dynamics of the L–H transition in tokamaks remains an enigma. Furthermore, even in OH and L-mode plasmas, electric fields and fluctuations are universally observed to coexist at the plasma edge [1, 2]. The central thesis of this paper is that any creditable theory of the phenomena must be intimately intertwined with an understanding of how expansion free energy (i.e.
A self-regulating system of turbulence and profiled flow. Thus, one must consider flow evolution and fluctuation evolution on an equal footing, with neither identifiable as 'cause' or 'effect'. The simultaneous tendencies of: (a) the profiled flows to suppress turbulence and (b) the turbulence to regulate the flow profile combine to define a self-regulating, dynamic non-linear system. Expansion free energy is thus shared between fluctuations and flows, the imbalance of the shares defining the confinement regime. States of dynamical stationarity and non-stationarity (defined by $\omega_{\text{fluctuation}}^{\text{flow}} > 1$ and $\omega_{\text{fluctuation}}^{\text{flow}} \leq 1$, respectively) are possible. Moreover, multiple final states are attainable in the course of temporal evolution. Since, at the edge, fluctuations are (universally) large [1] and local, fluctuation induced ambipolarity breakdown is easily attained, it is difficult to understand how any theory of the L→H transition which does not involve the concept of self-regulated shear flows is viable.

In this paper, we report on recent developments in the theoretical principles for components of such a theory. Potential applications to the L→H transition phenomenon are then discussed. Section 2 discusses the effects of turbulence on flow profile evolution. The basic mechanisms for shear ($V_\theta$) and curvature ($V_\phi$) amplification by fluctuations are identified. Here shear and curvature identify a more general class of fluctuation spectrum distortions, namely shifts ($V_\theta$) and dilatations (related to $V_\phi$). Curvature ($V_\phi$) amplification due to diamagnetic propagation of the turbulence is seen to be the dominant effect. Also, a more general discussion of flow generation by convective cell 'tilting instability' is included. Such considerations may have broad impact on the non-linear dynamics of Rayleigh–Benard convection and dynamo theory. In Section 3, the effects of flow shear and curvature on fluctuations are analysed. Results of non-linear calculations and theory indicate that shear is rather ineffective in suppressing turbulence, in contrast to popular wisdom based on heuristic arguments [3] or linear theory [4]. Curvature is a non-linearly robust effect. In Section 4, the coupled, non-linear dynamical equations for curvature and fluctuation level are analysed. A stationary state is determined, and the complex eigenfrequencies of oscillation around this state are calculated. We propose this stationary state as a novel form of dynamical equilibrium intrinsic to edge plasmas. In Section 5, the potential relevance of this work to L→H phenomenology is discussed. Possible future extensions of this programme are mentioned, as well.

2. EFFECTS OF TURBULENCE ON FLOW PROFILE EVOLUTION

Simple considerations of momentum balance in a cylindrical plasma indicate that the radial profile of poloidal flow evolves according to

$$\left(\frac{\partial}{\partial t} + \mu\right) \langle V_\theta(t) \rangle = -\frac{\partial}{\partial r} \left[ \langle \tilde{V}_r \tilde{V}_\theta \rangle - \frac{\langle \tilde{B}_r \tilde{B}_\theta \rangle}{4\pi n_0 m} \right]$$

(1)
Here, $\langle \tilde{V}_r \tilde{V}_\theta \rangle$ and $\langle \tilde{B}_r \tilde{B}_\theta \rangle$ are the Reynolds stress and the magnetic Reynolds stress, respectively, and $\mu$ represents a 'generic' collisional damping. It is apparent that flow evolution requires:

(a) a net imbalance between fluid stresses (related to vorticity transport) and magnetic stresses (related to magnetic flutter induced electron particle transport) associated with a local (i.e. not averaged over fluctuation correlation length) ambipolarity breakdown;
(b) a radially inhomogeneous fluctuation spectrum (i.e. local symmetry breaking);
(c) a local radial wave propagation mechanism active in the fluctuation dynamics.

Furthermore, it is clear from Eq. (1) that fluid and magnetic stresses only act (up to end point contributions and ionization effects) to locally redistribute poloidal momentum and cannot generate it. Thus, flow profile modification should be thought of as a mechanism for amplification of flow shear (i.e. a dipolar layer) or flow curvature (i.e. a quadrupole layer).

Tokamak edge plasmas in OH or L-mode are characterized by large fluctuation levels ($\delta n/n \sim e^/T \sim 1/2$) and strong radial symmetry breaking (i.e. boundaries or just the observation that $\Delta r/L_n \leq 1$, where $\Delta r$ is the radial correlation length and $L_n$ is the gradient scale). Thus, it is clear that edge flows will be regulated by fluctuations. Indeed, the only real question is the mechanism for radial propagation. The possibilities include:

(a) diamagnetically induced radial wave propagation in the ambient fluctuations;
(b) seed shears ($V'_\phi$) and dilatations ($V''_\phi$) which in turn induce radial propagation.

This concept is best illustrated by considering flow evolution in a simple, single helicity, electrostatic system. For that system:

$$\left( \frac{\partial}{\partial t} + \mu \right) \langle V_\phi(x) \rangle$$

$$= - \frac{\partial}{\partial x} \left( \left. \langle \tilde{V}_r \tilde{V}_\theta \rangle \right|_{V_{r,0}} + \left. \frac{\delta \langle \tilde{V}_r \tilde{V}_\theta \rangle}{\delta \tilde{V}_\theta} \right|_{V_{\theta,0}} \langle V'_\phi \rangle 
+ \left. \frac{\delta \langle \tilde{V}_r \tilde{V}_\theta \rangle}{\delta V''_\phi} \right|_{V_{\phi,0}} \langle V''_\phi \rangle + ... \right)$$

(2a)

Hereafter, a vertical bar denotes evaluation at $x = 0$, and $V'_\phi = 0$ or $V''_\phi = 0$.

Thus, mean shear and mean curvature evolve according to:

$$\left. \left( \frac{\partial}{\partial t} + \mu \right) \langle V'_\phi \rangle \right|_{x = 0}$$

$$= - \left( \frac{\partial^2}{\partial x^2} \left. \langle \tilde{V}_r \tilde{V}_\theta \rangle \right|_{V_{r,0}} + \left. \frac{\partial}{\partial x} \frac{\delta \langle \tilde{V}_r \tilde{V}_\theta \rangle}{\delta V'_\phi} \right|_{V'_\phi,0} \langle V'_\phi \rangle' + ... \right)$$

(2b)
This simple functional expansion establishes that both radial propagation and the spatial structure of the fluctuation spectrum determine the specific flow amplification mechanism. In particular, shear amplification occurs only if the fluctuation spectrum lacks reflection symmetry (i.e., \( x \rightarrow -x \) symmetry) about the rational surface \((x = 0)\), or if \( (\partial/\partial x)(\delta \langle \tilde{V}_r \tilde{V}_\theta \rangle / \delta \tilde{V}_\theta^\prime) \neq 0 \). In this regard, note \( \tilde{V}_\theta = (c/B_0) \partial \tilde{\phi}/\partial x \). On the other hand, curvature will be generated from rest by virtually any fluctuation propagating radially owing to diamagnetic effects (i.e., \( (\partial^3/\partial x^3)(\langle \tilde{V}_r \tilde{V}_\theta \rangle \neq 0 \) for any localized, radially propagating mode). Furthermore, seed curvature can be amplified if \( (\partial/\partial x)(\delta \langle \tilde{V}_r \tilde{V}_\theta \rangle / \delta \tilde{V}_\theta^\prime) \neq 0 \).

In view of these simple, general observations, the results of detailed numerical and analytical calculations discussed below are not surprising. For a simple drift wave mode [5], where \( \tilde{\phi}_k(x) = \tilde{\phi}_k(0) \exp(-\pi x^2/2) \) (here, \( \mu_k \) is real and an envelope varying slowly on the scale of \( x_i \) is understood), application of Eqs (2a, b) yields

\[
\frac{\partial}{\partial t} + \mu \langle V_\theta \rangle^\prime = 0 \quad (3a)
\]

\[
\left( \frac{\partial}{\partial t} + \mu \right) \langle V_\phi \rangle = -\rho_s c_s^2 \sum_k k \mu_k \frac{\partial^2}{\partial x^2} \left( \left| \frac{e \phi_k}{T_e} \right|^2 \right) \bigg|_{x = 0} \quad (3b)
\]

The prediction that net shear is not amplified while net curvature is generated is in good agreement with the results of numerical calculations [6]. Furthermore, as shown in Fig. 1, the results of the numerical calculation agree well with the quasilinear prediction given by Eq. (3b). Poloidal shear \( \langle V_\phi \rangle \) amplification may occur if \( x \rightarrow -x \) symmetry of the fluctuation spectrum is broken and a seed shear is ambient. In particular, for the (negative compressibility) parallel ion flow gradient driven (PIFGD) instability, in the limits of flat density and \( V_{lo} \ll c_s \), \( \tilde{\phi}_k(x) = \tilde{\phi}_k(0) \times \exp[-(\sigma_k/2)(x - b)^2] \), where \( \sigma = 2(1 - \rho_i)^{-1}(\Omega/V_{lo}) \) and \( b = L_s V_{lo}/2\Omega_i \), inclusion of a small, constant seed shear straightforwardly yields:

\[
\left( \frac{\partial}{\partial t} + \mu \right) \langle V_\phi \rangle \bigg|_{x = 0} = 2\rho_s c_s^2 \frac{\langle V_\phi \rangle^\prime}{V_{lo}} \bigg|_{x = 0} b_x \sum_k |\sigma_k| |\tilde{\phi}_k(0)/T_e|^2 \quad (4a)
\]

and

\[
\frac{\partial}{\partial t} \langle V_\phi \rangle^\prime = 0 \quad (4b)
\]
Thus, the seed shear is amplified (producing a dipolar layer), while no curvature generation (which produces a quadrupole layer) occurs. It is interesting to note that, in this case, the seed shear itself induced the radial propagation necessary for a non-zero Reynolds stress. Furthermore, in contrast to the case of curvature generation, Eq. (4a) naturally defines a critical $\epsilon_0/T_e$ necessary for the onset of amplification. Note also that $\langle V_\phi \rangle V_{I_0} > 0$ is required for curvature amplification. This constitutes a type of 'self-organization' criterion, which is here ultimately due to the fact that both $\langle V_\phi \rangle$ and $V_{I_0}$ shift the spectrum off $x = 0$. This observation typifies a number of considerations (some, such as spectral parity, noted above) which together strongly suggest that mean shear amplification is a significantly more 'delicate' process than mean curvature generation, which requires only diamagnetically induced radial propagation and a localized eigenmode. Thus, we suggest that mean curvature generation is the dominant flow profile modification mechanism relevant to confined plasma. Indeed, the sensitivity of mean shear amplification to issues of detail, such as the net imbalance in electrostatic and electromagnetic energies in resistive pressure gradient driven turbulence with diamagnetism, is nicely illustrated in Fig. 2. For this system, curvature generation is clearly the dominant flow profile modification process [7] as shown in Fig. 3.

While curvature generation is the process of greatest significance to tokamak plasma, mean shear amplification is an example of a type of self-organization process known for a long time in geophysical fluid dynamics [8–10] and recently encountered in the plasma community [11]. This process may be described as a vortex tilting instability, whereby a spatial symmetry breaking causes convection rolls to tilt and
align, thus generating a mean shear flow. Indeed, curvature generation is just an alternative manifestation of this type of 'Reynolds stress instability', which happens to be more 'natural' for localized fluctuations in a sheared magnetic field. Here, we discuss the detailed dynamics of such vortex tilting instabilities for the simple case of ambient rolls (i.e. no active buoyancy drive) in a 2-D fluid.
It is well known that in the case of sufficiently large Reynolds numbers sheared flow can evolve into cell turbulence by the Kelvin–Helmholtz instability. The possibility of an inverse process of sheared flow generation by a system of convective cells has been first recognized in connection with the problem of atmospheric circulation around Venus [12]. However, the instability investigated there was connected with finite viscosity. Here we report the existence of another type of instability of an ideal incompressible fluid. This instability is of purely kinematic nature. Let us start from the periodic array of the vertical counter rotating convective cells having a velocity potential in a form $\psi_0(x,y) = \psi_0 \cos(k_0 x) \cos(k_y y)$. This is the equilibrium corresponding to the solution of 2-D incompressible viscous fluid (Euler) equations. As usual in the case of incompressible flow the velocity of convection can be expressed through the velocity potential in the form $v_x = -\frac{\partial \psi}{\partial y}$, $v_y = \frac{\partial \psi}{\partial x}$. It is easy to see for such cells that there is no correlation between horizontal ($y$) and vertical ($x$) velocities and that the horizontal component of an average Reynolds stress, responsible for flow generation, is equal to zero, i.e. $R_y = \langle (\nabla \psi) \nabla \psi \rangle = 0$. A perturbation of the form $\psi_i = \delta \psi_0 \sin(k_y y)$ tilts cells. It is easy to see from this form of potential that there is a correlation between the velocities of horizontal and vertical convection and hence non-zero average Reynolds stress, $R_y$, for tilted cells. The stress results in the creation of a sheared flow that causes a subsequent increase in the inclination of the cells reinforcing the initial perturbation. This instability could be also interpreted as parametric coupling of two modes: $\delta \psi_1(x) \sin(k_y y)$, corresponding to the tilt of the cells, and $\delta \psi_0(\lambda)$, corresponding to the horizontal sheared flow. The coupling is caused by main convective motion $\psi_0 \sim \cos(k_0 x) \cos(k_y y)$. The system of equations describing this coupling in a linear approximation has the form

$$
-\omega \frac{d}{dx} \delta \psi_0 + \frac{k_y}{2} \psi_0 \cos(k_0 x) \left[ \frac{d^2 \delta \psi_i}{dx^2} + k_0^2 \delta \psi_i \right] = 0
$$

(5a)

$$
-\omega \left[ \frac{d^2}{dx^2} - k_y^2 \right] \delta \psi_i = k_y \psi_0 \cos(k_0 x) \left[ \frac{d^3 \delta \psi_0}{dx^3} + (k_y^2 + k_0^2)^2 \frac{d^2 \delta \psi_0}{dx^2} \right] = 0
$$

(5b)

Here, $\omega$ is the frequency of the perturbations, i.e. $\delta \psi_0$, $\delta \psi_i \sim e^{-i\omega t}$. In order to obtain the dispersion relation $\omega(k_y, k, k_0)$ we proceed, using the Floquet theorem, to the following Fourier expansions of the solution of Eqs (5a, b), i.e.:

$$
\frac{d}{dx} \delta \psi_0 = \sum_n V_{2n} e^{i(k + 2nk_0)x}
$$

(6)

$$
\delta \psi_i = \sum_n A_{2n+1} e^{i(k + (2n + 1)k_0)x}
$$
From Eqs (5a, b), it follows that the system of linear algebraic equations for the coefficients of the expansion is given by

$V_{2n} = \frac{k_y \psi_0}{4i\omega} \left\{ A_{2n+1}h_{2n+1} + A_{2n-1}h_{2n-1} \right\} \frac{k_0^2}{(k + 2nk_0)^2}$

(7a)

$A_{2n+1} = \frac{k_y \psi_0}{2ik\omega} \frac{1}{g_{2n+1}} \left\{ V_{2n}g_{2n} + V_{2n+2}g_{2n+1} \right\}$

(7b)

The following notation has been used: $h_{2n} = (k/k_0 + 2n)^2 - 1$, $n_{2n} = k_0^2/k_0^2 - h_{2n}$, $g_{2n} = h_{2n} + 1 + k_0^2/k_0^2$. The final set of equations for $A_{2n+1}$ that follows from Eqs (7a, b) defines a tridiagonal matrix relation:

$A_{2n+1} = -\frac{\omega^2}{k_0^2 \psi_0^2} g_{2n+1} + \frac{h_{2n+1} + \frac{k_0^2}{8} (r_{2n} + r_{2n+2})}{r_{2n}}$  

(8)

$A_{2n-1} = n = 0, 1, \ldots$

The condition of the determinant being zero defines the dispersion relation that we are looking for. It follows from the form of that matrix that $\omega$ is a periodic function of $k$ with period $2k_0$. Another symmetry property, $\omega(k) = \omega(2k_0 - k)$, also follows easily from the form of the dispersion matrix and the isotropy condition $\omega(-k) = \omega(k)$. Using these symmetry properties it is sufficient to find the solution of the dispersion relation in the interval $0 < k < k_0$. In spite of the absence of a true 'small parameter' in the system of Eqs (7a and b), the relative size of the factor $k_0^2/k_0^2 \sim 1/8 \ll 1$ (here, $k_0 \sim k_y$) justifies the truncation in these equations. Using the truncation procedure in which only $A_{\pm 1}$, $V_0$, $V_{\pm 2}$ harmonics are non-zero, i.e. taking $n = 0, +1, -1$ in Eqs (7a and b) and assuming $A_{\pm 3} = 0$, we can write the dispersion relation in the form of a biquadratic equation for $\omega$:

$\frac{\omega^4}{k_0^8 \psi_0^2} + \frac{\omega^2}{k_0^6 \psi_0^2} \left\{ h_1(r_2 + r_0) \frac{h_{-1}(r_{-2} + r_0)}{g_1} + \frac{h_{-1}(r_{-2} + r_0)}{g_{-1}} \right\}$

(9)

$+ \frac{1}{64} \frac{k_y^4}{k_0^4} \frac{h_1h_{-1}}{g_1g_{-1}} [(r_2 + r_0)(r_{-2} + r_0) - r_0^2] = 0$

It follows from this solution that the tilting instability is an aperiodic ($\gamma \sim \omega$), fast evolving instability with a characteristic growth time comparable with the period of fluid convection. The instability leads to the generation of vorticity $\omega = \Delta \psi$ from the
initial state of counter rotating cells in which the vorticity is equal to zero, in analogy with the $\alpha$-effect for magnetic field generation in the dynamo problem. This is the main difference between this analysis and that of Ref. [11]. There, the condition of vorticity conservation was imposed ab initio into the truncation procedure.

3. EFFECTS OF FLOW PROFILE STRUCTURE ON THE NON-LINEAR EVOLUTION OF FLUCTUATIONS

The concept of shear (or, more generally, flow profile) suppression of turbulence now seems to be viewed as a promising mechanism for the formation of the edge transport barrier so characteristic of H-mode plasmas. The original manifestation of this concept was in the context of enhanced decorrelation [3] due to the coupling of radial scattering to shear induced differential poloidal rotation. Predictably, this simple idea rapidly propagated via an epidemic of "linear stability of some mode in the presence of sheared rotation" calculations [4, 13]. However, when confronted with the (relevant!) problem of flow profile effects on finite amplitude fluctuations, the simple concepts required some re-examination [5].

Put simply, there are three classes of flow profile effects on fluctuations. These are:

(i) **shifts**, by which the $x \rightarrow -x$ symmetry of the $(m, n)$ spectrum for localized modes in a cylinder is broken. Shifts are induced by local flow shear ($V_\parallel$) and other odd derivatives of the flow profile. 'Shifts' may be complex, in that both a displacement of the spectrum from the rational surface and the introduction of a preferred direction of radial propagation are possible.

(ii) **dilatations**, which squeeze or dilate the eigenmode structure like an accordion. Dilatations are induced by local flow curvature ($V_\perp$) and other even derivatives of the flow profile. Dilatations may be complex, in that both spectral width and $(x \rightarrow -x$ symmetric) local $k(x)$ may be changed.

(iii) **absorption**, in which wave-flow resonance occurs within the eigenmode. For example, slab-like drift waves in a linear shear flow will be absorbed when $|x_s| < |x_i|$. Here, $x_s$ is defined by $\omega_c = k_\phi V_\parallel x_s$, and $x_i$ is the usual ion Landau damping point. Clearly, absorption requires very strong flow profile variation and severely distorts the eigenmode structure.

In practice, the effects of absorption and Kelvin-Helmholtz instability [14] require such a strong shear or curvature that they may be ignored when discussing edge turbulence, except possibly in the context of ELMs [15]. Also, for simplicity we limit consideration to lowest order shifts and dilatations, induced by local shear ($V_\parallel$) and curvature ($V_\perp$), respectively. This is, of course, equivalent to a truncation of the Taylor series expansion of $V_\phi(x)$ about the mode rational surface. Finally, all our discussions refer to simple drift wave [5] or RPGDT [16] turbulence.
In drift waves, the principal effect of shear induced shifts is to enhance ion dissipation by providing a 'net' \( k_t \) through the shift \( \Delta_s \), i.e. \( k_t \rightarrow k_t + \Delta_s \). In linear theory, this provides a strong damping decrement

\[
\frac{\gamma_k}{\omega_k} = -\Omega^{(1)}w_k^2/4\rho_s^2
\]  

(10a)

associated with the linear shift \( \Delta_k \), where

\[
\Delta_k = \Omega^{(1)}w_k^2/2\rho_s^2
\]  

(10b)

Here, \( \Omega^{(1)} = k_s\sqrt{\omega_s/\omega_r} \) is the normalized shearing frequency and \( w_k = \rho_s \sqrt{L_s/L_n} \times F(x_t, k_t, \rho_s) \) is the mode width, where \( F \) is a slowly varying function. However, as is shown in Fig. 4, the non-linear saturation level of drift wave fluctuations is clearly insensitive to changes in the level of flow shear. This counter-intuitive result has been shown to be related to changes in the drift wave mode structure induced by spatial gradients in the fluctuation spectrum [5, 6, 16]. Such gradients inhibit outgoing wave propagation and thus reduce ion damping and the \( \nu' \) enhancement thereof. Indeed, when the fluctuation decorrelation rate \( \Delta\omega_k \) exceeds the linear ion damping rate, i.e. \( \Delta\omega_k \geq (\rho_s^2/w_k^2)\omega_k \) at \((\bar{n}/n)_{\text{rms}} \geq (\rho_s/L_n) (1/k_s w_k \delta^{(0)}) \), the \( \nu' \) damping rate changes to its non-linear form:

\[
\frac{\gamma_k}{\omega_k} = -\frac{\Omega^{(1)}w_k^2}{4\rho_s^2 d_k}
\]  

(11)
Here $\bar{k}$ denotes a spatial average, $\delta^{(0)}$ denotes the non-adiabatic electron induced phase shift, and $d_{\bar{x}} = d_{\gamma}/\omega_x \rho^2_{eq}$, where $d_{\gamma}$ is the (non-Markovian) turbulent diffusivity, defined in the standard way. As $d_{\gamma}$ increases with $n/n$, it is clear that the non-linear $V_x$ damping decrement decreases with increasing amplitude, rendering it ineffective. A similar trend is observed in computational studies of resistive interchange turbulence [7]. Thus, it appears that local flow or electric field shear is not an effective mechanism for the suppression of turbulence. Hence, shifts are, in general, not likely to be dynamically significant.

Dilatations, by way of contrast, tend to squeeze or dilate the eigenmode structure. In linear theory, the curvature dilatation induced damping decrement is

$$\gamma_{\bar{k}} = -x_{\gamma} k^2 \omega^2_{\gamma}(1 + \epsilon_{\gamma}/2)$$  \hspace{1cm} (12a)$$

where

$$\epsilon_{\gamma} = \Omega^{(2)} w^2_{\gamma}/\omega^2_{eq}$$  \hspace{1cm} (12b)$$

Here, $\Omega^{(2)} = k_{\gamma} V_{xg}^2/\omega_{eq}$. Thus, it is readily apparent that the sign of $V_{xg}$ (and thus that of the resulting dilatation of the eigenmode structure) is crucial to whether or not turbulence will be suppressed. This contrasts with the case of shifts, where the trend is toward damping, independent of the sign of $V_{xg}$. Even more interesting is the

FIG. 5. Resistive interchange fluctuations: sensitive to flow curvature variation.
observation that a finite amplitude calculation [6] similar to the one discussed for
shifts reveals that the finite amplitude 'dispersion relation' becomes:

\[
-\omega_{ke} + 1 + k^2 \rho^2 - \frac{d\omega_{ke}^2}{2w_k^2}\bar{\varepsilon}_k - i\delta^{(0)} = 0
\]  

(13)

Here, \( \bar{\varepsilon}_k = \varepsilon_k/d_k \). Thus, in the non-linear regime:

(a) the direct result of curvature induced dilatation is a non-linear frequency
shift;
(b) for \( V_g^2 > 0 \), the non-linear frequency shift reduces \( \delta^{(0)} \) by increasing \( \omega_{ke}^2 \);
(c) the effect of curvature induced dilatation is non-linearly robust, i.e. the
stabilizing trend persists for \( \Delta \omega_{ke} > \rho^2 \omega_{ke}/w_k^2 \).

Thus, fluctuation levels will decrease with increasing curvature according to:

\[
(f/n)_{\text{rms}} = (f/n)_{\text{rms}} [1 - 4\Omega^2/\eta_n]
\]  

(14)

This trend is manifested in numerical calculations for drift wave and resistive inter­
change [7] turbulence. Indeed, Fig. 5 clearly shows the sensitivity of interchange
fluctuation levels to dilatations (i.e. \( \Omega^2 \sim V_g^2 \)), in marked contrast to the case of
shear. On the basis of these results we must therefore conclude that radial dilatations
of the fluctuation structure, induced by flow curvature, etc., are the dominant flow
profile induced modifications of turbulence [6].

4. NON-LINEAR DYNAMICS OF SIMULTANEOUSLY EVOLVING FLOW
CURVATURE AND FLUCTUATION LEVEL

Sections 2 and 3 have established that

(a) generic drift wave fluctuations in a sheared magnetic field will naturally gener­
ate a curved flow (or electric field), with \( V_g \neq 0 \);
(b) the principal flow profile effect on fluctuations occurs via radial dilatations
induced by flow curvature.

Together, observations (a) and (b) collectively suggest that drift wave turbulence in
a curved flow is a naturally self-regulating non-linear dynamical system. This system
is similar in structure to those encountered in population genetics [17], with flow cur­
vature analogous to the 'predator' species and fluctuation level analogous to the
'prey' species. This suggests that we should describe the evolution of the system by
coupled equations for the flow curvature and fluctuation spectrum. Here, we
approach this goal by examining the evolution of the mean curvature \( V_g(t) \) and
mean square fluctuation level \( \langle (f/n)^2 \rangle = \epsilon, \text{ different from } \bar{\epsilon} \) for a single helicity
system. The coupled equations are:

\[
\left( \frac{\partial}{\partial t} + \mu \right)V_g(t) = \left( \frac{\rho^2 c^2}{2} \sum_k (k, \alpha)\mid\alpha_\xi\mid F_k^{(0)} \right) (1 + \bar{\varepsilon}(t))\epsilon(t)
\]  

(14a)
\[
\frac{\partial}{\partial t} \eta(t) = \gamma_k^{(0)} \eta(t) - \left( \frac{4\omega_k^2}{V_{\omega_k}^2} \right) V_\theta''(t)\eta(t) - \frac{d_k(t)}{w_k^2} \eta(t)
\]

(14b)

Here, it is important to note that \( F_k^{(0)} \) is a spectral structure factor, \( \xi_k(t) \sim V_\theta''(t)\eta(t)^{1/2} \) and \( d_k(t) \sim \eta(t)^{1/2} \). The last term on the right hand side of Eq. (14b) represents fluctuation energy 'leakage' from the single helicity system to neighbouring helicities (where \( d_k/d_k > 1/w_k \)) and thus to dissipation. Such a term may be thought of as a means for introducing warfare with neighbouring cannibal groups (of prey) into our 'predator-prey' paradigm. For simplicity, Eqs (14a, b) may be rewritten in non-dimensional form as:

\[
\left( \frac{\partial}{\partial t} + \tilde{\mu} \right) V_{o0} = \alpha_1 V_\theta''(t)\eta(t)^{1/2}
\]

(15a)

\[
\frac{\partial}{\partial t} \eta(t) = \tilde{\gamma}^{(0)} \eta(t) - \alpha_2 V_\theta''(t)\eta(t) - \alpha_3 \eta(t)^{3/2}
\]

(15b)

Here time is normalized to \( \omega_k \), \( V_\theta''(t) \) is normalized to \( V_{\omega_k} w_k^2 \), and the definitions of \( \alpha_1, \alpha_2, \alpha_3 \) are obvious (compare with Eqs 14a, b)! Here, \( \eta(t) > 1 \) is assumed for times well past \( t = 0 \). Note that in this paradigm the width of the shear flow layer is set by the radial extent of the fluctuation spectrum about the single 'dominant' helicity.

Equations (15a, b) may now be straightforwardly analysed to determine stationary states, the eigenfrequencies for small oscillations about these stationary states, and the conditions for their stability. It thus follows directly that a fixed point (stationary state) exists for:

\[
\eta(t) = (\tilde{\mu}/\alpha_1)^2 \equiv \eta_0
\]

(16a)

\[
V_\theta''(t) = \frac{1}{\alpha_2} \left( \tilde{\gamma}^{(0)} - \alpha_3 \tilde{\mu}/\alpha_1 \right) = (V_\theta'')_0
\]

(16b)

Note that since \( V_\theta''(t) \) is required for flow curvature to damp turbulence, Eq. (16b) implies the consistency criterion that

\[
\tilde{\gamma}^{(0)} > \alpha_3 \tilde{\mu}/\alpha_1
\]

(16c)

for the existence of a fixed point. Linearizing Eqs (16a, b) about the fixed point \( \eta_0 \), \( (V_\theta'')_0 \) yields the eigenfrequency equation for small oscillations:

\[
\omega^2 + i\omega (\alpha_3 \epsilon_0^{1/2}/2) - (\alpha_1 \alpha_2 \epsilon_0^{1/2}(V_\theta'')_0/2) = 0
\]

(17)

Taking \( \omega = a + ib \) in Eq. (17) yields:

\[
a^2 = (\alpha_3 \epsilon_0^{1/2}/4)^2 + (\alpha_1 \alpha_2 \epsilon_0^{1/2}(V_\theta'')_0/2) - \alpha_3^2 \epsilon_0/8
\]

(18a)

\[
b = -\alpha_3 \epsilon_0^{1/2}/4
\]

(18b)
Thus, Eqs (18a and b) define a fixed point stability criterion that

\[
[(\alpha_3^2/16\alpha_1) + (1/2)(\gamma_0^{(0)}/\mu) + (\alpha_3/\alpha_1))]^{1/2} - \alpha_3/4\alpha_1 > 0
\] (19)

Thus, Eq. (19) identifies the 'accessibility criterion' for states of dynamically self-regulating drift wave-shear flow turbulence in our simple model. It is absolutely crucial to note that this accessibility criterion is intrinsic to the non-linear dynamical model of the flows and fluctuations. No external agent, such as orbit loss, is required! Equations (18, 19) also together imply that the physical system exists in states of 'dynamical equilibrium' characterized by oscillations of the flow and fluctuations about the fixed point \( e_0 \), \((V_x)_0\). Of course, the time-scale separation implicit to the derivation of Eqs (18, 19) requires that \( \tau_{\text{flow}} > 1 \), where \( \tau_{\text{flow}} \) is defined by \( a \) in Eq. (18a). Alternatively, dynamical 'non-equilibria' will exist if \( \tau_{\text{flow}} \omega \leq 1 \), or if Eq. (19) is not satisfied.

The model discussed in this section is quite crude. In particular, the fluctuations are characterized by a single root mean square level \( \epsilon \). In reality, the spectrum will evolve, too. This spectral evolution will select different \( k \) (dominant wave vector) in different final states, for a random distribution of initial conditions. Indeed, Fig. 6 shows that many such final states are accessible to resistive interchange turbulence. Figure 7 shows the probability distribution function for these different final states. Selection of the final state is akin to the phenomenon of 'pattern selection', well known in chemical catalysis systems [18].
5. DISCUSSION AND CONCLUSIONS

The central thesis of this paper is that, taken together, edge plasma fluctuations and flows constitute a self-regulating system, in which all constituents must be treated on an equal footing. This thesis is supported by the principal results, which are:

(a) The spatial symmetry of (generic) drift wave fluctuations in a sheared magnetic field suggests that mean flow curvature ($V_\ell^\prime$) will be spontaneously generated by fluctuation Reynolds stresses. It is unlikely that mean shear ($V_\ell$) will be amplified.

(b) Radial dilatations of the fluctuation spectrum due to flow curvature ($V_\ell^\prime$), etc. are the mechanism for flow profile induced suppression of fluctuations which is robust at finite amplitudes. Shear suppression is not robust.

(c) Together, the mean flow curvature and fluctuation level evolution equations define a non-linear, dynamical system with a well defined fixed point and natural eigenmodes. A condition for stable dynamical equilibrium has been determined. Satisfying these conditions constrains the relationships between plasma parameters and does not require an external agent, such as orbit loss.

The models discussed here, while relevant, are grossly oversimplified. Many issues of detail concerning plasma dynamics, magnetic geometry, poloidal asymmetry, etc. have been ignored. Two omissions, however, have greater impact than all the others. These issues are related to electric fields and flows in toroidal geometry and the width or extent of the ‘edge plasma’.
In a cylinder, flow and electric field are identical, via $V_\theta = -cE_r/B_0$. In a torus, however, the mean radial electric field (for electrostatic turbulence) is given by:

$$\frac{|e| \langle E_r \rangle}{T_e} = -\frac{|e|}{cT_e} [\langle V_\theta \rangle B_T - \langle V_\phi \rangle \langle B_\phi \rangle] + \frac{1}{n_i C_s^2} \frac{\partial \langle P \rangle}{\partial r}$$

$$\quad + \frac{\partial}{\partial r} \left( \frac{\langle \tilde{V}_r^2 \rangle}{C_s^2} \right)$$

(20)

Here $\langle P \rangle$ is the bulk ion pressure, $\tilde{V}_r$ is the fluctuating radial velocity of the bulk plasma and $C_s$ is the sound speed. Since it is the electric field profile which regulates fluctuation levels, Eq. (20) is the relevant ‘profile evolution equation’ in a toroidal system. Thus Eq. (20) immediately tells us that the simultaneous evolution of $\langle V_\phi \rangle$, $\langle P \rangle$ and $\langle \tilde{V}_r^2 \rangle$ will all determine the net evolution of $\langle E_r \rangle$. In particular, since poloidal and parallel flows are heavily damped by magnetic pumping in a torus and since $V_\phi/C_s < 1$ (usually), Eq. (20) suggests that $\langle V_\phi \rangle$ and $\langle P \rangle$ evolution will control $\langle E_r \rangle$ evolution. This, in turn, implies that the fluctuation induced Reynolds stresses in the toroidal flow evolution equation as well as the fluctuation induced transport fluxes will be the key regulators of $\langle E_r \rangle$ evolution. Also, it appears that $\langle V_\phi \rangle$ evolution is rather unimportant in a torus. Indeed, no conclusive evidence suggesting an increase in $\langle V_\phi \rangle$ or a change in its profile at the L→H transition yet exists [19, 20]. These issues will be discussed further in future publications.

The second principal weak point in the work to date is its reliance on studies of either single helicity or ‘few helicity’ systems. Thus, the issue of what determines the width of the good confinement region has not been addressed, as yet. The existing model pins this width to the radial width of the edge fluctuation spectrum, which is probably too small for consistency with observations [20]. This issue will remain an object of intense theoretical scrutiny for the foreseeable future.

REFERENCES

DISCUSSION

R.R. WEYNANTS: Do you think your theory can be extended to explain electrode biasing experiments? Doesn't your a priori assumption of zero radial current present a problem?

B.A. CARRERAS: The present studies are limited and do not include external sources. The numerical calculations are on the fluctuation time scale. The purpose is to isolate and to study some dynamic aspects of the self-regulated shear flow turbulence. In a model for the L to H transition, one should include the generation mechanism for the external radial electric field.

R.H. COHEN: In paper D-l-2, “Simulations for confinement in near-fusion experiments”, M. Kotschenreuther et al. reported that the Livermore gyrokinetic simulations (which are 3-D multiple helicity simulations) for $\eta_i$ with shear flow (including both $V_\phi$ and $V_Z$) show a $\chi$ which tracks surprisingly well with a mixing length estimate $(\gamma_{\text{max}}/k^2(\gamma_{\text{max}}))$; the coefficient is even within a factor of two of unity. Do your simulations show a similar tracking with mixing length estimates?

B.A. CARRERAS: For the turbulence models we have considered, the fluctuation level does not track the mixing length. This is obvious in the case of linear flow profiles. In the absence of flow, analytical theory has shown that the mixing length is not a relevant model. I do not see why it should be relevant in the presence of flow.

M. TENDLER: My question concerns the impact of toroidicity on the “shear Reynolds drive” for the poloidal rotation velocity. In a publication which I co-authored (TENDLER, M., ROZHANSKY, V., Phys. Fluids 4 (1992) 1877), it was shown that the impact is amplified by a large factor $(2q^2 + 1)$ due to toroidicity. Can you comment on this?

B.A. CARRERAS: In toroidal geometry, the structure of the Reynolds stress changes. I do not think that the overall effect can be simply given by a multiplicative factor, but I should read your paper.

J.W. CONNOR: Can you comment on the predictions of the model for the radial widths of the self-consistent solutions for the sheared flow region?

B.A. CARRERAS: The basic scale of the non-uniformity of the turbulence is the width of the mode, which is a few $\rho_s$ for the drift wave turbulence. The corresponding scale for the poloidal flow will also depend on the viscosity.
TOKAMAK EDGE TRANSPORT, L-H TRANSITION AND GENERATION OF VELOCITY SHEAR LAYERS

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Abstract

TOKAMAK EDGE TRANSPORT, L-H TRANSITION AND GENERATION OF VELOCITY SHEAR LAYERS.

An analytic and numerical investigation of turbulence and transport driven by drift-resistive ballooning modes in the tokamak edge is presented. The self-consistent generation of parallel and poloidal flows is included. The structure of the shear layer at the boundary between open and closed field lines at the edge is calculated and its role in controlling the L-H transition is explored. The feasibility of producing localized shear layers with external drivers for reducing turbulence and improving confinement is discussed.

1. TOKAMAK EDGE TRANSPORT, SHEARED FLOW AND THE L-H TRANSITION

There is now a significant body of evidence suggesting that the dynamics of the edge plasma controls the transition from L to H mode [1]. Edge fluctuations, which drive particles and energy transport in this region, are
dramatically reduced at the transition [2]. There is also now substantial observational evidence that sheared poloidal rotation in the edge may suppress the turbulence and therefore reduce transport. Poloidal rotation of impurity ions increases sharply at the transition [2], while the transition can be induced by imposing a radial electric field $E_r$ with a probe [3]. Finally, turbulence is actually suppressed in the naturally occurring shear layer in an Ohmic plasma [4]. Thus, a complete model of the L-H transition must include an understanding of edge turbulence and the mechanisms for the development of poloidal rotation. We present such a model.

There is substantial evidence that tokamak edge transport has a significant poloidal asymmetry and that it is peaked on the outside of the discharge [5, 6]. Such observations severely constrain the potential sources for anomalous transport. The drift-resistive ballooning (DRB) mode is essentially the only curvature-driven instability for the collisionality range of most tokamak edge plasmas [7]. Our conceptual model of the tokamak edge and the L-H transition centers on the poloidally asymmetric anomalous particle driven by DRB turbulence. Ohmic and L-mode discharges, in which magnetic pumping is large, are characterized by strong poloidal asymmetries. Large transport on the outside of the torus drives parallel flows ($V_|| \sim c_s$) which carry plasma from the outside to the inside of the machine [8]. Poloidal rotation rates are at the neoclassical diamagnetic level. In H-mode plasmas strong poloidal rotation ($V_\theta \sim e c_s / q$), amplified from neoclassical flows dominantly by the Stringer mechanism, symmetrizes the edge and shear stabilizes the edge turbulence. Edge transport is therefore reduced, allowing steeper edge density profiles to form. We present the results of 3-D toroidal simulations of DRB turbulence with the self-consistent generation of sheared flow in Sec. A and the results of 2-D toroidal axisymmetric transport simulations in Sec. B.

A. Toroidal Fluid Simulations of DRB Turbulence

A set of nonlinear fluid equations have been derived from the reduced Braginskii equations to model DRB turbulence, transport and sheared flow generation in toroidal geometry. Because of the high mode number nature of the fluctuations a novel rotation to a low order rational surface ($q_a$) in the edge region has been completed which allows us to treat the regions of favorable and unfavorable curvature while at the same time limiting the poloidal extent of the computational domain. The characteristic perpendicular ($L_0$) and parallel ($L_z$) scale lengths and time scale ($t_0$) of the turbulence are given by
The normalized toroidal equations are

$$\frac{d\tilde{n}}{dt} + \frac{\partial \tilde{\phi}}{\partial y} - \frac{2L_n}{R} C(\tilde{\phi} - \alpha \tilde{n}) + \frac{2L_n\alpha}{R} \nabla^2_{\|}(\tilde{\phi} - \alpha \tilde{n}) = D\nabla^2_1 \tilde{n}$$

(1)

$$\frac{d}{dt} \nabla^2_{\|} \tilde{\phi} + C \tilde{n} + \nabla^2_{\|}(\tilde{\phi} - \alpha \tilde{n}) = \mu \nabla^4_1 \tilde{\phi}$$

(2)

with $\nabla_{\|} = \partial/\partial z + 2\pi \tilde{s}(x - x_0)\partial/\partial y$, $C = (\cos 2\pi z - \epsilon)\partial/\partial y + \sin 2\pi z\partial/\partial x$, $d/dt = \partial/\partial t + \tilde{z} \times \nabla \tilde{\phi} \nabla$ with $\alpha = p_s c_{s0}/L_n L_0$ a measure of the diamagnetic frequency ($\sim 0.5$), $\tilde{s} = q'a/q$ the shear parameter and the normalizations defined by $\bar{n} L_n/n_0 L_0 \rightarrow \tilde{n}$ and $c_{s0} \bar{\phi}/B L_0^2 \rightarrow \tilde{\phi}$. Note that the poloidal variation of the curvature becomes a function of $z$ in the transformed coordinate system. The parallel flow $\tilde{v}_||$ is also included but is small. The characteristic transport rate is

$$D_0 = (2\pi q_a)^2 \rho^2 \nu_{ei} \left( \frac{R}{L_n} \right)$$

(3)

This is enhanced by the factor $R/L_n$ ($\sim 50$ in the edge) over the neoclassical value and is of order $3m^2/s$ in TEXT and DIII-D.

An equation for the generation of poloidal rotation can be obtained by averaging (2) over $y$ and $z$ and integrating along $x$,

$$\frac{\partial}{\partial t} \bar{v}_y + \frac{\partial}{\partial x} \bar{v}_x v_y + n \sin 2\pi z = 0$$

(4)

with $\bar{v}_y = \bar{\phi}/\partial x$ and the bar denoting an average over $y$ and $z$. The first term is the Reynolds stress [9] and the second the anomalous Stringer drive [10]. This last term is important if the transport is poloidally asymmetric (has a $\sin 2\pi z$ variation in our transformed system).

Equations (1) and (2) have been solved on a 3-D grid for a range of values of $\alpha$ and $\tilde{s}$ while either suppressing the sheared flow or allowing it to evolve self-consistently. In Figs. 1a and 1b we show contour plots of $\tilde{n}$ and $\tilde{\phi}$ at late time at the outer midplane from a simulation with $\alpha = \tilde{s} = 0.5$, $\epsilon = 0.25$, $L_n/R = 0.02$ and the sheared flow suppressed. The spectrum is essentially isotropic in the $x$-$y$ plane and has a characteristic scale length of order unity. The turbulence has a surprisingly strong ballooning character. This can be most clearly seen in the plot of particle flux versus $z$. 
in Fig. 2. The transport asymmetry is $\sim 3.5$ with the largest transport on the outside of the torus ($z = 0$). The turbulent spectrum and transport are surprisingly insensitive to magnetic shear. In a similar run with no magnetic shear ($s = 0$) the vortices were initially radially extended but later broke up into an isotropic state as in Figs. 1a,b. The diamagnetic parameter $\alpha$ has a much stronger influence on the transport. In Figs. 1a and b the contours of $\bar{n}$ and $\bar{\phi}$ are fairly strongly correlated with $\bar{\phi} \sim \bar{n}$, i.e., the turbulence is somewhat drift-wave like. Such phase locking of $\bar{n}$ and $\bar{\phi}$ reduces the transport. The transport increased by a factor of $\sim 2$ for $\alpha = 0.1$ and $\bar{n}$ and $\bar{\phi}$ were no longer in phase.

The sheared poloidal flow builds up as a result of the Reynolds stress and anomalous Stringer drives if it is not artificially suppressed. In Fig. 1c we show a contour plot of $\phi(x, y, 0)$ at late time from a simulation with the parameters as in Figs. 1a,b without suppression of $\bar{v}_y$. The sheared flow $\bar{v}_y$ is plotted versus $x$ in Fig. 1d. The sheared flow stretches the vortices
in the poloidal \( (y) \) direction, thereby reducing \( v_x \) and the particle flux. In this simulation the Reynolds and anomalous Stringer sources of rotation are comparable. We emphasize that the present simulations do not include the scrape-off-layer (SOL) physics which is necessary to properly model the development of parallel flows and the shear layer. A fully self-consistent H-mode transition cannot be simulated without this important physics.

**B. Parallel Flows and Sheared Rotation in the Tokamak Edge**

The tokamak edge is a complex region where the rate of transport of particles, energy and momentum across the magnetic field is comparable to the sound transit time \( t_s \sim \pi q R/c_s \) around the torus [8]. In this region density profiles are controlled both by anomalous transport and the rate of flow of plasma to limiters and divertors. The characteristic scale length \( L_x \) of this region is given by \( L_x^2 \sim D t_s \). For the DRB \( D \) itself depends on \( L_x \) (through \( L_n \sim L_x \)) in Eq. (1) so that self-consistently

\[
L_x = [(2\pi q)^2 \rho_e^2

Over a region of order \( L_x \) in the closed flux region the plasma dynamics differ greatly from the usual static equilibria of the interior of the discharge. In this region the shear layer, which controls the onset of the H-mode and reduced plasma transport in the edge, forms from a balance of the anomalous Stringer drive and magnetic pumping. To study the formation of the shear layer and the dynamics of the tokamak edge, we developed a set of 2-D nonlinear equations for density, parallel flow and poloidal rotation \( \bar{v}_y \) in an axisymmetric torus [11]. Included are poloidally asymmetric anomalous transport from DRB modes, magnetic pumping, diamagnetic drifts and a simple SOL. The SOL includes a particle sink and potential
\( \phi_0(x) = T_e(x) \ln(2m_i/\pi m_e)/e \), resulting from the Debye sheath of the limiter or divertor [12]. The anomalous transport \( D_\perp(\theta) \) is taken independent of \( r \) and is not reduced in regions of strong poloidal rotation. Thus, we are not simulating the L-H transition. Instead we investigate the poloidal structure of the density profile, and parallel and poloidal flows in the edge.

When magnetic pumping is large (Ohmic and L-mode discharges), large transport at the outside of the machine causes a pileup of plasma in this region and results in parallel flows \( V_\parallel \sim c_s \) carrying plasma from the outside to the inside of the machine [6,8]. Density asymmetries along a flux surface in the edge are of order unity. Because of the coupling of parallel flows to \( L_n \) the particle flux into the edge develops a mass dependence, \( \Gamma \sim D_\perp/L_n \sim L_n^{-2} \sim M_i^{-1/3} \). Transport is reduced with increased \( M_i \) even though \( D_\perp \) has no explicit \( M_i \) dependence. Such a scaling of particle flux with \( M_i \) is consistent with experimental measurements [13]. In Fig. 3b we show the radial dependence of the poloidal (\( \bar{v}_{Ey} \)) and diamagnetic (\( v_d \)) velocities (normalized to \( c_s a/qR \)) for \( \epsilon = 0.25 \). In the SOL (\( x > 0 \)) \( E_r > 0 \) is given by \( \phi_0(x) \), while in the closed flux region (\( x < 0 \)) \( E_r < 0 \) with \( \bar{v}_{Ey} \sim -v_d \), the usual neoclassical result [14]. In this case the Stringer drive is overpowered by magnetic pumping so no significant amplification of the neoclassical flows is possible.

When the magnetic pumping is weak (H-mode regime), the anomalous Stringer drive exceeds magnetic pumping and the seed neoclassical rotation is amplified until the poloidal rotation competes with the parallel flows in smoothing the edge density. Saturation of the rotation occurs when \( v_{Ey} \sim 0.7c_s \) as can be seen in Fig. 3a. The poloidal rotation now greatly exceeds the neoclassical diamagnetic level although \( E_r < 0 \) in the closed flux region. In spite of the large rotation rate no poloidal shocks [15] appear in the simulations. This is because the effective dissipation in the system
is large. The transport rate $D_z/L_x > c_s/\pi q R \sim t^{-1}$ so that the effective $Q$ of the system is unity, i.e., there is no resonance at the poloidal sound velocity.

Since the generation of poloidal rotation in a torus fundamentally depends on the poloidal asymmetry of the density [see Eq. (4)] plasma sources (from limiters, diverters, gas puffing or pellets) can impact the rotation. A localized plasma source has been incorporated into our equations. In Fig. 4 we show the saturated rotation rate $\bar{\nu}_{\phi}$ versus source position. The optimum location for driving rotation is around $\theta = 3\pi/2$, which corresponds to the direction of ion magnetic drift. This result is consistent with lower thresholds for the H-mode transition when the divertor throat (an effective source of plasma) is in this same position [16].

Finally we note that all significant physical effects in these 2-D toroidal simulations are being implemented in our 3-D DRB code so that the complete self-consistent L-H transition can be studied.

2. THEORY OF ACTIVE CONFINEMENT CONTROL BY EXTERNALLY-INDUCED GENERATION OF VELOCITY SHEAR LAYERS

The concept that a sheared electric field may suppress turbulence and thus allow access to enhanced confinement regimes is now well established, both theoretically and experimentally. We present ongoing theoretical work aimed at determining the comparative efficiency and feasibility of various methods for externally exciting electric-field shear layers. These methods of active confinement control include: radio frequency wave injection, neutral beam injection, the application of magnetic coils, and electric field generation by fast ion orbit loss.
A. Radio Frequency Wave Injection

A strong candidate for active control of the radial electric field \( E_r \) profile is the use of RF [17]. Advantages of such a mechanism include: (a) radial profile \( E'_r, E''_r, \text{etc.} \) and sign control; (2) non-invasive character; (3) the capability of combining \( E_r \) modification with current profile modification; (4) inward RF induced transport. Advantages aside, experimental evidence of \( E_r \) profile modification already exists for Alfvén wave heating on TCA [18] and ion Bernstein wave heating on PLT [19] and Alcator C. In the former, a strong \( E'_r \) was induced (but the transport response not examined), while in the latter case, indirect evidence for \( E_r \) exists (\( E'_r \) not measured) and significant confinement improvement was noted.

To gain physical insight into \( E_r \) drive, we use a cylindrical model \( E_r = V_B B_0 / c \) so that \( E_r \) drive is equivalent to poloidal flow drive, and examine low frequency (Shear Alfvén) and high frequency (Ion Bernstein) RF drive. For low \( \omega \), steady state poloidal flow arises from a balance of radially varying Reynold’s stress (both electrostatic and magnetic) and poloidal flow damping. An important consequence of Reynold’s stress drive is that little or no total momentum is imparted to the plasma, only the flow gradient is changed. In the case of high \( \omega \) (electrostatic) direct ion acceleration (similar to direct electron acceleration in current drive) acts as an additional local source of momentum (but only for poloidally asymmetric wave spectra).

A possible figure of merit for RF-induced suppression is found from Biglari, Diamond and Terry, i.e., \( V'_{g} = (cE'_r / B_0) > \Delta \omega / (k_\theta) \Delta r \), where \( \Delta \omega \) is the decorrelation rate, \( (k_\theta) \) is the rms poloidal wave number, and \( \Delta r \) is the radial correlation length. By substituting the RF wave fluctuation dependent power absorbed at the edge (for the particular wave), one can obtain specific criteria for turbulence suppression, i.e., \( P_{abs} > P_{crit} \), dependent on RF-specific parameters.

We analyze RF \( E_r \) profile modification for the two specific cases of kinetic Shear Alfvén waves (KSAW) and Ion Bernstein waves (IBW). For KSAW, the flow profile is [17]

\[
\langle V'_{g}(r) \rangle = \delta_{ek} \rho \left[ \frac{c^2 k^2}{\mu_0} \right] \frac{\omega^2 |\varepsilon_r(r)|^2}{k^2 \rho^2 c_s^2} \left( \frac{\omega^2}{k^2 V_A^2} - 1 \right)^2 e^{2k^2 r(r-r_s)}
\]

where \( \delta_{ek} \) is the electron dissipation, \( \varepsilon_r \) is the wave-induced displacement, \( k^2 r \) is the imaginary part of the KSAW wave number (due to electron
dissipation), \( \mu_\theta \) is neoclassical damping, and \( r_s \) is the Alfvén resonance surface. Note that the envelope is determined by electron dissipation at \( r_s \), \( k_\theta \) controls the sign of \( E_r \) and the profile is broader for high \( T_e \) plasmas (and hence easier to drive \( E_r \)). For IBW applications [20],

\[
\langle V_\theta(r) \rangle \simeq \frac{1}{\mu_\theta} \left[ k_\perp^2 \frac{c_i^4 k_{R,\perp} \omega_{ci}}{B} \left( \frac{c \phi}{\omega_{pi}} \right)^2 \frac{\Omega_{ci}}{\omega_{pi}^2} \right] \exp \left[ -k_\perp (r - r_s) \right]
\]

where \( k_\perp = k_{R,\perp}^i + i k_{\perp}^f \), \( \phi \) is the electrostatic potential and \( \omega_{pi} \) is the ion plasma frequency. We note that for IBW the sign of \( E_r \) is only inward and cannot be controlled (consistent with previous PLT results), and that \( k_{\perp}^f \) is determined by electron dissipation (in the edge) or ion cyclotron damping (in the core). For guidance, we offer the predictions that for TEXT (KSAW) \( P_{abs} \sim 300 \text{ kw} \) is necessary for a 8 cm suppression zone, for DIII-D (KSAW) \( P_{abs} \sim 300 \text{ kw} \) for 5 cm [17], and for PBX [20] (IBW) \( P_{abs} \sim 100 \text{ kw} \) for 3 cm. Of course, these estimates are crude and detailed feasibility studies of experiments are necessary. However, recent experimental results on PBX-M [21] have indicated improved core confinement during IBW heating, consistent with our theory. Finally, we will examine \( E_r \) modification in a torus, the effect of compressible modes, and ICRF in the future.

B. Neutral Beam Injection

The possibility for driving perpendicular rotation with NBI is assessed as follows. Two approaches can be considered: (1) poloidal injection with a significant off-axis component [22] and, (2) toroidal injection in which one relies on the rotational transform to impart a perpendicular component to the rotation [23]. The two cases are dissimilar, primarily in the role of magnetic pumping: the latter strongly damps poloidal rotation but does not affect toroidal rotation. In the sense, toroidal injection is favored. On the other hand, magnetic pumping acts as a friction, as opposed to being a diffusive mechanism. For this reason, poloidal injection is favored because the rotation will be sharply localized where the beam is deposited.

The basic equations that determine the efficacy of NBI to impart sufficient velocity shear to tokamak plasmas are momentum balance and energy balance [22]. The former determines the final rotation speed while the latter determines the final temperature. Here, we argue, the ratio \( V_{\perp} / c_s \) is what counts. For a given beam power \( P_b \) and beam speed \( V_b \), the ratio
$V/c_s$, where $V$ is the poloidal or toroidal speed depending on the injection geometry, is given by $V/c_s = (\tau_M/\tau_E)(c_s/V_b)(a/\Delta)$, where $\tau_E$ and $\tau_M$ are the energy and momentum confinement times and $\Delta/a$ is the fraction of the minor radius over which the momentum is localized. As discussed above, for poloidal NBI ($V \rightarrow V_\perp$), $\Delta$ can be controlled and $\tau_M$ is the magnetic pumping time. Since the latter time is shorter than $\tau_E$ and also since $V_b > c_s$, the final velocity shear is reduced but aided if $\Delta/a$ can be made small. We assume that the critical shear is $V_\perp > c_s/L_s$, a criterion that stems from several linear theories [22,23] and is not inconsistent with nonlinear theory. The required $\Delta/a$ is reasonable for present-day tokamaks, but restrictive for reactors. For toroidal NBI, $\Delta \rightarrow a$ and $\tau_M \sim \tau_E$. Using the critical shear criterion, we require $V_\perp > c_s$ for stabilization. Present-day rotation speeds are a fraction of the sound speed. Thus, the required speed is not unreasonable to contemplate for the future.

3. MAGNETIC ISLAND FLOW DRIVE

One means proposed to control the plasma rotation velocity profile, and thus $E_r$, involves the use of phased ac currents in external coils to establish and drive rotating surfaces of islands at selected radii inside the plasma. The plasma inside the moving islands tends to follow the islands and via viscous forces acts as a "stirrer." In this way one can in principle establish a desired rotation velocity profile, which implies control of the radial electric field and hence control of turbulence driven transport.

Models for such driven islands [24,25] suggest that surfaces of islands, thin enough to have only a negligible effect on transport, can apply significant torques to the plasma, and that the maximum torque scales as the coil current squared. These concepts are being studied in "magnetic braking" experiments in DIII-D [26] where stationary surfaces of islands are applied to the core region of an H-mode plasma kept rotating by co-injected NBI.

The braking experiments have demonstrated controllable changes of the plasma toroidal rotation velocities (to lower values) and gross alterations of the core $E_r$ profile. The edge region remained unaffected. Measured density and temperature profiles also remained unaffected ($|E'_r|$ was only reduced). This implies that externally driven rotating islands can apply significant torque to the plasma yet not by themselves adversely affect transport. Data from these experiments suggest that suitable islands located a few centimeters inside the edge and caused to rotate at 1-2 kHz may beneficially affect edge transport in the H-mode by increasing the field.
gradient and/or by broadening the high shear region, depending upon island location and rotation direction. Thus driven rotating islands may provide an attractive method of active confinement control.

4. NBI FAST ION ORBIT LOSS

Prompt fast ion loss due to NBI or ICRH can directly affect the radial electric field, \( E_r \). In turn, the effect of \( E_r \) on fast ion loss is found to be important. By carefully analyzing conservation laws, one can determine particle orbits, trapped-untrapped boundaries and squeezing factors in the presence of \( E_r \). Also, there can be significant \( E_r \) before NBI. This may be the reason why in some NBI experiments the expected (from orbit analysis without \( E_r \)) fast ion loss and concurrent change in toroidal rotation have not been observed. The loss of ions (fast or thermal), either by direct orbit loss or ripple loss, modifies \( E_r \) to generate a background return current which maintains quasineutrality. This return current spins up the plasma toroidally in the counter direction, since \( J_r \) is inward. A spontaneous toroidal spin-up in the counter direction has been frequently observed in the plasma edge region in DIII-D.

To further enhance the confinement along the H-mode line, it seems necessary to increase the radial depth of the radial electric field (\( E_r \)) shear layer. One way to enhance the shear layer's depth is to counter-inject a low energy, high current (50A) neutral beam. A low energy beam is chosen so that nearly all of the beam is ionized in an edge layer of suitable radial depth. Calculations show that tens of amperes of radial current will be generated from a region which extends several centimeters into the plasma. Radial currents of this magnitude drawn from inserted probes have driven an H-mode transition on CCT [3] and TEXTOR [27]. A refinement of this technique is to simultaneously apply high energy co- and low energy counter-injected beams. The more centrally deposited co-beam increases \( E_r \) shear and radially broadens the shear layer by driving co-rotation (\( E_r \) positive) inside of the edge region where the counter-beam is driving \( E_r \) negative.

REFERENCES


DISCUSSION

A.V. NEDOSPASOV: I agree with you that the origin of the edge turbulence is the dissipative interchange instability, but I believe that the dissipations due to fluctuating current flowing through the sheath potential near the surfaces dominate in the SOL or in the vicinity of the separatrix.
H.L. BERK: Mr. Drake, you used an ideal MHD estimate for the instability drive of a resistive-g driven mode. What is your justification for this?

J.F. DRAKE: Resistive ballooning modes have maximum growth rates at the ideal MHD rate. However, such large growth rates occur only for high mode numbers with scale lengths smaller than $L_0$ as defined in our paper. This situation must be distinguished from the one where the ideal MHD $\beta$ threshold is exceeded and low mode numbers grow at the fast ideal MHD rate.

F.B. MARCUS: To generate an edge electric field for shear flow stabilization, would it be more efficient to use MeV ion loss from ICRH or fusion alpha particles?

J.F. DRAKE: The most efficient way of generating a sheared electric field with ion loss would be to use 50–150 keV fast ions pushed into the loss cone at the plasma edge by edge-resonant ICH. Loss of MeV ions would cost too much energy and probably would not be as spatially localized as one would want in creating electric field shear.

S.D. SCOTT: It seems to me that the turbulence you simulate is collisional drift wave turbulence.

J.F. DRAKE: The parameter $\alpha$ defined in our paper determines the role of diamagnetic propagation. For $\alpha \ll 1$ diamagnetic rotation is negligible and the turbulence cannot be characterized as collisional drift waves. For $\alpha \simge 1$ the collisional drift wave characterization, destabilized by toroidal curvature, is correct.

S.D. SCOTT: But the turbulence does sense shear, since your results show near-adiabatic behaviour in the electrons!

J.F. DRAKE: Quasi-adiabatic behaviour of electrons can take place even without magnetic shear.

S.D. SCOTT: How large was the computational domain in $\rho_s$?

J.F. DRAKE: In our normalized variables, $\rho_s/L_0 = \alpha(2L_0/R)^{1/2} = 0.1$, so that $L_x = L_y = 40\rho_s$.

A. ROGISTER: Is it possible that the poloidal asymmetry of the density profile and of the losses you mentioned in TEXT and ALCATOR-C could simply be explained by the position of the limiter in these experiments?

J.F. DRAKE: The density asymmetry on ALCATOR-C was not interpreted as being caused by the limiter position by the experimentalists. Since I am not familiar with the limiter geometry on ALCATOR-C I cannot comment further on this question. Before the upgrade, TEXT had poloidal ring limiters. The distance to the limiter varies very little from the outside to the inside of the torus, so I doubt that the density asymmetry on TEXT is caused by the limiter.

H. CONRADS: I would like to draw your attention to the fact that in TEXTOR we measured the poloidal velocity for co- and co-counter-injection of neutral beams. The poloidal velocity is larger for balanced injection (7 km/s). My question is, do you expect a change in poloidal velocity when a rational surface/magnetic island hits the toroidal limiter? We have observed a change (+2 km/s) in TEXTOR when the limiter intersected with $q = 3$. (See CONRADS, H., EURINGER, H., RUSBÜLDT, D., "Poloidal rotation in the limb of the TEXTOR plasma measured

J.F. DRAKE: We have not considered the influence of low order islands on the rotation rate. If the q = 3 surface causes enhanced transport at the edge with a ballooning character, it could boost the rotation. On the other hand, you might also expect an increase in the effective viscosity, which would reduce the poloidal rotation.

M.G. HAINES: To get a change of poloidal rotation, an up–down asymmetry in pressure is needed. How did you provide this in your model?

J.F. DRAKE: The diamagnetic drift coupled with magnetic pumping provides the initial seed, which triggers Stringer spin-up. The up–down asymmetry is then self-generated.

M.G. HAINES: Instead of giving an up–down perturbation, including $\omega_*$ effects (Hall effect, $\nabla \rho_e$, etc.) leads naturally to up–down asymmetry (see HAINES, M.G., Phys. Rev. Lett. 25 (1970) 1480).
RECONNECTION AND TRANSPORT IN HIGH TEMPERATURE REGIMES*

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Abstract

RECONNECTION AND TRANSPORT IN HIGH TEMPERATURE REGIMES. Magnetic reconnection and transport are recognized as key issues on the path to proving ignition in magnetically confined plasmas. The excitation of \( m = 1 \) modes involving magnetic reconnection is considered, and collisionless regimes where these modes are suppressed or prevented from having significant growth rates (of the type that is consistent with the observed crash phase of sawtooth oscillations) are identified. An analytical model for the early non-linear topology and evolution of modes of this kind is given. The effective diffusion coefficients that can reproduce the observed canonical temperature profiles are assumed to result from the combined effects of modes excited in the body of the plasma column and at the edge of it. The impurity driven mode, analysed in the presence of magnetic shear, is considered to be a candidate for excitation at the edge. The transport resulting from this mode is assessed while it is pointed out that it can account for the observation of the ‘isotopic’ effect on the energy confinement time. A global transport model that covers both Ohmic regimes and regimes where injected heating prevails and can reproduce the observed temperature profiles is presented. In the injected heating regimes the enhanced diffusion process is attributed to the excitation of trapped electron ubiquitous modes in the body of the plasma. The adopted form of the relevant diffusion coefficient is suggested by the theory of these modes, and its temperature dependence \( (\propto T) \) is consistent with the most recent analyses of advanced experiments on transport. The evolution toward ignition of the high density plasmas for which the Ignitor-Ult experiment is designed is investigated using this transport model and the TSC 1 1/2-D code. The conditions under which ignition can be achieved are found to depend on the evolution of the temperature and current density profiles that can be controlled in order to avoid the onset of large scale sawtooth oscillations. The envisioned favourable characteristics of high density, high magnetic field experiments to investigate fusion burn regimes are confirmed by this analysis.

1. RECONNECTION IN HIGH TEMPERATURE REGIMES

Despite great progress in experimental studies of sawtooth oscillations in toroidal plasmas, many basic characteristics of this phenomenon remain unexplained theoretically. The one-fluid MHD theory, for example, although it has given the \( m = 1 \) reconnected configuration [1], has failed to explain the crash time, stabilization conditions and triggering of sawteeth and is inadequate in the case of contemporary high

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temperature experiments. Here, we first study a kinetic model for the linear \( m = 1 \) mode and compare the results with those obtained from a two-fluid description. The non-linear evolution of the mode is then studied in the context of the latter model.

1.1. Linear \( m = 1 \) mode and necessary stability condition

The large aspect ratio approximation with \( B = B_z > B_g, R > r \) (\( R \) being the major radius) will be used, and we shall consider plasma equilibria with \( \beta \ll 1 \) for the marginal stability condition for ideal MHD, \( m = 1 \) modes. To summarize our notation, we point out that, in addition to the usual parameters such as the shear value \( q' \) at the \( q = 1 \) resonant surface \( r = r_1 \), the characteristic Alfvén transit time \( \tau_A = R \sqrt{4 \pi n/m}/B \), and the characteristic resistive time \( \tau_{\text{res}} = 4 \pi \sigma r_i^2/c^2 \), in the high temperature regimes, the following parameters are also essential: the collisionless skin depth \( d_e = c/\omega_p e \), the effective 'Larmor radius' \( \rho_s = \sqrt{(T_e + T_i)/m_i}/\Omega_i \) (where \( T_e, T_i \) are the plasma temperatures and \( \Omega_i \) is the ion cyclotron frequency), the 'drift frequencies' \( \omega_d = cT_n'/(neB_i) \), \( \omega_i = -cT_i n'/neB_i \), and the temperature gradient parameters \( \eta_{e,i} = T_{e,i}/n' \). We shall also refer to the more familiar scale distances \( \rho_{se} = \sqrt{T_e/m}/\Omega_i \) and \( \rho_{si} = \sqrt{T_i/m}/\Omega_i \).

First, we consider a simplified kinetic model for the collisionless \( m = 1 \) mode in which a kinetic description of the electrons is used, while the response of the ions is treated in the Padé approximation, which accurately reproduces the kinetic terms in the limits \( \rho_e^2 \partial^2/\partial x^2 \gg 1 \) and \( \rho_e^2 \partial^2/\partial x^2 \ll 1 \). Working in terms of the helical flux perturbation \( \psi' = \tilde{\psi}'(r) \exp(-i\omega t - i\theta + iz/R) \), we obtain a single equation \[1.2\] obeyed by the function \( \chi = x^2 (\psi'/x)' \), \( x = r - r_1 \), in the resonant region:

\[
\frac{d}{dx} \left\{ h(x) \frac{d}{dx} \chi \right\} = \left\{ 1 - \frac{\tau_A^2 \omega (\omega - \omega_n (1 + \eta_1))}{q^2 \chi^2} \right\} \chi
\]

where

\[
h(x) = \rho_e^2 \left[ \frac{\omega - \omega_n (1 + \eta_1)}{\omega - \omega_i} \right] \left[ \frac{T_i}{T_e} \frac{1}{\omega - \omega_i} + \frac{1}{\omega \rho_{se}} \right]
\]

and the electron kinetic response is contained within

\[
\Psi_e = - \frac{\eta_e}{2} \omega_e - W_e \left[ 1 - \frac{\omega_e}{\omega} + \frac{\eta_e \omega_e}{2 \omega} \left( 1 - \frac{\tau_A^2 \omega_2 d_e^2}{q^2 \rho_{se} x^2} \right) \right]
\]

\[
W_e = - \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \xi \exp \left( -\tau_A \omega d_c \sqrt{2q^2 \rho_{se} x} \right) e^{-\xi^2} d\xi
\]

In addition, using a Padé approximation for \( \Psi_e \) leads to the simplification

\[
h(x) = h_{\text{MHD}}(x)
\]

\[
= \frac{\omega [\omega - \omega_n (1 + \eta_1)]}{(\omega - \omega_e) (\omega - \omega_i)} \rho_s^2 - \frac{\tau_A^2 \omega_2}{q^2 x^2} \frac{\omega - \omega_n (1 + \eta_1)}{\omega - \omega_e (1 + \eta_c)} d_e^2
\]
It is found that this form coincides with the result obtained [3] from a two-fluid MHD model, which includes the Hall term in Ohm’s law and electron parallel compressibility and uses an isothermal equation of state both for the ions and the electrons.

The distinction between these models is relatively minor in the case \( \omega_{\perp i} = 0 \) and \( \eta_{e} = 0 \) [3]. Note that the normalized growth rate in the fluid model \( \omega = \Gamma_{0} \sqrt{\omega_{\perp i} (1 + \eta_{e}) / \omega_{n} (1 + \eta_{i})} \), depends only on \( \rho_{i} / d_{e} \), and in the most realistic range \( \rho_{i} / d_{e} \approx 1-7 \) it can simply be approximated [2] as

\[
\Gamma_{0} (\rho_{i} / d_{e}) = 1 + 0.3 \frac{\rho_{i}}{d_{e}} \quad (1.5)
\]

In the presence of \( \omega_{\perp} \) effects, both models predict the existence of stability conditions. In the fluid model, the dispersion relation for the marginally stable case, \( \gamma = \text{Im}(\omega) = 0 \), may be expressed in terms of the same function \( \Gamma_{0} \) introduced above (but with a different argument) as

\[
[\omega + i \omega_{\perp} (1 + \eta_{e}) (1 + \eta_{i}) - \omega] = \frac{q_{i}^{2} d_{e}^{2}}{\tau_{A}} \Gamma_{0}^2 \sqrt{\frac{\omega + i \omega_{\perp} (1 + \eta_{e})}{\omega + i \omega_{\perp}} \sqrt{\frac{\omega_{n} (1 + \eta_{i}) - \omega}{\omega_{n} - \omega}}}
\]

Taking as an example \( \eta_{e} = \eta_{i} = 0 \), the stability condition resulting from this is simply

\[
\Omega_{\perp} = (\tau_{A} / q^{2} d_{e}) (1 + \omega_{\perp} / \omega_{n}) / 2 \geq \Gamma_{0} \left( \frac{\rho_{i}}{d_{e}} \right)
\]

or, in practical units, keV, cm\(^{-3}\), T:

\[
0.06 \sqrt{\mu} \frac{(T_{e} + T_{i}) n R}{B^{2}} > q^{2} r_{1} \left( 1 + 0.8 \sqrt{\frac{\mu n (T_{e} + T_{i})}{B^{2}}} \right)
\]

where \( \mu = m_{i} / (2 m_{p}) \) is the ratio of ion to deuterium mass and all parameters are evaluated at the \( q = 1 \) surface. Note that the density gradient serves as a stabilizing factor while the shear \( q^{2} r_{1} \) has a destabilizing influence on the reconnection mode we are considering.

A comparison between the kinetic and fluid model results is given in Fig. 1, which shows the behaviour of the normalized growth rate \( \Gamma \) obtained numerically as a function of \( \Omega_{\perp} \), with \( \rho_{i} / d_{e} \) held fixed at a realistic value \( \rho_{i} / d_{e} = 5 \) (for simplicity, the ion temperature gradient is assumed to be negligible, \( \eta_{i} = 0 \)). The two sets of curves with various values of \( T_{i} / T_{e} \), one with \( \eta_{e} = 0 \) and another with \( \eta_{e} = 2 \), demonstrate an additional stabilizing effect of the electron temperature gradient.

It is also seen that the growth rates obtained from the two models are in reasonable agreement; they are also in agreement with the results of Ref. [4] when large values of \( \rho_{i} / d_{e} \) are considered. Furthermore, both models predict the delocalization of the mode near marginal stability when the real part of the frequency is negative (in the fluid theory this occurs typically when \( \omega_{\perp} > \omega_{n} \)). In the opposite case the fluid model leads to a well localized \( (x = \rho_{i}) \) mode while the kinetic model still predicts delocalization near marginal stability.
1.2. Geometry and evolution of non-linear reconnection

As in the linear theory, the magnetic configuration outside the 'dynamic layer' is well described by equilibrium equations, even in the non-linear phase. For the early non-linear stage when the central core displacement $\xi_0$ is small with respect to $r_1$ but greater than the largest characteristic scale of the layer, a relatively simple set of equations determining this configuration has been obtained by Waelbroeck [5].

A magnetic configuration resulting from the numerical solution of these equations is given in Fig. 2. The separatrix encloses an island and separates the shifted central core from a nearly unperturbed outer zone. The corners of the island form two Y-points at $\theta = \pm \theta_Y$ which are connected by a 'ribbon'. A singular current flowing in the direction opposite to the equilibrium toroidal current is situated along the separatrix. This configuration obeys the Kadomtsev conservation laws and equilibrium equations everywhere except in the interior of the singular layer comprising the separatrix.

The configuration has a set of general relationships: $\theta_Y = 60^\circ$, $w = 1.84 \xi_0$, $\psi_Y = 0.08r_1 Bq' \xi_0^2/R$, where $w$ is the island width and $\psi_Y$ is the helical flux inside the island. The helical magnetic field and the plasma velocity normal to a flux surface have the form

$$B_\theta^* = \pm 0.34 \frac{r_1}{R} Bq' \sqrt{f(\psi^*) - g(\theta)} \xi_0, \quad V_n = \frac{0.47}{\sqrt{f(\psi^*) - g(\theta)}} \frac{d\xi_0}{dt} \quad (1.9)$$
where \(g(\theta)\) and \(f(\psi^*)\) are eigenfunctions of Waelbroeck's equation which have been determined but are not presented here. This solution provides the boundary conditions for a separate analysis of the singular layer.

Given an expression for the layer width \(2\Delta\), the above results allow the early non-linear evolution of \(\xi_0(t)\) to be obtained. For any \(\Delta\), we have, from the approximate incompressibility of the plasma motion, the integral relationship

\[
\langle V_\theta \rangle \Delta = r_1 \int_0^{\psi_y} V_\theta d\theta, \quad \langle V_\phi \rangle = \frac{\sqrt{B_{x_{\Delta}}(0) - B_{x_{\Delta}}(\phi_T)}}{\sqrt{4\pi n m_i}} = 0.43 \frac{r_1}{R} V_A q_i \xi_0
\]

(1.10)

where \(\langle V_\phi \rangle\) is the average velocity of the plasma flowing from the singular layer at \(\theta = \theta_T\) and is estimated as the characteristic Alfvén speed. Using Eq. (1.9) for \(V_n\), we obtain the crash evolution equation for the early non-linear phase:

\[
\xi_0(t) = 0.96 \frac{R}{V_A q_i \Delta} \frac{d\xi_0(t)}{dt}
\]

(1.11)

The only unknown quantity here is the characteristic layer width \(\Delta\), which determines the time evolution.

For resistive (collisional) reconnection the width of the dynamic layer depends on the plasma velocity, \(\Delta = \Delta_{res} \sim c^2/(4\pi \sigma V_n)\), which leads to algebraic growth [5], \(\xi_0(t) = 0.42r_1t^2/\tau_{Kad}^2\), with Kadomtsev's characteristic time, \(\tau_{Kad} = \sqrt{\tau_{res} T_A/(q_i r_i)}\) [1].

In the collisionless case we may assume that \(\Delta\) is time independent. Then the reconnecting mode grows exponentially

\[
\xi_0(t) \propto e^{\nu t}, \quad \tau = \frac{R}{V_A q_i \Delta}
\]

(1.12)
For this mode the characteristic poloidal velocity scale-length is $\Delta = \rho_s$, which gives the fastest estimate for the reconnection time:

$$
\tau_{rec} = \tau_{\rho_s} = \frac{R}{V_A q_i \rho_s} = 43 \frac{r_i R \sqrt{n}}{q_i r_i \sqrt{T_e + T_i}} \left[ \frac{m^2 \sqrt{10^{19} \text{ m}^{-3}}}{\sqrt{\text{keV}}} \right] \text{ ms} \tag{1.13}
$$

Note that this time is a factor of $\rho_s/d_e$ smaller than the Wesson estimate [6] based on the collisionless skin depth $\Delta = d_e$ (which may be applicable in the low beta case, $\beta < 2m_e/m_i$).

2. TRANSPORT AT THE EDGE AND THE ISOTOPIC EFFECT

It is well accepted that, in order to reproduce the experimentally observed temperature profiles, the thermal diffusivities used in transport codes must have characteristic radial profiles that increase monotonically with the radius. On the other hand, transport models that rely on collective modes whose driving factors are localized within the main body of the plasma generally do not yield such diffusivities. Thus we assume that there also exists a class of modes that are excited at the periphery, providing the 'channel' for energy transfer to the edge. Furthermore, we argue that these modes should be sensitive to the presence of impurity ions and, in particular, to the ratio of their mass to that of the primary ions (e.g. deuterium). We relate this to the observation of the 'isotopic effect', i.e. the improvement of the energy confinement time when the main ion species is changed from hydrogen to deuterium.

In our model a second ion population, the 'impurity species' with a mass number $A_i$ and a charge number $Z_i$, is assumed to be present at the edge of the plasma column as a consequence of its interaction with the first wall. For simplicity, we refer to a plane configuration with a sheared field, $\vec{B} = B_0 (e_z + e_x/L_x)$, and to electrostatic modes represented by $\vec{E} = -\nabla \phi$, where $\phi(x, y, z, t) = \phi(x) \exp(-i\omega t + ik_y y)$. Thus, $k_x = k_y x/L_x$. The densities of electrons, main nuclei and impurity species are indicated by $n_e(x), n_i(x)$ and $n_i(x)$, respectively. We define the ion diamagnetic frequency as $\omega_i = (k_z c T_i/Z_e B_n) \times (dn/dx)$ and notice that 'impurity driven modes' [7] can be found in the frequency range $k_i^2 V_{th}^2 < \omega^2 \leq k_i^2 V_{th}^2$.

In particular, we consider a weakly collisional regime where $\omega < \nu_i k_i^2 \lambda_i^2 = k_f V_{th} (k_f \lambda_i), \lambda_i = V_{th}/\nu_i$ is the mean free path and $k_f \lambda_i < 1$. The perturbed density of the main nuclei is given by the momentum conservation equation, which, to lowest order in $(\omega/k_i V_{th})^2$, can be expressed as: $-i \kappa_i \nabla \Phi_i + n_i \dot{T}_i + \alpha_i n_i \dot{T}_i + Z_i e n_i \phi = 0$, where $\alpha_i n_i \dot{T}_i$ accounts for the thermal force due to collisions between the main nuclei population and the impurity species. In general, we find that the radial width of the mode is of order $\rho_i$. Thus, the expressions for $n_i$ and $\dot{T}_i$ involve an integral operator in $\phi_i$ [8]. The solution of the relevant integral equation [8] can be avoided in two limits: for large impurity density gradients [9], or near
marginal stability (i.e. for $\eta_i \equiv d \ln T_i / d \ln n_i = 2/3$), when the radial width of the mode is larger than $\rho_i$ and the relevant finite Larmor radius terms can be kept to lowest significant order [10].

Since we consider the limit $\omega^2 > k_i^2 V_{thi}^2$, we may ignore the longitudinal motion of the impurity population and obtain, from the relevant mass conservation equation, $\omega \dot{n}_i + (k_i c \dot{\phi}/B) (dn_i/dx) = 0$. The perturbed electron density is $\hat{n}_e = (e \dot{\phi}/T_e) n_e$. Given that $\hat{n}_e$ and $\hat{\phi}$ are in phase, no electron transport is produced by these modes. Thus, if hot impurity nuclei are transported out, cold main nuclei are transported in (or vice versa). The dispersion equation (differential in $x$) resulting from the quasi-neutrality condition $Z_f \hat{n}_f + Z_i \hat{n}_i = \hat{n}_e$ is solved, yielding:

$$\omega \approx -Z_f k_f D_B \frac{1}{n_e} \frac{dn_i}{dx} \left[ 1 + Z_i \frac{T_e}{T_i} \right]^{-1} \left[ 1 + \frac{6}{5} \frac{Z_f}{\chi^0} \frac{T_e}{T_i} \frac{\eta_i (\omega c_i c_f / k_i c_f)}{(k_i c_f)^2} \eta_i \left( \frac{T_e}{T_i} \frac{\eta_i}{\eta_e} - 2/3 \right) \right]^{-1}$$

(2.1)

where $D_B \equiv c T_e/e B$ and $k_i \equiv k_i \rho_i / L_x$, $c_i^2 \equiv T_e / m_i$, $\rho_i \equiv c_i / \Omega_i \equiv \rho_i \sqrt{T_i / 2 T_e}$, and $\chi^0$ is a known numerical coefficient. In particular, we find (see Ref. [9] for the constant temperature case) that the characteristic scale distance for the spatial variation of this mode, $\Delta x \approx \rho_i \sqrt{\eta_i / (\eta_i - 2/3)}$, is larger than the ion gyroradius only near marginal stability. Furthermore, the eigenmodes are only localized when they are unstable, $\eta_i > 2/3$.

An associated problem is whether the thermal energy can be transported over a distance larger than the width of these modes. A possibility is that a combination of these modes [8] or their non-linear evolution produces convective cells of macroscopic dimensions. In fact, non-linear simulations [11] of ion temperature gradient modes (that are similar to those discussed here) in the presence of magnetic shear have shown the existence of such convective cells, for realistic combinations of relevant parameters.

Then we may argue that the effective energy diffusion coefficient resulting from the impurity modes should be: $D_i \sim \gamma / k_i^2 \sim (Z_f n_f / n_e) (Z_i \Lambda_i / r_{ni}) (c T_e / e B) F_i$, where we take $\omega_i \equiv k_i c (dT_i / dx)/(e B) \sim k_i V_{thi} k_i / \lambda_i$, while $k_i^2 \sim k_i / \lambda_x$, and $\lambda_x$ is the scale distance of the resulting convective cell, and $1/r_{ni} \equiv 1 d \ln n_i / dx$ l. The function $F_i$ represents the width of the excited spectrum of modes, is a decreasing function of $A_i$ for hydrogenic plasmas and depends on other plasma parameters such as $\eta_i$. In particular, when the main ion species is changed from deuterium to hydrogen, the window of instability for impurity modes ($k_i^2 V_{thi}^2 < \omega^2 < k_i^2 V_{thi}^2$) is broadened, allowing for larger radial transport. We suggest [12] that this broadening may account for the isotopic effect. On the other hand, when helium is substituted for deuterium, the fact that $D_i$ has an unfavourable dependence on $Z_i$ should explain the lack of significant improvement in energy confinement that is observed.
3. TRANSPORT MODEL FOR IGNITION REGIMES

A model \cite{13} that combines the effects of transport processes originating in the main body of the plasma with those occurring in the periphery has been adopted to identify the optimal approach to fusion burn conditions by compact ignition experiments. The observed (canonical) electron temperature profiles \cite{14}, both in the cases where Ohmic heating is dominant and where other forms of heating are prevalent, are reproduced. In the Ohmic case, we obtain the values of the toroidal loop voltage at the edge of the plasma column, V_e \sim 1 \text{ V}, that are commonly measured in experiments. The adopted electron thermal ‘diffusion’ coefficient combines a variant of the Coppi-Mazzucato-Grüber (CMG) coefficient \cite{15}, which has been shown to simulate well a variety of Ohmic discharges, with an additional component \( D_e^I \) when injected heating is applied. In particular, we write

\[
D_e = D_e^{\text{OH}} + [(P_{\text{HEAT}} - P_{\text{OH}})/P_{\text{HEAT}}]D_e^I,
\]

where \( P_{\text{HEAT}} \) is the total input power, \( P_{\text{OH}} \) the Ohmic heating power, and \( D_e^{\text{OH}} = D_e^{\text{CMG}} (V_e^*/V_e)^{2/3} \) extends \( D_e^{\text{CMG}} \) to regimes in which Ohmic heating is no longer dominant. Here, \( V_e^* \) is the canonical value of \( V_e \) under steady state conditions when only Ohmic heating is applied.

Specifically, we take

\[
D_e^{\text{CMG}} \approx \frac{7.76 \times 10^8 I_e}{n_e^{4/5} T_e} \left( \frac{Z_i \ln \Lambda}{A_i} \right)^{2/5} \left( \frac{\pi^{3/2} V_e^2}{A_i^{3/2}} \right) \left( \frac{1}{\langle |\nabla V|^2 \rangle} \right) \text{(m}^2/\text{s})
\]

(3.1)

Here, \( \langle \rangle \) denotes a magnetic surface average, \( \Phi \) is the toroidal magnetic flux, \( V(\Phi) \) the plasma volume, \( A_i \) the area of the plasma cross-section, \( V_a \) the total volume, \( \ln \Lambda \) the Coulomb logarithm, \( Z_i = \sum_j n_j Z_j/n_i A_j \), summed over all ions \( j \); \( \bar{A}_i \) is the mean ion atomic weight, \( I_e(\Phi) \) the toroidal plasma current, and \( n_e(\Phi) \) the electron density (in MKS units, except for \( T_e \), which is in keV).

The component \( D_e^I \) accounts for the degradation of confinement that is observed when injected heating is applied. In particular, we take \( D_e^I \approx D_{\text{UB}} \), where \( D_{\text{UB}} \) is a ‘diffusion’ coefficient that (i) is non-linear in the pressure gradient, (ii) leads to temperature profiles that are rather insensitive to the energy deposition profile, and (iii) is based on the properties of the trapped electron (‘ubiquitous’ \cite{16}) mode that is most likely to be excited in the main body of the plasma column combined with some of the properties of the impurity driven mode that can be excited at the edge of the plasma. In particular,

\[
D_{\text{UB}} \approx \frac{cT}{eB_0} \left( \frac{r^2}{r_p^2} \right) \frac{c}{\sqrt{\Omega_{\phi_0}\Omega_{\phi_e}}} \frac{1}{q} \left( \frac{r}{R} \right)^{1/2} \frac{Z_i}{A_i}
\]

(3.2)

where \( r_p^{-1} = -(dp/dr)/p \), \( p \) is the total plasma pressure, \( Z_i \) and \( A_i \) are the charge and mass number of the main ion species, and \( \Omega_{\phi} \) are the poloidal cyclotron frequencies.
The actual form that we have used is

$$\mathcal{D}_{UB} \approx C_{UB} \left( \frac{1}{n_i(\Phi)} \right)^2 \frac{1}{\sqrt{T_e T_i}} \frac{r}{q(\Phi)} \left( \frac{r}{R} \right)^{1/2} \frac{Z_i}{A_i}$$

(3.3)

where \( r(\Phi) = \sqrt{\Phi/\pi B_0} \).

Correspondingly, when considering the ions, we take:

$$\mathcal{D}_i = \mathcal{D}_i^{NEO} + \gamma_i \times [(P_{\text{HEAT}} - P_{\text{OH}})/P_{\text{HEAT}}] D_i^0$$

where \( \gamma_i \) is a numerical coefficient that, given the nature of the ubiquitous mode, we consider in the range 0.5 to 1. Here we assume that the density profiles are maintained as reasonably peaked (e.g. by pellet injection techniques) so as to avoid the excitation of \( \eta_i \) modes. Thus their effect is not included. The numerical coefficient \( C_{UB} \) in \( \mathcal{D}_{UB} \) is evaluated so that the confinement times of present day L-mode regimes with injected heating are reproduced once an appropriate value for \( \gamma_i \) is chosen.

The reference parameters of the Ignitor-Ult [13] device have been considered \((R_0 \approx 1.3 \text{ m}, a \approx 0.47 \text{ m}, b \approx 0.87 \text{ m}, I_p \leq 12 \text{ MA}, B_T \leq 13 \text{ T})\). By using an upgraded version of the free boundary TSC code [17], the initial plasma current rise is programmed, together with that of the plasma column cross-section and the plasma density, in such a way as to maintain the current density profile within stable bounds. The loop voltage is minimal at the centre of the plasma column and increases monotonically up to the edge, where \( V_o \approx 1 \text{ V} \) at the end of the current ramp. This allows the Ohmic heating to remain strong at ignition conditions \((n_0 \approx 10^{15} \text{ cm}^{-3}, T_0 \approx 12 \text{ keV})\) and avoids the full degradation of confinement that occurs in present day experiments when injected heating is dominant. A comparable degradation of \( \tau_E \) due to \( \alpha \)-heating is by no means certain, as this heating is internal to the plasma and is distributed axisymmetrically, two features it shares with Ohmic heating that has optimal confinement. We note that the \( \mathcal{D}_{UB} \) given by Eq. (3.3) degrades linearly with temperature while the form that we had used earlier [13] degrades as \( T^{3/2} \). This improvement reflects the results of the most recent experimental analyses on the scaling of \( \mathcal{D}_c \). The volume where \( q \) is below unity (hence potentially subject to sawtooth oscillations) has been maintained at quite low values until ignition, by programming the current rise. At the same time, the best possible margin against the onset of the \( m = 1 \) modes that are involved in sawtooth oscillations is the low value of \( \beta_p \) (\( \approx 0.15 \)) at which Ignitor can achieve ignition, an important characteristic of this experiment.

In the numerical simulation, the plasma current density diffuses according to a neoclassical electrical resistivity. Specified density profiles of the main ion species and impurities are imposed and recycling from the walls and energy losses due to ionization and charge exchange are neglected. Typical results for a variety of current and magnetic field scenarios are shown in Table I, with and without injected (ICRH) heating applied during and after the initial current ramp. The maximum values of the current and fields are given in the first part of the table. The plasma is started on either the outside of the torus at large R ('out') or the inside at small R ('in'), and the length of the current ramp \( t_R \) varies with the maximum plasma current. For these
TABLE I. ALPHA HEATING RESULTS FOR DIFFERENT IGNITOR PLASMA SCENARIOS

<table>
<thead>
<tr>
<th>Case</th>
<th>1</th>
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<th>3</th>
<th>4</th>
<th>5&lt;sup&gt;a&lt;/sup&gt;</th>
<th>6</th>
<th>7</th>
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<tbody>
<tr>
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<tr>
<td>$B_T$ (T)</td>
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<td>13</td>
</tr>
<tr>
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<td>3.0</td>
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<td>3.5</td>
<td>3.5</td>
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<td>$10^9$</td>
<td>0</td>
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<td>0</td>
</tr>
<tr>
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<td>5.6</td>
<td>5.0</td>
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<td>5.0&lt;sup&gt;c&lt;/sup&gt;</td>
<td>4.3&lt;sup&gt;c&lt;/sup&gt;</td>
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<tr>
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<td>8.0</td>
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<td>10.5&lt;sup&gt;c&lt;/sup&gt;</td>
<td>10.5&lt;sup&gt;c&lt;/sup&gt;</td>
</tr>
<tr>
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<td>9.7</td>
<td>17.2</td>
<td>13.2&lt;sup&gt;c&lt;/sup&gt;</td>
<td>11.8&lt;sup&gt;c&lt;/sup&gt;</td>
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<td>0.28</td>
<td>0.57&lt;sup&gt;c&lt;/sup&gt;</td>
<td>0.59&lt;sup&gt;c&lt;/sup&gt;</td>
<td>0.66&lt;sup&gt;c&lt;/sup&gt;</td>
</tr>
<tr>
<td>$P_a$ (MW)</td>
<td>2.3</td>
<td>10.3</td>
<td>10.8</td>
<td>38.0</td>
<td>20.0&lt;sup&gt;c&lt;/sup&gt;</td>
<td>21.0&lt;sup&gt;c&lt;/sup&gt;</td>
<td>17.8&lt;sup&gt;c&lt;/sup&gt;</td>
</tr>
<tr>
<td>$P_{OH}$ (MW)</td>
<td>6.1</td>
<td>4.0</td>
<td>7.1</td>
<td>4.0</td>
<td>5.8&lt;sup&gt;c&lt;/sup&gt;</td>
<td>8.9&lt;sup&gt;c&lt;/sup&gt;</td>
<td>9.5&lt;sup&gt;c&lt;/sup&gt;</td>
</tr>
<tr>
<td>$Q$</td>
<td>2.0</td>
<td>4.0</td>
<td>8.0</td>
<td>14.0</td>
<td>$\infty$&lt;sup&gt;e&lt;/sup&gt;</td>
<td>$\infty$&lt;sup&gt;e&lt;/sup&gt;</td>
<td>$\infty$&lt;sup&gt;e&lt;/sup&gt;</td>
</tr>
</tbody>
</table>

<sup>a</sup> Same as case 3, except $D_I^I = D_{UB}/3$.
<sup>b</sup> $P_j = 5$ MW at $t \geq 15.5$ s; 10 MW at $t = t_R$.
<sup>c</sup> At ignition, $P_a = P_{LOSSE}$.
<sup>d</sup> Time of maximum $P_a$ or ignition.
<sup>e</sup> $Q = P_a/(P_{OH} + P_j - dW/dt)$.

Cases, $n_0/\langle n_e \rangle = 2$ and $Z_{eff} = 1.2$. Results are given at ignition or, for subignited cases, at the time when the fusion heating $P_a$ reaches its maximum. The $\alpha$ heating level is measured by the parameter $Q$, given in the last line of the table. The positive dependence on $I_p$ of the Ohmic temperature and the L-mode confinement times means that higher plasma current favours ignition, as is shown by the simulation results.

REFERENCES

A TRIGGERING MECHANISM OF FAST CRASH IN SAWTOOTH OSCILLATIONS

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Abstract

A TRIGGERING MECHANISM OF FAST CRASH IN SAWTOOTH OSCILLATIONS.

Full torus, compressible, resistive MHD simulations have been performed to study the fast crash mechanism in sawtooth oscillations. The simulation results reveal that the q-value, which first decreases in accordance with current peaking due to Ohmic heating, starts increasing in the q < 1 region, because of strong excitation of non-linear modes, and becomes flattened. When the q-profile is flattened in the q < 1 region, the plasma flow pushes the magnetic surface radially outwards, and the poloidal magnetic field lines are driven to reconnect rapidly with each other across the q = 1 surface. Consequently, the central hot plasma is pushed out towards the wall to crash the confinement. It turns out that the m = 1 plasma flow induced by the kink instability, rather than the pressure gradient, plays a decisive role in the crash process.

1. INTRODUCTION

The mechanism of fast crash in the sawtooth oscillation phenomenon is still unclear although a good deal of theoretical research has been reported [1–3]. The temperature distribution after the crash in recent large tokamak experiment data [4] seems to support Kadomtsev’s resistive model [5] rather than Wesson’s interchange mode model [6] as far as the geometrical change of the plasma is concerned, but it remains difficult to account for the rapid time-scale of the fast crash.

Here, we propose a triggering mechanism for the fast crash by means of a self-consistent three-dimensional compressible resistive MHD simulation for the torus geometry. For this purpose, we have elaborated the previous model [7] and reconstructed a more easily feasible simulation model representing an ohmically heated tokamak plasma.

2. SIMULATION MODEL

The basic equations are:

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0
\]  \hspace{1cm} (1)

\[
\rho \frac{d\vec{v}}{dt} = \vec{j} \times \vec{B} - \nabla p
\]  \hspace{1cm} (2)
\[
\begin{align*}
\frac{\partial \mathbf{B}}{\partial t} &= \nabla \times (\nabla \times \mathbf{B}) - \nabla \times (j/S) \\
\frac{\partial p}{\partial t} \times \nabla \cdot (p \mathbf{v}) &= (\gamma - 1) \left(-p \nabla \cdot \mathbf{v} + \frac{j^2}{S} + \kappa \Delta T\right)
\end{align*}
\]  

where the magnetic Reynolds number \( S \) is classical and proportional to a power of \( 3/2 \) of the temperature, and the thermal conductivity coefficient \( \kappa \) is constant. The thermal conduction effect, \( \kappa \Delta T \), is only switched on when the shift of the temperature axis from the initial plasma centre becomes more than 20\% of the minor radius, owing to the temperature crash. This effect is merely introduced to recover the initial plasma pressure profile after the crash, on the assumption that a certain anomalous heat transport mechanism would operate, which is beyond the scope of the present MHD model.

The tokamak device is modelled by a torus surrounded by a conducting wall with a square cross-section, where cylindrical co-ordinates \((R, \theta, Z)\) are adopted; \(R\) is the major radius, \(\theta\) the toroidal angle, and \(Z\) the vertical axis. The initial equilibrium configuration without resistivity is obtained by solving the Grad–Shafranov equation in the above geometry. The initial on-axis \( S \) value is set to be 40,000; its distribution is calculated from the temperature.

3. SIMULATION RESULTS

The simulation results show that the ramp-up phase and the ensuing crash are formed in a way to be described in this section. The evolution of the magnetic field line mapping in a poloidal plane and the q-profile are shown in Fig. 1.

The q-value at the plasma centre which was initially set to be 1.03 \((t = 0; \text{Fig. } 1(a))\) decreases gradually in accordance with current peaking caused by Ohmic heating. As the on-axis q-value falls below 1, an \( m = 1/n = 1 \) ideal kink mode appears near the axis. After the magnetic field of the \( n = 1 \) mode has developed up to a certain amplitude, q stops being reduced from its on-axis value and turns to increasing in the \( q < 1 \) region and to becoming flattened \((t = 374 \tau_{PA}; \text{Fig. } 1(b))\). As time elapses, the q-profile becomes more and more flattened towards \( q = 1 \) in the whole \( q < 1 \) region. When the q-profile is nearly flattened \((t = 480 \tau_{PA}; \text{Fig. } 1(c))\), the plasma flow pushes the magnetic surface radially outwards so that the poloidal magnetic field lines are driven to reconnect rapidly with each other across the original \( q = 1 \) surface [8] and the energy deposited in the region \( q \leq 1 \) is rapidly released to the outside region. Consequently, the central hot plasma is pushed out towards the wall, and the temperature distribution crashes \((t = 488\tau_{PA}; \text{Fig. } 1(d))\).

In Fig. 2 the time evolution of the magnetic field energy of the \( n = 1 \) mode and the non-linear modes is shown by the solid lines \((a)\) and \((b)\), respectively. When the
FIG. 1. Evolution of magnetic field line mapping in a poloidal plane and the $q$-profile at (a) $t = 0$, (b) $t = 374 T_{PA}$, (c) $t = 480 T_{PA}$, (d) $t = 488 T_{PA}$. 
system reaches a turning point (T.P.) where the on-axis q-value changes from decrease to increase in the ramp-up phase, non-linearly excited modes grow drastically. In order to examine what causes the turning of the q-profile change, two artificial simulations are executed where both the magnetic field and the plasma flow of the n = 1 mode are artificially suppressed at a time before the system reaches the turning point and at a time before the q-profile is flattened. The time evolution of the magnetic field energy of the n = 1 mode for these cases is shown by the dashed lines (c) and (d) in Fig. 2, respectively. The results show that while the on-axis q-value continues to decrease, non-linear modes start growing drastically after a while, and the system reaches the turning point. In Fig. 2, we observe the interesting fact that the magnitude of the energy at the turning point in each case (lines (a), (c) and (d)) is almost the same, which suggests that the magnitude of the n = 1 magnetic field determines the turning point.

Another artificial simulation is carried out where both the magnetic field energy and the kinetic energy of non-linear modes are fixed to their values at t = 309τpA (before the turning point). Then, the simulation results show that the q-value keeps decreasing instead of being flattened; thus no crash appears.

These facts certainly indicate that the turning point is the moment of time where the system reaches a strongly non-linear phase and that both the n = 1 magnetic

\[\text{FIG. 2. Time evolution of magnetic field energy of: (a) } n = 1 \text{ mode; (b) non-linear modes; (c) } n = 1 \text{ mode for the case where the } n = 1 \text{ mode is artificially suppressed at a time before the turning point; (d) the } n = 1 \text{ mode for the case where the } n = 1 \text{ mode is suppressed before flattening.} \]
mode and the non-linearly excited modes play leading roles in q-profile flattening and the ensuing crash process.

Magnetic reconnection driven kink flow plays a decisive role in the destruction of the magnetic surface, while no apparent magnetic reconnection occurs through the ramp-up phase in contrast to results of a previous study [7] where a peripheral vacuum region was modelled by a medium with artificially high resistivity. When we carried out a simulation in which the term $-\nabla \times (\mathbf{j}/S)$ is removed from Eq. (3) at a time before the q-profile is flattened, thus magnetic reconnection being inhibited, the q-profile flattening continued but no crash occurred.

In order to clarify the role of plasma pressure in triggering the crash, we have performed a simulation where the pressure gradient force was removed from the equation of motion, Eq. (2). The result is that a crash did occur in the same way as in the case with pressure gradient force. This simulation indicates that the crash is not due to a pressure driven instability but, rather, to plasma flow driven reconnection.

Driven magnetic reconnection occurs at the head point of the kink flow on the $q = 1$ surface. Thus, the geometrical features of magnetic surface destruction resemble those of Kadomtsev's model rather than those of Wesson's. An important and essential difference from Kadomtsev's model is, however, that the destruction takes place on the MHD rather than on the resistive time-scale.

In Fig. 3, the temperature equi-contours at the times corresponding to Fig. 1 and at (e) $t = 521\tau_{PA}$ are shown. The temperature axis stays at its initial position until the time of the turning point (Fig. 3(a) and (b)). At a time just before magnetic surface disruption (Fig. 3(c)), the temperature axis shifts slightly, owing to the strong kink flow, while the magnetic axis does not. When the crash occurs, the high temperature spot in the central region slides towards the wall, leaving a temperature plateau region in the central domain (Fig. 3(e)). These features are in good agreement with experimental results such as those of TFTR [4].

The on-axis temperature is plotted versus time in Fig. 4, where the thermal conduction effect is switched on at $t = 537\tau_{PA}$ for the first sawtooth oscillation. Return to the initial state of the pressure profile is completed at $t = 602\tau_{PA}$, and the thermal conduction is then switched off. The system then returns to the ramp-up phase, and a similar sawtooth feature shows up again. In the present work, the heat release mechanism is not specified, but certainly plays an essential role in retrieving a normal state from a crash phase.

As can be seen in Figs 1 and 4, the time-scale of the magnetic surface disruption due to magnetic reconnection is 20–30$\tau_{PA}$ and the temperature on the plasma axis drops on a time-scale of 50–100$\tau_{PA}$. In a compressible plasma the time-scale of the driven magnetic reconnection is almost independent of the resistivity, but depends strongly on the magnitude of the plasma flow [9]. Therefore, when a higher magnetic Reynolds number is chosen as the initial on-axis value, the time-scale of the crash does not differ much as long as the kink flow velocity develops into the same order of magnitude, while the time-scale of the ramp-up phase becomes longer because the
FIG. 3. Temperature equi-contours at (a) $t = 0$, (b) $t = 374\tau_{PA}$, (c) $t = 480\tau_{PA}$, (d) $t = 488\tau_{PA}$, corresponding to Fig. 1, and at (e) $t = 521\tau_{PA}$.

FIG. 4. On-axis temperature versus time.
q-profile change is strongly depending on the S-number and the crash depth becomes smaller.

Let us suppose that a simulation is carried out in which the density continuity equation is not solved, i.e. an incompressible plasma. Then, the magnetic Reynolds number will seriously affect the reconnection rate and, hence, the time-scale of the crash. This may explain why the time-scale of the crash in our simulation is of the order of $100R_{pa}$ and different from that in other simulations such as Aydemir's [1, 2].

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DISCUSSION

A. BONDESON: The resistivity in your simulations is very high: $S = 1/q$ ranges from $10^2$ to $3 \times 10^2$. Have you made simulations using S-values characteristic of tokamak experiments ($10^6$ to $10^9$), and if so, is the reconnection rate independent of S?

K. WATANABE: The S-value we adopted in our sawtooth crash simulation was $S = 4 \times 10^4$, which is not quite as small as $10^2$ but is certainly still too small to represent the S-value of actual tokamak experiments. However, other model simulations dealing with flow driven reconnection (Sato et al., Phys. Fluids B 4 (1992) 450) have shown that the reconnection rate is almost independent of S, but proportional to the magnitude of the driving plasma flow.

We have made a simulation of the sawtooth crash where $S = 8 \times 10^4$. The result shows that the time-scale of the ramp-up phase becomes proportionally longer while that of the crash phase remains more or less the same. This is consistent with our conclusion that the ramp-up time-scale is governed by the q-profile change which, in turn, is strongly dependent on the S-value, whereas the reconnection rate is almost independent of the S-value. Note, however, that a finite resistivity, no matter how small it may be, is definitely required for reconnection.

B. COPPI: The ideal MHD $m = 1/n = 1$ mode is driven by the pressure 'excess' (gradient) within the $q = 1$ surface. You have stated that this mode is the
one that is excited. How do you explain your conclusion that the pressure gradient plays no role in the evolution of this mode?

K. WATANABE: You are right, the excited ideal MHD $m = 1/n = 1$ mode is driven by the pressure gradient which is produced by the Ohmic heating. In our artificial simulation to examine the role of the pressure gradient in triggering the crash, the pressure gradient force was removed from the momentum conservation equation just before the appearance of the crash in the normal simulation with the pressure gradient operating, i.e. just before the q-profile flattening. Thus at the time when the pressure term was removed, the $m = 1/n = 1$ mode had developed sufficiently and non-linear modes had also been strongly excited. These modes keep modifying the magnetic structures even if the pressure term is discarded just before the crash. We believe that the crash is due to dynamic pressure driven reconnection.
STABILITY OF TAE MODES IN DIII-D*

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Abstract

STABILITY OF TAE MODES IN DIII-D.

TAE modes driven by neutral beam injection have been observed in DIII-D. The measured frequency agrees very well with theoretical predictions for DIII-D discharges. At large amplitude these instabilities can lead to loss of over 50% of the beam power, as well as large loss of non-resonant MeV fusion products. The threshold value of fast ion beta for destabilization and the observed range of unstable mode numbers are in reasonable agreement with predictions for the mode growth rate. Continuum damping dominates at low mode numbers, while damping by electron kinetic effects dominates at high mode numbers. Preliminary experiments suggest that TAE modes can be stabilized by current profile control.

1. INTRODUCTION

The toroidicity induced Alfvén eigenmode (TAE mode) is predicted to result from the toroidal coupling of shear Alfvén waves [1], and can be destabilized by a large, centrally peaked population of fast ions with velocities near the Alfvén speed [2] such as fusion alpha particles or energetic ions from neutral beams used for heating or for non-inductive current drive. The instability has been observed in existing tokamak experiments [3,4] where the mode can be driven by neutral beam ions. At large amplitude this instability can cause loss of up to 50% of the neutral beam ions, as well as large loss of non-resonant fusion product species. Experimental studies of TAE modes in DIII-D provide the opportunity to test various theories for the driving and damping of these instabilities in a shaped plasma, and to develop techniques to stabilize these modes in future fusion reactors.

As previously reported [3], MHD oscillations which we have identified as TAE modes are observed in DIII-D discharges. The observed frequencies are in the range 50-200 kHz, and the largest amplitudes are typically associated with toroidal mode numbers $n \sim 3-5$. [A typical magnetic spectrum is shown in Fig. 1(a).] These modes are clearly distinguished from

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ordinary MHD activity by having a non-zero velocity of propagation in the plasma rest frame, as expected for Alfvén waves. Soft x-ray and magnetic measurements suggest that the mode is located between the $q = 1$ and $q = 2$ surfaces, within the region of large fast ion pressure gradient. The observed modes are destabilized when a threshold fast ion pressure is reached, and when the fast ion speed $v_f$ is comparable to or greater than the Alfvén speed $v_A$, both characteristics expected of beam ion-driven TAE modes. The threshold at $v_f/v_A = 1$ is not a sharp one, consistent with the theoretical prediction [5] that sideband coupling also allows the mode to be driven by fast ions having velocities $v_f \sim v_A/3$.

2. MODE FREQUENCY

The frequency of the observed instability agrees well with theoretical predictions for TAE modes, at moderate values of beam power and beta. As shown in Fig. 1(b), the measured frequency in the plasma frame
increases linearly with the Alfvén speed, in good quantitative agreement with the simple TAE mode theory prediction $\omega = V_A/2qR$ evaluated at $q = 3/2$, and also with the more exact prediction from stability calculations for these specific discharges. [Here the Doppler shift of the observed frequency due to toroidal rotation of the plasma is inferred by assuming that it is the cause of the equal separation of peaks with different toroidal mode number $n$ seen in Fig. 1(a), and that the frequency in the plasma frame is independent of $n$ as expected for TAE modes [3].] In contrast, for the same discharges the expected frequency for kinetically destabilized ballooning modes [6], estimated as $\omega_{\text{kin}} = k_\theta \rho_i v_{ti} p_i^{-1} dp_i/dr$, decreases weakly with increasing Alfvén speed. (Here $k_\theta = m/r$ is the poloidal wave number, $\rho_i$ is the ion gyroradius, $v_{ti}$ is the ion thermal speed, and $p_i$ is the ion pressure.) This poor fit to the experimental data shows clearly that the observed instabilities are not kinetic ballooning modes.

The mode structure and frequency are calculated in DIII-D geometry with the CONT [7] and GATO [8] stability codes, using reconstructions of experimental equilibria which incorporate temperature, density and current profile data as well as magnetic measurements. These calculations show the existence of a wide toroidicity-induced spectral gap and isolated eigenmodes within the gap. The mode structure is broad, typically with a peak in the vicinity of $q = 3/2$. The real frequency [Fig. 1(b)] agrees well with experiment. GATO calculations have predicted the existence of multiple TAE modes with the same toroidal mode number but slightly different frequencies; for the discharge corresponding to $v_A \approx 3.2 \times 10^6$ m/s in Fig. 1(b), two groups of TAE-like modes were found with closely-spaced frequencies near the two values plotted. Careful analysis of experimental data has resolved similar multiple modes in some cases. Several other features of the TAE spectrum are still under investigation, including changes in the observed spectrum associated with a close approach to the Troyon beta limit, and the calculation and possible observation of ellipticity induced Alfvén eigenmodes [9].

3. FAST ION LOSS

At large mode amplitudes, anomalous loss of more than 50% of the fast ions is observed, as determined from the 2.5 MeV neutron emission and corroborated by edge diagnostics [3,10]. Large losses typically occur during combined TAE and fishbone activity, but can also occur during pure TAE activity. The total stored energy calculated from kinetic profile measurements agrees much more closely with the MHD equilibrium and diamagnetic loop measurements when the fast ion pressure obtained from classical slowing-down calculations is reduced according to these estimates.
Fig. 2. Fast ion loss and TAE mode amplitude during a scan of neutral beam power (discharge 71524). (a) Beam ion beta. Dashed line: predicted from classical slowing down. Solid line: measured from 2.5 MeV neutron emission. (b) Triton burnup fraction. Dashed line: predicted from classical slowing down. Solid line: measured from 14 MeV neutron emission. (c) RMS amplitude of TAE activity, $60 < f < 250$ kHz, at the outboard vacuum vessel wall.

of fast ion loss [10]. In power scans at various values of toroidal field, the fast ion beta is found to saturate at $\beta_f \sim 1.2\%$, as seen in Fig. 2. This saturation coincides with a rapid rise in the observed TAE mode amplitude, indicating that the instability has a threshold in $\beta_f$, and that ejection of the fast ions is the primary mechanism for nonlinear saturation of the instability.

The confinement of fusion products is also drastically reduced during strong TAE activity [11], as shown by decreases of up to an order of magnitude in the 15 MeV proton and 14 MeV neutron signals from the burnup of 0.8 MeV $^3$He ions and 1.0 MeV tritons [Fig. 2(b)]. This interaction of the TAE mode with non-resonant MeV fusion products could have significant consequences for ignition in future devices.

4. COMPARISON TO LINEAR GROWTH RATE THEORIES

In order to gain insight into the scaling and relative importance of the various physical mechanisms which govern the behavior of this insta-
bility, several representative theories for the linear driving and damping rates have been evaluated using DIII-D data [12]. The theories to be discussed here have employed the simplifying assumptions of large aspect ratio, low beta, circular cross-section, isotropic fast ion velocity distribution, and in some cases large toroidal mode number. These may not be good approximations for the DIII-D discharges studied here, and therefore the quantitative values obtained must not be taken too seriously. Nevertheless, the results should at least provide qualitative guidance, showing which theories should be emphasized in future work, and suggesting ways to control or avoid the instability.

In this analysis the plasma is modeled in one dimension, and the theoretical expressions are applied locally. The TAE mode is assumed to be radially localized, so that the theories can be evaluated using only the local values of the experimental profiles and their gradients. Profiles of density, temperature, safety factor, etc. are obtained from equilibrium reconstructions with complete profile data.

There is reasonable quantitative agreement of the predicted growth rate with experimental measurements, given the many approximations mentioned above. The various contributions to the total growth rate are compared in Fig. 3, evaluated for the marginally unstable 5 MW case of the power scan in Figure 2. These include fast ion drive [5] with an additional factor \((1 + k_s \rho_f)^{-1}\) to estimate the effects of the fast ion Larmor radius \(\rho_f\) [13], electron and ion Landau damping [5] (with ion Landau damping dominated by the \(v_i \sim v_A/3\) sideband term), and coupling to the stable continuum of Alfvén waves [14]. The only damping rate which exceeds the fast ion driving rate in the region of \(1 < q < 2\), where the mode is believed to be resonant, comes from a detailed treatment of the electron dynamics including coupling to kinetic Alfvén waves [15]. Ion Landau damping may also contribute in the center of the discharge, and continuum damping at larger minor radius. The predicted driving rate and total damping rate agree to within about a factor of 2, good agreement considering the simplifying assumptions used in the theories.

These estimates of the damping rate can account for the observed range of toroidal mode numbers. The fast ion driving term increases with \(n\), but saturates because of the FLR term, in this example for mode numbers \(n \gtrsim 10\). The Landau damping terms are independent of \(n\). The continuum damping rate decreases roughly as \(n^{-3/2}\), as the mode becomes more localized at high \(n\), while the electron kinetic damping term increases as \(n^{2/3}\) due to improved coupling to the kinetic Alfvén wave. Therefore continuum damping becomes large for low mode numbers, while electron kinetic damping becomes large for high mode numbers, leaving an intermediate range of mode numbers as the most unstable.
5. CONTROL OF TAE MODES

A preliminary experiment suggests that current profile control is a promising approach for stabilization of TAE modes. Increasing the magnetic shear should increase both the continuum damping and electron kinetic damping terms. In a discharge where a centrally peaked plasma current profile is produced transiently by ramping the current down rapidly from a higher value, the onset of TAE mode growth is delayed until about 200 msec after the end of the current ramp (Fig. 4). Immediately after the current rampdown the predicted continuum damping rate is about twice as large as in a constant current case. Then as the current profile relaxes, shown by the slow decay of the internal inductance, the TAE mode grows to the same amplitude as in the constant current discharge.

A shift in the spectrum of toroidal mode numbers consistent with the damping theories is also observed. The dependence on shear is strongest for continuum damping, and this mechanism is most important for low mode numbers. Consequently, the maximum total growth rate (evaluated
Fig. 4. Time traces for discharges with current rampdown (solid lines, discharge 71531) and constant current (broken lines, discharge 71517). Traces include: plasma current $I_p$, internal inductance $\ell_i$ from MHD equilibrium fits, and time-averaged MHD oscillation amplitude $\langle dB/\ell /dt \rangle$ at frequencies $90 < f < 250$ kHz. ($B_t = 1.0$ T, $I_p = 0.6$ MA, $P_{NB} = 10$ MW, $n_e = 4 \times 10^{18}$ cm$^{-3}$, L-mode.)

at $q = 3/2$) shifts to $n = 4 - 6$ in the current rampdown case, as compared to $n = 3 - 4$ in the constant current case [Fig. 5(a)]. In a similar discharge, immediately after the rampdown the TAE spectrum consists primarily of mode numbers $n = 6 - 8$. Then as the current profile relaxes toward lower shear and lower internal inductance, the TAE spectrum evolves toward lower mode numbers, $n = 3 - 5$, and larger amplitudes [Fig. 5(b)].

6. CONCLUSIONS

Experiments show that the TAE mode exists, and can cause significant loss of fusion products. The threshold value of fast ion beta for destabilization is an order of magnitude larger than initial estimates which considered only electron Landau damping. However, this threshold may still be reached in ITER by alpha particles or by fast ions injected for noninductive current drive, so it is important to understand the stability properties of this mode. The good agreement between measured and calculated frequencies gives credence to the theoretical description of this
Fig. 5. (a) Calculated total growth rate vs. mode number, evaluated at $q = 3/2$. Solid line: after current rampdown, $\ell_i = 1.33$ (discharge 71531, t=2325 msec). Dash line: constant current $\ell_i = 1.08$ (discharge 71517, t=2200 msec). (b) Evolution of the TAE spectrum following a current rampdown. Solid line: after current rampdown, $\ell_i = 1.32$. Dash line: after current profile relaxation, $\ell_i = 1.13$. (Discharges 73281 and 72383, $B_t = 0.8$ T, $I_p = 0.6$ MA, $P_{NB} = 10$ MW, $n_e = 4 \times 10^{13}$ cm$^{-3}$, L-mode.)

instability. A semi-quantitative comparison of experimental data with various predictions for the growth rates shows that high mode numbers are stabilized by electron kinetic effects, while low mode numbers are stabilized by continuum damping and possibly by ion Landau damping. The dependence of continuum damping and electron kinetic damping on magnetic shear implies that current profile control can help to stabilize TAE modes, and preliminary experiments with a transiently peaked current profile support this conclusion. In the future, the development of growth rate theories for realistic experimental geometries will allow the required current profiles to be determined, and profile control with rf current drive will allow such profiles to be sustained. Experimental and theoretical efforts will also be directed toward understanding the influence of discharge shape on TAE mode stability.
REFERENCES


DISCUSSION

M.E. MAUEL: Have you been able to compare the fast ion loss rates for the low $\ell$ lower $n$ TAE modes and the high $\ell$ higher $n$ TAE modes observed after the $I_p$ ramp-down?
E.J. STRAIT: In the high $\ell$ case where the TAE modes are partially stabilized we no longer see the small, frequent drops in neutron emission correlated with bursts of TAE activity. However, larger, less frequent drops occur which are associated with fishbones and sawteeth. The net fast ion loss is almost as large in the second case, suggesting that the threshold in fast ion beta for the fishbone instability is only a little higher than for the TAE instability.

J. JACQUINOT: Do you observe degradation of current drive efficiency with NBI when TAE modes are excited?

E.J. STRAIT: We have no information, at present, on the effects of TAE modes on current drive efficiency, but this is an important issue which we plan to investigate in the future.

T.N. TODD: You commented that the changes in the spectral character of the TAE modes which were observed when comparing a higher elongation discharge with a lower elongation one were similar to the changes seen when operating at unusually high $\ell$. Is it possible that $\ell$ was higher in the higher elongation shots? What parameters were held constant?

E.J. STRAIT: In order to isolate the effects of discharge shaping as far as possible, this comparison was made between two discharges with the same safety factor at the edge. The q-profiles were very similar, so it is likely that the differences in TAE behaviour were due to shaping rather than current profile effects. However, more realistic theoretical calculations are needed to answer this question definitively.

A. GIBSON: You mentioned that these discharges also have large fishbone induced fast ion loss. Does this also lead to $\beta_{\text{fast}}$ limitation and can you distinguish this from TAE $\beta_{\text{fast}}$ limitation?

E.J. STRAIT: The losses due to TAE modes and fishbones can be distinguished by comparing the detailed time behaviour of the neutron emission and the MHD activity. TAE modes alone cause some loss, while a burst of simultaneous TAE and fishbone oscillations causes a significantly larger loss than either mode on its own. However, fishbones and other low frequency MHD modes can also cause a significant loss of fast ions and should not be ignored.
TRANSPORT OF ENERGY AND PARTICLES IN JET PLASMAS

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Abstract

TRANSPORT OF ENERGY AND PARTICLES IN JET PLASMAS.

Four series of recent JET L-mode experiments are used to elucidate the basic transport processes in a tokamak. Experiments with strong on/off axis heating show that the energy transport can be well described by a local diffusive model, \( Q = -\chi n VT \), for a large range of \( VT \), (including \( VT > 0 \)). Larmor radius scaling experiments show that the thermal diffusivity scales in a Bohm-like fashion which suggests that the turbulence responsible for the transport has a large radial correlation length compared to the Larmor radius, which is confirmed by reflectometer measurements of the density fluctuations. Further support for a Bohm-like scaling comes from the very weak dependence of the energy confinement on the plasma species observed in mass scaling experiments. Finally, the scaling impurity transport is examined in a series of experiments with varying current, toroidal field, density and auxiliary power.

1. INTRODUCTION

During the JET 1991-92 experimental campaign a series of experiments has been undertaken to improve our understanding of the transport of energy and particles in L-mode plasmas. In this paper a brief account of four of these experiments and their analysis is given.

The objective of the first series of experiments [1,2] with off and on axis ICRH following pellet injection has been to determine whether the heat flux \( Q \) can be described by a diffusive model of the form

\[
Q = -\chi(\nabla T, T) n \nabla T
\]

and in particular to investigate the behaviour of Eq.(1) for a large range of \( n \nabla T \).

The injection of the pellet prior to off-axis ICRH allows us to study both positive and negative values of \( -n \nabla T \) in the same pulse.

Some authors [3,4] have questioned the validity of Eq.(1), suggesting that \( \chi \) seems to respond to the power input profile rather than the temperature gradients, when on and off axis heating results are compared. In contrast from the JET experiments in which strong ICRH (\( P_{\text{CRH}} > 9 \ P_{\text{OH}} \)) is applied to a high density (\( n > 6 \times 10^{19} \ \text{m}^{-3} \)) reactor relevant plasma, it is found that the heat flux can indeed be well described by Eq. (1) for both positive and negative \( VT \); possible forms for \( \chi \) as a function of \( \nabla T \) and \( T \) are given. The JET current
ramp experiments [5] can also be described by a local diffusive model of the form of Eq. (1) and the dependence of $\chi$ on the safety factor $q$ and shear has been derived. The objective of the next set of experiments [5] described in section 3 has been to determine the scale size of the turbulence that is responsible for the transport; the scaling of $\chi$ with the ion Larmor radius $\rho_i$ is investigated. A set of dimensionally similar discharges with central ICRH have been developed in which all dimensionless parameters such as $\beta$, $v^*$, $q$ etc., except the normalised radius $\rho^* (= \rho_i /a)$, have been held constant.

An extensive local transport analysis reveals that a long wavelength "Bohm" scaling ($\lambda \sim$ plasma radial scale size) gives the best description of the data, rather than the short wavelength ($\lambda \sim \rho_i$) gyro-Bohm scaling of most theoretical models. This result is also in accord with correlation reflectometry measurements [6,7] where radial correlation lengths $L_c > 10 \rho_i$ are measured in L-mode plasmas. Such measurements are briefly reviewed in section 4.

Further support for the long wavelength turbulence model of transport comes for a series of experiments, described in section 5, in which the scaling of energy confinement upon the plasma ion species mass [8] is investigated. It is found that the dependence of the thermal energy confinement time on mass is very weak indeed, indicating the absence of the ion Larmor radius in the scaling.

Finally in section 6 further experiments on the diffusion of trace impurities, injected into the tokamak by the laser blow off technique, are reported [9]. The data exhibit the same strong radial dependence of the impurity diffusion coefficient $D_{imp}$ with plasma radius as reported earlier [10]: very low and close to neoclassical in the centre and high and anomalous on the outside. In the most recent experiments the scaling of $D_{imp}$ with power and density has been investigated. $D_{imp}$ has been shown to behave in a very similar manner to $\chi$ in the outer region suggesting that the same mechanism is responsible for the transport of both heat and impurities.

2. ON/OFF AXIS HEATING EXPERIMENTS

The original off axis heating experiments following pellet injection were completed in 1988 (1). During 1992 these experiments were repeated and supplemented with on axis and mixed on and off axis ICRH discharges. Fig.1 shows the time behaviour of electron temperature on axis versus time for the three types of discharges. The total power is the same in all three pulses, in the mixed heating pulse the on axis heating is applied one second after the off-axis heating. The difference in the response of the axial electron temperature to the on and off axis heating is very clear in Fig.1 and even clearer in Fig.2(a) where the off and on axis temperature profiles are compared at three different times. In the outer region beyond the off axis heating location the profiles are similar as would be expected but in the inner region the profiles are markedly different.

The evolution of the electron temperature for the off axis heated pulse, is given in Fig.2(b), note that the profile remains hollow for approximately one second and only becomes slightly peaked some 2 seconds after the start of heating. The reason for this behaviour can be understood from the power balance. Initially the off axis heating produces a hollow temperature profile with
FIG. 1. Time development of axial electron temperature and ICRH power for three discharges with purely off axis, purely on axis and mixed heating.

A shape close to the shape of the heating profile. Heat is then conducted away from the peak of the temperature profile towards the centre and edge. Due to the large thermal inertia of the plasma centre it takes a long time for the centre to reach the temperature of the region of heating. Eventually the ohmic heating dominates over the thermal inertia term and the electron temperature profile becomes marginally peaked in the centre with an outward heat flux.

A simple method of exhibiting the response of the total heat flux Q to positive and negative temperature gradients is to display the data in the Q versus -nVT space at different radii and this is shown in Figs. 3(a) for the off axis case and in Fig. 3b for all three pulses. The curves at radial positions close to the plasma centre in the off axis heating case pass close to the origin (Fig.3(a))
FIG. 2. (a) Electron temperature profile at three different times for purely on and off axis heated discharges. (b) Electron temperature profile for off axis heated case.
showing that the heat flux is approximately zero when $nV_T$ is zero. This implies that any heat pinch term would have to be very small. The higher radii curves have a steeper slope indicating that $\chi$ increases with radius, a common feature of all L-mode plasmas.

In Fig.3(b), the inner radial position $x = 0.25$ is shown for all three pulses, it can be seen that, as $-nV_T e$ increases, $Q$ also increases more strongly than linearly (roughly parabolically). The most obvious interpretation of this is that $\chi$ is simply proportional to $|\nabla T|$ since the density profiles are very similar in all three pulses. Other possible forms for $\chi$ such as the critical temperature gradient form proposed by RLW give an equally good fit (2). One can not rule out $\chi$ being a function of temperature only, since the temperature increases weakly with $\nabla T$; however $\chi$ would have to be quite a strong function of $T$, scaling as $T^{2.5}$.

All three forms for $\chi$ lead to the usual degradation of $\tau_e$ with power, the first $\chi \sim |\nabla T|$ giving $\tau_{e\text{th}} \propto P^{-1/2}$, $\chi \propto T_e^{2.5}$ giving $\tau_{e\text{th}} \propto P^{-0.7}$ and the critical temperature gradient mode giving the offset linear form $\tau_{e\text{th}} = A + B/P$.

3. NON-DIMENSIONAL SCALING EXPERIMENTS

It has been shown by Kadomtsev [11] and Connor and Taylor [12] that, provided the wavelength characterizing plasma turbulence is greater than the Debye length, then the local diffusivity can be expressed in the following dimensionally correct form:

$$\chi = \chi_{\text{Bohm}} F\left(\rho^*, v^*, \epsilon, \beta, \kappa, q, L_i, L_\kappa, \cdots\right)$$  \hspace{1cm} (2)

where $\chi_{\text{Bohm}} = cT_e/Be$ (the Bohm form) and the dimensionless parameters in $F$ are the normalised local Larmor radius $\rho^* (\equiv \rho_i/a)$, normalised local collisionality $v^*$, plasma beta $\beta$, inverse aspect ratio $\epsilon$, safety factor $q$, elongation $\kappa$, $L_i$, $L_\kappa$ are profile scale lengths such as $(\nabla n/n)^{-1}$. The dimensionless variables of Eq. (2) are given in ascending order of magnitude, ranging from the smallest $\rho^* \approx 10^{-4} \sim 10^{-3}$. Since tokamak experiments have not shown any abrupt changes in the dependence of $\chi$ upon $\rho^*$ we can expand Eq. (2) without loss of generality as

$$\chi = \chi_{\text{Bohm}} \rho^{x_p}_* F(\nu^*, \epsilon^*, \cdots)$$  \hspace{1cm} (3)

The value of the exponent $x_p$ determines the type of plasma turbulence responsible for the transport:

1. $x_p = 1$, corresponds to the turbulence having a characteristic scale length of the order of the Larmor radius (gyro-Bohm scaling).
2. $x_p = 0$, corresponds to a radial scale length of the plasma turbulence of the same order as the plasma radius (Bohm scaling).
3. $x_p = -1$, will result from a stochastic magnetic field throughout the tokamak.

It is important to establish the dependence of $\chi$ upon $\rho^*$ in Eq. (3): extrapolation from JET to ITER with $x_p = 1$ (gyrobohm) gives a fusion product $n\pi T$ 3 times the value obtained with $x_p = 0$ (Bohm).
FIG. 3. (a) Total heat flux $Q$ versus $-n\nabla T_e$ for off axis heated case at different values of normalised radius $x (= r/a)$. (b) Total heat flux $Q$ versus $-n\nabla T_e$ for on/off and mixed heated discharges at normalised radius $x = 0.25$. In the analysis of these pulses, the electron temperature data from the ECE diagnostic (Figs 1 and 2) have been normalised to the LIDAR Thomson scattering data.
The determination of $x_p$ is made by nondimensional scaling experiments in which all of the parameters of the function $F$ in Eq. (3) are held constant. In such experiments on TFR, DIII-D[13] and JET [5], $p^*$ has been varied by a factor $\sim$1.5 to 2.0. The experimental strategy adopted in the JET experiments is as follows. A series of pulses with $I_0 = 2, 3$ and 4 MA and $B_0 = 1.7, 2.6$ and 3.4T have fixed $q_\Psi (x = 1)$ and plasma shape. On axis ICRH is ensured by choosing frequencies 27, 41 and 54 Mhz. In these pulses the target densities are scaled as $n \sim I_0^{4/3}$, which should lead to

$$v^* \beta^2 = \text{const.}$$

where $v^*$ and $\beta$ are radial averages (see Christiansen et al.[14]). The power $P$ is varied in a predetermined pattern from 3 to 14 MW to cover the predictions of (3) for $x_p = -1, 0, +1$. The data on $v^*$ and $\beta$ from 16 "almost dimensionally similar" pulses confirms the relation (4) very well. A 2MA, 3MA and a 4MA pulse each with $v^* = 3 \times 10^{-3}$, $\beta = 0.25$ has been selected for local transport analysis.

The total conducted heat flux $Q$ is then compared with the Bohm and Gyro-Bohm scaling by normalising it with $Q_{\text{Bohm}} = nT_e^{-2}/aB$ and $Q_{\text{G-Bohm}} = nT_e^{5/2}/a^2B^2$. These comparisons are shown in Fig.4 for the three currents: clearly, the data is best grouped by the Bohm scaling. This

![Fig. 4.](image-url)
conclusion has to be somewhat qualified though, since as is shown in Fig. 5 not all of the dimensionless parameters were kept absolutely constant. Although $\beta$, $q$, $v^*$ were the same in the three pulses, the density and temperature radial scale-lengths were slightly different. It would however require a thermal conductivity model which had a strong dependence on $L_n$ (say $\chi \propto L_n^{-3}$) to improve the Gyro-Bohm model.

4. DENSITY FLUCTUATION MEASUREMENTS USING CORRELATION REFLECTOMETRY

The correlation reflectometry technique has been used to measure the scale-length and motion of the density fluctuations in the radial direction and the motion of the perturbation in the toroidal direction [6, 7]. In the radial direction, four independent reflectometers operating at different frequencies probe the plasma along the mid-plane. The reflectometers probe layers
separated by distances typically between 5 and 30 mm. The fluctuating signals are recorded with a wide-bandwidth, typically 100 kHz, during a time of interest in the plasma pulse. The measurements are analysed in pairs using standard data analysis techniques. The autopower spectra, $G_i(\omega)$, $i = 1, 2$, cross power spectrum, $G_{12}(\omega)$, crossphase spectrum $\theta_{12}(\omega)$, and the coherence function, $\gamma_{12}(\omega) = \frac{|G_{12}(\omega)|}{G_1(\omega) G_2(\omega)}$ $\frac{1}{2}$, are determined. The phase difference between the two signals is assumed to be due to the propagation of a density perturbation between the two sampled layers and thus gives information on the motion of the perturbations. The decrease in the coherence function with interlayer separation gives an estimate of the scale-size of the perturbations.

In the toroidal direction two independent reflectometers probe the plasma on the mid-plane at two different toroidal locations. The reflectometers operate at the same frequency and therefore probe the same density layer. The toroidal motion of the perturbations is determined from the slope of the crossphase spectrum.

Measurements with both pairs of devices have been made in three different plasma conditions (ohmic, L-mode, and H-mode) and for different radial positions of the reflecting layers ($0.5 < r/a < 0.95$). The measurements are interpreted in terms of fine scale density structures in the plasma. In ohmic and H-mode plasmas the structures have a radial extent $< 5$ mm. In L-mode plasmas they have a radial extent which depends on the level of additional heating power and for high power levels (15 MW) it is $> 30$ mm i.e. $\sim 10 \rho_i$ (Fig.6). The measurements also show that perturbations do not propagate radially but rather toroidally and/or poloidally. In L-mode plasmas, comparison of the reflectometer measurements with charge exchange measurements shows that the perturbations rotate toroidally with the plasma.

![FIG. 6. Variation of coherence function (average over 0-62.5 kHz) with interlayer separation for three different values of additional heating power. $\gamma_r$ is the lowest level of significance set by the finite size of the data set.](image_url)
5. THE DEPENDENCE OF CONFINEMENT ON THE PLASMA ION SPECIES

A series of discharges with fixed geometric configuration, current (3MA) and toroidal field (2.8T) but with different ion species H, D and He³ have been made. To avoid mixing of species the neutral beam heating system injected the background species. The time evolution of several parameters of discharges in deuterium and hydrogen are compared in Fig.7. The total energy content measured by the diamagnetic loop is ~ 25% larger in deuterium. The only other significant difference between the D and H discharges is in the sawtooth period which is a factor of two larger in deuterium.

A full transport analysis has been completed for all the data, and in Fig.(8a) both the total and thermal energy confinement time are shown as a function of density. The thermal energy confinement time in deuterium is some 15% larger than hydrogen at all densities, this difference being somewhat smaller than the difference in the total confinement times which is 25% due to the larger fast ion component with deuterium injection. Interestingly, the particle confinement time \( \tau_p \) (Fig.8(b)) in deuterium is 50% larger than that of hydrogen. In fact the difference in particle confinement times and hence the increased convective losses with hydrogen is partly responsible for the poorer energy confinement time in hydrogen.

Comparisons between helium-3 and deuterium have been made at higher densities and up to higher powers (12 MW) than the hydrogen-deuterium comparison. No significant difference is observed in the thermal energy confinement time and there is no evidence for any difference in particle confinement either.

In summary, the JET results point to a very weak dependence of the thermal energy confinement time on the ion species (\( \tau_e \propto A_i^{0.2 \pm 0.1} \)), and do not support the hypothesis used in most scaling laws of \( \tau_e \propto A_i^{0.5} \). In hydrogenic plasmas, particle confinement and sawtooth activity display the strongest isotope dependence.

6. THE SCALING OF IMPURITY TRANSPORT

The initial experiments on transport of trace impurities, injected by the laser blow-off technique (10), have been supplemented with a large set of experiments in which the scaling of the impurity diffusion coefficient \( D_{imp} \) with power P, current I, toroidal field B and density n has been investigated. The same measurement and analysis techniques described in reference (10) have been used and again the measurements show clearly the existence of two distinct transport regimes with a narrow transition region. An example is given in Fig.(9a) where the soft x-ray emissivity shows that some 80 ms after the injection of Ni impurity there is very little change in the central emissivity. There is only a significant increase in the central emission after a sawtooth crash.

The evolution of the soft X-ray emission is modelled using a time dependent impurity transport code. It is found that a \( D_{imp} \) having the radial form of Fig.9(b) gives a good fit to the data. In the inner region the transport is
FIG. 7. Time evolution of main plasma parameters in deuterium (solid line) and hydrogen discharges.
FIG. 8 (a) Thermal energy confinement time versus volume average density; (b) total energy confinement time (thermal + fast) versus volume average density; (c) particle confinement time versus density.
FIG. 9. (a) Evolution of X-ray emissivity profile along a horizontal chord following Ni injection; (b) Ni diffusion coefficient for discharge shown in (a).
close to neoclassical and in the outer region transport is strongly anomalous being some 20 times larger than that of the inner region.
By varying the current and field it can be seen from Fig. (10(a)) that the width of the low diffusion region appears to be related to the profile of the safety factor $q$ in the central region. There appears to be no scaling of $D_{\text{imp}}$ in the internal or external region with current as seen in the scaling of energy confinement. The density scan and power scan experiments at fixed current (3 MA) and field (3.2T) show that $D_{\text{imp}}$ increased with total power per particle $P_{\text{tot}}/n_e$ in the outer region (Fig. 10(b)), which suggests that $D_{\text{imp}}$ increases with $\nabla T$ or $T$, similar to the behaviour of $\chi$. In the inner region no correlation of $D_{\text{imp}}$ with power or density was found.
Thus, in summary it appears there is a fairly strong similarity between $D_{\text{imp}}$ and $\chi$, both in their radial behaviour and in their scaling with power per particle, however the scaling with current is different and further more detailed comparisons are presently being undertaken.

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REFERENCES


DISCUSSION

J. WEILAND: Was there fuelling at the centre during the pinch flux experiment or was there a particle outflux during the temperature evolution? It seems that this highly collisional experiment cannot rule out the presence of off-diagonal elements of the transport matrix proportional to $\nabla n$.

J.G. CORDEY: After the initial pellet fuelling no further fuelling takes place. There is very little change in the density profile during the evolution of the temperature profile. Convective heat loss terms are very small, and there is no evidence of any other large off-diagonal term.

R.J. TAYLOR: The $10_1$ result of radial correlation reflectometry only applies to L-mode plasmas. In H-mode plasma you may come to a different conclusion about the role of small scale turbulence.

J.G. CORDEY: Yes, this is correct. In the H-mode we observe much shorter radial correlation lengths, i.e. $\sim 2\rho_1$. 

M. KIKUCHI: In your isotope experiments, is internal inductance ($\ell$) kept constant? In JT-60U we have seen $\ell$ differ for hydrogen and deuterium. In addition, we have had 10–15% deuterium plasma in the hydrogen discharge (and vice versa). Did you incorporate a correction for such a dilution effect in your $A_{i0}^{0.2\pm0.1}$ conclusion?

J.G. CORDEY: Our $\ell$ was constant at the same value in all three shots: $D \to D$, $H \to H$ and $^3$He $\to ^3$He. The hydrogen concentration in deuterium was $<5\%$ and so no correction was made for this small effect.

S.D. SCOTT: Can you comment on whether the observed difference in sawtooth period between hydrogen and deuterium discharges is caused by the difference in beam stored energy?

J.G. CORDEY: Yes, we believe the difference in the sawtooth behaviour between $D$ and $H$ may indeed be due to the increased fast ion energy with deuterium over hydrogen. We have assessed this effect quantitatively.

B. COPPI: Have you tried to see whether a diffusion coefficient $D \propto T_e/r^2_p$, where $1/r_p = -(1/p)dp/dr$, can reproduce the JET data? Please note that since $D \propto T_e$, this does not involve excessive degradation with power.

J.G. CORDEY: The form you suggest would reduce to $\chi = |\nabla T_e|^2/T_e$. This would perhaps be a little strong in $|\nabla T_e|$ but certainly could not be excluded when the errors were taken into account.

M.C. ZARNSTORFF: For off-axis heating, your Fig. 3 shows $Q \propto n\nabla T$ with no offset at all radii, in particular for $\chi \geq 0.35$. How does $Q$ vary for these outer radii for on-axis and mixed heating?

J.G. CORDEY: The behaviour at the outer radii for the on-axis and mixed heating cases is very similar to that of the off-axis heating case.

M.C. ZARNSTORFF: Are the outer radii consistent with the composite curve shown for $\chi = 0.25$?

J.G. CORDEY: Yes, the outer radii can be made consistent with the $\chi = 0.25$ curve by including a radial dependence in $\chi$ of the form $q_p^{1/2}$. 
Abstract

The relationship between naturally-occurring large-scale (m=2) Mirnov oscillations, plasma density and potential profiles, and small-scale turbulence in the Texas Experimental Tokamak (TEXT) is described. It is experimentally observed that as the magnetic and density islands grow in size they rotate more slowly. Variations in this relationship between TEXT and TEXT-Upgrade are presented. Island widths are observed to be significantly broader on the high field side than on the low field side, in agreement with theoretical expectations. Microturbulence dependences on plasma profiles (e.g. density and potential), which are strongly perturbed in the vicinity of the magnetic islands, are unfolded. Theoretical expectations for tearing modes and electron drift waves are compared with the characteristics of the measured mode amplitudes and frequency spectra.

1. Introduction

The ability to induce intrinsic Mirnov oscillations of varying amplitude during the plateau phase of TEXT and TEXT-Upgrade discharges is used to investigate the relationship between MHD activity, plasma profiles and broadband microturbulence. Modifications to the plasma potential are measured with a heavy-ion-beam-probe [HIBP] while a
high-resolution interferometer is employed to characterize changes in the electron density. By relating changes in these parameters to those associated with magnetics, insight into the nature of these perturbations and their effect on the plasma is obtained. Correlation of broadband density fluctuations, measured via collective scattering, with changes in local parameters and their scale lengths, e.g. radial electric field and density scale length, assists in elucidating the nature of the turbulence. This paper will describe the details of these measurements and their interpretation.

2. Experimental Results

2.1. Plasma Profile Modifications

Ohmically heated discharges with large \( m=2, n=1 \) tearing modes can be produced in TEXT, in which the amplitude and frequency of the magnetic perturbation remains constant to within 10% for \( \leq 300 \) ms. This time is long compared to any resistive or confinement time scale. The mode structure is determined from poloidal and toroidal magnetic coil arrays and from soft X-ray or density tomography. The measured frequency \( f_{MHD} \) as a function of magnetic island width \( w \) is shown in Fig. 1(a). Here \( w \) is computed from the perturbed magnetic field at the resonant surface derived by extrapolating the perturbation measured at the wall radius back to the \( q=2 \) surface assuming that in the absence of a nearby conducting wall the field falls off as \( r^{-m+1} \). Because of the approximately 2 cm thick stainless steel wall of TEXT-Upgrade (versus 0.08 cm for TEXT), it was necessary to use internal magnetic coils to calibrate the external coils located outside of the insulating break. The island full width can then be calculated from the expression[1]

\[
w = 4 \left( \frac{\tilde{b}_r r_s q}{B_{\text{sat}} (dq/dr)} \right)^{1/2},
\]

where \( \tilde{b}_r \) is the oscillating radial field amplitude, \( r_s \) is the resonant surface minor radius, \( m \) is the poloidal mode number, and \( q \) is the safety factor; all quantities are evaluated at \( r_s \). The safety factor \( q \) and its spatial derivative \( dq/dr \) are derived from Thomson scattering electron temperature profiles and Spitzer resistivity. The results show a clear inverse relationship between \( w \) and \( f_{MHD} \), that is, island size increases with decreasing rotation frequency. Notice that the island width for a given frequency is significantly reduced in TEXT-Upgrade. This is thought to be due to the stabilizing influence of the thick conducting wall.
The measured density profile in TEXT exhibits a significant flattening in the vicinity of the q=2 surface for a particular orientation of the rotating magnetic island. From the relative phase and toroidal location of the Mirnov coil signals we conclude that the local density profile is flattened across island O-points but not across X-points. Typical density profiles viewing the island X-point and O-point are shown in Fig. 1(b), for a large-amplitude MHD discharge with $f_{\text{MHD}}=1$ kHz. At the O-point, the density profile is strongly modified by the dominant $m=2,n=1$ tearing mode as flattening occurs in the region of the q=2 surface. Variations in the O-point electron density profile with MHD frequency are shown in Figs. 1(b-d). Island induced density profile modifications are seen
to vary inversely with $f_{MHD}$. Time-averaged density profiles also tend to broaden with increasing magnetic island size.\[2\] From theory, when an island grows beyond a critical width $3\rho_s L_s / L_n (\simeq 3 \text{ cm for TEXT})$, sound waves propagating along the field lines tend to flatten the density profile.\[3,4\] The partial flattening of the density profile for $f_{MHD} > 4 \text{ kHz}$ is at least partly due to the spatial resolution limits of the interferometer ($\simeq 1.5 \text{ cm}$). Incomplete density equalization on a flux surface in the neighbourhood of the island separatrix may also play a role.\[3\]

The radial extent of the flattened region in the density profile can be defined as the density island width. The island width on the high-field side of the torus is measured to be $\simeq 10 \text{ cm}$ while that on the low-field side is $\simeq 6 \text{ cm}$ for $f_{MHD} = 1 \text{ kHz}$. The $m=2$ widths determined from profile flattening of soft x-ray emissivity and electron density are found to be comparable to estimates of the magnetic island width. Both the density island width and the maximum perturbation to the line-averaged density at approximately $r_q=2$, as shown in Fig. 1(e), are larger on the high-field side for all values of $f_{MHD}$. The smaller difference between the high- and low-field sides at large $f_{MHD}$ is likely due to spatial resolution limits of the interferometer. The observed flattening of temperature [175 eV for X-point and 125 eV for O-point at $q=2$ surface], density and soft x-ray emission profiles across the islands is greater on the torus high magnetic field side by a factor of $\simeq 1.5$. Such an asymmetry, required by poloidal flux conservation, was predicted by Finn.\[1\] A 3-D numerical magnetohydrodynamic (MHD) code, developed by Aydemir\[5\], which solves the full, nonreduced, resistive MHD equations in toroidal geometry, has been used to simulate the growth of these magnetic islands in TEXT. For our experimental conditions, the code predicts the ratio of the island widths from the high to low field side to be $w_H / w_L = 1.3$.

An analytical expression has been derived for the magnetic island shape resulting from a helical deformation of the toroidal Shafranov-like equilibrium assuming that the helical currents responsible for islands on a resonant surface $r = r_s$, are centered on that surface and are deformed with the magnetic surfaces. Writing down the components of $\mathbf{B} \cdot \nabla \Psi = 0$ and expanding $\Psi$ in the small parameters, for finite $\epsilon$ we obtain

$$x^2(1 + 2 \Delta' \cos \theta) = (1 - \cos(m\theta)) \frac{w^2}{8} ,$$

where $\Delta'(r) = (\beta_p + l_i/2)\epsilon$, $\epsilon = r/R_0$, and $x \equiv r - r_s$, which for $\cos(m\theta) = 0$ simplifies to the following expression for the toroidal island width

$$w_T^2 = \frac{w^2}{1 \pm 2\Delta'} .$$
This implies that the ratio of the island widths on the high to low field sides is
\[
\frac{w_H}{w_L} = \frac{1 + 2\Delta'}{1 - 2\Delta'} .
\]  
(4)

With \(\epsilon = 0.18\) at the \(q=2\) surface and an experimentally determined value of \(\beta + l_i/2 = 1.05\pm0.15\) from a combination of a horizontal saddle coil and a cosine coil, we find \(\Delta'(r_{q=2}) = 0.19\). Thus we estimate \(w_H/w_L = 1.5\), which is in good agreement with the other determinations.

The HIBP has been used to measure the plasma potential \(\phi\) from which the electric field is derived \([E_r = -(d\phi/dr)]\) across the minor radius. The central potential which is normally negative \((\approx -1000 \text{ V})\) becomes increasingly positive as the island width increases and approaches zero for the largest islands.[6] Only a small \(\approx \pm 50 \text{ V}\) oscillation in \(\phi\) is

![Graphs showing the relationship between island rotational frequency and radial electric field or inverse electron density scale length.](image)

**FIG. 2.** Island rotational frequency \(f_{\text{MHD}}\) as a function of (a) measured radial electric field \(E_r\) and (b) inverse electron density scale length. The density scale length is for the island region \(0.6 < r/a < 0.8\). (c) Comparison of \(f_{\text{MHD}}\) empirical scaling law with data from various tokamaks.
observed at the MHD frequency for large islands. In Fig. 2(a), the relationship between $f_{\text{MHD}}$ and $E_r$ is shown at the island radius $r/a=0.7$. As the island width increases, the frequency decreases along with $E_r$. The solid line represents a best fit to the data, while the broken line is the frequency expected if $f_{\text{MHD}} = E_r m /(2 \pi r B_\phi)$, with $B_\phi$ being the toroidal field and $m=2$. The difference between the two lines is approximately the electron diamagnetic drift frequency for an $m=2$ mode. For discharges with small islands, the measured $E_r$ is consistent with that given by its neoclassical value.[7] In addition, as implied by Fig. 2(b), $f_{\text{MHD}}$ scales proportional to the inverse density scale length measured in the island region. An empirical scaling law for the frequency of small islands is then found: $f_{m=2} \propto T_{e0} / (B_\phi a^2)$ with $T_{e0}$ being the electron temperature and $a$ the minor radius. In Fig. 2(c), it is observed that this expression well represents the data published from many different ohmically heated tokamaks.

2.2. Plasma Turbulence Modifications

In order to investigate the relation between MHD activity (i.e. tearing modes) and plasma microturbulence, the density fluctuation frequency spectra for poloidal wavenumbers $k_\theta$ at varying levels of Mirnov activity have been examined. The time-averaged (i.e. over approximately 10 cycles) turbulent spectra, as measured via heterodyne far-infrared laser scattering during discharges with low ($b_\theta/B_\theta \leq 10^{-3}, f_{\text{MHD}} = 7.1$ kHz), moderate ($b_\theta/B_\theta \approx 5 \times 10^{-3}, f_{\text{MHD}} = 5.0$ kHz) and large ($b_\theta/B_\theta \geq 10^{-2}, f_{\text{MHD}} = 2.4$ kHz) amplitude Mirnov oscillations, are shown in Figs. 3(a-c), for $k_\theta = 7$ cm$^{-1}$ [$k_\theta \rho_e \approx 0.5$]. As previously reported[2], for gas-fueled plasmas in TEXT with densities $2 \times 10^{19}$ m$^{-3} \leq n_e \leq 4 \times 10^{19}$ m$^{-3}$ and quiescent MHD activity (i.e. low amplitude), the frequency spectra [see Fig. 3(a)] are typified by a distinct large-amplitude broadband peak at negative frequencies, which corresponds to propagation in the electron diamagnetic drift direction. The measured dispersion relation for the turbulent density fluctuations agrees well with that predicted for electron drift waves when an $E_r \times B_\phi$ spectral shift is included:

$$\omega' = \omega_k + \omega_{E \times B},$$

where $\omega'_k$, $\omega_k$ and $\omega_{E \times B}$ ($\omega_{E \times B} = k \cdot v_D$, where $v_D = E_r/B_\phi$) are the frequency of the measured fluctuations, the turbulence dispersion relation, and the Doppler shift due to $E \times B$ plasma rotation, respectively. Since the $E_r \times B_\phi$ correction accounts for the bulk of the spectral shift, unambiguous identification of the electron drift wave dispersion is difficult.
FIG. 3. Time-averaged frequency spectra for $k_\theta = 7 \text{ cm}^{-1}$ with (a) low, (b) moderate, and (c) large-amplitude Mirnov oscillations. Negative (positive) frequency corresponds to the electron (ion) diamagnetic drift direction in the laboratory frame of reference. (d) Temporal evolution of the magnetic and density fluctuations. (e) Time-averaged fluctuation amplitude $S_k$ (triangles) and $S_G$ (squares), and island region density scale length $L_n$ (circles) for $k_\theta = 7 \text{ cm}^{-1}$.

However, a roll-over in the dispersion relation for $k_\theta > 7 \text{ cm}^{-1}$ is consistent with finite ion temperature and mass effects which are theoretically expected.[8] In addition, the spectral width $\Delta \omega/\omega^*$ $\geq 4$ appears to be a real feature of the turbulence and not due to a radially varying electric field across the sample volume.

At moderate to high levels of MHD activity, the frequency spectra at each wavenumber measured are significantly modified, as shown in Figs. 3(b) and (c). A notable feature readily distinguished from these broadband spectra is that the large-amplitude negative frequency peak shifts towards zero frequency as the MHD amplitude increases. Like
$f_{MHD}$, the fluctuation mean frequency $\bar{\omega}$ varies linearly with $E_r$ and $L_n$ for all $k_\theta$ measured. Time-resolved frequency spectra show virtually no change in either $\bar{\omega}$ or $\Delta \omega$ when looking at island X and O points. This is consistent with the measured $m=2$ component of the radial electric field, which is small with an upper limit of 3 V/cm. Within error bars, the $m=2$ component of $E_r$ may be zero.\[6\]

Changes in the fluctuation amplitude which correlate with $f_{MHD}$ and consequently the plasma parameter $L_n$ indicate that an increase in the turbulence magnitude, $S_k \propto \bar{n}_k^2$, directly correlates to a decrease in the density scale length. From the time-resolved density fluctuation amplitude data (integrated over all frequencies), as shown in Fig. 3(d) for $k_\theta = 9$ cm$^{-1}$, clear maxima and minima are observed which correlate with the island X-point and O-point, respectively. The density fluctuation amplitude, which is typically a few percent, is modulated by $\Delta \bar{n}_k / \bar{n}_k \simeq \pm 20\%$ during the MHD cycle for large magnitude islands. Similar observations are made at the other values of $k_\theta$ investigated. Recalling the density profile information provided in Fig. 1, one can readily see that $\bar{n}_k$ and $L_n$ at the island radius are inversely proportional. Mixing length estimates of the density fluctuation amplitude give the saturated limit as

$$\frac{\bar{n}}{n_e} \simeq \frac{1}{k_\theta L_n},$$

(6)

for the case of isotropic turbulence. Since $n_e$ changes are small and $k_\theta$ does not appear to change, one would expect $\bar{n}$ to scale inversely with $L_n$. The relationships between the time-averaged fluctuation amplitude, $L_n$ at the island ($q=2$) radius, and $f_{MHD}$ are provided in Fig. 3(e). Here $S_G$ refers to the portion of the total fluctuation level $S_k$ which can be attributed to electron drift wave type turbulence originating in the island region. While both $S_k$ and $S_G$ decrease with increasing $L_n$, better agreement with the mixing length estimate is achieved by using the parameter $S_G$.

REFERENCES

DISCUSSION

B. COPPI: How is it possible that you have an (ordinary) electron drift mode, given the finite values of $\eta_e$ you appear to have?

D.L. BROWER: The electron drift wave that I have been referring to pertains to a toroidal, collisional plasma with impurities. Under such conditions, drift waves are unstable even for large values of $\eta_e$, i.e. $\eta_e > 2/3$.

F. ALLADIO: To which range of $q(a)$ values does the correlation you observe between the MHD frequency and the peakedness of the density profile apply? In particular, can you also observe it at low $q(a)$ values?

D.L. BROWER: TEXT plasmas, which exhibit the large amplitude Mirnov oscillations, are limited to a narrow range of discharge parameters, i.e. $I_p$, $B_T$ and $n_e$. Consequently we have not been able to perform any type of $q(a)$ or density scan.

F.C. SCHÜLLER: You found an enhancement of the fluctuation level at the X points of the 2/1 mode and attributed it to the smaller value of $L_n$ at the X mode. However, magnetic turbulence models indicate that stochasticity of the magnetic field is largest at the tip of the islands, so you could equally attribute this particular observation to magnetic turbulence.

D.L. BROWER: While it is true that the density and magnetic fluctuations scale together at island X and O points, one must recall that the average density fluctuation level decreases as the islands become larger, i.e. as magnetic fluctuations increase. This would seem to indicate that the measured density fluctuations are not linked to magnetic turbulence.
PROFILE CONTROL FOR A STABLE HIGH $\beta_p$ TOKAMAK WITH A LARGE BOOTSTRAP CURRENT

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Abstract
PROFILE CONTROL FOR A STABLE HIGH $\beta_p$ TOKAMAK WITH A LARGE BOOTSTRAP CURRENT.

A stable high $\beta_p$ plasma with a large fusion power gain is numerically studied by making the pressure profile steep and employing an off-axis beam driven current. MHD equilibria are obtained that are consistent with the bootstrap current and the neutral beam driven current, and they are tested for ideal MHD stability, i.e. the $n = 1$ kink mode, the infinite $n$ ballooning mode and the infernal mode. Large fusion power gain can be obtained in an equilibrium with large bootstrap current owing to the steep pressure gradient. As a large bootstrap current creates a hollow $q$ profile, which makes the plasma unstable, further increase of the bootstrap current by pressure profile steepening is prohibited. However, by controlling the neutral beam current profile and shifting the pitch minimum position outwards, the ideal MHD modes can be stabilized and the bootstrap current fraction is enhanced further owing to the decrease of $B_p$ near the high pressure gradient. The results suggest a guideline for current profile control suitable to the pressure gradient profile.

1. INTRODUCTION

In a steady state tokamak reactor, the bootstrap current accompanying a high poloidal beta ($\beta_p$) discharge is a key issue for achieving a large fusion power gain by reducing the externally driven current [1]. Since the bootstrap current density is proportional to $-\sqrt{\epsilon (1/B_p)} (\partial \rho / \partial r)$, however, the current profile due to the bootstrap current is essentially hollow. In order to suppress MHD instabilities caused by this hollow current profile [2], active control of the current density and pressure profiles is required, while keeping a large bootstrap current fraction. The purpose of the profile control is to improve the stability of the equilibrium as well as to reduce the amount of the externally driven current.

Though a hollow current profile is unfavourable for MHD stability, the current profile should be kept hollow to reduce the externally driven current. Previous work [2] showed that MHD instabilities for the hollow current profile are caused by the large pressure gradient at the pitch minimum, i.e. zero shear region. One of the methods for solving this problem is to shift the position of the pitch minimum outwards, far from the location of the maximum pressure gradient, by controlling the
current density profile. In this paper, from the viewpoint of stable steady state operation of a tokamak reactor, we analyse the MHD stability in a high $\beta_p$ plasma dominated by the bootstrap current, and show the possibility to obtain a stable high $\beta_p$ plasma by the control of the pressure and current profiles.

2. SELF-CONSISTENT ANALYSIS

For the analysis of the MHD stability of a steady state high $\beta_p$ plasma, it is necessary to include the bootstrap current and the externally driven current in a self-consistent way. Here, for the self-consistent equilibria obtained by using the ACCOME code [3], ideal MHD stabilities, i.e. the $n = 1$ kink mode, the infinite-$n$ ballooning mode and the infernal mode, are studied to establish a steady state plasma with a large bootstrap current fraction.

In ACCOME, the geometric parameters and the profile data ($T_s(\rho)$, $T_e(\rho)$, $n_s(\rho)$ and $Z_{\text{eff}}(\rho)$) are required for the input data. Here $\rho$ is the normalized effective minor radius ($= \sqrt{V(\psi)/V_{\text{tot}}}$). MHD equilibrium is obtained by an iterative calculation for $\rho' (= dp/d\psi)$, including the $\alpha$ particle pressure and the beam pressure, and the current density $\langle \mathbf{J} \cdot \mathbf{B} \rangle$ of the bootstrap current and the beam driven current. The bootstrap current for multispecies ions, including fast ions, is numerically evaluated by using the Hirshman–Sigmar moment approach in the neoclassical theory [4]. The beam driven current is evaluated by analytic eigenfunctions of the Fokker–Planck equation.

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<th>TABLE I. REFERENCE PARAMETERS FOR CALCULATIONS</th>
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For a steady state plasma, the MHD equilibrium is solved with a restriction that the total plasma current is kept constant by the regulation of NBI power at every iteration step. The stabilities to the infinite n ballooning mode and low n modes are studied for a set of equilibria by using the numerical codes BETA [5] and ERATO-J [6], respectively. Machine parameters used in this analysis are shown in Table I [1].

3. PROFILE EFFECTS FOR STABLE HIGH $\beta_p$ PLASMAS WITH LARGE Q

3.1. Pressure profile control

First we study the role of the pressure profile in obtaining plasmas with a large fusion power gain, Q, which is the ratio of fusion power, $P_f$, to neutral beam power, $P_{NB}$. Here we consider two types of profiles of the plasma temperature and density. As a standard case we choose the following profile:

Type A: $T_i = T_e \propto \{(1 - \rho^2)^{1.5} + 0.01\}$

$n_e \propto \{(1 - \rho^2)^{0.5} + 0.01\}$

Type A has a relatively peaked temperature profile and a broad density profile. One possible way to obtain a stable high $\beta_p$ plasma is to create a monotonic current profile by means of a large fraction of externally driven current near the plasma centre. By combining central current drive with type A, a monotonic q profile (case A) can be produced, as shown in Fig. 1(a, b). In order to increase Q by reducing the beam driven power, it is necessary to increase the fraction of bootstrap current by choosing a pressure profile with a steeper pressure gradient near the halfway point on the plasma radius as follows:

Type B: $T_i = T_e \propto n_e \propto 0.6 \times (1 - \rho^2) + \exp\{-(\rho^2/0.55)^{1.8}\}$

In type B a hollow current profile (case B) is produced by the large bootstrap current, which causes a weakly hollow q profile even with the central current drive, as shown in Fig. 1(c, d).

Figure 2 (lines A and B) shows the results of the stability analysis of the n = 1 kink mode and Q as a function of $\beta_p$. As $\beta_p$ increases, the bootstrap fraction linearly increases, and the minimum value of q, $q_{\text{min}}$, rises in both types of profile. The equilibrium of case B has a higher $q_{\text{min}}$ and larger Q owing to the larger bootstrap current for the same $\beta_p$ value. Owing to the hollow q profile, the $\beta_p$ limit against the n = 1 kink mode without a conducting shell is reduced from $\beta_p = 1.6$ ($g = 3.0$) for case A to 1.45 ($g = 2.7$), and the limit of the ballooning mode also becomes lower. Here g is the Troyon factor of $\beta_t/(\Omega/\alpha_B)$. The infernal mode reduces the stability boundary further in both case A and case B.
FIG. 1. Pressure and current profiles of plasmas with $\beta_p = 1.6$. (a, b) Case A equilibrium with the normal temperature and density profiles of type A; (c, d) case B equilibrium with the temperature and density profiles with steep gradient of type B; (e, f) case C equilibrium with the temperature and density profiles of type B and off-axis beam driven current.
3.2. Current profile control

Next we try to control the current profile in addition to the pressure profile. MHD stabilities for the hollow current profile strongly depend on the relative location of the high pressure gradient and the pitch minimum [2]. Thus, stabilization can be expected by shifting the position of the pitch minimum outwards and keeping it apart from the high pressure gradient by off-axis current drive. In this analysis the beam line is shifted in the vertical direction, keeping the tangential injection. By the combination of off-axis current drive with type B, a more hollow q profile (case C) can be produced with the high pressure gradient inside the pitch minimum, as shown in Fig. 1(e, f).

Figure 3 shows changes of parameters characterizing the equilibrium and the MHD stability for the vertical position, \( \delta_z \), of the beam line with \( \beta_p = 1.6 \) (\( g = 3.0 \)). With increasing \( \delta_z \), \( q_{\text{min}} \) increases and the hollow profile becomes strong.
As the location of the maximum pressure gradient $p'_{\text{max}}$ relatively shifts to the negative shear region (the top graph in Fig. 3), the infinite $n$ ballooning mode becomes stable above $\delta_{x}/b \sim 0.2$ and the $n = 1$ kink mode becomes stable above $\delta_{x}/b \sim 0.45$ after destabilization owing to the existence of $p'_{\text{max}}$ on $q_{\text{min}}$ around $\delta_{x}/b \sim 0.25$. Here it should be noted that the bootstrap current increases further owing to the decrease in $B_{P}$ by the off-axis current drive.

As shown in Fig. 2 (lines C), by means of off-axis current drive ($\delta_{y}/b = 0.51$) in profiles of type B, $q_{\text{min}}$ is kept at a high value ($\sim 2.3$). Because of the reduction of the destabilizing factor due to the pressure gradient in the zero shear region, the
critical $\beta_p$ is improved in comparison with the previous case (line B in Fig. 2). The infinite n ballooning mode is stable even in the region with the steep pressure gradient because of the stabilization of the negative shear, and the infernal mode disappears. Larger $Q$, more than two times greater than the normal profile of case A, is obtained. The improvement is more pronounced when we consider the conducting shell stabilizing effect (open and closed triangles in Fig. 2). Fourier mode analysis of the radial displacement of the $n = 1$ kink mode for $\beta_p = 2.0$ is shown for cases A and C in Fig. 4. In the monotonic q profile (case A) the dominant $m = 2$ mode is located on the $q = 2$ rational surface, though it is strongly coupled with higher poloidal modes (Fig. 4(a)), while in the hollow q profile (case C) the dominant mode of $m = 3$ is located on the pitch minimum (Fig. 4(b)). Since the pitch minimum is shifted to the outside of the plasma by current profile control, the location of the

**FIG. 4.** Radial dependence of the normal displacement $\rho \xi_p$ (solid lines) and the q profile (broken lines) for the plasmas of $\beta_p = 2$ on (a) line A and (b) line C in Fig. 2.
dominant mode is also shifted outwards. Therefore, if we could expect wall stabilization even in the steady state plasma, a greater stabilization effect could be expected in the hollow current profile of case C.

In the above analyses, the off-axis current drive is carried out by the vertical shift of the neutral beam line. The same effect can be produced by the reduction of the beam energy. When the beam energy is reduced from 2 to 0.6 MeV, by keeping the beam line through the plasma centre, the deposition profile of the beam is changed near the plasma surface and the position of the pitch minimum shifts outwards. Consequently, stable high $\beta_p$ plasmas with large Q are obtained similarly, as shown by point D in Fig. 2. The lower beam energy is desirable from the viewpoints of reducing the engineering difficulty and improving the efficiency of the neutralization.

4. CONCLUSION

A high $\beta_p$ and large Q tokamak stable against ideal MHD modes was found to be possible by the control of the pressure and current profiles. For the hollow current profile resulting from a large bootstrap current fraction, the relative position of the maximum pressure gradient and the minimum q is essential for the MHD stability. Off-axis current drive, by means of off-axis neutral beam injection or the reduction of neutral beam energy, was shown to be effective both for the stabilization of MHD modes and the enhancement of the bootstrap fraction. This scenario is a realistic and attractive method to realize a steady state tokamak.

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IONIZATION INSTABILITY AND SELF-CONTAINED PLASMA EDGE MODEL

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Abstract

IONIZATION INSTABILITY AND SELF-CONTAINED PLASMA EDGE MODEL.

A self-contained equilibrium model of the plasma edge is proposed whereby: (i) ionization of the neutrals released by the material structure not only is an electron/ion source, but also is the driving mechanism for a drift-like mode; (ii) marginal stability of the mode is brought about by electron Landau damping (the reason why collisional damping is inadequate is mentioned); and (iii) plasma is conducted to the target plates along open field lines which, owing to the mode's radial magnetic field component, penetrate the plasma edge to a depth of a few centimetres. The theory provides expressions for the electron temperature at the separatrix (more precisely on the last closed equilibrium magnetic surface) and for the particle confinement time. The latter, interestingly, is almost identical to the Kaye-Goldston energy confinement time; these expressions are compared with measured or estimated values. The theory also predicts bifurcation to an equilibrium with higher energy content when a certain heating power is exceeded. Because it overlaps a region in which the equilibrium profiles change substantially (the plasma edge region), occurrence of the drift eigenmode requires the presence of a self-consistent radial electric field given (except for a constant) in dominant order by the equation \( \omega_E + \omega^* = \omega = \text{const} \) (\( \omega_E \) is the \( E \times B \) Doppler frequency and \( \omega^* \) is the electron diamagnetic frequency; \( \omega \) is the mode's angular frequency). It is shown that the electric field profile required by the theory is consistent with some observations.

1. EQUILIBRIUM CONSTRAINTS FOR GLOBAL DRIFT EIGENMODES

The abscissa of the conventional drift eigenmode turning point \( x_{t.p.} = a_s \left( L_s/L_N \right)^{1/2} \) with the conventional notation, see below) being at least as large as the equilibrium length scale, the plasma edge equilibrium has to fulfill certain compatibility conditions in order to allow this global drift structure to coexist. These are

\[
\partial \left( \omega_E^{(0)} + \omega_e^{* (0)} \right)/\partial x = 0, \tag{1}
\]

where \( \omega_E = - k_B c E_x/B \) and \( \omega_e^* = k_B (c T_e/e_e B) \partial \ln N_e/\partial x \) (\( k_B \) is the wave vector

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component in the direction $-\hat{a}_x \times \vec{B}/B$ of the binormal to the equilibrium magnetic field), and

$$v_E = (\pi/2)^{1/2} \eta_e \omega_e^{*2}/2|k_1|c_e,$$  \hspace{1cm} (2)

where $v_E = \langle \sigma v \rangle_e N_0$ [$\langle \sigma v \rangle_e$ is the electron ionization rate coefficient, $N_0$ is the neutral density, $\eta_e = (\partial \ln T_e/\partial x)/(\partial \ln N_e/\partial x) = L_n N_e/L_T$, $k_1 = k_q x/L_s$ is the parallel component of the wave vector (the mode's rational surface is at $x=0$), $L_s$ is the shear length, $c_e = (T_e/m_e)^{1/2}$]. The first condition arises from the fact that the eigenmode's frequency $\omega(0) = \omega(0) + \omega^{*(0)}$ must be space independent [the correction $\omega - \omega^{(0)}$ to this expression is of order $(a/L)^2$, i.e., typically $< 10^{-2}$; here $a = c_e/\Omega_e$, $c_e = (T_e/m_e)^{1/2}$, $\Omega_e = eB/m_e c$]; it provides the self-consistent equilibrium radial electric field profile which is a plasma internal degree of freedom (it has to be noted that a constant radial electric field does not affect the stability and is thus left arbitrary). The second condition is the mode's marginal instability condition; it is also required for local ambipolarity of the quasi-linear cross-field fluxes; the left-hand side is the destabilizing (primarily electron) ionization rate; the right-hand side is the stabilizing (for $\eta_e > 0$) electron Landau damping rate. [The compelling physical reason why Landau damping is preferred to collisional damping ($\gamma_L/\gamma_C = 0.77 |k_1|c_e \tau_e$) is that the latter decreases as $T_e^{-1/2}$ (the former increases as $T_e^{3/2}$); a related equilibrium would therefore be thermally unstable]. It has to be noted that Eq. (2) requires either $\partial \ln T_e/\partial x$ or $\partial \ln N_e/\partial x$ to vanish at the mode's rational surface; interestingly, the highly resolved DIII-D edge density profiles display a narrow plateau (that we identify with the rational surface) within an otherwise monotonic trend in the L (low confinement) regime. One can show (see Fig. 1 and Ref. 3) that Eq. (1) also is qualitatively consistent with DIII-D density, temperature and electric field profiles if it is noted, on the one hand, that the radial electric field inferred experimentally $\langle E_x \rangle_{exp}$ includes a contribution $- \langle n_i/N_i \rangle \partial \Phi/\partial x$ arising from the correlation of the probe impurity density and electric field oscillations and, therefore, is not the electric field appearing in Eq. (1): rather,

$$\langle E_x \rangle_{exp} = E_x + e_x \partial \Phi |z^2/2T_e \partial x$$  \hspace{1cm} (3)
FIG. 1. The first graph, where $V_1^* = \omega_1/k_1$, has been derived by the authors from the data of the DIII-D group (Ref. 2). The second graph is essentially a plot of $\varepsilon d^2\Phi/2T_2 \delta x$ [cf. Eq. (3)] for a suitable choice of the mode amplitude ($\varepsilon/T_2 \approx 0.4$) and of the mode width. Summing the two yields $V_E + V_1^*$ approximately constant, in compliance with Eq. (1).

(this expression, however, is valid only if $n_l/N_1 = n_e/N_2$); and, on the other hand, that the potential $\Phi$ vanishes at the rational surface in view of $[\partial \ln n_e/\partial x]_{x=0} = 0$ (cf. above) and of the eigenvalue equation

$$\left[ -\frac{(\omega_1 - \omega_e - \omega_e^*)^{(f)}}{\alpha \omega_e^{(0)}} - k_2^2 c_2^2 + \frac{k_1^2 c_2^2}{\omega_e^{(0)}^2} \right] \Phi + \frac{a_s^2}{N_e \alpha \omega_e^{(0)}^2} \frac{\partial}{\partial x} \left( N_e \alpha \omega_e^{(0)} \frac{\partial}{\partial x} \omega_e^{(0)} \Phi \right) = 0$$

(4)
where \( \alpha(x) = 1 + (1 + \eta) \frac{T_i}{T_e} \). Additional checks that Eqs. (1) and (2) are generally verified would confirm that the heating power indeed forces its way out of the discharge by setting up the kind of eigenmode considered here.

2. SELF-CONSISTENT EDGE PROFILES

The destabilizing role of electron ionization may lead to a self-contained theory of the plasma edge if it can be argued that the excited mode induces ionized particles and energy fluxes to the walls: recycling would provide neutrals; ionization of the latter as they enter the plasma both would provide electron/ion pairs and drive the mode to bring them back to the plates where they would be recycled. The ambipolar quasi-linear fluxes across the magnetic field, however, turn out to be directed towards the core [as a result of the electron stabilizing role in Eq. (2)]. This result suggests an alternative trail to explain the rapid rise observed in the peak plasma density when neutral gas is blown into the vessel (see also Ref. 4), but is not compatible with steady state operation. The quasi-linear results, however, are of little use if the particle’s orbits intersect the walls. This is precisely what is going to occur in the presence of an instability either if the radial cross-field oscillation (owing to the mode poloidal electric field or pressure gradient) or the radial component of the mode’s magnetic field guide a sufficiently large amount of plasma beyond the last closed equilibrium magnetic surface (or separatrix). (If the ion collisionality parameter \( \nu_i^* > 1 \), as is the case in DIII-D \(^5\) and in TEXTOR \(^6\), these are actually the sole relevant orbit losses). These considerations have led us to study the solution of the following system of steady state equations:

\[
\begin{align*}
\frac{\partial n_e}{\partial x} &= -\frac{\partial n_i}{\partial x} = v_k N_e; \quad \frac{\partial Q_e}{\partial x} = 0, \tag{5a}
\end{align*}
\]

and, cf. Eq. (2):

\[
v_k = \left( \frac{\pi}{2} \right)^{1/2} \left( \frac{c T_e}{e B} \right)^{1/2} \left( \frac{\partial n_e}{\partial x} \frac{\partial n_e}{\partial x} |k_2| L_z \right) \tag{5b}
\]
assuming that \( Q_e^* = \lambda \Gamma_e^* T_e \) [where \( \lambda = (m_e T_e / m_i T_i)^{1/2} \gg 1 \) for losses along "open" magnetic field lines], \( \Gamma_0 = -v_0 N_0 \) (\( v_0 > 0 \) is the radial velocity component of the incoming neutrals) and \( \Gamma_e^* + \Gamma_0 = 0 \). The marginal stability condition (5b) closes the system which is amenable to a single second order nonlinear equation (the spatial variations of \( \lambda, v_0 \) and \( \langle \sigma v \rangle_e \) are neglected, as well as the coupling to the ion temperature). If the explicit \( x \) dependence of Eq. (5b) is replaced by a characteristic length \( (|x|) \), the solution is

\[
N_e = N_e \tanh[(x_s - x)/\lambda_0] \quad T_e = T_e \cosh^{\lambda_0^2}[(x_s - x)/\lambda_0] \quad (6a)
\]

and, from \( v_0 N_0 = T_e^* = Q_e^*/\lambda T_e \) and \( Q_e^* = P_e/4\pi^2 a R \),

\[
N_0 = P_e/4\pi^2 a R \lambda v_0 T_e(x); \quad (6b)
\]

here, \( x_s \) is the abscissa of the separatrix, \( N_e(x=x_s) = 0 \), \( \lim(x \to -\infty) N_e(x) = \bar{N}_e \), \( T_{e,s} = T_e(x=x_s), \lambda_0 = (4/5) v_0 / \langle \sigma v \rangle_e \bar{N}_e \), \( P_e \) is the power carried away by the electrons (\( P_i \ll P_e \) should be expected on the basis of the model since \( c_i \ll c_e \)), and \( a \) and \( R \) are the plasma minor and major radii. The divergence of the electron temperature and the flatness of the density profiles for \( x \to -\infty \) follow from the neglect, in the calculation, of transport mechanisms appropriate to the tokamak "confinement zone"7.

From the physical meaning of Eq. (2) [or Eq. (5b)] and its \( k_\phi \) dependence, it is to be expected that the equilibrium will be held together by the mode with the lowest mode number, all the others being actually damped to their thermal level. In support of the model are thus the facts that an estimate of \( k_\phi a \) from Eq. (5b) yields a value of only a few unities upon introduction of the DIII-D edge profile parameters (the angle average of the neutral density being estimated from the equation \( \tau_p = a \bar{N}_e/2 \), \( \Gamma_{e,s}^* = a \bar{N}_e/2 N_0 v_0 \) where \( \tau_p \) is the particle confinement time), and that a helical instability with \( k_\phi a \sim q_a \) known as the Toi mode has persistently been observed in ASDEX L discharges 8,9.
The solution of Eqs. (5a) and (5b) also provides the expression for $T_{e,g}$; replacing $k_B$ by ($\sim$) $q/a$ where $q$ is the edge safety factor, as just argued, and $x_e$ by $a/2q^2$ (i.e., $x_e$ characteristically is half the distance between the surfaces $q_a$ and $q_a-1$), one finds with a proper prescription for $(|x|)$:

$$T_{e,g} = \left[0.5 \times 10^4 \lambda^{-1} (A_e/2)^{-1/2} T_0^{1/2} P_e B^2 q^{-3} N_{e,20}^{-2} a R^{-2} \right]^{2/5}$$  \hspace{1cm} (7)

where $A_e$ is the atomic mass (we note that $\lambda \propto A_e^{1/2}$) and the various physical parameters are expressed in the following units: $[T_e,0] = 10$ eV; $[P_e] = 1$ MW, $[B] = 1$ T, $[N_{e,20}] = 10^{20}$ m$^{-3}$, $[a,R] = 1$ m; $q$ can be eliminated in terms of the plasma current via the expression $q = 5a^2B/RI$ where $[I] = MA$.

3. CONSEQUENCES OF THEORY AND EXPERIMENTAL ASPECTS

3.1. The expression of the particle confinement time can be obtained from these results and the definition $\tau_p = a\bar{N}_e/211_e$: in the above practical units:

$$\tau_p = 6.3 \times 10^{-2} B^{-2/5} P^{-3/5} N_{e,20}^{1/5} R^{7/5} A_e^{1/10}$$ \hspace{1cm} (8)

(if $T_0 = 10$ eV and $\lambda = 10A_e^{1/2}$); Eq. (8) is almost identical to the empirical Kaye-Goldston expression of the energy confinement time$^{10}$ (even the normalization factors do agree if both formulae are converted to the same units). Moreover, solution of the exact system of equations (5a), (5b) yields an almost linear dependence between the exact confinement time and expression (8), except at low densities and/or high $q$'s, i.e. typically for $x_e/\lambda_0 = (5/8)$ a $\langle oV \rangle_e \bar{N}_e q^2 \nu_0 \leq 1$. Introducing finally the experimental parameters of Ref. (11) in Eq. (7) yields $\tau_p = 16$ ms (we have added an ohmic power of 0.3 MW to the neutral beam power of 1.6 MW) to be compared with the reported estimation of 20 ms.

3.2. The electron temperature predicted on the last closed magnetic surface and its density dependence are quite reasonable. We obtain $T_{e,s} = 54$ eV for the N.B. heated plasma of Ref. (11) and, respectively, $T_{e,s} = 42$ eV and $T_{e,s} = 32$ eV for
the ohmic hydrogen and deuterium plasmas of Ref. (12) (V = 1 V and N_e = 0.3 x 10^{20} m^{-3}): these values compare well with those reported. As in most tokamaks, the bulk temperature however is higher in deuterium than in hydrogen^{12}; this can be explained by theory since the edge length scale \( \lambda_0 \) is proportional to \( A_i^{1/2} \) through \( v_0 \).

3.3. The drift wave amplitude and profile have to adapt to the level of power to be exhausted. The radial component of the energy flux, indeed, can be written

\[ P_e/4\pi^2aR = Q_e^* = \chi_e^c (b_x/B) B^{-1} (\hat{B} + \hat{b}) \cdot \nabla T_e \]

where \( \chi_e^c = 3.16 N_e T_e \tau_e/m_e \) is Braginskii's electron thermal conductivity along the field lines and \( \hat{b} \) is the mode's magnetic oscillation. We thus obtain, for example,

\[ (b_x/B)_{xs} = (1.4/n) \times 10^{-2} Z_{ef} P_e a^{-1} T_{e,s}^{-7/2} \]  

(9)

at the separatrix [where \( B^{-1} (\hat{B} + \hat{b}) \cdot \nabla = nm/\pi qR = n/\pi R \), \( n \) being the number of toroidal collectors around the cross-section]. Inserting again the parameters of Ref. (11), we find \( (b_x/B)_{xs} = 1.5 \times 10^4 \). The half width of the magnetic island belt which brings the confined plasma beyond the separatrix \( [w/2 \sim (4b_x L_s/|k_b| B)^{1/2} \sim (4aRb_x/B^2)^{1/2}] \) is accordingly \( \sim 1.5 \) cm. We note however that \( (b_x/B)_{xs} \propto P_e^{2/5} \) since \( T_{e,s} \propto P_e^{2/5} \) [Eq. (7)]. It is thus to be expected that the width of the magnetic islands will decrease if the heating power is increased! Above a certain power level they will no longer carry plasma to the other side of the separatrix; the neutral source will then dry up, leading to collapse of the ionization rate (H\( _{\alpha}/D_{\alpha} \) signal) and, therefore, of the island belt itself. As in the L-H equilibrium bifurcation^{13}, the profiles will steepen until another exhaust mechanism is turned on. It is interesting to note that the Toi mode also disappears at the transition^{8,9}.

3.4. A final point of interest is that one can show, qualitatively, that the mode tends to produce a foot in the edge current density profile, as observed on TEXTOR^{14}; this result might open the way to a novel interpretation of major disruptions depending, like Gibson's^{15}, but for different reasons, primarily on atomic processes.
REFERENCES

SELF-CONSISTENT MODELS
OF DRIFT WAVE TURBULENCE:
IMPLICATIONS FOR TRANSPORT SCENARIOS

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Abstract

SELF-CONSISTENT MODELS OF DRIFT WAVE TURBULENCE: IMPLICATIONS FOR TRANSPORT SCENARIOS.

Recent results from an ongoing study of drift wave turbulence in a sheared magnetic field are described. The key point is a self-consistent treatment of the electron dynamics: density and temperature fluctuations evolve independently according to their own equations rather than being tied in a 'linear response' to the fluctuating potential. Consequences follow from the differing non-linear dynamical character of these quantities as well as their coupling through parallel electron dynamics and collisions involving electrons. Results are both positive and negative: positive in the sense that many exotic phenomena of tokamak transport have simple explanations in the context of this model; negative in the sense that the validity of many if not all of the customary treatments of turbulence and of drift waves in turbulence become highly questionable in the light of these results.

1. INTRODUCTION

Most work, even in recent years, on anomalous transport in fusion devices has concentrated on turbulence as a random walk or diffusive process. It has been concentrated on a search for an instability or a small set of instabilities which can serve as the drive for turbulence of the familiar variety subject to mixing length rules: there must be a given drive rate, $\gamma$, and a preferred scale, $k_1^{-1}$ (perpendicular to $B$), such that particles and energy are transported down-gradient proportionally to $D \sim \gamma/k_1^2$. The mainstream preoccupation has been with deriving transport scalings arising from such models for comparison with actual transport fluxes, given the gradients measured in the devices. This paradigm, like any other, has a set of validity conditions behind it, especially that there should be only one important space- or time-scale and that the dynamics should behave as in a random process in the thermodynamical sense. The point of this paper is to show that these conditions can be badly violated, even within the physics of a rather simple model: that of collisional drift wave turbulence in a sheared slab geometry. The model contains, however, ingredients which are common to all modes of the electron based fluid dynamics at drift scales. This and the fact that it nevertheless reproduces many important properties observed in tokamak fluctuations and transport, especially those which, at least superficially, do not fit within the usual perspective, should provide strong
motivation for moving beyond existing practices and rebuilding transport scenarios, even those based on familiar modes, in a more realistic way.

In the second part of this paper, trapped electron instabilities of the drift wave type are reconsidered, using a drift kinetic equation with a Lorentz collision operator for the electrons and a cold fluid operator for the ions. It is found that these instabilities are rather weak, especially when the temperature gradient is retained, to the extent that non-linear instability may be dominant in realistic situations. It is highly advisable that this whole class of transport candidates be readdressed with self-consistent treatments. Here, as well, it is time to discard the body of assumptions which has hampered progress in understanding anomalous transport for many years.

2. COLLISIONAL DRIFT WAVE TURBULENCE

The most basic form of the model used herein is collisional, electrostatic electron dynamics with cold (inertia providing) ions in a two-dimensional sheared slab magnetic field [1, 2]. Braginskii equations with parallel collisional effects are incorporated into a numerical simulation of $\bar{n}$, $\bar{T}$, $\bar{\phi}$ (density, temperature and potential fluctuations) as well as the parallel ion flow, $\bar{u}_i$, all self-consistently with $E \times B$ non-linearities in separate equations. The collisional Ohm's law relates the current, $\bar{J}_i$, to all three of $\bar{n}$, $\bar{T}$, $\bar{\phi}$, so that the central ingredients are $E \times B$ advection of $\bar{n}$ and $\bar{T}$ in the presence of their gradients, with shear induced, dissipative coupling between all three quantities through the parallel dynamics. The former represent the sources; the latter refer to the sinks.

Two scales play important roles in the resulting turbulence: the longest available wavelength on the resonant surface, with poloidal wavenumber $k_y = k_0$, and the width, $\Delta_D = \sqrt{0.51 \omega_{pe} L_y/k_0 V_e}$, of a layer at the resonant surface in which departures from electron adiabaticity, hence non-trivial electron dynamics, are allowed [3]. Modes with $k_y \sim k_0$ and $k_y \Delta_D \sim 1$ are in direct contact, with inverse spectral (non-linear) transfer in $\bar{\phi}$ maintaining low $k_y$ and direct transfer in $\bar{n}$ and $\bar{T}$ feeding higher $k_y$. All properties of the mode structure and the results follow therefrom. As an example, Fig. 1(a) shows the spectra of the energy sources and sinks: the net surplus at an intermediate spectral region with net deficit at both higher and lower $k_y$ is evidence that both directions of interscale transfer are important. The most important result is the fact of a non-linear instability: linear waves are all damped, but the ability of isotropic fluctuations to persist in the longest wave's $\Delta_D$ channel allows strong enough enhancement of free energy tapping to sustain the turbulence. Hence, the most basic ingredient of most transport models — the linear growth rate or its absence — is of sharply diminished relevance. The bidirectional spectral transfer makes clear the incorrect nature of the popular 'i-delta' model, in which $n_i/n$ is represented by $(\epsilon \bar{\phi}_k/T)(1 - i\delta_k)$: such a model would force all quantities to have identical non-linear dynamics. It is easy to show further that the spectra of two independent non-adiabatic quantities cannot both be represented with
FIG. 1. (a) Energy source ($\Gamma_+$) and sink ($\Gamma_-$) rate spectra for $C = 5$ and $\eta_s = 0.7$. Non-linear transfer in both directions out of the $k_y \Delta_0 \sim 1$ region is indicated. (b) The $\delta$ needed in the 'i-delta model' to reproduce the non-adiabatic density spectrum (solid) or the $\nabla n$ energy source rate (dashed). (For $C = 5$ and $\eta_s = 0.2$.) The lack of resemblance indicates the ill posed nature of such a model.

the same $\delta_s$ spectrum (Fig. 1(b)): the 'i-delta' model cannot even represent correct results, let alone dynamics! Moreover, the relative phase distributions are narrow, the amplitude distributions of non-adiabatic quantities are not Gaussian, etc., so the turbulence is clearly not of a diffusive or random nature.

3. TRANSPORT ISSUES

Isotope mass scaling in which transport weakens with heavier ions may be understood in terms of the relative importance of the $\Delta_0$ layer to the whole, or $C = \Delta_0^2/\rho_s^2$, where $\rho_s = \sqrt{M_i T_e/Z(c/eB)}$ measures the ion scales. Changes in the ion $M_i/Z$ affect only $\rho_s$, so that heavier ions reduce the relative strength of non-adiabatic dynamics. Since mixing length concepts are irrelevant, one need only note that lower $C$ leads to weaker turbulence and transport. This is shown in Fig. 2.

Sheared equilibrium $\mathbf{E} \times \mathbf{B}$ flows ($v_0$) are incorporated into the basic system to test recent ideas on the possible suppression of turbulence [4]. It is found that the turbulence may be either driven or damped by the flows, for energetic reasons. Short wavelength $\eta$ are tilted by the flow, so that $\delta$ is excited by $\eta$ into an energy losing mode structure with respect to the flow. At long wavelengths, the whole mode structure changes in order to tap the flow energy directly. The $\mathbf{E} \times \mathbf{B}$ flow therefore drives at low $k_y$ and damps at $k_y \Delta_0 \sim 1$. Further, it drives where it affects the mode structure and damps where it does not. Both modes of behaviour are directly counter to naive two-point correlation arguments [5] and can be taken to rule them out. The energetic arguments, verified by direct measurement of the results, can also explain why turbulence is never suppressed at larger $C$: the driving tendency at low
FIG. 2. Isotope effect. Mode widths (a) follow $\Delta \phi$ but $\Delta_\phi$ for $\tilde{\phi}$. Transport (b) drops with increased $M_i$. (Heat flux $q_H$ is in terms of 'gyro reduced Bohm': $q_B = \frac{1}{4} nT_e \times \rho_t^2/L_x$.)

$k_y$ introduces an instability before the damping tendency at higher $k_y$ becomes sufficient to suppress the turbulence. Turbulence is, however, suppressed at the $C \approx 3$ appropriate to ASDEX and DIII-D L-mode edges; the results do fit the ion orbit loss models of Shaing and collaborators. It is important to note here the strongest evidence yet obtained against the two-point correlation model: two of the cases with different $C$ (3 and 10, respectively) were given the same velocity shear, $V = (L_n/c_s) (dv_0/dx) = 0.3$. Both suffered the same structural shift to longer wavelength as a result of the suppressive tendency at $k_y \Delta_D \sim 1$. Mode widths in both cases at $k_y \Delta_D \sim 1$ were reduced since the adiabatic region, unable to acquire support from the gradients, became an energy losing region. However, for $C = 10$, the drive tendency at long wavelength was able to compensate, and this case maintained its amplitude while the $C = 3$ case was suppressed (Fig. 3). No mixing length, two-point, quasi-Gaussian, or other similarly non-self-consistent arguments can stand in the face of this result. But it is a natural result of the energetic consequences of the sheared flow on drift wave mode structure, provided of course that the structural freedom between $\tilde{\phi}$ and $\tilde{\phi}$ is respected.

Up-gradient transport can result from collisional drift wave turbulence if there is a large disparity in the relative density and temperature gradient strengths ($|\log \eta_e|$ large, where $\eta_e = d \log T_e/d \log n$). The fluctuation corresponding to the weaker gradient is excited mainly through electron compression, which results directly in a negative phase shift relative to $\tilde{\phi}$, hence transport in the up-gradient direction. For example, if $\nabla n$ is weak, the $\tilde{n}$ will be forced by the parallel compression and the turbulence:

$$\frac{\partial \tilde{n}}{\partial t} \approx -\langle \nabla \cdot \nabla n \rangle - \langle k_T^2 \rangle \langle V^2/n_e \rangle \left(1.71 \bar{T} + \bar{n} - \bar{\phi} \right)$$

in suitably normalized units, where the angle brackets indicate averages over the mode structure. Turbulence acting by itself tends to randomize phases (force the
average to zero) [6], so the average particle flux is determined by the only capably competing effect left — the parallel compression, which forces the phase of \( \bar{n} \) towards that of \(-\left(1.71 \bar{T} - \phi\right)\). Since \( \nabla T \) is forcing \( \bar{T} \) ahead of \( \phi \), this will turn out negative if \( \nabla n = 0 \).

In an edge fuelled discharge, an initially flat or hollow density profile 'backs up' as a result of this up-gradient transport, until the competition of the density gradient and these compression effects finds a balance in which the average \( \bar{n} \rightarrow \phi \) phase shift is zero. This was seen in the computations [4], in which \( \eta_e \) was varied at constant \( \nabla T \) for \( C = 5 \) and 20 (Fig. 4). Both cases show the drives corresponding to \( \nabla n \) and \( \nabla T \) (\( \Gamma_n \) and \( \Gamma_T \), respectively) dropping to zero as these gradients become relatively weak. The particle pinch at \( \eta_e \approx 1.5 \) (defined as a vanishing \( \Gamma_n \) — hence particle flux also — at finite \( \nabla n \)) is consistent with the experimental situation [7]. The mechanism is not unlike the original proposal by Coppi and Spight [8], but that was a linear response model with the turbulent \( \phi \) taken as given. The present results have the advantage of having emerged self-consistently from a model free of dynamical assumptions.

The validity of the resistivity gradient driven turbulence (RGDT) model [9], which has received wide attention, has been checked by incorporating an \( \bar{n} J_z \) term into Ohm's law [10]. It was found that RGDT's incorrect neglect of parallel pressure and thermal forces in the Ohm's law are responsible for its results. And when the correctly self-consistent system is simulated, rippling instability effects are restricted to the linearly unstable region of drift rippling parameter space, as originally argued by Hassam and Drake [11]. Since no tokamak edge is in this regime at present, the RGDT model can be safely ruled out as a viable scenario for edge turbulence.
FIG. 4. Pinch effect in collisional drift wave turbulence, for $C = 5 \text{ (a)}$ and $C = 20 \text{ (b)}$. (Here, $C$ is defined at fixed $L_p$, and the abscissa is linear in $L_p/L_T$.) Note that cases with $\eta_e > 1.5 \text{ (b)}$ had to be driven externally.

4. DYNAMICS OF TRAPPED ELECTRONS

Popular models of dissipative trapped electron (DTE) effects for drift wave turbulence misrepresent important properties of the collision process. Use of Krook-type operators [12] underestimates the effects of the circulating particles. Sensitivity to collision operator modelling has been extensively investigated by Rewoldt et al. [13] in slab geometry with no trapping, and a pitch angle dependent Krook operator was used for toroidal geometry [14]. Although it was concluded that pitch angle dependences could be modelled with Krook-type operators [13], this was not tested in the presence of trapping, which gives rise to sharp boundaries in velocity space (in the simplest sense, the parallel velocity for trapped electrons may be declared to vanish because of bounce averaging). The key ingredient of small angle collisions is velocity space diffusion, which is captured at its simplest by the Lorentz operator, $C_L(f)$. This effect is what transfers energy gained by the trapped electrons to parallel dissipation in passing electrons. In fluid treatments, a term such as $-n_{\text{eff}}(a_1 n_\perp - a_2 \phi)$ in the trapped particle density ($n_{\text{tr}}$) equation should be balanced by an equal term of opposite sign in the $\phi$ equation so that the overall $\dot{n}/\dot{t}$ is not changed by collisional dissipation. (Here, $a_1$ and $a_2$ are appropriate constants, and $n$ and $\phi$ are in suitably normalized units.) A similar procedure can be used to ensure energy conservation if temperature fluctuations are followed. To test these ideas, a kinetic simulation has been developed for simple magnetic geometry (straight field, $k_i = \text{constant}$), treating the electrons in the drift kinetic approximation and the ions as a cold fluid, as before. The collision operator is $C_L(f)$. Difficulties in treating $C_L(f)$ at the boundaries in velocity space are surmounted by expanding $f$ in Legendre polynomials, $P(\zeta)$, where $\zeta = v_{||}/v$. Parameters are collisionality, $\nu = v_{||}/\omega_*$, transit ratio, $q = \sqrt{2k_i V_e/\omega_*}$, trapped electron fraction, $g$, gradient ratio,
FIG. 5. Dissipative trapped electron mode (DTEM) growth rate, $\gamma_L$, as a function of $\nu$ for $\eta_e = 0$ (a), and of $\eta_e$ for $\nu = 0.1$ (b), each with $k_y \rho_s = 0.5$ and $g = 0.5$. Notable are (a) the smooth transition from collisional drift wave to DTEM as $\nu$ drops, and (b) the stabilizing effect of $\nabla T$.

$\eta_e$, and mode wavenumber, $K = k_y \rho_s$. Infinite bounce frequency is assumed and $q$ is set to 10.

In general, the passing electrons indeed play an important role in the energetics, numerically, about one third the strength of free energy access (and note that the velocity space diffusion is what holds the latter to small values). Two very interesting results are obtained: (1) At $\eta_e = 0$ there is a gradual transition between proper collisional drift waves at large $\nu$ and the DTE mode at $\nu < 0.1$, perhaps the kernel of an explanation for the absence of an obvious transition in the character of transport in tokamaks from edge to interior. (2) The longer-wavelength modes ($K \leq 0.3$) are completely stabilized by $\eta_e$ (maximum unstable $\eta_e$ is at $\nu \lesssim 1$). At $\nu_e = 0.01 c_s/L_n$, the $K \sim 1$ modes are still unstable, but with $\gamma \sim 10^{-2} c_s/L_n$. Further, non-adiabatic circulating electron dynamics have an important role in limiting the $\gamma$ values. The contrast with conventional viewpoints does not need emphasis. In general, the DTE instabilities are rather weak: the maximum growth rate seen for experimentally relevant $\eta_e$ was $0.06 c_s/L_n$ for $K = 2$ at $\nu = 0.1$, $g = 0.5$ and $\eta_e = 2$. At $\eta_e = 1$ the longest-wave unstable mode was $K = 0.7$, with $\gamma = 0.02 c_s/L_n$. With such weak growth rates, linear stability is unlikely to have much effect on turbulence due to these modes (Fig. 5).

An indicative test of the idea that the non-linear instability should determine the turbulent state even in the presence of a weak linear instability is provided by adding a constant 'curvature' of $\mathbf{B}$ in the collisional drift wave model. The fluid equations were rederived with $\nabla \mathbf{B}$ a constant co-direction with $\nabla p$. The non-zero $\nabla \mathbf{B}$ serves mainly to drive the phases of $\mathbf{n}$ and $\mathbf{T}$ ahead of $\mathbf{\phi}$, exacerbating instability. The controlling parameter is $\epsilon_n = L_n/L_B$. Runs were done for $C = 3.36$ and $\eta_e = 1$, with results shown in Fig. 6. With $\epsilon_n = 0$, one has the usual collisional drift wave case with no linear instabilities, as discussed above. With $\epsilon_n = 0.06$, every Fourier
component is linearly unstable, but the effect on the turbulence — mode structure as well as amplitude — is negligible. When $\epsilon_n \approx 0.12$, strong enough to overcome the collisional dissipation at the longest wavelength, $k_y = k_0$, the linear instability takes over and the turbulent state is dominated by the single component at $k_y = k_0$. Although this is no model for toroidal curvature, it does serve to show that linear instabilities need not be important. A rough guide may be the linear growth rate as compared to the energetic throughput of the non-linear instability acting alone. By this rule, linear instabilities as strong as $0.1 k_y \rho_s c_s/L_n$ are important, and weaker ones might not be.

The $\bar{n}$ have the same non-linear properties for both trapped and circulating electrons. So DTE driven turbulence may as a result be qualitatively similar to collisional drift wave turbulence, albeit with different basic scales. Self-consistent treatment of $\bar{n}$ is definitely called for; such a model is under development.

5. CONCLUSIONS

The turbulence results arise from fully self-consistent computations of a model with broad validity, which shares properties not only with other drift wave models (e.g. DTE) but even with other microinstability based transport scenarios. Many widely assumed properties of turbulence are simply not tenable, most importantly because drift type turbulence is forced turbulence in which different components have differing non-linear dynamical properties. Simple models based on inertial 3-D fluid turbulence are simply not applicable to this situation, a fact which should enjoy wider appreciation.
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RADIAL PROPAGATION OF TURBULENCE IN TOKAMAKS

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Abstract

RADIAL PROPAGATION OF TURBULENCE IN TOKAMAKS.

Radial propagation of turbulence could explain a non-local dependence of the fluctuation spectra and transport coefficients on plasma parameters. It is shown in the paper that such a propagation could be due to toroidal coupling or non-linear mode coupling. In the first case, the propagation is characterized by a group velocity whereas both ballistic and diffusive types of behaviour are possible when non-linearities are involved. A numerical simulation shows that a strong perturbation radially diffuses in the non-linear regime.

1. INTRODUCTION

Although it is well known that microturbulence is responsible for anomalous transport in tokamaks, there is no theory providing satisfactory transport coefficients. Theories based on drift turbulence predict a scaling with plasma parameters that is in agreement with the experiment [1]. However, the predicted radial profiles of heat diffusivities decrease generally at the edge, in contradiction to profiles given by local power balance analysis. This drawback is generally overcome by involving a specific turbulence at the edge [2] or by adjusting the radial profiles of the transport coefficients while maintaining drift mode scaling [3].

In this paper we propose a quantitative model based on the concept of radial turbulence propagation, which can explain this behaviour. Such a model describes how turbulence at a given radius is coupled to turbulence elsewhere, leading to diffusion coefficients which do not only depend on local plasma parameters. Thus, the bulk turbulence in a tokamak could be partly induced by the edge turbulence, which is known to be strong. Radial propagation can be induced by mode coupling due to toroidal or non-linear effects. These two processes have been investigated and compared by using a unified Lagrangian formalism. A numerical calculation has been carried out to test this model.
2. TOROIDAL COUPLING

For the sake of simplicity, only electrostatic perturbations will be considered in the present paper. In cylindrical approximation, an electrostatic perturbation $U(\vec{x}, t)$ can be expressed as

$$U(\vec{x}, t) = \mathcal{U}(r - r_L) \exp\{i(m\theta + n\varphi - \omega t)\}$$  \hspace{1cm} (1)

where $\theta (\varphi)$ is the poloidal (toroidal) angle, $r$ the minor radius and $L = (m, n, \omega)$. The radius $r_L$ corresponds to the $n, m$ resonant surface, where the safety factor $q$ is equal to $-m/n$. The radial eigenfunction $\mathcal{U}$ is assumed to be even. In toroidal geometry, an eigenmode is characterized by a frequency $\omega$ and a toroidal number $n$ and can be written as a sum of poloidal harmonics:

$$U(\vec{x}, t) = \sum_m U_m(r - r_L) \exp\{i(m\theta + n\varphi - \omega t)\}$$ \hspace{1cm} (2)

If the gradient lengths are larger than the scale of $U_L$, a WKB approach is possible and the Fourier components can be written as

$$U_L(r - r_L) = f(r_L) \exp(im\delta) \mathcal{U}(r - r_L)$$ \hspace{1cm} (3)

where $\delta$ is the ballooning angle (weakly dependent on $m$), and the envelope $f(r)$ depends slowly on $r$. Let $\rho(U)$ be the plasma charge response to $U$. The self-consistency (i.e. plasma quasi-neutrality) can be expressed by stating that the Lagrangian $\mathcal{L}_w(U, U^*) = -\int d^3x \rho_w(U) U^*$ is extremum with $U^*$. The energy content of a mode is obtained by expanding $\mathcal{L}_w(U, U^*)$ around a zero growth rate state ($\gamma = \text{Im} \omega \ll \omega$), i.e.

$$\gamma \frac{\partial}{\partial \omega} (\omega \text{ Re } \mathcal{L}_w) + \omega \text{ Im } \mathcal{L}_w = 0$$ \hspace{1cm} (4)

Since the quantity $\omega \text{ Im } \mathcal{L}_w$ is the energy exchanged between waves and particles, the first term represents the growth rate of $W = (1/2)(\partial/\partial \omega) \text{ Re } (\omega \mathcal{L}_w)$, then identified as the energy content of a mode. The energy flux $\Phi$ carried by a mode $L = (m, n, \omega)$ around the resonant surface $r = r_L$ is linked to the part $\mathcal{L}_w^{\text{prop}}$ of $\mathcal{L}$ which depends on the gradient of the envelope $f(r_L)$. It may be deduced from the energy balance equation that $\partial\Phi/\partial r = \omega \text{ Im } \mathcal{L}_w^{\text{prop}}$ (calculated per volume unit). This part of the functional is linked to the toroidal mode coupling, and the ion contribution is dominant, i.e. near an $n, m$ resonant surface we have:

$$\mathcal{L}_w^{\text{prop}}(U, U^*) = \sum_{\epsilon = -1, +1} (f(r_L + \epsilon d/n) f(r_L) - f^2(r_L)) \exp(i\epsilon\delta) I_i$$ \hspace{1cm} (5)

$$I_i = -\frac{n_e e^2}{T_i} \int d\rho \left\langle \frac{\omega - m\Omega^*}{\omega + (n/dqR) \rho v_i} \Delta(v_i) \right\rangle$$

$$\times \left[ r \frac{\partial}{\partial \rho} (-\epsilon m) \right] J_0 \mathcal{U}(\rho - \epsilon d) J_0 \mathcal{U}^*(\rho)$$ \hspace{1cm} (6)
where \( \rho = |r - r_L| \), \( d/n \) (\( d = -qr/s \)) is the distance between two adjacent resonant surfaces, \( R \) the major radius, \( S = 4\pi^2 R \) the tore surface, \( s \) the magnetic shear, \( J_0 \) accounts for finite Larmor radius effects, and \( m\Omega_i^* \) is the ion diamagnetic frequency. The coupling contains two terms \( \epsilon = \pm 1 \) corresponding to the interaction of the mode \( m \) with its two neighbours. The toroidal coupling strength is determined by the parameter \( \Delta(v_i) \), which is the horizontal shift of the trajectory normalized to the minor radius (\( \Delta \approx q\rho_{\text{th}}v_i/2rv_{\text{th}} \)). Since \( \mathcal{U}(\rho) \) is even and \( \Delta(v_i) \) is odd the two integrals \( I_\pm \) in \( \mathcal{L}_{\text{prop}}^\text{e} \) are equal. Hence,

\[
\text{Im} \mathcal{L}_{\text{prop}}^\text{e}(U, U^*) = \frac{d}{dr} \left( f^2(r_L) \Re I_\pm \frac{d}{n} \sin \delta \right) \tag{7}
\]

which can be expressed as the divergence of an energy flux \( \Phi \). Dividing \( \Phi \) by the energy density \( W \), a group velocity can be obtained:

\[
V_{\text{group}} = \frac{d}{n} \frac{\partial \mathcal{L}_L}{\partial \delta} \left/ \frac{\partial \mathcal{L}_L}{\partial \omega} \right.
\]

was already found in Ref. [4]. Using Eqs (5-7), we find that a rough estimate of \( V_{\text{group}} \) is a fraction of the diamagnetic velocity \( V^* \):

\[
V_{\text{group}} \approx \frac{q}{s} k_{\theta\text{th}} \sin \delta V^* \tag{8}
\]

Thus, modes with \( \delta = 0 \) or \( \pi \), i.e. ballooned in the equatorial plane, do not propagate.

3. NON-LINEAR COUPLING

The turbulence can be described as a superposition of modes localized around resonant surfaces \( r_L \). By analogy with the ballooning representation, each Fourier harmonic is assumed to be the product of a standard radial shape \( \mathcal{U}(r - r_L) \) by an envelope term \( f(r_L) \) and a complex amplitude \( u_L \):

\[
U(x,t) = \sum_L U_L(r - r_L) \exp\{i(\mathcal{M} \theta + n\varphi - \omega t)\}
\]

\[
U_L(r - r_L) = f(r_L) u_L \mathcal{U}(r - r_L) \tag{9}
\]

For a given mode \( L \), the non-linear part of \( \mathcal{L}_{\text{prop}}^\text{e} \) near an \( n, m \) resonant surface is given by

\[
\mathcal{L}_{\text{prop}}^\text{e}(U, U^*) = \sum_{L',L''} \left( f(r_L) f(r_{L'}) f(r_{L''}) - f^2(r_L) \right) u_{L'}^* u_L u_{L''} \Gamma_{L,L',L''} \tag{10}
\]

\[
\Gamma_{L,L',L''} = -\frac{S}{T_i} \frac{ne^2}{4\pi^2} \int \frac{dK dK' dK'' \Gamma_{L,L',L''}}{dK dK'} \mathcal{U}^*(K) \mathcal{S}(K') \mathcal{U}(K')
\]

\[
\mathcal{S}(K') = m'K' - m''K' \mathcal{U}^*(K) \mathcal{S}(K') \mathcal{U}(K')
\]

\[
\mathcal{U}(K') = m'K' - m''K' \mathcal{U}^*(K) \mathcal{S}(K') \mathcal{U}(K')
\]

\[
\mathcal{S}(K') = m'K' - m''K' \mathcal{U}^*(K) \mathcal{S}(K') \mathcal{U}(K')
\]
\[ A_{L,L',L''}(K',K'') = \exp\{-i[K'(r_L - r_{L'}) + K''(r_L - r_{L''})]\} \delta(K - K' - K'') \]

where

\[ \Gamma_{L,L',L''} = \frac{i}{2B_r} \text{P.P.} \left\{ \frac{1}{R_{L''}} \left( \frac{\omega' - m'\Omega^*_r}{R_{L'}^2} - \frac{\omega - m\Omega^*_r}{R_L} \right) \right\} \times \delta(L - L' - L'') \]

\[ R_L = \omega - \left( n + \frac{m}{q(r)} \right) \frac{v_L}{R} \]

Using an expansion of \( f \) around \( r_L \) in expression (10) we obtain

\[ \text{Im} \mathcal{L}_{\text{prop}}^f = \frac{d}{dr} \left( \frac{f^2(r_L)}{3} \sum_{L',L''} (r_{L'} + r_{L''} - 2r_L) I_{L,L',L''} \text{Im}(u^*_{L'} u_{L''}, u_{L''}) \right) \]

This equation is similar to Eq. (7), except that it involves a sum over many modes. The main difference is that the phase between interacting modes is not so well defined as in the toroidal case, the term \((r_{L'} + r_{L''} - 2r_L) u^*_{L'} u_{L''} \) replacing the quantity \( d/n \sin \delta \) in Section 3. If this phase term does not vanish, the propagation is convective.

By using the mixing length rule to estimate the turbulence amplitude, a group velocity which is a fraction of the diamagnetic velocity is found. Nevertheless, the constant phase matching has not been proven in the present paper, and it is also possible that the ‘average phase’ in Eq. (11) vanishes because of random phases. In this case, the energy flux propagation is expected to be diffusive.

4. NUMERICAL CALCULATION

It can be shown [5] that the extremum of the Lagrangian \( \mathcal{L} \) gives the Hasegawa-Mima equations [6] when modes are in the marginal situation near the linear dispersion relation. Looking for the phases \( u_M(t) (M = (m, n)) \), which give the \( u_L \) after a Fourier transform in time, the following equations are obtained:

\[ i \frac{\partial u_M}{\partial t} = \omega_M u_M + \frac{1}{2} \eta \varepsilon^2 \sum_{M'} g_{M,M'-M'} \frac{(m - m')^2 - m'^2}{m^2} u_{M-M'} u_{M'} \]

On the assumption of a Gaussian radial shape, \( U(x) = U_0 \exp(-K_0^2 x^2) \), the coupling elements are

\[ g_{M',M''} = \left( \frac{m'}{n'} - \frac{m''}{n''} \right) \frac{n''m' + n'm'' + nm}{n} \times \exp \left\{ - \left[ \frac{K_0^2}{3} \frac{d^2}{n^2} \left( \frac{m'}{n'} - \frac{m''}{n''} \right)^2 (n^2 + n'^2 + n''^2) \right] \right\} \]

\[ ^1 \text{P.P. stands for 'principal part'.} \]
\[ \eta = -\left(\frac{2}{3}\right)^{3/2} \frac{q \gamma K_0}{s \bar{m} \Omega_s B} \quad \epsilon = \frac{\bar{m}}{r} \rho, \quad d = \frac{q}{s} r_0 \]

where \( \bar{m} \) is a reference poloidal number and the time is normalized to \( \bar{m} \Omega_s \). The linear drift frequency \( \omega_M \) is given by

\[ \omega_M = \frac{m}{m} \left[ 1 - \epsilon^2 \left( \frac{m}{m} \right)^2 \right] \]

The grid for the numerical simulation contains all modes \( m, n \) in a cone determined by \( 1 \leq n \leq 12 \) and \( 1.1 \leq m/n \leq 1.9 \) (Fig. 1). Note that the slope \( q = -m/n \) gives the safety factor, i.e. the radial position. At \( t = 0 \), the amplitudes are initialized to one in an upper cone bounded by \( 1.7 \leq q \leq 1.9 \). This forced turbulence then propagates towards lower angles. The average time necessary for modes on a magnetic surface to reach a level of \( 10^{-4} \) as a function of \( q(r) \) is given in Fig. 1. The shape is clearly parabolic, indicating a diffusive behaviour in this case.

**FIG. 1.** (a) Grid for the numerical simulation where black points are modes set to one at \( t = 0 \). (b) Propagation time of a turbulence as a function of the safety factor for \( \epsilon = 0.3, \eta = 1, \bar{m} = 12, K_0 \bar{d} = 3 \). The dashed line indicates a parabolic fit \( q = 1.6 + 0.1Nt \).
5. CONCLUSIONS

The turbulence level profile in tokamaks is determined by two processes: the local energy source (or sink) and the energy flux associated with the radial propagation of the turbulence. This propagation can be due to the toroidal coupling as well as to non-linear coupling of modes localized on neighbouring magnetic surfaces. Toroidal coupling convects energy while non-linear coupling may create a convection or a diffusion since phase matching between adjacent modes makes both types of behaviour possible. It should be noted that, as this is the case for standard waves, propagation can be hampered by impedance mismatch due, for example, to a shear layer. Thus, the heat conductivity may not depend on local parameters but rather on a weighted stability of the whole profile. For example, the turbulence level at the plasma edge could be dominated by the influence of turbulence propagating from the scrape-off layer (SOL) rather than by local stability.

REFERENCES

LONG WAVELENGTH ELECTROSTATIC MODES AND STABILIZING EFFECTS OF DISSIPATIVE TRAPPED ELECTRONS ON MHD MODES

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Abstract
LONG WAVELENGTH ELECTROSTATIC MODES AND STABILIZING EFFECTS OF DISSIPATIVE TRAPPED ELECTRONS ON MHD MODES.
In the long wavelength regime, \((k_\phi)^2 \approx O(10^{-2})\), electrostatic modes propagating in the electron diamagnetic drift approach the ion acoustic transit mode. Even when \(T_i = T_e\), the ion Landau damping is effectively suppressed by the ion magnetic drift. A hybrid Bohm-like diffusivity, \(\chi_e \propto T_i^{1/2}B_iB_e\), emerges from the instability. MHD ballooning mode and high energy ion modes are subject to effective stabilization by dissipative trapped electrons. A collisionality parameter of order \(\nu_e = 10^{-2}\) can stabilize the high energy ion modes.

1. Introduction
In the long wavelength limit, the drift type electrostatic modes approach the ion acoustic transit mode, \(\omega_s = c_s/qR\). Since in tokamaks \(T_i \approx T_e\) holds, such modes are expected to be strongly Landau damped by ions. However, as recently pointed out [1], ion Landau damping can be mitigated by the ion magnetic drift provided the modes propagate in the electron diamagnetic drift (\(\omega > 0\) in the conventional formulation). In Ref. [1], it has been shown that long wavelength electrostatic modes in tokamaks tend to acquire a constant frequency, \(\omega_r \approx \omega_s\), without suffering ion Landau damping. The marginally stable ion acoustic transit mode can easily be destabilized by electron dissipation, e.g., inverse Landau damping on transit electrons and magnetic drift resonance of trapped electrons.
The latter mechanism is dominant in high temperature collisionless discharges. In this paper, results will be presented of linear stability analysis and quasilinear estimates for the electron thermal diffusivity caused by the ion acoustic transit instability. The diffusivity is Bohm-like (rather than gyro-Bohm) and also inversely proportional to the poloidal magnetic field, $B_\theta$, indicating confinement improvement with the toroidal plasma current.

In burning tokamak reactors, the stability of the Alfvén mode in the presence of energetic ions ($\alpha$ particles) will be a major concern. The recent $\alpha$ simulation experiments and successful tritium injection in JET appear to indicate that in reactor grade tokamaks $\alpha$-driven energetic ion modes may not be as dangerous as generally conjectured. Recent studies [2,3] have revealed significant stabilizing roles played by dissipative trapped electrons. Stabilization is effective particularly for low $m$ modes ($m \lesssim 10$), which can be stabilized by a relatively small electron collisionality parameter, $\nu_{ee} \gtrsim O(10^{-2})$.

2. Destabilized Ion Acoustic Transit Mode and Anomalous Transport

In the gyro-kinetic formulation of drift-type low frequency modes, the ion kinetic resonance occurs at a velocity $v$ satisfying

$$\omega + \omega_{Di}(v) - k_{||}v_{||} = 0$$

where

$$\omega_{Di}(v) = \frac{cM}{eB^3} \left( \frac{v_\perp^2}{2} + v_{||}^2 \right) (\nabla \times B) \cdot k$$

is the ion magnetic drift frequency with the standard notation. When $\omega > 0$, that is, when the modes propagate in the electron diamagnetic drift, the velocity range of resonance is severely restricted or nullified even when the frequency is low and of the order of the ion transit frequency [1]. Ion Landau damping is therefore effectively mitigated for modes propagating in the electron diamagnetic drift, and the lower limit of the unstable range in the poloidal wavenumber $k_\theta$ is extended to a smaller value (longer wavelength). When the ion kinetic resonance is unimportant, ion dynamics can be described by fluid approximation. From the ion continuity equation and momentum balance equation parallel to the magnetic field, one obtains the following ion density perturbation:

$$n_i = \frac{\left( \omega + \omega_{Di} \right) \left[ \omega_{ee} - \omega_{De} - (\omega + \omega_{ee})b_\theta \right] + \omega_{ei}^2 \frac{e\phi}{T_e}}{\left( \omega + \omega_{Di} \right)^2 - \omega_{ei}^2 / \tau} n_0$$
where \( \omega_a = k_{\|} c_s \) is the ion acoustic transit frequency, \( \tau = T_e / T_i \), and \( b_a = (k_{\perp} \rho_s)^2 \) with \( \rho_s = c_s / \Omega_i \). It is noted that in the limit \( \omega_D \to \infty \) and \( \omega_a(k_{\|}) \to \infty \), the ion density approaches the adiabatic limit, \( n_i = -e \phi n_0 / T_i \), as expected from the gyro-kinetic formulation. Also, for simplicity, the ion temperature gradient \( (\eta_i) \) is ignored, for it is only weakly stabilizing, and plays no significant roles for positive \( \omega \) modes. For the trapped electrons, we employ the two-temperature approximation [3]. The electron density perturbation is

\[
n_e = F_e \frac{e \phi}{T_e} n_0 = \left( 1 - \sqrt{\frac{\omega - \omega_{ee}}{\epsilon}} + \sqrt{\frac{\epsilon \omega_{ee} \omega_{De}}{(\omega - \omega_{De})(\omega - 3 \omega_{De})}} \right) \frac{e \phi}{T_e} n_0
\]

The resultant mode equation, when \( \tau = 1 \), is

\[
\left( \frac{c_v}{q R} \right)^2 \frac{d}{d\theta} \left( \frac{1}{\omega + \omega_D(\theta)} \frac{d\phi}{d\theta} \right) + [\omega + \omega_D(\theta)] V(\theta) \phi = 0
\]

where \( \omega_D(\theta) = 2 e_n \omega_a (\sin \theta + s \theta \cos \theta) \), and

\[
V(\theta) = \frac{F_e}{1 + F_e} - \frac{\omega_e - \omega_D - (\omega + \omega_a)(k_{\theta} \rho_s)^2 (1 + s^2 \theta^2)}{(1 + F_e)(\omega + \omega_D)}
\]

The mode equation has been numerically integrated, and the eigenvalue \( \omega \) compared with that from the semilocal kinetic dispersion relation. The maximum growth rate can be approximately represented by

\[
\gamma_{max} \simeq \frac{e}{\sqrt{s}} \left( \eta_e + \frac{1}{2} - \frac{5}{2} e_n \right) c_s L_n \sqrt{b_0}
\]

and occurs at

\[
(k_{\theta} \rho_s)^2 = b_0 \simeq \frac{1 - \sqrt{e}}{30} \frac{1 + s}{\sqrt{e}} \frac{1}{\left( \frac{2}{3} + \frac{5}{9} s \right)^2} \frac{1}{q^2} \simeq O(10^{-2})
\]

The resultant electron thermal diffusivity based on the mixing length estimate is

\[
\chi_e = \frac{\sqrt{e} \gamma_{max}}{k_{\perp}^2} = 4 \frac{e^{3/2}}{e_n} \left( \eta_e + \frac{1}{2} - \frac{5}{2} e_n \right) \left( \frac{2}{3} + \frac{5}{9} s \right) \frac{q c_s}{R} \rho_s^2
\]

which is Bohm-like in the dependence on the toroidal magnetic field, and also inversely proportional to \( B_\theta \), the poloidal magnetic field. This favorable dependence on the plasma current originates from the effects of
the ion acoustic transit time yielding an instability with a growth rate
\( \gamma \propto c_s/qR \). It is noted that the diffusivity resembles the neoclassical ion
thermal diffusivity in the plateau regime.

3. Effects of Trapped Electron Collisions on the Energetic
Ion Modes

The earlier investigations [4,5] of the effects of dissipative trapped
electrons on the energetic ion modes have indicated a relatively weak sta­
bilizing role. The problem has recently been revisited for the energetic ion
drift Alfvén mode [2] and the toroidicity induced global Alfvén mode [3],
in terms of the bounce averaged trapped electron response independent of
the magnetic perturbation [6].

In order to see qualitatively the stabilizing influence of dissipative
trapped electrons on MHD modes, we consider the ballooning mode. The
electron density and parallel current may be approximated by

\[
n_e = \left( 1 - \sqrt{\frac{\omega - \omega_{ce}}{\omega - \omega_{De} + i \nu_{eff}}} \right) \frac{e \phi}{T_e} n_0 - \left( 1 - \sqrt{\frac{\omega - \omega_{ce}}{c k || T_e}} \right) \frac{e A ||}{T_e} n_0
\]

\[
J_{\parallel e} = -\frac{n_0 e^2}{k || T_e} (1 - \sqrt{\epsilon}) (\omega - \omega_{ce}) \left( \phi - \frac{\omega - \omega_{De}}{c k ||} A || \right)
\]

where \( \nu_{eff} = \nu_{ei}/\epsilon \), and for circulating electrons, \( |\omega| \lesssim k || v_A \ll k || v_{Te} \) is
assumed. The ion density perturbation in the compressible limit \( |\omega| \gg
k || v_{Ti} \) is electrostatic, and given by

\[
n_i = \left( -1 + \frac{\omega + \omega_{ei}}{\omega + \omega_{Di}} (1 - b) \right) \frac{e \phi}{T_i} n_e
\]

where \( b = (k || \rho_i)^2 \). The charge neutrality and Ampere’s law yield the
following dispersion relation in the MHD limit \( |\omega| \gg \omega_e, \omega_D \):

\[
\omega^2 = (k || V_A)^2 - \frac{2 \omega_D (\omega_e - \omega_D)}{b} - i \frac{\sqrt{\epsilon} \nu_{eff}}{b} \omega
\]

Although complete stabilization is not achieved in the fluid approximation,
a significant stabilizing role of dissipative trapped electrons can be seen.
When \( b = O(10^{-2}) \), \( \nu_{eff} = O(10^{-2}) \), the growth rate of the ballooning mode
that is well into the unstable regime in the collisionless limit is reduced to
a small fraction of \( \omega_{ce} \).
Energetic ions effectively modify the MHD eigenvalue, and the contribution from the dissipative trapped electrons remains essentially unchanged. In the weak collisional limit, energetic ions tend to destabilize the drift Alfvén mode, but stabilize the ideal ballooning mode [7]. That is, energetic ions are destabilizing at lower $\alpha$ (the ballooning parameter) and stabilizing at higher $\alpha$. However, a small electron collisionality parameter $\nu_e \simeq 10^{-2}$ is sufficient to stabilize the high energy ion driven Alfvén mode.

4. Conclusions

Long wavelength electrostatic modes propagating in the electron diamagnetic direction are not subject to ion Landau damping and acquire the constant transit frequency $\omega \simeq c_s/qR \gg \omega_e$. The growth rate scales with the ion acoustic transit frequency, and an electron thermal diffusivity proportional to $1/B_\phi B_\theta$ emerges.

Collisions of trapped electrons have effective stabilizing effects on the long wavelength MHD modes. A relatively small collisionality parameter $\nu_e \simeq O(10^{-2})$ has been found to be sufficient to stabilize the high energy ion modes.

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POLOIDAL FLOW GENERATION BY RESISTIVE PRESSURE-GRADIENT-DRIVEN TURBULENCE

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Abstract

POLOIDAL FLOW GENERATION BY RESISTIVE PRESSURE-GRADIENT-DRIVEN TURBULENCE.

The radial structure of poloidal flows and radial electric fields in the tokamak plasma edge plays an important role in determining global confinement properties. In a turbulent plasma, poloidal flow can be generated through the Reynolds stress. At the same time, this poloidal flow controls the level of fluctuations. Therefore, self-consistent calculations of plasma turbulence in the presence of flows are needed to identify the mechanism of confinement improvement. Here, we study the interaction between flows and turbulence for the case of resistive pressure-gradient-driven turbulence. Linearly, both the shear flow, $V'_\theta \neq 0$, and the curvature flow, $V''_\theta \neq 0$, are stabilizing. Nonlinearly, the effect of the shear flow is weak, while the curvature flow is robust and survives in the nonlinear regime.

1. INTRODUCTION

Poloidal flows at the edge of a tokamak play a critical role in the overall plasma confinement. At the plasma edge in Ohmic-type discharges, a velocity shear layer is present. In neutral-beam-heated discharges and for low confinement mode (L-mode), the shear in the poloidal flow velocity at the plasma edge seems to decrease. However, there is a strong surge of the poloidal flow in high confinement mode (H-mode) discharges. Within the time resolution of present experiments, the increase in the poloidal shear flow
occurs simultaneously with the decrease in the fluctuation level at the plasma edge and with the improvement in confinement, the L to H transition.

A poloidal shear flow in a plasma confined by a sheared magnetic field has, in general, a strong stabilizing effect [1]. Linear stability theory shows this stabilizing effect for many plasma instabilities. One might expect that the shear flow would have a similar effect on a nonlinear stability. A general scaling model developed by Biglari et al. [2] shows that when the shearing frequency dominates over the diffusive time scale, fluctuation levels should be reduced. The poloidal shear flow can be generated by the plasma turbulence via Reynolds stress [3, 4]. In this paper, we look at the interaction between flows and turbulence within the framework of the resistive pressure-gradient-driven turbulence model [5].

2. POLOIDAL FLOW GENERATION

The poloidal flow generation equation is derived by taking the flux surface average of the momentum balance equation [3, 4]. The resulting equation gives the conservation of momentum:

\[
\frac{d\langle V_\theta \rangle}{dt} = -\frac{\partial}{\partial r} \left( \langle \dot{V}_r \dot{V}_\theta \rangle - \frac{1}{\rho_m \mu_0} \langle \dot{B}_r \dot{B}_\theta \rangle \right) + \mu \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial}{\partial r} \langle \dot{r} \langle V_\theta \rangle \rangle \right),
\]

where \( \rho_m \) is the mass density, \( \mu \) is the viscosity, and the angular brackets, \( \langle \rangle \), represent average over a magnetic flux surface. The nonlinear terms in the equation generate the nondiagonal \( r\theta \) terms of the Reynolds stress tensor, \( S_{ij} \equiv \langle \dot{V}_i \dot{V}_j \rangle - \frac{1}{\rho_m \mu_0} \langle \dot{B}_i \dot{B}_j \rangle \).

For profile modification of an average poloidal flow, the Reynolds stress tensor must be nonzero, \( S_{r\theta} \neq 0 \). This requires a symmetry-breaking effect, such as that induced by radial propagation of the fluctuations. The symmetry breaking can be produced by several mechanisms. Examples are an initial "seed" flow with radial structure [6, 7], that is, \( V'_0 \neq 0 \) and/or \( V''_0 \neq 0 \), and radial propagation induced by electron diamagnetic rotation.

When the turbulence is electrostatic, \( \langle \dot{B}_r \dot{B}_\theta \rangle = 0 \), and the Reynolds stress tensor is given by \( \langle \dot{V}_r \dot{V}_\theta \rangle \). The electrostatic approximation is a good approximation for the resistive pressure-gradient-driven turbulence when neither flows nor diamagnetic effects are included. However, this is not the
case when the flows and/or the real frequency are large. The most important difference is in the generated flow. Figure 1 compares the time evolution of \( \langle V_0 \rangle \) at the resonant surface for a numerical nonlinear calculation in the single-helicity limit using the full set of equations with the time evolution using the electrostatic approximation.

3. SHEAR FLOW EVOLUTION AND ITS EFFECTS ON TURBULENCE

We begin by studying the case of an initial poloidal flow with no diamagnetic effects included. At \( t = 0 \), \( \langle V_0 \rangle = V_0(r) \). In the case of a sheared poloidal flow, \( V_0 \neq 0 \) at the resonant surface. The linear eigenvalue problem can be solved analytically in the weak shear flow limit, \( \omega_s/\gamma_0 \ll 1 \). Here \( \omega_s = k_0 V_0 \Delta \) is the shearing rate, and \( \gamma_0 \) is the linear growth rate of the resistive interchange mode. The linear eigenfunction of resistive interchange modes with no diamagnetic effects and with a weak external shear flow is

\[
\Phi(x, \theta, \zeta, t) = \Phi_{mn} \exp[-i\omega t + i(m\theta + n\zeta)] \exp\left[-\alpha_r(x + i\delta)^2/2\right].
\]

Here, we use the shear slab approximation with \( x = r - r_s \), where \( r_s \) is the singular surface of the mode considered, \( m \) is the poloidal mode number, and \( k_0 = m/r_s \) is the corresponding wave number. The dispersion relation is such that \( \alpha_r \) and \( \delta \) are real. Here, \( \alpha_r = \Delta^{-2} \) and \( \Delta \) is the radial width of the mode.
The radial shift is proportional to the gradient of the velocity, \( \delta = \Delta \omega_x / \gamma_0 \).

The linear growth rate is

\[
\gamma = \gamma_0 \left( 1 - \frac{5}{3} \left( \frac{\omega_x}{\gamma_0} \right)^2 \right).
\]

Using the linear eigenfunction for modes with the same helicity, it is possible to calculate the contribution of such modes to the Reynolds stress tensor. From this contribution, we have the following evolution equation for the velocity shear:

\[
\frac{\partial \langle V_0^\alpha \rangle}{\partial t} \bigg|_{x=0} = \sum_{m,n} k_0 \Phi_{mn} \left( \frac{\omega_x}{\gamma_0} \right)^2 \delta \alpha^2 \propto V_0',
\]

and the evolution equation for the flow curvature \( \langle V_0^\beta \rangle \) is

\[
\frac{\partial \langle V_0^\beta \rangle}{\partial t} \bigg|_{x=0} = 0.
\]

Since the shear flow is proportional to the initial shear, the effect of the turbulence is to amplify the shear flow. In the electromagnetic case, we can only make an estimate of the flow generation equation. The magnetic contribution can dominate, and the shear flow can be reduced (Fig. 1). In both cases, there is no flow curvature generation.

We have done nonlinear single- and multiple-helicity numerical calculations. Using the nonlinear three-dimensional code KITE [8], we numerically solve the set of evolution equations for the poloidal flux, stream function, and electron density [4]. The input parameters are the same as in Ref. 5, for the helicity resonant at \( q = 3/2 \). The nonlinear calculation was initialized with a small value of equilibrium shear flow to act as a seed.

Different values of the seed flow have been used in the nonlinear calculations. The values of \( V_0 ' \) at the resonant surface ranged from 0 to 10000 in units of resistive time \( \tau_R \). The largest value used causes a reduction in the linear growth rate of the \((m = 3; n = 2)\) mode by a factor of 4, and the unstable linear spectrum is reduced to the lowest two modes. In all cases considered, the seed flow slows down the evolution of the modes, as can be expected from the linear stability properties. However, the saturation level of fluctuations is not significantly affected by the shear flow (Fig. 2).

The fluctuation spectrum at saturation is dominated by the lowest \( k \) modes in the calculation, the \( m = 3 \) and \( m = 6 \) modes. These modes are not
affected by the shear flow. However, for larger values of \( m \), there is an effect. In Fig. 2, at \( t \approx 0.027 \tau_R \), the case with \( V_0' = 0 \) has a shoulder that corresponds to the saturation of modes in the range \( m = 21 \) to \( m = 27 \). This shoulder is not visible for \( V_0' = -5000 \), because these higher-\( m \) modes are suppressed.

4. CURVATURE FLOW EVOLUTION AND ITS EFFECTS ON TURBULENCE

In the case of flow curvature, \( V_0'' \neq 0 \) at the resonant surface. In this case, the shearing rate is defined as \( \omega_z'' = k_0 V_0'' \Delta^2 \). In the weak curvature flow regime, \( \omega_z'' \approx \gamma_0 \), the linear eigenfunction is

\[
\Phi(x, \theta, \zeta, t) = \Phi_{mn} \exp[-i \omega t + i(m \theta + n \zeta)] \exp[-\alpha x^2/2].
\]

The coefficient \( \alpha = \alpha_r + i \alpha_i \) is complex, with \( \alpha_r = \Delta^{-2} \) and \( \alpha_i = -\alpha_r \omega_z''/\gamma_0 \). The instability acquires a real frequency through the curvature flow, \( \omega_r = \alpha_r \omega_z''/\gamma_0 \), and is damped by the curvature flow. The linear growth rate is

\[
\gamma = \gamma_0 \left[ 1 - \frac{1}{2} \left( \frac{\omega_z''}{\gamma_0} \right)^2 \right].
\]

With \( V_0 = V_0'' \Delta^2 / 2 \) used as a seed flow, the flow profile modification equations in the electrostatic limit, but including the magnetic stress term, are
\[
\frac{\partial (V_0^a)}{\partial t} \bigg|_{x=0} = 0 ,
\]
\[
\frac{\partial (V_0^a)}{\partial t} \bigg|_{x=0} = 3 \sum_{m,n} k_0 \frac{\Phi_{mn}}{B_0^2} \left[ \frac{k_0 V_0''}{\Delta^2 \gamma_0} \left( 1 - \frac{\beta_0}{2 \epsilon^2} \frac{a^2}{L_n r_c} \right) \right] .
\]

Here, \( a \) is the minor radius, \( L_n \) is the characteristic density scale length, and \( r_c \) is the radius of curvature of the magnetic field line. The electrostatic approximation gives flow curvature amplification and no change of flow shear. However, the electromagnetic results are different. The magnetic Reynolds stress is such as to cancel the seed flow in the cases that we have considered.

Although the electrostatic approximation is not a realistic model for the flow profile evolution, we can use it as a test model for determining the effects of curvature flow on the resistive pressure-gradient-driven turbulence. We model the curvature seed flow with a gaussian,

\[
V_0(x) = \frac{r}{r_s} V_0 \exp(-x^2/2L_E^2).
\]

In these calculations, \( L_E = 0.1a \ll \Delta \). A factor \( r/r_s \) is included to reduce the \( V_0 \) induced by the cylindrical geometry effects. For the parameters of Ref. 5, we have calculated the 3/2 single-helicity evolution for different values of \( V_0 \). The results are plotted in Fig. 3. It is clear that the fluctuation level at saturation decreases with increasing \( \Omega''_s = \omega''_s/\gamma_0 \). For the largest value of

![Graph](image)

**FIG. 3.** Time evolution of the electrostatic potential fluctuations for different values of the shearing rate \( \Omega''_s \).
the curvature seed flow, \( \tau_R V_0^\alpha a |_{x=0} = 7 \times 10^5 \) (\( \Omega_{s0}^\alpha = 1.86 \)), the fluctuation level at saturation is reduced by a factor of 1.6. In Fig. 3, the higher \( \Omega_{s0}^\alpha \) values have been initialized with a larger \((m=3; n=2)\) amplitude. The reason is to ensure that the final state is dominated by the same Fourier component, \((m=3; n=2)\). The fluctuation levels at saturation have been fitted with a quadratic function of \( \Omega_{s0}^\alpha \), and the result is

\[
\left( \left( \frac{\Phi}{B_0} \right)^2 \right)^{1/2}_0 = \left( \left( \frac{\Phi}{B_0} \right)^2 \right)^{1/2}_0 \left( 1 - 0.103\Omega_{s0}^\alpha \right)
\]

The reduction of the fluctuation level tracks the reduction in the linear growth rate, which can be fitted by \( \gamma/\gamma_0 = (1 - 0.185\Omega_{s0}^\alpha) \). This result is in sharp contrast to the shear flow effect discussed in Sec. 3. These results confirm that the turbulence reduction is caused by the flow curvature and that the flow curvature damping is effective in the nonlinear regime.

We can make a similar analysis when electron diamagnetic effects are included, and there is no initial seed flow. In this case, a curvature flow is naturally induced by diamagnetic effects, and a reduction in the fluctuation levels is obtained [9].

5. MULTIPLE STEADY STATE SOLUTIONS

By changing the initial perturbation, different saturated states can be reached. They are associated with different dominant Fourier components. The higher the \( m \) of the dominant component, the lower the saturation level of the fluctuations. The existence of multiple final states is one consequence of the nonlinear stabilizing effects that a mode exercises on all the others. The case \( \Omega_{s0}^\alpha = 1.86 \) in Fig. 3 has been initialized with a perturbation two times the perturbation of the case with \( \Omega_{s0}^\alpha = 1.6 \). If it were initialized with the same perturbation, we would obtain a final state dominated by the \((m=6; n=4)\) component, and the turbulence level would be reduced by a factor of 1.6.

To study these nonlinear stabilization mechanisms, we have calculated the \((m=9; n=6)\) single–helicity evolution by initializing the modes with random perturbations. For each seed flow, we have done 40 nonlinear calculations with an initial random condition such that \(|\Phi| < 0.1\) for each Fourier component. We have obtained different saturated states, which are characterized by the dominance of a given \(m/n\) helicity. From the numerical results, we can estimate the probability of reaching a given saturated state.
The nonlinear evolution of the electrostatic potential fluctuations for the cases with no flow is plotted in Fig. 4. The 9/6 helicity is dominant only in one case, while the 27/18 helicity is dominant in 75% of the cases. We can define a probabilistic saturation level by averaging over the different cases. The value of this saturation level is 2.4. When an initial sheared poloidal flow is included in the calculations, the saturated states corresponding to low-$m$ helicities are more probable. For $V_0' = 5000$, the 9/6 helicity is dominant in all the cases.

When the seed flow is a curvature flow, the results are the opposite. We have calculated the nonlinear evolution for values of $V_0$ going from 3000 up to 12000. The 27/18 helicity is still dominant, but the 36/24 is more probable as $V_0$ increases. For $V_0 = 12000$ ($\Omega_{s0}'' = 2.28$), all the saturated states but one are characterized by the dominance of these two helicities. The results of the probabilistic saturation levels are plotted in Fig. 5. It is clear that the fluctuation level at saturation is reduced with $\Omega_{s0}''$. The fluctuation reduction is related with the level of flow curvature at saturation. This can be seen in Fig. 6, where the evolution of the curvature flow and the potential fluctuation are plotted for two cases with $V_0 = 3000$ ($\Omega_{s0}'' = 0.57$). The solid line corresponds to a case in which the 36/24 helicity is dominant at saturation, and the broken line to a case in which the 27/18 helicity is dominant. The higher the flow curvature at saturation, the lower the potential saturation level.
FIG. 5. Probabilistic saturation level versus curvature seed flow.

FIG. 6. Time evolution of the potential fluctuation and the curvature flow for two cases with $\bar{\nu}_0 = 3000$.

6. CONCLUSIONS

Turbulence can generate flows through the Reynolds stress tensor. The radial structure of these flows depends on the nature of the turbulence and on the symmetry-breaking mechanism, which generates a nonzero Reynolds stress. In the case of resistive interchange turbulence with self-generated flows, the effect of shear flow on the turbulence saturation level is weak,
although it can cause changes in the fluctuation spectrum. However, the effect of flow curvature on the turbulence saturation is strong, and produces a reduction of the fluctuation level.

Multiple steady state solutions exist with different fluctuation levels. These solutions are characterized by the dominance of a given helicity at saturation. The consequence is that the effect of flow curvature on the saturation is stronger.

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SOME ASPECTS OF EDGE TURBULENCE IN TOKAMAKS

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Abstract

SOME ASPECTS OF EDGE TURBULENCE IN TOKAMAKS.

In the paper several linear and nonlinear aspects of tokamak edge instabilities, including the influence of ionization and sheared poloidal flow, are investigated. Particle diffusion damping effects are shown to overwhelm the ionization instability, which can be recovered only when additional physics effects are retained. The stability characteristics of resistive ballooning modes are found to be significantly influenced by linear coupling to rippling instabilities in the edge region. Nonlinear coherent vortex structures (appropriate for drift like modes and also for curvature induced Rayleigh–Taylor like fluctuations) are found to be strongly influenced by poloidal sheared flows, particularly when their propagation velocity matches the poloidal rotation speed. The authors also demonstrate that under suitable conditions parallel currents, perpendicular shear of parallel velocity, nonrigid rotation of the core, etc., associated with the vortex can drive secondary instabilities of ion acoustic waves, drift waves, shear Alfven waves, etc. These instabilities prevent accumulation of energy at the long wavelength and also provide a basis for intermittency. Preliminary calculations based on a mapping closure technique are presented to give an interpretation of observed non-Gaussian features of the probability distribution function of tokamak edge fluctuations.

1 Introduction

Currently there is a great deal of interest in linear and nonlinear theories of instabilities responsible for edge fluctuations in tokamaks. It is generally believed that low-frequency \( \omega < \omega_{ci} \) plasma instabilities driven by unfavourable magnetic curvature and/or pressure, current, resistivity, poloidal velocity gradients etc. are responsible for most of the observed phenomena [1]. Atomic physics effects such as ionization of neutrals, electron energy loss by radiative effects play an important additional destabilizing role in many of these instabilities [2–3]. In this paper we have re-examined the linear theory of drift-like ionization instabilities. We demonstrate in contrast to earlier work, that inclusion of particle diffusion effects and/or the equilibrium time dependence (which naturally follow in a self-consistent equilibrium with an ionization source) in the perturbation calculation, is extremely crucial. In fact, these effects overwhelm the ionization drive in the simplest ionization instability calculation. Retention of additional physical effects such as the dependence of ionization rate on electron temperature, is necessary to recover an instability driven by ionization effects. We have also examined the
linear stability of $m = 1, 2$ type resistive ballooning modes in the presence of resistivity gradients and find an interesting linear coupling to rippling modes which mixes parities and significantly influences the stability characteristics. Among nonlinear studies on edge fluctuations, we present the results of two investigations. In the first calculation we show the influence of sheared poloidal rotation on coherent vortex structures, which may be naturally generated by the nonlinear interaction of edge fluctuations in the tokamak edge plasma [4]. The second calculation is motivated by recent observations [5] of intermittency in tokamak edge turbulence. We adopt the point of view [6] that intermittency arises because the nonlinearly generated coherent long scale vortex structures are themselves unstable to secondary fine scale instabilities [7]. We demonstrate that under suitable conditions, parallel currents, perpendicular shear of parallel velocity, non-rigid rotation of the core etc. associated with the vortex can drive strong secondary instabilities of ion-acoustic waves, drift waves, shear Alfven waves etc. These instabilities prevent accumulation of energy at the long wavelength end and also provide a basic source of intermittency for the plasma turbulence. To give an interpretation of the observed non-Gaussian features of the probability distribution functions [4], one may use a mapping closure technique recently developed for Navier-Stokes equations by Kraichnan and others [8]. We present some preliminary calculations on this topic for the nonlinear edge plasma turbulence models.

2 The Basic Equations

We consider a cold ion plasma and write nonlinear plasma equations viz. equations of continuity, parallel ion motion, parallel current and electron energy. These are coupled to Maxwell’s equations viz. the charge neutrality condition $\nabla \cdot J = 0$ and Ampere’s law $\nabla \times B = J$. The resulting equations are

$$\frac{dN}{dt} - \nabla \cdot \frac{d}{dt} \Phi + \nabla_n u_n + \dot{z} \times \nabla \Phi \cdot \nabla \ln n_0 = S_1$$

$$\left( \frac{d}{dt} - \mu \nabla_n^2 + s_2 \right) u_n = -T_0 (\nabla_T T + T \nabla N)$$

$$J_n = \sigma [\nabla_{||} \Phi - \frac{\partial a_{||}}{\partial t} + T_0 (1.71 \nabla_T T + T \nabla N)]$$

$$\left( \frac{d}{dt} - J_n \nabla_T \right) \left( \frac{3}{2} \ln T - N \right) + \dot{z} \times \nabla \Phi \cdot \nabla \left( \ln \frac{T_0^{3/2}}{n_0} \right) = \frac{1}{T} \nabla_{||} \chi_{||} \nabla_{||} T + S_3$$

$$\nabla_n \cdot \frac{d}{dt} \Phi = T_0 \dot{z} \times \nabla B \cdot (\nabla_T T + T \nabla N) + \nabla_{||} J_{||}$$

$$\nabla_{||}^2 a_{||} = -\beta J_{||}$$
where
\[ N = \ln \frac{n}{n_0(x)}, \quad \Phi = \frac{e\phi}{T_{00}}, \quad T_0 = \frac{T_0(x)}{T_{00}}, \quad T = \frac{T}{T_0(x)}, \quad u = u/C_s, \quad \nabla = a_s\nabla, \]
\[ \frac{\partial}{\partial t} = \frac{1}{\omega_{ei}} \frac{\partial}{\partial t}, \quad J_\| = \frac{J_\|}{enC_s}, \quad a_\| = \frac{C_s}{T_{00}}, \quad \sigma^{-1} = \frac{0.5\nu_{ei}}{\omega_{ci}}, \quad \beta = \frac{4\pi nT_{00}}{B_0^2} \]
and \( S_1, S_2, S_3 \) are the normalized ionization and charge exchange sources in the three equations.

### 3 Linear Instability Studies

We first examine the ionization instability of drift-interchange and collisional drift modes in a tokamak edge plasma [3]. The former is likely to be stable on closed surfaces because \( q > 1 \), but may be unstable for that part of the scrape-off layer plasma which samples only open field lines with bad curvature. Our basic results are:

1. In the hydrodynamic limit \( \omega k^2 \gg \chi ||^2 \), we get the perturbative expression for the growth rate, \( \gamma = (\epsilon_n^{1/2}(1 + \eta_e)^{1/2})/(k_\perp \rho_s)\omega_\ast + 1/2(1 + \eta_e)\gamma_{TIc} + \gamma_{ITc} - \chi_{2\perp}k_\perp^2 - \gamma_{ex} - \gamma_{ex}(1 + \eta_e) \) where \( \gamma_{in} = N_n < \sigma >_i n, \gamma_{ITc} = T_c(\partial_{\gamma_{in}}/\partial T_c) \). Note that the direct ionization effect is always opposed and overwhelmed by the diffusive damping term \( D_0 k_\perp^2 \), because \( k_\perp L_n > 1 \) and \( \gamma_{in} \approx D_0/L_n^2 \) for steady equilibrium. Earlier workers [3] have neglected these diffusive damping terms without any justification. At the same time, an ionization instability effect arises through the \( \gamma_{ITc} \) terms, which take account of the fact that the ionization rate itself can increase and provide a feed-back if temperature fluctuations with the right phase are produced.

2. To examine the collisional drift branch, we ignore sound wave coupling and look at the limit \( \omega \ll \chi ||^2 k_\perp^2 \). Defining \( \gamma_{in} = (\gamma_{in} - D_0 k_\perp^2) \), \( \kappa = k_\perp a_s \) the frequency and growth rate are given by \( \omega_r = \omega_\ast/(1 + \kappa^2) \) and \( \gamma = \gamma_{in}/(1 + \kappa^2) + [\omega^2/\chi || k_\perp^2(1 + \kappa^2)^2][1.71\eta_e + 3.53k^2/(1 + \kappa^2)] \) and again show that ionization effects further stabilize the mode.

3. We also examine the effects of the momentum charge-exchange term, parallel viscosity and parallel thermal conductivity on the stable parallel velocity shear mode [3]. It is found that the mode is destabilized with a growth rate \( \gamma = (\gamma_{ex} + \mu k_\perp^2)|\omega_\ast \omega_{ex}|/\omega_\ast^2 + \gamma_{in} + \omega^2/\chi || k_\perp^2[3.53k^2 - 1.71\eta_e + 1.14\alpha|\omega_\ast \omega_{ex}|/\omega_\ast^2] \).

We next examine the question of ionization instability in a plasma with an ionization source but no diffusion damping to balance it. The equilibrium is initially time-dependent, and we wish to examine if an instability can arise which, on saturation, gives a diffusion term to generate a self-consistent
steady-state. Taking an equilibrium time variation given by $n_0 \propto \exp(\gamma_{in} t)$ and carrying out a WKB type analysis one finds [9] that $\phi \sim \exp[\alpha_{in}^2/(1 + n_0)]^{1/2}/k + \gamma_{iT}/2t$ for the interchange mode, so that only the $\gamma_{iT}$ ionization source is seen as destabilizing, as above. The drift mode is always found to be stable.

Finally we investigate the effect of resistivity gradients on the resistive ballooning mode [10]. For the $m = 1$ type mode, we obtain a dispersion relation of the form

$$\omega/\omega_A^2 + (i\nu_R/\omega)(\alpha_0^2/2s^2) = (i/\nu_R)(\omega_R^4/(2s)^4 \omega^3),$$

where $\omega$ is the mode frequency, $\omega_A = sV_A/qR$ is the poloidal Alfven frequency, $\alpha_0^2 = \alpha^2/(1 + \omega_R^2/\omega^2)$, $\alpha = -8\pi q^2 R_p^0/B_0^2$, $p_0^0 = dP_0^0/dr$, $s = d\ln q/d\ln r$, $\omega_R = -(keqRc_{\Omega}^{||}/B_0)dn_0/dr$ and $\nu_R = \eta_0 c^2 k^2_d/4\pi$ and other notations are standard. The term on the r.h.s. represents the new contribution due to linear coupling of rippling modes with ballooning modes. We have looked at the physically admissible roots of this equation in various limits. In the limit of weak coupling, treating the rippling mode term as a small parameter (i.e. $|\omega| >> \omega_R$), the dispersion relation reduces to $\omega = i\omega_0(1 + \omega_R^2/\omega_0^2)$, where $\omega_0 = (\alpha^2/2s^2)^{1/3} \omega_A^{2/3}$. Hence, in this limit, we find that the resistivity gradient parameter enhances the ballooning mode growth. When $|\omega| << \omega_R$ (i.e. the rippling mode coupling is significant), the dispersion relation gives a stable mode, $\omega = -(\omega_R/\nu_R)^{1/2} \omega_R^{-2/3}(\alpha_0^2) - 1/2$. Next considering the limit $(\omega/\omega_A)^2 << |\nu|\alpha_0^2/2s^2$ and for $\omega \sim \omega_R$, there arises a new unstable mode whose dispersion relation is given by $\omega^2 = 0.5\omega_R^2(\kappa - \sqrt{\kappa^2 + 4\kappa})$, where $\kappa = 2^{-3}(\omega_0^2/2s^2)^{1/3} \omega_R^{-2/3}$. We note, however, that the growth rate of the new unstable mode is considerably diminished compared to its classical growth. Similar results are also found for the $\Delta'$ driven modes ($m=2$ type resistive ballooning modes) corresponding to the frequency regime, $\omega > \omega_s/s$. The dispersion relation is found to be $(1 - 7\mu\Delta'/30)/\Delta'\sqrt{\lambda} = (\nu/2s^{3/2})(\Gamma(1/4)/\Gamma(3/4))(e^{-i\pi/4}/\lambda^{3/4})$ where $\mu = \omega_R/s\omega$, $\lambda = \nu(\omega^2/\omega_A^2 + \nu\alpha_0^2/2s^2)$, and $\nu = (i\nu_R/\omega)$. For $\omega_R << |\omega|$, the term $1 - 7\mu\Delta'/30$ is found to provide a destabilizing influence on the $\Delta'$ driven resistive ballooning mode. In the opposite limit, $\omega_R > |\omega|$ however it is possible to find a stable window in the parameter space. We conclude that parity mixing terms can play an important role in the linear evolution of resistive ballooning modes in the edge region.

4 Effect of Velocity Shear on Dipole Vortices

We consider a cold ion plasma in a sheared external magnetic field $B = B_0(\hat{z} + \hat{y}S_1x/L_S)$, with a density profile $n_0 = N_0 \exp(-ex)$ and an equilibrium potential profile which may be approximated as $\phi \simeq n_0e + 0.5v_0^2x^2$. For $\sigma, \chi_|| \to \infty, a_|| = 0$, we see from the electron dynamics (Eqns.(3) and (4)) that $n_0 = n_0 \exp(\Psi)$. Using the ion dynamics given by equations (1) and (2), transforming to a frame moving with phase velocity $u$ (viz. $y = Y + S_0x - ut$), we may write the final equations in the form of Poisson brackets as: $[\Psi - \vec{u} x + v_0^2x^2/2, \nabla_\perp^2 \Psi - \Psi + x] = (S_0 + S_1x)\partial n_0/\partial y$, ...
\[
[\Psi - \bar{u}x + v_0'x^2/2, \ v_\parallel - (S_0x + S_1x^2/2)] = 0, \text{ where we have defined } \bar{u} = u - v_0
\]
and have rescaled the variables as \( \epsilon \rightarrow a_\phi/L_n, \Psi \rightarrow \epsilon \Phi, v_\parallel \rightarrow u_\parallel/\epsilon, \ n_\parallel \rightarrow \epsilon^{-1}\nabla_\|, \partial/\partial t \rightarrow \epsilon \partial/\partial t \). Eliminating \( v_\parallel \) from the above equations, we get

\[
\nabla_\|^2 \Psi - \Psi = \left(\bar{u}/v_0' - x\right) - \psi^*/v_0' + S_1/2v_0'\{(S_0v_0' + S_1\bar{u})/\psi^* - S_1\} \times
\]

\[
\{(\bar{u}/v_0' + S_0/S_1 - \psi^*/v_0')^2 - (x + S_0/S_1)^2\}
\]

where \( \psi^* = \sqrt{2\Psi v_0' + (\bar{u} - v_0'x)^2} \). For \( S_1 = 0 \) and \(| \bar{u} - v_0'x | \gg \Psi v_0' \), we recover the Meiss-Horton equations [4] with corrections of order \( v_0' \).

On the other hand, when \(| \bar{u} - v_0'x | \approx \Psi v_0' \), the square-roots play an essential role and Eqn.(7) cannot be simplified further. This indicates that when the phase velocity of the vortex locally matches the mean fluid velocity, interesting resonant effects can strongly modify the vortex structure. When \( v_0' \) is small, we can solve Eqn.(7) perturbatively. The results indicate a mild distortion of the vortices by \( v_0' \) terms, generation of \( \cos 2\theta \), \( \sin 2\theta \) components (i.e. quadrupole deformations of the vortex) and a modification of the eigenvalue conditions as a function of \( v_0' \). For arbitrary \( v_0' \) (including the above mentioned resonant case), the equations have to be solved numerically.

5 Intermittency in Edge Turbulence

We consider excitation of secondary fine scale instabilities by the free energy sources associated with the vortex as a source of intermittency in tokamak edge turbulence. The basic free energy sources associated with the vortex are: (a) parallel currents, (b) pressure gradients, (c) velocity shear effects, (d) ellipticity of vortex core etc. We go to the frame of the vortex and use a local approximation for the instability calculation; this is valid whenever \(| k_\perp L_\parallel | \gg 1 \) (i.e. secondary waves are much shorter than the vortex structure lengths). No discussion of convective vs. absolute nature of the secondary instabilities is presented here. It may be mentioned that many of the instabilities discussed below depend on drift-like terms \( \hat{z} \times \nabla \phi \cdot \nabla n \) coming in the continuity equation. If \( n \sim \phi \) is true both for the vortex and perturbation, then these terms vanish nonlinearly [11]. This requires that \( n \sim 1 \) should be violated for the vortex and/or for the perturbations; the former is more likely because the vortex \( L_\parallel \) is much larger than secondary wave \( \lambda_\parallel \) and we may have \( L_\parallel > \lambda_\parallel \). Of course the regime \( L_\parallel > \lambda_\parallel > \lambda_\perp \) is also of interest since both the vortex and the secondary modes are collisionally modified. Earlier workers [7] have not emphasized this point.

We first investigate excitation of collisional drift—acoustic waves by parallel currents associated with a vortex. Noting that \( \nabla \cdot J = 0 \) for the vortex we can estimate \( J_\| \) as \( J_\|/enC_s \simeq (\epsilon \phi/T_0)[(L_\parallel a_{\phi} / L_\perp L_n) \left(a_{\phi}^2 / L_\perp^2 \right) + a_n V_d / R u)] \), where the two side terms come from polarization current and curvature effects respectively. Following Coppi and Mazzucato [12] for the excitation of current driven ion—acoustic instability in a collisional plasma
one finds a growth rate $\gamma = -\left(\omega_2^2/k_{||}^2\chi_{||}\right)(1 + k_{||}^2J_{||}|e\nu_0\omega_0)$ giving a threshold electron current of $J_{||}/en_0C_\ast > 1$. This can be directly used to get a critical $(e\phi/T_\ast)$ for the vortex. Assuming that for the vortex $L_{||}^2 \sim \nu_{ch}^2/\nu_{gi}\omega$, so that collisionality plays a role, we find that $e\phi/T > (m_\ast/m_i)^{1/2}(L_{||}/\lambda_f)$ which can readily be satisfied for vortices with density fluctuations of a few percent. If $k_{||}v_d > k_{||}C_\ast$ for the secondary waves, we have excitation of drift-acoustic waves with a growth rate $\gamma \sim 0.5(k_{||}^2\omega k_{||}^2\omega \lambda_f)^{-2}(k_{||}u_{e0} - 1.5\omega_\ast T)$. The mode is harder to excite in this case as $1.5\omega_\ast T > k_{||}C_\ast$. Note that $\omega_\ast$ and $\omega_\ast T$ have to use the scale-length definitions $1/L_n \equiv (1/L_n + (1/L_\perp)(\delta n/n))$ and $1/L_T \equiv (1/L_\perp+1/L_\perp)(\delta T/T)$ where the latter terms give the contributions due to the vortex.

We next consider the excitation of secondary instabilities by perpendicular shear of parallel velocity. Following Ware et al. [3] one may write the dispersion relation $\omega = (\omega_{e0}/2)[1 \pm (1 + k_{||}^2\omega_{e0}\omega_{e0}^2)^{1/2}]$ which leads to instability if $-4k_{||}C_\ast\omega_{e0}\omega_{e0}^2 > 1$. This leads to a critical amplitude for the vortex, $e\phi/T > (L_{||}/L_{||})(4k_{||}/k_{||})(L_{||}2/\nu_{gi}^2)$, which could again be satisfied for a vortex with few per cent density fluctuations.

We finally study stability of elliptical vortex cores using methods of Floquet theory advocated by Bayly [13] for the Navier-Stokes problem. Taking a model equilibrium vortex flow near the core given by $u_0 = \Omega (\delta y x/\epsilon - \epsilon x \delta y)$, where $\Omega = (c/B)\phi_0^0$ is the angular frequency of rotation of the vortex core and $\epsilon - 1$ is a measure of ellipticity, we consider excitation of secondary ion-acoustic and shear Alfven pertubations, using equations (1) to (6). The resulting Floquet equations have been solved by multiple time scale methods and using numerical techniques. It is found that shear Alfven waves may be driven unstable with a growth rate of order $\gamma \approx \Omega |(1 - \epsilon^2)/(1 + \epsilon^2)|$ provided the vortex rotation frequency resonantly matches the Alfven frequency i.e. $\Omega \approx k_{||}V_A$. This gives the requirement that $(e\phi/T_\ast)vortex \simeq (L_x/qR)(L_y/a_\ast)(m/\sqrt{\beta_T})$ where $L_x, L_y$ are perpendicular scale-lengths of the vortex, $m$ is the poloidal mode number of excited Alfven wave and $\beta_T$ is the toroidal beta. This condition could be satisfied at the tokamak edge where $e\phi/T_\ast$ is as high as 50 per cent. The ion-acoustic waves are found to be stable for the simple model described above because of certain symmetries in the Floquet operators [7]. However, introduction of magnetic shear term breaks these symmetries and may modify this conclusion.

We would now like to draw further conclusions regarding intermittency and non-Gaussian pdf's in tokamak edge turbulence [5]. Recently Chen et al. [8] have proposed a systematic method for closing the diffusion term, in the pdf equations for turbulent flows. This closure is based on mapping of the stochastic scalar field $\psi$ to a reference Gaussian field $\psi_0$ through $X$ (known as the mapping function). The pdf of $\psi$ can then be known by the relation $P(\psi, t) = P_0(\psi_0)(\partial X/\partial \psi_0)^{-1}$. Chen et al. [8] also give a prescription for constructing the evolution equation of the mapping function $X$. 
The set of basic equations given earlier describe the scenario of inverse cascade (associated with pseudo 2 dimensionality of the low frequency modes in the plasma) and the secondary instabilities of the coherent vortex structure leading to intermittency. Hence one expects that the dynamic evolution of pdf from this set of equations would display distinct non--Gaussianity. Here we present the results of our pdf studies. Eliminating T and N from the linearized set of the basic equation where ion sound coupling and polarization effect has been dropped yields \( \frac{\partial \phi}{\partial t} = -i\alpha \partial \nabla_\parallel^2 \phi / \partial t + a_2 \nabla_\parallel^2 \phi \), where \( a_1 = \chi || / 1.71 \omega_n^2 (1.5 \omega_g - \omega_n) \) and \( a_2 = \chi || \omega_n / 1.71 (1.5 \omega_x - \omega_n) \).

We seek a mapping \( \phi = X(\phi_0, t) \), where \( \phi_0 \) is a multivariate Gaussian distribution. Following Chen et al. [8] \( \partial X / \partial t = -ia_1 \partial[\nabla_\parallel^2 \phi] / \partial t + a_2 [\nabla_\parallel^2 \phi]_{\omega \phi} \).

The conditional average \( [\nabla_\parallel^2 \phi]_{\omega \phi} \) can be expressed in terms of averages of \( \phi_0 \) and its derivatives. Choosing \( X = X_0 \exp(-i \omega t) \) and defining \( \alpha = 1 / < \phi_0^2 > \), \( \beta = i \omega (a_2 - a_1 \omega) < \xi_\omega > \), \( \psi_\omega = \sqrt{\alpha / 2 \phi_0} \), where \( \xi_\omega = \nabla_\parallel \phi_0 \), we get \( d^2 X_0 / d \psi_\omega^2 - 2 \psi_\omega (dX / d \psi_\omega) + (2 \beta / \alpha) X = 0 \). This equation is satisfied by the Hermite function of order \( n \) if \( \omega_n < \phi_0^2 > / (a_2 - a_1 \omega_n) < \xi_\omega^2 > = n \). Thus if one starts with an initial distribution such that \( \phi(t = 0) = X(t = 0) = \sum_n a_n(0) H_n(\psi_\omega) \), then \( X(t) = \sum_n a_n(0) \exp(-i \omega_n t) H_n(\psi_\omega) \), indicating that all Hermite modes decay but their time constants vary. For values of \( n \) up to which the second term in the denominator dominates \( |\omega_n| \propto n \), whereas in the other limit \( |\omega_n| \propto 1/n \). Thus preliminary calculations seem to suggest that for large parallel thermal conductivity an initial non--Gaussian pdf persists for a long time. Similar calculations were done by neglecting temperature fluctuations but retaining the parallel dynamics with viscous terms. Here one observes approach towards Gaussianity. These linear results need to be verified using numerical methods. For the full nonlinear equation the mapping equation reads \( \psi_\omega(x, t) = X(\phi(x), t) \), where \( \phi(x) \) is the standard multivariate Gaussian reference field residing in coordinate \( x \) and \( \psi_\omega(x, t) \) is a surrogate field in \( x \) which satisfies the same governing equation as the stochastic variable. In general the coordinate system \( \tilde{x} \) and \( x \) are related by the equation \( \partial \tilde{x}_i / \partial x_j = J_{ij}(t) \), due to the nonlinear term. She and Kraichnan [8] have taken \( J_{ij} \) to be isotropic and have developed a heuristic model for the evolution of \( J \). The model for \( J \) can also be provided using direct numerical simulation results. We are in the process of carrying out a similar heuristic analysis for the plasma equations discussed here.

REFERENCES

BOUNDARY CONDITIONS ON THE ELECTRIC FIELD PROFILE IMPOSED AT THE SEPARATRIX

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Abstract

BOUNDARY CONDITIONS ON THE ELECTRIC FIELD PROFILE IMPOSED AT THE SEPARATRIX.

An electric field is obtained at the separatrix in a tokamak by numerical methods. An analysis is performed for modern tokamaks with low collisionality. The kinetic approach is developed, accounting for the effect of the electric field on trapped and untrapped particle orbits. Thus, the 'squeezing' effect is considered. The ion density as a function of the electric field is computed by integrating the distribution function for ions in velocity space over the volume with confined trajectories. Therefore, the ion density close to the separatrix is governed by an electric field and depends on the poloidal coordinate. In contrast, there is almost no deviation of the electrons from the magnetic surfaces, and therefore the electron density is governed by completely different effects such as anomalous transport and ionization of neutrals. For a given electron density profile, by imposing quasi-neutrality a self-consistent electric field is obtained. It is shown that there is an important causal relationship between the density profile gradient and the value of the electric field potential at the separatrix. Steeper profiles imply stronger electric fields and vice versa. Poloidal electric fields stemming from the inhomogeneity of the loss cones are shown to emerge, resulting in two convective cells close to the poloidal angles of 0 and $\pi/2$ in the upper half of a torus. The double-null configuration is symmetric with respect to the midplane. It is concluded that the electric field at the separatrix is mainly governed by peculiarities of the ion orbits. The boundary condition on the electric field emerges to be fundamental in determining a regime of confinement. It is found to be not too sensitive to the tokamak parameters.

1. INTRODUCTION

Recently, the radial electric fields were observed to have pronounced effects on the confinement in a tokamak [1, 2]. In order to assess the results of experiments, the neoclassical theory of plasma rotation in a tokamak is revisited in Refs [2, 3] because the usual assumptions invoked in it become invalid if very large gradients exist at the plasma edge. It has been shown that the model described in Refs [2, 3] yields both steep and gradual profiles for the poloidal rotation velocity at the edge corresponding to the H- and L-regimes of confinement, respectively.

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The bifurcation is shown to be sensitive to a boundary condition imposed on the radial electric field at the separatrix. The boundary condition depends on processes occurring both within the scrape-off layer (SOL) [4] and within the approximate distance of a poloidal Larmor radius from the separatrix. The latter issue was first addressed by Berk and Galeev [5] and later by Itoh and Itoh [6] and Shaing and Crume [7]. In Refs [6, 7] the non-ambipolar flux of ions on loss orbits has been invoked to explain L–H transitions by employing fluid quantities within the range of a poloidal Larmor radius. However, it appears that only the kinetic approach is truly warranted within such a short range from the separatrix. In contrast, addressing the issue of ions on loss orbits, Hazeltine [8] used the kinetic approach assuming, however, that (a) the transport at the edge is entirely neoclassical resulting in large toroidal rotation, (b) the electric field is poloidally uniform and (c) 2-D processes governing the electric fields within the SOL were not considered. In the present study, we do not make these assumptions and address the issue by numerical methods.

2. MODEL

In general, the problem of correctly modelling the interface between closed and open field lines is fundamental but very complex [9]. Therefore, the existing approaches are bound to remain simplified, maybe even simplistic. They are often based on the hydrodynamic equations and on the assumption that particles having crossed the separatrix are lost to the divertor plates or the limiter, because of an outflow on the time-scale of \( \tau \approx qR/C_s \).

In reality, physics is more complicated. For example, if the plasma is sufficiently hot in the vicinity of the separatrix (the mean free path is of the order of the tokamak circumference), new kinds of particle trajectory emerge (mainly corresponding to ions deviating strongly from a magnetic surface) which either intersect or do not intersect plates or limiter. Examples of typical trajectories of both kinds are shown schematically in Fig. 1.

Note that in the vicinity of a divertor plate at \( |\theta| = \pi/2 \) no particle intersects a divertor plate, owing to the \( \nabla \cdot \mathbf{B} \) drift. Therefore, there are no loss orbits in that region enhancing the ion density at these locations when compared to \( \theta = 0 \).

The ion densities on various trajectories are determined by the following factors: (a) anomalous radial flux into these trajectories; (b) sources due to ionization by neutrals and (c) collisions with ions filling the trajectories from the other ones. The resulting ion density on loss orbits is lower than the density of ions on confined orbits. The number density on the former orbits versus the latter ones depends drastically on the electric field in the vicinity of the separatrix. It is well known that the electric field affects the banana orbits by ‘squeezing’ them close to the separatrix [5].

Furthermore, there is a significant poloidal dependence of the ratio of both kinds of orbit. Hence, the ion density close to the separatrix is governed by an electric
field and depends on the poloidal co-ordinate. In contrast, there is almost no deviation of the electrons from the magnetic surfaces, and therefore the electron density is governed by completely different effects such as anomalous transport and ionization of neutrals.

For a given electron density profile, self-consistent radial and poloidal components of the electric field result in providing the quasi-neutrality given by $n_e (r, \theta) \approx n_i (r, \theta, \Phi (r, \theta))$. Both components of the emerging electric field acquire new and unexpected features. Here, the ion density and the distribution function are found numerically, and the electric field results from quasi-neutrality. The filling time into the confined trajectories is assumed to exceed the time-scale of $\tau \approx qR/C$. Therefore, the ion density on the loss orbits vanishes. On the other hand, the distribution function on confined orbits is given by

$$f_i = n_0 m_i^{3/2} / (2\pi T_i)^{3/2} \exp \left[ -m_i V_i^2 / (2T_i) - e\Phi / T_i \right]$$

where $\Phi (r, \theta)$ is the electric field potential.

Equation (1) is strictly valid on the trajectory. Thus our model is justified within the range of one or two poloidal Larmor radii from the separatrix. The algorithm for this problem has been obtained as follows: For given co-ordinates $r$, $\theta$, velocity $V_i$, and electric field $\Phi (r, \theta)$, the program checks whether an orbit intersects (and is lost thereby) or does not intersect the plate. The divertor plate (loss region) is at $|\theta| = \pi/2$ and $r \geq a$, where $a$ is the minor radius. The ion density is obtained by
integrating in velocity space over the volume corresponding to the ions in confined orbits. An example of the integration volume is shown in Fig. 2 for \( r/a = 0.98, \theta = 0 \) and \( \Phi_0(r) \) given by Eq. (3). The computed density unfolds to be poloidally inhomogeneous. The perturbed potential \( \Phi_1(r, \theta) \) yields the Boltzmann distribution of the electrons on a magnetic surface:

\[
\ln \left( \frac{n_{le}}{\langle n_e \rangle} \right) = e\Phi_1(r, \theta)/T_e = \ln \left( \frac{n_i}{\langle n_i \rangle} \right)
\]

The potential \( \Phi_1(r, \theta) \) affects the poloidal distribution of the ion density because, in Eq. (1), \( \Phi(r, \theta) = \Phi_0(r) + \Phi_1(r, \theta) \). Iteration has been carried out until quasi-neutrality provided \( n_{le} = n_{li} \). The potential profile \( \Phi \) is determined by a given profile of the electron density \( n_e(r) \), taken from measurements on different tokamaks.

The zero order potential has been approximated by

\[
e\Phi_0(r) = \gamma T_i(a) \exp \left\{ -[(r - a)^2/(\lambda a)^2] - \gamma T_i(a) \right\} \quad r < a
\]

\[
e\Phi_0(r) = \gamma T_i(r) - \gamma T_i(a) \quad r \geq a
\]

Here, \( T_i = T_e \) is assumed and \( \lambda \) is a parameter. Note that near the plate for \( r > a \) and \( ||\theta| - \pi/2| < \delta \), where \( \delta \) is a small parameter, Eq. (2) has been modified to provide the constant potential at the plate \( \Phi_{\text{plate}} = -3T_e(a) \). The electric field within the SOL is governed by the electron outflow, as is shown in Ref. [4]. An arbitrary
FIG. 3. H-mode density profiles for $\lambda = 0.04$ and 0.05, $T_i = 500$ eV at the separatrix. Poloidal Larmor radius: $\rho_{Li} = 1.1 \text{ cm} = 0.01a$. Experimental points are shown by black dots.

potential constant is chosen to read $\Phi(a) = 0$ at the separatrix and $\Phi = -3T_e$ at the plate. Since measurements of the temperature in the SOL are scarce, the value has been approximated by Eq. (3) also for $r > a$, instead of the more accurate Eq. (4). The model electric field is positive in the SOL and negative inside the separatrix. The governing dimensionless parameter $\lambda$ is chosen to model the real experimental measurements performed on TEXT, JFT-2M and other tokamaks.

3. RESULTS

For the measured steep and gradual profiles of the electron density in both the H- and L-regimes, the ion density has been shown to yield large and small electric fields in the vicinity of the separatrix. The corresponding values of the parameter $\lambda$ (the measure of the steepness) are $\lambda = 0.04$–0.05 for the H-mode and $\lambda = 0.4$–0.6 for the L-mode in Ref. [10] (Figs 3 and 4). The profiles shown are at the outer midplane at $\theta = 0$. The ion temperature is $T_i = 500$ eV at the separatrix in the H-mode. The agreement between measured and computed values appears to be rather good, in particular close to the separatrix, within the range of validity of the model. As is shown in Fig. 5, the electric field is strongly reduced by at least a factor of ten in transition from the H- to the L-regime. In summary, it is shown that steep profiles automatically imply large electric fields at the separatrix. Note that the electric field further inside is mainly governed by anomalous inertia and viscosity and neoclassical parallel viscosity (see Eq. (17) of Ref. [3]). The boundary condition for this equation is determined by the present calculation. Hence, the following scenario of transition
into an improved confinement regime emerges: steep density profiles cause the large electric field to arise at the separatrix. Electric field profiles with a large shear result from the boundary condition $E_{\text{sep}} > E^{(\text{NEO})}$. Electric field profiles with a large shear enhance the confinement, thereby maintaining the steepness of density and temperature profiles by formation of a 'thermal' barrier. In conclusion, the boundary condition on the electric field is fundamental in determining a regime of confinement, and two steps of transition emerge naturally. It is found to be not too sensitive to the tokamak parameters.

As to the poloidal dependence of the loss cones, ions on orbits close to a plate affect the poloidal electric field pattern. As was already mentioned in Section 1, these orbits do not intersect a plate. Therefore, the ion density increases at these locations, thereby causing a positive potential to peak there. The resulting strong electric field evolves self-consistently to provide quasi-neutrality (attracting electrons and repelling ions) by pushing out excessive orbits from these locations. Note that this physical picture, in fact, describes the iteration procedure employed to obtain self-consistent radial and poloidal electric fields. Hence, a self-consistent poloidal electric field emerges with large maximum values around the plates. In Fig. 6 the poloidal variation of the potential $\Phi_1$ is shown. It is positive close to plates and negative elsewhere. Two rather pronounced convective cells emerge at the upper half, one located close to the midplane with negative potential and the other one close to a plate with positive potential, rotating in opposite directions. The physical explanation for two cells stems from two effects competing poloidally: (a) the mentioned enhancement of
FIG. 5. Electric potential profiles for H- and L-modes, corresponding to the density profiles shown in Figs 3 and 4. The parameter \( \lambda \) varies from 0.04 to 0.4 in the transition from H to L.

the number of untrapped ions peaking sharply at \( |\theta| = \pi/2 \) and (b) the well known reduction of the number of banana ions with increasing poloidal angle. The competition results in two peaks: one minimum with negative potential close to \( \theta = 0 \) and one maximum with positive potential close to \( \theta = \pi/2 \). The picture is the same below the midplane, because of the symmetry assumed. For the L-regime, the effect is qualitatively similar for the L- and H-modes, but in view of the much weaker radial electric field for the L-mode, the relative importance of the convective cells is greater in the L-regime [11]. Finally, we have also computed the density and electric field profiles for JFT-2M [12]. There was a high degree of agreement in the vicinity of the separatrix.

4. CONCLUSIONS

The issue of ions on loss and confined orbits has been addressed by numerical methods. It is demonstrated that steep density profiles in the vicinity of a separatrix automatically imply strong radial electric fields at the separatrix and vice versa. These large values of the electric field used as the boundary condition result in poloidal rotation velocity profiles with a large shear which are considered beneficial to improved confinement. On the contrary, gradual density profiles close to the separatrix do not cause any significant radial electric fields and therefore unfold to yield the L-regime of confinement. It is shown that significant poloidal electric fields
FIG. 6. Computed contours of $e$/$T_i$. Two convective cells emerge within the range $0 < \theta < \pi/2$. 
emerge as well, due to peculiarities of orbits in the vicinity of plates and quasi-neutrality. The radial electric fields result in poloidal rotation, and the poloidal electric fields result in flux surface average radial fluxes. In summary, the electric field at the separatrix is mainly governed by the peculiarities of the orbits and not by neoclassical viscosity.

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MHD STABILIZATION USING RADIOFREQUENCY CURRENT DRIVE

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Abstract

MHD STABILIZATION USING RADIOFREQUENCY CURRENT DRIVE.

The paper extends previous theoretical work on MHD stabilization using radiofrequency current drive. Stability calculations are carried out for realistic toroidal equilibria, a 3-D Fokker–Planck code is used to estimate radiofrequency driven current perturbations and calculations of the stabilizing influence of modulated radiofrequency drive are presented. Practical issues such as application of the scheme in a feedback loop are addressed.

1. INTRODUCTION

Localized radiofrequency (RF) current drive is one of the most promising methods for the suppression of magnetic islands in tokamaks formed by unstable current profiles or by external error fields. In particular, its use to control $m = 2$ activity and thus for disruption avoidance has been proposed [1, 2]. Such a scheme using electron cyclotron current drive (ECCD) is planned for ITER and will be tested on the COMPASS-D tokamak at Culham. The work described here extends previous theoretical work in several respects. First, stability calculations have been carried out for realistic, non-circular toroidal equilibria using the FAR code and are for both Gaussian and bipolar RF driven current perturbations, $J_{RF}(r)$, the latter being appropriate for modelling recent JET ion cyclotron current drive (ICCD) results [3]. Secondly, a 3-D Fokker–Planck code (BANDIT-3D [4]), which calculates power absorption and current drive profiles using a self-consistent ray tracing/Fokker–Planck model, has been used to estimate the likely $J_{RF}(r)$ for realistic launch spectra allowing for the broadening of $J_{RF}(r)$ due to radial transport of the heated electrons. Finally, calculations of the stabilizing influence of modulated RF current drive are presented which are more comprehensive than previous work [5], including realistic current drive and power absorption profiles in the island evolution equation and also addressing practical issues such as application of the scheme in a feedback loop.

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FIG. 1. Growth rate $\gamma$ of the $n = 1$ (predominantly $m = 2$) tearing mode for a monopolar $J_{RF}$ as its location ($\tilde{\psi}_0$), width ($\bar{w}$) and magnitude ($\delta I/I$) are varied. $q = 2$ is at $\tilde{\psi} = 0.76$, and $\gamma$ is normalized to the inverse of the resistive diffusion time.

2. MHD CALCULATIONS

The effect on $n = 1$ (predominantly $m = 2$) stability of the monopolar current perturbation produced by LH (or EC) current drive in large tokamaks has been studied using the FAR code [6]. A Gaussian profile, $J_{RF}(r) = C \exp \left(-b(\tilde{\psi} - \tilde{\psi}_0)^2\right)$, was applied to an equilibrium unstable to the $m = 2$ tearing mode, having an aspect ratio of 3, an elongation of 1.7, an edge $q$ of 4.1 (parameters typical of COMPASS-D, JET and ITER) and a current profile given by $R\phi \propto (1 - \tilde{\psi}^2) - 0.525 (1 - \tilde{\psi}^4)$ (with $\tilde{\psi}$ the poloidal flux normalized to $\tilde{\psi} = 1$ at the edge). In all cases the self-consistent equilibrium using the total current was calculated and zero $\beta$ was assumed to allow unambiguous study of the tearing mode. The value of the stability parameter $\Delta_0^c$ of the unperturbed equilibrium was obtained from the growth rate using the expression given in Ref. [7], with the result $\Delta_0^c = 7.7$. The FAR results are summarized in Fig. 1, where the $n = 1$ growth rate ($\gamma$) is shown as a function of the radial position ($\tilde{\psi}_0$) for various half-widths in minor radius ($\bar{w}$) and relative proportions of RF to total current ($\delta I/I$). Complete stability to the $n = 1$ tearing mode is obtained when $J_{RF}$ is located near $q = 2$ ($\tilde{\psi} = 0.76$), with smaller amounts needed for narrower $J_{RF}$. In the cylindrical limit, Westerhof [2] has analytically calculated the current required for marginal stability:
\[
\frac{\delta I}{I} = \frac{\bar{w}^2}{8} \left[ \frac{1}{q} \frac{dq}{dr} \right]_{\Delta_0^*}
\]

Comparing the marginal stability results from FAR with Eq. (1) (using the value of \(\Delta_0^*\) deduced from FAR) shows reasonable agreement, despite the finite aspect ratio and shaping effects which are not included in Eq. (1), and confirms that \(\delta I\) (and hence the required RF power) scales as \(\bar{w}^2\):

<table>
<thead>
<tr>
<th>(\bar{w})</th>
<th>(\delta I/I) (code)</th>
<th>(\delta I/I) (Eq. (1))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.16</td>
<td>5.7%</td>
<td>6.1%</td>
</tr>
<tr>
<td>0.32</td>
<td>19.4%</td>
<td>24.4%</td>
</tr>
</tbody>
</table>

Calculations have also been carried out [8] for bipolar \(J_{RF}(r)\) profiles relevant to recent ICCD experiments on JET. These indicate that, for the same overall width of \(J_{RF}(r)\), bipolar profiles have a greater stabilizing effect than monopolar profiles. If, instead, the comparison is made with the width in each lobe of the bipolar profile equal to the width of the monopolar profile, which is probably more relevant to experiment, stabilization using the monopolar profile needs approximately 75\% of the absolute current required for the bipolar profile.

3. NON-INDUCTIVE CURRENT DRIVE PROFILES

In many circumstances, Fokker-Planck calculations which treat each flux surface separately are not valid since the collisional and transport time-scales can be comparable. The BANDIT-3D code [4] allows proper treatment of such situations by solving for particle distributions in non-circular, axisymmetric tokamaks as a function of speed, pitch angle and flux surface radius, \((v, \theta, r)\), with full treatment of electron trapping effects. The code treats collisions, heating (Ohmic, ECR, LH and fast wave) and radial transport of the particles, yielding both kinetic information (current drive, non-thermal distributions, etc.) and profile information \((T(r), n(r), j(r))\), obtained as moments of \(f(v, \theta, r)\), and thus acts as a unified kinetic and transport code. An ad hoc diffusive and convective radial transport operator is used, although transport based on a particular model is also possible. (A future code will include the full 3-D neoclassical operator [4, 9] for electrons or ions. This will give the neoclassical fluxes and currents associated with non-Maxwellian distributions and also allow self-consistent treatment of fast ions (e.g. alpha particles) whose drift orbits differ substantially from flux surfaces.)

BANDIT-3D has been used to simulate the ECCD experiments on CLEO [10], in which the driven current was only one third of that predicted by theories
which neglect radial transport. The experimental plasma and ECRH parameters were used together with an imposed transport model with diffusive and convective transport chosen to maintain the observed density profile. Only the level of transport was adjusted, and it was found that one value \( (n_e(r)D(r) = 1.5(1 + 3(r/a)^3) \times 10^{19} \, \text{m}^{-1} \cdot \text{s}^{-1} \), with the diffusivity \( D \) taken independent of \( v \) and \( \theta \) gave both the magnitude and shape of the observed temperature profile and the observed current drive. Figure 2 shows that, because of the radial transport of the heated electrons, the current profile is significantly broader than the power deposition profile.
BANDIT-3D has also been used to calculate the ECRH driven $\delta J(r)$ for COMPASS and ITER mode stabilization conditions. For COMPASS ($R/a = 0.56 \text{ m}/0.2 \text{ m}, n_e = 1.7 \times 10^{19} \text{ m}^{-3}, T_{e0} = 1.5 \text{ keV}$), a diffusivity typical of Alcator or ITER-L scalings ($n_e(r)D(r) = 5 \times 10^{19} \text{ m}^{-1} \cdot \text{s}^{-1}$) broadens $J_{RF}(r)$ significantly [11], though the power calculated for stabilization is still within the available power. For ITER conditions, radial transport only gives a small broadening of $\delta J(r)$ if thermal diffusivities consistent with ITER scaling laws are used since the energy confinement time is much longer than the collision time of the heated electrons.

4. PHASED CURRENT DRIVE

A cylindrical non-linear model for the evolution of the width, $W$, of an island has been developed for phased heating and current drive, and the results have been compared with the steady state current drive approach discussed above. It is found that stabilization using current drive phased with the island motion can be much more efficient and less sensitive to localization than either steady state current drive or phased heating. However, the scheme only works if the current drive is poloidally positioned so that the current carrying electrons are localized in a helical band and thus is appropriate to ECCD but not to LHCD. The influence of an RF driven current on the island width is described by [12]:

$$\tau_R \frac{d(W/a)}{dt} = a (W/a) + \frac{a}{W} \frac{1}{s} \frac{J_1}{J_{0,av}} C_J$$

where $\tau_R$ is the resistive time-scale, $a$ is the minor radius, $s$ is the shear, $J_1$ and $J_{0,av}$ are the perturbed and average current densities, and $C_J$ (negative for O-point drive) depends on the radial profile and poloidal extent of $J_{RF}(r)$ [13]. The optimum scheme for stabilization is to drive current parallel to the equilibrium current at the island O-point, this being a factor of $\sim 2$ more efficient than using CD in the opposite direction at the X-point. When the width of the heating/current drive profile is greater than $W$, the behaviour is not sensitive to the localization of the driven profile, but more power is required as the profile broadens. Use of phased ECCD in a feedback loop could restrain the island to a harmless (though non-zero) size [13]. For unstable $J_2(\hat{\psi})$ similar to that in Section 2, phased current drive would require only 20% of the power required when steady ECCD is used and thus appears very promising with $\leq 5 \text{ MW}$ of power needed for ITER.

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IMPORTANCE OF THE
DETAILED MAGNETIC STRUCTURE IN TOKAMAKS:
ENERGY TRANSPORT AND
INTERNAL DISRUPTIONS

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Abstract

IMPORTANCE OF THE DETAILED MAGNETIC STRUCTURE IN TOKAMAKS: ENERGY TRANSPORT AND INTERNAL DISRUPTIONS.

Experimental and theoretical results on the importance of the detailed magnetic structure in a tokamak are presented, and it is discussed how these results throw some light on anomalous transport and internal disruptions. The paper, although containing several new results, is also a summary of a number of works done by the authors in the field of tokamak magnetic turbulence.

1. \textit{q}-PROFILES AND MAGNETIC TURBULENCE

1.1. Detailed \textit{q}-profiles are obtained from \textit{H\textsubscript{a}} striations observed during pellet injection [1–3]

Strong striations observed on the \textit{H\textsubscript{a}} signal during pellet injection can in many cases be explained by the intersection of resonant surfaces, where, because of the closed nature of the field lines, the energy reservoir is lower than on irrational field lines.

In a previous paper [1] it was shown that the rational \textit{q}-values should be associated with a shear plateau to yield a striation. Moreover, the fact that striations do not appear systematically on every strong rational surface suggests that the shear plateaus are due to pre-existent structures which the pellet may or may not intersect, depending on the poloidal phase (X- or O-point) and on the turbulence activity (width of the islands). Therefore we have tried to associate every important striation with shear plateaus due to small scale magnetic islands and to use the \textit{H\textsubscript{a}} modulation to determine the \textit{q}-profile with high accuracy. To fulfil this goal, we have used a simulated annealing technique; we describe our method in detail in Ref. [3].

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We shall present results for two Ohmic discharges with pellets injected at 610 m/s: (i) shot No. 4159, with a safety factor at the edge $q(a) = 3.8$, and (ii) shot No. 6435, for which $q(a) = 2.8$. During shot No. 4159, the inductance $\beta + \ell'/2$ varies and therefore the position of rational surfaces is expected to vary; we present in Fig. 1(a) the $q$-profile for one pellet of shot No. 4159 (with the $H_a$ signal and the envelope used to compute the $\alpha$). The same $q$-profile (on which each plateau is labelled by its corresponding $m$ and $n$ values) is compared in Fig. 1(b) with the profile given by IDENTC. Shot No. 6435 is more interesting since the inductance stays constant; Fig. 2 displays the $q$-profiles deduced from the three last pellets injected during this shot. The $q$-profile given by the IDENTC code is identical, and the profiles determined from the striations analysis are within a 5% range. A careful analysis of the signal shows that for the last pellet all striations corresponding to rational surfaces with $m \leq 9$ are present. We also present recent results of a Tore Supra Ohmic discharge with fast (2200 m/s) injector. The $H_a$ signal is essentially similar, but the pellet reaches inside the $q = 1$ surface. A tentative $q$-profile is given in Fig. 3. The central $q$-value is $q(0) \approx 0.7$. 

FIG. 1(a). $q$-profile superimposed on the $H_a$ signal.

FIG. 1(b). $q$-profile with shear plateaus for shot No. 4159.
1.2. These q-profiles exhibit shear plateaus indicating magnetic islands [1–3]. The cores of these islands have good confinement properties [4].

Except for the $q = 1$ striation, it is unlikely that the shear plateaus are poloidally symmetric: this would need unrealistic negative current layers. As discussed in Ref. [4], it is much more plausible that they are due to small scale magnetic islands. A direct visualization of these structures is sometimes obtained when the luminous $H_\alpha$ filaments are curved instead of being parallel to the field lines. It is argued that the electrostatic potential inside well confined island cores is responsible for the transverse velocity of the $H_\alpha$ emitting matter [4]. These observations yield constraints on the possible theoretical mechanisms.

1.3. Interpreting these structures as island remnants in a stochastic sea, one can derive electron energy transport coefficients [5, 6].

Making a very simple hypothesis, namely that the field line diffusion coefficient can be computed in the same way as in the noisy standard mapping [5] and that the electron distribution function can be approximated by a monokinetic population at
thermal velocity, we can make an order of magnitude estimate of the electron energy transport coefficients which these islands would induce [6]. Figures 4(a) and 4(b) show, for shots No. 4159 and No. 6435, three $\chi_e$ profiles obtained by this method for three different pellets and a fit of the averaged value. In fact, only averaged values can be compared to energy balance results since magnetic turbulence creates intermittent percolation paths rather than a smooth, constant value of $\chi_e$. The averaged value of the order of 1 m$^2$ s$^{-1}$ and the tendency of $\chi_e$ to increase slowly with $r$ are in good agreement with the value of $\chi_e$ obtained by transport analysis (Fig. 4(c)).

1.4. A theoretical analysis of the non-linear self-consistency of such a system of islands has been attempted [7]. A dynamical treatment of the tokamak magnetic turbulence regimes is under way [8]

Writing the energy balance of an island core embedded in a stochastic sea, one can find a stable static solution where such an island can be sustained [7]. Using the discrete nature of magnetic turbulence, where the active sites are localized on resonant magnetic surfaces and realize intermittent percolation paths, one can study the non-linear dynamical behaviour by using a Von Neumann object representation such as the 'beast's model' [8]. Adaptation to tokamaks is under way.

2. INTERNAL DISRUPTIONS

Let us consider the still unresolved problem of internal disruptions. We will principally address the simplest sawtooth with a resistive $m = 1$ precursor, as is the case, e.g. in Ohmic discharges.

2.1. It has been shown [9] that the $m = 1$ island can evolve non-linearly on a resistive time-scale only if the $q = 1$ surface is in a shearless region, and, moreover, that during the resistive evolution the angle at the X-point of the separatrix is decreasing and tending towards zero.

If the shear plateau is too large, the resistive tearing mode is stabilized and, conversely, if it disappears (or if the angle at the X-point tends to zero), the inertial effects become important, and the evolution is no longer a resistive sequence of MHD quasi-equilibria [9, 10].

2.2. Shearless regions around $q = 1$ have been measured in tokamaks [2]; they seem to be quite general.

The observation by pellet striations analysis or Faraday rotation of a wide shear plateau on the $q = 1$ surface in recent experiments such as JET, TEXTOR and TS,
FIG. 4(a). $\chi_e$ profile for three pellets of shot No. 4159.

FIG. 4(b). $\chi_e$ profile for the last three pellets of shot No. 6435.

FIG. 4(c). Typical $\chi_e$ profile from transport analysis.
as well as the existence of an \( m = 1 \) precursor in Ohmic sawteeth led us to investigate the effects of this plateau and of the separatrix evolution on the disruption mechanism and, specifically, on the stochasticity onset at the X-point.

2.3. In slab geometry, a null angle at the X-point of the separatrix lowers the stochastic threshold \([11]\). This effect is enhanced if a shearless zone exists near the resonant surface.

We use the following Hamiltonian:

\[
H = \frac{y^2}{2} - \frac{\beta}{\pi^2} \cos(\pi y) + \epsilon \cos(x - t - \alpha \sin(x - t)) \\
+ \epsilon \cos(x + t - \alpha \sin(x + t))
\]  

(1)

Figure 5 shows that, for the same amplitude of the perturbation, the stochastic region is much more extended in the presence of a shearless zone and with a null angle at the X-point than in the standard case (the stochastic threshold amplitude is \( \epsilon = 0.12 \) in the standard case and only \( \epsilon = 0.045 \) when the two effects are present).

2.4. This effect is also observed in toroidal geometry for the \( m = 1 \) island \([12]\)

In toroidal geometry, we use a Hamiltonian representation of the field line configuration in MHD equilibrium, to which we add an \( m = 1 \) perturbation with a non-harmonic component; we are therefore able to adjust the separatrix shape of the \( m = 1 \) island and to vary continuously the angle at the X-point. We compute the stochastic zone extension by numerical integration of the field flow. This calculation is done for a continuously growing \( q \)-profile and also in the presence of a shear plateau on \( q = 1 \). A more complete description is given in Ref. \([12]\).

In Fig. 6 we show the four cases: finite angle and monotonous profile, null angle and monotonous profile, finite angle and inflexion point at \( q = 1 \), and null angle and inflexion point at \( q = 1 \), with the same amplitude of the perturbation; we clearly see that both the shape of the separatrix (non-harmonicity of the perturbation) and the shape of the \( q \)-profile (shear plateau at \( q = 1 \)) have a strong effect on the stochasticity onset.

2.5. A coherent scenario for the internal disruption can thus be proposed

We suggest the following scheme for the sawtooth phenomenon (at least in simple cases): the crash of the internal disruption is due to the sudden appearance of stochasticity in a roughly annular region, corresponding to the \( q = 1 \) surface. After the crash, this region is completely shearless. The remainder of the \( m = 1 \) island decreases as it is stable in these conditions \([9]\). Ohmic (and, possibly, additional) heating peaks the profile while resistive diffusion slowly erodes the shear
plateau. At some point, the non-linear resistive growth of the $m = 1$ tearing becomes energetically favourable, and the precursor starts growing again. The angle at the X-point decreases slowly while the island width increases. Meanwhile, the resistive erosion of the shear plateau tends to leave an inflexion point at $q = 1$, thus accelerating the growth rate of the $m = 1$ island and, therefore, the decrease of the angle at the separatrix. These two effects combine to yield an extremely fast transition to stochasticity, and, as the crash is due to the electron random walk in the ergodized region, it takes place in a time $\tau_c$, scaling as $R T_c^{-1/2}$; it is thus practically the same for small and large tokamaks whereas the sawtooth duration typically scales as a resistive time, i.e. with $R^2 T_e^{3/2}$.

We thus retrieve a picture quite similar to the one initially suggested by Samain in 1976 [13, 14]. The main difference is that the field line ergodization which yields

FIG. 5. Cases corresponding to the same perturbation amplitude ($\epsilon = 0.045$).
FIG. 6. Field line sections in toroidal geometry with the same perturbation amplitude ($K = 0.006$) and parameters close to those of Tore Supra: $R = 2.3\text{ m}$, $a = 0.75\text{ m}$, $r_{q=1} = 0.2$, $q(0) = 0.93$, $q(a) = 2.8$.

enhanced transport is intrinsic (Hamiltonian chaos) [15] while in the original suggestion it is caused by plasma microinstabilities. These may nevertheless be present because of the current images forming at the edge of the stochastic region, thus helping it to expand faster.

It is possible that additional phenomena play a role in some cases, e.g. enhancement of the $m = 1$ perturbation growth rate [16]. Powerful numerical codes [17] could be useful to analyse the details of the crash even if they cannot give an overall description of the complete sawtooth.
3. CONCLUSIONS

The puzzle of electron energy transport and internal disruptions in tokamaks seems to take the shape of a macroscopic (global) coherence of magnetic turbulence. We have been trying to find some of the pieces of the puzzle and have presented them in this paper. The restrictive list of references does not mean that we ignore or neglect similar works.

REFERENCES

THEORY OF THE L–H TRANSITION IN TOKAMAKS WITH A BIASED ELECTRODE

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Abstract

THEORY OF THE L–H TRANSITION IN TOKAMAKS WITH A BIASED ELECTRODE.

A new theory of the L–H transition in tokamaks is put forward. It successfully explains the experiments with bias in CCT, TEXTOR and HL-1 and asserts that negative bias is slightly more efficient than positive bias for inducing the H mode, a conclusion also reached by the TEXTOR group after several experiments.

Recently the L–H transition has been observed in the CCT, TEXTOR and HL-1 tokamaks with bias [1–3]; that is, edge microturbulence is suppressed and markedly enhanced plasma confinement is achieved in these devices as soon as the bias (especially its shear) has gone beyond a certain value. These experiments, together with those on ASDEX, DIII-D and JFT-2M [4–6], greatly strengthen the confidence of the nuclear fusion research community in the tokamak. There exist several heuristic theory models [7–9] for interpreting the experiments on ASDEX, DIII-D and JFT-2M. The significance of the CCT, TEXTOR and HL-1 experiments also consists in the fact that imposing the bias is an active measure for inducing the H mode. However, finding an explanation for these experiments with bias on CCT, TEXTOR and HL-1 has always puzzled the theorists. In the present paper, we first propose a physically intuitive picture to explain these experiments.

We note that in tokamaks the bias can drive the plasma to flow poloidally (this flow tangential to the magnetic surface cannot destroy the plasma confinement), and we think that during the time the bias is imposed the edge turbulence and the poloidal flow of plasma driven by the bias in the edge compete with each other. From the MHD point of view the turbulence is an irregular flow and the poloidal flow is a laminar regular flow. The regular flow is, figuratively speaking, just like a 'barrier' against the development of turbulence. In brief, the L–H transition process is just the process of competition between the two kinds of flow. The perturbation causes the plasma column to deviate from the original equilibrium position and brings it into contact with the limiter and the vacuum chamber if and only if the perturbation sufficiently disturbs the edge laminar flow. Doubtless in this process the perturbation energy is consumed; namely, the amplitudes of all perturbed quantities drop and hence the system enters the H operating mode.
On the basis of the physical picture presented above we derive an analytical expression for the linear growth rate $\gamma$, explicitly showing the competition between the irregular and regular flows, from the following set of equations:

$$
- \nabla \phi + \vec{\nabla} \times \vec{B} = \eta \vec{J} - \left( T_e/\text{en} \right) \nabla n - \left( 1/e \right) \nabla T_e + \left( 1/\text{en} \right) \vec{J} \times \vec{B} \tag{1}
$$

$$
nM(d\vec{V}/dt) = -(T_e + T_i) \nabla n - n \nabla (T_e + T_i) + \vec{J} \times \vec{B} \tag{2}
$$

$$
\partial n/\partial t + \nabla \cdot (n \vec{V}) = 0 \tag{3}
$$

$$
\nabla \cdot \vec{J} = 0 \tag{4}
$$

where all notations are conventional. We work in the toroidal co-ordinate system $(r, \theta, z)$, where $ds^2 = dr^2 + r^2 d\theta^2 + (1 + \epsilon \cos \theta)^2 dz^2$. Therefore, the gradient operator $\nabla$ can be expressed as:

$$
\nabla = \hat{r} \frac{\partial}{\partial r} + \hat{\theta} \frac{\partial}{\partial \theta} + \hat{z} \frac{\partial}{\partial z} \left( 1 + \epsilon \cos \theta \right) \frac{\partial}{\partial z}
$$

where $(\hat{r}, \hat{\theta}, \hat{z})$ are the orthogonal unit vectors. Here $\epsilon = r/R$ is the inverse aspect ratio. The confined magnetic field is assumed to be given: $B_r = 0$, $B_\theta = (\epsilon/q)B_0/(1 + \epsilon \cos \theta)$, $B_z = B_0/(1 + \epsilon \cos \theta)$, with $q$ being the tokamak safety factor. Then $r = \text{const}$ denotes a flux surface and other equilibrium quantities are functions only of $r$. Equations (1-4) represent, respectively, Ohm's law, the momentum equation, the continuity equation and the charge conservation equation, in the electrostatic approximation. In the procedure of linearization we assume that (with no perturbations of the temperatures and the magnetic field):

$$
n = n_0(r) + \tilde{n} \\
\phi = \phi_0(r) + \tilde{\phi} \\
\vec{V} = \vec{V}_0(r) + \tilde{\vec{V}} \approx V_0(r) \hat{\theta} + \tilde{\vec{V}} \\
\vec{J} = \vec{J}_0(r) + \tilde{\vec{J}}
$$

where $\vec{V}_0(r) = \pm E_0(r)\hat{r} \times \vec{B}/B^2 \equiv V_0(r)\hat{\theta}$ and $V_0(r) = \pm E_0(r)/B_0$ (positive and negative correspond to positive and negative bias) is the poloidal rotation velocity with $E_0(r)$ being the absolute value of the radial electric field induced by the biased electrode introduced into the tokamak edge region. Also, we postulate that the perturbed quantities (e.g. $\tilde{n}$) vary as:

$$
\tilde{n}(r, \theta, z; t) = \tilde{n}(m, k) \exp[i\omega t + i(m\theta + kz)]
$$

i.e., in differentiating $\tilde{n}(m, k, r)$ with respect to $r$ we approximately treat $\tilde{n}(m, k, r)$ as a constant (local approximation). Assume that $\epsilon \ll 1$. In this paper, $\epsilon$ is taken to be a basic small parameter and all perturbed quantities are ordered as $\epsilon$. In addition, we require all results to be accurate to the first order of $\epsilon$. To a reasonable approxima-
tion, from Eqs (1-4), via lengthy manipulation, we obtain \((\tilde{V}_r, \tilde{V}_\theta, \tilde{V}_z)\) and \((\tilde{I}_r, \tilde{I}_\theta, \tilde{I}_z)\), expressed in terms of \(\tilde{n}(m, k)\) and \(\tilde{\varphi}(m, k)\), respectively. Substituting the two sets of expressions just obtained into the perturbed component of the linearized Eq. (3) and Eq. (4), respectively, we arrive at the following system of equations:

\[
\left(\omega_1 - \frac{2m}{rL_n} \rho_i V_i - \frac{kk_i c_i^2}{\omega_1}\right)\tilde{n} - \frac{m n_0}{rL_n B_0} \tilde{\varphi} = 0
\]

\[
\left[i \frac{\omega_1}{B_0 r L_n} \rho_i V_i + i \frac{m^2}{B_0 r^2} \rho_i V_i \omega_1 + i \frac{m}{B_0 r^2} \rho_i V_i \left(\frac{dV_0}{dr} + \frac{V_0}{r}\right) + i \frac{2 m V_0}{B_0 r^2 L_n} \rho_i V_i - i \frac{m}{B_0 r^2} V_0^2 + i \frac{m}{B_0 r L_n} c_i^2 - \frac{kk_i T_e}{\eta \epsilon n_0 M}\right] \tilde{n}
\]

\[
+ \frac{1}{M} \left[i \frac{m}{r^2 \mu_i c_i^2 A} \left(m \omega_1 + \frac{dV_0}{dr} + \frac{V_0}{r}\right) \right] \tilde{\varphi} = 0
\]

where \(\omega_1 = \omega + (m V_0/r)\), \(k_i = k + (\epsilon m/qr)\) and \(L_n > 0\) and \(L_s > 0\) are the scale length for the density and \(c_s = [(T_e + T_i)/M]^{1/2}\), respectively. The dispersion relation drawn from Eqs (5, 6) will be \(\omega\)-cubic, making our analysis very difficult. Fortunately, we find that over a considerably wide range of parameters:

\[
\frac{|i (\rho_i V_i / r B_0 L_n)\omega|}{|kk_i T_e / \eta \epsilon n_0 M|} \gg 1 \quad \text{and} \quad \frac{|kk_i / \eta|}{|i (m^2 \omega / r^2 \mu_i c_i^2 A)|} \gg 1 \quad \text{(for } m \leq 10\text{)}
\]

Then, after neglecting the terms:

\[
\frac{kk_i T_e}{\eta \epsilon n_0 M} \quad \text{and} \quad \frac{m}{r^2 \mu_i c_i^2 A} \left(m \omega_1 + \frac{dV_0}{dr} + \frac{V_0}{r}\right)
\]

in Eq. (6), the dispersion relation is reduced to be \(\omega\)-quadratic and yields the following expression of the linear growth rate:

\[
\gamma = \frac{m^2 \eta}{2 kk_i r_i^2 L_n \mu_i c_i^2 A} \left(\frac{r_e}{L_n} c_i^2 \pm \frac{4 \rho_i}{L_n} V_i \frac{E_0}{B_0} - \rho_i V_i \frac{dV_0}{dr}\right)
\]

where \(k_i = k + (\epsilon m/qr)\), \(r_e\) is the radius of the plasma edge and the rest are conventional. Here it should be noted that \(dV_0/dr\) is always positive.

Clearly, Eq. (7) yields four significant conclusions: (i) In the case of \(E_0 = 0\), Eq. (7) reduces to the usual result for low frequency drift wave instability in tokamaks and the contributions to \(\gamma_0\) come from \(T\) and \(n\) gradients and curvature effect.
Substituting typical tokamak parameters, $\gamma_0$ is estimated to be $10^4 \, \text{s}^{-1}$, agreeing with the experimental value [3, 10]. In this case, the instability rapidly grows and the plasma has L mode confinement. (ii) Usually, the absolute value of the second term in Eq. (7) is much less than that of the first; thus suppressing the edge perturbation relies on the shear of $E_0$ rather than $E_0$ and large shear (making $\gamma$ small) can induce the H mode, confirming the experiments on CCT, TEXTOR and HL-1 [1–3]. In Eq. (7) the first and third terms, representing the effect of irregular and laminar regular flows, respectively, compete with each other. (iii) The positive or the negative bias will induce the H mode, provided $\text{d}E_0/\text{d}r$ is large enough, in accord with the experiments [2, 3], but the former is less efficient than the latter. This correct conclusion was reached by the authors of Ref. [2] only after repeated experiments. (iv) As previously stated, the absolute value of the second term in Eq. (7) is much less than that of the first; thus the second term, compared with the first, can be omitted, and setting $\gamma = 0$, we find the shear of $E_0$ necessary to quench the instability entirely as follows:

$$\frac{\text{d}E_0}{\text{d}r} \sim \frac{r_0 B_0}{L \rho_1 V_1} c_s^2$$

Condition (8) is, in view of the smallness of $L_s$ and $\rho_\perp$ and the largeness of $c_s$, difficult to satisfy. It indicates that it is by no means easy to quench the instability thoroughly. Therefore, we think that the H mode is characterized by a substantial drop in the amplitudes of all perturbed quantities and by markedly enhanced plasma confinement but not by thorough quelling of the instability.

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RAY PATHS IN THE ELECTRON CYCLOTRON RESONANCE REGION

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Abstract

RAY PATHS IN THE ELECTRON CYCLOTRON RESONANCE REGION.

The influence of finite temperature and relativistic effects on the ray behaviour in the electron cyclotron resonance region has been investigated. The term in the ray equations which generates differences between the thermal and cold plasma ray descriptions is analysed first. Then, the numerical solutions of the ray equations in a tokamak equilibrium are presented. It is found that the ray path may be strongly affected by thermal effects in the electron cyclotron resonance region. For oblique propagation they induce an additional ray refraction (as compared to the ray paths obtained in the cold plasma limit) or even a ray reflection from the resonance region. This has important consequences on the wave absorption and current drive efficiencies, in particular in the finite density regime.

1. INTRODUCTION

The ray tracing technique is used routinely to study the propagation and absorption of waves near the electron cyclotron frequency and its harmonics in various plasma devices. It is generally accepted that the wave dispersion relation in the cold plasma limit provides a good approximation for the real part of the wave refractive index, the ray path and the wave polarization. The cold plasma dispersion relation is real, and the ray path represents the position and the wave vector of a ray in a real, physical space. When we allow the plasma to have a finite temperature, the dispersion relation governing the waves becomes complex and its anti-Hermitian part may be of a magnitude comparable to the Hermitian part. The imaginary part of the wave refractive index, which governs absorption (or emission) of wave energy by the plasma, is usually determined within the weak damping approximation of the hot plasma dispersion relation. In order to restrict the ray path to real values, the wave absorption is assumed to be weak [1], i.e. \(|\mathbf{g}N| \ll |\mathbf{g}\tilde{N}|\), where \(\tilde{N}\) is the wave refractive index. Weitzner and Batchelor [2] have shown that when the electromagnetic waves are weakly damped at cyclotron resonance, ray paths can be defined and traced through the resonance by using the cold plasma dispersion relation. Accordingly, one does not need to solve the hot plasma ray tracing problem. The solutions of the hot plasma dispersion relation near the particle (cyclotron) and wave (hybrid) resonances differ, however, from those obtained in the cold plasma limit [3]. We have reconsidered this problem and found that thermal effects are essential not
only near the wave resonances where mode conversion occurs, but also near the cyclotron resonances. The results obtained indicate that for oblique propagation these effects induce an additional ray refraction (as compared to the ray paths in the cold plasma model) or even a ray reflection from the resonance layer.

2. DERIVATIVES OF THE DISPERSION RELATION

The pair of the Hamiltonian equations for the ray path, with the local dispersion relation $D(N, \omega, \vec{r}, t) = 0$ playing the role of Hamiltonian, has the form

$$\frac{d\vec{r}}{ds} = -\frac{\partial D/\partial \vec{N}}{|\partial D/\partial \vec{N}|}$$

$$\frac{d\vec{N}}{ds} = \frac{\partial D/\partial \vec{r}}{|\partial D/\partial \vec{r}|}$$

Here, $\vec{r}$ is the spatial co-ordinate and $s$ is the arc length along the ray. The dispersion relations in the cold plasma ($D_C$), hot plasma ($D_H$) and relativistic plasma ($D_R$) approximation are readily available in the established wave literature [4-6] and need not be repeated here. In general, the dispersion relation has an infinite number of roots, including the two fundamental electromagnetic modes (the ordinary and extraordinary modes) and the plasma (quasi-electrostatic) mode. Here, we confine ourselves to the consideration of weakly damped waves, $|\partial \vec{N}| \ll |\partial \vec{N}|$, that is, waves whose characteristic absorption distance is large compared to their wavelength. At the low temperature plasma edge these modes go continuously into the aforementioned fundamental electromagnetic modes. However, in the hot plasma core and, in particular, near the cyclotron and wave hybrid resonances, the solutions of the dispersion relation in thermal plasma differ from those obtained in the cold plasma limit. This generates differences between the thermal and the cold plasma ray descriptions.

Let us examine the behaviour of the spatial derivative of the dispersion relation near the electron cyclotron (EC) resonance. Its general form is given by

$$\frac{\partial D}{\partial \vec{r}} = \frac{\partial D}{\partial X} \frac{\partial X}{\partial \vec{r}} + \frac{\partial D}{\partial Y} \frac{\partial Y}{\partial \vec{r}} + \frac{\partial D}{\partial \gamma} \frac{\partial \gamma}{\partial \vec{r}}$$

$$+ \frac{\partial D}{\partial \vec{N}_1^2} \frac{\partial \vec{N}_1^2}{\partial \vec{r}} + \frac{\partial D}{\partial \vec{N}_1} \frac{\partial \vec{N}_1}{\partial \vec{r}}$$

where the normalized quantities $X$, $Y$ and $\gamma$, which determine the plasma equilibrium, are defined as $X = \omega_p^2/\omega^2$, $Y = \omega_c/\omega$ and $\gamma = v_i/c$, with $\omega_p$ and $\omega_c$ being the plasma and EC frequency, respectively, and $v_i$ the electron thermal velocity. In thermal plasma the spatial derivative of the dispersion relation contains an additional term $(\partial D/\partial \gamma)(\partial \gamma/\partial \vec{r})$ whose contribution is, however, small at relatively low electron temperatures ($T \leq 20$ keV). Out of the EC resonance region the derivatives
\( \frac{\partial D_H}{\partial X}, \frac{\partial D_H}{\partial Y}, \frac{\partial D_H}{\partial N_1^2} \) and \( \frac{\partial D_H}{\partial N_1} \) coincide with those obtained in the cold plasma limit and \( \frac{\partial D_H}{\partial T} = \frac{\partial D_c}{\partial T} \). In the cold plasma approximation, the derivatives \( \frac{\partial D_C}{\partial X}, \frac{\partial D_C}{\partial Y}, \frac{\partial D_C}{\partial N_1^2} \) and \( \frac{\partial D_C}{\partial N_1} \) are proportional to \( (1 - Y)^{-1} \). We note in passing that it is not difficult to solve the ray equations near and through the resonance because the right hand sides of the ray equations (1) and (2) are not singular at \( Y = 1 \). So, as the wave approaches the resonance, the absolute value of the aforementioned partial derivatives increases. With the exception of \( \frac{\partial D_C}{\partial Y} \), all these derivatives are of the same order of magnitude. For the extraordinary electromagnetic mode, the absolute value of \( \frac{\partial D_C}{\partial Y} \) is larger, while for the ordinary mode it is usually smaller than the other derivatives.

The most strongly temperature dependent partial derivative of \( D_H \) (or \( D_R \)) is that with respect to the static magnetic field. For oblique wave propagation, in the EC resonance layer (\(|1 - Y| \leq |\gamma N_1|\)), this derivative is proportional to \( X/(N_1^2 \gamma^2) \). We recall that the plasma contribution to the dielectric tensor in this layer is of the order \( \eta = X/(2Y^2 N_1 \gamma) \). So, for equilibrium plasma parameters such that \( \eta < 1 \), i.e. in the tenuous plasma limit [5], the plasma effects in the wave propagation are weak, \( \frac{\partial D_H}{\partial Y} \) (and \( \frac{\partial D_R}{\partial Y} \)) are relatively small and the hot plasma ray paths of the ordinary mode tend to those obtained in the cold plasma limit. As the parameter \( \eta \) is increased, the plasma contribution to the dielectric tensor increases, and the hot plasma ray paths of the ordinary mode start to deviate from the cold plasma ray paths. In fact, the relatively large values of the derivative \( \frac{\partial D_H}{\partial Y} \) in the resonance region cause changes of sign of the equations governing the poloidal wavenumber \( N_\theta \), the toroidal and the poloidal angles (see Eqs 8, 6, 5). The values of the poloidal wavenumber differ significantly from those obtained in the cold plasma limit. This results in a substantial ray refraction (as compared to the rays obtained in the cold limit). The considered effect is pronounced, particularly for ordinary waves propagating in decreasing magnetic field, where it can lead to a ray reflection from the resonance layer.

The rays initialized to the extraordinary mode exhibit a similar behaviour at the resonance layer. Here, the refraction is effective in both, the tenuous plasma (\( \eta < 1 \)) and the finite density (\( \eta > 1 \)) regimes. Specifically, thermal effects affect the propagation of extraordinary waves in both these regimes. Moreover, the wave polarization changes in the EC resonance layer.

### 3. THE RAY TRACING CODE

To verify the previous qualitative picture we have developed a fully three-dimensional ray tracing code. The code is written in the usual toroidal co-ordinates \( \{r, \theta, \phi\} \), where \( r \) and \( R = R_0 + r \cos \theta \) are the minor and major radial positions, and \( \theta \) and \( \phi \) are the poloidal angle toroidal angles, respectively. It should be noted that right handed Cartesian co-ordinate systems are used throughout this paper. By
applying a canonical transformation and by implicit differentiation of the dispersion relation we have obtained the ray equations in the following form:

\[
\frac{dr}{ds} = -F \frac{\partial D}{\partial N_r} = -F^2 N_r \frac{\partial D}{\partial N_\perp} \tag{4}
\]

\[
\frac{d\theta}{ds} = -\frac{F}{r} \frac{\partial D}{\partial N_\theta} = -\frac{F}{r} \left( 2b_\theta N_u \frac{\partial D}{\partial N_\perp^2} + b_\theta \frac{\partial D}{\partial N_\perp} \right) \tag{5}
\]

\[
\frac{d\phi}{ds} = -\frac{F}{R} \frac{\partial D}{\partial N_\phi} = -\frac{F}{R} \left( -2b_\phi N_u \frac{\partial D}{\partial N_\perp^2} + b_\phi \frac{\partial D}{\partial N_\perp} \right) \tag{6}
\]

\[
\frac{dN_r}{ds} = F \frac{\partial D}{\partial r} = F \left\{ \frac{\partial D}{\partial X} \frac{\partial X}{\partial r} + \frac{\partial D}{\partial \gamma} \frac{\partial \gamma}{\partial r} - \frac{Y \cos \theta}{R} \frac{\partial D}{\partial Y} \right\} 
+ \frac{df}{dr} \left[ Y_\phi b_\theta \frac{\partial D}{\partial Y} + N_u b_\phi^2 \left( -2N_\perp \frac{\partial D}{\partial N_\perp^2} + \frac{\partial D}{\partial N_\perp} \right) \right] \tag{7}
\]

\[
\frac{d(N_\perp)}{ds} = F \frac{\partial D}{\partial \theta} = r \sin \theta \left( \frac{FY}{R} \frac{\partial D}{\partial Y} + N_\phi \frac{d\phi}{ds} \right) \tag{8}
\]

\[
\frac{d(RN_\phi)}{ds} = F \frac{\partial D}{\partial \phi} = 0 \tag{9}
\]

Here, \( F = 1/|\partial D/\partial \bar{N}| \), \( \bar{N} = \bar{Y}/Y = \{0, b_\theta, b_\phi\} \), and \( f = B_\theta/B_\phi = r/(R_0 q(r)) \). The code is applied to toroidal (i) a cold plasma, (ii) a hot Maxwellian plasma and (iii) a weakly relativistic Maxwellian plasma. The magnetic surfaces are assumed to have circular and concentric meridian cross-sections. We have assumed that the equilibrium tokamak magnetic induction has a toroidal component \( B_\theta(r, \theta) = B(0)R_0/r \) and a poloidal component \( B_\phi(r, \theta) = f B_\phi(r, \theta) \), where the safety factor is \( q(r) = q(0) + \left[ q(a) - q(0) \right](r/a)^2 \), and \( a \) is the plasma radius. Furthermore, the axisymmetric equilibrium is modelled by the following electron density and temperature radial profiles: \( n(r) = n(0)[1 - (r/a)^2] \) and \( T(r) = T(0)[1 - (r/a)^2]^2 \). The components of the wave refractive index in the orthogonal frame of reference tied to \( \bar{B}(\bar{r}) \) are determined by the following relations: \( N_\parallel = b_\parallel N_\theta + b_\parallel N_\phi, \ N_u = -b_\parallel N_\phi + b_\parallel N_\phi \) and \( N_\perp = (N_\parallel^2 + N_\phi^2)^{1/2} \). The dielectric tensor components \( e_{ij} \) of a cold plasma and a hot Maxwellian plasma are defined elsewhere (see, e.g. Refs [4, 5]). Since in most cases of interest in fusion research the electron temperatures are not too high \( (T_e \approx 20 \text{ keV}) \), we have used the dielectric tensor of a weakly relativistic thermal plasma in the relativistic description [6]. The imaginary part of the wave refractive index is determined from the roots of the complete (ii) hot plasma or (iii) weakly relativistic dispersion relation.
4. RESULTS AND DISCUSSIONS

We assume that the waves are launched from the plasma boundary at the position \( \{a, \theta_a, \phi_a\} \) in a direction characterized by the angle \( \chi \) between the wave refractive index \( N \) and the unit vector \( \vec{e}_p \), and the angle \( \psi \) between \( \vec{e}_p \) and the projection of \( \vec{N} \) in the poloidal plane. Since our main attention is focused on the ray refraction induced by thermal effects, the numerical code is not applied to a specific toroidal device. Instead, we have chosen parameters characterizing a medium size tokamak device (for example, RPT), \( a = 0.24 \text{ m}, R_0 = 0.72 \text{ m}, q(0) = 1 \) and \( q(a) = 3 \), and vary the central electron density and temperature in wide ranges: \( n(0) = 10^{16} - 10^{20} \text{ m}^{-3} \) and \( T(0) = 1 \text{ eV} - 20 \text{ keV} \), respectively. The frequency of the launched waves is assumed to be \( f = 60 \text{ GHz} \). For simplicity, in all figures illustrating our discussion, the EC resonance layer is placed at the plasma centre. The magnetic induction at the plasma axis is, in that case, \( B(0) = 2.14 \text{ T} \).

We first consider rays initialized to the ordinary mode and injected from the low magnetic field side of the torus. They suffer radial reflection with \( N_r = \pm \sqrt{N_i^2 - N_H^2} \) = 0. Two distinct topologies of ray reflection can occur. At large values of the parallel wave refractive index, \( Y/(1 + Y) < N_i < 1 \), that is, for \( \chi \leq 60^\circ \) the rays are reflected near the high density (or left hand) cut-off, \( X_c = (1 - N_i)/(1 + Y) \). In fact, the ray undergoes reflection before the cut-off is reached, that is, at the electron density \( X_r \), which is smaller than \( X_c \). Note that \( X_r - X_c \) only when \( N_i \rightarrow 1 \). As \( N_i \) is decreased, the perpendicular wave refractive index of the ordinary mode increases, and the rays reflect well before \( N_i^2 \) itself vanishes. This is illustrated graphically in Fig. 1, where we show the poloidal and toroidal projections of rays launched from the point \( \{a, 0^\circ, 0^\circ\} \) in the direction of the plasma centre (\( \psi = 180^\circ \)) at various angles \( \chi \). The central plasma density is \( n(0) = 2 \times 10^{19} \text{ m}^{-3} \), while the central electron temperature in the weakly relativistic description is assumed to be \( T(0) = 1 \text{ keV} \). Obviously, for \( \chi < 50^\circ \) the rays do not reach the EC resonance layer, and the ray paths in the weakly relativistic hot plasma coincide with those obtained in the cold plasma limit. In the range of intermediate values of the angle \( \chi (50^\circ \leq \chi \leq 60^\circ) \), the rays are refracted near the plasma centre and reflected at the high magnetic field side of the torus. The length of the ray paths in the toroidal direction becomes large. The wave launching in this \( \chi \)-range has the advantage that the ray paths cross the absorption region twice. We see that the ray paths in the weakly relativistic and cold plasma descriptions differ significantly. Finally, for large values of the angle \( \chi (\chi > 60^\circ) \) the radial ray reflection disappears, and the rays reach the plasma edge at the high field side. Here, as the angle \( \chi \) approaches 90°, the rays tend to stay in the equatorial plane and to coincide with those obtained in the cold plasma model.

Similar behaviour of the rays initialized to the ordinary mode is observed for wave launching out of the equatorial plane. In fact, for large \( N_i \) values the radial reflection is associated with the formation of a focal point at the low magnetic field side. For small \( N_i \) values, the incident rays still converge to a real focus but pass
FIG. 1. Poloidal (a) and toroidal (b) views of ray paths for $\theta_0 = 0^\circ$, $\psi = 180^\circ$, $n(0) = 2 \times 10^{19} \text{ m}^{-3}$, $T(0) = 1 \text{ keV}$; (1) $\chi = 20^\circ$, (2) $\chi = 50^\circ$, (3) $\chi = 52^\circ$, (4) $\chi = 60^\circ$, (5) $\chi = 80^\circ$, (6) $Y = 1$. 
FIG. 2(a). Poloidal view of ray paths for $\theta_a = 45^\circ$, $\chi = 60^\circ$, $\psi = 180^\circ$, $T(0) = 1 \text{ keV}$; (1) $n(0) = 10^{19} \text{ m}^{-3}$; (2) $n(0) = 10^{18} \text{ m}^{-3}$; (3) $n(0) = 10^{17} \text{ m}^{-3}$; (4) $T(0) = 0 \text{ eV}$, $n(0) = 10^{17} \text{ m}^{-3}$; (5) $Y = 1$.

FIG. 2(b). Poloidal view of ray paths for $\theta_a = 45^\circ$, $\chi = 60^\circ$, $\psi = 180^\circ$, $n(0) = 10^{19} \text{ m}^{-3}$; (1) $T(0) = 100 \text{ eV}$; (2) $T(0) = 1 \text{ keV}$; (3) $T(0) = 10 \text{ keV}$; (4) $T(0) = 0 \text{ eV}$; (5) $Y = 1$. 
FIG. 3. Poloidal views of ray paths for $\theta_a = 135^\circ$, $\chi = 45^\circ$, $T(0) = 1$ keV; (1) $\psi = 140^\circ$, (2) $\psi = 160^\circ$, (3) $\psi = 180^\circ$, (4) $\psi = 200^\circ$, (5) $\psi = 220^\circ$, (6) $\gamma = 1$; (a) $n(0) = 10^{17} \text{ m}^{-3}$ and (b) $n(0) = 10^{18} \text{ m}^{-3}$. 
FIG. 4. Poloidal (a) and toroidal (b) views of ray paths for $\theta_s = 135^\circ$, $\chi = 45^\circ$, $n(0) = 10^{19} \text{ m}^{-3}$, $T(0) = 1 \text{ keV}$; (1) $\psi = 140^\circ$; (2) $\psi = 160^\circ$; (3) $\psi = 180^\circ$; (4) $\psi = 200^\circ$; (5) $\psi = 220^\circ$; (6) $\psi = 180^\circ$ for $T(0) = 0 \text{ eV}$; (7) $Y = 1$; (8) $X + Y^2 = 1$.

through the EC resonance. In the tenuous plasma regime ($\eta \equiv X/(2Y^2N_T\gamma) < 1$), the hot and cold plasma ray paths almost coincide. When, however, $\eta$ is increased they start to deviate. This effect can be understood qualitatively from Fig. 2(a), (b), in which the ray paths obtained for (a) different central electron densities and (b) different central electron temperatures are compared with those obtained in the
cold plasma limit. We see that up to the resonance layer the aforementioned ray paths coincide. Here, the refraction of rays in thermal plasma increases for increasing $\eta$.

The effect considered is particularly pronounced for waves propagating in a decreasing magnetic field. In the finite density regime, the equation governing the poloidal wavenumber changes sign at the resonance layer. This induces further modifications of the ray path, as compared to the cold one. Specifically, $d\phi/ds$, $N_p$ and $d\theta/ds$ change sign so that the ray is reflected from the resonance layer. The ray reflection may take place right before or behind the EC resonance as is the case for the parameters of the ray paths represented in Fig. 3(b). The waves are launched from the point $\{a, 135^\circ, 0^\circ\}$ in the direction $\chi = 45^\circ$ at various angles $\psi$. The central electron temperature is $T(0) = 1$ keV. For $n(0) = 10^{17}$ m$^{-3}$ (Fig. 3(a)) the rays cross the EC resonance, and their paths almost coincide with those obtained by using the cold plasma dispersion relation. If the central electron density is increased to $n(0) = 10^{18}$ m$^{-3}$, we go over to the finite density regime, and the ray path changes completely at the resonance. With the exception of that labelled ‘I’, which does not reach the resonance, the rays are reflected from the EC resonance layer.

The previous results imply important modifications of the ordinary wave absorptions and current drive characteristics which are determined by using the cold plasma ray paths. Obviously, in the finite density regime the power deposition profiles are substantially altered. From the known scaling of the absorption coefficient (or the optical thickness) of the ordinary mode, for example, one would expect that the ray paths in Fig. 3(b) are characterized by significantly stronger wave absorption than those in Fig. 3(a). In fact, the maximum single pass wave absorption of the ray paths represented in these figures is of the same order of magnitude.

We now turn to the rays initialized to the extraordinary mode. In the cold plasma limit, propagating in the decreasing magnetic field the X-mode passes unhindered through the EC resonance and reaches the upper hybrid resonance. A typical ray path obtained within this approximation is shown in Fig. 4 (curve labelled ‘6’). In this figure the poloidal and toroidal views of rays initialized to the X-mode and launched from the point $\{a, 135^\circ, 0^\circ\}$ in the direction $\chi = 45^\circ$ are represented. When thermal effects in the resonance layer are taken into account the ray behaviour changes completely. Specifically, after reaching the EC resonance layer, the polarization of the wave changes. Depending on the launching angles $\psi$ and/or the parameter $\eta$, the ray then is refracted or reflected from the resonance layer. So, this part of the ray path shows the features of an O-mode. The effect described has important consequences for X-mode plasma heating and current drive.

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MHD CONSTRAINTS FOR
ADVANCED TOKAMAK OPERATION*

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Abstract

MHD CONSTRAINTS FOR ADVANCED TOKAMAK OPERATION.
Advanced tokamaks are characterized by having a ratio of bootstrap driven to total current of near
unity, or by having extreme shaping to maximize the plasma beta limits. The paper reports on the theo­
retical equilibrium and stability properties of such configurations.

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Foundation.
Detailed studies of the economic tradeoffs of fusion reactor designs[1] have shown us that economic viability depends on operating a tokamak reactor plasma in a regime where most of the current is produced by the bootstrap effect. We have carried out intensive theoretical studies of the stability properties of such plasmas, of the physics parameters needed to optimize them, and of the scaling relations which allow us to prototype certain advanced tokamak reactor features in smaller experiments such as PBX-M.

The bootstrap fraction, \( I_{bs}/I_p \), and the shape of the bootstrap current density, \( J_{bs} \), can depend sensitively on the shape of plasma profiles. We have expanded Hirshman's collisionless form[2] for \( J_{bs} \) in powers of \((a/R)^{1/2}\). We find that if the plasma is within the first regime stability limit (Troyon Limit), then \( I_{bs}/I_p \) can be maximized by peaking both the current and pressure profiles, but poor alignment prevents values of \( I_{bs}/I_p > 0.5 \) with moderately flat density profiles, or 0.7 for peaked profiles from being achieved. Second stability operation with concomitant high \( q_0 \) makes possible configurations with \( I_{bs}/I_p > 0.9 \).

We derive analytically a new scaling of the poloidal plasma beta, \( \beta_p \), with the bootstrap fraction, \( f_b \), for a steady-state plasma in the banana regime with a fixed seed current profile. This scaling, which assumes that \( n(r)/n(0) = [T(r)/T(0)]^{\lambda} \), is of the form \( \sqrt{\epsilon} \beta_p = A(\lambda, Z_i)[a_1 f_b - a_2 f_b^2] \), where \( \epsilon \) is the aspect ratio, \( Z_i \) is the ionic charge, \( A(\lambda, Z_i) \) is a monotonically decreasing function of \( \lambda \), and \( a_1 \) and \( a_2 \) are numerical coefficients of order unity. For plasma operation in the first stability regime, we have obtained scaling relations by combining the bootstrap formula discussed above, the Troyon limit, and the L-mode scalings.

We have performed extensive parameter studies of the stability of equilibria with high bootstrap fraction using the ideal MHD codes PEST, BALLOON and CAMINO. For aspect ratio \( A = 4.5 \), we have focused attention on a first stability regime with central safety factor \( q_0 = 1.3 \), \( I_{bs}/I_p = .70 \), Troyon factor \( C_t = 3.0 \), and \( \beta = 1.95\% \), which is stable to all ideal MHD modes with a wall at infinity, and a second stability regime with \( q_0 = 2.4 \), \( I_{bs}/I_p = .98 \), \( C_t = 4.15 \), and \( \beta = 1.61\% \), requiring a wall at \( b = 1.29 \) for kink mode stabilization and a second stability regime with \( q_0 = 4.0 \), \( I_{bs}/I_p = .98 \), \( C_t = 3.3 \), and \( \beta = 0.9\% \), which is stable to all ideal MHD modes without a conducting wall.

At very high \( \beta \), asymptotic scaling relations and increased physical insight come from investigating the stability and bootstrap current properties of analytic very high \( \beta \) large aspect ratio equilibria.[3] It is found that there are regimes at arbitrarily large \( \beta \) that are stable to high-\( n \) ballooning modes. These equilibria

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have greatly modified neoclassical transport properties primarily due to a large reduction in the fraction of trapped particles on a surface at very high $\beta$. The presence of a magnetic well leads to the enhanced stability of some trapped particle modes at high $\beta$.

We have carried out TSC/LSC simulations to study the degree to which we can prototype advanced plasma configurations in PBX-M and/or in the proposed SSAT experiment. LSC (Lower-hybrid Simulation Code) does multiple ray tracing in arbitrary numerically specified equilibria, and a 1D quasilinear solution for the parallel velocity distribution function in each flux shell. Then, using the response functions[4] and the dc electric field specified with the equilibrium, it calculates the current driven in each flux shell.

There is a correlation between the appearance of MHD instabilities in TFTR 'supershoot' plasmas and subsequent deterioration of their confinement. This phenomenon is variously referred to as 'beta-saturation' or 'beta-collapse' depending on the severity of the confinement degradation. A 3/2 instability is excited most frequently, though modes with $n = 1, 3$ and 4 are also observed. At the largest values of $C_t \sim 2.7$ some 90% of all plasmas are unstable. To elucidate the nature of these instabilities, a detailed study of the predictions of MHD theory based on equilibrium profiles obtained from TRANSP analysis of the experimental data has been performed.

Analysis of equilibria constructed from unstable shots identifies conditions that are near marginal to either low-$n$ ideal ballooning modes or to the infernal modes, depending on the peakiness of the current profile. In each case, the limiting $\beta$ follows a Troyon-like scaling $\beta_{\text{max}} = C_t(I/aB)$, but with $C_t$ sensitively dependent on the peakiness of the pressure profile; the more peaked pressure profiles are more severely limited. The infernal modes are excited only when the shear within the rational surface of interest is small, as illustrated in Fig. 1 and, therefore, $q_0$ is close to a rational value. On the other hand, the ideal ballooning mode does not rely on elevated values of $q_0$ and can be excited when $q_0$ is reduced to unity.

2. IDEAL-MHD BETA LIMITS AT HIGH ELONGATION

The construction of strongly shaped and elongated tokamaks is to a large extent motivated by the theoretical prediction that the beta limit is proportional to the plasma current [5]. Experimental results confirm the advantageous effects of elongation; $\beta \equiv 2\mu_0(p)/B_0^2 = 11\%$ was reached with an elongation $\kappa = 2.34$ at aspect ratio $A \approx 2.96$ in DIII-D [6]. The understanding of beta and other operational limits at very high elongation is, however, rather incomplete. It is not clear whether the beta limit can be further improved by increasing the elongation or by other types of shaping, and to what extent control of current and

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FIG. 1. Limiting $\beta_N$ for ideal, pressure driven modes as a function of the shear. When the shear is small $\beta$ is limited by the $3/2$ infernal mode. At large values of the shear the $\beta$ limit is determined by the $n = 2$ ballooning modes. The two curves correspond to pressure profiles with different peakedness, spanning the range of supershot data studied; the lower curve has the more peaked pressure profile.

pressure profiles is needed. Notably, at high elongation, the vertical instability ($n = 0$) gives an upper limit to the internal inductance, $l_i$, which may reduce the beta limit due to the kink ($n = 1$) [6] and ballooning ($n = \infty$) [7] modes.

We have studied the operational limits imposed by $n = 0, 1$ and $\infty$ stability for highly elongated tokamaks with shapes accessible to the TCV tokamak in Lausanne [8]. The plasma boundary is parametrized as

$$R/a = A + \cos(\theta + \delta \sin \theta + \lambda \sin 2\theta) , \quad Z/a = \kappa \sin \theta$$

(1)

Positive $\lambda$ broadens the "tips" and the case $A = 3.7, \delta = 0.5, \lambda = 0.2$ is referred to as a TCV dee. Vertical stability has been calculated, assuming a resistive wall shaped as the TCV vacuum vessel, and that the active feedback stabilization is effective if the growth time in the absence of feedback is longer than 0.5 ms. No wall stabilization is assumed for the kink. To calculate beta limits, we choose the current profiles and optimize the pressure profiles for ballooning in the first region of stability. Subsequently, the pressure limits for vertical and kink stability are computed for pressure profiles that are scaled versions of those at the ballooning limit.

When vertical stability is disregarded and only kink and ballooning modes are considered, the highest beta limits result for current profiles with a high internal inductance [6]. We have studied the dependence of the kink-ballooning limit on geometry, Eq. (1), using current profiles with high $l_i$ [8]. The beta
limit increases when the elongation increases from 2 to 2.5, if the boundary is sufficiently triangular. However, the limit decreases again when \( \kappa \) is further increased to 3. Concerning the current limit for the \( n = 1 \) kink, we find clearly different behavior at \( \kappa = 3 \) than for near circular cross sections; at \( \kappa = 3 \) the current limit is no longer correlated with integer \( q_\psi \) but is strongly dependent on the current profile. Typical values are \( q_{95} \geq 3.2 \) at \( \kappa = 3 \) while \( q_\psi = 2 \) can be reached at \( \kappa = 2.5 \) [8]. The current limit is almost identical for \( \kappa = 2.5 \) and \( \kappa = 3 \).

While kink stability is favored by high internal inductance, vertical stability is favored by low \( l_i \). With increasing elongation, the set of current profiles that are stable to both modes becomes more and more restricted. At \( \kappa = 2.5 \), the requirements of \( n = 1 \) and \( n = 0 \) stability are quite easily reconciled and we find completely stable equilibria with TCV dee shape and \( \beta \approx 7.5\% \). Increased triangularity is beneficial; we find \( \beta \approx 9.2\% \) for \( \delta = 0.8 \) and \( \lambda = 0 \).

At \( \kappa = 3 \), vertical stability requires sufficiently low \( l_i \), and this clearly reduces the beta limit for the kink mode. We have found the most advantageous results for \( \kappa = 3 \) by using current profiles with \( q_0 \approx 1 \) and “shoulders” near the edge of the plasma, around the \( q = 2 \) surface [8]. In TCV dee configurations with \( \kappa = 3 \), such profiles remain completely stable for beta up to about 4.5%.

Figure 2 shows the beta limits as \( \hat{\beta}_{\text{Troyon}} = 2\mu_0(p)/(B^2) \) vs. normalized current \( I_N = \mu_0I_p/aB_0 \) for ballooning (dotted), kink (solid) and vertical stability (dashed) curves for a TCV dee. Three sequences of current profiles have been tested. In order of decreasing \( l_i \) at fixed \( I_N \) these are: 1 – standard profile without shoulders, and two profiles with 2 – weak and 3 – large shoulders in the current density. Figure 3 shows \( l_i \) vs. \( I_N \) at the \( n = 1 \) beta limit for these equilibrium sequences.
As shown by Fig. 2, vertical stability is favored not only by low inductance but also, in a significant way, by high pressure (the $n = 0$ mode is stable above and to the right of the marginal curves in Fig. 2). The pressure stabilization appears to be due to an outward shift of the maximum in the current density which redistributes the eddy currents in the wall towards the outboard side. In fact, at high elongation, the window in $I_i$ for stability to both $n = 0$ and $n = 1$ modes is wider for moderate pressure than for zero pressure.

The beta limit for kink modes is highest for the equilibrium sequence with high inductance (curve 1). However, for these equilibria, vertical stability requires $\beta > 10\%$, clearly above the kink limit, so the high-$I_i$ sequence is always unstable. The sequences with medium (curve 2) and low (curve 3) $I_i$ give a maximum $\beta_{\text{troyon}}$ close to $4\%$. The vertical stability is better for the low inductance profile (curve 3). These equilibria are stable to all modes over a fairly large range of plasma current. The sequence of intermediate inductance (curve 2) gives a smaller operational window because of poorer vertical stability.

Figure 2 shows that at $\kappa = 3$, the ballooning limit is only weakly dependent on the inductance, as opposed to the near-circular case, where the ballooning limit increases with inductance [7]. We conclude from Fig. 2 that, at elongation 3, the beta limit is set entirely by the $n = 0$ and $n = 1$ modes, the ballooning limit being less restrictive.

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STUDIES OF CONVECTION IN THE EDGE TOKAMAK PLASMA

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Abstract

In the H mode, tokamak plasma confinement crucially depends on the edge localized mode (ELM) activity. This MHD phenomenon resembles damping of high density and high temperature plumes, leading to quasi-periodic oscillations of the particle and energy fluxes [1-3]. Recent data from ASDEX show that the edge turbulence is essentially two dimensional perpendicular to the magnetic field. Flute-like instability leads to the convection of flux bundles at the edge [1]. In a similar case the TFTR data [2] demonstrate the damping of electrons: a radially localized electron temperature perturbation appears at a radius of ≈0.2 m inside the plasma and moves radially outward at a velocity of ≈2 × 10^3 m/s. When this disturbance reaches the wall, the D_n signal begins to increase, which is assumed to be correlated with ELM activity.

To elucidate the nature of the ELM the following model is proposed. We consider the modified dissipative flute-like instability at the plasma edge, where the parallel dissipation (along the magnetic field lines) is taken into account for flux tubes with ends at the divertor plates or at the limiter. The lack of closed magnetic surfaces near the plasma edge permits the manifestation of instabilities which are forbidden in the bulk plasma. In this case, the flute-like instability in tokamaks has the characteristic features of flute-like instabilities in open systems: first, the magnetic field line curvature is not averaged as it usually is in tokamaks and it gives a contribution to instability of the first order in $\epsilon = r/R$ (r and R are the minor and major tokamak radii), rather than an $\epsilon^2$ effect as in the ballooning modes; secondly, the inclusion of longitudinal dissipation leads to a stabilizing effect. The result is determined by the competition of these two tendencies.

In order to describe the edge plasma convection, a set of equations is derived. The equilibrium magnetic field is assumed to be constant and in the z direction,
y corresponds to the poloidal angle and the equilibrium pressure gradient is in the x direction. Applying rot\(_e\) (1/B) to the plasma equation of motion:

\[
M_n \frac{d\vec{V}}{dt} + \nabla P = \frac{1}{c} (\vec{j} \times \vec{B}) + M_n \mu \Delta_\perp \vec{V}
\]

we obtain the equation for potential:

\[
\frac{cM_n}{B^2} \frac{d}{dt} \Delta_\perp \varphi + \left( \nabla P \times \nabla \frac{1}{B} \right) = \frac{1}{c} \left( \vec{b} \nabla \right) j_x + \frac{cM_n}{B^2} \mu \Delta_\perp \varphi
\]

where \( \vec{B} = B_0 \vec{e}_z - (\vec{e}_z \times \nabla A_z) \), \( B = B_0(1 - (x/R)) \), \( x = R - R_0 \), \( j_x = -(c/4\pi) \Delta_\perp A_z \), \( d/dt = (\partial/\partial t) + (\vec{v}_\perp \nabla) \), \( \vec{v}_\perp = (\vec{e}_z \times \nabla A_z) \) and \( \mu \) is the perpendicular viscosity. Substituting the parallel electric field \( E_p = -(1/c)(\partial A_x/\partial t) \) into the parallel component of the electron equation of motion we obtain:

\[
\frac{\partial A_x}{\partial t} + c(\vec{b} \nabla) \varphi = -c \frac{j_x}{\sigma} + \frac{c}{en} (\vec{b} \nabla) P_e
\]

The first term in this equation can be omitted in the case of electrostatic fluctuations. Equations (2, 3) must be completed with the continuity equation:

\[
\frac{\partial n}{\partial t} + (\vec{v}_\perp \nabla) n = \frac{1}{e} (\vec{b} \nabla) j_x + D \Delta_\perp n \]

and with the energy equation:

\[
\frac{\partial T}{\partial t} + (\vec{v}_\perp \nabla) T = \chi_1 \nabla^2 T + \chi_\perp \Delta_\perp T
\]

Here \( \nabla_\perp = (\vec{b} \nabla) \), instead of electron and ion temperatures the effective temperature \( T \) is introduced and the amount of \( \nabla (1/B) \) is assumed to be constant. In this paper we consider \( T = \text{const} \) or \( n = \text{const} \) separately. When \( T = \text{const} \), so that the density gradient drives the instability, the corresponding equations are:

\[
\frac{\partial \Delta \varphi}{\partial t} + (\vec{e}_z \times \nabla \varphi) \nabla \Delta \varphi + \frac{\partial n}{\partial y} + \sigma_1 \nabla^2 \left( \varphi - \frac{1}{\alpha} n \right) = \Delta \mu \Delta \varphi
\]

\[
\frac{\partial n}{\partial t} + (\vec{e}_z \times \nabla \varphi) \nabla n + \frac{\partial \varphi}{\partial y} + \sigma_2 \nabla^2 \left( \varphi - \frac{1}{\alpha} n \right) = \nabla D \nabla n
\]
Here \( \sigma_1 = (\tau_1/\tau_A) \beta^{-1/2}(\alpha R)^{1/2}/L_4 \), \( \sigma_2 = (\tau_1/\tau_A)c/\omega_p L_4 \), \( \alpha = \epsilon^{1/2}a/\rho \), \( \tau_4 = 4\pi \sigma a^2/c^2 \), \( \tau_A = a/C_A \), \( \varphi = \alpha \phi / T \), \( n = n/n_0 \) and the \( \alpha \) terms introduce the drift effects. Averaging Eqs (6) over the field lines we obtain:

\[
\begin{align*}
\frac{\partial \Delta \varphi}{\partial t} + (\bar{\varepsilon}_x \times \nabla \varphi) \nabla \Delta \varphi + \frac{\partial n}{\partial y} + \frac{1}{\tau_1} \left( \varphi - \frac{1}{\alpha} \right) n = \Delta \mu \Delta \varphi \\
\frac{\partial \tilde{\varphi}}{\partial t} + (\bar{\varepsilon}_x \times \nabla \varphi) \nabla \tilde{\varphi} + \frac{\partial \tilde{n}}{\partial y} + \frac{1}{\tau_2} \left( \varphi - \frac{1}{\alpha} \right) \tilde{n} = \nabla \varphi \frac{\partial \tilde{\varphi}}{\partial y}
\end{align*}
\]

(7)

The corresponding equations for the temperature gradient driven instability \((n = \text{const}, \alpha \gg 1)\) derived from Eqs (2, 3, 5) and averaged along the field lines take the form:

\[
\begin{align*}
\frac{\partial \Delta T}{\partial t} + (\bar{\varepsilon}_x \times \nabla \varphi) \nabla \Delta \varphi = & -g_B \frac{\partial \tilde{T}}{\partial y} + \sigma (\varphi - \tilde{\varphi}) + \mu \Delta_\perp \varphi \\
\frac{\partial \tilde{T}}{\partial t} + (\bar{\varepsilon}_x \times \nabla \varphi) \nabla \tilde{T} = & g_T \frac{\partial \varphi}{\partial y} - \frac{\tilde{T} - \tilde{T}}{\tau_1} + \chi \Delta_\perp \tilde{T}
\end{align*}
\]

(8)

where \( T = \langle T \rangle + \tilde{T}, \langle T \rangle = 1 - x/2\pi, g_T = \partial \langle T \rangle / \partial x = -1/2\pi, g_B = \rho/R, \) angle brackets indicate averaging over one full period, bars indicate averaging in the \( y \) co-ordinate and tildes indicate variables which are periodic in space. The following dimensionless variables were introduced in (8): \( \varphi = e \varphi / T_0, T = T / T_0, t = \omega t, \omega = e B_0 / mc, x, y = x, y / \rho, \rho^2 = \omega_c^2 T_0 / M, \mu_X = \mu / \mu_X, \sigma = \sigma_{k^2} B / n c e, \) and \( \tau_1 = \rho^2 \omega_c / \chi k_1^2 \).

To compare Eqs (8) with the well known Boussinesq equations for thermal convection it is useful to put them in the form:

\[
\begin{align*}
\frac{\partial \Delta \varphi}{\partial t} + (\bar{\varepsilon}_x \times \nabla \varphi) \nabla \Delta \varphi = & -\text{Ra} \frac{\partial \tilde{T}}{\partial y} + \sigma (\varphi - \tilde{\varphi}) + \text{Pr} \Delta_\perp \varphi \\
\frac{\partial \tilde{T}}{\partial t} + (\bar{\varepsilon}_x \times \nabla \varphi) \nabla \tilde{T} = & -\frac{\partial \varphi}{\partial y} - \frac{\tilde{T} - \tilde{T}}{\tau_1} + \Delta_\perp \tilde{T}
\end{align*}
\]

(9)

where \( \text{Ra} = g_B g_T H^4 / \mu_X \) is the Rayleigh number, \( \text{Pr} = \mu / \chi \) is the Prandtl number, and the following scale transformations have been used: \( x \rightarrow x / H, t \rightarrow t \chi / H^2, \varphi \rightarrow \varphi / \chi, \sigma \rightarrow \sigma H^4 / \chi, \tau_1 \rightarrow \tau_1 \chi / H^2 \). Obviously, Eqs (9) coincide with the Boussinesq equations when \( \sigma = 0, \tau_1 \rightarrow \infty \). However, these terms with parallel conductivity and parallel thermal conductivity are especially important in the scrape-off layer plasma, where magnetic field lines are disconnected. The linear analysis of the
system shows that the intrinsic spatial scale length of the fluctuations appears in the region where parallel dissipation takes place. This spatial scale is determined by the scale of the most unstable mode. The main contribution to transport is made by hot plumes which come to the plasma surface.

Introducing the characteristic scales of the increment $\gamma_0 = \omega_g$, the perpendicular wave vector $k_0^2 = C_L^2/L_1^2 D_M \omega_g$ and the poloidal wave vector $k_1^2 = 1/L_1^2$, where $L_1$ is the distance between the upper and lower x points along the field line,

$$\omega_g^2 = \frac{1}{RM} \frac{d T_0}{dx} = \frac{C_L^2}{a_T R}$$

$$\frac{1}{a_T} = \frac{1}{T_0} \frac{d T_0}{dx}$$

$$C_L^2 = \frac{B^2}{4 \pi M n}$$

Then in the dimensionless variables $\gamma \rightarrow \gamma/\gamma_0$, $k_+ \rightarrow k_+ / k_0$, $\mu \rightarrow \mu k_0^2 / \gamma_0$, $\chi_+ \rightarrow \chi_+ k_0^2 / \gamma_0$, $\tau_{1}^1 \rightarrow \chi_1 k_0^2 / \gamma_0 = \chi_1 / L_1^2 \omega_g$ the dispersion relation for (8) takes the form:

$$(\gamma + k_+^2 \mu + 1/k_0^2)(\gamma + \tau_{1}^1 + k_+^2 \chi) = k_+^2 k_0^2$$  \hspace{1cm} \text{(10)}$$

The term $1/k_0^2$ is due to the stabilizing influence of open ends on the ideal instability. The main parameters here are $k_0$ and $\chi_1$, because they are well estimated.

For the typical ASDEX parameters $B = 3$ T, $T = 0.1$ keV, $R = 1$ m, $q = 3$, $n = 10^{13}$ cm$^{-3}$, $a_T = |T(dT/dr)|^{-1} = 0.01$ m, $A = 1$, $Z_{\text{eff}} = 3$, $L_1 = \pi q R$, $k_0$ and $\chi_1$ can be estimated as:

$$k_0^2 = C_L^2/L_1^2 D_M \omega_g \approx 2.85 \times 10^{22} \frac{B^2 T_e a_T^{1/2}}{n R^{3/2} q^2 Z_{\text{eff}} A^{1/2}} \text{ cm}^{-2} \approx 1.4 \times 10^2 \text{ cm}^{-2}$$  \hspace{1cm} \text{(11)}$$

(The corresponding wavelength or typical vortex size in a turbulent flow is $\lambda_+ = 2\pi/k_0 \approx 0.45$ cm.)

$$\tau_{1}^1 = \chi_1 / L_1^2 \omega_g = \frac{1}{\pi q} \left( \frac{a_T}{R} \right)^{1/2} \approx 8 \times 10^{-3} \text{ s}^{-1}$$  \hspace{1cm} \text{(12)}$$

where an upper bound for $\chi_1$, $\chi_1 = L_1 C_s$, is used, i.e. the plasma is assumed to flow along the field lines with the sound speed. A Bohm-like estimate for the viscosity and thermal conductivity $\mu \approx \chi_+ \approx \eta C_s \rho_s$ gives:
FIG. 1. Instability threshold versus wavenumber $k_y$ for $k_x = 1$, $\tau_1^{-1} = 5 \times 10^3$ and $Pr = 5$ (a), 1 (b), 0.2 (c) and 0.05 (d); broken line: $k_x = 4$.

$$\mu, \chi_\perp = \eta C_\perp x_i k_y^2/\gamma_0 \approx 70 \eta$$

where $\eta \ll 1$. Figure 1 shows that the mode $(1, 1)$ becomes unstable if $70 \eta \leq 0.25$. This figure represents the dependence of the stability threshold of $\chi_\perp$ on $k_y$. The threshold $\chi_{\perp cr}$ is determined by Eq. (11) with $\gamma = 0$, $\mu = Pr x_i$:

$$\chi_{\perp cr} = \{-1 - k_y^2 Pr r_1 - [(1 - k_y^2 Pr r_1)^2 + 4 Pr k_y^2 x_i^2]^{1/2} \} (2 Pr k_y^2)^{-1}$$

In the presence of the poloidal rotation $V(x) = \partial \phi/\partial x$ ($V' \neq 0$) one must consider an eigenvalue problem in $x$. In a dissipationless plasma the potential equation takes the form:

$$\left(\frac{d^2}{dx^2} - k_y^2\right) \phi_k + \frac{k_y V'' \phi_k}{\omega - k_y V} = \frac{g_0 g_\tau k_y^2 \phi_k}{\omega - k_y V}$$

When hard walls are taken as a boundary condition, the shear flow stabilizes linearly convective modes for $V'^2 > g_0 g_\tau/2$ [4]. On the other hand, as was mentioned in Ref. [5], the presence of an infinitesimal shear flow creates a non-zero Reynolds stress and amplifies itself. But when the shear becomes larger than critical, the convective modes appear to be suppressed, which in turn leads to the decrease of the
poloidal velocity. Then the process is repeated. Thus the quasi-periodic oscillations of the poloidal velocity and the intensity of turbulent fluctuations are the inherent features of our model, which is completely confirmed by the results of numerical simulations.

For solving Eqs (8) numerically we used the de-aliased spectral method [6]. The variables \( \varphi \) and \( T \) are expanded in the Fourier series:

\[
\varphi(x, t), T(x, t) = \sum_{(k_1, k_2)} \varphi(k_1, k_2, t), T(k_1, k_2, t) \exp(2\pi i k_1 j/N) \tag{16}
\]

where \( N \) is an integer chosen as a power of 2 and the fast Fourier transform is applied. The stream function \( \varphi \) and the temperature \( T \) fields are represented by the values at the \( N^2 \) points, \( \mathbf{k} = 2\pi j/N \) with \( j = (j_1, j_2) \) as well as the wave vector \( \mathbf{k} = (k_1, k_2) \), which are two-dimensional vectors with integer components. The summation in (21) is made over the wavenumbers which satisfy the condition:

\[
k_1^2 + k_2^2 < (N/3)^2 \tag{17}
\]

in order to remove the aliasing errors that would appear when evaluating the non-linear terms.

In physical space \( \varphi \) and \( T \) fields of a two dimensional flow satisfy rigid free-slip boundary conditions on the planes \( x = 0, \pi \) and periodicity \( 2\pi \) in \( y \). These boundary conditions are equivalent to symmetry conditions of the form:

\[
T(0, ky; t) = 0, \quad \text{Re} \ T(k_x, 0; t) = 0, \quad T(-k_x, ky; t) = T(k_x, ky; t) \tag{18}
\]

(\( \varphi \) has the same symmetry as \( T \)) that are preserved in every case by the equations of motion (8), provided they are satisfied initially.

The results of numerical simulations reveal a new effect of sheared flow generation by convection cells due to Rayleigh-Taylor instability. As far as we know, this effect has not previously been observed in the Boussinesq equations, because Rayleigh-Taylor convection has usually been studied experimentally and theoretically for models bounded in the \( y \) direction. For the tokamak edge plasma the periodicity constraint in the \( y \) direction and free-slip rigid planes at \( x = 0, \pi \) are more adequate boundary conditions. Thus an effect of spontaneous generation of the stable sheared flow (with the dimensionless amplitude of the order of 1) was observed in the numerical simulations of Eqs (8). When the system is far above the stability threshold (\( \text{Ra} \gg 1 \)), the velocity profile is close to the linear one and the shear value \( V' \) experiences quasi-periodic oscillations in time corresponding to the spikes on the heat (or particle) flux, which can be interpreted as ELMs [2, 3]. For ordinary Rayleigh-Taylor convection the flow generation emerges only in transition to the stationary state or in strong turbulent regimes. The presence of parallel dissipation in the \( \varphi \) equation (9) seems to be crucial for flow generation. Figure 2 illustrates
the influence of the parameter \( \sigma \) on the amplitude of the heat flux and the sheared velocity. The period of the heat flux oscillations is weakly dependent on \( \sigma \) and is mainly determined by the viscosity; it increases with the decrease of \( \mu \) (Fig. 3). The direction of the flow is arbitrary in our model and depends on the initial conditions, although it should be noted that the seed sheared flow is not necessary.

In spite of the similarity of the magnetized plasma and the two dimensional fluid, there is an important difference. Namely, the direction of the fluid flow is casual, but in a plasma it is determined by the edge particle transport and has a definite direction. Thus, at the edge of the tokamak a negative electric field is more probable because of the fast loss of electrons along the field lines in the scrape-off layer. So our model based on the edge plasma convection naturally and self-consistently explains the origin of the electric field in the separatrix region.

Now we propose a qualitative explanation of the numerical results. The solution of (7) is determined by the structure of vorticity and particle fluxes: \( P = \langle v_x \Omega \rangle \), \( \Omega = \Delta \varphi \) and \( \Gamma = \langle v_x n \rangle \). A qualitative understanding of \( P \) and \( \Gamma \) properties can be obtained using quasi-linear theory. For the variables averaged over \( y \) we obtain:
FIG. 3. Heat flux \( q \) over time for viscosity \( Pr = 1 \) (a) and 2 (b); \( \sigma_1 = 20; \sigma_2 = 0; Ra = 1024. \)

\[
\frac{\partial}{\partial t} \langle \Omega \rangle + \frac{\partial}{\partial x} P = 0
\]

\[
\frac{\partial}{\partial t} \langle n \rangle + \frac{\partial}{\partial x} \Gamma = 0
\]

where

\[
P = \text{Re} \sum_{(k_x, k_y)} i k_y \frac{\Delta_\alpha}{\Delta} \varphi_{\xi} \varphi_{-\xi}
\]

\[
\Gamma = \text{Re} \sum_{(k_x, k_y)} i k_y \frac{\Delta_\alpha}{\Delta} \varphi_{\xi} \varphi_{-\xi}
\]

\[
\Delta_\alpha = \begin{pmatrix}
  k_y \frac{\partial \langle \Omega \rangle}{\partial x} & k_y - \frac{\mu}{\tau_1 \alpha} \\
  -k_y \frac{\partial \langle n \rangle}{\partial x} & \dot{\omega} + i \nu_n
\end{pmatrix}
\]
\[ \Delta_n = \begin{pmatrix} -(\omega + iv_\alpha) & k_y \frac{\partial \langle \Omega \rangle}{\partial x} \\ \frac{i}{\tau_2 k_\perp^2} & -k_y \frac{\partial \langle n \rangle}{\partial x} \end{pmatrix} \]

\[ \Delta = \begin{pmatrix} -(\omega + iv_\alpha) & k_y - \frac{\mu}{\tau_1 \alpha} \\ \frac{i}{\tau_3 k_\perp^2} & \omega + iv_n \end{pmatrix} \]

\[ \tilde{\omega} = \omega - k_y V(x), \quad \nu_\alpha = \mu k_\perp^2 + \frac{1}{\tau_1 k_\perp^2}, \quad \nu_n = D k_\perp^2 \]

To simplify this model we consider the fluid limit \( \tau_{1,2} \to \infty \), when (25) reduces to:

\[ P = -(\mu + \mu_n) \frac{\partial \langle \Omega \rangle}{\partial x} + b \frac{\partial \langle n \rangle}{\partial x} \]

\[ \Gamma = -(D + D_n) \frac{\partial \langle n \rangle}{\partial x} + 0 \]

where

\[ \mu_n = \sum_{(\alpha, k_y) \omega} \frac{k_\perp^2 \varphi \varphi_{-} \varphi_{+} \nu_\alpha}{\tilde{\omega}^2 + \nu_\alpha^2} \]

\[ D_n = \sum_{(\alpha, k_y) \omega} \frac{k_\perp^2 \varphi \varphi_{-} \varphi_{+} \nu_n}{\tilde{\omega}^2 + \nu_n^2} \]

\[ b = \sum_{(\alpha, k_y) \omega} \frac{k_y \varphi \varphi_{-} \varphi_{+} (\nu_\alpha + \nu_n) (\omega - k_y V(x))}{[(\omega - k_y V(x))^2 + \nu_\alpha^2] [(\omega - k_y V(x))^2 + \nu_n^2]} \]

All variables here are dimensionless. Note that the cross-gradient term is zero in density flux, but in vorticity flux the cross-term vanishes \( b = 0 \) only in the case of shearless flow \( V = \text{const} \) because of the spectrum \( \varphi \varphi_{-} \varphi_{+} \) symmetry in the rest frame. Obviously, for the large scale modes the transition into the moving frame of reference is impossible in the case of shear flow \( V(x) \approx V' x \). Because the large scale modes play the main role in energy transfer dynamics (being the most unstable, they
gain energy from the non-equilibrium density distribution), a more adequate description is to consider the amplitude of the most unstable mode as the control parameter. Owing to non-linear mixing processes there are two means of energy dissipation: by means of cascade to the stable small scale modes and by means of shear flow generation. Thus two parameters arise: the averaged spectral intensity \( I \) and the characteristic velocity of the shear flow \( U \). To provide an energy source for the instability, the average density gradient \( N \) must be incorporated into the model. The following set of model equations can be written:

\[
\frac{\partial U}{\partial t} = - \left( \frac{1}{\tau_{OU}} + \frac{1}{\tau_{IU}} \right) U + \beta U I N
\]

\[
\frac{\partial N}{\partial t} = - \frac{N}{\tau_{ON}} - \frac{I}{\tau_{IN}} (N - N_0)
\]  \hspace{1cm} (22)

\[
\frac{\partial I}{\partial t} = \left( \frac{\gamma_0}{1 + U^2} - \frac{1}{\tau_i} - DI \right) I
\]

Very rough substitutions have been used here, for example:

\[
\frac{\partial}{\partial x} \left( \mu + \mu_u \right) \frac{\partial \langle \Omega \rangle}{\partial x} - \left( \frac{1}{\tau_{OU}} + \frac{1}{\tau_{IU}} \right) U
\]

FIG. 4. Velocity \( U \) (solid line) and averaged spectral intensity \( I \) (broken line) over time.
because \( \langle \Omega \rangle = d^2 U/dx^2 \to U/a^2 \). This term represents the viscous damping of the shear flow due to ordinary and turbulent viscosity. The last term in the velocity equation is responsible for the flow generation. In the spectrum equation the first term describes both the linear instability and the stabilization effect due to the shear flow, the second term gives small linear damping and the last term corresponds to the nonlinear stabilization. Because Eqs (22) have been developed from the dimensionless Eqs (7), it is natural to assume that \( \tau_{IU}, \tau_{IN}, \beta \) and \( D \) are of the order of 1, and \( \tau_{OU}, \tau_{ON} \) and \( \tau_i \) are much greater than 1. This model demonstrates a variety of phenomena (regime with saturation, periodic oscillations of parameters, doubling of period, etc.) which are in surprisingly good agreement with the behaviour of the full set of equations (8) (Fig. 4). So we believe we have gained an insight into the phenomenon of shear flow generation on the plasma periphery.

Thus a model of spontaneous generation of the shear flow and the treatment of ELMs as the quasi-periodic pattern of particle or energy fluxes have been proposed in this paper. The turbulence due to the modified dissipative flute-like instability has been studied analytically and numerically.

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NEOCLASSICAL CURRENT AND RELATED MHD STABILITY, GAP MODES AND RADIAL ELECTRIC FIELD EFFECTS IN HELIOTRON AND TORSATRON PLASMAS

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Abstract

NEOCLASSICAL CURRENT AND RELATED MHD STABILITY, GAP MODES AND RADIAL ELECTRIC FIELD EFFECTS IN HELIOTRON AND TORSATRON PLASMAS.

By developing the neoclassical theory for parallel flow, it is found that if $v^* \neq v^*$ a parallel current directly generated by the radial electric field, which does not exist in axisymmetric systems, exists in non-axisymmetric systems. This newly found current has the potential of reducing the bootstrap current under suitable conditions even in the opposite direction in the Heliotron/Torsatron. The differences between the rotations of bulk ions and impurities are discussed in the case where the impurities are in the Pfirsch–Schlüter collisionality regime. In connection with the bootstrap current the effects of the net toroidal current on the ideal interchange instability are numerically studied by taking the Large Helical Device (LHD) as an example. The direction of the current is crucial to the Mercier criterion. Helicity induced shear Alfven eigenmodes (HAE) attributed to the helicity of helical coils are considered for the first time in the Heliotron/Torsatron and compared with toroidicity induced shear Alfven eigenmodes (TAE). Spectral gaps and eigenmodes significantly depend on the finite-β effects. The radial electric field producing a poloidal shear flow also plays a role in anomalous edge transport in the Heliotron/Torsatron. The electrostatic resistive interchange mode which is considered to be a case of anomalous transport can be stabilized linearly by the poloidal shear flow. However, its effect on the turbulent state practically disappears by non-linear simulation for resistive drift wave and interchange turbulence. It is pointed out that the poloidal shear flow destabilizes the electromagnetic ideal interchange mode.
1. NEOCLASSICAL CURRENT AND ROTATION

Neoclassical theories for parallel flow are extended to a multispecies plasma in general toroidal systems, in which each species lies in a regime of different collisionality [1]. As a result, the bootstrap current is for a simple plasma consisting of electrons and protons given by

\[
\langle B J \parallel \rangle = L_{11} \langle G_{BS} \rangle_e \left( \frac{dP_e}{d\psi} + e n_e E_\psi \right) + L_{11} \langle G_{BS} \rangle_i \left( \frac{dP_i}{d\psi} - e n_e E_\psi \right) 
- L_{12} \langle G_{BS} \rangle_e n_e \frac{dT_e}{d\psi} + L_{11} L_{34} \langle G_{BS} \rangle_i n_e \frac{dT_i}{d\psi}
\] (A.1)

where \(L_{ij}, \langle G_{BS} \rangle_{ei}, \) and \(E_\psi\) are the transport coefficients, the geometric factor [2] and the radial electric field, respectively. The direction of flow damping due to the parallel viscosities is, in terms of the geometric factor, given by

\[
\nabla \theta^*_e \equiv \nabla \left[ (I + \langle G_{BS} \rangle_e) \theta + (J - \varepsilon \langle G_{BS} \rangle_e) \zeta \right]
\] (A.2)

where \(\theta\) and \(\zeta\) are the poloidal and toroidal angles in Boozer co-ordinates, respectively, and \(I, J, \) and \(\varepsilon\) are the toroidal current inside the flux surface, the poloidal current outside the flux surface and the rotational transform, respectively. In axisymmetric systems, as is seen from Eq.(A.2), the flow dampens only in the poloidal direction, regardless of the collisionality of each particle species, i.e. \(\langle G_{BS} \rangle_e = \langle G_{BS} \rangle_i = J/\varepsilon\). Therefore, the current proportional to \(E_\psi\) vanishes in Eq.(A.1), which is a direct result of symmetry, momentum conservation of friction forces and charge neutrality. On the other hand, in non-axisymmetric systems, lack of symmetry allows the flow in any direction to damp, and the damping direction becomes to depend on the collisionality, which makes a current directly generated by \(E_\psi\) exist if \(\nu_e^* \neq \nu_i^*\) (\(\langle G_{BS} \rangle_e \neq \langle G_{BS} \rangle_i\)). This newly found current can be comparable with the conventional pressure driven neoclassical current since \(e \phi \approx T \left( E_\psi = -d\phi/d\psi \right)\). In the region where \(\mid \langle G_{BS} \rangle_{ei} \mid \) increases as \(\nu_e^* \) decreases, if \(\nu_e^* < \nu_i^*\), then \(\mid \langle G_{BS} \rangle_e \mid > \mid \langle G_{BS} \rangle_i \mid \) and \(E_\psi > 0\) would be realized according to the neoclassical theory. In such a situation the first term with \(\langle G_{BS} \rangle_e\) in Eq.(A.1) dominates and the current proportional to \(E_\psi\) tends to cancel the conventional pressure driven current. In the opposite case of \(\nu_e^* > \nu_i^*\) where \(\mid \langle G_{BS} \rangle_e \mid < \mid \langle G_{BS} \rangle_i \mid \) and \(E_\psi < 0\), the resultant current would also be reduced. If \(\mid E_\psi \mid\) is enough large we can expect even an inverted bootstrap current in the Heliotron/Torsatron.
The extended neoclassical theory is applied to the poloidal and toroidal rotations in a plasma consisting of electrons, ions and impurity ions in the Pfirsch-Schlüter regime [3]. It is found that the differences between bulk ions and impurities come from the different diamagnetic flows and the ion temperature gradient in the $1/\nu$ regime, but depend strongly on the field structure in the Heliotron/Torsatron, in contrast to the tokamak case [4]. For the experimental parameters of CHS (Compact Helical System), the differences are small and of the order of bulk ion diamagnetic flow.

2. IDEAL MHD STABILITY WITH NET TOROIDAL CURRENT

The currentless condition is often violated by a net toroidal current such as the bootstrap current. We first consider the effects of the net toroidal current on the Mercier criterion systematically, using the three dimensional equilibrium code called VMEC. As an example, we take the standard configuration of the Large Helical Device (LHD), where $R = 3.9$ m, $B = 3$ T, $\gamma_c = 1.25$ ($\gamma_c$ is the pitch parameter of helical coils), $\alpha = 0.1$ ($\alpha$ is the pitch modulation parameter), $\Delta_{axis} = -15$ cm ($\Delta_{axis}$ is the magnetic axis shift in the vacuum field), and the toroidally averaged magnetic surfaces are nearly circular. We find the second stability for the currentless equilibrium with a pressure profile of $P = P_0(1 - \Phi_T)^2$, where $\Phi_T$ is the toroidal flux. The unstable region is so small that the growth rates of low-$n$ interchange modes are expected to be very small. The additive current, which increases the central rotational transform $\epsilon^0_\nu$ in the vacuum field and makes the magnetic well shallow, is unfavourable to MHD stability. On the other hand, the subtractive current decreasing $\epsilon_\nu^0$ allows the large Shafranov shift to extend the well region and increases the shear near the edge as $\beta$ increases. Thus subtractive current improves the MHD stability against interchange modes. For the same pressure profile as in the above currentless case, the additive current of 50 kA (the current density is $J = J_0(1 - \Phi_T)^2$) expands the unstable Mercier region, but the second stability persists for $< \beta > \gtrsim 3\%$. The subtractive current of 50 kA can stabilize the plasma completely against the Mercier criterion; hence, the configuration is stable to ideal low-$n$ interchange modes.

3. GAP MODES (SHEAR ALFVÉN EIGENMODES)

In the Heliotron/Torsatron and other helical systems the helicity of helical coils, in addition to the toroidicity, can cause the poloidal and toroidal mode couplings result in a breakup of the shear Alfvén continuous spectra into small bands of continuous spectra. We first consider the high $n$ helicity induced shear Alfvén eigenmodes (HAE) and the high $n$ TAE modes in the Heliotron/Torsatron, where $n$ is the toroidal mode
number. The high-\( n \) HAE modes are identified in a low \( \beta \) straight helical system, and it is found that the polarity of the helical coil influences the structure of spectral gaps through the shape of the flux surface [5]. The existence of high \( n \) HAE and TAE modes in the finite \( \beta \) toroidal Heliotron/Torsatron is confirmed numerically. The continuous spectral gaps of high \( n \) HAE and TAE modes are essentially determined by \( |\nabla \psi| \) and the lowest ones appear, respectively, around \( \Omega^2_{HAE} = [(L - M/\ell)/2]^2 \) and \( \Omega^2_{TAE} = [1/2]^2 \), where \( \psi \) is the flux function, \( \Omega = \omega/\omega_A \), \( \omega_A \) is the poloidal Alfvén transit frequency, and \( L \) and \( M \) are the polarity of helical coils and the toroidal pitch number of the helical coils, respectively. The gap structure and the eigenvalues of the high \( n \) TAE modes change widely with the flux surface when a shearless region appears as a result of finite \( \beta \) effects. In the shearless region, a great number of discrete eigenmodes exist with local or global structures along the magnetic field line. Although the localized high \( n \) HAE modes appear to be independent of the \( \beta \) value, the localized high \( n \) TAE modes appear only at finite \( \beta \). The \( \theta_k \) dependences of \( \Omega_{HAE} \) and \( \Omega_{TAE} \) are stronger than the \( \alpha \) dependence, where \( \theta_k \) and \( \alpha \) are the radial wavenumber and the label of the magnetic field line, respectively. In the Heliotron/Torsatron with \( |L - M/\ell| > 1 \), high \( n \) HAE modes with high frequencies may be irrelevant to the alpha particle losses. The high \( n \) TAE modes may, however, be relevant to them. We also consider the low \( n \) TAE and HAE modes, placing emphasis on poloidal mode coupling. As in the case of the high \( n \) TAE modes, the poloidal mode coupling gives rise to the spectral gap in the shear Alfvén continuum in the Heliotron/Torsatron.
1. INTRODUCTION

Recently, it has been recognized from the study of \( L/H \) transition phenomena that the radial electric field plays an important role in tokamak confinement \[6,7\]. We study the radial electric field effects on heliotron and torsatron plasmas from several points of view, i.e. trapped particle confinement \[8\], neoclassical ripple transport \[9\], ideal and resistive MHD stabilities, and resistive drift wave and interchange turbulence \[10\]. The latter two subjects are discussed in this paper.

2. POLOIDAL SHEAR FLOW EFFECTS ON LINEAR MHD STABILITY

Since the most probable candidate for anomalous transport in Heliotron/Torsatrons is resistive interchange turbulence, we have studied effects of poloidal shear flow, driven by the radial electric field, on the electrostatic resistive interchange mode in the slab model \[11\]. From linear stability analysis \( (i) \ k_y v_0 > \gamma_L L_E / \Delta \) and \( (ii) \ \Delta < \Delta_E \) are required for suppressing this instability, where the flow velocity profile is assumed to be \( v_E = v_0 \tanh(x / L_E) \), \( k_y \) is the wavenumber in the poloidal direction, \( \gamma_L \) is the growth rate at \( v_0 = 0 \) and \( \Delta \) is the radial mode width at \( v_0 = 0 \). The origin \( x = 0 \) corresponds to a mode resonant surface. According to these conditions, a stabilizing effect is expected in Heliotron E for \( v_0 > 3 \times 10^5 \text{ cm/s} \) and \( L_E \approx 0.5 \text{ cm} \). Experimentally, \( v_0 \gtrsim 5 \times 10^5 \text{ cm/s} \) was already observed; however, the shear flow width was not clear in the experiment. When \( v_0 \) is sufficiently large, the usual Kelvin-Helmholtz (K-H) instability appears.

Next, we study the poloidal shear flow effect on the ideal interchange mode or the Suydam mode. In the slab model, the stability criterion becomes \( D_v = -k_y^2 P'\Omega' / ((k_\parallel')^2 - (k_y v'_E)^2) < 1/4 \), where the primes denote derivatives with respect to the radius, \( P \) is the equilibrium pressure, \( \Omega' \) is related to the average curvature of Heliotron and Torsatron, and \( k_\parallel \) is the parallel wavenumber \[12\]. Here, \( v_E \) is normalized to the Alfvén velocity. Usually, \( \Omega' \) is positive and destabilizing. If \( v'_E = 0 \), this criterion reduces to the Suydam criterion. The linear growth rates of the \((m, n) = (1, 1)\) mode in the Heliotron E model configuration \[13\] are shown in Fig. 1 for various poloidal shear flow cases, where \( \omega_{E0} = k_y v_0 \) is normalized to the Alfvén transit time and \( L_E \) is normalized to the radius. The pressure profile is assumed to be given by \( P(r) \propto (1 - r^2)^2 \). For \( v'_E \neq 0 \), the shear
flow has a destabilizing effect on the beta limit. Thus, the poloidal shear flow is unfavourable to obtaining a high beta stable plasma. For $\omega_{E0} = 0.2$ and $L_E = 0.1$, clearly the K-H instability appears, and the growth rate becomes finite even at $\beta_0 = 0$. When $\beta < \beta_c^\alpha = 1.62\%$, the instability destabilized by the poloidal shear flow has characteristics similar to those of the K-H instability, where $\beta_c^\alpha$ is determined by $D_v = 1/4$ for $v'_E = 0$.

3. RADIAL ELECTRIC FIELD EFFECT ON RESISTIVE DRIFT WAVE AND INTERCHANGE TURBULENCE

When $\Omega' > 0$ or the average curvature is unfavourable, both the resistive drift wave and the resistive interchange modes become unstable. We have studied the non-linear evolution of these instabilities in a cylindrical plasma with magnetic shear by using two field model equations for the density fluctuation and the potential fluctuation [14]. We showed that the radial electric field is self-generated by the turbulent fluctuations and this electric field affects the particle transport.

When the adiabatic parameter $(\Omega_\ast/\nu_\ast)(\rho_\ast^2/R^2)/(\kappa\rho_\ast)$ is small, a trend towards dual cascades, normal cascades of the density fluctuations and inverse cascades of the potential fluctuations is seen in the wavenumber...
FIG. 2. Time evolution of the energy of electric field fluctuation, $E_k$, and the energy of density fluctuation, $E_n$, for $E_r(r) = -4r^2$ and $E_r(r) = 4r^2$.

spectra, producing a large particle flux proportional to $\nu_e^{1/3}$. Here, $R$ is the major radius, $\rho_s$ the ion Larmor radius at the electron temperature, $\Omega_e$ the electron cyclotron frequency, $\nu_e$ the electron ion collision frequency and $\kappa$ an inverse scale length of the background density gradient. Parallel wavenumbers are represented by $1/R$. In this case, a deviation from the Boltzmann relation is seen clearly [10]. For large $(\Omega_e/\nu_e)(\rho_s^2/R^2)/(\kappa \rho_s)$, the electrons become adiabatic, with a significant reduced particle flux proportional to $\nu_e$.

We have also studied the effect on the turbulence of the externally radial electric field imposed on the ambipolar electric field. We assume $E_r(r) = \pm 4r^2$ and $\nu_e/\Omega_e = 2.1 \times 10^{-4}$ here. The background density is taken to be Gaussian and fixed in the non-linear calculation. Figure 2 shows the time evolution of the energy of the electric field fluctuation, $E_k = \int |\nabla \phi|^2 dv$, and the energy of the density fluctuation, $E_n = \int |\vec{n}|^2 dv$, for cases with positive and negative radial electric fields. The case with
$E_r = 0$ lies between the two lines. In the presence of this type of electric field with negative (positive) polarity, $E_r < 0$ ($E_r > 0$), the poloidal velocity shear in the $\mathbf{E} \times \mathbf{B}$ drift motion suppresses (enhances) the fluctuation level in the growth phase; however, these effects practically disappear in the saturated state. Here, we note that the polarity of the electric field affects the result since our model equation includes the electron drift frequency $\omega_{ce}$ [15]. This result suggests the importance of including the mechanism of radial electric field generation self-consistently in the turbulence study as shown in Ref. [14].

It may be possible to find that the poloidal shear flow induced by the radial electric field suppresses the electrostatic resistive interchange modes and does not excite electromagnetic interchange modes. The next subject is to find a way to produce such a radial electric field consistently.

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NONLOCAL TRANSPORT PHENOMENA AND LONG WAVELENGTH TURBULENCE IN TOKAMAKS

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Abstract

The paper discusses aspects of long wavelength drift wave turbulence as it pertains to recent observations on transport scalings and properties, measured correlation lengths, and profiles. A variety of simple models are employed in order to study in detail, both analytically and computationally, the dynamics of trapped ion convective cells and long wavelength trapped electron modes. These studies indicate that many of the experimental observations that have proved most difficult to reconcile with the properties of traditional drift waves \((k_p \rho_s \leq 1)\) are consistent with robust features of long wavelength electrostatic turbulence.

I. Introduction

The problem of explaining anomalous transport in the core of tokamaks remains unsolved and, indeed, quite poorly understood. For a long time, the leading hypotheses have invoked drift waves or ion temperature gradient driven microinstabilities. Such theoretical models have the generic feature that the basic turbulent "eddy" scale is of the order of a few gyro-radii, i.e., \(k_p \rho_s \leq 1\).
Recently, however, a number of experimental results have called the conventional wisdom of ‘gyro-Bohm’ transport into serious question. These results include:

a) The observation\(^1\) that net confinement scaling is consistent with Bohm, not gyro-Bohm scaling. This suggests that the transport agent is characterized by a comparatively larger scale.

b) The recent beam emission spectroscopy results from TFTR which indicate the presence of large scale density fluctuations\(^2\) (i.e., correlation lengths of \(\Delta r \sim 2-3\) cm.).

c) The recent findings of choppy, step-like temperature profiles, with step location correlated with low \(q(r)\) rational surfaces, observed on TFTR.

d) The ubiquitous observation of nonlocal fluctuation and transport phenomena, such as those noted on TEXT\(^3\).

e) The continued universal observation of the effects of the \(q(r)\) and current profiles on confinement. All of the above results suggest that larger scale fluctuation phenomena co-exist with drift wave microturbulence and play an important role in transport processes.

The central thesis of this paper is that trapped ion convective cells are the best available candidate for explaining the origin of these “macroscale” fluctuations. However, apart from linear theory, convective cell dynamics is very poorly understood. Hence, we here focus on the nonlinear dynamics of trapped ion convective cells and the closely related problem of long wavelength dissipative trapped electron mode turbulence. In particular:

a) A simple model of ion temperature gradient driven convective cells is presented. Simple statistical arguments are used to support mixing length estimates of the fluctuation levels and transport. Results indicate likely consistency with Bohm scaling.

b) A theoretical model of long wavelength dissipative trapped electron modes has been analyzed in depth. Results explain the obstruction of low \(k\) condensation and illustrate the nonlocal transfer in wavenumber space that is characteristic of modulational type interactions. Flow visualizations in real space and wavenumber space are presented.

c) A detailed theoretical study of modulational interaction between drift waves and convective cells is discussed. Results indicate that convective cell destabilization by decay instability may exceed linear growth. Also, modulational interaction is shown to have a strong effect on convective cell structure.

A second theme of the paper is that low \(q(r)\) surfaces may support highly nonlinear drift modes that are insensitive to shear damping. These results offer a dynamically plausible link between \(q(r)\) structure and confinement. In particular, magnetic islands of rather vague etiology need not be invoked.
II. Long Wavelength Electrostatic Turbulence

a) Trapped Ion Temperature Gradient Driven Convective Cell Turbulence

In this subsection, trapped ion temperature gradient driven convective cells are analyzed. Among other characteristics, these are distinguished from dissipative trapped ion convective cells (DTICC) by propagation in the ion diamagnetic direction. In previous work, turbulence and transport associated with resonantly destabilized ion temperature gradient driven convective cells were examined. (The destabilizing resonance is \( \omega = \omega_d \), where \( \omega_d \) is the bounce averaged precessional drift frequency.) In the presence of strong auxiliary heating, however, it is likely that the plasma system is pushed strongly off resonance. Therefore, the nonlinear dynamics of the hydrodynamic branch of these modes is taken up here.

These modes are modeled by the following equations:

\[
\frac{\partial \delta n}{\partial t} - v_{i,1}^T \frac{\partial \delta n}{\partial y} + \frac{v_{i,1}^T}{4v_{\text{eff},e}} \frac{\partial^2 \delta n}{\partial y^2} + v_d \frac{\partial \delta T}{\partial y} - \frac{D_B}{v_{\text{eff},e}} \hat{e}_\parallel \nabla \frac{\partial \delta n}{\partial y} \times \nabla \delta n = 0, \tag{1}
\]

\[
\frac{\partial \delta T}{\partial t} - v_{i,1}^T (1+\eta_i) \frac{\partial \delta n}{\partial y} + D_B \hat{e}_\parallel \nabla \delta n \times \nabla \delta T = 0, \tag{2}
\]

where

\[
(\delta n, \delta T) = \left[ e^{1/2}/2 \right] (\delta n_i^T, \delta T_i^T),
\]

\[v_{i,1}^T = e^{1/2} \omega_i/k_y, \quad v_d = \omega_d/k_y,
\]

\( \delta n_i^T \) and \( \delta T_i^T \) are trapped ion density and temperature fluctuations, \( e \) is the trapped ion fraction, \( D_B = cT_i/eB \), \( \omega_i \) is the diamagnetic drift frequency, and \( T_i/T_1 \) has been set to unity for simplicity. This quasi-2-D, coupled set of equations for two scalar fields represents the next level of complication beyond the single-field Kadomtsev-Pogutse model of dissipative trapped ion convective cell turbulence, and exhibits a number of interesting features. First, curvature couples the two fields together and is ultimately responsible for linear instability. Second, the nonlinearities are of different strength, with the self-adveective density nonlinearity reduced in magnitude because of the constraint imposed by quasineutrality. Third, the two nonlinearities are of different character, the first being self-advection of density, while the second represents the "passive" advection of heat and density fluctuations.

Performing a mixing length analysis, the levels of density and temperature fluctuations, and the scalings of ion and electron thermal transport are found to be:
b) Dynamics of Dissipative Trapped Electron Turbulence

The dynamical evolution of long wavelength drift wave turbulence and its interaction with shorter wavelength fluctuations has been the focus of a series of computational studies designed to sort out the interrelationships of different physical processes. These processes include the interplay of the conservative polarization drift nonlinearity (ExB advection of vorticity) and the enstrophy conservation-breaking ExB nonlinearity (ExB advection of density); the feedback on saturation dynamics of nonlinear frequency shifts; and the role of coherent structures in nonlinear dynamics. These computational studies are based on fluid models for dissipative trapped electron mode turbulence (DTEM) and collisional drift waves.

The simplest model for DTEM turbulence utilizes a single field model based on the iô approximation. Consistent with strong electron collisionality, electron inertia, both linear $(\partial \mathbf{n}/\partial t)$ and nonlinear $(\nabla \times \mathbf{B} - \nabla \psi_e)$ is neglected. In this approximation it is possible to examine the wavenumber space transfer dynamics of the ion ExB and polarization drifts, and thereby isolate from other physical processes the coupling of long and short wavelengths through the transfer induced by these nonlinearities.

From numerical solution of the one-field model equation, it is found that wavenumber space transfer is characterized by two regions in wavenumber space, corresponding to long (low k) and short (high k) wavelength extremes. In the long wavelength regime, the ExB nonlinearity dominates, and transfer of energy is toward shorter wavelengths. The direction of spectral transfer, and the associated generation of enstrophy (mean squared vorticity) in successively smaller scales is consistent with the fact that the ExB nonlinearity conserves energy, but not enstrophy. Because the ExB nonlinearity requires a nonadiabatic electron response, transfer is anisotropic in wavenumber space with transfer to high k_x enhanced relative to the k_y transfer rate. Physically, the enhanced k_y transfer is produced by a robust nonlocal transfer which deposits energy directly on short wavelength fluctuations, bypassing intermediate scales.
In the short wavelength regime, the polarization drift nonlinearity dominates. Consistent with the conservation of energy and enstrophy, there is a significant transfer of energy to the lowest wavenumbers of the high wavenumber regime and enstrophy is transferred conservatively to shorter wavelengths. The transfer is isotropic and local in wavenumber space, proceeding primarily to adjacent scales.

The transfer in both regimes is graphically illustrated by subjecting the saturated spectrum to a large, localized perturbation (in wavenumber space) and observing the subsequent relaxation of the spectrum to its original stationary configuration. Such pulses, placed in either the low k or high k regimes, result in markedly different relaxation properties. In Fig. 1, a low k pulse explosively fills k-space consistent with nonlocal transfer to high k. In contrast, a pulse at high k undergoes a diffusive relaxation, spreading energy locally, as shown in Fig. 2. In this case an increase in long wavelength energy in the transition region between long and short wavelength regimes results from energy transfer to lower k from the wavenumber of the pulse.

In the intermediate wavenumber regime the two nonlinearities are comparable. In this regime a large shift in the frequency spectrum peak is observed. The peak arises from the cross coupling of the two nonlinearities through the driven fluctuations. Since the ExB nonlinearity has one less spatial derivative than the polarization drift nonlinearity, the cross coupling term is 90° out of phase with the eddy damping decrement and thus enters as a shift in the spectrum peak (as opposed to broadening). Frequency shifts potentially affect linear stability through the electron inertia.

Because the one-field model neglects electron inertia, observation of the effect of the frequency shift on mode fluctuation levels requires an extended model. Accordingly, nonlinear solution of a two-field DTEM model has been undertaken, by treating the evolution of electron and ion densities separately and linking them via quasineutrality. In this way electron inertia, both linear and nonlinear, is incorporated into the fluctuation dynamics. With such a model, extension to the collisionless trapped electron regime is readily accomplished by varying the electron collisionality.

A visualization of the real space evolution of the two-field DTEM model is presented in Fig. 3. Elongated structures are apparent. Their long axis is in the x direction and they propagate in the y direction. The anisotropy apparent in these structures is consistent with the ExB nonlinearity, which efficiently removes long wavelength energy to large \( k_y \), via nonlocal transfer, but not to large \( k_x \). Moreover, the linear instability also preferentially feeds the small \( k_x \) wavenumbers. Indeed similar structures appear in the linear growth phase prior to saturation and the onset of significant nonlinear coupling. It is evident that the elongated structures break up, forming smaller isotropic structures. Comparison with the one-field model results suggests that the smaller structures arise from the transfer to high k that is driven by the ExB nonlinearity. They are isotropized as they reach the smaller scales where the polarization drift nonlinearity becomes active. After the breakup of the elongated structures, they reappear at a later time. Indeed, these structures grow, break up, and reappear in a cyclical fashion with a period given by many nonlinear correlation times.

A clue to the cyclical behavior is given in the time history of total fluctuation energy and the root mean square wavenumber \( \langle k_x^2 \rangle^{1/2} \) shown in
FIG. 1. Evolution of the saturated DTEM energy spectrum after an impulse of energy localized to a rectangular annulus at low $k_y$ where the $E \times B$ nonlinearity dominates. Time goes sequentially from left to right and top to bottom. The $(0,0)$ mode is at the center bottom of each snapshot. (41 by 7. Both nonlinearities with $E \times B$ dominating and pulse at low $k_y$.)
FIG. 2. Evolution of the saturated DTEM energy spectrum after an impulse of energy localized to a rectangular annulus at high $k$ where the polarization drift nonlinearity dominates. (41 by 11. Both nonlinearities comparable, large pulse at high $k$.)
Fig. 3. Flow visualization of the DTEM potential field in real space. Time goes sequentially from left to right and top to bottom, showing formation and break-up of elongated structures. Approximately ten temporal correlation times are covered by this sequence.

Fig. 4. It is seen that the cyclical appearance of the filaments, manifested by the minima of $\langle k_x^2 \rangle^{1/2}$, is correlated with large amplitude cyclical excursions of the energy having the same period. Maxima in the energy correspond to minima of $\langle k_x^2 \rangle^{1/2}$. A plausible explanation is provided by the effect on the instability process of the frequency shift (which is proportional to the fluctuation energy). As the energy increases toward a maximum, the frequency shift also increases and quenches the linear instability above some threshold. Consequently, the energy turns over and falls. With the linear drive turned off, energy in the filaments is not replenished as it cascades to small scales. This results in an increase in $\langle k_x^2 \rangle^{1/2}$ that is correlated with the decrease in energy. As the energy falls, the frequency shift decreases until it is no longer sufficient to quench the linear drive. At this point a new cycle begins as the linear instability starts to replenish the low k part of the spectrum, restoring the filaments, and making the energy grow. This cycle is clearly evident in the spectra, which show a collapse of a low k plateau as the energy decreases (and $\langle k_x^2 \rangle^{1/2}$ increases) and its re-emergence a half cycle later when the energy grows. The observation of this cyclical behavior is believed to be the first identifiable instance of fluctuation levels being affected by frequency spectrum shifts, a long ignored aspect of drift wave turbulence.
Numerical solution of a two-field collisional drift wave model\textsuperscript{11} has been undertaken in order to study the existence of long-lived coherent structures and their interplay with the dynamics controlling turbulent transport. The model equations\textsuperscript{9} exhibit markedly different behavior based on the parameter $\alpha = \frac{k_{||}^2 V_t^2}{2 v_e \omega}$ which determines the degree of adiabatic electron response. In the adiabatic regime, corresponding to $\alpha > 1$, the turbulent flow is homogeneous and amorphous. In the hydrodynamic regime, corresponding to $\alpha > 1$, the flow supports long-lived coherent structures lasting ten or more eddy turnover times. The appearance of such structures is of dynamical importance because of implied effects on calculated transport. Indeed, in the hydrodynamic regime, radial particle transport driven by the $E\times B$ advection of density fluctuations is much smaller than that predicted by standard estimates based on quasilinear or resonance broadened theories. Moreover, a weak negative correlation between kurtosis and transport is observed. These results suggest that coherent structures, when they exist, inhibit cross field transport in inhomogeneous plasmas.

c) Modulational Theory

There are several key questions concerning the modulational interaction of long and short wavelength electrostatic fluctuations. First, linear stability calculations reveal a rather murky story in parameter space, i.e. the location in
parameter space is sensitive to profiles $s(r)$, $q(r)$, and poloidal gyroradius. Nevertheless, Bohm scaling appears to be universal. Second, even the relevance of linear or conventional quasilinear predictions appears dubious, given that long wavelength convective cells exist in a background of short wavelength, higher frequency drift wave turbulence. This environment can exert significant stress on the cells via nonlinearity, thus influencing the cell structure. Simultaneously, the short wavelength drift waves must 'ride' on the longer wavelength convective cells and be modulated by them. Thus, trapping of drift wave packets may occur. This interaction of disparate scales strongly calls into question conventional assumptions made in stability calculations concerning equilibria. Such questions, in turn, indicate a need for fundamental study of the nonlinear dynamics of drift wave-trapped ion convective cell turbulence in the spirit of previous work on Langmuir turbulence.

Here the origin and dynamics of modulational interaction between short wavelength, broad-band drift waves ($k_{\perp}a_s \sim 1$) and large scale dissipative trapped ion convective cells (DTICC) are examined in the context of a very simple model. The principal results are that 1) modulational instability of convective cells interacting with a background of either broad-band or coherent wavelength drift wave turbulence occurs. The modulation-induced growth can exceed linear growth rates typical of DTICC. Modulational interaction also shifts (nonlinearly) the DTICC frequency. 2) A simple set of model 'envelope equations', akin to the Zakarov equations of Langmuir turbulence, has been derived and solved numerically. A quasi-universal final state of dipolar, strongly anisotropic convective cell trapping by drift wave Reynolds stresses is indicated. 3) Drift wave packets with $k_{\perp}a_s \sim 1$ are trapped by the DTICC and thus undergo highly nonlinear modulations.

The basic equations account for drift wave evolution via polarization drift and E×B nonlinearities, and DTICC evolution via the E×B nonlinearity alone, i.e.,

$$\frac{\partial}{\partial t} n^p + \frac{\partial}{\partial x} \rho_s^2 \nabla_{\perp}^2 n^p + \mathbf{V} \cdot \frac{\partial n^p}{\partial y} + D_0 \frac{\partial^2 n^p}{\partial y^2}$$

$$= - \rho_s^2 C_s \nabla_{\perp} n^c \cdot \mathbf{z} \cdot \nabla_{\perp}^2 n^p + L_0 D_0 \nabla_{\perp} \left( \frac{\partial n^p}{\partial y} \right) \cdot \mathbf{z} \cdot \nabla_{\perp} n^c \quad (3)$$

$$\frac{\partial n^c}{\partial t} + \nabla \cdot \mathbf{v} \cdot \frac{\partial n^c}{\partial y} + \frac{\nabla D_0}{2} \frac{\partial^2 n^c}{\partial y^2}$$

$$= - \rho_s C_s \left( \nabla_{\perp} n^p \cdot \mathbf{z} \cdot \nabla_{\perp}^2 n^p \right) \quad (4)$$

where $n^p$ and $n^c$ refer to drift wave and DTICC respectively. Analyzing these equations, the renormalized dissipative trapped ion convective cell frequency is

$$\omega_k = + \frac{\nabla e}{\omega_{\omega} + i \delta_{k}^{(0)}} - i \nabla e \rho_s^2 C_s^{2} \sum_{k'} \left\{ \left( k k' \times z \right)^2 \left[ k_{\perp}^2 a_s^2 - i \delta_{k}^{(0)} \right] \Delta \omega_{k'} \right\}$$

$$\ast \left[ k_{\perp}^2 a_s^2 + 2 k_{\perp} \cdot k_{\perp} a_s^2 \right] k \frac{\partial \langle N^p \rangle}{\partial k'} \quad (5)$$
where $N_{D}k'$ is the drift wave action density. This result is easily interpreted. First, it is clear that for $\partial(N_{D}k')/\partial k' < 0$ (consistent with virtually all models of drift wave turbulence), modulational instability results in DTICC cell growth due to polarization drift nonlinearity. Similarly, the E×B nonlinearity results in a nonlinear frequency upshift. Second, it is interesting to compare the linear and nonlinear dissipative trapped ion convective cell growth rates. To this end, since $\gamma_{k}^{(0)}$ increases rapidly with $k$, it is sensible to approximate drift wave levels by their mixing length values (i.e., a typical nonlinear estimate). In estimating $\gamma_{k}^{NL}/\gamma_{k}^{(0)}$, straightforward algebra gives

$$\gamma_{k}^{NL}/\gamma_{k}^{(0)} = \left( \frac{L_{n}L_{T}}{a^{2}} \right) \frac{\varepsilon^{2} (v_{\text{eff}})}{C_{s}L_{n}} ,$$

indicating that it is easy for the nonlinear growth to exceed the linear growth. This result may partly explain the apparently ubiquitous character of convective cell turbulence. In the nonlinear modulation regime, the structure of the DTICC is strongly distorted, as shown in Fig. 5.

d) Modulational Experiments

Experimental identification of modulational interactions between approximately Gaussian processes of very different spatial and temporal scale...
lengths is possible using a new correlation technique. This technique is discussed and application to X-mode scattering data taken on TFTR is reported.

Restricting the discussion to locally homogeneous turbulence, consider a contribution to the fluctuating part of the electron density of the form $\tilde{n}_m = \tilde{n} \cdot m$ where $\tilde{n}$ and $m$ are approximately independent Gaussian processes. It is assumed that $\tilde{n}$ is an underlying short scale feature modulated by the much larger temporal and spatial scale length feature $m$. The issue considered is whether the correlation properties of $m$ may be determined from the correlation analysis of $\tilde{n}_m$. To second order, given $\rho_{mn}(\Delta r, \tau) = \langle \tilde{n}_m(r_1,t_1) \tilde{n}_m(r_2,t_2) \rangle$, then $\rho_{mn}(\Delta r, \tau) = \rho_n(\Delta r, \tau) \rho_m(0,0)$, where $(\Delta r, \tau) = (r_1-r_2, t_1-t_2)$ and $\rho_n$ and $\rho_m$ are the second order correlations of $\tilde{n}$ and $m$ respectively. From this expression, it is apparent that to second order, only the correlation properties of the underlying short scale process are recovered. However, defining the local intensity fluctuation of the field by the expression $\tilde{\Gamma}_{nm}(r,t) = \tilde{n}_m^2(r,t) - \langle \tilde{n}_m^2(r,t) \rangle$, the second order intensity correlation is $\langle \tilde{\Gamma}_{nm}(r_1,t_1) \tilde{\Gamma}_{nm}(r_2,t_2) \rangle = 6\rho_n^2(\Delta r, \tau) \rho_m^2(0,0) + 2\rho_n^2(0,0) \cdot 2 \rho_m^2(\Delta r, \tau)$. From the second term of this expression, it is clear that the correlation properties of the modulator may be recovered from the intensity correlations in the field. Note that the fourth and second order scale lengths can then be very different, which is an indication of strongly non-Gaussian behavior. The disparity between second and fourth order scale lengths is also a strong indicator of multiscale interactions in the turbulent field.

For microwave scattering data, it is necessary to consider the correlation properties of high spatial and/or temporal frequency components in the turbulent field. Taking $\tilde{\Gamma}_k(t) = |\tilde{N}_m(k,t)|^2 - \langle |\tilde{N}_m(k,t)|^2 \rangle$ as the fluctuations component of the intensity of a spatial Fourier component of the modulational field, then $\langle \tilde{\Gamma}_{k_1}(t_1) \tilde{\Gamma}_{k_2}(t_2) \rangle \approx \langle |\tilde{N}(k_1)|^2 \rangle \langle |\tilde{N}(k_2)|^2 \rangle \cdot \int \rho m^2(r,\tau)$. Thus, for intensity correlation in the scattered field, the time scale of the random modulator is recovered. Assuming further that the modulator is of very large scale compared with the region of integration, then for the normalized intensity correlation,

$$\mu_1(\tau;k_1,k_2) = \frac{\langle \tilde{\Gamma}_{k_1}(t_1) \tilde{\Gamma}_{k_2}(t_2) \rangle}{\langle |\tilde{\Gamma}_{k_1}|^2 \rangle^{1/2} \langle |\tilde{\Gamma}_{k_2}|^2 \rangle^{1/2}},$$

the simple expression is obtained:

$$\mu_1(\tau;k_1,k_2) \approx \frac{\rho_m(0,0)}{\rho_m(0,0)}.$$  (7)

It may further be shown that the intensity correlation between two distinct frequency bands for $k_1=k_2$ is also a strong indicator of non-Gaussian modulational behavior.
FIG. 6. Normalized correlation between two independent oscillators of frequency 400 kHz and 200 kHz in the core of an L-mode plasma clearly recovers the modulation of the scattered signal due to the m=1 sawtooth precursor.

To show how modulational properties of short scale fluctuations may be identified by the method of intensity interferometry, consider scattering measurements of core fluctuations in L-mode plasmas taken during the m=1 precursor to the sawtooth crash. Figure 6 illustrates the correlation in power fluctuations between two distinct frequency bands corresponding to a $k_{pol} = 7$ cm$^{-1}$ in the plasma core taken using the 60 GHz X-mode scattering system on TFTR. Note that for frequency bands separated by 200 kHz, significant correlation is obtained with the period of the m=1 precursor, suggesting that variations in $n_e/n$ at high $k_{pol}$ in the core are produced by perturbations in $L_n$ in the presence of the precursor.

However, except in only a small number of cases, no significant modulational characteristics of short scale structures have been identified in the absence of clear MHD activity. This observation suggests that the scale length of any modulational property of small scale structures is significantly shorter than the extremely large (~40 cm) dimension of the scattering volume. Noting that a 40 cm scattering region would engulf most trapped ion convective cells, further investigation of the modulational properties of short scale structures should be performed with more spatially localized diagnostic measurements.

III. $q(r)$-Profile Effects on Drift Wave Turbulence

The reconciliation of a dynamically plausible model of electrostatic drift wave microturbulence with the observed sensitivity of transport to microscopic magnetic structure [via $q(r), I_p$, etc.] remains one of the outstanding problems in tokamak confinement theory. In particular, the measured scaling of confinement with plasma current $I_p$ has long been at odds with the robust scaling properties of drift wave turbulence. Moreover, the possible existence
of choppy equilibrium temperature profiles with flat regions near rational surfaces presents a further challenge that is difficult to accommodate within the traditional framework of electrostatic microturbulence. Previous theories have largely ignored these issues, or produced predictions that exhibit extreme and unrealistic sensitivity to the details of the magnetic structure.

Recent theoretical and computational studies of a simple model of drift wave turbulence in a sheared cylinder have revealed intrinsic spatial intermittency whose characteristics depend on local magnetic properties\textsuperscript{1}, and thereby shed new light on this problem. These effects occur in a paradigmatic model describing ion density fluctuations driven by the inverse dissipation from trapped electron collisions, and subject to magnetic shear through parallel ion dissipation. The essential features of the spatially intermittent turbulent steady state supported by this model appear in numerical solutions of the evolution of undriven finite amplitude modes near low order rational surfaces. It is observed that if these modes are initialized above a critical amplitude threshold, they do not decay to zero under the influence of magnetic shear damping, but remain at finite amplitude. The lack of decay can be attributed to a finite amplitude-induced alteration of the radial fluctuation structure, so as to suppress out-going wave radiative damping.

This fluctuation structure modification has its origin in the radial variation of the nonlinear diffusivity \( d_k \) and growth rate \( \gamma_{NL} \) which govern the evolution and saturation of finite amplitude fluctuations:

\[
d_k(x) = (L_n D_0)^2 \sum_{k'} k_y'^2 |\tilde{n}_{k'}|^2 R_{k', k''} \left[ k_y'^2 + \left( k_y'^2 - k_y^2 \right) \right],
\]

\[
\gamma_{NL} = k_y^2 (L_n D_0)^2 \sum_{k'} |\frac{\partial \tilde{n}_{k'}}{\partial x}|^2 R_{k', k''} \left[ k_y'^2 + \left( k_y'^2 - k_y^2 \right) \right],
\]

where \( (L_n D_0)^2 \left( 2k_y'^2 - k_y^2 \right) \) is the nonlinear coupling strength of the ExB nonlinearity, \( R_{k, k', k''} \) is the three-wave correlation time, and \( \tilde{n}_{k'}(x) \) is the nonlinear eigenfunction of the \( k' \) mode. These expressions describe the coupling of all modes \( k' \) that are radially accessible to the mode \( k \) and satisfy the three-wave sum rule \( k = k' + k'' \). In principle, the coupling involves modes on different rational surfaces. If the rational surfaces are uniformly spaced and sufficiently close that eigenfunctions of adjacent surfaces overlap, the sum over \( k' \) samples eigenfunctions centered at different radial locations and smoothes the spatial variation of individual eigen-structures. In this case the diffusivity and growth rate are essentially uniform in space. If the mode \( k \) is on a low order rational surface, however, coupling is predominantly to modes on the same surface, i.e., modes of the same helicity. Because such modes are all centered at the same surface, thereby peaking at the same location, the differential and nonlinear growth rate are spatially nonuniform. Strong spatial nonuniformity in the vicinity of low order surfaces is apparent in Fig. 7, obtained from the numerical solution of the model equations for steady state (saturated) turbulence.

The spatial structure of the diffusivity creates an effective well for the mode \( k \), provided the scale length of the diffusivity is smaller than the mode width \( d_k^{-1}(x)(d/dx)[d_k(x)] \geq w_k^{-1} \). Localization of the mode in the effective
well circumvents magnetic shear damping and ultimately increases the level of the turbulence near a low order surface, providing a self-bound spike in the fluctuation level profile. This effect is significant only when the amplitude of the diffusivity is sufficiently large to allow the nonlinear diffusion to compete with the polarization drift, \( \frac{d}{dx} (d_k(x) \frac{df}{dx}) > i \omega \rho_s^2 \left( \frac{d^2}{dx^2} \right) \rho_k \), i.e. when the nonlinear decay time of a fluctuation of scale \( \rho_s \) is shorter than the linear time scale \( \omega^{-1} \approx \omega_s^{-1} \).

When the above conditions are satisfied, a representation of the radial variation of the diffusivity in terms of a Gaussian, \( d_k(x) = d_k(o) \exp(-\alpha x^2) \), yields a consistent solution of the fluctuation level and diffusivity (specification of \( \alpha_d \)). The effect of magnetic shear damping, on both the localization of the mode, and the rate of extraction of free energy, is found to be subdominant to the role played by the diffusivity profile by a factor of \( \omega_s \rho_s^2 / d_k(o) \). These results provide a simple understanding of previously observed finite amplitude stationary states that exist in the presence of magnetic shear damping without any linear damping\(^{13}\). It is worth noting that the blocking of magnetic shear damping by finite amplitude-induced mode structure modification is a general process occurring near low order rational surfaces in 3-D geometry and does not require a hydrodynamic layer. Moreover, the recent assertion\(^{14}\) that these processes cannot be modeled with turbulent diffusivities is patently incorrect.

---

**FIG. 7.** Radial profile of the turbulent diffusivity for saturated DTEM turbulence. Large spikes occur at low order rational surfaces.
What is required is proper accounting of the radial variation of the turbulent diffusivities.

Pragmatically speaking, the conditions for magnetic shear suppression are met when \( \langle (m^2)/m^2 \rangle (\delta p_s L_s / r L_n) > 1 \), where \( (m^2) \) is the mean square mode number and \( \delta \) is the nonadiabatic electron shift. Clearly, \( q(r) \) structure, in terms of \( L_s \) and \( \langle m^2 \rangle / m^2 \), enters this expression as the dominant variable determining when this criterion is met. The resulting self-bound spikes at low order surfaces in turn couple energy to radial modes, thus profoundly influencing drift wave transport both with regard to magnitude and radial profile. Indeed the \( L_s \) scaling of the above criterion and the probable linkage of fluctuation spikes with increased transport and local equilibrium profile flattening are consistent with the measured current scaling of plasma confinement and the possible existence of bumpy profiles.

**IV. Summary**

This paper has discussed aspects of long wavelength drift wave turbulence as it pertains to recent observations on transport scalings and properties, measured correlation lengths, and profiles. A variety of simple models were employed in order to study in detail, both analytically and computationally, the dynamics of trapped ion convective cells and long wavelength trapped electron modes. These studies indicate that many of the experimental observations that have proved most difficult to reconcile with the properties of traditional drift waves (\( k_L p_s \leq 1 \)) are consistent with robust features of long wavelength electrostatic turbulence.

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PROFILE CHARACTERISTICS OF H-MODE BIFURCATION MODELS, AND TURBULENCE SIMULATIONS WITH GYRO-LANDAU FLUID MODELS IN SLAB AND TOROIDAL GEOMETRY*

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Abstract

PROFILE CHARACTERISTICS OF H-MODE BIFURCATION MODELS, AND TURBULENCE SIMULATIONS WITH GYRO-LANDAU FLUID MODELS IN SLAB AND TOROIDAL GEOMETRY.

A study of the sensitivity of the width of the H-mode transport barrier to the energy, particle and momentum sources has led to a possible explanation for the very high confinement regime (VH-mode) presented in Part A of the paper. In Part B recent progress in the development of gyro-Landau fluid (GLF) models is described. GLF models with a few moment equations encompass with high accuracy both linearly and non-linearly the effects of finite gyroradius to all orders and of the Landau resonances in both parallel field and curvature drift motion. Numerical simulation of turbulence with a novel application of a non-linear ballooning mode representation of these models is given.

Part A: Profile Characteristics of H-Mode Bifurcation Models


This is a brief report on a particular result which has come out of a more extensive study of the relationship between the sources and the profile characteristics of H-modes. We have studied the dependence of the density, temperature and velocity profiles on the particle, energy and momentum sources, using a model for turbulent transport which simulates the suppression of turbulence by a gradient in the radial electric field of the plasma. The model extends previous

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work [1,2] by including the momentum transport equations. One of the features which we set out to understand was the width of the transport barrier just inside the separatrix, where the density and ion temperature gradients are large. In this report, we present a simplified version of the transport model, which illustrates how the particle source at the plasma edge fixes the width of the transport barrier. A testable explanation for the VH-mode phenomenon [3], in which the transport barrier becomes larger than for a usual H-mode, will be given.

It is now well established theoretically that an electric field gradient can suppress certain micro-turbulent instabilities. Moreover, the transport barrier region of H-mode is observed to possess a steep electric field gradient. The radial ion force balance determines the electric field through

\[ E_r = \frac{B_\theta}{c} U_\phi - \frac{B_\phi}{c} U_\theta + \frac{1}{e n_i} \frac{\partial P_i}{\partial r} \]

If we neglect the U x B contribution to the gradients of the electric field, the momentum transport equations become decoupled from the turbulence suppression and the H-mode bifurcation. In a forthcoming paper we will argue that data from DIII-D supports this approximation, and that momentum transport is not a major factor in spontaneous H-modes. For now we will make the assumption that \( dE_r/dr \sim \frac{\kappa_0}{n_i} dE_r/dr \), neglecting \( d^2 P_i/dr^2 \) and the velocity terms.

In order to illustrate how the width of the transport barrier depends upon the sources we will use a variant of the model of Ref. [2]. In this model the combined electron and ion energy and particle fluxes are chosen to have the form

\[ Q = - \left[ \kappa_0 + \frac{\kappa_1}{1 + \alpha \left( \frac{dE_r}{dr} \right)^2} \right] \frac{dP_i}{n_i dr} \]

\[ \Gamma = - \left[ D_0 + \frac{D_1}{1 + \alpha \left( \frac{dE_r}{dr} \right)^2} \right] \frac{dn_i}{n_i dr} \]

The simplest model is to take \( \kappa_0, \kappa_1, D_0, D_1, \) and \( \alpha \) as constants. Then taking the product of \( Q \) and \( \Gamma \) yields a function which only depends upon \( dE_r/dr \), reducing the non-linear transport problem to one dimension. Figure 1 shows a plot of \( Q \Gamma \) versus \( dE_r/dr \). If \( Q \Gamma \) is above the critical value \( \gamma_{\text{max}} \), the electric field gradient must jump to a large value for a new stable equilibrium to occur. An H-mode occurs when the separatrix value of \( Q \Gamma \) exceeds \( \gamma_{\text{max}} \). Moving in from the separatrix the gradients remain high until the local value of \( Q \Gamma \) drops below \( \gamma_{\text{min}} \) in Fig. 1, at which point a bifurcation back to the shallow gradient solution occurs. Profiles similar to the experimental profiles of density and temperature can be obtained from this model [2]. The distinctive feature of the profiles is the formation of a pedestal where the density and temperature gradients discontinuously change due to the jump in electric field gradient. The width of the steep gradient zone is determined by the pedestal position at which \( Q \Gamma = \gamma_{\text{min}} \). The sources are calculated from DIII-D data by the ONETWO transport code. The energy and toroidal momentum flux densities (mostly due to the neutral beams), obtained by integrating the sources, are only weakly changing within the transport barrier. The particle flux density has a slowly varying part, due to neutral beam fueling,
and an edge fueling component which strongly increases towards the separatrix within the transport barrier. The edge fueling component of the particle flux is therefore the important quantity in fixing the width of the transport barrier. As long as the position where $\Gamma T = \gamma_{\min}$ falls within the region where the edge fueling is the dominant particle source the width of the transport barrier will depend on the neutral ionization length and be fairly insensitive to the heating power [2]. However, if the neutral beam particle source is large enough, and the edge fueling source is small enough, $\Gamma T = \gamma_{\min}$ can occur in the region dominated by the neutral beam particle source. In this case the relatively shallow slope of $\Gamma T$ can dramatically increase the width of the transport barrier and make it much more sensitive to the neutral beam heating and fueling profile. The condition for this to happen is roughly $\Gamma_{\text{beam}}(Q)_{\text{separatrix}} > \gamma_{\min}$. This may be the explanation for the improved confinement (over H-mode) regime VH-mode observed on DIII-D. This regime was discovered only after boronization of the vacuum vessel reduced impurity and particle sources from the walls [3]. The width of the transport barrier in VH-mode is observed to be noticeably wider than for H-mode, and the neutral beam power threshold for VH-mode is higher than for H-mode.

A phase diagram for the three confinement regimes L, H and VH can be derived from this simplified model. We do not expect this to be quantitatively accurate but it illustrates the basic physics. First we break down the particle flux at the separatrix ($\Gamma_a$) into a neutral beam component and an edge fueling component $\Gamma_a = \Gamma_{\text{beam}} + \Gamma_{\text{edge}}$. Similarly we decompose the energy flux at the separatrix into a neutral beam part (neglecting Ohmic heating), and an energy loss due to radiation which is taken to be proportional to the edge fueling part of the particle flux $Q_a = g_{\text{beam}} - \epsilon_1 \Gamma_{\text{edge}}$. Noting that $\Gamma_{\text{beam}} = g_{\text{beam}}$, we can derive an L/H phase boundary in $(g_{\text{beam}}, \Gamma_{\text{edge}})$ space from the threshold condition $Q_a \Gamma_a = \gamma_{\max}$, as shown in Fig. 2 (solid line). The H/VH boundary in Fig. 2 is determined from the condition $Q_a \Gamma_{\text{beam}} = \gamma_{\min}$ (dash-dot line in Fig. 2). This is not
a bifurcation boundary as was the L/H boundary, but demarks the region where a deeper transport barrier can be present. The edge fueling flux is determined by recycling with the walls. Denoting the recycling coefficient by \( r = \frac{\Gamma_{\text{edge}}}{T_a} \), a given discharge will fall along one of the constant recycling lines \( \Gamma_{\text{edge}} = r_1 \varepsilon_2 q_{\text{beam}}/(1-r) \) shown as dashed lines in Fig. 2. For large wall recycling even H-mode may be impossible due to the large radiated power, whereas VH-mode requires the lowest recycling conditions. This behavior is qualitatively similar to the importance of a divertor for H-mode and boronization for VH-mode. The radiated power fraction for this model is \( f_{\text{rad}} = \varepsilon_1 \frac{\Gamma_{\text{edge}}}{q_{\text{beam}}} = \varepsilon_1 \varepsilon_2 r/(1-r) \). For the parameters used in Fig. 2, the \( r = 0.9 \) line gives \( f_{\text{rad}} = 22\% \), an H-mode threshold of 3.5 MW and a VH-mode threshold of 9 MW, all of which are similar to experiment despite the crudeness of the model. A very important constraint on this phase diagram is the absence of edge localized modes (ELMs). If ELMs occur then the width of the transport barrier, and the degree of confinement improvement, will be limited by them. This may be the reason that the ideal ballooning mode limit and magnetic configuration are important restrictions on the observation of VH-mode.

Work is underway to try and constrain the profile and gradient dependence of the coefficients held constant in the present transport model. We are in the process of analyzing the time evolution of the profiles in H-modes and VH-modes. The dependence of the width of the transport barrier on the particle source is a testable prediction of this theory which is not strongly dependent on the details of the flux-gradient model. A simple test would be to gas puff during a VH-mode to try and cause a transition back to H-mode.
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Part B: Turbulence Simulations with Gyro-Landau Fluid Models in Slab and Toroidal Geometry

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1. INTRODUCTION

Fluid models have found wide use in simulating plasma turbulence because of their numerical simplicity and tractability. Such models time advance a few velocity moments of a kinetic equation rather than the full velocity distribution function. Until the recent work of Hammett and Perkins [1], it was not widely appreciated that kinetic effects such as Landau damping could be accurately represented by truncating the fluid moment hierarchy with the addition of a collisionless heat diffusivity $\chi = (2/\sqrt{\pi})v_{th}/|k|^\parallel$. Taking moments of the nonlinear electrostatic gyro-kinetic equation, this earlier $E \times B$ and parallel motion result has been generalized to include large gyro-motion [2,3] and in particular the curvature drift motion and resonance effect [3]. This yields nonlinear fluid equations for the gyro-density, parallel velocity, and parallel and perpendicular pressures. The procedure for truncating the higher moments in terms of the lower is highly constrained by modeling the deviation from a perturbed Maxwellian (with reactive as well as dissipative elements) to fit the kinetic linear response function at both small and large values of the kinetic parameters for parallel, gyro- and curvature motion: $k^\parallel v_{th}/\omega$, $b = (k^\perp \rho)^2/2$, and $\omega_D/\omega$. Taking adiabatic electrons, such gyro-Landau fluid (GLF) models for the ions give good agreement with kinetic theory for ion temperature gradient (ITG) mode local (constant $k^\parallel$ and $\omega_D$) linear stability [3].

This paper describes further refinements of such models and their application to numerical simulation of ITG mode turbulence in both slab and toroidal geometry. The slab simulations employ conventional $(m,n,r)$ 3-D nonlinear code representations. For toroidal geometry we develop and apply a more efficient 3-D nonlinear ballooning mode representation (BMR) code. All codes run on CRAY and CONNECTION MACHINE computers.

2. GLF MODEL FORMULATION

There are two procedures for taking moments of the gyro-kinetic equations. The first [3] results in a "simple gyro-average" model which is linearly exact for shearless geometry but which contains approximate nonlinear and shear terms with errors of $O(b)$. The second [2] or "full gyro-average" model regains nonlinear
accuracy through $O(b)$ (thus containing additional FLR-induced nonlinearities) even with shear, but at twice the computational expense. Most importantly, both models are well-behaved at large $b$. For completeness the “simple gyro-average” model is formulated in ion units:

$$
\frac{id}{dt} N_k = -\omega_*[(1 - \eta) \phi_{1k} + \eta \phi_{2k}] + \omega_D \phi_{12k} + k\| v_{th} U_k + \omega_D (P_{\|k} + P_{\perp k})/2
$$

$$
\frac{id}{dt} U_k = (1/2) k\| v_{th} (P_{\|k} + \phi_{1k}) + \omega_D \left[ (\Gamma_{\|} + \Gamma_{\perp})/2 - i \sigma_1 \mu \right] U_k
$$

$$
\frac{id}{dt} P_{\|k} = -\omega_*\left[ \phi_{1k} + \eta \phi_{2k} \right] + k\| v_{th} (\Gamma_{\|} U_k - i \sigma_1 \chi_{\|} T_{\|k}) + X_{\|} \omega_D \phi_{12k} + \omega_D \left[ X_{\|} P_{\|k} + \omega_D \left( X_{\|} P_{\perp k} + (1/2) T_{\perp k} \right) - i \sigma_1 \left( \nu_{\perp} T_{\|k} + \nu_{\|} T_{\perp k} \right) \right]
$$

$$
\frac{id}{dt} P_{\perp k} = -\omega_*\left[ (1 - \eta) \phi_{2k} + 2 \eta \phi_{3k} \right] + k\| v_{th} (\Gamma_{\perp} U_k - i \sigma_1 \chi_{\perp} T_{\perp k}) + X_{\perp} \omega_D \phi_{23k} + \omega_D \left( X_{\perp} P_{\perp k} + (1/2) T_{\perp k} \right) - i \sigma_1 \left( \nu_{\perp} T_{\|k} + \nu_{\|} T_{\perp k} \right)
$$

where $\Gamma_{\|} = 3, \Gamma_{\perp} = 1, X_{\|} = 2, X_{\perp} = 3/2$ are the compression indices for parallel and curvature drift motion. $T_k = P_k - N_k$. To model the deviations from a perturbed Maxwellian $\chi_{\|} = 2 \chi_{\|} = 2/\sqrt{\pi}, \nu_{\|} = \nu_{\perp} = (1 - i \sigma_1), \nu_{\|} = \nu_{\perp} = 0$, and $\mu = (0.80 - i 0.57 \sigma_1)$ where $\sigma_1 = k_{\|}/|k_{\|}|$ and $\sigma_1 = \omega_D/|\omega_D|$. In terms of Bessel functions $\phi_{1k} = \Gamma_{0} \phi_{1k}, \phi_{2k} = [\Gamma_{0} - b (\Gamma_{0} - \Gamma_{1})] \phi_{1k}, \phi_{3k} = (1/2) [(2 - 4b + b^2) \Gamma_{0} + (5b - 2b^2) \Gamma_{1} + b^2 \Gamma_{2}] \phi_{1k}$ where $\phi_{1k} = (e/T_{\perp}) \phi_{1k}$ with $\phi_{12k} = (\phi_{1k} + \phi_{2k})/2$, and similarly $\phi_{23k} = (\phi_{2k} + \phi_{3k})/2$. The density perturbation is given by $N_k/n_0 = N_k - (\phi_k - \phi_{1k})$ which equals $(T/T_{\perp}) \phi_{1k}$ for adiabatic electrons. To include the gyro-shear effect, $k_{\|}$ is replaced by $k_{\|} = k_{\|} - i L_s^{-1} (k_x/k_y) b \Gamma_{0} (b)/\Gamma_{0}(b)$. The curvature drift frequency is $\omega_D = 2 (L_n/R) \omega_* [\cos(\theta) + (k_x/k_y) \sin(\theta)]$. In applications $d/dt$ is taken to be the total time derivative with a simple nonlinear $E \times B$ convection.

The “full gyro-average” equations have a similar structure to the “simplified gyro-average” equations above, but differ somewhat because moments of the gyrokinetic equation are taken directly in guiding center coordinates with transformation to particle position delayed until the Poisson equation. For example, considering only the first moment and ignoring the toroidal terms here, integrating the gyrokinetic equation over velocity leads to a conservation law for the guiding center density $\hat{n}$

$$
\frac{\partial \hat{n}}{\partial t} + \nabla_{\|}(\hat{n} u_{\|}) + \nabla \cdot (\hat{n} (J_0) \vec{v}_E) = 0
$$

where $\vec{v}_E$ is the $E \times B$ drift. We approximate $(J_0)$ as $\Gamma_{0}^{1/2} (\hat{b})$ plus a dissipative term to be discussed below. Here $\hat{b} = \hat{\rho}^2 k_{\perp}^2$ with $\hat{\rho}^2 = \rho^2 + \rho_1^2 = (T_{\perp,0} + T_{\perp,1})/(m_i \Omega_i^2)$ contains both equilibrium and perturbed components. Expanding the last term above in gyrokinetic ordering:
The $\nabla \dot{n}_1$ term gives the nonlinear $E \times B$ convection of the perturbed density with a gyroaveraged $\vec{v}_E$. The $\nabla \rho^2_0$ leads to part of the usual linear FLR corrections and the $\nabla \rho^2_1$ term represents the nonlinear effect of perturbations in the gyroradius. Additional FLR effects arise from the transformation of guiding center density and perpendicular temperature to the perturbed particle density, $n_1/n_0 = N - (1 - \Gamma_0)e\Phi/T$, where $N \approx \left( \Gamma_0^{1/2} \dot{n}_1/n_0 + \frac{1}{2} \nabla^2 \Gamma_0^{1/2} \rho^2_1 \right)$, with $\frac{1}{2} \nabla^2 \Gamma_0^{1/2} = \partial\left( J_0 \right)/\partial \rho = \frac{1}{2} \nabla^2 \left( 1 - \Gamma_1/\Gamma_0 \right) \Gamma_0^{1/2}$. Finally, the FLR modified $E \times B$ drift must include a dissipative term to model the perpendicular phase mixing caused by ions drifting at different speeds. The essential features of this can be captured by adding to $\partial\dot{n}/\partial t$ the terms $\nu \frac{1}{2} (\nabla^2 \Gamma_0^{1/2} \vec{v}_E) \cdot \nabla \dot{n} + \lambda \frac{1}{2} \nabla^2 \left( \Gamma_0^{1/2} \vec{v}_E \right) \cdot \nabla \dot{n}$ where $(\nu, \lambda) = (0.5, 0.8)$. At typical mixing length turbulence levels for $\vec{v}_E$, this hyperviscosity-like nonlinear term can provide an important and natural sink of turbulent energy for $k_\perp \rho > 1$.

Figure 1 compares linear eigenfrequencies from both GLF models with exact kinetic calculations for ITG modes in sheared toroidal geometry from Ref. [4]. These results were obtained from a BMR code as described in Section 4. The agreement is fairly good at higher $\eta_i$ or lower $k_\theta \rho$, but there is some disagreement at low $\eta_i$. For these parameters, the kinetic $\eta_{\text{crit}} \approx 1.0$ while the GLF $\eta_{\text{crit}} \approx 1.3$. 

FIG. 1. Growth rate versus wave number. $\eta_i = 3, 2, 1.5, L_\phi/R = 0.2, q = 2, \delta = 1$, and $T_i/T_e = 1$. Solid line: Fig. 3 of Ref. [4]; dashed line: "simple gyro-averaged" model; dotted line: "full gyro-averaged" model.
3. NUMERICAL SIMULATION OF TURBULENCE IN SLAB GEOMETRY

Using a conventional (m,n,r) code in sheared slab geometry (neglecting curvature terms) several topics are investigated: scaling with gyro-radius, insensitivity to numerical representation, scaling with shear in the magnetic field and shear in the $E \times B$, and temperature gradient threshold behavior. The scaling with relative gyro-radius $(\rho/a)$ over a two-fold variation using a band of helicities between $q = 1$ and $2$, shows that the transport is gyro-Bohm-like $[\chi \propto c_s \rho (\rho/a)]$ rather than Bohm-like ($\chi \propto c_s \rho$). By going to a sparse mode spacing ($\Delta n = 5$ and 10) and operating over a narrower band of helicities, it is found that the gyro-Bohm scaling persists over a ten-fold variation in $(\rho/a)$ independent of the $k_\perp \rho_{max}$ cut-off. This contradicts the Beklemishev-Horton [5] hypothesis that transport is “per singular surface” which leads to a density of states factor converting gyro-Bohm scaling to Bohm scaling. There is no evidence of condensation in the longest wavelength modes which might lead to Bohm scaling, although the peak of the nonlinear spectrum is usually downshifted from the most unstable mode. Our transport fluxes in slab geometry are significantly smaller than previous simulations with the “standard” fluid models [6], but they show qualitatively similar trends: the transport increases with magnetic shear length and increases with moderate values of electric field shear before finally vanishing just beyond the linear stability point. The linear threshold is well known to be sensitive to the Landau resonance, thus GLF models are well suited to demonstrate that there is no significant subcritical ITG turbulence and we have found none in slab geometry.

4. SIMULATIONS WITH A NONLINEAR BMR IN TOROIDAL GEOMETRY

In general we have found (m,n,r) representations to be not well suited for high-n toroidal simulations of nearly radially homogeneous turbulence. A nonlinear 3-D ballooning mode representation (BMR) appears to be more practical. We use a Fourier transform of the twisted eddy basis of Cowley et al. [7] $(k_y, k_x, z')$ where we identify $k_y = n q/r$ with the toroidal mode label, $k_x/(ky) = \phi = \theta$ as the ballooning angle, and parallel field distance $z'/(R q) = \theta$ as the extended poloidal angle. $k_y = k_y', k_x = k_y, s(\theta - \phi)$, and $k_y = -(i/R q)\phi/\partial \phi$. This basis is particularly convenient for accurate evaluation of the gyro-effects and the dissipative elements of the toroidal GLF truncation. In contrast to previous BMR formulations, the $E \times B$ nonlinearity at each $\phi$ has the simple standard form of a 2-D convolution in $k_w$ and $k_y'$ [7]; most importantly we retain the $k_w$ or $\phi$ variable which is crucial to obtaining a true 3-D nonlinear representation. A priori equal phase space weighting is given to the unstable outward ($\phi = 0$) and stable inward ($\phi = \pi$) ballooning modes.

As an example BMR simulation we consider a case with TFTR L-mode parameters: $\gamma = 4$, $L_n/R = 0.4$, $q = 2.4$, $s = 1.5$, $T_i/T_e = 1$, $R/a = 2.8$. The “simple gyro-average” model was used with only primary modes $-\pi < \phi < \pi$ retained. [Interaction with cyclic “image” modes at $\phi + 2\pi$ and $\phi + 2\pi$ remains to be explored.] The outside to inside asymmetry in the transport flow is very
pronounced at 10:1, although the asymmetry in turbulence level is at most 2:1. The flux surface average diffusion is 11 gyro-Bohm units \([c_s \rho_s (\rho_s/a)]\) compared to the experimental value of 8 units. The corresponding "slab" simulations gave only 0.4 units illustrating the importance of treating toroidal geometry.

This work is part of the Numerical Tokamak Project (NTP).

REFERENCES

KINETIC STUDIES OF MICROINSTABILITIES IN TOROIDAL PLASMAS: SIMULATION AND THEORY*

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Abstract

KINETIC STUDIES OF MICROINSTABILITIES IN TOROIDAL PLASMAS: SIMULATION AND THEORY.

A comprehensive program for the development and use of particle simulation techniques for solving the gyrokinetic Vlasov-Maxwell equations on massively parallel computers has been carried out at the Princeton Plasma Physics Laboratory. This is a key element of the ongoing theoretical efforts to systematically investigate physics issues vital to understanding tokamak plasmas. The paper focuses on spatial-gradient-driven microinstabilities. Their importance is supported by the recent progress in achieving a physics-based understanding of anomalous transport in toroidal systems, which has been based on the proposition that these drift-type electrostatic modes dependent on ion temperature gradient (ITG) and trapped particle effects are dominant in the bulk ("confinement") region. Although their presence is consistent with a number of significant confinement trends, results from high temperature tokamaks such as TFTR have highlighted the need for better insight into the nonlinear properties of such instabilities in long-mean-free-path plasmas. In addressing this general issue, the paper reports important new results, including the first fully toroidal 3D gyrokinetic simulation of ITG modes and realistic toroidal eigenmode calculations demonstrating the unique capability to deal with large scale kinetic behavior extending over many rational surfaces. The effects of ITG modes on the inward pinch of impurities in 3D slab geometry and on the existence of microtearing modes in 2D slab geometry are also discussed. Finally, sheared toroidal flow effects on trapped-particle modes are presented.

With the advent of simulation techniques [1-3] for the gyrokinetic Vlasov-Maxwell equations and the progress made in simulation algorithms utilizing massively parallel architecture, we have developed an efficient 3D gyrokinetic particle code in toroidal (x=rcosθ, y=rsinθ, ξ=-R_0φ) geometry implemented on the Connection Machine (CM200) for the systematic kinetic investigation of ITG instabilities. In order to gain a better understanding of the linear and nonlinear behavior of these modes, we have

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carried out three types of simulations: linearized [2], partially linearized [2] and fully nonlinear [3]. In the linearized simulation the most unstable modes with ballooning-type mode structures of moderate \((\text{m, n})\) with \(k_r \ll k_\theta\) have been observed to dominate the time evolution of the instability as shown in Fig. 1. This is basically a benchmarking procedure in that it involves comparisons with results from realistic linear toroidal eigenmode calculations [4]. Good agreement is found with respect to both the magnitude of the eigenvalues (within 25%) and the characteristic features of the eigenfunctions, which exhibit strong ballooning in the poloidal direction and radially extend over many rational surfaces. In the partially linearized simulation, the \(E\times B\) advection is the only nonlinearity kept and is the mechanism responsible for the nonlinear saturation of the instability. The results in the nonlinear stage of development indicate that (i) the fluctuations become much more isotropic, i.e., \(k_r = k_\theta\) as shown in Fig. 1; (ii) the energy cascades to both longer and shorter wavelength modes; (iii) the fluctuation spectrum is dominated by a relatively flat distribution at small k's (long wavelength modes); and (iv) the energy flux from the toroidal simulation, which is greatly enhanced over its slab counterpart, scales like gyro-Bohm and peaks slightly toward the outside of the region of maximum growth. Fully nonlinear simulations are also carried out with results compared to those from the partially linearized simulation. The objective here is to look for insights into the

\begin{figure}
\centering
\includegraphics[width=\textwidth]{fig1.png}
\caption{Poloidal cross-section of the electrostatic potential during the linear phase \(\Omega t = 500\) (left) and the saturated turbulent state \(\Omega t = 12\,500\) (right) of the ITG instability for the 128 \(\times\) 128 \(\times\) 64 grid with 1\,048\,576 particles \((L_T = 50\rho_i, \Delta x = \Delta y = 1.5\rho_i, a = 96\rho_i)\).}
\end{figure}
possible origins of ITG transport in steady state. A recent 1D simulation, in which only the velocity-space nonlinearity is kept, has shown the importance of resonance overlap and particle stochasticity in relation to anomalous transport and entropy production in the steady state. This observation is consistent with the previous 2D simulation, where particle stochasticity due to the combined ExB and velocity space nonlinearities has been investigated as the possible cause for the transport [5].

In addition to providing valuable self-consistent benchmarks for the aforementioned gyrokinetic simulations, realistic kinetic toroidal eigenmode calculations have been carried out to support a proper assessment of the influence of long-wavelength microturbulence on transport in tokamaks. This is a very relevant issue in that scattering diagnostics have generally indicated that the largest fluctuation amplitudes occur at the longest measurable wavelengths. New diagnostics [beam emission spectroscopy (BES) and reflectometry] have recently been implemented on TFTR to address this problem. Motivated by these considerations, we have calculated the fully 2D (r,θ) mode structures for long-wavelength trapped-ion and toroidal ITG modes using a comprehensive toroidal finite-element code with the unique capability for evaluating large scale kinetic behavior extending over many rational surfaces. Improvements over previously used numerical procedures [1] were necessary in order to tractably evaluate eigenvalues for realistic tokamak parameters. An important new result from the present studies is that, in contrast to the long-accepted picture predicted by 1D radial eigenmode calculations and supported by earlier idealized (very weak magnetic shear) 2D calculations [4], we find that trapped-ion eigenmodes are not strongly localized between rational surfaces. In fact, even for very long wavelengths the eigenfunctions generally exhibit a strong ballooning character with the associated radial structure relatively insensitive to ion Landau damping at the rational surfaces. This trend is further supported by results obtained from the most comprehensive existent 1D kinetic ballooning code [6] which shows agreement within less than 50% with the eigenvalues from the 2D code. The linear growth rate spectrum for a representative TFTR plasma is displayed in Fig. 2. Results of this type demonstrate that employed together, our 2D and 1D codes provide a unique capability for calculating the linear properties of toroidal microinstabilities over the entire spectral range of interest. It is worthwhile to note that very recent BES measurements in TFTR plasmas have
indicated a possible peaking of the spectrum just beyond $k_0 \rho_i = 0.1$ -- a trend qualitatively similar to that displayed in Fig. 2. Results from the present studies of particular relevance to the development of nonlinear models of microturbulence include a significantly revised picture of the radial extent of long-wavelength instabilities such as trapped-ion modes and a demonstration of the capability to provide a realistic picture of the driven versus damped (inertial) regions of the spectrum.

Impurity transport properties in the presence of microturbulence have also been studied using gyrokinetic simulations. In particular, results from 3D sheared slab simulations have provided insight into the physics responsible for the inward pinch of impurities in the presence of ITG modes. It is found that the parallel acceleration in the poloidal direction and the $E \times B$ advection in the radial direction give rise to the observed inward pinch of the impurity ions in both the linear and nonlinear stages of development. This is true even when both the bulk ions and impurity ions have a flat density profile. However, the pinch is sensitive to the temperature gradient of the impurities as well as the $\eta_1 (=L_n/L_{Ti})$ value for the bulk ions. Because particle diffusion
is intrinsically ambipolar, the inward flux for the impurity ions is compensated by the
outward flux of the primary ions for an instability in which the electron response is
nearly adiabatic. Since the concentration for the impurities is usually very low, their
average inward velocity is, therefore, much higher than that of the primary ions.
Simulations with heavier impurities have also confirmed this trend as well as the Z/M
scaling predicted by our theory. Comparisons with results from ongoing toroidal
simulations of this type as well as with those from experimental studies of impurity
transport [7] are encouraging.

In order to make further progress in simulating realistic tokamak plasmas, it is
necessary to include finite-β physics as well as non-adiabatic trapped-electron
dynamics in our toroidal code. Electromagnetic codes in 1D and 2D slab geometry
have been developed to study the influence of magnetic perturbations on the
stability and transport of microinstabilities in general and on ITG modes in particular.
The results have confirmed the theoretical prediction of finite-β stabilization due to
finite-Larmor-radius effects. Furthermore, we have found that higher radial harmonics
of the ITG eigenmodes cannot be fully stabilized by the finite-β effects, and,
consequently, we have observed the existence of microtearing modes near the
rational surfaces in the simulation. The investigation of microtearing effects on
anomalous transport and the building of a finite-β toroidal gyrokinetic code are
ongoing projects. With respect to non-adiabatic trapped-electron dynamics, a
bounce-averaged response is the most efficient representation since the
characteristic frequencies of interest are well below the electron bounce frequency.
For collisionless trapped-electron modes we have developed a fluid-type model in
which energy and pitch-angle distributions are fixed but the dispersion of toroidal
precession velocities with energy is retained. This captures the main destabilizing
driving mechanism, retains the dominant ExB nonlinearity, and is computable in a
three dimensional space.

In light of their large radial extent, it is important to examine the sensitivity of
trapped-ion eigenmodes to sheared toroidal flow effects. We begin by noting that
valuable insight into the single-particle ion orbit properties can be gained by using the
action-variational method in an extended phase space. Sheared flow is shown to
induce ellipticity of the Larmor orbit which modifies the first adiabatic invariant μ. The
gyrokinetic equation is thereby also modified and can be expressed in terms of the effective potentials, $A^* = A + (m/q)\mathbf{v}_b$, and $\Phi^* = \Phi + (m/2q)\mathbf{v}_b^2$. New analytical formulas for the second adiabatic invariant $J$, the bounce frequency, the precession frequency, the banana radial width, the transit frequency, and the fraction of trapped particles are obtained for flow values up to Mach one. These results indicate that in the context of the simplest local theory, toroidal plasma rotation can increase both the fraction of trapped particles and their precession drift velocity and thereby tend to enhance the destabilization of trapped-ion modes [8]. With respect to trapped-electron modes, we find that toroidal shear flow can enhance the nonlinear ion Landau damping of moderately long-wavelength dissipative trapped-electron modes not only by modifying the beat wave-ion resonance condition but also by changing the radial dependence of the linear susceptibility. At saturation the spectral intensity of the fluctuations is found to scale with flow shear as $(\partial \psi / \partial r)^2$ and decays according to a power law $k_0^{-1}$ [9]. Toroidal rotation can also increase the precession drift frequency influencing the collisionless trapped-electron mode. Here the enhanced trapped-electron Compton scattering results in a stronger forward spectral transfer. Therefore, the longer wavelength part of the spectrum is expected to be suppressed. The implications of these new trends are under investigation.

REFERENCES

MODELLING OF EDGE LOCALIZED MODES AND DYNAMIC RESPONSES OF THE H-MODE

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Abstract

MODELLING OF EDGE LOCALIZED MODES AND DYNAMIC RESPONSES OF THE H-MODE.

On the basis of the electric bifurcation model of the H-L transition, models of the edge localized modes (ELMs) are developed and the dynamic responses of the H-mode are analysed. The spatial and temporal evolution of the H-L transition is formulated in a form of the time dependent Ginzburg-Landau equation which includes transport processes. The equation governs the development of the density and the radial electric field. By solving this equation for the given boundary condition, three classes of solutions pertaining to the H-, L- and ELMy-H states are identified. The ELMy-H state is characterized by the self-generating oscillations under the condition of a constant source. The parameter regions of these states are identified. The radial extent of the transport barrier during the H- and ELMy-H states is characterized by the diffusion Prandtl number. Transient responses of the H-states to the external perturbations are also analysed. A sudden change of the flux from the core causes transient ELMy oscillations. When sinusoidal oscillations are imposed (simulation of sawtoothing), ELMy oscillations with mode locking to the external oscillation frequency (or the subharmonics) appear. This suggests a possible scenario for the ELM control. A model for the giant ELMs is also developed. A ballooning instability triggers the pulsive loss flux to the edge region. If its magnitude exceeds the threshold, the transport barrier is destroyed for a short period and a large solitary pulse is generated in the outflux.

1. Introduction

Edge localized modes (ELMs) usually follow the L-to-H-mode transition, show a variety of appearances in the magnitude and frequency of the bursts and occur in a restricted parameter space of the H-phase [1]. The H-mode with small and frequent ELMs is a candidate for the standard operation in the experimental tokamak reactor because, at present, improved confinement

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compatible with the efficient ash exhaust is only found in this operation mode. The giant ELMs are associated with the large heat pulse and affect the plasma facing components unfavourably. The characterization of ELMs has just started, and the research aiming at an understanding of the ELMs and at controlling them is an urgent task. Extending the bifurcation model of the H-mode[2], we discuss the model of small and frequent ELMs and its parameter range. The dynamic responses of the H-mode to the external perturbation and the giant ELM model are presented.

2. Model

We are interested in the slab region near the plasma edge, \(-L<x<0\), \((x=r-a;\) r is the minor radius\). The radial structures of the density and the radial electric field (or poloidal rotation) are studied, the poloidal and toroidal variations being neglected. The Poisson equation, combined with the equation of ion motion, is written as \(\varepsilon \varepsilon_0 \partial E_r / \partial t = \varepsilon (\Gamma_e - \Gamma_i)\), where \(\varepsilon_1\) is the perpendicular dielectric constant. The effective diffusivity is usually written as \(D \equiv D_0 [E_r] \nabla n\). The model equations consist of the radial transport equations for the density and the normalized radial electric field \(Z\) with viscous diffusivity \(\mu\). The equations are given in dimensionless form by [3]:

\[
an/\partial t = a/ax D(Z) an/ax \tag{1}
\]

\[
\varepsilon aZ/\partial t = -N(Z, g) + \mu a^2 Z/\partial x^2 \tag{2}
\]

The parameter \(\varepsilon\) is a small coefficient \((\rho_p/\rho)\), \(\rho_p\) and \(\rho\) are the poloidal and toroidal ion gyroradii, respectively, showing that Eq. (2) has a faster timescale than Eq. (1) when \(\mu\) and \(D\) have similar magnitudes. The non-linear term \(N\) corresponds to the local part of \(\Gamma_i - \Gamma_e\), which arises from ion orbit loss, drift wave convection, ripple loss and ion parallel viscosity[2,4]. The variable \(g\) corresponds to \(\lambda d\), where \(\lambda = \rho_p n'/n\), \(d = D_0 / \nu \rho_p^2\), \(\nu\) is the ion collisionality, and \(D_0\) is a typical electron diffusivity at the \(L\)-phase. Length and time are normalized to \(\rho_p\) and \(\rho_p^2 / D_0\), \(D\) and \(\mu\) to \(D_0\), and \(\Gamma\) to \(D_0 n_0 / \rho_p\), respectively. To obtain analytic insight, we use in
FIG. 1. (a) Model of effective diffusivity $D$ as a function of gradient parameter $g$ ($\alpha = \beta = 1$, $D_{\text{max}} = 3$, $D_{\text{min}} = 0.01$, $g_0 = 1$). See the text for definitions. (b) Temporal evolution of outflux from the plasma surface ($\alpha = \beta = 0.2$, $\mu = 1$, $\Gamma_{\text{in}} = 3$, $\lambda(0) = 4/5$). (c) Spatial profile of $D$ in high (solid line) and low (dashed line) confinement phases. $\Delta = \sqrt{2}$ in this case. (See arrows in (b) for specification of time.)

the model $S$-figure curve for $N$ the form $N(Z,g) = g - g_0 + [gZ^0 - \alpha Z]$ and $D$ is assumed as $D(Z) = D_{\text{max}} + D_{\text{min}}$ tanh$Z$, where $D_{\text{max}} = (D_{\text{max}} + D_{\text{min}})/2$. $n_0$ is chosen to satisfy $g_0 = 1$. The parameters $\alpha$, $\beta$, $g_0$, $D_{\text{max}}$, and $D_{\text{min}}$ are treated as constants. An example of $D(Z,g)$ for $N(Z,g) = 0$ is shown in Fig. 1(a).

The following boundary conditions at $x=-L$ and $x=0$ are chosen: (1) the particle flux $\Gamma_{\text{in}}$ is given at $x=-L$ and (2) $n'/n$ is constant at $x=0$, following the result of the transport simulation in the scrape-off layer[5].

3. $H$-, $L$- and ELMy-H states

For the case of constant influx $\Gamma_{\text{in}}$, three classes of solutions of Eqs (1) and (2) are found. Two of them are constant in time and attributed to the $H$- and $L$-states. The former occupies a layer near the edge ($x=0$) in which the effective diffusivity is low, even though a steep gradient near the edge is formed. This is the transport barrier of the $H$-mode. Its thickness, $\Delta$, is given by $\Delta = \sqrt{2}\beta/\alpha$. In this barrier, $D$ takes an intermediate value between $D_{\text{max}}$ and $D_{\text{min}}$; this layer is called the mesophase. Non-linear solutions of self-generated oscillations of $n$ and $\Gamma_{\text{out}}$ are found for constant supply of $\Gamma_{\text{in}}$. This oscillation appears near the $L$ and $H$ phase.
boundary. Figures 1(b) and (c) illustrate the temporal evolution of the pulsed outflux and the spatial variation of D. The thickness of the transport barrier is also given by $\Delta = \sqrt{2} \beta \mu / \alpha$. This oscillatory solution is found in the parameter region of $D_m / g_m < \Gamma_{in} \lambda(0)^2 < D_M / g_M$. The interpretation in terms of physical parameters depends on the model of the ion flux $\Gamma(E)$. An extension of Ref. [2] gives $\alpha = 3 \beta = 3 \sigma_+ / 2 \sigma_- \left( \sigma_\pm = 1 \pm \sqrt{n(2e/d)} \right)$. $D_m = d$, $D_M = d / 2(\ln 2e / d)$. Figure 2 shows the regions of L, ELMy-H and H states in the $(d, \lambda(0)/\sqrt{\Gamma_{in}})$ plane. The dashed lines denote the contours of the oscillation frequency normalized to $\rho_p^2 / D_0$. The shaded area indicates the bistable region where both H- and L-solutions are possible and the stationary solution depends on the initial condition.

The gain in confinement time by pedestal formation depends strongly on the thickness $\Delta$. The result shows the importance of the Prandtl number, $\mu / D_0$.

4. Transient response

A variety of dynamic responses of these states to external perturbations are found. The case near
FIG. 3. Driven oscillation in $\Gamma_{\text{out}}$ by external oscillation in $\Gamma_{\text{in}}$ ($\Omega = 2\pi$). $\lambda(0) = 1$, $\Gamma_{\text{in,0}} = 3$, (a) $\Gamma = 5.5$ and (b) 6.0. Period doubling (a) and mode locking (b) are observed.

the ELMy-H region in the d-\(\lambda\) plane is studied. The density pulses, which are caused by sawtoothing or MHD Mirnov oscillations, are simulated by the temporal change of $\Gamma_{\text{in}}$. A sinusoidal oscillation is added to give $\Gamma_{\text{in}} = \Gamma_{\text{in,0}} + \Gamma \sin(\Omega t)$. Depending on the amplitude $\Gamma$, mode locking to the applied frequency $\Omega$ and to its subharmonics are found (Fig. 3). By this perturbation, the region of the periodic oscillation solution is broadened in the (d, \(\lambda\)) plane. This suggests the possibility of enlarging the operational region for the mode with small and frequent ELMs.

A step-like change in $\Gamma_{\text{in}}$ can induce transition from the L- to the H-phase, if the $\Gamma_{\text{in}}$ in the initial and final states are those in the L and H-states, and vice versa. A few oscillations in $\Gamma_{\text{out}}$ preceding the transition are shown in Fig. 4. This illustrates that an L-to-H-mode transition takes place, passing through the ELMy-H state.

5. Giant ELMs

As transient response of H-states, the giant ELM model is developed. This giant ELM is due to a pulsive change in $\Gamma_{\text{in}}$ caused by the ballooning instability. The magnetic shear is strong near the edge, and the mode can become unstable inside the transport
barrier($x_1^-\Delta$). The dynamics of the ballooning mode near critical beta is studied, and pulsive growth and decay of the perturbation are shown[6]. This enhanced loss process within the finite radial extent is modeled by superposing an additional diffusivity, $D_{\text{add}}$, on $D$ in the region $x_1^-x<x_2^-\Delta$ for a time interval of $t_1^-t<t_2^-$. $x_1$, $x_2$ and $t_2^-t_1$ are treated as parameters in this article.

When the magnitude of $D_{\text{add}}$ exceeds a certain threshold for given $x_1$ and $x_2$, the flux impulse caused by local density flattening in the $x_1^-x<x_2$ region destroys the edge H-states for a short period and generates the large pulse in $\Gamma_{\text{out}}$ (Fig.5). The delay of the burst in $\Gamma_{\text{out}}$ is some fraction of the period of the ELMy-H oscillation. The existence of the threshold in $D_{\text{add}}$ may explain the experimental observation that the MHD perturbation must exceed a certain value to cause the giant ELW [7].

6. Summary

The theoretical ELWs model is developed as an extension of the H-mode bifurcation model. A self-generated oscillation of the density $n$ and the loss flux $\Gamma_{\text{out}}$ is found. We have identified the parameter ranges for three states and attributed them to the L- H- and ELMy-H modes. The dynamic responses of the
states to external perturbations in the influx from the core are investigated. The occurrence of mode locking to an applied frequency and of a transition triggered by the impulse are shown. The giant ELM model is also presented. These analyses provide the basis for exploring the method of controlling the ELMs in future experiments.

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REFERENCES

**K–ε MODEL OF ANOMALOUS TRANSPORT IN RESISTIVE INTERCHANGE TURBULENCE**

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**Abstract**

*K–ε MODEL OF ANOMALOUS TRANSPORT IN RESISTIVE INTERCHANGE TURBULENCE.*

A K–ε anomalous transport model for resistive interchange turbulence is presented and applied to the transport analysis of ECH plasmas in Heliotron E. In this model, the turbulent kinetic energy $K = \frac{1}{2} \langle \vec{v}^2 \rangle$ and its viscous dissipation rate $\epsilon$ characterize the local turbulence and the anomalous transport coefficient is given by $D \sim K^2/\epsilon$, which has some nonlocal properties not included in the conventional expressions since their temporal and spatial variations are determined by taking into account the transport of the turbulent energy itself. In the case of the homogeneous turbulence where the anomalous transport may be described in terms of the local plasma parameters, the dimensional analysis applied to the model yields the two types of local parameter expressions of the anomalous diffusivity in the high and low collisional cases. The authors find a familiar diffusivity for the resistive interchange turbulence derived in the high collisional case and another one, similar to the gyro-reduced Bohm (GRB) diffusivity, in the low collisional case. However, it is shown from the transport simulation using the model that, in the region where the turbulence inhomogeneity is significant, the anomalous diffusivity deviates from the local parameter expression due to the transport terms in the K–ε equations. The model explains the experimental results consistently in that it gives the GRB or LHD scaling for the energy confinement time and reproduces the experimentally obtained profile of the anomalous diffusivity, which has large values in the peripheral region, in contrast with the GRB model.

**1. INTRODUCTION**

Conventional treatments for anomalous transport have been based on the local transport coefficients ($D$ or $\chi$) which are expressed as functions of local plasma parameters such as local density $n$, temperature $T$, magnetic field $B$ and a number of gradient scale lengths $L_n, L_T, L_s, \cdots$:

$$D \text{ or } \chi = F(n, T, B, L_n, L_T, L_s, \cdots).$$

These treatments assume that the mixing length $l$ and the time scale $\tau$ of the turbulence responsible for the anomalous transport are determined by
the local plasma parameters. However, it is possible that the turbulence structure has a nonlocal nature and the validity of the expression for the local transport coefficients as given above is limited.

Here we present a $K$-$\epsilon$ type model for the analysis of anomalous transport in the resistive interchange turbulence. A $K$-$\epsilon$ model was originally proposed for modeling the turbulent (or eddy) viscosity of the large Reynolds number turbulent shear flow [1]. In the $K$-$\epsilon$ model the turbulent kinetic energy $K \equiv \frac{1}{2} \langle \vec{v}^2 \rangle$ and its viscous dissipation rate $\epsilon$ characterize the local turbulence spectral structure and their temporal and spatial variations are governed by transport equations. The turbulent transport coefficient is given by $D \sim K^2/\epsilon$, which has some nonlocal properties not included in the conventional expressions since the mixing length $l \sim K^{3/2}/\epsilon$, the turbulent time scale $\tau \sim K/\epsilon$ and the turbulent transport coefficient $D \sim l^2/\tau \sim K^2/\epsilon$ are determined not locally but globally by the solution of $K$-$\epsilon$ transport equations.

The resistive interchange turbulence has been extensively studied as a cause of anomalous transport in the peripheral region of stellarator plasmas [2,3]. The $K$-$\epsilon$ equations for the resistive interchange turbulence have the turbulent energy production terms, which are given by the pressure gradient multiplied by the average magnetic curvature, the viscous and Joule dissipation terms, and the transport terms. In the case of the homogeneous turbulence, the transport terms vanish and the anomalous transport may be well described in terms of the local plasma parameters. However, when the turbulence is significantly inhomogeneous, the anomalous diffusivity deviates from the local parameter expression due to the transport terms in the $K$-$\epsilon$ equations.

2. $K$-$\epsilon$ ANOMALOUS TRANSPORT MODEL

The $K$-$\epsilon$ model for anomalous transport in resistive interchange turbulence was derived in Ref.[4] by applying two-scale direct-interaction approximation (TSDIA) [5] to the resistive magnetohydrodynamics (MHD) equations. Assuming that the mean velocity vanishes $\langle \vec{v} \rangle = 0$ and that there is the inhomogeneity of turbulence only in the minor radial direction, statistical analyses of the resistive MHD equations show that the equations governing the turbulent kinetic energy $K \equiv \frac{1}{2} \langle \vec{v}^2 \rangle$ and its dissipation rate $\epsilon$ are written as follows,

$$\frac{\partial K}{\partial t} = P_K - \epsilon - \epsilon J + T_K$$  \hspace{1cm} (1)

$$\frac{\partial \epsilon}{\partial t} = C_1 \frac{\epsilon}{K} P_K - C_2 \epsilon^2 \frac{\epsilon}{K} - C_3 \epsilon J + T_\epsilon.$$  \hspace{1cm} (2)
where \( \epsilon_J \) denotes the Joule dissipation term and the turbulent energy production term \( P_K \), the transport terms \( T_K \) and \( T_\epsilon \) are given by

\[
P_K = \langle \tilde{p} \tilde{v}_r \rangle \cdot \frac{1}{\rho_m} \frac{d\Omega}{dr}
\]

\[
T_K = \frac{1}{r} \frac{\partial}{\partial r} \left( r C_K \frac{K^2}{\epsilon} \frac{\partial K}{\partial r} \right), \quad T_\epsilon = \frac{1}{r} \frac{\partial}{\partial r} \left( r C_\epsilon \frac{K^2}{\epsilon} \frac{\partial \epsilon}{\partial r} \right)
\]

Here \( \rho_m \) denotes the mass density, \( c \) the light velocity in a vacuum, \( \eta \) the resistivity, \( B \) the magnitude of the magnetic field, and \( d\Omega/dr \) the average magnetic curvature. As seen from (3), the turbulent energy production is in the form of the product of the flux and the centrifugal force due to the average magnetic curvature. The turbulent pressure flux \( \langle \tilde{p} \tilde{v}_r \rangle \) is expressed in terms of the mean pressure gradient \( dP/dr \) and the turbulent diffusivity \( D_p \) as

\[
\langle \tilde{p} \tilde{v}_r \rangle = -D_p \frac{dP}{dr} = -C_p \frac{K^2}{\epsilon} \frac{dP}{dr}.
\]

In the same manner as in (5), the turbulent diffusivity for passive scalars in the turbulent velocity field is given by

\[
D_\theta = C_\theta \frac{K^2}{\epsilon}.
\]

\( C_K, C_\epsilon, C_{e1}, C_{e2}, C_{eJ}, C_p, \) and \( C_\theta \) are non-dimensional constants, which are determined empirically or theoretically from TSDIA.

With the electrostatic approximation, the Joule dissipation term \( \epsilon_J = \eta J^2 / \rho_m \) can be expressed in terms of \( K \) and \( \epsilon \) as follows. For high collision frequencies such that \( L_s^2 m_e \nu_e T_e > \tau (\sim K/\epsilon) \), we have

\[
\epsilon_J = C_J \frac{B_0^2}{\rho_m \epsilon^2 \eta L_s^2} \frac{K^4}{\epsilon^2} \quad \text{for} \quad L_s^2 m_e \nu_e T_e > \tau
\]

where \( L_s \) is a magnetic shear length and \( C_J \) a non-dimensional numerical constant. Here we used the Ohm's law \( J \sim \eta^{-1} k_\parallel \tilde{\phi} \) and estimated that \( K \sim \tilde{v}^2 \sim (c k_\perp \tilde{\phi} / B)^2 \) and \( k_\parallel / k_\perp \sim l / L_s \). On the other hand, for low collision frequencies such that \( L_s^2 m_e \nu_e / T_e < \tau \), we need to take account of the adiabaticity of electron response to potential fluctuations to evaluate \( \epsilon_J \). From the balance between the time scales of the electron parallel conduction and the potential fluctuation \( k_\parallel^2 T_e / (m_e \nu_e) \sim \tau^{-1} \) with \( k_\parallel \sim k_\perp \Delta / L_s \), the width of the non-adiabatic layer \( \Delta \) is given by \( k_\perp^2 \Delta^2 \sim L_s^2 (m_e \nu_e / T_e) \tau^{-1} \). Therefore, in this low collisional case, \( \Delta \) becomes smaller than the turbulent mixing length \( l \sim K^{3/2} / \epsilon \). Using the
generalized Ohm’s law \( \mathcal{J} \sim (T_e/e\eta)k_\parallel(\bar{n}/n_0 - e\phi/T_e) \) and noting that we have the Boltzmann relation \( n/n_0 \sim e\phi/T_e \) outside the non-adiabatic layer and \( n/n_0 < e\phi/T_e \) inside it, we obtain \( \mathcal{J}^2 \sim \eta^{-2}(\Delta/L_s)^2(k_L^2\phi)^2 \). From these relations, we have

\[
\epsilon_I = C'_I \frac{1}{\rho_s^3} \frac{K^3}{\epsilon} \quad \text{for} \quad L_s^2 \frac{m_e \nu_e}{T_e} < \tau
\]

where \( C'_I \) is a numerical constant and \( \rho_s = c\sqrt{m_iT_e}/(eB_0) \) the ion Larmor radius at the electron temperature.

If the profiles of the mean pressure gradient \( dP/dr \) and the magnetic curvature \( d\Omega/dr \) are given, we can obtain the turbulent diffusivity (6) by solving the \( K-\epsilon \) transport equations (1) and (2) with proper boundary conditions imposed. The self-consistent model for the analysis of anomalous transport is obtained by combining the \( K-\epsilon \) equations with the transport equations for the density and temperature. The profiles of the density and temperature affect the turbulence through the production and dissipation terms in the \( K-\epsilon \) equations while the turbulence has dominant effects on the transport of the density and temperature through the anomalous diffusivities. The diffusion terms given by (4) describe the radial propagation of the turbulence which have not been taken into account by the conventional anomalous transport model.

3. SCALING IN TERMS OF LOCAL PARAMETERS

Here, we consider the stationary case in which the transport term in the turbulent energy can be neglected so that the energy production and dissipation terms are balanced with each other. Even then, the turbulent energy flux does not need to vanish although it should have a constant value. Then we obtain from (1), (3) and (5),

\[
C_p \left( \frac{1}{\rho_m} \frac{dP}{dr} \frac{d\Omega}{dr} \right) \frac{K^2}{\epsilon} - \epsilon - \epsilon_I = 0
\]

In this case, as seen from (7)-(9), it can be assumed that the turbulence property is characterized locally by two parameters, which are for high collision frequencies \( (L_s^2 m_e \nu_e/T_e > \tau) \),

\[
\frac{-1}{\rho_m} \frac{dP}{dr} \frac{d\Omega}{dr}, \quad \frac{\rho_m c^2 \eta L_s^2}{B_0^2}
\]

and for low collision frequencies \( (L_s^2 m_e \nu_e/T_e < \tau) \),
In the case considered here, the turbulence is regarded as locally homogeneous and these parameters need to be nearly constant. When the variation of the parameters is large, the turbulence can no longer homogeneous and the turbulent energy transport term becomes important so that the assumption given above is not valid. The first parameter of (10) or (11) is written as \(-P'\Omega'/\rho_m \sim c_s^2/(L_p L_c)\) and has a dimension of square frequency, which gives the characteristic time scale \( \sqrt{L_p L_c}/c_s \) of the turbulence driven by the pressure gradient and the magnetic curvature. Here \(dP/dr = P' = -P/L_p\), \(d\Omega/dr = \Omega' = 1/L_c\) and \(c_s = \sqrt{T_e/m_i}\) are used and \(T_e \geq T_i\) is assumed.

We have the following scaling in terms of the above local parameters from the dimensional analysis. First, in the high collisional case,

\[
K \sim c_s^2 \eta \rho_m^{-1/2}(-P'\Omega')^{3/2} L_p^3/B_0^3, \quad l \sim K^{3/2}/\epsilon \sim c_s \eta^{1/2}(-\rho_m P'\Omega')^{1/4} L_p^2/B_0^2, \\
\epsilon \sim c_s^2 \rho_m^{-1}(-P'\Omega')^2 L_p^4/B_0^4, \quad \tau \sim K/\epsilon \sim (-P'\Omega'/\rho_m)^{-1/2} \sim (c_s/\sqrt{L_p L_c})^{-1}.
\]

Then we obtain the anomalous diffusivity \(D\) as

\[
D \sim \frac{K^2}{\epsilon} \sim \frac{l^2}{\tau} \sim \frac{c_s^2 \eta(-P'\Omega') L_p^2}{B_0^2} \sim \frac{D_d L_p^2}{L_p L_c}
\]

where we used the classical diffusivity \(D_d = c^2 \eta P/B_0^2\). Equation (13) is the same expression as that of the anomalous diffusivity for the resistive interchange turbulence obtained from the reduced MHD equations using the dimensional analysis or the scale invariance technique. Using the time scale \(\tau \sim \sqrt{L_p L_c}/c_s\), we can write the condition for the high collisional case as \(L_p^2 m_e \nu_e/T_e > \sqrt{L_p L_c}/c_s\).

Next, in the same way as above, we obtain the scaling in the low collisional case as follows

\[
K \sim \rho_s^2 c_s^2 L_p L_c, \quad l \sim K^{3/2}/\epsilon \sim \rho_s, \quad \\
\epsilon \sim \rho_s^2 c_s^2/(L_p L_c)^{3/2}, \quad \tau \sim K/\epsilon \sim (c_s/\sqrt{L_p L_c})^{-1}.
\]

which gives the anomalous diffusivity \(D\) as

\[
D \sim \frac{K^2}{\epsilon} \sim \frac{l^2}{\tau} \sim \frac{\rho_s^2 c_s}{\sqrt{L_p L_c}} \sim \frac{D_B \rho_s}{\sqrt{L_p L_c}}
\]

where we used the Bohm diffusivity \(D_B = \rho_s c_s = c T_e/(e B_0)\). Equation (15) has the form of the gyro-reduced Bohm (GRB) diffusivity [6] which
is the Bohm diffusivity multiplied by the factor \( \rho_s/\sqrt{L_pL_c} \). The condition for the low collisional case is written as \( L_p^2 m_e \nu_e / T_e < \sqrt{L_pL_c}/c_s \).

4. TRANSPORT SIMULATION OF ECH PLASMAS IN HELIOTRON E

We have combined the K-\(\epsilon\) anomalous transport model with the transport code for stellarators [7] to simulate ECH plasmas in Heliotron E (\(R = 2.2m, a = 0.2m\)) [8,9]. The anomalous particle diffusivity \( D \) is given by (6). We have assumed that both the electron and ion thermal diffusivities are given by the same expression \( \chi_e = \chi_i = \frac{\kappa}{2}D \). In our simulations, the averaged electron density \( \bar{n}_e \), the ECH absorbed power \( P_{abs} \) and the magnetic field strength \( B \) were scanned in the following ranges: \( 1 \leq \bar{n}_e \leq 3 \times 10^{19}m^{-3}, 96 \leq P_{abs} \leq 288kW \) and \( B = 0.95, 1.9T \), respectively. We have done numerically the time integration of the electron and ion temperatures, \( T_e, T_i \) as well as the turbulent energy \( K \) and its dissipation rate \( \epsilon \) while the electron density profile was fixed. We gave the profiles of the electron density and the absorbed power density as \( n_e(r) = n_e(0) [0.95 (1 - (r/a)^6) + 0.05] \) and \( p_{abs}(r) = p_{abs}(0) (1 - (r/a)^4)^2 \) which fit the experimentally observed results. The average magnetic curvature \( d\Omega/dr \) was given from the vacuum magnetic field configuration of Heliotron E since the beta values of the ECH plasmas simulated here are very low. The neoclassical diffusivities were included in our simulations although the effects of the radial electric field and the neutral particles are assumed to be negligible in order to clarify the effects of the anomalous transport. In the parameter regime of the ECH plasmas in Heliotron E, the whole plasma is considered to satisfy the low collision frequency condition and therefore we employed the Joule dissipation term given by (8). The numerical constants used here are \( C_K = 0.09, C_\epsilon = 0.07, C_{\epsilon 1} = C_{\epsilon 2} = C_{\psi 1} = 1.7, C_p = C_\phi = 0.135 \) and \( C' = 0.05 \).

After adequate time steps, we obtained the stationary states in which the radial profiles of \( T_e, T_i, K \) and \( \epsilon \) did not depend on the initial conditions. Figure 1 shows the radial profile of the anomalous thermal diffusivity obtained by the K-\(\epsilon\) model, \( \chi_e^{K-\epsilon} = \frac{\kappa}{2}C_\phi K^2/\epsilon \), in the stationary state for \( \bar{n}_e = 1 \times 10^{19}m^{-3}, P_{abs} = 192kW \) and \( B = 1.9T \). In this case, the boundary conditions for \( K \) and \( \epsilon \) were given such that the energy confinement time took the experimentally observed value. There also shown is the profile of the anomalous diffusivity expressed in terms of the local parameters as in (15), \( \chi_e^{local} \equiv C (\rho_s/\sqrt{L_pL_c}) c(T_e+T_i)/eB \), where the value of the numerical coefficient employed in Fig.1 is \( C = 0.57 \). It can be seen that both of the diffusivities have the same radial dependence in the region \( 0.1 < r/a < 0.6 \).
while the discrepancy between their profiles becomes large in the other regions. Especially, in the peripheral region, $\chi_e^{K-\varepsilon}$ increases in approaching the boundary whereas $\chi_{e,local}^{local}$ decreases. Figure 2 shows the radial profiles of the turbulent energy production, viscous and Joule dissipations in the same case as in Fig. 1. It is found that the production and dissipation occur mostly near the peripheral region where the average magnetic curvature becomes large. In this case, the inward transport of the turbulent energy appears. The ratio of the Joule dissipation to the viscous one increases in the peripheral region due to the decrease in the temperature, which is correlated with the deviation of $\chi_e^{K-\varepsilon}$ from $\chi_{e,local}^{local}$ since the ratio between
the production, viscous and Joule dissipations needs to be homogeneous or constant in order to ensure the validity of the scaling by the local parameters. Thus the local parameter expression poorly predicts the anomalous transport coefficient in the peripheral region where the inhomogeneities of the local parameters in (11) are significant.

In Fig. 3, the experimentally obtained thermal diffusivity $\chi_e^{\text{exp}}$ in the case corresponding to Fig. 1 is compared with the numerically predicted total diffusivity $\chi_e^{\text{total}} = \chi_e^{K-\epsilon} + \chi_e^{\text{neo-ax}} + \chi_e^{\text{ripple}}$, where $\chi_e^{\text{neo-ax}}$ and $\chi_e^{\text{ripple}}$ denote the neoclassical axisymmetric and non-axisymmetric (ripple) thermal diffusivities, respectively. The disagreement between $\chi_e^{\text{total}}$ and $\chi_e^{\text{exp}}$ seems to be within the accuracy of the experimental results although the predicted diffusivity $\chi_e^{\text{total}}$ may be relatively smaller than $\chi_e^{\text{exp}}$ in the inner region $r < 0.3a$ since there the magnetic curvature is quite small and other turbulence sources are not taken into account in our model. It is seen that the anomalous diffusivity is a dominant contribution to the whole plasma confinement although $\chi_e^{\text{ripple}}$ is comparable to $\chi_e^{K-\epsilon}$ at $0.3 < r/a < 0.5$ and $\chi_e^{\text{neo-ax}}$ is the largest at $r/a < 0.1$.

We have scanned the electron density, the absorbed power and the magnetic field strength in the ranges mentioned earlier. Since we have seen in Fig. 1 that the local parameter expression (15) is valid in the regions except for the peripheral and central regions, we adjusted the boundary conditions for $K$ and $\epsilon$ in all the simulations in the above ranges such that the $K-\epsilon$ anomalous diffusivity coincides with the local expression at $r = a/2 : \chi_e^{K-\epsilon}(a/2) = \chi_e^{\text{local}}(a/2)$. In Fig. 4, the energy confinement times $\tau_{E}^{K-\epsilon}$ obtained from the simulations are compared with the LHD scaling

![FIG. 3. Radial profiles of the neoclassical axisymmetric thermal diffusivity, $\chi_e^{\text{neo-ax}}$, the neoclassical ripple diffusivity, $\chi_e^{\text{ripple}}$, the $K-\epsilon$ anomalous diffusivity, $\chi_e^{K-\epsilon}$, the total diffusivity, $\chi_e^{\text{total}} = \chi_e^{K-\epsilon} + \chi_e^{\text{neo-ax}} + \chi_e^{\text{ripple}}$, and the experimentally obtained diffusivity, $\chi_e^{\text{exp}}$.

\begin{align*}
0 & \quad 0.5 & \quad 1 \\
\log_{10}(\text{m}^2/\text{s}) & \quad \chi_e^{\text{total}} & \quad \chi_e^{\text{exp}} \\
10^{-1} & \quad \chi_e^{K-\epsilon} & \quad \chi_e^{\text{neo-ax}} \\
10^{-2} & \quad \chi_e^{\text{ripple}} & \quad \chi_e^{\text{local}} \\
10^{-3} & \quad \frac{r}{a} \quad 0.5 & \quad 1 
\end{align*}
FIG. 4. Comparison between the energy confinement times obtained from the simulations using the $K$-$\varepsilon$ model, $\tau_{E}^{K-\varepsilon}$, and those of the LHD scaling [10], $\tau_{E}^{LHD}$.

It is found that the simulation results are in good agreement with the LHD scaling. This seems to be natural since the thermal diffusivity imposed at $r = a/2$ obeys a type of GRB scaling which gives almost the same energy confinement time as the LHD scaling. Thus our model predicts the experimental results consistently in two aspects: the first is that it gives the energy confinement time following the GRB or LHD scaling and the second is that it overcomes the drawback of the GRB diffusivity, i.e., it reproduces the experimentally observed profile of the anomalous diffusivity which has large values in the peripheral region.

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STOCHASTICITY DRIVEN DISRUPTIVE PHENOMENA IN TOKAMAKS

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Abstract

STOCHASTICITY DRIVEN DISRUPTIVE PHENOMENA IN TOKAMAKS.

A theoretical model of the generic mechanism in disruptive phenomena of tokamak plasma such as sawtooth oscillations and major disruptions is presented. The MHD helical instability combined with toroidicity gives rise to an overlapping of the secondary magnetic islands near the separatrix of the primary islands. If the perturbation amplitude exceeds a critical value, global stochasticity sets in and the region of the braided field lines expands. In this region, the transport coefficients are strongly enhanced and change the mode stability itself. Enhanced electron viscosity (i.e. the current diffusivity) destabilizes the mode further, leading to an acceleration of the mode growth. This mechanism is proposed for the magnetic trigger observed before a sawtooth crash and major disruption. The enhanced thermal conductivity causes a thermal collapse. The mode can also be stabilized by enhanced ion viscosity. Variations in the phenomena of partial/full temperature collapse with partial/full magnetic field reconnection depend on the widths of the stochastic region and the magnetic island. — A dynamic sawtooth model which includes \( m = 1 \) mode growth and enhanced transport coefficients is introduced. This model describes sawteeth with \( q(0) < 1 \). The effect of finite beta is also studied. The major disruption is explained by the \( m = 2 \) mode interacting with the toroidicity induced stochasticity. The role of stochasticity for both major disruption and sawteeth increases with the strength of magnetic shear.

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1. INTRODUCTION

Disruptive phenomena in tokamak, such as sawtooth oscillations and major disruptions, are characterized by slow initial growth of a magnetic perturbation followed by rapid mode growth and fast temperature crash. We propose a new theoretical view of these phenomena based on enhanced transport induced by the stochastic magnetic field near the island separatrix. We find that the enhancement of electron and ion viscosities strongly modifies the physics picture of these disruptive phenomena.

2. ROLE OF STOCHASTICITY

In a sheared magnetic configuration, helical magnetic perturbations with different periodicity give rise to stochasticity of the field lines. When the amplitude of the perturbation exceeds a threshold value, a layer of stochastic field lines is formed near the separatrix of a magnetic island. In a toroidal plasma, a single helicity perturbation interacting with toroidicity produces a stochastic layer [1]. A typical Poincaré map of the field lines for $m/n = 1/1$ perturbation is shown in Fig. 1.

![Poincaré map of the field line in a tokamak with $m = 1$ magnetic perturbation ($r_i/R = 0.1$, $s = 0.4$, $\tilde{B}_n = 0.025$).](image)

FIG. 1. Poincaré map of the field line in a tokamak with $m = 1$ magnetic perturbation ($r_i/R = 0.1$, $s = 0.4$, $\tilde{B}_n = 0.025$).
In the stochastic layer, the radial diffusion of the magnetic field lines with diffusivity, $D_M$, strongly enhances the electron and ion viscosities as well as the electron thermal diffusivity, $\chi_e = (3/\sqrt{\pi})v_{te}D_M$, over the classical collisional values. The current diffusivity, $\lambda = (\sqrt{\pi}/2)(\mu_0 c^2/\omega_p^2)v_{te}D_M$, due to the enhanced electron viscosity, $\mu_e = (1/\sqrt{\pi})v_{te}D_M$, drives fast reconnection and further destabilizes the MHD mode [2]. This positive feedback mechanism leads to sudden acceleration of the growth of the helical perturbation, observed as a 'magnetic trigger', before the sawtooth crash in large tokamaks.

The enhanced radial heat diffusion flattens the temperature profile in the stochastic region and leads to a temperature crash. Although the current diffusion seriously affects MHD mode stability, the flattening of the current profile takes place more slowly by a factor of $\lambda/\mu_0\chi_e(\Delta t)^2 \sim c^2/\omega_p^2(\Delta t)^2$.

When $D_M$ becomes large enough, the stabilizing effect due to the enhanced ion viscosity $\mu_i$, even though it is a factor of $v_{ti}/v_{te}$ less than $\mu_e$, may overcome the current diffusivity driven instability. In such a case, complete flattening of the temperature profile is prevented, which is observed as a partial temperature crash.

3. SAWTOOTH OSCILLATION

3.1. Model of triggering

When the safety factor on axis, $q(0)$, is less than unity, the $m/n = 1/1$ MHD mode can be unstable and a magnetic island grows on the $q = 1$ surface ($r = r_1$). We develop a simple dynamical model of the sawtooth oscillation, including a stochastic trigger to an enhanced growth rate and stabilization by ion viscosity [2]. Assuming that the linear growth rate is not strongly modified in the presence of a finite amplitude magnetic island [3], we obtain the growth rate in the stochasticity-dominant phase [2]:

$$\gamma = \gamma_0 - \frac{\mu_i}{w^2}$$

where the time is normalized by $\tau_A = R/v_A$, $\gamma_0 = \gamma_\lambda = 5(2\epsilon s)^{4/5} \tilde{\lambda}^{1/5}$, $w = (2\epsilon s)^{-1/3} \times (\tilde{\lambda}^2 \gamma_0)^{1/6} r_1$, $\tilde{\lambda} = \lambda \tau_A/\mu_0 r_1^4$, $\epsilon = r_1/R$, where $v_A$ is the Alfvén velocity and $s$ the shear parameter.

Figure 2(a) shows typical trajectories on the shear and perturbation amplitude space with a parabolic $q$-profile ($s = [1 - q(0)]/q(0)$, $\tilde{B}_s = R_0 \tilde{B}_s/r_1 B_0$). The solid line (1) corresponds to values for which the island separatrix touches the magnetic axis (i.e., complete magnetic reconnection). The line (2) indicates values for which the stochastic layer reaches the axis (i.e. central temperature crash). The dashed line (3) designates the critical value for the onset of the stochasticity enhanced reconnection rate (the growth of the MHD mode is accelerated). The ion viscosity stabilizes the mode above the dot-dashed line (4). Our model predicts that four types
FIG. 2. (a) Schematic trajectories of the sawtooth evolution in the shear parameter $s$ and normalized magnetic perturbation $\tilde{B}_n$ space. (b) Typical temporal evolution of sawtooth oscillation of type (I). ($r_i/R = 0.1$, $\mu_i/\lambda = 1.8 \times 10^4$, $\omega_p^2 r_i^2/c^2 = 7.8 \times 10^4$, and $\nu_T \sim \nu_A$).
[(I)-(IV)] of the sawtooth oscillations are possible even for a plasma with monotonic $q(r)$ profile. These possible variations in oscillation type explain the fast crash, the magnetic trigger and varieties of the sawtooth oscillation in high temperature plasmas.

An example of the time evolution in case (I) is illustrated in Fig. 2(b). The change in the shear parameter $s$ is calculated by taking account of the current peaking due to the neoclassical resistivity and the enhanced current diffusivity. Initially, the perturbation amplitude grows with the growth rate of the resistive MHD mode. When $\hat{B}_n$ reaches a critical value $\hat{B}_c < 0.003/s$, the growth rate is strongly enhanced until the stochastic layer reaches the geometrical axis. Then we assume that the shift of the magnetic axis is reset $q$ being kept constant on the axis, and $\hat{B}_n$ is reduced to the initial value.

The order of the threshold displacement of the magnetic axis (2 cm) as well as the growth rate in the stochasticity driven phase (20–30 $\mu$s) agree with the experimental observations on JET [4].

3.2. Effect of pressure profile flattening

Pressure profile flattening may affect the dynamics of the mode. The ideal MHD theory for the pressure driven $m = 1$ mode yields a growth rate of $\gamma_{\text{MHD}} \approx (\pi/\sqrt{2})\nu^{-1}(\nu^2 - 13/144)$ [5], where $\nu_{\text{pl}} = 1/(e^2 r^2) \int_0^r r^2 (-d\beta/dr)dr$ is a measure of the pressure gradient. We employ a connection formula between the current diffusive tearing and current diffusive kink modes for the MHD stable case, $\gamma_0 = \gamma_{\text{pl}}/(1 + \xi)$, where $\xi = \gamma_{\text{pl}}[(2e\nu)^{1/3} 3^{1/3} |\gamma_{\text{MHD}}|^{-1/3}]$. We assume that the transport coefficients $(\lambda, \chi, \mu_i)$ are finite for $\hat{B}_n < \hat{B}_c$ and write them as $(\lambda, \chi, \mu_i) = (\lambda, \chi, \mu_i)_E + (\lambda, \chi, \mu_i)_M$. The subscript $M$ denotes the contribution of the magnetic stochasticity, and $E$ stands for the contributions made by the electrostatic modes. We assume $\chi_E = \mu_{\text{IE}}$, $\chi_E/\lambda_E = \chi_M/\lambda_M = (6/\pi)\omega_0^2/(\mu_0 e^2)$. The stability boundary including these effects, obtained from Eq. (1), is shown in Fig. 3(a).

The development of $\beta_{\text{pl}}$ is modelled by the equation $d\beta_{\text{pl}}/dt = H - \gamma_{\text{pl}}\beta_{\text{pl}}$, $\gamma_{\text{pl}} = (r_i^2/r_i^2)\chi_E + (1 - r_i^2/r_i^2)\chi_M$, where $H$ is the heating rate and $r_* < r < r_i$ the stochastic region. Figure 3(b) illustrates an example of the decay of $\hat{B}_n$ due to pressure flattening. The trajectory in $\beta_{\text{pl}}$-$\hat{B}_n$ space is also shown in Fig. 3(a). During this period of the burst, the change of $s$ due to current profile flattening is small.

4. MAJOR DISRUPTION

The trigger mechanism of major disruptions is investigated. Toroidicity creates a stochastic layer near the separatrix of an $m = 2$ island which is produced by the $m/n = 2/1$ tearing mode [6]. Figure 4 shows the domain of stochasticity and fast growth in the $s$-$\hat{B}_n$ plane ($s = r q^3/q$ at $r = r_2$, where $q(r_2) = 2$). If the magnetic
FIG. 3. (a) Stability region in the space of integrated pressure gradient $\beta_{pl}$ and normalized magnetic perturbation $\tilde{B_n}$ and a typical cyclic trajectory. (b) Typical evolution of $\beta_{pl}$ and $\tilde{B_n}$ ($\lambda_c/\lambda_M|\tilde{B}_n=2b_c=5 \times 10^{-3}, H = 10^{-5}$; the other parameters are the same as in Fig. 2).
perturbation exceeds the threshold, line (3), the enhanced current diffusion in the stochastic layer results in an accelerated growth of the tearing mode. The lower solid line (2) shows the condition that the stochastic layer extends near the axis, and line (1) denotes that the island reaches the axis. Stochasticity can be important for the high shear region, $s > 0.3$.

The dynamics of the mode is described by the growth rate in the Rutherford regime with the enhanced current diffusivity. If stochasticity is switched on, explosive growth occurs, which in the quasi-linear limit of $D_M$ is given as

$$\bar{B}_n = \frac{\tilde{B}_{n0}}{(1 - \sqrt{\tilde{B}_{n0}gt})^2}$$

where $g = s^{3/2}(c^2/\omega_p^2r_0^2)(\pi^{3/2}v_{Te}/v_A)r_2\Delta'$ and $t = 0$ is taken when the trigger starts. The characteristic growth time $1/g$ is independent of resistivity. A large value of $\Delta'$ has been reported [7] when the $q(r)$ profile is hollow and $\Delta q = q(0) - 1$ exceeds a critical value $\Delta q_c$, satisfying $r_2\Delta' = r_2\Delta'_{cyl} + D(\Delta q_c)^{3/2}/[(\Delta q)^{3/2} - (\Delta q_c)^{3/2}]$, where $\Delta'_{cyl}$ is the value prevailing in the cylindrical calculation and $D$ is a constant. This is due to the toroidal coupling between the $m/n = 2/1$ and $m/n = 1/1$ modes. The particular phase relation between $m = 1$ and $m = 2$ modes has been found in experiments [8], confirming the importance of toroidicity.

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SELF-CONSISTENT DESCRIPTION OF THE L-H MODE TRANSITION DYNAMICS

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Abstract

SELF-CONSISTENT DESCRIPTION OF THE L-H MODE TRANSITION DYNAMICS.
A self-consistent theoretical model of the L-H mode transition in a tokamak is presented. Numerical simulation and analytical treatment show that this model adequately describes the main experimentally observed features of the L-H transition dynamics. It is also shown that under certain conditions the transition can have an auto-oscillatory character.

1. INTRODUCTION

One of the most promising modes of fusion facility operation is the H mode, i.e. the mode with improved plasma confinement. This is shown in a number of recent studies devoted to separate problems of the L-H mode transition dynamics [1-5]. The present study is the first attempt to combine the results in this field in order to obtain a self-consistent representation of the bifurcation stage of the L-H transition. Besides the mechanisms of ambipolar potential emergence at the plasma edge, proposed previously, the particle losses along the magnetic field lines are also taken into account, and the role of suprathermal electrons emerging under auxiliary plasma heating in realization of the bifurcation is considered.

2. SELF-CONSISTENT L-H TRANSITION MODEL

Multiple experimental realizations of the L-H transition have shown three main processes accompanying the stage of bifurcation:

— An abrupt drop in the $H_\alpha$ intensity, confirming a reduction in the particle and energy losses from the discharge;
— Growth of a radial electric field $E_r$ in a narrow peripheral zone;
— Stabilization of drift and magnetic fluctuations near the SOL.

If it is assumed that bad plasma confinement in the L mode is related to an anomalous diffusion of particles from the discharge due to their interaction with plasma instabilities, stabilization of these instabilities in a natural way will result in
the transition into the mode with improved confinement. The reason for stabilization of instabilities can be related to an increased shear induced electric field [1]. Thus the self-consistent representation of the bifurcation stage in the L–H transition calls for taking these processes into account simultaneously. A set of equations corresponding to the self-consistent model should include the equations of radial diffusion for each plasma component (the index ‘b’ refers to suprathermal electrons, ‘e’ to thermal electrons and ‘i’ to ions):

\[
\begin{align*}
\frac{\partial n_\alpha}{\partial t} + \text{div } \Gamma_{\perp\alpha} + \frac{\Gamma_{\text{pl},\alpha}}{L_1} + \frac{\Gamma_{\text{edge},\alpha}}{L_1} &= S_\alpha \quad \alpha = \begin{cases} 
b \cr e \cr i \end{cases} \\
\text{(1)} \\
\text{(2)} \\
\text{(3)}
\end{align*}
\]

the condition of quasi-neutrality (determining the plasma potential \(\varphi\)):

\[
n_i = n_e + n_b
\]

and the equation for energy density in oscillations:

\[
\frac{dW}{dt} = \gamma_0 W - \gamma_1 |\varphi'| \left( \frac{W}{W_0} - \frac{W}{W_0} \right) - \gamma_2 W^2
\]

The right hand sides of continuity equations (1–3) represent the sources of particles for each plasma component:

\[
S_b = \frac{P_{bf}}{VT_b} - \nu_{eb} n_b \\
S_e = \nu_{eb} n_b + \langle \sigma v \rangle_i n_i n_e - \frac{P_{bf}}{VT_b} \\
S_i = \langle \sigma v \rangle_i n_i N
\]

In the derivation of these expressions it is assumed that the auxiliary plasma heating (e.g. by EC waves, \(P_{bf}\) is the heating power) is a source of fast electrons (with the energy \(T_b\)), the losses of which occur owing to collisions with thermal electrons and the subsequent Maxwellization. The source of thermal electrons and ions is related to the ionization of neutrals having the density \(N\). Other designations are standard. The fluxes of particles from the discharge (the second, third and fourth terms in the left hand sides of Eqs (1–3)) are provided by the following processes:

(a) Anomalous transverse diffusion of particles, caused by their interaction with plasma instabilities. The particle flux \(\Gamma_\alpha\) in the quasi-linear approximation has the form [6]:

\[
\Gamma_{\perp\alpha} = -D_\alpha(W)n_\alpha \left( C_{n,\alpha} \frac{\nabla n_\alpha}{n_\alpha} + C_{T,\alpha} \frac{\nabla T_\alpha}{T_\alpha} \right. \\
\left. + C_{\varphi,\alpha} \frac{e_\alpha}{T_\alpha} \nabla \varphi (r - r_w) \right)
\]

\[
\text{(6)}
\]
where $D_a$ is the coefficient of quasi-linear diffusion, coefficients $C_{\beta,\alpha}$ are described in Ref. [6], $r > r_w$ is the region at the plasma edge, where diffusion is not ambipolar, and the potential $\varphi$ is not equal to zero.

(b) The longitudinal escape of banana ions near the wall [2] is one of the generally adopted mechanisms of particle loss in the consideration of the L–H transition. The boundary region, the width of which is comparable with the width of the banana trajectory, is assumed to be a region of escape. Here we shall give an expression for the longitudinal particle flux more strict than that in Ref. [2], taking account of the finiteness of both the particle confinement time in the banana region, $\tau = 1/\nu = e/\nu_{\text{li}}$, and the time of longitudinal particle escape, $\tau_L = qR/c_s$ ($c_s$ is the sound speed).

Since the plasma parameters are not changed along the field line, we shall use the diffusion equations integrated with respect to the longitudinal co-ordinate $L = 2\pi R$.

Moreover, assuming the ion distribution function to be Maxwellian, the following relationship can be obtained for the dependence of the magnitude of $\Gamma_{\text{lpl, }a}$ on the radius ($a_1$ is the radius of the limiter):

$$\frac{\Gamma_{\text{lpl, }a}}{L} = \frac{n_e}{\tau_L} \left( \frac{1 - \exp(-\xi)}{\xi} + \int_{\xi_1}^{\infty} \exp(-y^2)dy \right) \frac{\tau_1}{\nu^{-1} + \tau_1} \tag{7}$$

where $\xi = q\rho_1/a_1 - r$, $\rho_1$ is the ion Larmor radius. It is clear that a similar relationship can be written for the electrons; however, the electron flux can be neglected, since the electron Larmor radius is much less than that of the ions.

Relationship (7) is produced under the assumption that electrons and ions do not undergo transverse diffusion during their longitudinal motion. This process results in the extension of the longitudinal escape region by $\Delta r \sim (DqR/c_s)^{1/2}$. The longitudinal draining off of particles towards the region beyond the limiter (separatrix) results in the emergence of a positive potential at the plasma edge, equalizing the electron and ion fluxes up to the level:

$$\frac{\Gamma_{\text{edge, }a}}{L} = v_e n_e F_a(\varphi) \tag{8}$$

where

$$v_e,b = v_{T_e,b}, \quad v_i = c_s, \quad F_a(\varphi) = \begin{cases} \exp(-e\varphi/T_b) & \alpha = b \\ \exp(-e\varphi/T_e) & \alpha = e \\ 1 & \alpha = i \end{cases}$$

The behaviour of the energy density of oscillations (Eq. (5)) represents qualitatively the processes of exciting the oscillations and their non-linear saturation, on the basis of the previous studies [1–4]. The first term in the right hand side represents the linear drive of oscillations with the increment $\gamma_0$ (of the order of the drift frequency). The second term phenomenologically represents the suppression of oscillations due to the emergence of a high SOL potential, $\varphi \sim T_b$, taking account of a non-linear relationship between the decrement and the energy level of oscillations,
in accordance with the conclusion from numerical simulations [4] that with a rise in the amplitude of oscillations, the capability of stabilizing them through shear rotation is reduced. And finally, the third term in the right hand side of (5) represents (again qualitatively) the oscillation level saturation due to non-linear processes of the wave redrive within the spectrum.

As noted above, the condition of quasi-neutrality (4) virtually is an equation for the plasma edge potential \( \varphi \). Using Eqs (1-4) and the ambipolarity of transverse plasma motion at \( r \leq r_w \), it can be represented in the form:

\[
\sum_{\alpha = i,e,b} \mathcal{D}_\alpha \left( C_{n,\alpha} \nabla n_\alpha + C_{T,\alpha} T_\alpha \frac{\nabla T_\alpha}{T_\alpha} \right) \bigg|_{r = r_w} \int_{r_{w,i,e,b}}^r \sum_{\alpha = i,e,b} \Gamma_{E,\alpha} \sigma_\alpha \frac{\nabla \varphi}{\sigma_\alpha} \frac{1}{T_\alpha} \sum_{\alpha = i,e,b} \Gamma_{E,\alpha} \sigma_\alpha \frac{\nabla \varphi}{\sigma_\alpha} \frac{1}{T_\alpha} dr
\]

where \( \sigma_i = 1 \), \( \sigma_{e,b} = -1 \), \( \Gamma_{E,\alpha} = \Gamma_{E,i,e,b} + \Gamma_{E,\text{edge},\alpha} \).

The set of equations (1-3, 5, 9) was analytically studied in the plane geometry approximation and numerically solved in cylindrical geometry and has been used for the self-consistent representation of the bifurcation stage in the L–H transition. Before discussing the results of solving such a set, it is necessary to note the following. A number of experiments for the study of an H mode with both positive and negative potentials on the plasma surface (H\(_+\) mode and H\(_-\) mode) have recently been carried out. Since the flux in (7) results in the emergence of a negative plasma potential and the flux in (8) of a positive one, it is clear that their combination should adequately represent all the experimentally realized situations. In this study we limit ourselves to an analysis of the second mechanism for the potential emergence only. Therefore, all the cases considered below refer to the H\(_+\) mode produced mainly by the departure of electrons.

3. STANDARD L–H TRANSITION; HINTON’S HYSTERESIS

Let us consider the bifurcation related to the diffusion of suprathermal electrons as an illustration of the possibilities of the proposed model. Let us assume that the density distributions of thermal electrons and ions along the radius are not changed (parabolic distributions have been used in the calculations given here) upon the background of a rapidly diffusing suprathermal electron component. Such a situation is represented by the set of equations (5, 9), the results of the numerical integration of which are given in Fig. 1. As follows from Fig. 1, the stationary solution to the set of equations (5, 9), at a small launched power, is the L mode characterized by a low potential at the edge, \( \varphi \approx T_e \) (Fig. 1(b)), incapable of stabilizing an initially high level of oscillations, \( W \) (Fig. 1(a)), and by a small fraction of fast electrons at the plasma edge (Fig. 1(c)). The increase of \( P_{\text{hf}} \) results in the increase of the amount of
fast electrons both in the core and in the edge plasma that results in the edge potential growth. With an increase in the power above a certain limit the growth of $\varphi$ will result in dominance of the shear stabilization of oscillations over their non-linear damping. This in turn will cause a decay in $W$ down to the thermal noise level in the peripheral zone (its width is an external parameter; in our calculations it has been adopted to be equal to 10% of the plasma radius). The values of the parameters in Eq. (1) are chosen so that the losses of fast electrons, due to their Maxwellization and longitudinal departure, are small, i.e. the flux $\Gamma_{\perp b} \approx \text{const}$ in the stationary state. Therefore a reduction in $W$, and in the quasi-linear diffusion coefficient $D(W)$, results in the growth of $n_b$ at the periphery, and hence in the further rise in $\varphi$, in accordance with expression (8). Thus the stationary solution in the case described here is an H mode with a high edge potential, a low level of oscillations at the periphery and a suprathermal electron beam density profile pedestal, characteristic for the model given here.
The time dependence of $W(t)$ allows one to give a self-consistent representation of another model for the L–H transition, related to heating of the main electron component, thermobifurcation [5]. Equations (5, 9) should be completed by the addition of the equation for electron temperature to represent this process:

$$\frac{3}{2} \frac{\partial n_e T_e}{\partial t} = P_{hf} + \text{div}(\chi n_e \nabla T_e)$$

where the heat conduction coefficient $\chi$ is determined by an anomalous heat transfer. Let us consider the electron and ion densities to be given and $n_i = n_e$, $n_b = 0$. The stationary solution to the set of equations (5, 9, 10) (Figs 2, 3) is very similar to that obtained in Ref. [5]. Thus the L–H transition is accompanied by the emergence of a pedestal in the temperature profile (Fig. 2), and the energy confinement time dependence on the deposited power shows hysteresis (Fig. 3). However, in contrast to Ref. [5], our result has been obtained, first, without involving neoclassical heat conduction, which even at a small level of $W$ can turn out to be lower than anomalous heat conduction, and secondly, by solving a self-consistent one dimensional set of

![Graphs showing radial distributions of oscillation energy densities $W(r)$, edge potential $\varphi(r)$, and electron temperature $T_e(r)$ for the L mode (solid curves, $P_{hf}/VT_b = 8.9 \times 10^{17} \text{ m}^{-3} \text{s}^{-1}$) and for the H mode (dashed curves, $P_{hf}/VT_b = 18 \times 10^{17} \text{ m}^{-3} \text{s}^{-1}$) without electron beam.](image)
FIG. 3. Relation between heating power and energy confinement time in the case of (a) power rise and (b) power decrease.

equations with due regard for temporal dependences of the plasma parameters under study.

4. AUTO-OSCILLATORY L–H TRANSITION

The proposed model, along with the stable L–H transition, can have a solution with a periodic auto-oscillatory nature. Let us consider, for example, the L–H transition caused by suprathermal electron beam diffusion. As noted, fast electrons, owing to anomalous diffusion, reach the chamber walls, which results in the growth of the SOL potential \( \varphi \). The emergence of an inhomogeneous radial electric field, stabilizing the instability, in its turn reduces the anomalous diffusion and the flux of particles to the plasma periphery, leading to a reduction in the magnitude of \( \varphi \). It is evident that under certain conditions such a process can have a periodic nature. Let us make some simplifications for a more illustrative representation of auto-oscillatory L–H transitions. If we assume that the anomalous radial particle flux (7) is an intrinsically ambipolar one, and Eq. (9) includes longitudinal fluxes only:

\[
n_e V_T \exp(-e \varphi/T_e) + n_b V_T \exp(-e \varphi/T_b) = c_s (n_e + n_b)
\]  

(11)

Secondly, let us use the zero dimensional fast electron diffusion equation:

\[
\frac{dn_b}{dt} = \frac{P_{bf}(W/W_0)}{VT_b} - \nu_{eb} n_b - \frac{n_b W}{\tau_1 W_0}
\]  

(12)

which is valid in a very narrow SOL region, \( a - \Delta r \leq r \leq a \). Equation (12) takes into account the fact that the diffusive flux from the centre, provided by the presence
of oscillations and hence proportional to $W$, is a source of fast particles. The second term in the right hand side of Eq. (12) takes the Maxwellization of fast particles into account and the third term the anomalous diffusion of particles in their interaction with oscillations (the confinement time for fast electrons is assumed to be $\tau_0 = \tau_1 W_0 / W$, where $\tau_1$ is a parameter independent of $W$).

The set of equations (5, 11, 12) was solved numerically. It has been shown that the discharge makes the transition, in succession, from the L mode into an auto-oscillatory state and then into the H mode (Fig. 4) with an increase in deposited power, $P_{th}/n_e$. The time dependence of the quantities $W$, $n_b$, and $\varphi$ in a typical auto-oscillatory mode is given in Fig. 5. It can be seen that a high level of oscillations leading to enhanced anomalous transport results in a rise in the fast particle density in the SOL region (Fig. 5(c)). The increased entry of beam particles into the limiter causes a steep rise in the plasma surface potential (Fig. 5(b)), $e\varphi \approx T_b$, and hence in the suppression of oscillations to the thermal noise level (Fig. 5(a)). The anomalous particle diffusion is abruptly reduced and the electron flux and the potential at the edge drop, which in turn causes oscillation growth. Thus the described

![Graphs](image-url)
process is periodically repeated in time. The obtained result can serve as an interpretation of the phenomenon observed on the T-10 tokamak [7]. Namely, under current drive by EC waves, when the parameter $\beta = P_{n_e} \frac{a}{n_e}$ exceeded $\beta_{cr} = 2 \times 10^{-6}$ W/cm$^3$, periodic expulsions of fast electrons, registered by the soft X ray probe (similar to the results shown in Fig. 5(c)), were observed. The emergence of fast electrons, born at the discharge centre, can be caused by their anomalous diffusion along the radius and therefore can be described by the above proposed model. The range of changes in the specific power inside which an auto-oscillatory mode of operation exists (Fig. 4) satisfies the inequality $4.2 \leq P_{n_e} \tau_1 \leq V_n T_b \leq 10^3$, and for the T-10 parameters it turns out to be rather wide.

Note that the described mode of operation is not the only auto-oscillatory solution which follows from the proposed model. So the thermal bifurcation described in Section 3 can, under certain conditions, also have an auto-oscillatory nature that can be interpreted as an ELM.
5. CONCLUSION

The proposed model makes possible a self-consistent representation of the bifurcation stage in the L–H transition. The study of this phenomenon, using a self-consistent closed set of equations for the main plasma parameters which are changed under the bifurcation, has allowed us to detect the intermediate stage between the L mode and the H mode, an auto-oscillatory mode of operation characterized by the periodic escape of electrons to the chamber walls. The role of the fast electrons in the realization of both a stable L–H transition and the auto-oscillatory transition has been studied.

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MODEL OF L-MODE CONFINEMENT IN TOKAMAKS

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Abstract

MODEL OF L-MODE CONFINEMENT IN TOKAMAKS.

A model of L-mode confinement in tokamaks is developed on the basis of the microscopic ballooning mode instability. The anomalous transport coefficients determine the stability below the critical beta against the ideal MHD ballooning instability. The current diffusivity has a strongly destabilizing effect while the thermal transport $\chi$ and the ion viscosity tend to stabilize the mode. The stability boundary for the least stable microscopic ballooning mode determines the anomalous transport coefficients. The obtained formula is compared with experimental observations on the L-mode confinement. The predictions on (1) the radial shape of $\chi$, (2) the temperature profile resilience, (3) the global confinement characteristics such as power degradation and dependence on the plasma current, current profile and mass number, and (4) the ratio of perturbative $\chi$ to energy balance $\chi$ are consistent with the experimental database.

1. Introduction

L-mode confinement has been observed in all tokamaks [1,2]. Microscopic fluctuations have been confirmed to play an important role in anomalous transport[3]. The power degradation is modelled by the relation $\chi \sim T^{1.5}/aB^2$, which has been derived for
drift wave theories[4]. However, this form of $\chi$ contradicts the radial form of $\chi$. Ohkawa's model, $\chi = \delta^2 v_A^2 / qR$ ($\delta$ is the collisionless skin depth, $R$ the major radius, $q$ the safety factor and $v_A$ the Alfvén velocity), is one of the few models explaining that $\chi$ is large at high temperature and becomes larger towards the edge, as well[5]; it could not, however, fully explain the dependences of $\tau_B$.

Here we present a new model of L-mode confinement based on transport MHD analysis. Shear stabilized plasma can become unstable by fluctuation driven transport[6,7]. We analyse the ballooning instability by taking into account anomalous cross-field transport. Below the critical beta of the ideal MHD mode, the microscopic ballooning mode has a large growth rate due to the small but finite current diffusivity $\lambda$. This mode is destabilized by $\lambda$ but stabilized by other transport coefficients, $\chi$ and the ion viscosity $\mu$. If the anomalous transport is substantially enhanced by this mode, then the mode is stabilized. The marginal stability condition for the least stable mode determines the transport coefficients. The results on $\chi$ are compared to the experimental observations and show agreement.

2. Stability Analysis

We analyse the circular tokamak with toroidal coordinates ($r$, $\theta$, $\zeta$). We use the reduced set of equations and keep the $\lambda$ term in Ohm's law, $\nabla \times B = J/\sigma - \nabla^2 \lambda$ [8], where $\sigma$ is the conductivity. The ballooning transformation is applied, and the linearized equation can be written as [9]

$$\frac{d}{d\eta} \frac{F}{1 + \Sigma F/T + \Lambda F^2/T} \frac{d\phi}{d\eta} + \frac{\alpha [\kappa + \cos\eta + (s\eta - \alpha \sin\eta) \sin\eta]}{1 + \lambda F/T} \frac{d\phi}{d\eta} = 0 \quad (1)$$

with the following notations: $\Sigma = n^2 q^2 / \delta$, $\Lambda = \lambda n^4 q^4$, $\chi = \lambda n^2 q^2$, $\Theta = \Omega n^2 q^2$, $\Lambda$ is the growth rate, $F = 1 + (s\eta - \alpha \sin\eta)^2$, $\kappa$ is the average well and $\kappa = -(r/R)(1 - 1/q^2)$. 


\[ B_p = B_0 + qR, \quad \alpha = q^2 \rho' / \varepsilon, \quad \varepsilon = a/R, \quad \beta \]
is the pressure divided by the magnetic pressure, and \' denotes the derivative with respect to \( r/a \). We use the normalizations: \( r/a \rightarrow r, \quad t/a \rightarrow t, \quad a \tau_A / a^2 \rightarrow \xi, \quad \mu \tau_A / a^2 \rightarrow \mu, \quad \tau_A / a^2 \rightarrow \alpha, \quad a \tau_A / a^4 \rightarrow \lambda, \quad \tau_A = Rq / \psi_A, \quad \tau_A \rightarrow \tau \). If we neglect \( \lambda \), \( \xi \) and \( \alpha \), the equation is reduced to the resistive ballooning equation, and the ideal MHD mode equation is recovered by further taking \( 1/\alpha = 0 \).

Equation (1) predicts that the current diffusive ballooning mode has a large growth rate even with a very small value of \( \lambda \). We take \( 1/\alpha = 0 \), for simplicity. The growth rate of the short wave-length mode, driven by the \( \xi \) term, is given analytically by \( \tau \sim \lambda^{1/5} (nq)^{4/5} \alpha^{3/5} s^{-2/5} \) [9]. This large growth rate is confirmed by numerical calculation [10].

If the transport coefficients increase substantially, the stabilizing effects of \( \xi \) and \( \lambda \) overcome the destabilizing effect of \( \alpha \). The stability boundary is derived by setting \( \tau = 0 \) in Eq.(1). For the ballooning mode which is destabilized by the normal curvature, and not by the geodesic curvature, i.e., \( 1/2 + \alpha s \), the stability boundary for the least stable mode is obtained as [9]

\[ \alpha^{3/2} \lambda = f(s) \sqrt{\alpha} \]

where \( f(s) = \sqrt{6s} \) or 1.7 \((s \rightarrow 0)\).

3. Transport Coefficient

From the stability analysis, we derive the formula for the anomalous transport coefficient. When the mode amplitude and the associated anomalous transport are small, Eq.(1) predicts instability. When the mode develops and the transport coefficient reaches condition Eq.(2) for a given pressure gradient \( \alpha \), a self-sustaining turbulent state is realized. This state is thermodynamically stable: the excess growth of the mode and enhanced transport coefficients lead to mode damping. When the mode amplitude and the transport coefficients are below Eq.(2), the mode continues growing.
From Eq. (2) we express $\tau$ in terms of the Prandtl numbers $\mu/\tau$ and $\lambda/\tau$ (note that $\lambda$ is proportional to the electron viscosity in the plasma frame[8]) as $\tau = \alpha^{3/2} (\lambda/\tau) \sqrt{\tau/\mu} / f(s)$. The ratios $\lambda/\tau$ and $\mu/\tau$ are given to be constant. We have $\lambda/\tau = 9^2/4^2$ and $\mu/\tau = 1$ for electrostatic perturbations[11]. The formula for $\tau$ is finally obtained in an explicit form as

$$\tau = f(s)^{-1} q^2 (R_B' / r)^{3/2} \sigma^2 v_A / R$$  \hspace{1cm} (3)$$

The typical perpendicular wavenumber of the most unstable mode satisfies $k_B \approx 1/\sqrt{\alpha}$. The typical correlation time of the mode is estimated to be $\tau = 1/R, \tau = \sqrt{\alpha/\delta} (v_A / qR)$.

4. Comparison with Experiments

This form of $\tau$ is consistent with the experimental results known for the L-mode. In the following, we list the predictions from our theory and compare them with the experimental database, by choosing $n_i = n_e$ and $T_i = T_e$ for simplicity.

(i) $\tau$ has a dimensional dependence of $[T]^{1.5} / [\mu][B]^2$. Note that $\tau$ is not enhanced by the local value of $T$, but by the pressure gradient.

(ii) The density and $q$ profiles govern the radial profile of $\tau$. Equation (3) indicates that $\tau$ increases towards the edge for the usual plasma profiles in the L-mode. Figure 1 shows a typical example of a predicted $\tau$ profile.

(iii) The dependence on the dimensionless quantities is predicted to be $\tau \approx q^2 / f(s) \sqrt{A}$, where $A$ is the ion mass number. This is consistent with the experimental result with respect to local $\tau$ that $\tau \approx B_p^{-1/2}$, with $1 < y < 2$ and $0 < z < 1$ [12], and with the favourable mass dependence.

(iv) The point model argument of the energy balance, $\tau_e = a^2 / \chi$ and $2\pi^2 a^2 R_e T = \tau_e P$, provides the scaling law

$$\tau_e = C a^{0.4} R_1 1.2 I_p 0.8 P - 0.6 A 0.5 f - 0.4 (n_e \& P / \sqrt{A}) 0.6$$  \hspace{1cm} (4)$$

\hspace{1cm}
FIG. 1. Predicted radial profile of χ as a function of r/a. The solid line refers to Eq. (3) and the dashed line to $5 \times$ Eq. (3). Parameters: $B = 4$ T, $R = 3$ m, $R/a = 4$, $q(r) = 1 + 2(r/a)^2$, $T(0) = 10$ keV, $p(r) = p(0)h(r)$, $n(r) = n(0)\sqrt{h(r)}$, $h(r) = (1 - (r/a)^2 + \Delta)$, $\Delta = 0.01$, and $A = 1$.

where C is a numerical coefficient and $\lambda_p$ is the gradient scale length $(nT)/|\nabla(nT)|$. This result is consistent with the L-mode scaling law, including the dependences on a, R, $I_p$, P, A and internal inductance\[13\]. A slight difference is seen in the final term within the parentheses. We note, however, that the density and the gradient scale length show collinearity in the database. In L-mode plasmas, the density profile is much steeper than the temperature gradient near the edge and $\lambda_p$ in Eq. (4) would be replaced by $\lambda_n = |n_i/Vn_i|$. The high density plasma has a steeper edge density; Tsuji has reported that $\lambda_n n_e$ is a weak function of density\[14\]. For some data set of JT-60, Takizuka found the density dependence to be $\tau_p(\text{thermal}) \propto n_e^{0.5}$, suggesting that a careful classification of the database with respect to the profile is necessary\[15\].

(v) Because of the dependence of χ on VT, the temperature profile depends weakly on the location of the power deposition peak. The peaking factor, $T(\text{at } q=1)/\langle T \rangle$, is predicted to scale as $q(a)^{0.6}$. These results explain the profile resilience.
(vi) Since \( \tau \sim (\nabla (nT)/n)^{1.5} \), the thermal diffusion coefficient deduced from the pulse propagation, \( \tau_{HP} \), is larger than that evaluated by the power balance \( \tau \). If \( |\nabla n_i|/n_i | \ll |\nabla T/T| \) holds, we have, for instance, \( \tau_{HP} = 2.5 \tau \).

(vii) The typical wavelength and correlation times of the mode are calculated. \( k_p \rho_i \) is of the order of 0.1 or less (\( \rho_i \) is the ion gyroradius.) It should be noted that, though the dimensional relation \( k_i \propto [B]/[\sqrt{T}] \) holds, \( k_i \) does not scale with the local gyroradius. The collisionless skin depth would be a more relevant length. The typical time-scale \( \tau_C \) depends on the average temperature, not explicitly on \( n_e \) and \( B \).

(viii) The estimate \( \pi/n \sim 1/k_p \rho_p \) shows that the mode amplitude is larger near the edge and larger for high heating power.

5. Summary and Discussion

A model of the L-mode in tokamaks was developed. The stability of the microscopic ballooning mode is investigated under the influence of the self-generated anomalous transport coefficients \( \chi \), \( \lambda \) and \( \mu \). The transport coefficient is determined from the marginal stability condition for the least stable mode. (We note here that the usual method for the estimation of \( \chi \) by \( \tau/k_p^2 \) for the most unstable mode gives the same results as \( \tau \sim \alpha^{3/2} (\chi/\lambda) \).) The predictions as to \( \chi \) are compared with experiments. The major part of the observations on the L-mode can be explained by this model simultaneously.

Here we use simplifications for the analytic treatment. A study on the case where the geodesic curvature drives the instability gives similar results[10]. The result, Eq.(3), contains an uncertainty in the numerical factor. Non-linear simulation would give this numerical factor and test the assumption for \( \chi \) that the transport coefficients affecting the microscopic mode are equated to those for global quantity. Also necessary is a study of effects such as the diamagnetic drift for kinetic
corrections. This should be the subject of future studies.

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NON-TOKAMAK CONFINEMENT SYSTEMS

(Sessions C-1 to C-3 and Poster Session C-4)

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Paper IAEA-CN-56/C-1-5-2 was presented
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Paper IAEA-CN-56/C-3-4-2 was presented
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Abstract

DIMENSIONLESS SCALING OF CONFINEMENT IN ATF.

The results of dimensionless parameter modulation and configuration control experiments in the Advanced Toroidal Facility (ATF) are presented. The global energy confinement time fits gyro-Bohm scaling better than Bohm-like scaling. An additional dependence was determined by modulation of single dimensionless parameters (collisionality, $\nu$, and beta, $\beta$), yielding $\tau_E/\tau_B \propto \nu^{\alpha_1} \beta^{\alpha_2}$, where $\alpha_1 = 0.18 \pm 0.03$ and $\alpha_2 = 0.3 \pm 0.1$. Application of this formula to NBI and ECH data significantly improves the fit, implying that improved confinement will result from increasing the heating power. Little change in confinement occurred for wide variations in the confined trapped particle fraction at constant magnetic well radius and shear. This may be explained by reduction of the helically trapped particle loss.


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region due to the radial electric field. Configuration modulation experiments showed that the energy confinement time improves as the magnetic well radius expands, the result expected from some stabilization of resistive interchange instabilities. This is consistent with fluctuation measurements and may explain the positive dependence of confinement on $\beta$.

1. INTRODUCTION

The Advanced Toroidal Facility (ATF) was designed to investigate improvement of toroidal confinement concepts in a stellarator device. The device [1,2] is an $\ell = 2$, $m = 12$ sheared stellarator (torsatron) with major radius $R_0$ of 2.1 m, average minor radius $a$ of 0.27 m, and maximum magnetic field on axis $B_0$ of 2 T. In the standard configuration, the rotational transform $\pm \equiv \pm 1/q$ varies from 0.3 on the axis to 1.0 at the edge, and a modest magnetic well extends out to about the $\pm = 1/2$ surface at the normalized radius ($\rho = \bar{r}/a$) of $\approx 0.6$. The earlier studies used up to 1.5 MW of neutral beam injection (NBI) from two opposing tangential 40 kV, 0.3 s injectors. The plasma performance in this device is similar to that in a tokamak of the same minor radius (e.g. ISX-B) [2,3]. The maximum parameters achieved (not simultaneously) are $T_i(0) = 1.0$ keV, $T_e(0) = 1.5$ keV, $n_e = 1.5 \times 10^{20}$ m$^{-3}$, $\tau_E = 30$ ms, and $\langle \beta \rangle \approx 1.7\%$. Sets of simultaneous plasma parameters for four operating regimes are given in Table I [2,4]. Since the last IAEA Conference, ATF has been operated primarily with 0.4 MW of electron cyclotron heating (ECH) from two cw 53.2-GHz gyrotrons (except for a brief period when operation with combined NBI and ion cyclotron range of frequencies (ICRF) heating was used for a proof-of-principle demonstration experiment of CO$_2$ scattering as an alpha particle diagnostic [5]). Although

<table>
<thead>
<tr>
<th>Regime</th>
<th>High stored energy</th>
<th>High $\beta$</th>
<th>Low $v_*$</th>
<th>Long ECH pulse</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shot number</td>
<td>11740</td>
<td>11186</td>
<td>14514</td>
<td>16654</td>
</tr>
<tr>
<td>$B_0$ (T)</td>
<td>1.9</td>
<td>0.45</td>
<td>1.9</td>
<td>0.95</td>
</tr>
<tr>
<td>$n_e$ ($10^{19}$ m$^{-3}$)</td>
<td>11</td>
<td>4.3</td>
<td>0.53</td>
<td>0.5</td>
</tr>
<tr>
<td>$P$ (MW)</td>
<td>0.96</td>
<td>0.98</td>
<td>0.79</td>
<td>0.25</td>
</tr>
<tr>
<td>$\tau_E$ (ms)</td>
<td>26</td>
<td>6.1</td>
<td>6.4</td>
<td>5.0</td>
</tr>
<tr>
<td>$T_e(0)$ (keV)</td>
<td>0.4</td>
<td>0.3</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>$T_i(0)$ (keV)</td>
<td>0.4</td>
<td>0.3</td>
<td>1.0</td>
<td>0.2</td>
</tr>
<tr>
<td>$\langle \beta \rangle$ (%)</td>
<td>0.4</td>
<td>1.7</td>
<td>0.1</td>
<td>0.15</td>
</tr>
<tr>
<td>Duration (s)</td>
<td>0.25</td>
<td>0.2</td>
<td>0.15</td>
<td>20</td>
</tr>
</tbody>
</table>
2. DIMENSIONLESS SCALING EXPERIMENTS

Confinement time in a toroidal device is described in dimensionless parameters [8–12],
\[ \tau_E \Omega = \rho_* \alpha_p \cdot \nu_* \alpha_v \cdot \beta \alpha_\beta \cdot C \]

where \( \tau_E \) is the energy confinement time, \( \Omega \) is the gyrofrequency, \( \rho_* \) is the ratio of the gyroradius (\( \rho_s \)) to the average plasma radius (\( \bar{a} \)), \( \nu_* \) is the collisionality, and \( \beta \) is the plasma beta. The coefficient \( C \) can be a function of other dimensionless parameters, including those related to magnetic configurations, as discussed later. We first focus on the dependence of the first three variables in a fixed (standard) configuration. The leading term (\( \rho_* \) dependence) can be compared with two limiting forms: Bohm-like (\( \alpha_p = 2 \)) and gyro-Bohm-like (\( \alpha_p = 3 \)) scaling, which are characterized by long-wavelength (of scale \( \bar{a} \)) and short-wavelength (of scale \( \rho_s \)) turbulence, respectively. Any remaining \( \nu_* \) and \( \beta \) dependence can help explain the source of the turbulence by comparison with theoretical scaling [12].

The earlier regression analyses [3] of the global energy confinement time \( \tau_E \) in ATF were made from data in a multishot database representing diverse conditions with both NBI and ECH plasmas. These suggested that the gyro-Bohm scaling was more closely followed than the Bohm-like scaling, although the distinction was not strong (Table II). We also observed that the LHD empirical expression [13] based on past stellarator data agrees well with the gyro-Bohm scaling [10]. Regression analysis of the ATF data for the above dimensionless form yields the exponents \( \alpha_p = -3.1 \) (with additional exponents of \( \nu_p = -0.45 \) and \( \alpha_p = +0.83 \)), indicative of gyro-Bohm-like scaling. However, collinearity of the control variables and the presence of hidden variables (e.g., heating power profile shape) make the distinction less certain.

<table>
<thead>
<tr>
<th>Scaling</th>
<th>Constant</th>
<th>( \alpha_\Omega )</th>
<th>( \alpha_B )</th>
<th>( \alpha_p )</th>
<th>( \alpha_Ai )</th>
<th>( R^2 )</th>
<th>( \sigma^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>LHD</td>
<td>0.0150</td>
<td>0.69</td>
<td>0.84</td>
<td>-0.58</td>
<td>0</td>
<td>0.938</td>
<td>6.7%</td>
</tr>
<tr>
<td>Gyro-Bohm</td>
<td>0.0140</td>
<td>0.6</td>
<td>0.8</td>
<td>-0.6</td>
<td>0</td>
<td>0.954</td>
<td>4.8%</td>
</tr>
<tr>
<td>Gyro-Bohm</td>
<td>0.0145</td>
<td>0.6</td>
<td>0.8</td>
<td>-0.6</td>
<td>-0.2</td>
<td>0.952</td>
<td>5.0%</td>
</tr>
<tr>
<td>Bohm</td>
<td>0.0139</td>
<td>0.5</td>
<td>0.5</td>
<td>-0.5</td>
<td>0</td>
<td>0.944</td>
<td>5.9%</td>
</tr>
</tbody>
</table>
FIG. 1. Time evolution of plasma parameters with $v_*$ modulation and constant $\langle \beta \rangle$.

FIG. 2. Polar diagram of the relative modulation of global (shown by bold arrows) and local (fine arrows) plasma parameters for the $v_*$ scan.
Since gyro-Bohm scaling gives the leading term variation, the more recent experiment was aimed at determining the additional dependence of $v_*$ and $\beta$ in the standard magnetic geometry (with a fixed coefficient $C$). The experiment involved modulation of a single dimensionless variable ($v_*$ or $\beta$) while keeping the other constant. This was accomplished by simultaneously modulating ECH power $P$ and electron density (with gas feedback) such that $\bar{n}/n = (-2/3)\bar{P}/P$ for $v_*$ modulation and $\bar{n}/n = (4/9)\bar{P}/P$ for $\beta$ modulation, where both equations are based on the gyro-Bohm scaling. Figure 1 shows the time evolution of the plasma parameters in the $v_*$ modulation. For a given ECH power modulation ($\bar{P}/P = 25\%$), the amplitude and phase of the density feedback demand were adjusted to give the specified line-average density modulation ($\bar{n}/n = 15\%$). The modulation period (80 ms) was chosen to be longer than both energy and particle confinement times, and the modulation was applied (at 0.16 s) after the quasi-stationary discharge was established.

The polar diagram in Fig. 2 shows relative modulation amplitudes and phases of several global (magnetic-based) and local (at a radius of $\rho = 0.64$ in the confinement region) plasma parameters based on Fourier analysis at the modulation frequency (12.5 Hz). A modulation amplitude of 50\% in $v_*$ with a negligible $\langle \beta \rangle$ modulation (3\%) resulted in a modulation of 8\% in the normalized confinement time $\tau_E/\tau_{2B}$, implying that $\alpha_v = -1/6$. It is noteworthy that

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig3.pdf}
\caption{Development of the $\langle \beta \rangle$ dependence of confinement time using the ATF database with $\alpha_v = -0.18$.}
\end{figure}
modulation of $\chi/\chi_{gB}$ (the ratio of the local heat diffusivity ($\chi \propto P/nVT_e$) to the gyro-Bohm prediction ($\chi_{gB} = (1/16)(T/eB)|\rho_s|$)) is out of phase with $\tau_E/\tau_{gB}$, and its amplitude (3.5 times $\tau_E/\tau_{gB}$) is roughly the expected value (2.5 times, also out of phase) [12]. This may imply that the global scaling derived is representative of local phenomena in the confinement region ($\rho \approx 2/3$).

Regression analysis for both this $v^*_s$ scan and the orthogonal $\beta$ scan yields $\alpha_v = -0.18$ and $\alpha_\beta = +0.36$ for the modification to the gyro-Bohm scaling. Since regression of $\tau_E/\tau_{gB}$ in the overall ATF database (which includes data with NBI) yields $\alpha_v = -0.17$ and $\alpha_\beta = +0.31$, we conclude that $\alpha_v = -0.18 \pm 0.03$. Based on this $v^*_s$ dependence, Fig. 3 shows variation of the exponent $\alpha_\beta$ as a function of $\langle \beta \rangle$ in the overall database, indicating $\alpha_\beta = 0.3 \pm 0.1$ over the range of $\langle \beta \rangle$ up to 1.7%. Use of both the $v^*_s$ and $\beta$ dependences significantly improves the fit over the ordinary gyro-Bohm scaling, as shown in Fig. 4.

Of the theoretical models compared with tokamak data in Ref. [12], dissipative trapped electron mode (DTEM) models (developed for tokamaks) give an opposite dependence on $v^*_s$ ($\alpha_v = +0.4$), implying that DTEM may not play a significant role in global confinement in these ATF experiments. A simple tokamak-based resistive MHD turbulence model shows that the $v^*_s$ dependence is close to the data ($\alpha_v = -0.4$), but the $\beta$ dependence is opposite. The indicated favorable trend of confinement with increasing $\beta$ may result from the $\beta$ self-stabilization effect on interchange modes [14], as discussed below.

Though the range over which this scaling can be extrapolated is uncertain, it suggests excellent prospects for improved confinement in ATF with increased heating power, since adding power both increases $\beta$ and decreases collisionality.
3. MAGNETIC CONFIGURATION SCANS

The flexible magnetic configuration control in ATF allows independent control of parameters that are fundamental to toroidal confinement and stellarator optimization [1]. The three degrees of freedom afforded by the three vertical field coil sets enable us to prescribe the dipole and quadrupole moments (specifying the magnetic axis shift and plasma shaping, respectively) while the linked flux is kept constant (to avoid driving an inductive current). The variation of the poloidal field permits external control of physical properties of magnetic confinement on a flux surface, such as magnetic field curvature (and thus control of magnetic well and hill), rotational transform (shear), and l|B| spectrum (neo-classical viscosity) and trapped particle population. Figure 5 shows contour plots of three configuration-related (dimensionless) parameters in the dipole-quadrupole configuration space: (1) contained trapped particle fraction $f_{TPC}$, which is related to the helically trapped particle loss region modeled without the electric field; (2) average magnetic shear, $s = -(\rho/\tau)(d\tau/d\rho)$, at normalized radii $\rho$ between 0.4 and 0.6; and (3) magnetic well radius $\rho_{well}$ inside which the magnetic well resides for low-$\beta$ plasmas ($\langle \beta \rangle = 0.1\%$, typical for ECH plasmas). Contours of constant magnetic shear and well radius nearly coincide for these low-$\beta$ plasmas, but contours of constant $f_{TPC}$ are nearly orthogonal to the other two over much of the range of interest. This enables separating the effects of the trapped particle population from those of magnetic shear and well/hill.

A full-range dynamic variation (along path 1 in Fig. 5) of the parameter $f_{TPC}$ was accomplished with two long-pulse discharges while keeping constant the shear and magnetic well and the line-average electron density (by density feedback control). Figure 6 shows results of such a scan. Changing $f_{TPC}$ has little effect on plasma parameters until this quantity becomes greater than 0.8. Although the ECH power deposition profile changes somewhat with increasing $f_{TPC}$, the deposition profiles tend to remain broad and rather insensitive to the variation of the configuration (except possibly for the extreme value of $f_{TPC}$) with the present ECH launching scheme [4]. One possible explanation for the insensitivity of the confinement is the radial electric field that tends to reduce the orbit loss region. Heavy ion beam probe (HIBP) measurements of plasma potential in configurations other than the standard one are rather sparse, but data are available for a quadrupole scan (path 2 in Fig. 5) [15]. Figure 7 shows that the electric field measured by the HIBP increases as the loss region expands (i.e. as $f_{TPC}$ decreases) in this scan. The simultaneous increase in the global energy confinement time in the quadrupole scan may hint at an improvement of confinement with increasing radial electric field, as seen in the L-H transition in tokamaks. The role of the electric field in reducing the loss region is important for stellarator optimization, so further studies are clearly needed. Implications of this scan related to the DTEM instability are discussed in the companion paper [16].

Figure 8 shows the plasma response to sinusoidal modulation (along path 3 in Fig. 5) of the magnetic well radius. The modulation substantially affects the
FIG. 5. Contour plots of physics parameters in the dipole-quadrupole configuration space and paths followed in the dynamic configuration scans.

FIG. 6. Plasma parameters as a function of the confined trapped particle fraction during a dynamic configuration scan in which the well radius and the shear are kept constant.
FIG. 7. Variations of measured electric potential ($\Phi$ at $\rho = 0.6$ relative to the plasma edge), confined trapped particle fraction and global energy confinement time in a quadrupole scan.

FIG. 8. Time evolution of plasma parameters with sinusoidal modulation of the magnetic well radius and constant contained trapped particle fraction.
stored energy and core electron temperature. This result is consistent with resistive interchange modes influencing energy confinement, since interchange modes are stabilized by a magnetic well. Density fluctuation measurements showed radial behavior of $\tilde{n}/n$ consistent with theoretical predictions of resistive interchange modes [13]. Difficulties in separating the effects of magnetic shear and well/hill can be mitigated by magnetic well broadening in higher-$\beta$ plasmas. Indeed, the earlier experiments with centrally peaked pressure profiles (due to field errors) showed suppression of interchange modes at relatively low $\beta$ ($\beta_0 \leq 1.5\%$), consistent with theoretically predicted $\beta$ self-stabilization [16].

4. CONCLUSIONS

The favorable $v_*$ and $\beta$ dependences ($\alpha_v = -0.18 \pm 0.03$, $\alpha_\beta = 0.3 \pm 0.1$) experimentally observed suggest excellent prospects for improved confinement in ATF with increased heating power. Long-pulse and modulation techniques have been used to establish the dependence of confinement on critical stability factors: magnetic shear, well/hill, and trapped particle fraction. It is anticipated that at some time in the future these experiments will be extended to higher heating power to provide useful information on stellarator optimization and a better understanding of toroidal transport.

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DISCUSSION

K. ITOH: In performing experiments to determine $\alpha_v$ and $\alpha_\beta$, the parameter $p^*$ is also modulated. Therefore, it seems to me, the evaluations obtained for $\alpha_v$ and $\alpha_\beta$ are somewhat premature. (If one substitutes $\alpha_v = -0.18$, $\alpha_\beta = 0.36$, we have $T \sim T^*_{-0.9}$ suggesting a large deviation from dependence.)

M. MURAKAMI: In the modulation experiment, there was some variation of $\rho_*$, as shown in Fig. 2. With the wisdom of hindsight we should have included a third modulation experiment with a constant $\rho_*$ (by adjusting ECH power and plasma density such that $\tilde{n}/n = \tilde{P}/P$). According to my calculation, substitution of $\alpha_v$ and $\alpha_\beta$ leads to $\tau \sim T^{0.78} B^{1.28} n^{0.18}$. It is almost a matter of opinion whether the $T$ dependence is said to be closer to Bohm or gyro-Bohm, and the $B$ and $n$ dependences should be taken into account.

J.G. CORDEY: You stated that in ATF the confinement was gyro-Bohm rather than Bohm. In a tokamak it is quite difficult to determine the $\rho_*$ dependence owing to the collinearity between $\rho_*$ and $\beta$. What was the range of $\rho_*$ investigated in ATF and how constant were the remainder of the dimensionless variables such as $\nu_*$, $\beta$, and so on?

M. MURAKAMI: I agree that the distinction between gyro-Bohm and Bohm scaling is difficult. However, there is an important difference between stellarator and tokamak operation. There is stronger plasma density and field dependence in stellarator confinement, which shows a better fit with gyro-Bohm than with Bohm scaling. We failed to complete our planned $\rho_*$ scan with ECH at different field values because of limited ATF operation before shutdown. The range of $\rho_*$ in the overall...
ATF database is nearly an order of magnitude (0.003 to 0.020). Attempts to select data sets at constant $\nu_*$ and $\beta$ values did not lead to any reliable $\rho_*$ dependence. Regression analysis of $\tau_\rho \Omega$ with $\rho_*$, $\nu_*$ and $\beta$ yielded a gyro-Bohm-like (-3) rather than Bohm-like (-2) $\rho_*$ dependence, but remains less conclusive owing to the collinearity with $\beta$ and $\nu_*$. 

O. KARDAUN: In your regression analysis with the dimensionless variables, did you (a) do the regression directly on the calculated dimensionless variables, or (b) do the regression on the engineering variables and then apply the dimensionless constraints, as in the paper by Christiansen et al. (Nucl. Fusion 31 (1991) 2117)?

M. MURAKAMI: In our regression analysis we used the calculated dimensionless variables (that is (a) rather than (b)). Collinearity of $\rho_*$ with $\nu_*$ and $\beta$ made determination of the $\nu_*$ and $\beta$ dependence less reliable than we had hoped for. The collinearity was in fact the reason why we undertook the 'orthogonal' modulation experiments to isolate the $\nu_*$ and $\beta$ exponents. The modulation results independently corroborated the regression results.
CONFINEMENT STUDIES IN RECENT HELIOTRON-E EXPERIMENTS

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Abstract

CONFINEMENT STUDIES IN RECENT HELIOTRON-E EXPERIMENTS.

A further extension of Heliotron-E confinement improvement studies has been performed using (i) magnetic configuration control with additional free coil parameters, (ii) density profile and heating profile control with pellet injection and with highly focused 106 GHz ECH, and (iii) edge plasma control with biased or non-biased conditions. Measurements of NBI plasmas with limiter insertion suggest that the global energy confinement scaling is not contradictory to the $a^2$ scaling, despite a decrease in magnetic shear, while drift orbit optimization by means of the inward shift of the magnetic axis isstill effective, indicating the importance of the restoration of helical symmetry. A peaked density profile using pellet injection was observed to improve the energy confinement by 20–30% as compared with gas-puffing-only plasmas in a moderate density range. Highly focused second harmonic X mode ECH could induce a detached plasma with a peaked temperature profile in a low recycling state. By using a negatively biased limiter inserted well inside the separatrix, fast reciprocating Langmuir probe measurements have revealed a sharp drop in the floating potential in the limiter shadow, accompanied by the reduction of edge density fluctuations (ECH) and by the improvement of particle confinement (NBI). Related results for controlling the confinement from the edge are discussed from the viewpoint of maintaining the good particle confinement regime.

1. INTRODUCTION

The Heliotron-E vacuum magnetic field has a rotational transform ($0.52 < \psi/2\pi < 2.5$) sensitive to the low mode perturbation field, and therefore one of the targets for the improvement of Heliotron-E plasma confinement is to explore means of controlling these potentially dangerous resonances in addition to the built-in strong magnetic shear as a countermeasure. Until now, the experimental evidence has suggested

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the most favourable confinement configuration in Heliotron-E to be $a^* = 0.05$ and $\beta^* = -0.192$ ($\alpha^* = B_t/B_h$ and $\beta^* = B_v/B_h$) [1-3] in the natural divertor operation regime. In this configuration, the experiments have shown energy transport improvement as well as MHD stability improvement due to the toroidal field shifting the $\nu/2\pi = 1$ surface towards the outer region of strong shear, accompanied by a shallow well in the core region. The Heliotron-E magnetic configuration control studies have recently been extended to combined studies with density profile and/or heating profile effects and limiter biasing effects. A new 106 GHz X mode ECH system was operated to investigate its absorption efficiency and the resultant unique plasma performance. A study of the capabilities of the edge region to control the core region was performed using an externally biased limiter to induce an edge radial electric field. A change of impurity transport and MHD activity in this regime is also discussed.

2. CONFIGURATION STUDIES

The resistive interchange mode (turbulence) is always unstable in the outer plasma region of Heliotron-E, where the configuration feature is inevitably the magnetic hill. The earlier regression analysis of the global energy confinement time $\tau_E$ and the electron thermal diffusivity $\chi_e$ suggested that the anomalous transport in the outer plasma region closely follows a scaling like the gyro-reduced Bohm scaling [4], provoking interest in the nature of the turbulence. With regard to the anomaly in the outer plasma region, vacuum magnetic surface mapping was carried out in a full two dimensional scan [5-7] and the results suggested a low level magnetic braiding in the $\nu/2\pi \geq 2$ region, i.e. the peripheral zone. To study the effects of such braiding as well as to study the experimental differences in their role in confinement improvement in the core region and the edge region, a material limiter was inserted in order to investigate the aspect ratio dependences as well as the magnetic axis shift dependences.

The insertion of a non-biased limiter drastically decreased the thermal load onto the vacuum chamber as well as the bolometric loss, CX loss, $H_a/D_a$ emissions, etc. [8]. The thermal load onto the limiter, on the other hand, was found to reach almost 50% of the absorbed ECH/NBI power for limiter plasmas. For NBI plasmas, it had already been reported [9] that the global energy confinement scaling with respect to the plasma radius $a_p$ is not contradictory to the $a_p^2$ scaling for natural divertor configurations where the plasma radius is varied ($0.7 \leq (a_p/a_l)^2 \leq 1.2$) with the toroidal field parameter, $\alpha^* = B_t/B_h$, where $a_l$ is the plasma radius in the standard configuration. For material limiter configurations this size scaling could be checked again in the much wider range of $0.4 \leq (a_p/a_l)^2 \leq 1$ under different boundary conditions where the dominant plasma–wall interaction area is mostly localized near the limiter head. These two types of experiment are mutually complementary, and the results in Fig. 1(a) give a greater confidence in expecting the $a_p^2$ dependence. Recent Monte Carlo simulations using the HELIOS code [10] have
FIG. 1. (a) Global energy confinement scaling as a function of $a_s^2$ in NBI experiments. (b) Improvement of global energy confinement with $\Delta_s = -2$ cm for limiter NBI plasmas.
shown that the orbit loss with perpendicular injection is highly sensitive to the conditions of the loss boundary of fast ions. However, in the normalized plot of Fig. 1(a), the ratio $\frac{\tau_{E_{\text{itr}}}}{\tau_{E_{\text{exp}}}}$ is independent of the model of orbit loss, although $\tau_{E_{\text{exp}}}$ is estimated using a simplified orbit loss model for the limiter plasmas.

When the material limiter position $a_L$ was fixed inside the separatrix ($a_L = 2a_s/3$) to maintain an almost constant plasma radius $a_p = a_L$, edge rotational transform $\frac{1}{2\pi}(a_p)$ and practically unchanged plasma-wall interactions, the inward magnetic axis shift experiments with ECH/NBI plasmas showed the same behaviour of confinement improvement as that of the natural divertor configuration. Assuming the $a_s^2$ scaling, the limiter NBI plasma confinement under the fixed radius $a_p = 15$ cm and $\Delta_n = -2$ cm at $B_n = 1.9$ T is enhanced by 40% as compared with the predicted standard scaling of $\tau_{E_{\text{exp}}}$ [4], as shown in Fig. 1(b). This improvement is reflected also in the local power balance analysis. Thus it is found that the decrease in the magnetic shear in the outer region of limiter plasmas has only small effects on confinement as compared with the confinement improvement of bulk trapped particles due to the restoration of helical symmetry. The experimental comparison between plasmas with and without limiter showed an apparent upgrade of confinement improvement especially for the limiter ECH plasmas, which might be interpreted as being due to the removal of the possible deleterious effects caused by the transformed plasma-wall interaction locations as well as the conceivable edge magnetic braiding. However, even in the drift orbit optimized configuration, the anomalous transport still dominates the plasma confinement in the outer plasma region, and a most probable candidate is still the resistive interchange turbulence. From the viewpoint of stabilizing the resistive interchange turbulence [11], the effects of the radial electric field which can be introduced by a biased limiter are particularly interesting.

3. PROFILE CONTROL STUDIES

3.1. Pellet induced density profile effects

Deuterium/hydrogen pellet injection (up to six pellets in one discharge) is used for investigating the density profile effects on confinement and MHD properties. In the case of NBI plasmas, the diamagnetic internal plasma energy $W_{\text{dia}}$ of pellet injected plasmas is 2–3 kJ higher than that of gas puffed plasmas, which represents a 20–30% improvement over the gas puffed case (which obeys the LHD scaling [12]) in a moderate density range. This confinement improvement is considered to be due to the peaked density profile produced by pellet injection (as observed in tokamaks), as shown in Fig. 2. It is confirmed that $W_{\text{dia}}$ is larger in the case of more peaked profiles for the same $n_e$. Moreover, the operational density range is widened in the case of pellet injection. The bolometric power is decreased in the case of the peaked density profile, which may be attributed to the lower density in the peripheral region.
for the same $\bar{n}_e$. In the case of ECH plasmas, since a peaked density profile was difficult to obtain owing to the shallow penetration of the pellet (presumably attributable to rapid ablation by suprathermal electrons), the confinement properties of pellet injected ECH plasmas remained essentially unchanged. In the case of high $\beta$ ($\beta_0 > 2\%$) plasmas at $B = 1.9$ T, the peaked pressure profile produced by pellet injection without gas puffing tends to destabilize the plasma for $P_{nh}$ of about 4 MW. The mode of $m/n = 3/2$ at $\nu/2\pi = 2/3$ seems to be the major MHD instability which results in internal disruption. As the temperature profile is very flat for NBI plasmas, the large density gradient at $\nu/2\pi = 2/3$ should be avoided from the viewpoint of MHD stability for high $\beta$ plasmas.

3.2. Centrally localized ECH absorption profile effects

A new ECH system has been introduced using a 106 GHz gyrotron. The purpose is to investigate the properties of second harmonic X mode heated plasma and to produce a high density plasma (R cut-off density is $7.0 \times 10^{13}$ cm$^{-3}$). The HE$_{11}$ beam is launched from the top of the torus and is focused by the parabolic mirror before injection into the torus. Measurements of the radiation pattern by the thermal
imaging method showed that the 1/e beam power radius at the magnetic axis was well focused to 2 cm and 3 cm in the radial and toroidal directions, respectively. Figure 3(a) shows the time history of a 106 GHz heated plasma, where $B_h$ is 1.96 T and the limiter is shallowly inserted into the plasma ($a_L/a_s \sim 0.9$). Although the resonance layer does not access the centre region ($r/a_L \leq 0.26$) at this field strength, the injected microwave can surely cross the resonance layer. The 200 kW, 106 GHz power is injected after the plasma production by 53 GHz fundamental heating. In the afterglow phase of the 53 GHz heating, the limiter current goes down to zero and the ion saturation current goes down by half. The 106 GHz ECH is injected in this afterglow phase, and it was found that these currents did not rise again, which was quite different from the case of fundamental heating at the same power level. $D_{\alpha}^{\text{wall}}$ at the wall also decreased. These results suggest that the outflux has drastically decreased as compared with the case of fundamental heating. This low recycling state
is maintained until density clamping occurs. The line density increases during this low recycling phase, accounting for the improvement in the particle confinement. The density increase becomes larger as \( n_e \) becomes lower. The electron temperature measured by Thomson scattering is \( T_e(0) \sim 1.3 \text{ keV} \) at \( n_e \sim 1.0 \times 10^{13} \text{ cm}^{-3} \). As shown in Fig. 3(b), the \( T_e \) profile is peaked while \( T_e \) is less than 100 eV at \( r/a_L \sim 0.7 \). Measurements by Langmuir probe also showed that \( T_e \sim 10 \text{ eV} \) over \( r/a_L \sim 0.8 \). From these results, almost all the power is considered to be deposited in the central region in the first pass absorption, resulting in improved confinement where the 106 GHz heated plasma is detached from the limiter.

4. EDGE CONTROL STUDIES

As the first step to externally control the plasma potential (or edge electric field), a bias voltage was applied between the carbon limiter and the vacuum chamber wall by an external power source. With insertion of the limiter into the core plasma with a constant bias voltage \( V_b \), the bias current \( I_b \) decreases for either polarity of \( V_b \). Taking account of the field structure and the mechanical size of the limiter, the change of current from parallel to perpendicular flow can cause the decrease of \( I_b \). The original 'natural' divertor discharge changes into a limiter discharge by the reduction of the limiter radius \( z_{lim} \), so that the decrease of \( I_b \) is also effected by the reduction of the divertor flow. Figure 4 shows examples of radial profiles of the floating potential \( V_f \) in the limiter shadow from a fast reciprocating probe. While \( V_f \)}
is gradually increased from the wall potential to the limiter potential for the positive bias case ($V_b > 0$). $V_f$ for $V_b < 0$ suddenly drops at a certain position. This position seems to shift with a change of the limiter position, but such an abrupt potential drop was not observed for shallow limiter insertion.

For ECH plasmas ($n_e < 0.5 \times 10^{15} \text{ cm}^{-2}$), the limiter bias increases the line density for either polarity of $V_b$. Under the condition of $n_e \equiv \text{const}$, the intensity of $D_\alpha$ emission far from the limiter section, $D_{\alpha, \text{wall}}$, is increased for $V_b > 0$ and decreased for $V_b < 0$, whereas the electron temperature is almost the same. The increase of soft X ray emission $I_{\text{sx}}$ and the peaking of its chord profile were also observed for both polarities. This suggests that the limiter bias changes the impurity transport. In the limiter shadow, the reduction of the density fluctuation was observed with the probe measurement for $V_b < 0$. As for the potential fluctuation, no clear difference between the two polarities was observed. As mentioned previously, the limiter insertion reduces the divertor flux. Although the residual divertor flux $\Gamma_{\text{div}}$ is also affected by the limiter bias, the response shows toroidal asymmetry. In addition, it is found that limiter biasing is effective for prolonging the low recycling state of
the 106 GHz ECH plasma. Figure 5 shows the time evolution of the 106 GHz heated plasma with and without limiter biasing. The bias is imposed throughout the discharge. The density clamping is suppressed by the limiter biasing (∼−300 V), and the low recycling phase becomes longer.

For NBI plasmas, \( n_e f \) maintained a steady increase \( (n_e f \approx (2-4) \times 10^{15} \text{ cm}^{-2}) \) during injection even for \( V_b = 0 \). As the limiter was biased, the level of the radiation power and the impurity line emissions such as Fe(XVI) became larger. Contrary to the ECH case, the response of \( D_a \text{wall} \) showed toroidal asymmetry for \( V_b > 0 \), i.e. \( D_a \text{wall} \) at \( \phi = 128^\circ \) (far from the beam lines) decreased, whereas \( D_a \text{wall} \) at \( \phi = 317^\circ \) (near one beam line) increased. For \( V_b < 0 \), however, both signals decreased. For \( V_b = 0 \), the time behaviour of \( D_a \text{lim} \) is strongly correlated with that of \( n_e f \), which is usually expected for the limiter configuration. In the \( V_b < 0 \) case, \( D_a \text{lim} \) also decreased and the correlation with \( n_e f(t) \) weakened. Thus, this reduction of \( D_a \) can suggest the improvement of \( T_P \). The inhomogeneous response of \( \Gamma_{CX} \) observed for ECH plasmas becomes clearer for NBI plasmas, especially in the shallow limiter insertion and positive bias case. This result suggests a change of the divertor flow pattern due to the local change of the edge plasma potential [13].

The \( I_{sx} \) profile becomes rather broader under limiter bias conditions, and in some cases sawtooth-like oscillation of \( I_{sx} \) due to MHD activity was diminished by the limiter biasing. This can be related to the broadening of the pressure profile as suggested by the \( I_{sx} \) profile. On the other hand, it was also observed that the CX flux \( \Gamma_{CX} \) near the injection energy range changed its value in accordance with the oscillation of \( V_b \) (in the \( V_b > 0 \) case). Since such a response of \( \Gamma_{CX} \) was not observed at other energy ranges, the population of the fast ions itself is considered to be modulated by the limiter bias.

5. CONCLUSION

Magnetic configuration studies using the limiter revealed the special importance of drift orbit optimization for confinement improvement in Heliotron-E compared with the shear effects, beyond the overall confinement properties represented by the \( a_2^2 \) scaling. Density profile and heating profile control successfully provided other tools to improve the confinement in Heliotron-E. Highly focused second harmonic ECH could produce a detached plasma with a peaked temperature profile at significantly reduced outflux conditions, and this was first achieved in Heliotron-E. With regard to edge control, negative limiter biasing could produce a sharp drop of the potential in the limiter shadow, suggesting that there should be a sufficient poloidally directed \( \mathbf{E} \times \mathbf{B} \) drift in the vicinity of the limiter. This might cause a rearrangement of the core confinement as well as the edge transport as the boundary condition. Negative biasing for limiter NBI plasmas could introduce an improvement of the particle confinement. For ECH plasmas, this also accompanied the reduction of the edge electron density fluctuation and was also effective for prolonging the low recycling state of the 106 GHz ECH plasma.
REFERENCES


DISCUSSION

F. WAGNER: You have shown that confinement improves in Heliotron-E with peaking of the density profile (as in tokamaks). What is your understanding of the physical basis for this?

T. OBIKI: Possible explanations for the confinement improvement by peaked profile are (i) the realization of a low recycling state at the plasma edge, and (ii) utilization of inner magnetic surfaces where no magnetic braidings exist.
FLUCTUATION STUDIES OF ECH PLASMAS IN
THE ATF TORSATRON AND
THE IMS STELLARATOR*

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Abstract

FLUCTUATION STUDIES OF ECH PLASMAS IN THE ATF TORSATRON AND THE IMS STELLARATOR.

The results of experimental studies of fluctuations and transport in ECH plasmas in the Advanced Toroidal Facility (ATF) torsatron and the Interchangeable Module Stellarator (IMS) stellarator are presented. In ATF, a variety of diagnostics have been used to measure turbulence amplitudes and spectral characteristics throughout the plasma. In the plasma core, density fluctuations with characteristics like those expected for drift waves (possibly connected with trapped electron effects) are seen. In the gradient region, the density fluctuation levels are consistent with those expected for resistive interchange turbulence. In the edge, fluctuations appear to be correlated with density gradients, and the fluctuation induced particle flux and global particle confinement can be influenced by positive limiter biasing. In IMS, whose parameters model those in the edge of larger devices, two-dimensional (2-D) probe measurements show substantial poloidal variations in the fluctuation induced particle flux that are most pronounced at the plasma edge; as in ATF, the particle flux can be reduced by positive limiter biasing. The 2-D probe measurements also show good agreement between the plasma poloidal flow velocities determined from the Reynolds stress and force balance.

1. INTRODUCTION

Electron cyclotron heated (ECH) plasmas in stellarator devices are macroscopically quiet, but exhibit anomalous outward transport very much like that seen in tokamaks, suggesting that the underlying anomalous transport mechanisms in these toroidal confinement devices share features that are independent of the source of the confining rotational transform [1]. This paper presents results from the Advanced Toroidal Facility (ATF) torsatron (R = 2.1 m, \( \bar{a} = 0.27 \) m, \( \iota_0 = 1/q_0 = 0.3 \), \( \iota_s = 1/q_s \approx 1 \), \( B \leq 1.9 \) T) and the Interchangeable Module Stellarator (IMS) (R = 0.40 m, \( \bar{a} = 0.04 \) m, \( \iota_0 = 0.0 \), \( \iota_s = 0.6 \), \( B \leq 0.6 \) T). While the magnetic configurations of these two devices differ in detail, the general structures of rotational transform, shear, magnetic well and helical field ripple are broadly similar.

2. FLUCTUATION STUDIES IN ATF

A principal goal of experiments on the ATF torsatron in 1991 was the study of turbulent fluctuations in long pulse (1–20 s), low collisionality ECH discharges (\( B = 0.94 \) T, \( P_{\text{ECH}} = 300–400 \) kW, \( \bar{n_e} \approx (4–7) \times 10^{12} \text{ cm}^{-3}, T_{e0} \approx 1 \text{ keV} \)). The flexible magnetic configuration of ATF makes it possible to change properties — rotational transform, magnetic well, shear, trapped particle confinement — that are expected to influence turbulence and transport even within a single discharge.

The array of fluctuation diagnostics installed on ATF includes a heavy ion beam probe (HIBP), a microwave reflectometer, a fast reciprocating Langmuir probe (FRLP) and a 2 mm microwave scattering diagnostic. Together, these diagnostics make it possible to measure fluctuation amplitudes and other characteristics from the
plasma core to the edge. During the 1991 operating period, fluctuation data from the various diagnostics were combined to yield a picture of the turbulence as a function of radius which can be compared with theoretical concepts (Fig. 1).

The edge plasma fluctuations show characteristics similar to those seen in tokamaks, provided the radial position is normalized to the radius of the velocity shear layer [2]. This supports the hypothesis that drive mechanisms such as radiation, ionization, etc. that are independent of the details of magnetic configuration are responsible for the observed edge turbulence. For constant electron temperature profile in the edge region, increases in $\nabla n$ are correlated with increases in fluctuation amplitudes and fluctuation induced particle transport [3]. Positive limiter biasing at $+120$ V [4] shifts the shear layer inward in minor radius, decreases the measured fluctuation induced particle transport by a factor of ten, and increases the global particle (but not the energy) confinement time.

In the gradient region, $0.7 \leq r/a \leq 0.9$, the fluctuations exhibit significant radial correlation lengths $L_c = 2-5$ cm and no radial propagation ($k_r \approx 0$) [5]. The measured fluctuation amplitudes $\tilde{n}/n$ are smaller than those predicted by simple mixing length estimates by a factor of 2–10, but are comparable to amplitudes predicted theoretically [6] for resistive interchange turbulence driven by the unfavourable average magnetic curvature in this region. Single shot configuration variations which vary the magnetic curvature during a discharge indicate that this region governs the overall energy confinement in ATF; global confinement improves as the volume of the magnetic well region expands. This supports the conclusions of early high $\beta$ experiments on ATF [7], which demonstrated transient access to the second stability regime for interchange instabilities, and could be related to the enhancement of confinement with $\beta$ observed in ATF during dimensionless parameter variation experiments [8].
FIG. 2. (a) Variation of line average density (2 mm interferometer) and central electron temperature (ECE) during 'density step' experiment in ATF ECH plasma; (b) electron density profiles at three times during the density step (indicated by arrows in (a)).

The properties of density fluctuations in the plasma core have been probed by using 2 mm microwave scattering [9]. The geometric configuration of the four scattering channels on ATF permits measurements of fluctuations with $k_x \rho_s \approx 1-4$ with sufficient spatial resolution to separate signals from the plasma core and edge.

The scattered signals from the turbulent density fluctuations have frequency components in the range of 20–250 kHz, with a power weighted mean frequency $\approx 50$ kHz. Measurements from a dual homodyne scattering configuration show that the turbulence propagates unidirectionally in the electron diamagnetic direction; an ion feature is not present.

For discharges in which the density is held constant, the fluctuation levels do not vary strongly as the magnetic configuration is varied (by changing the quadrupole moment of the magnetic field) so as to change the fraction of helically trapped particles that are confined in the vacuum stellarator field. The fluctuation level
(S/n)\(^{16}\) \(\propto\) \(\bar{n}/n\) determined by summing over the four scattering channels varies by only 30% as the confined trapped particle fraction is changed from 98% to 49%. Similarly, variation of the confined trapped particle fraction has no significant effects on global energy confinement [8]. It must be pointed out, however, that changes in the electric field measured during the configuration scans could improve the trapped particle confinement over that expected for the vacuum field alone; this possibility needs further investigation.

ECH discharges in ATF typically show flat or hollow density profiles (determined by inversion of FIR interferometer data) and peaked electron temperature profiles (measured by Thomson scattering and ECE). By puffing in additional gas during a discharge, it is possible to modify the form of \(n_e(\rho)\) so as to change the sign of \(\nabla n\) in at least the region \(\rho \geq 0.65\). Figure 2 illustrates the temporal evolution of the central electron temperature and line-average density and density profiles for such a discharge. During this density step, the scattering signals change markedly. Figure 3

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig3.png}
\caption{Variation in density gradient at \(\rho = 0.7\) during ATF density step experiment and scattered power for two values of \(k\).}
\end{figure}
shows the time variation of two scattering channels at $k = 7.5$ cm$^{-1}$ and 12.4 cm$^{-1}$ with the time variation of $\nabla n$ ($\rho = 0.7$). The scattered power rises sharply as $\nabla n$ decreases and becomes negative in response to the density step; $S(k)$ then falls as $\nabla n$ increases after the density step. Note that because of the relatively poor radial resolution of the scattering diagnostic ($\Delta \rho = 0.3$–0.5), the scattering signals contain contributions from minor radii well inside of $\rho = 0.7$.

The behaviour shown in Fig. 3 is seen in signals from regions of the plasma on the inside, top and bottom of the torus, where trapped particles are well confined [10]. Signals from the plasma region on the outside of the torus, where trapped particles are poorly confined, remain nearly constant.

These observations could be evidence of turbulence driven by electrons trapped in the helical ripples of ATF. Theoretical calculations for dissipative trapped electron modes (DTEM) [10] indicate that such instabilities should be larger in radial extent than in tokamaks because of localization of the instabilities in the helical field ripples (which have length $2\pi R/12$ in ATF) and can become unstable when both $\nabla T_e$ and $\nabla n < 0$. The instabilities have a relatively large radial width of $\Delta r \sim 0.1a$ and so could lead to significant transport losses. The strength of the instabilities might also account for the relative insensitivity of the observed fluctuations to variations in the confined trapped particle fraction. Operation near the marginal stability condition for trapped electron instabilities could thus conceivably account for the persistence of flat and hollow density profiles in ECH stellarator plasmas.

3. FLUCTUATION INDUCED TRANSPORT AND FLOWS IN IMS

Current studies on IMS in the area of fluctuation induced transport are designed to address key issues in determining the role of this plasma loss mechanism in toroidal confinement devices.

Measurements of the plasma potential, plasma density and fluctuations in these quantities, over an entire poloidal cross-section of ECH plasma ($B = 0.26$ T, $P = 1$ kW, $n_e = (5–11) \times 10^{10}$ cm$^{-3}$, $T_e = 5–10$ eV, $T_i = 1–4$ eV) have been made by using an array of emissive and Langmuir probes. The spatial orientation and spacing of the four emissive probes provide a measurement of the plasma potential and its three-dimensional gradient over a small cell of the plasma volume. From these measurements, one can compute the Reynolds stress $\langle \nabla_v \nabla_p \rangle$ and, along with the corresponding local density fluctuations, the electrostatic fluctuation induced transport $\langle n_e \nabla_p \rangle$.

These parameters are presented for two distinct conditions created in IMS discharges of 10 ms duration: (1) an electrode placed 1 cm inside the separatrix is biased to $+80$ V, which creates a strong positive radial electric field (3–4 kV/m) and drives a large poloidal rotation of the edge plasma (10–15 km/s); (2) the electrode is electrically disconnected from its source, and the plasma is allowed to relax to its nominal state.
FIG. 4. Comparison of Reynolds stress driven poloidal flow and the flow computed from the electric field and ion pressure profiles.

FIG. 5. Fluctuation induced particle transport in IMS for biased and unbiased conditions, decomposed onto a magnetic flux surface for three minor radii: (a) r/a = 1.0; (b) r/a = 0.7; (c) r/a = 0.4.
From the measured radial electric field and pressure profiles, IMS exhibits a net poloidal rotation in the unbiased case, where there is no external driving force. It has been suggested that fluctuations are coupled to poloidal flow through the Reynolds stress tensor [11]; such a mechanism might be involved in the L−H confinement transition. The Reynolds stress driven poloidal flow is estimated by

\[ V_\theta = \gamma \left\{ \frac{\partial \langle \tilde{v}_r \tilde{v}_\theta \rangle}{\partial r} \right\} \]

where \( \gamma \) is the poloidal damping time, the braces \{ ... \} indicate an average over the magnetic flux surface, and the angular brackets \( \langle ... \rangle \) indicate an ensemble average of the velocity components \( \tilde{v}_r \) and \( \tilde{v}_\theta \). A poloidal damping time of 19−24 \( \mu s \) is determined from the measured time response of poloidal plasma flow to pulsed electrode biasing. The estimated Reynolds stress induced flow of 600 \( \pm 350 \) m/s in the IMS edge region agrees well with the value of 500 \( \pm 150 \) m/s determined by force balance, as is shown in Fig. 4.

The two-dimensional measurements show that there are significant asymmetries in the fluctuation induced transport on a magnetic flux surface. Figure 5 shows the radially directed flux density decomposed onto magnetic flux surfaces for the biased and unbiased conditions at three minor radii. At the separatrix (top graph), the flux is directed mainly on the outside (\( \theta = 0^\circ \)) and inside regions of the torus for the unbiased plasma, while the biased case shows significant reduction in transport for the entire surface. The transport at \( r/a = 0.7 \) shows inward transport on the top of the device (\( \theta = 90^\circ \)) for both conditions. The bottom graph is indicative of the transport for \( r/a < 0.5 \).

Comparisons with measurements of particle confinement times show that the fluctuation induced transport is a major particle loss mechanism in the nominal (no electrode bias) discharges. Estimates of this loss rate have been compared to the total particle loss rate, measured from the sum of divertor particle fluxes, and indicate that the fluctuation induced transport represents 46% of the total particle losses in nominal IMS discharges. However, the significant reduction of the fluctuation induced transport during electrode biasing does not result in enhanced global particle confinement in IMS since the current required to maintain the electrode voltage represents a significant loss channel for the electrons. The magnitude of the electrode current is \( \approx 50\% \) of the equivalent current of the total divertor particle flux and is thus comparable to the fluctuation induced particle flux of the unbiased plasma.

4. CONCLUSIONS

Fluctuation measurements in ATF ECH plasmas have yielded evidence for operation of multiple instability mechanisms. While there remain many uncertainties in the detailed interpretation, the present results are generally consistent with an overall picture in which the density profile is constrained by trapped electron instabilities,
the global energy confinement is determined by curvature driven instabilities in the gradient region and the global particle confinement is regulated by edge turbulence.

In IMS, the Reynolds stress driven poloidal flows compare well with flows computed from plasma equilibrium profiles; this mechanism has been proposed as the drive for the poloidal rotation seen in the L–H transition [11]. Induced radial electric fields and poloidal flows from the electrode biasing experiments cause the fluctuation induced transport to be significantly reduced, although the overall confinement properties of the device were not improved.

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DISCUSSION

R.J. TAYLOR: In ATF, $\tau_p$ improves by a factor of ten with bias. You state that $\tau_E$ does not improve with bias, but in paper C-1-1, M. Murakami seems to claim that it does. If $\tau_E$ does not, like $\tau_p$, improve with bias, can shear suppression theories possibly explain your results (and those from the continuous current tokamak (CCT) of the University of California at Los Angeles)?

J.H. HARRIS: The improvement in $\tau_p$ with bias in ATF is definitely the stronger effect — for the modest bias voltages (~100 V) that were applied. Since
\( \tau_E \) in ATF seems to be most strongly affected by the magnetic well radius deeper inside the plasma, we may simply not have applied enough \( E_r \) to affect \( \tau_E \) yet.

J.A. WESSON: You suggest that the confinement in ATF is determined by two types of turbulence: the trapped electron mode in the central region and resistive MHD in the outer region. Is this consistent with Murakami's finding that the confinement time is given by a single-term expression?

J.H. HARRIS: Our observations of fluctuations so far seem consistent with a working hypothesis that energy confinement is governed by the gradient region (interchange) while the form of \( n(r) \) is constrained by trapped particle instability. The interchange stability should improve with \( \beta \), which is supported by Murakami's findings. The core region could well enter more directly into the determination of \( \tau_E \) when one has low collisionality and high \( \beta \) simultaneously.

R.J. GOLDSTON: You showed spectra with nice peaks at low \( k \rho_s \) during the period of \( \nabla n < 0 \). Could you comment on the fluctuation spectra versus radius during normal operation?

J.H. HARRIS: Our radial resolution is restricted by geometry to \( \Delta \tau/a \sim 0.3-0.5 \) at \( k \rho_s \geq 2 \), and is generally even worse at \( k \rho_s \leq 1 \). Hence we have concentrated so far on simply separating core from edge fluctuations.
IMPROVED MODELS OF BETA LIMIT, ANOMALOUS TRANSPORT AND RADIAL ELECTRIC FIELD WITH LOSS CONE LOSS IN HELIOTRON/TORSATRON

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Abstract

IMPROVED MODELS OF BETA LIMIT, ANOMALOUS TRANSPORT AND RADIAL ELECTRIC FIELD WITH LOSS CONE LOSS IN HELIOTRON/TORSATRON.

The physics mechanisms determining beta limit, anomalous transport, radial electric field and loss cone are studied theoretically. A new theory is developed analysing the stability boundary against the interchange mode in high aspect ratio toroidal helical plasmas, taking into account transport processes. The stability beta limit is given at finite beta, and its dependence on the plasma parameters and the transport coefficient is investigated. It is found that in hot plasmas the current diffusive interchange mode is more important than the resistive mode. In the range of experimental observation the beta limit is predicted for anomalous transport. The dynamics of pressure gradient and mode amplitude around this stability boundary is analysed. As the heating power is increased, the dynamics changes from monotonic saturation through saturation with overshoot, towards sawtooothing. By the mean field theory approach of statistical physics for the microscopic current diffusive interchange mode, an anomalous transport theory is developed. An expression for the thermal transport coefficient is obtained. The pressure gradient, rather than the temperature itself, increases the transport coefficient. A comparison with experimental observations from various aspects is made, and the model is found to explain the experimental observations. A method of obtaining a self-consistent picture of the radial electric field $E_r$ and the loss cone loss is explored. The structures of $E_r$ and the loss cone are obtained, and the direct ion loss is confirmed to make $E_r$ more negative near the edge. The effects of other non-classical losses are also evaluated.

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1. INTRODUCTION

In order to understand the confinement and perform the future optimization in heliotron/torsatron configurations, the physical mechanisms determining beta limit and anomalous transport must be studied theoretically. The importance of the interaction between the radial electric field and the loss cone is also recognized. In this article, we present (1) new theoretical results on the beta limit of high temperature plasmas in the presence of the transport processes, (2) a theory on anomalous transport which can explain the experimental observations and (3) a self-consistent picture of the radial electric field and the loss cone.

2. FINITE BETA STABILITY OF DISSIPATIVE PLASMA

We study the interchange instability by taking into account transport processes governed by quantities such as thermal conductivity $\chi$, ion viscosity $\nu$, current diffusivity $\lambda$ and resistivity $\eta$. The reduced set of equations, and a cylindrical model are used. The case of a high aspect ratio system with a magnetic hill is studied.

By linearizing the basic equations, an eigenvalue equation is obtained [1]. It has been shown that the mode is always unstable when only resistivity is taken into account. If, however, other transport coefficients are kept, a critical beta value for stability, $\beta_c$, is found such that the mode is stable below $\beta_c$, as is shown in Fig. 1. The stability condition is given by $D < D_c$, where

$$D_c = \min \left( \left( \frac{3\pi s}{2} \right)^2 \frac{\chi}{\eta} \left( \ln \left( \frac{3\pi s}{2} \right)^2 \frac{1}{\nu^2 k_0^4} \right) - 2 \ln \left| \ln \left( \frac{\chi^3 k_0^8}{64} \right) \right| \right)^{-2},$$

$$C^3 \lambda^{3/2} k_0^{-1/2} \psi^{-3/4} \chi^{1/4}$$

(1)

![FIG. 1. $n = 1$ mode growth rate in H-DR ($\eta = 10^{-4}$, $\chi = 5 \times 10^{-4}$). Previous results are shown by dashed lines (resistive and ideal MHD calculations).]
for a given transport coefficient. The notations are as follows: $D = -\beta'\Omega'/2\epsilon^2$, $\Omega$ is the average curvature of the field line, $' = d/d\rho$, $\epsilon = a/R$, $\rho = r/a$, $k_\rho = m/\rho_1$, $\rho_1$ is the mode rational surface, $m$ the poloidal mode number, $s = \rho t'$, $t$ is the rotational transform, $\eta = \eta \tau_A/\mu_0 a^2$, $\lambda = \lambda \tau_A/\mu_0 a^4$, $\tau_A = R/\gamma_A$, and $C$ is a numerical constant; the other notations are standard.

Equation (1) shows the dependence of $\beta_c$ on the electron temperature $T_e$. When $T_e$ is low and $\eta$ is large, $\eta^5 D > k^2 \lambda^3$, the low $m$ resistive interchange mode ($\eta$ mode) determines the stability beta limit. As $T_e$ increases and $\eta$ is reduced, $\beta_c$ increases for a given value of $\chi$. In the high $T_e$ case, $\eta^5 D < k^2 \lambda^3$, the current diffusivity plays a more important role indestabilizing the mode. The low $m$ current diffusive interchange mode ($\lambda$ mode) determines $\beta_c$. It is also noted that larger values of $\chi$ and $\nu$ for higher $T_e$ (as are observed in experiments) can also increase $\beta_c$ in hot plasmas. The value of $\beta_c$ is in the range of the experimental data for anomalous transport coefficients observed at present.

3. BETA LIMITING PHENOMENA

The evolutions of the pressure gradient and the mode amplitude near $\beta_c$ is studied by taking into account the background modification effect. Introducing the radial localization width of the mode around the rational surface, $\ell$, we have the model equation describing the dynamics of the normalized pressure gradient $D$ and the mode amplitude $K$ as [2]

$$\frac{dD}{dt} = (D_{heat} - D) - 3D/[2(1 + \tilde{\gamma})], \quad \frac{dK}{dt} = 2\tilde{\gamma}K$$  

(2)

**FIG. 2. Temporal evolution of (a) pressure gradient and (b) amplitude. $D_{heat} = 2.7$, $\tilde{\gamma}k^2 = 0.1$, $\ell = 2$.**
where $\mathcal{D} = D/D_0$, $\mathcal{K} = (\dot{\ell}/\chi^2) \langle |\nabla|^2 \rangle$, $\dot{\gamma} = \gamma \ell^2/\chi$ ($\gamma$ is the growth rate), and $t$ is normalized to $\ell^2/\chi$. $D_{\text{heat}}$ denotes the contribution of the external heating and corresponds to the gradient which would be sustained in the absence of this low $m$ activity. Note that $\gamma = 0$ at $\mathcal{D} = 1$.

When the heating power is increased and $D_{\text{heat}}$ exceeds unity, a dynamical evolution of $D$ and $K$ takes place. The stationary solution of Eq. (2) is given by $(\mathcal{D}^*, \mathcal{K}^*) = (1, 2D_{\text{heat}} - 2)$. For a given value of $z = [\partial \dot{\gamma}/\partial \mathcal{D}]$ (at $\mathcal{D} = 1$), the critical heating rate is found. For the high heating case, $D_{\text{heat}} > D = 1 + 1/(z - 1)$, a sawtooth-like periodic relaxation of the pressure gradient is found, associated with repeated bursts of the mode amplitude. Figure 2 shows the case of the $\eta$ mode. The time average of the pressure gradient is limited to the critical value.

4. ANOMALOUS TRANSPORT

The stability criterion for the interchange mode, Eq. (1), indicates that the $\lambda$ mode is most dangerous for the high $m$ mode (high $m$ $\eta$ modes are stabilized below the beta limit). The anomalous transport is determined by the high $m$ $\lambda$ mode [3]. For the high $m$ mode, the growth rate $\gamma$ approaches that of the fast interchange mode. The highest $\gamma$ is evaluated for $\gamma T_A \sim \sqrt{D}$, for which the radial mode number satisfies $a^2 k_r^2 \sim s/\sqrt{\lambda \gamma T_A}$. The thermal transport coefficient is estimated by the formula $\chi = \gamma/k_r^2$. Here we employ the mean field theory of statistical physics stating that this value of $\chi$ is identical with the thermal transport coefficient governing the stability of the high $m$ modes. This closure assumption gives the result $\chi = (\lambda/\chi)D^{3/2} \gamma s^{-2}$. Noting the relation $\lambda/\chi \sim (\delta_s/a)^2$, where $\delta_s$ is the collisionless skin depth, we obtain the formula for $\chi$ in dimensional form:

$$\chi = F(\rho) \left\{ d\beta/d\rho \right\}^{3/2} \delta_s^2 v_A/R$$
where \( F = \left\{ (N/2)L d(\psi')/d\rho \right\}^{3/2} \cdot s^{-2} \cdot \rho^{-3} \), \( N \) is the toroidal pitch number and \( \ell \) the multipolarity.

The following results are obtained from this formula: (i) \( \chi \) has the dimensional dependence of \( T(0)^{3/2}/B^2RVA \) (\( A \) is the mass ratio). (ii) \( \chi \) has the radial dependence of \( \left\{ \beta^* n(0)/n(\rho) \right\}^{3/2} \) and can be larger near the edge although \( T \) itself is lower near the edge (Fig. 3). (iii) \( \tau_E \sim F^{-0.4}A^{0.2}B^{0.8}n^{0.2}a^{2}R^{-0.6} \) is predicted, which explains the so-called LHD scaling, including the weak dependence on \( A \) and \( \ell \). (iv) \( \chi \) as derived from the heat pulse, \( \chi_{HP} \), is larger than \( \chi \) itself; in the region of \( |\nabla T/T| > |\nabla n/n| \), we have \( \chi_{HP}/\chi = 2.5 \). (v) It can also be shown that the relative density fluctuation is smaller than the potential fluctuation. These results explain the experimental observations [4].

5. RADIAL ELECTRIC FIELD AND LOSS CONE LOSS

We have developed a model analysing the radial electric field \( E_r \) and the loss cone loss in a self-consistent manner [5]. By using the formula for the loss cone boundary as a functional of the radial electric field, \( \rho(\rho) \), the radial current of the loss cone, \( c_i \), is calculated [6]. Taking the neoclassical current \( \Gamma_{NC} \) [7] and other non-classical loss such as due to charge exchange (cx), \( \Gamma_{cx,i} \), we solve the equation

\[
\Gamma_{eNC} = \Gamma_{NC} + \Gamma_{cx,i}
\]

with the boundary condition \( E_r = 0 \) for \( r = 0 \). The radial electric field and the loss cone are obtained simultaneously. Figure 4 illustrates the profile of \( E_r \) and the self-consistent loss cone boundary. It is found that \( E_r \) is more negative than the neoclas-
sical prediction near the boundary. The effect of the loss cone is small for $\rho < 0.7$ for the Compact Helical System (CHS) parameters. The experimental results on CHS indicate a strong electric field, suggesting the importance of the ex losses [8].

6. SUMMARY

Progress was made in the theoretical studies of the physical mechanisms determining beta limit, anomalous transport, radial electric field and loss cone. Results were obtained with analytical insight. These results provide a firm basis for an understanding of confinement and for a future optimization of the torsatron/heliotron configurations. They need quantitative estimates and a more careful comparison with the experiments. We emphasize that these analyses can be extended to more general toroidal plasmas; results will be reported elsewhere.

ACKNOWLEDGEMENTS

The authors acknowledge discussions with the CHS and Heliotron-E groups. One of the authors (KI) is grateful to Drs H. Zushi and M. Yagi for several discussions. The work is partly supported by a grant in aid for scientific research of the Ministry of Education, Japan.

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DISCUSSION

R.W. CONN: Your theory of the current diffusive mode yields a prediction for thermal diffusivity that transforms into an $nT_{\text{e}}$ formula which scales strongly with
B and n, namely as $B^{1.6}n^{1.2}$. This strong density (and field) scaling is absent in scalings for $nT_\tau E$ in tokamaks. What is your view on the absence of the density in the scaling laws for tokamaks?

K. ITOH: In heliotron/torsatron configurations this density dependence has been observed experimentally. The principal role in these configurations is played by the interchange mode, whereas in tokamaks it is played by the ballooning mode — for which the key parameter is the connection length between the regions of good and bad curvature, meaning that q appears in $\chi$. This explains the roles of B and $I_p$ in the $\tau_E$ of heliotron/torsatron systems and tokamaks, respectively. Regarding the density dependence in tokamaks, I notice that $n$ and the gradient scale length $\ell_n$ are linked. As density increases, $\ell_n$ becomes shorter (if we may dispense with a theory explaining this observation here) and the improvement in $\tau_E$ by $n$ is cancelled out by the reduction in $\ell_n$. I would like to emphasize the important influence of the 'profile shape'.

R.J. TAYLOR: You have developed an approach in which $\chi$ depends mostly on turbulence and $\Gamma_n$ depends mostly on the radial electric field. Is this a fair summary?

K. ITOH: It will take some time before we can judge whether the summary is 'fair' or not. We are at present extending the model to include the influence of the radial electric field, which is a key concept for toroidal configurations. We hope to have explicit energy and particle results in the near future.
EQUILIBRIUM AND STABILITY PROPERTIES OF THE FOUR PERIOD HELIAC TJ-II

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Abstract
EQUILIBRIUM AND STABILITY PROPERTIES OF THE FOUR PERIOD HELIAC TJ-II.
The equilibrium and stability properties of the flexible Heliac TJ-II under construction in Spain are described. Equilibrium is studied by using the full 3-D equilibrium code PIES that permits the evolution of islands to be followed as the pressure is increased. The stability analysis was conducted for Mercier, ballooning and dissipative trapped electron modes.

1. EQUILIBRIUM STUDIES

It has long been recognized that finite aspect ratio, high rotational transform heliacs could develop equilibrium problems at finite pressure owing to the beating of the pressure induced toroidal Shafranov shift with helical harmonics, but traditional production equilibrium codes such as the 3-D VMEC assume nicely nested flux surfaces [1] and hence cannot account for these effects directly. However, there are now some new 3-D codes that are not subject to the restriction of nested magnetic surfaces. One of these is PIES [2], which determines MHD equilibrium configurations by direct integration along the field lines and whose predictions have been compared with those of VMEC in cases without islands, with very good agreement [3].

The four period TJ-II is a device that is particularly well adapted to studying the evolution of the magnetic islands as the pressure is increased, because of its flexibility and its small shear. It is designed to have nearly shearless configurations with the rotational transform ranging from 0.96 to 2.48. Therefore it can, and will normally, be operated in iota windows free of low order resonances, but one can also use this flexibility to generate large islands inside the plasma. We are now conducting a systematic study of some special configurations with large islands, using the PIES code. These configurations are, of course, exceptional in the sense that they will not occur under normal operational conditions but they are nevertheless inside the range of operation. Specifically, we are studying configurations such that the rotational transform per period takes the value 1/3 somewhere inside the plasma, and therefore is close to 1/3 over the entire volume of the plasma, because of the small shear. Typically, the rotational transform per period varies between 0.331 at the magnetic axis
and 0.342 at the plasma boundary. As should be expected, there is a large third order island (Fig. 1).

We first consider the zero pressure case without plasma currents. In this case the structure of the magnetic field can be obtained by following the field lines of the magnetic field created by the currents in the external coils (we neglect the effect of the vacuum chamber wall and other conducting parts of the apparatus). These external currents are chosen so that we obtain the kind of configuration mentioned above, with an appropriate bean shaped cross-section for the plasma boundary. This boundary is given as an input to the PIES code. In this calculation, we assume that both the pressure and the current density are constant inside the islands and that they are continuous across the separatrices. The choice of the number of Fourier modes to be used by the code is very important. In most of our initial runs we had:

\[ 0 \leq m \leq 5 \]

\[ -10 + m \leq n/N \leq 10 + m \]

where \( m \) is the poloidal number and \( n/N \) is the toroidal number per period. This non-rectangular mode selection is chosen to take advantage of the approximate helical symmetry. The fundamental resonant term, \( m = 3, n/N = 1 \), is selected, but not the higher order terms \( m = 3k, n/N = k \), with \( k > 1 \). With this mode selection, the general structure of the magnetic field given by PIES agreed reasonably well with that obtained from the currents in the external coils, with the shape and the size of the island fairly well reproduced. But there is one aspect in which the agreement was quite poor: the distribution of the magnetic surfaces inside the island. PIES gave sur-
faces pressing inwards inside the island, while the field created by the coils gave surfaces pressing outwards. On the other hand, another run with

\[ 0 \leq m \leq 7 \]

\[ -10 + m \leq n/N \leq 10 + m \]

which includes the resonant term \( m = 6, n/N = 2 \) as well as the fundamental term, \( m = 3, n/N = 1 \), yields the correct structure inside the island [4].

In the case of finite pressure the same plasma boundary as for the zero pressure case was used. A series of runs with different values of central beta and with the above mentioned \( 0 \leq m \leq 5 \) mode selection seemed to indicate that the central width of the island decreases with increasing beta, until the island splits when \( \beta(0) = 6 \times 10^{-4} \). But from some recent runs with the \( 0 \leq m \leq 7 \) mode selection it appears that either there is no splitting or the splitting takes place at much higher pressures. At \( \beta(0) = 1 \times 10^{-3} \) only a slight narrowing of the island is observed. More work is under way to provide a better understanding of the island evolution when the pressure increases.

2. STABILITY STUDIES

2.1. Ballooning modes

Ballooning modes are pressure driven instabilities, strongly localized in 'bad curvature' regions of the confining magnetic field that limit the achievable pressure in tokamaks. In stellarators with iota profile gradients opposite to tokamaks, it was widely believed that Mercier rather than ballooning modes were the limiting instabilities to the maximum beta achievable. However, recent studies have shown that, at least in some class of stellarators, ballooning modes are, like in tokamaks, the limiting instabilities [5]. In this paper a study attempting to find out what effect, if any, ballooning modes have on the previously obtained beta limits for the Heliac TJ-II under construction at CIEMAT [6] is presented. After obtaining a TJ-II equilibrium with the VMEC code, we changed it to a Boozer co-ordinate system to solve the ballooning equation. The strong indentation of the bean shaped magnetic surfaces of the TJ-II and the pronounced helicity of its magnetic axis required a great number of modes (138) to correctly describe its magnetic surfaces in a Fourier expansion:

\[
R(s, \theta, \xi) = \sum R_{mn}(s) \cos (m\theta - n\xi)
\]

\[
Z(s, \theta, \xi) = \sum Z_{mn}(s) \sin (m\theta - n\xi)
\]

Typically, \( m \leq 6, -12 \leq n \leq 12 \) were needed to obtain, with VMEC, equilibria that correctly reproduced the vacuum magnetic surfaces and magnetic properties such
as iota profile and magnetic well that were previously obtained from a field line following code. This large number of toroidal and poloidal modes becomes even larger when we try to describe the equilibrium in Boozer co-ordinates needed for the stability analysis. For a correct reconstruction of the magnetic surfaces more than 750 modes were needed for a TJ-II configuration, characterized by $\langle \beta \rangle \approx 2\%$, and even with this number of modes for the most external surfaces the reconstruction, when compared in detail with the original VMEC magnetic surfaces, was not good enough. The number of modes needed for a satisfactory reconstruction grows with $\beta$, making it more difficult, from a numerical point of view, to obtain a good representation of the surfaces as $\beta$ increases. This problem restricted our study to configurations with $\beta$ values such that the surfaces were well reconstructed and therefore not all cases previously considered could be analysed. By using these equilibria, the ballooning equation

$$\frac{\partial}{\partial \xi} \alpha(s, \xi) \frac{\partial}{\partial \xi} \Psi + D(s, \xi) \Psi = \Gamma^2 b(s, \xi) \Psi$$

was solved in Boozer co-ordinates, where the bending term $\alpha$ is given by

$$\alpha(s, \xi) = \left(1/g^{\uparrow}\right) \left[1 + \left[(\Phi'G'^{\uparrow}/B)\xi + (-I_{ge} - J_{gb})/(\sqrt{gB})\right]\right]$$

and the driving term $D$ is given by

$$D(s, \xi) = -\left[p'/\xi'/(\Phi'[J_{X'} - I\Phi'])\right](-I\partial\sqrt{g}/\partial \theta$$

$$-I\partial\sqrt{g}/\partial \xi)\xi + \left(1/\Phi'\right)^2\left[\sqrt{g}p^{\uparrow}/B^2 - p'\partial\sqrt{g}/\partial s\right]$$

$$+ (p'/B^2)(J_{X''} - I\Phi'') + (p'/B)^2(\sqrt{g} - V')$$

$$+ p'\sqrt{g} B_\xi \cdot \nabla \sqrt{g}(1/[J_{X'} - I\Phi'])$$

and in this co-ordinate system the term $b(s, \xi)$ is written as:

$$b(s, \xi) = \left(2/B_{\theta}\right)\sqrt{g} p^{\uparrow}/(\Phi' A^{\uparrow}) \alpha(s, \xi)$$

Figure 2 shows the stabilizing bending term $\alpha$ and the driving term $D$ for a TJ-II configuration that is stable to ballooning modes. Notice the very different scales in both plots. Figure 3 shows the strongly localized eigenfunction obtained from the ballooning equation in an unstable case.

We have applied this procedure to a shearless TJ-II configuration characterized by $\iota(0)M \approx 0.30$ and a vacuum magnetic well of about $1.3\%$. Our results, summarized in Fig. 4, indicate that, for this configuration, ballooning and Mercier modes impose similar limits on the achievable $\beta$ values although numerical problems in the convergence to Boozer co-ordinates limit the confidence in the results to average
FIG. 2. Variation of driving term (above) and stabilizing term $\alpha$ along a field line.

FIG. 3. Eigenfunction of the ballooning equation in an unstable TJ-II configuration showing the localization of the ballooning mode.
FIG. 4. Maximum $\beta$ values calculated for a TJ-II configuration with Mercier criterion and ballooning equation versus minor plasma radius.

To clarify this point, numerical and analytical studies of the Mercier criterion and ballooning modes are under way.

2.2. DTEM

To study the dissipative trapped electron mode in TJ-II, we follow the formalism developed by Antonsen et al. [7]. It is formulated in the ballooning representation, and the drift wave problem is posed as an eigenvalue equation along a magnetic field line. The change in frequency due to the non-adiabatic electron contribution is
calculated perturbatively [8]. We have only considered low \( \beta \) TJ-II equilibria. We limit the calculations to the plasma core and to regions in the parameter space where localized solutions can exist. We solve the ballooning equation for only one field line starting at half field period. The relevant parameters are \( \omega_n \) and \( \rho L_n/a^2 \), where \( \omega_n \) is the ratio of electron diamagnetic to sound frequency, \( \rho/a \) the normalized radius, \( L_n \) the electron density scale length and \( a \) the minor radius.}

In Fig. 5, we plot the eigenfunctions for different values of the parameter \( \rho L_n/a^2 \). The configuration is one of the deep well scan with peak beta equal to 1.33%. The flux surface is located at a radius of 0.683a, and the parameter \( \omega_n \) is set equal to 10. As the gradient of the density decreases, the mode changes from a helically induced to a toroidicity induced mode. The change in the eigenfunction with \( \omega_n \) is illustrated in Fig. 6. The electron density scale length in 5 for the three cases. The eigenfunction becomes more localized as \( \omega_n \) increases.

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EQUILIBRIUM, STABILITY AND TRANSPORT OF AN $\ell = 1$ COMPACT HELICAL MAGNETIC AXIS TORSATRON, AND INFLUENCE OF MAGNETIC ISLANDS ON PLASMA CONFINEMENT IN A TOROIDAL HELIAC

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Presented by C. Alejaldre

Abstract

EQUILIBRIUM, STABILITY AND TRANSPORT OF AN $\ell = 1$ COMPACT HELICAL MAGNETIC AXIS TORSATRON, AND INFLUENCE OF MAGNETIC ISLANDS ON PLASMA CONFINEMENT IN A TOROIDAL HELIAC.

The $\ell = 1$ torsatron modified to improve the magnetic hill enhances the stability beta limit without changing significantly the equilibrium beta limit and has the attractive transport features of smaller Pfirsch–Schlüter current and field ripple compared with the standard $\ell = 1$ torsatron, besides a favourable helical symmetry. The effects of magnetic islands in the Tohoku University Heliac are presented. The $n = 3, m = 2$ islands are produced by an artificially applied perturbation field. The magnetic islands are measured with the fluorescent mesh method and compared with computed drift surfaces. The island structure can also be seen in the radial profiles of the ion saturation current. The magnetic island affects ECRH plasma confinement and consequently decreases the particle confinement time in proportion to the island size.

1. INTRODUCTION

The compact $\ell = 1$ torsatron with a small excursion of the helical magnetic axis has the simplest coil structure among the helical confinement systems. This system has a high positive magnetic shear; however, it has a large magnetic hill. Two modifications to improve the magnetic hill were proposed [1, 2]. In this paper, the equilibrium, stability and transport of these modified $\ell = 1$ torsatrons are described.
The recent favourable outlook for the stellarator approach in the fusion programme has led to the exploration of new helical axis configurations that offer the hope of high beta confinement. To better understand the confinement properties of such devices, we constructed a small heliac, the Tohoku University Heliac (TU Heliac) in Sendai. The TU Heliac has four periods, a 48 cm major radius, a maximum toroidal field of 3.5 kG and an average plasma radius of 6-8 cm. The swing radius of the TF coils is 8 cm [3].

2. MODIFICATION OF THE $\ell = 1$ TORSATRON

The first modification of the $\ell = 1$ torsatron (called the $\ell = \pm 1$ torsatron [1]) to improve the magnetic hill ($V'' > 0$) is the superposition of a relatively weak $\ell = -1$ torsatron field on the standard $\ell = 1$ torsatron, where the excursion amplitude of the magnetic axis is larger in the major radius direction than in the up-down direction. The balance of the two torsatron fields enables the system to close the magnetic field lines with no rotational transform [4]. The second modification is the negative pitch modulation of the coil winding law, $\theta = N\phi + \alpha^* \sin(N\phi)$, where $\theta$ and $\phi$ are the poloidal and toroidal angles, $N$ is the period number and $\alpha^* (< 0)$ is the pitch modulation parameter (called the negative pitch $\ell = 1$ torsatron [2]). Here the pitch angle of the coil is larger inside than outside the torus. These modifications strengthen substantially the good local curvature of the magnetic field lines and improve the magnetic hill. The vertical field to shift the magnetic axis is adjusted so that the average position of the magnetic axis becomes nearly equal to the coil minor axis. The first modification improves the magnetic hill, increasing the magnetic positive shear ($t' > 0$) associated with the decrease of the rotational transform on the magnetic axis, $t(0)$. The second forms a local magnetic well ($V'' < 0$), decreasing $t'$, which is associated with the increase of $t(0)$, and also reduces the variation of $\dot{\psi}dl/B$ and the field ripple. For comparison we select the coil current ratio $I_{-1}/I_{+1} = -0.3$ and the coil minor radii 0.4 m/0.3 m for the $\ell = \pm 1$ torsatron, and $\alpha^* = -0.4$ for the negative pitch $\ell = 1$ torsatron. The typical period number in the case of major radius $R_0 = 2.1$ m and minor radius $a = 0.3$ m is $N = 17$, where the magnetic hill is improved without significantly decreasing $t(0)$. The three magnetic configurations under consideration are summarized in Table I, where the aver-

<table>
<thead>
<tr>
<th>\ell = \pm 1</th>
<th>9.4</th>
<th>0.61</th>
<th>1.46</th>
<th>1.39</th>
<th>0.057</th>
<th>0.42</th>
<th>0.28</th>
<th>0.10</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\ell = 1$</td>
<td>7.7</td>
<td>0.38</td>
<td>1.07</td>
<td>1.82</td>
<td>0.013</td>
<td>0.48</td>
<td>0.49</td>
<td>0.14</td>
</tr>
<tr>
<td>$\alpha^* &lt; 0$</td>
<td>11.6</td>
<td>0.71</td>
<td>1.03</td>
<td>0.45</td>
<td>0.0</td>
<td>0.15</td>
<td>0.20</td>
<td>0.025</td>
</tr>
</tbody>
</table>
age shear \( \tau'(a) \) is defined as \( (\tau(a) - \tau(0))/\tau(0) \) and the average magnetic hill \( \bar{V}''(r) \) as \( (V'(r) - V'(0))/V'(0) \), and \( \Delta V'(a) \) means the variation of \( \tau d\tau /B \) on the plasma boundary and \( \Delta B(0) \) the field ripple on the magnetic axis. The standard \( \ell = 1 \) torsatron has an unmodified filamentary coil.

3. FINITE BETA EQUILIBRIUM AND STABILITY

The effects of these modifications on the finite beta equilibrium and stability are investigated by using the BETA code [5] for fixed boundary and net toroidal current free plasma. The outward shifts of the magnetic axis are smaller than half the plasma radius in the beta range calculated of \( \sim 4\% \) for the three \( \ell = 1 \) torsatrons mentioned above, so there appears to be an equilibrium beta limit \( (\beta_{\text{eq}}) \) larger than \( 4\% \). The observed outward shift is the smallest for the negative pitch \( \ell = 1 \) torsatron because it has the largest value of \( \tau(0) \) of the three configurations. This shift has the scaling of \( \beta^2 A/\ell^2(0) \), where \( A = R_0/a \). The helical excursion amplitude of the magnetic axis, \( \Delta \), is 0.13a at \( \beta = 4.2\% \) for the standard \( \ell = 1 \) torsatron, indicating the helical curvature to be larger than the toroidal curvature. The beta dependence of \( \Delta \) is weak, but the excursion amplitude of the magnetic axis for the \( \ell = \pm 1 \) torsatron becomes larger in the major radius direction. As \( \beta \) increases, the distribution of \( \tau \) flattens out and the magnetic well deepens. The \( \ell = \pm 1 \) torsatron with \( \tau \) in the range \( 0.38 \leq \tau \leq 1.07 \) becomes shearless at \( \beta \sim 10\% \). The local magnetic well at half the plasma radius is formed at \( \beta \sim 4.5\% \), lower than for the standard \( \ell = 1 \) torsatron. The dangerous instability is the global mode with \( m = 1, n = 1 \), and the \( m = 2, n = 1 \) mode associated with the resonant surface \( \ell = 0.5 \). The stability calculation shows that both \( m = 1 \) and \( m = 2 \) modes go directly into the second stability region before developing any instability for finite \( \beta \). For the standard \( \ell = 1 \) torsatron, on the other hand, the \( m = 1, n = 1 \) mode appears to become unstable at \( \beta \geq 3\% \), as shown in Fig. 1. This stability improvement of the \( \ell = \pm 1 \) torsatron is explained by the improved magnetic hill, which forms a local magnetic well at lower \( \beta \), and the large outward shift of the magnetic axis, which causes plasma self-well. However, our calculation of the Mercier local stability criterion shows instability, especially near the magnetic axis. The growth rate is the largest for the \( \ell = \pm 1 \) torsatron. The main contribution to the growth rate comes from the Pfirsch–Schlüter current term. The superposition of the \( \ell = -1 \) torsatron field leads to an asymmetry of the configuration, since it makes the magnetic axis excurse with a larger amplitude in the major radius direction than in the up–down direction. Moreover, as \( \beta \) increases, the magnetic axis shifts outward and the plasma digs its own magnetic well. At the same time, the Pfirsch–Schlüter current increases and contributes to a destabilizing term. In contrast, the Mercier criterion near the magnetic axis can be significantly improved by considering a flatter pressure profile.

The negative pitch \( \ell = 1 \) torsatron with \( \tau \) in the range \( 0.71 \leq \tau \leq 1.03 \) becomes shearless at \( \beta \sim 5.3\% \). The local magnetic well at half the plasma radius
is formed at $\beta \sim 2.7\%$, and this value is the lowest among the three $\ell = 1$ torsatrons. The stability beta limit ($\beta_{st}$) for the global $m = 1, n = 1$ mode is reduced to 1.3\% or thereabouts, in spite of the deeper local magnetic well. The comparison of the $\beta_{st}$ value for the three configurations suggests that the magnetic shear dominates over the local magnetic well for the stabilization in the first stability region. Therefore, it is required for higher $\beta_{st}$ confinement by negative pitch modulation to attain higher shear keeping the improved hill. In comparison with the filamentary coil, we find that the thickness of the coil has the effect of forming a local well and also increasing the shear associated with the decrease of $\kappa(0)$. Our results for the coil parameters desired for stabilization are: the combination of $N = 17$, $\alpha^\ast = -0.2$, and the width and height of the coil cross-section, 0.14 m and 0.035 m, respectively, lead to $R_0/a = 7.6$, $\kappa(0) = 0.35$, $\kappa(a) = 1.65$, $\gamma'(a) = 3.71$, $\gamma''(0.5a) = 0$, $\gamma''(a) = 0.66$, $\Delta V'(a) = 0.17$ and $\Delta B(0) = 0.027$. These values are preferable to those for the $\ell = \pm 1$ torsatron.

4. TRANSPORT

The negative pitch $\ell = 1$ torsatron has the attractive transport features of reducing the variation of $\delta \delta l/B$ and the field ripple, as shown in Table I. The appearance
of the magnetic island is due to the resonant pressure driven currents in three-dimensional MHD equilibria, which are associated with the variation of $\frac{\delta d\ell}{B}$ on the corresponding rational surface $\ell = n/m$. The island width is expressed in terms of the Fourier components $\delta_m$ on the magnetic field lines. For the $\ell = 1$ torsatron, $\delta_m$ on the vacuum magnetic field lines is much larger than that on the finite beta field lines because of the large number of field periods ($n \geq N = 17$). Therefore the critical beta value $\beta_0$ above which the island width becomes larger than half the plasma radius is evaluated for the resonance caused by the variation of $\frac{\delta d\ell}{B}$ in the vacuum field. The value of $\beta_0$ is given to be much larger than $\beta_{eq}$, indicating no effect of the magnetic island on the equilibrium beta limit, in contrast to the heliac with a small period number. The largest components correspond to $(n, m) = (17, 0), (0, 1)$ and $(17, 1)$. The $\delta_{17,0}$ component corresponds to the field ripple, the $\delta_{0,1}$ to the toroidal curvature and the $\delta_{17,1}$ to the helical curvature. For the standard $\ell = 1$ torsatron, $\delta_{17,0} = 0.045, \delta_{0,1} = 0.04$ and $\delta_{17,1} = 0.14$ on the magnetic axis. Since $\delta_{17,1} > \delta_{0,1}$, there exists a favourable helical symmetry in the $\ell = 1$ torsatron system.

The particle drift orbits have been obtained by using Boozer's magnetic coordinates, which are automatically transformed from the Cartesian co-ordinates. Figure 2(a) shows typical particle drift orbits for the standard $\ell = 1$ torsatron, where each particle has a kinetic energy of 10 keV and a pitch ($= v_T/v$) of 0.6. Figures 2(b–d) show the loss cone region in velocity space at $\theta = \phi = 0$ for several magnetic surfaces $\psi$. The magnetic field strength at the magnetic axis is about 4.4 T. In this case we confirm that all particles which have kinetic energy less than 16 keV and are started from the surface $\psi < 0.3$ (normalized by the outermost surface) are confined in the standard $\ell = 1$ torsatron.

5. EXPERIMENTAL RESULTS IN THE TU HELIAC

The topics which we present here concern the experimental results on the effect of magnetic islands produced by artificially applied perturbations on plasma confinement in the TU Heliac. Many papers have discussed theoretically magnetic island formation and the destruction of flux surfaces in three dimensional MHD equilibria [6, 7]. However, no clear experimental understanding of these problems has emerged.

As the sources of the perturbation fields we used a pair of dipole coils at toroidal angle $\phi = 180^\circ$. In Figs 3(b, d) we show contours made by the fluorescent mesh method [8] of the drift surfaces without and with a perturbation field. Figures 3(a, c) show the calculated drift surfaces under the same conditions as in Figs 3(b, d). In Fig. 3(d) we see the $m = 2$ islands as shown in Fig. 3(c). We have also observed closed magnetic surfaces surrounding these islands.

The plasma is produced by ECRH with a 2.45 GHz microwave source with a 4 ms pulse of 3 kW. The working gas is argon at a pressure of $3.7 \times 10^{-3}$ Pa. Figures 4(a, b) show the typical radial profiles of the ion saturation current by a Langmuir probe. We can see from these results that there is no apparent difference
FIG. 2. (a) Projection of typical particle drift orbits of the standard $\ell = 1$ torsatron ($N = 17, v_\parallel/v = 0.6$) onto the $\psi, \theta$ plane. (b–d) Particle loss cone in velocity space at $\theta = \phi = 0$ for several values of $\psi$, where $\psi = 1$ corresponds to the outermost surface. The velocity is normalized by the thermal velocity with $T_i = 10$ keV.
FIG. 3. Computed drift surfaces (a) without islands and (c) with islands, and drift surfaces observed with the fluorescent mesh method (b) without islands and (d) with islands. Drift surfaces A and B correspond to observed drift surfaces. The ratio of external perturbation coil current is $I_p/I_e = 0.12$ in (c) and (d).
FIG. 4. Typical radial profiles of ion saturation current (a) without and (b) with islands. In (b) the island exists in the range $R = 0.035-0.06$ m.
between the case with islands (a) and that without islands (b) during plasma production and that the difference between the cases appears during the afterglow (no plasma production). This saturation current profile clearly dips at the inner boundary of the island so as to have the same current value along the separatrix of the island.

In Fig. 5 particle confinement time $T_P$ versus magnetic island half-width is plotted. This half-width is calculated numerically at toroidal angle $\phi = 0^\circ$, where the magnetic island is symmetrical with the equatorial plane, and is proportional to $(I_{ex}/I_c)^{1/4}$, where $I_{ex}$ is the external perturbation coil current and $I_c$ is the centre conductor coil current. The confinement time is estimated from the decay of the electron density measured by a Langmuir probe in the afterglow soon after the microwave is switched off. In Fig. 5 the confinement time evidently decreases with the increase of magnetic island half-width in the range over 5 mm but hardly changes in the range where the islands have a somewhat narrower width. In the configuration without artificial perturbation the TU Heliac may have very narrow natural islands along the rational surface due to unexpected error fields. It may be necessary to measure more precisely magnetic surfaces near rational surfaces with the R/C method [3].
6. CONCLUSION

As a result of the modification of the standard $\ell = 1$ torsatron the plasma stability against the global MHD instabilities has been improved, indicating no stability beta limit in the beta range calculated of $\sim 4\%$. Furthermore, the negative pitch $\ell = 1$ torsatron has the attractive transport features of a favourable helical symmetry, as well as small field ripple and Pfirsch–Schlüter current.

The artificially produced magnetic island affects ECRH heliac plasma confinement and consequently decreases the particle confinement time in proportion to the island width.

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DEVELOPMENT OF DIVERTOR CONCEPT FOR OPTIMIZED STELLARATORS

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Abstract

DEVELOPMENT OF DIVERTOR CONCEPT FOR OPTIMIZED STELLARATORS.

A divertor concept has been developed for optimized stellarators which is in keep­ ing with their geometrical characteristics on the one hand and with the open divertor concept of tokamaks on the other hand.

1. INTRODUCTION

Helias configurations [1, 2] as optimized [3] for the Wendelstein 7-X (W7-X) stellarator [4] exhibit interesting properties also for physics issues which were not original objectives of the optimization procedure. One of these is their adaptation to divertor operation, which plays an important role with respect to power and particle exhaust, impurity control, and minimization of erosion and impurity production. Stellarators are different from axisymmetric tokamaks in this respect because they lack magnetic surfaces outside the separatrix, i.e. the last closed magnetic surface (LCMS), so that the ordered field-line structure property of the axisymmetric scrape-off layer of tokamaks has been considered to be unlikely. Stellarators have similarities with non-axisymmetric tokamaks which explore the concept of an ergodic divertor [5, 6] or which are three-dimensional because of construction imperfections [7]. Stellarators – in particular optimized stellarators – look geometrically complicated, which has led to the expectation that the leading-edge problem – i.e. the vertical impingement

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of scrape-off-layer field lines on material objects, which results in excessive heat and particle deposition – is insolvable.

The diversion properties of Helias configurations have therefore been investigated [8] in order to assess these difficulties and search for a divertor concept exploiting specific properties of these configurations favourable for this purpose. These investigations have indeed led to such a concept; a summary of results reached so far was given in [9]. Section 2 gives a short review of this approach. Section 3 describes an example analyzed with the various methods developed. Section 4 presents an outlook.

2. OVERVIEW OF APPROACH TO A DIVERTOR STELLARATOR

Here, the following aspects of stellarator divertor considerations are briefly reviewed [9]: configurational aspects, field line tracing as a method of investigating the diversion properties, one-dimensional estimates of SOL parameters, SOL simulations with a Monte Carlo diffusion model, the definition of divertor troughs as a result of these considerations, and the investigation of the field line map connecting the divertor plates [10].

Important configurational aspects of optimized stellarators as far as the plasma boundary is concerned are: i) the existence of helical edges at the plasma boundary, ii) a significant positive shear (i.e. increasing rotational transform towards the boundary) and the persistence of a magnetic well in the region of the LCMS, iii) stability with respect to Mercier, peeling, resistive interchange, and ballooning modes, iv) the possibility for approximately constant locations of the helical edges in an experimental-flexibility space mainly characterized by a range of edge values of rotational transform, v) near invariance of the helical edges and positions of fixed points in the vacuum field region with variation of $\beta$ [11].

Field line tracing is considered to be the basic method for investigating the flow of energy and particles outside the confinement region. It reveals the basic diversion structure: plasma-boundary magnetic field lines intersect a suitable enclosing control surface in helical strips following the helical edges.

One-dimensional SOL parameter estimates exploiting particle and power balances show that the density and the temperature at the plasma boundary as well as the SOL thickness are determined by the cross-field (anomalous) transport characteristics $D_p n$ and $\chi_e n$, the characteristic field line length, and the particle and power outflow density. In particular, the temperature-decay thickness and the temperature itself have relatively weak dependencies on field line length and power density.
With these estimates and an appropriately chosen 'diffusion' of field lines (representing cross-field mass and energy transport) simplified SOL simulations can be obtained which measure the broadening of deposition patterns of mass and energy as compared to those obtained by mere field line tracing.

With these results, it becomes possible to define 'helical troughs' (as parts of the control surface) which satisfy the following conditions: i) they do not act as limiters, but as divertor plates because they do not scratch intact magnetic surfaces; instead, only open field lines (and particle and energy flows on them) reach them; ii) they lead to a complete separation of the magnetic field lines starting at the LCMS and ending at the troughs from the field lines starting at the first wall [8], so that the interaction of the charged particles leaving the plasma is completely removed from the first wall and managed on the troughs designed for this purpose; iii) this separation of interactions is independent of the detailed structure beyond the LCMS (see [8] and the results given here). Leading edges do not exist because the deposition patterns do not extend to the boundaries of the troughs.

With the divertor plates defined, the field line structure in the outer neighbourhood of the LCMS may be investigated from a complementary point of view: the map of the troughs onto themselves which is given by the field lines starting on the troughs can be investigated and correlated with the Monte Carlo simulations yielding an estimate of the SOL extent.

One-dimensional fluid solutions can the be obtained with the help of power and particle sources in this SOL and lead to tentative distributions of temperature, density, and velocity in the SOL which allow discussion of improvements of the description obtained.

3. RESULTS FOR AN EXAMPLE

Here, a specific example representative of the W7-X standard configuration is described by results obtained with the procedure given in Sec. 2.

Figure 1a shows the vacuum field case selected [12] (the flexibility of W7-X will allow various other choices). The LCMS occurs at a value of rotational transform above but very close to unity. Note that this has implications for the structure of ergodicity of field lines beyond the LCMS due to the sparseness of low-order rational numbers around unity.

Figure 1b shows the intersection in the form of helical strips of field lines starting close to and outside the LCMS with a control surface at about 0.1 m distance from the LCMS. The homogeneity along the strips can be further improved, see below.
FIG. 1a. Poincaré plot of a configuration designed for W7-X (average major radius 5.5 m), Fig. 1a from Ref. [12]. Note the preferential escape of the ergodic field line close to the helical edges.

FIG. 1b. Intersection pattern of field lines on a control surface at about \(1/5\) plasma radius distance from the LCMS which in the case shown here is nearly identical with the outer separatrix of the \(5/6\) islands; so, at the top and at the bottom of the cross-section shown, the control surface is at about 0.1 m distance from this LCMS. \(u\) poloidal parametrization, \(v\) proportional to angle \(\varphi\) of cylindrical coordinates, \(u = 0, v = 0\) is the outboard point on the symmetry axis of the left figure. Grazing incidence of field lines prevails: \(\langle \Psi \rangle \approx 3^\circ\).
One-dimensional SOL estimates obtained with 10 MW power and $3 \cdot 10^{22}$ particles into the SOL, standard assumptions on $\chi_n$ and $D_vn$ as cross-field transport characteristics, and the assumption (see below) of a characteristic field line length of 75 m from the plasma boundary region to the control surface lead to estimates of the SOL temperature and density determining the mean free path $\lambda$ for the Monte Carlo simulations of electrons and ions shown in Fig. 2. The effective field line lengths found here are in rough accordance with the input estimate, so that this consistency requirement can easily be improved. The helical troughs resulting from these calculations are also indicated in Fig. 2; further extension of the patterns along the troughs while still avoiding leading edges is possible, see below.

A three-dimensional view of the troughs is seen in Fig. 3, and Fig. 4 shows the complete separation of ‘plasma’ field lines (with diffusion taken into account) from the ‘wall’ field lines. Note that no stellarator built as yet has observed this principle of construction.

Investigation of the field line map between the troughs, see Figs. 5 and 6, reveals the following properties. Characterization of the field lines according to their lengths shows that i) the foot points of the field lines on the troughs form ordered areas up to $O(10^2 \text{ m})$ length; ii) there is a clear correlation between field line lengths and minimum distance to the LCMS so that an ordered-layer structure prevails up to lengths of $O(10^2 \text{ m})$. iii) field lines of several hundred meter lengths form an inner area enclosing the LCMS and are mapped to a line-like region on the troughs. Homogeneity of this ‘line’ can easily be improved by iteration of the trough positions according to the information provided by the field line map between the troughs.

Inspection of the Monte Carlo simulations, see Fig. 7, allows the approximate definition of a SOL plasma region (as opposed to the plasma region in front of and close to the divertor plates) with the help of the free-boundary MHD-equilibrium code NEMEC [14] which constructs surfaces nearly tangential to the field lines in the region beyond the LCMS. This result leads to estimates of an average SOL thickness of approximately 0.02 m (corresponding to $\approx 2 \text{ m}^3$ of SOL volume) and associated estimates of approximately 5 MW/m$^3$ of power and $10^{22} \text{ s}^{-1}\text{m}^{-3}$ particle flow densities.

With these quantities – together with the information on the field line positions relative to the SOL plasma area – one-dimensional fluid solutions along the field lines mapping the troughs onto themselves can be obtained. The simplest possible choice for computing a temperature distribution corresponding to a high-density situation in front of the targets
FIG. 2. Intersection patterns of 'diffusing' field lines on a control surface. Above: electrons, $\chi_e = 5m^2/s$; below: ions, $D_p = 0.25m^2/s$; $\lambda \approx 4m$ ($T = 0.07keV, n = 0.2 \cdot 10^{20}m^{-3}$); this mean free path is short compared with half the connection length (which is the appropriate normalization length), so that 3D ripple effects are unimportant; even particles with $E \approx 4kT$ are only at the transition from the plateau to the $1/\nu$-regime; in the whole range of such mean free paths neoclassical cross-field electron-heat and ion-particle diffusivity coefficients are of the order of or below the ion plateau value $[O(10^{-3}m^2/s)]$ [13]. The framed areas indicate the 'helical troughs', see below. Effective field line lengths from the LCMS are $(L) \approx 2.7 \cdot 10^2$ and $1.3 \cdot 10^2$ m.
FIG. 3. Plasma and divertor troughs.

FIG. 4. Separation of interactions. Plasma at the indented cross section, SOL plasma obtained by diffusing-field-line simulation (ions), divertor region, and field lines starting at the first wall.
FIG. 5. Characterization of the field line map between the troughs. Field line foot points on the troughs are coloured according to the field line lengths: $0 \leq 9$ m light blue, $9 \leq 36$ m dark blue, $36 \leq 144$ m green, $144 \leq 324$ m yellow, $324 \leq 576$ m red, $\geq 576$ m white. Discretization of trough area: $\approx 0.02$ m (poloidal), $\approx 0.04$ m (toroidal).

FIG. 6. Characterization of the field line map between the troughs. Poincaré plot in part of the triangular cross-section. Field lines are coloured according to the field line length, with the same colour coding as in Fig. 5. The dark-blue area marks the helical trough, the yellow area shows the plasma, and the dashed line indicates the first wall.
FIG. 7. Characterization of the SOL by the Monte Carlo calculation for the electrons, compare Fig. 2. Poincaré plot of magnetic field lines (black dots) in part of the triangular cross-section. The yellow area represents the plasma inside the LCMS, while the red line denotes the NEMEC [14] surface which is estimated as outer SOL boundary. The dark-blue area marks the helical trough, while the dashed line indicates the first wall.

FIG. 8. Characterization of the temperature distribution in the SOL. Poincaré plot in part of the triangular cross-section. Field lines are coloured according to the temperature distribution along their lengths: $0 \leq 18$ eV dark blue, $18 \leq 36$ eV light blue, $36 \leq 54$ eV green, $54 \leq 72$ eV red, $72 \leq 90$ eV yellow. The black solid line shows the LCMS.
is energy transport by pure electron heat conduction along the field lines with boundary conditions corresponding to small temperatures at the plates. With this approach approximate temperature profiles along the field lines shown in Figs. 5 and 6 have been obtained and are shown in Fig. 8. Again an ordered structure seems to prevail. Investigations of this type will serve to assess the applicability of one-dimensional modeling along three-dimensional field lines.

4. SUMMARY AND OUTLOOK

A concept for divertor operation in optimized stellarator devices has been presented. It may be characterized as of the open-divertor type, such as the ASDEX Upgrade and JET divertor configuration [15, 16], and is the first attempt to allow divertor operation – exploiting tokamak experience – in modular stellarators. In particular, it has been shown that long divertor troughs with both — grazing incidence of field lines and no leading edges — can be defined in optimized stellarators. These divertor troughs lead to a complete separation of ‘plasma’ from ‘wall’ field lines. Field lines connecting the troughs and approaching the LCMS are long. W7-X [4] will provide experimental experience for this concept.

A system of codes is envisaged which is shown below. Heavy arrows connect areas where work in progress has been sketched in this paper. However, in all areas work has begun.
REFERENCES


DISCUSSION

R.W. CONN: You present strong parallel T_e gradients along field lines. This implies that for constant pressure the density rises towards the plates. How are the particles and neutrals trapped near the plate?

J. NÜHRENBERG: The preliminary calculations outlined here were not intended to be a self-consistent simulation of a high density regime in front of the plates (although this is planned). Instead, these boundary conditions were used to investigate the ensuing temperature distribution in the 3-D scrape-off layer. These investigations (and similar ones for density and velocity distributions) will serve to clarify the validity of an approach which combines 1-D fluid equations along the field lines with their 3-D structure.
R.W. CONN: Do the alphas in this configuration diffuse out or are they lost — at least in part — along loss orbits?

J. NÜHRENBERG: Briefly, at finite $\beta$, there is no loss cone for the $\alpha$ particles. References [3, 17, 18] deal with this in more detail.

R.J. GOLDSTON: Could you explain the concept for pumping impurities and helium ash in W7-X and a reactor?

J. NÜHRENBERG: In W7-X the divertor hardware structure will be approximately 0.1 m thick to allow for the use of cryopumps. The projected W7-X stellarator reactor (see paper G-1-2) involves a linear enlargement by approximately a factor of four, so there will be sufficient space for pumping.
FORMATION OF AN H-MODE-LIKE TRANSPORT BARRIER IN THE CHS HELIOTRON/TORSATRON


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Abstract

FORMATION OF AN H-MODE-LIKE TRANSPORT BARRIER IN THE CHS HELIOTRON/TORSATRON.

Control of the rotational transform profile with a small OH current to increase the rotational transform has led to a continuous rise in the line averaged electron density with a reduction of H\textsubscript{n} lines around the torus in a heliotron/torsatron plasma heated by neutral beams. During the density rise a transport barrier with steep temperature and density gradients near the plasma edge has been formed. ELM-like activity is also observed. Incoherent magnetic fluctuations in the frequency range \( f > 20 \) kHz are clearly suppressed during the density rise phase, while the low frequency coherent part remains almost unchanged. The global energy confinement time in the density rise phase is improved by \(-15\%\) compared with that of a discharge without OH current. These experimental observations are very similar to those for the H mode in tokamaks. There is a current threshold for the transition, where the increment of rotational transform produced by the threshold current corresponds to \(-0.12\) at the edge. No obvious change in the poloidal rotation velocity, however, is observed between L mode and H mode type discharges.

1. INTRODUCTION

The global energy confinement time of net current free plasmas produced in heliotrons, torsatrons and stellarators is well expressed by the LHD scaling\cite{1} or slightly modified versions of it\cite{2,3}. The scalings indicate an obvious improvement with electron density and toroidal magnetic field, while the power degradation is very similar to that of the L mode or H mode ITER scaling for tokamaks\cite{4}. Study of the differences and similarities between the respective scaling laws for tokamak and helical plasmas may improve understanding of anomalous transport in toroidally confined plasmas and may elucidate the roles of net plasma current and magnetic field ripple in plasma confinement. Recently many studies of confinement in tokamaks have been addressed to the improved confinement regime, i.e. the H mode.

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Experimental and theoretical studies on the H mode discuss the effect of the edge radial electric field $E_r$ and its shear $E_r'$ on the L-H transition and confinement improvement [5, 6]. However, a cause and effect relationship between $E_r$ or $E_r'$ and the L-H transition has not yet been established. To search for the H mode in helical devices is very significant for extending the parameter range in helical plasmas and is also helpful for understanding the physical mechanism of the H mode. In helical devices [7] a trial for obtaining the H mode with a biased electrode is being carried out, but the results so far are obscure. Another promising way to trigger the H mode is to control the rotational transform profile by introducing toroidal plasma currents. This approach is motivated by the H mode experiments based on control of the edge toroidal current density profile in the JIPP T-IIU tokamak [8-10]. We have achieved, for the first time, an H-mode-like transition in the low aspect ratio heliotron/torsatron type device CHS [11] with control of the rotational transform profile.

2. EXPERIMENTAL RESULTS AND DISCUSSION

At toroidal magnetic field $B_t \sim 0.94$ and 1.56 T, hydrogen plasmas initiated by ECH (53 GHz) are heated by co-injected hydrogen beams (NBI) of about 600-900 kW (through a port) with a beam energy of 32-36 keV. The line averaged electron density $n_e$ is adjusted to be less than $\sim 4 \times 10^{13} \text{ cm}^{-3}$ by gas puffing. The OH current is generated during NBI heating by induction of a small loop voltage of less than $\sim 2 \text{ V}$ using a set of poloidal field coils [12]. In this experiment the position of the magnetic axis in the vacuum field is adjusted to be $R_{ax} = 92-94 \text{ cm}$. Radial profiles of the electron temperature $T_e$ and density $n_e$ are obtained by Thomson scattering, and those of the ion temperature $T_i$ and poloidal rotation velocity $v_\theta$ are measured with $\sim 16 \text{ ms}$ time resolution by CXR spectroscopy [13]. Particle recycling is monitored by an 11 channel poloidal fan array [14] and by a single channel $\text{H}_\alpha$ detector at eight toroidal locations.

Figure 1 shows the temporal behaviour of various plasma parameters in an NBI heated plasma with OH current, where the beam driven current is estimated to be $\sim 4-6 \text{ kA}$. The OH current is driven to increase the external rotational transform. The plasma confinement is obviously degraded in the case of current driven in the opposite direction. The rotational transform produced by the current at the last closed flux surface is $\sim 0.18$ for $B_i = 0.94 \text{ T}$ and that produced by the vacuum magnetic field is $\sim 0.9$. At $t \sim 95 \text{ ms}$ the line averaged electron density begins to increase continuously in the course of the ramping phase of the OH current, while the gas puffing rate is kept constant. During the density rise phase the $\text{H}_\alpha$ lines observed around the torus are reduced. This behaviour suggests the reduction of outward particle loss flux and the improvement of particle confinement. In the discharge shown in Fig. 1 the reverse transition, the so-called H-L transition, is not clearly seen. Radial profiles of $T_e$, $n_e$ and $T_i$ just before the density rise phase (at $t = 80 \text{ ms}$ for $T_e$ and $n_e$ measurement, or 67-83 ms for $T_i$ measurement) and during the rise phase
FIG. 1. Temporal evolution of plasma parameters in the case with OH current. Plasma current rises to 27 kA with a time-scale of ~100 ms at $B_n = 0.94$ T, where the beam driven current is estimated to be ~4-6 kA. The total NBI power injected through a port is ~900 kW with a beam energy $E = 36$ keV. The H-mode-like transition occurs at $t \sim 95$ ms, when $n_e$ begins to rise continuously with an appreciable reduction of the $H_\alpha$ line. The gas puffing rate is kept constant before and after the transition.
(at t = 120 ms or 117-133 ms) are shown in Fig. 2, together with the profiles for a discharge without OH current. Just before the density rise the profiles are nearly parabolic. After entering the density rise phase, the \( n_e \) profile evolves to a hollow profile with a very steep gradient just inside the last closed flux surface, and the \( T_e \) profile also evolves to a profile with a steep gradient there. Also, the \( T_i \) profile becomes broader during this phase. The \( T_e, n_e \) and \( T_i \) profiles in the density rise phase are all broader than those in the OH current free discharge with similar \( n_e \).

Figure 2 clearly shows the formation of an edge transport barrier very similar to the H modes observed in tokamaks. During the density rise phase (t ~ 120 ms) the deposited NBI power is estimated to be ~540 kW, which is much higher than the OH power of ~40 kW, and to be ~740 kW for the discharge without OH current. The global energy confinement time in the H mode type discharge is improved by about 15% compared with the L mode type discharge without OH current. In contrast to the tokamak H modes, no obvious change in edge poloidal rotation is found between the discharges with and without OH current, where \( v_\theta \) near the edge (R ~ 104 cm) for both cases is ~3-6 km/s (in the electron diamagnetic direction). This result seems to indicate that the connection between the formation of the edge transport barrier and edge poloidal rotation or edge radial electric field is weak. The ELM-like spike is observed in the \( H_a \) lines during the density rise phase of the discharge shown in Fig. 1. We have obtained a discharge with clearer ELM-like activity (Fig. 3). This ELM-like spike ends the continuous density rise, exhibiting a small dip in \( \bar{n}_e \). In the density rise phase the incoherent part of the poloidal magnetic fluctuations (f > 20 kHz) detected by magnetic probes is suppressed by more than a factor of 2 compared with that without OH current. But the low frequency part (f ~ 3-10 kHz), which dominates the amplitude of the magnetic probe signals, remains unchanged for both cases.

We have studied the relationship between this H-mode-like transition and the required plasma current for two cases of different toroidal magnetic fields, changing the OH current. The transition, which is defined by a rapid change in the density rise rate, is triggered at \( I_p = 14-22 \) kA for \( B_t = 0.94 \) T and at 25-33 kA for \( B_t = 1.56 \) T. For both toroidal magnetic fields, the transition occurs at nearly the same value of edge rotational transform produced by \( I_p \) (i.e. \( \tau_e(a)/(2\pi) \sim 0.12 \)), although the fine structure of the rotational transform or magnetic shear may be a more important factor for the transition [15]. The existence of the threshold in \( I_p \) for the transition suggests that the control of the magnetic field structure near the edge is most essential for triggering the H-mode-like transition.

3. CONCLUSION

An H-mode-like transition has been discovered in the heliotron/torsatron type device CHS through control of the rotational transform profile with a small OH current. The observed phenomena are very similar to H modes in tokamaks except that
FIG. 2. Radial profiles of electron temperature, electron density and ion temperature for three cases. Open circles and dotted curves denote the profiles just before the transition at $t = 80$ ms (for $T_e$ and $n_e$ profiles) or 67-83 ms (for the $T_i$ profile) in Fig. 1. Solid circles and solid curves denote the profiles during the density rise phase $t = 120$ ms or 117-133 ms in Fig. 1. The profiles at $t = 120$ ms or 117-133 ms for the discharge without OH current are shown by open squares and broken curves. During the density rise phase all the profiles have a pedestal or steep gradient near the edge.
FIG. 3. Time behaviour of H-mode-like discharge with clear ELM-like activity, where the ELM-like spikes are recognized at $t = 114$ and 121 ms.
no change in edge poloidal rotation is observed. In a future helical device the attractive confinement regime can be extended under the condition of intense plasma heating, by the combination of smaller OH current with non-inductive currents.

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DISCUSSION

Y. KAMADA: Have you observed a power threshold for achieving H-mode-like discharges and what is the dependence of the threshold power on $B_t$ or rotational transform?

K. TOI: In our experiment the applied loop voltage is very low (less than 2 V) and the Ohmic heating power is less than 40 kW. If we reduce NBI power in order to establish the threshold power, the plasma current does not increase up to the level required for the H-mode-like transition. So far we have not yet studied the heating power threshold.
M. MURAKAMI: Is the improvement in confinement (by ~15%) a result of the density increase expected from the LHD scaling?

K. TOI: The improvement of about 15% is estimated from comparison between the H-mode-like discharge with an OH current and a discharge of similar density without the current. This improvement comes from broadening of the temperature and density profiles.

W.M. NEVINS: If I analyse your machine as a circular tokamak with $B_0^2 = 1$ T, $R = 1$ m and $a = 20$ cm, I conclude that at a plasma current of 40 kA this tokamak would have a safety factor of 5 ($q = 5a^2B_0/RI$). Applying JET/DIII-D H mode energy confinement scaling, I would expect $\tau_E = 0.1(I_pR^{1.5}/P^{0.5})s$ ($I_p$ in MA, $P$ in MW, $R$ in m) at $I_p = 40$ kA and $P = 600$ kW. This gives $\tau_E \approx 5$ ms. How does this estimate of $\tau_E$ compare with the experimentally observed $\tau_E$?

K. TOI: In this experiment, $\tau_E \sim 2$ ms owing to the low $B_0$ of $\approx 0.9$ T. Our results are still very preliminary and experimental studies are continuing.

G. GRIEGER: Proper iota adjustment is a well known optimization procedure for the confinement of stellarators (distribution and width of islands). Since you use iota variation to establish the H mode, how did you distinguish the H mode occurrence from the usual iota optimization?

K. TOI: We have shown that the H-mode-like transition sensitively depends on the increment of rotational transform by the plasma current. However, we have not yet identified any special profile of rotational transform and will try to investigate this profile optimization in the near future.
H-MODE LIKE TRANSITIONS
IN THE W7-AS STELLARATOR
WITH HIGH POWER 140 GHz ECRH

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Abstract

H-MODE LIKE TRANSITIONS IN THE W7-AS STELLARATOR WITH HIGH POWER 140 GHz ECRH.

H-mode like transitions were observed at W7-AS during high density operation with 140 GHz electron cyclotron resonance heating under boronized wall conditions. Such spontaneous transitions from low to high energy and particle confinement and correlated characteristic phenomena have been well known from tokamaks since 1982 and are now for the first time observed in a stellarator. The increase of the energy confinement time in W7-AS is less pronounced (up to 30%) than in typical divertor tokamaks, but the transition shows all major features characterizing H-mode transitions in tokamaks. In particular, a steepening of the density and temperature gradients at the plasma edge, enhanced poloidal rotation, impurity radiation increase and edge MHD activity (ELMs) are observed after the transition. The experimental observations from W7-AS are presented and discussed in the light of the well established database from H-mode tokamak transitions. An outline of the existence regimes of the transitions at W7-AS is given, which is somewhat preliminary because of the limited database obtained so far. The general problem of weighting the merits of the H-mode like phase in stellarators is discussed because a 'normalization standard' such as L-mode- or OH-confinement as in tokamaks is not defined in stellarators.

1. INTRODUCTION

The phenomenon of spontaneous abrupt transitions from low (L-mode) to high (H-mode) confinement properties of both energy and particle transport was first observed at ASDEX in 1982 [1] and later on confirmed in several other tokamak experiments [2-5]. The improvement in energy confinement correlated with such transitions ranges from a factor of 1.1 in limiter dominated devices [4] up to a factor of 3.5 in divertor configurations under VH-mode conditions [6]; an improvement by a factor of two is typical. Such transitions have seemed to be a particular feature of tokamak physics up to now because no physically similar events were observed in other magnetic confinement systems such as stellarators. In recent experiments at the W7-AS stellarator with high density ECRH at 140 GHz we observed H-mode like transitions for the first time in a non-tokamak device. The experimental observations correlated with this transition in W7-AS will hereafter be discussed in the light of the well established tokamak results obtained during one decade of H-mode research [7]. Although there is a large database available from several tokamaks with significant differences in size and experimental boundary conditions (limiter, divertor), there is still a substantial lack of theoretical understanding of the basic physics driving these transitions. Additional information on stellarators with the inherent absence of high currents, the high flexibility in modifying the magnetic configuration and hence the plasma edge conditions for the development of radial electric fields and poloidal/toroidal flow may contribute to a better understanding of the H-mode transition physics.
2. EXPERIMENTAL SET-UP

W7-AS is a partially optimized stellarator with five field periods and an aspect ratio of 11–16, which can be controlled by the limiter position and the rotational transform. The W7-AS standard configuration has almost vanishing vacuum shear and is provided by a modular coil set with \( \ell = 2 \) and \( \ell = 3 \) as dominant poloidal field components. The ‘built-in’ standard rotational transform of 0.39 can be changed from 0.2 to 0.6 by positive or negative superposition of a separate toroidal field. The shear of the configuration changes with increasing rotational transform from -5\% to +5\%. The cross-section of the magnetic surfaces varies strongly from a nearly elliptical to a nearly triangular shape within one half field period; the average minor and major radii are 0.18 m and 2.0 m, respectively. Two movable instrumented limiters can be inserted to control the plasma edge or can be removed outside the last closed flux surface for sufficiently high rotational transform (\( \tau > 0.5 \)) as is shown in Fig. 1. In this case, the plasma is limited by a natural separatrix, which is shaped by the remainders of 5/m natural magnetic islands originating from the non-axisymmetry of the configuration. The experiments reported here were performed at the electron cyclotron (EC) resonant magnetic induction of \( B_0 = 2.5 \) T, with \( \tau(a) = 0.5 \). The plasma net current was feedback controlled by compensating the bootstrap current with the OH

\[ Z(cm) \]

\[ Z(cm) \]

\[ \varphi = 0^\circ \]

\[ \varphi = 36^\circ \]

\[ R(cm) \]

**FIG. 1.** Magnetic topology change within half a field period at \( \tau > 0.5 \) and extreme positions for the movable instrumented limiter.
A new 140 GHz ECRH system with 0.5 MW and 0.5 s pulse duration in second harmonic X-mode was operated for the first time. The cut-off density for the 140 GHz system is $1.2 \times 10^{20} \text{ m}^{-3}$ and allows an extension of the accessible density range in ECR heated plasmas by a factor of two over to the 70 GHz ECRH system available so far. The experimentally obtained density is $n_{eo} = 1.1 \times 10^{20} \text{ m}^{-3}$. All experiments were performed with boronized vacuum vessel walls and graphite armour of the exposed in-vessel components.

3. H-MODE LIKE TRANSITIONS AND CORRELATED PHENOMENA

The plasma is generated and heated by a 70 GHz ECRH prepulse with sufficient pulse duration to ramp up the density and establish a quasi-steady-state phase at typically $(4-5) \times 10^{19} \text{ m}^{-3}$ by feedback controlled gas puffing. Then the 70 GHz heating pulse is turned off and followed by 140 GHz ECRH with approximately the same heating power of 0.42 MW. As is seen from Fig. 2, the density is ramped up further in this phase until a preprogrammed central density of $0.9 \times 10^{20} \text{ m}^{-3}$ is reached. The external gas flux drops as the density tends to level off. The 70 GHz prephase helps to save flat-top time during the limited 140 GHz ECRH pulse length (0.5 s), because the density has to be ramped up slowly enough to avoid serious deterioration of the discharge by strong edge cooling. In the high density flat-top phase a transition to a plasma state with better global confinement occurs as is seen from the total stored plasma energy trace $W$ in Fig. 2. The density starts to increase further after the transition although the external gas feed is turned off by the feedback system, which indicates improved particle confinement. The time and smoothness of the transition are dependent on the strength of the gas puffing in the density ramp-up phase. The most pronounced signature of the transition is a drop of the $D_\alpha$ line emission signals from both limiters (only one signal is displayed), indicating reduced particle recycling. The $D$-flux measured by deposition probes well outside the separatrix drops typically by a factor of two during the transition, and the ion saturation current from Langmuir probes (~3 cm outside the separatrix) drops by the same amount. A reduction in the magnetic turbulence level is measured by Mirnov coils. As in the H-mode of tokamaks, there is a backtransition indicated by the sudden rise of the $D_\alpha$ emission after the ECRH pulse has been turned off (the large spike is due to recombination in the afterglow). Some erratic bursts appear during this reduced recycling phase, which are correlated with magnetic fluctuations and soft X ray (SX) emission fluctuations as seen from Fig. 3. The SX measurements indicate that this activity is located at the plasma periphery at $r_{eff} \approx 14 \text{ cm}$ with a radial extension of about 3 to 4 cm. The single bursts, which are also detected by probe measurements, have a duration of, typically, 0.2–0.3 ms and cause fast energy and particle losses. At sufficiently high intensity of the eruptions, the density loss is high enough to cause a reaction of the gas puff feedback system. This type of instability is identified as the well known edge localized modes (ELMs) appearing in tokamak H-modes. Quiescent
FIG. 2. Time evolution of total stored plasma energy $W$, $H_\alpha$ emission intensity at the top limiter, line integrated density $\int ndt$ for the central chord, gas flux, central electron temperature from SX diagnostics and ECRH input power. The dotted line indicates the transition.

ELM free H-mode like phases (denoted as H* mode in tokamaks) of over 100 ms are observed.

The electron temperature profile as a whole increases after the transition and, in particular, a pronounced steepening of the edge temperature gradient is seen from Fig. 4. The edge ion temperature measured by impurity line broadening also increases after the transition as seen from Fig. 5(a), which is confirmed by low
FIG. 3. ELM activity from Mirnov coils (top), soft X ray emission (centre) from a chord close to the plasma boundary and correlated Hα signal (bottom) at the top limiter during the H-mode like phase. The magnetic background fluctuation level increases at the transition from the ELM free prephase to the ELMy phase of the discharge.

FIG. 4. Electron temperature half-profile from ECE diagnostics. The position of the separatrix is indicated. H-mode like transition occurs at about 0.5 s.
energy time of flight neutral diagnostics. These findings, together with the measured steepening of the edge density gradients, give clear evidence of the development of a transport barrier close to the plasma edge.

An enhanced poloidal plasma rotation velocity of up to 3.5 km/s was measured in the H-mode like phase by B IV impurity line Doppler measurements and indicates an increase of the radial electric fields from, typically, 30 to 100 V/cm. The time development of the poloidal impurity rotation velocity at the plasma periphery
(r/a \approx 0.9) is displayed in Fig. 5 for one out of very few discharges, where a back-transition is observed before the heating pulse is turned off. The $D_n$ trace is also given in Fig. 5 as an indication for the transition. It should be noted that the rotation measurement gives a lower velocity limit induced by the complex observation geometry. A comparatively slow increase of the rotation velocity is followed by a sudden jump, as the discharge switches into the quiescent phase. The backtransition is smoothed out by a series of ELMs with a gradual slowing-down of the rotation velocity until the pretransition level is reached.

The total power emitted by impurity radiation rises after the transition and does not level off during the H-mode like phase of the discharge. The total radiated power measured by a ten-channel bolometer amounts up to about 50% of the input power. The hot central impurity line emission (e.g. O VIII, Fe XVI) increases, whereas the cold edge impurity line emission (e.g. O IV, C III), which is a measure of the impurity influx, remains essentially constant. The impurity line emission from Al XII, which was inferred by laser flow-off techniques showed no drop within the available heating pulse duration. The spectroscopic measurements during the H-phase are compatible with impurity accumulation effects; a conclusive interpretation is, however, difficult because of the non-stationary profiles of $n_e$ and $T_e$.

4. PARAMETER RANGE FOR H-MODE LIKE TRANSITIONS

An examination of the experimental range for the achievement of the H-mode like transitions indicates that the rotational transform, the density and the edge condition (limiter position) are leading parameters. We have varied both $B_z$ and $B_0$ in a limited number of shots, which seem to be of minor influence on the transition (note that the power deposition region for ECRH changes from on- to off-axis heating if $B_0$ changes).

Below a line integrated density of $2 \times 10^{19}$ m$^{-2}$ ($n_{w0} \approx (4-5) \times 10^{19}$ m$^{-3}$) no transitions were observed, which a posteriori explains why no clear H-mode like phenomena were found in previous ECR heated discharges, i.e. because of the density limitations given by the cut-off condition for 70 GHz heating. The transition correlated phenomena tend to fade away rather than showing a hard threshold if the density becomes low enough. At sufficiently high density the transition occurs every time. A thorough analysis of all previous W7-AS data exhibited a few discharges (heated with ECRH and NBI as well) with some H-mode similarity but weakly (if at all) established characteristics. All these discharges were operated with the limiter removed from the last closed flux surface at $t \approx 0.5$ and with densities approaching or exceeding the limiting numbers given above.

The choice of the rotational transform is closely interlinked with the magnetic topology at the plasma boundary. Magnetic islands significantly influence the edge structure because of the low shear in W7-AS. The total stored plasma energy is plotted versus the total rotational transform in Fig. 6 and shows the well known feature
FIG. 6. Total stored plasma energy (diamagnetic loop) as a function of total rotational transform from shot to shot measurements. The marked discharges show the H-mode like transition.

of optimum confinement in the vicinity of the \( t = 0.5 \) resonance [6] (the range with optimum confinement around \( t = 0.3 \) has not yet been investigated). The confinement degrades towards higher and lower \( t \), which is partially explained by the appearance of natural islands \( (t = 5/11, 5/9) \) deteriorating the magnetic boundary configuration. In all cases investigated so far (keeping in mind the limitation in the available heating power), the H-mode like transitions occur only in this optimum confinement regimes. Discharges at \( t = 0.52 \) between the two confinement maxima show no clear transition, although some ELM activity is present, and thus seem to run at the borderline of the existence regime.

The transition is restricted to cases where the limiter is not or only marginally affecting the plasma edge (for the topology, see Fig. 1). The power flow to both limiters is typically \( \leq 30\% \) of the total power in these cases. The limiter loading is concentrated to a small strip of about 2 cm width along the limiter surface indicating that the power flow is mainly through the X-point of the separatrix. The transition was completely suppressed if the limiter was inserted at about two centimetres from the last closed flux surface. It should be noted that the separatrix position and the shape depend strongly on the exact value of the edge rotational transform and are known with sufficient accuracy for the vacuum magnetic field only. Finite pressure effects and the bootstrap current modify the edge topology; therefore, some uncertainty has to be accepted in the relative position of limiter and separatrix. The experimental boundary conditions for the achievement of the transitions suggest, however, that an unperturbed and separatrix dominated plasma boundary is required to establish the H-mode like transitions with the available heating power. This may
be a straightforward explanation for the fact that the transition is only observed at high rotational transform. Please note that the limiter is always (even in the outermost position) in contact with the bulk plasma for $\iota < 0.4$, because the plasma cross-section increases with decreasing rotational transform.

5. DISCUSSION

The improvement in energy confinement by, typically, a factor of two is the most important figure of merit of the H-mode in divertor tokamaks as compared to the L-mode. In general, the scaling laws for the energy confinement time in L-mode remain valid for the H-mode but with an enhancement factor. The W7-AS global scaling law obtained from regression analysis is based on optimum confinement discharges, i.e. confinement degradation by deterioration of the magnetic configuration was excluded:

$$\tau_E \propto a^{1.9} B_0^{0.5} n_e^{0.7} T^{-0.5} \iota^{0.26}$$

A comparison of the measured data with the scaling law is plotted in Fig. 7. Within the experimental data scatter, the LHS scaling or the Lackner–Gotthardi scaling fits

\[\text{FIG. 7. Experimental confinement time versus W7-AS scaling from multiple regression analysis (mlr).}
\text{Only ECRH data are used. The high density discharges with H-mode like transitions are indicated by dots.}\]
equally well. The confinement improvement in the H-mode like phase of up to 30% lies within the data scatter. It should be noted that the energy confinement time plotted here is not corrected for the impurity line radiation losses, which are in the range of 10–20% of the heating power for ECR heated discharges. In case of high density discharges with H-mode like transitions, the impurity radiation losses are significantly higher (up to 50%). For the H-mode experiments only the density could be varied within a factor of two; all other parameters such as heating power, magnetic induction, minor plasma radius or rotational transform were bound to a narrow parameter window. As a consequence, no conclusion can be drawn at present about the confinement scaling of the H-mode like plasma state. It is likely, although somewhat speculative, that this parameter window can be broadened if a higher heating power is available.

High enhancement factors of the H-mode confinement were only measured in divertor tokamaks, whereas only a moderate increase in the range of 10–30% was observed in limiter dominated tokamaks. From this point of view the achievements at W7-AS, where no divertor configuration is implemented at present, compare well with the tokamak results.

The existence of a power threshold for the achievement of an H-mode is a common result in tokamak research. In the W7-AS experiments there was not much room for a heating power reduction to investigate this field. The launched power of 0.42 MW was already marginal in view of the high impurity radiation level at high density operation. We note that for constant impurity concentration the radiation is expected to increase with $n^2$. For the experiments reported here the plasma was sustained by ECRH alone, in contrast to tokamaks where OH heating is still present if the additional heating power is varied.

6. SUMMARY AND CONCLUSIONS

We have observed spontaneous transitions to improved confinement of energy and particles (including impurities) under boronized wall conditions at W7-AS, which exhibit all major features of the H-mode transitions in tokamaks. In particular, an increase in edge temperatures and densities and a steepening of the edge gradients were observed, together with reduced recycling, indicating the development of a transport barrier at the plasma periphery. An increase of the radial electric fields was measured and seems to play an important role in the transitions observed. ELMs were identified during the H-mode like phase, which affect the energy and particle confinement. The H-mode like transitions where only observed at densities above $(4-5) \times 10^{19} \text{ m}^{-3}$ and under optimum confinement conditions in the vicinity of $t = 0.5$, where the plasma is separatrix dominated. The H-mode like transitions are quenched with the limiter inserted and with deteriorated edge magnetic configuration. These findings support the explanation that sufficiently long connection lengths between plasma edge and wall (limiter) and thus a good isolation of the bulk plasma
from the walls are required to provide the starting conditions for the H-mode like
transitions under the given heating power restrictions. In general, the transitions in
W7-AS show a smoother behaviour and seem to develop more slowly than in tokamak H-mode transitions. Further effort will concentrate on a broadening of the database and on an investigation of the edge conditions for the H-mode-like transitions with an improved set of edge diagnostics. Here we can benefit from the W7-AS flexibility to change the magnetic configurations and thus the edge topology over a wide range. A crucial question, which remains unanswered at present, is the physical process driving the transition and the link between edge phenomena and bulk plasma confinement.

REFERENCES


DISCUSSION

R.J. TAYLOR: I agree that all of the physics properties of your H mode are identical to the tokamak H mode, but I disagree with your historical perspective of your experiment being the 'first' stellarator H mode — in view, for example, of the W7-A accumulative data and orbit loss effects.

V. ERCKMANN: The statement that the H mode was seen for the first time in a stellarator in our experiment is correct in the sense that all the major signatures of the H mode in tokamaks were verified on W7-AS. This was not the case for previously measured improved confinement modes in stellarators.

E.J. DOYLE: You gave an example of the poloidal rotation clearly changing before the L–H transition. Was this measured using passive emission spectroscopy or active charge exchange recombination (CXR) spectroscopy? On DIII-D, when passive spectroscopy was used in the past, the inferred rotation was indeed thought to change before the L–H transition. However, using CXR spectroscopy, the rotation is observed only to change directly at the transition. The previous passive emission result was probably due to movements in the ionization layer.

V. ERCKMANN: The impurity rotation velocity was measured independently by two diagnostics from B IV emission. Measurements by active CXR spectroscopy are also available.
R. BARTIROMO: Unlike tokamaks, stellarators claim gyro-reduced Bohm scaling, therefore their turbulence should have a short wavelength scale. Hence higher poloidal rotation shear should be needed to produce a transport barrier. Have you compared the velocity shear of W7-AS with that of ASDEX?

V. ERCKMANN: We have not yet measured velocity shear because of the limited spatial resolution of the diagnostics. The measurement I presented shows the poloidal impurity rotation velocity at about 2 cm inside the last closed flux surface. The diagnostic system is at present being improved to tackle the question of velocity shear.
FLUCTUATIONS AND TRANSPORT IN
THE W7-AS STELLARATOR

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Abstract

FLUCTUATIONS AND TRANSPORT IN THE W7-AS STELLARATOR.

W7-AS is a versatile stellarator, allowing variations of both rotational transform $\iota = 1/q$ and shear. Coherent fluctuations in the plasma core are often observed in experiments performed with low shear. In particular, a neutral beam driven mode is investigated and discussed in terms of global Alfvén eigenmodes. In high $\beta$ discharges a low frequency mode is found with a mode pattern reflecting the Shafranov shift of the flux surfaces. So far, instabilities do not impose any limitation on $n_e$ or $\beta$. Higher shear can be produced by electron cyclotron current drive. The confinement in these tokamak like discharges is not significantly changed. Modes with $m = 2$ were detected which show many similarities to tearing modes, including mode locking. Recently, H-mode like transition phenomena have been observed in the W7-AS stellarator in discharges heated by a high power 140 GHz gyrotron.

1. INTRODUCTION

W7-AS is a partly optimized modular stellarator ($R = 2$ m, $a \leq 18$ cm, vacuum field with low shear and variable rotational transform) with improved equilibrium and confinement properties [1]. For the results reported in this paper electron cyclotron resonance (ECR, 70 and 140 GHz) and neutral beam injection (NBI) tangential injection, balanced or unbalanced) were used for plasma heating. Usually, the experiments were performed under the condition $I_p = 0$ (net current free) by applying a small external loop voltage. However, by means of the OH transformer or by employing EC current drive (ECCD) in addition to the bootstrap current, substantial toroidal current can be produced leading to a (tokamak-like) medium shear.

In W7-AS, with low shear and vanishing net toroidal current, mode characteristics different from those observed in tokamaks are to be expected. In fact, different types of coherent MHD fluctuation ($f \approx 2$–100 kHz) were observed, being driven partly by the plasma pressure and partly by neutral beam ions. A few of these will be discussed. The coherent fluctuations are superimposed on a turbulent background with amplitudes which decay roughly with $1/f$ and a root mean square value which typically scales inversely with the confinement properties. For the fluctuations studies in the plasma core, ECE, reflectrometry, Mirnov coil and soft X ray measurements were used. The spatial mode structure is analysed by using the magnetic coil and the soft X ray imaging systems.

The transport coefficients are generally of the same order of magnitude as in tokamaks and behave similarly (e.g. dependence on radius and heating power). In addition, the global confinement depends on $\iota$, especially for low shear [1], and only in very few cases is deteriorated by instabilities. However, the observed coherent fluctuations usually show no effect on confinement. All experimental limits in W7-AS such as the maximum achievable density ($n_e(0) \leq 2.5 \times 10^{14}$ cm$^{-3}$) or $\beta(\langle \beta \rangle \leq 1.1\%$ at 1.25 T) are not determined by instabilities but are due to a slow thermal collapse [2].
2. H-MODE LIKE TRANSITION PHENOMENA

Spontaneous transitions to improve energy and particle confinement have occasionally been observed in W7-AS [3]. Recently, during the first high power (420 kW), long pulse 140 GHz ECR heating experiments, an increase in the energy content (up to 30%) accompanied by H-mode like transition phenomena was measured [4]. Figure 1 shows time traces of the energy content, the line density, an \( H_a \) signal and a Mirnov coil signal. The transition to better confinement is correlated with a drop in the \( H_a \) radiation and in (turbulent) Mirnov coil fluctuations. Electron cyclotron emission (ECE) measurements indicate a broadening of the \( T_e \) profile, and from \( B^{3+} \) emission an increase in the poloidal plasma rotation can be inferred, indicating a change in radial electric field. Later on, ELM like bursts in the \( H_a \) light emission, the magnetic fluctuations, the soft X ray emission and other edge diagnostics can be seen. A high repetition rate of these fluctuation bursts is accompanied by a degradation of the energy and particle confinement. So far, these H-mode like transitions have been observed under conditions where a magnetic separatrix was close to the limiter (\( t(a) \approx 0.52 \)). It has to be noted that W7-AS is not supplied with a divertor.

3. POSSIBLE EXCITATION OF GLOBAL ALFVÉN EIGENMODES BY NBI

In the low and medium \( \beta \) range, very pronounced coherent fluctuations in W7-AS with NBI were found [5]. They are characterized by sharp lines (\( f \approx 12 \) to \( 45 \) kHz) in the spectrum and a fast decay (\( \approx 300 \) \( \mu \)s) after switch-off of the neutral beam (Fig. 2). Sometimes a single frequency was observed, sometimes satellite lines with downshifted frequency were present. The poloidal rotation of the modes (typically, \( m = 2 \) or \( 3 \) with \( n = 1 \)) is in the ion diamagnetic drift direction. This is opposite to the direction of rotation usually observed for pressure or current driven MHD modes. This direction changes if the magnetic field is reversed, but does not depend on co-/counter injection.

These modes are evidently driven by the fast beam ion component. This can be inferred from the much shorter decay time compared to the slowing-down time of the injected ions and the energy replacement time. The fast ion velocity \( v_b \) (45 keV full energy protons) marginally reaches the Alfvén velocity, \( v_A = B/\sqrt{4\pi n_i m_i} \), at the location where the mode is centred. Typically, one finds \( v_b/v_A \approx 0.35-1.0 \). Therefore, global Alfvén eigenmodes (GAE) are discussed as a possible cause of the observed fluctuations. They have been examined both theoretically [6–9] and experimentally [10–12] in tokamaks. The investigation is motivated by the concern that in future devices these waves could be driven unstable by fast particles (\( \alpha \) particles or particles from NBI), finally causing increased heating power losses. Although the fluctuation level in W7-AS can be relatively high (\( \bar{B}/B_0 \approx 10^{-5} \), \( \bar{P}_{sx}/P_{sx} \leq 50\% \), \( \bar{T}_e/T_e \leq 20\% \)) and the modes seem to be excited preferentially under poor
FIG. 1. Gas flux, central line density, diamagnetic energy content, $H_{\alpha}$ emission close to the upper limiter and a Mirnov coil signal as a function of time for a discharge with an H-mode like transition at about 410 ms ($B = 2.5$ T, $i(a) = 0.53$, 420 kW central ECR heating at 140 kHz). The feedback controlled line density is programmed constant for $t > 400$ ms.
FIG. 2. A Mirnov coil signal (upper left) and the spectrum of $\dot{B}$ (lower left) at the end of an NBI heated discharge ($B = 2.5$ T, $i(a) = 0.35$). The beam power of 350 kW is switched off at about 700 ms. On the right are shown from the bottom the density profile and the approximate $i$ profile used to calculate the Alfvén frequency $f_A$. The horizontal bars below the minimum of this curve approximately give the expected mode frequency. Their radial extension marks the range where co- (solid line) and counter-injected ions (broken line) are expected to resonate with the wave.
confinement conditions, no effect on fast particle losses or global plasma confinement has been observed, so far.

Toroidicity induced Alfvén eigenmodes (TAE) are not expected in W7-AS, because of the low shear. For the same reason GAE like modes [6, 9], which are damped more strongly in tokamaks because of toroidal mode coupling effects, have to be considered in W7-AS [13]. They are predicted just below the shear Alfvén continuum, which is given by the dispersion relation \( \omega = k_B v_A \) with \( k_B = (m \ell - n)/R \). Therefore, the resonant values \( \ell = n/m \) (with \( n, m > 0 \) in W7-AS) have to be excluded from the plasma in order to provide a gap below the Alfvén continuum. This is consistent with the estimated profiles of \( \psi(r) \). As they depend on the internal currents they are not known with high precision. By making realistic assumptions and using the expression for the fundamental frequency \( \omega_0 \) for GAEs (Eq. (6)) in Ref. [6] with \( \omega_0 \leq \omega_{A,\text{min}} \), the predicted frequencies are consistent with the observed values in the whole parameter range of \( B \) (1.25 and 2.5 T), \( \ell \) and density. An example is given in Fig. 2. The mode rotation expected in the diamagnetic drift direction of the fast ions also agrees with the experimental observation.

Radial mode displacement vectors have been deduced from \( T_e \) fluctuation measurements by ECE. Usually, they show no radial nodes (all \( T_e \) in phase), but in the case of multiple frequencies the weaker components have a radial node (180° phase shift of \( T_e \)). This observation seems to be consistent with GAE theory [7], which predicts frequency shifted modes corresponding to different numbers of radial nodes.

4. MODE ACTIVITY AT HIGH PLASMA BETA

Low frequency coherent mode activity (e.g. \( m = 3, n = 1, f = 5 \text{ kHz} \)) in the range of \( 0.8\% \leq \langle \beta \rangle \leq 1.1\% \) (high power NBI at 1.25 T) was detected in the soft X ray emission close to resonant \( \ell \) values. The signals do not show localized phase inversions, which could be used for mode identification, but rather a continuous phase shift across the plasma cross-section. This observation could be explained by simulation calculations taking into account the outward shift of the inner flux surfaces with respect to the outer ones due to the plasma pressure (Shafranov shift). Although this shift is reduced by a factor of 2 in W7-AS [14] it can reach \( \Delta a \approx 0.25 \) at low \( \ell \) values according to equilibrium calculations and a tomographic reconstruction of soft X ray profiles. Because of flux conservation, the mode structure is therefore expected to be squeezed radially and extended poloidally at the low field side and vice versa at the high field side [15].

The modes can be suppressed by small changes of \( \ell \) without any effect on global parameters. This proves that they do not affect the confinement. Global ideal MHD stability calculations [15, 16] for singular cases indicate stability due to shear and magnetic well stabilization in the whole range of magnetic configurations and plasma parameters accessible at present. It is therefore conjectured that the observed fluctuations are due to resistive interchange modes.
FIG. 3. Left: time traces of several soft X ray chords and of an integrated Mirnov coil signal (below). Lower right: simulation of $B$ fluctuation for a situation where the rotation velocity is modulated by the interaction with the vacuum field island. Upper right: shape of $T_e$ profile measured in the presence of the $m = 2$ mode. The radius of the $i = 1/2$ surface is about 10.5 cm.
5. CONFINEMENT AND m = 2 MODES IN DISCHARGES WITH ECCD

With ECCD adding to the uncompensated bootstrap current, toroidal currents of 18 kA, corresponding to a tokamak safety factor $q \approx 5$, were achieved. Including the vacuum field $\iota$, the total rotational transform $\iota(a)$ was almost $1/2$. Both current components heat the plasma only negligibly. The centred heating is entirely due to three gyrotrons (550 kW total). The current needs almost 2 s to reach a stationary value because of the plasma inductivity. During the current rise time the feedback controlled line density and the energy content remained almost constant. This indicates that the energy confinement is not strongly affected by the significant change in the magnetic field configuration and the transition from rather low shear to (tokamak like) medium shear.

At about half the maximum current, $m = 2$ modes, with $\tilde{B}/B_0 \leq 2 \times 10^{-4}$, $f \approx 2-4$ kHz and time traces very similar to tearing modes, were observed. At high amplitudes even mode locking phenomena occurred (Fig. 3). To check further the hypothesis of tearing modes a $\Delta'$ code [17] was applied. The driving terms are the EC driven current (centred) and the electron bootstrap current (two maxima according to $\nabla T_e$, Fig. 3). The code predicts instability, which is linked to the local minimum of $j(r)$ around $\iota = 1/2$, and a saturated island width of about 0.2a. This value agrees very well with the measured $\tilde{B}$ and fits the width and radial position of the local $T_e(r)$ (Fig. 3). Furthermore, the interaction between a rotating tearing mode and the static $m = 2$, $n = 1$ component of the vacuum field perturbation [1, 19] was studied according to Ref. [18]. In this way the non-sinusoidal shape of the Mirnov coil signal can be reproduced (Fig. 3) and, in accordance with force balance, the observation of mode locking at high amplitudes can be understood qualitatively. Difficult to understand, however, is the observed mode rotation in the ion diamagnetic direction. A possible explanation is offered by the observation $\nabla p_e \geq 0$ around $\iota = 1/2$, which follows from the broad and slightly hollow density profile.

6. CONCLUSIONS

In the W7-AS transition, phenomena with a moderate increase in confinement and features with a striking similarity to $L \rightarrow H$-mode transitions in tokamaks were observed. Coherent fluctuations driven by injected beam ions are discussed on the basis of Alfvén eigenmodes. Their saturated amplitudes remain sufficiently small so that no effect on hot ion slowing-down could be detected. Tearing mode like fluctuations could be produced in discharges with EC driven currents only.

REFERENCES

DISCUSSION

K.M. McGUIRE: In your H-mode discharges, you observe ELM activity with precursor modes. Are you able to identify the precursor mode and establish what \( m \) or \( n \) number it has?

R. JAENICKE: No. The ELMs observed in W7-AS do not usually show a clear precursor mode. Furthermore, with the present Mirnov coil system it is not possible to identify the high \( m \) numbers \( (m = 10-12) \) which are expected from tokamaks.

K. TOI: I would like to ask how the frequency spectra of the Mirnov signals change in the transition from the L to the H phase, given that magnetic fluctuations still persist to a certain extent during the H-phase.

R. JAENICKE: The frequency spectrum of the background magnetic fluctuations does not significantly change at the H-mode transition. It remains a turbulent spectrum with amplitudes which decay roughly at \( 1/f \).
MHD AND CONFINEMENT CHARACTERISTICS IN THE HIGH-β REGIME ON THE COMPACT HELICAL SYSTEM LOW ASPECT RATIO HELIOTRON/TORSATRON

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Abstract

MHD AND CONFINEMENT CHARACTERISTICS IN THE HIGH-β REGIME ON THE COMPACT HELICAL SYSTEM LOW ASPECT RATIO HELIOTRON/TORSATRON.

The MHD and confinement characteristics associated with the finite-β effect have been investigated in the low aspect ratio heliotron/torsatron Compact Helical System (CHS). <β> values of 1.8% in the quasi-steady state and 1.9% in the transient phase have been realized, and β0 has reached 5.8%. The experimentally observed equilibrium characteristics are in good agreement with current 3-D analysis. Significant instabilities suppressing the β value or terminating the discharges have not yet been observed in standard operation. This is supposed to be due to self-stabilization of the ideal interchange mode by spontaneously generated magnetic wells, which is a distinguishing feature of the low aspect ratio. Magnetic fluctuations with a frequency range of less than 100 kHz have reached as much as 0.1% of the poloidal equilibrium field. The predominant modes are globally coherent with the low mode number and do not affect confinement. The scaling of β follows the prediction from LHD scaling. The thermal diffusivity depends on the local scale length of β, the characteristics of which are emphasized for the magnetic hill region. The present β value is limited by saturation of confinement with increasing density and degradation of confinement with increasing power rather than by the MHD characteristics.

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1. INTRODUCTION

Toroidal helical systems (stelarator/heliotron/torsatron) have traditionally had large aspect ratios, as seen in a series of Wendelstein and Heliotron devices (for a review, see Ref.[1]). Recently, low aspect ratio helical systems have attracted much interest because of favourable MHD aspects as well as their potential use as reactors[2,3]. In this paper, we have investigated the characteristics of MHD equilibrium and stability, and the confinement properties associated with finite-β effects in a low aspect ratio heliotron/torsatron configuration, i.e. the Compact Helical System (CHS)[4] (\(lm = 2/8, R/a \sim 1 \text{ m} / 0.2 \text{ m}\)). The CHS has an aspect ratio as low as 5, which is uniquely low for toroidal helical devices and induces strong breaking of helical symmetry. The Shafranov shift, \(\Delta_{\text{Sh}}\), is estimated for the low β case to be \(\Delta_{\text{Sh}} = \beta_0 A_p a / \tau_a^2\). Since \(\tau_a\) is approximately proportional to \(A_p\), \(\Delta_{\text{Sh}}\) is enhanced in a low-aspect-ratio configuration. The profile of \(\tau\) which is opposite to that of the tokamak facilitates a large Shafranov shift (\(\tau_0 \sim 0.3\) and \(\tau_a \sim 1\) in CHS). Efficient generation of a magnetic well due to a large Shafranov shift tends to stabilize the interchange instability, but a large distortion of the magnetic surface structure might cause deterioration of confinement. The experimental results obtained here are of importance in demonstrating the advantage of a low aspect ratio configuration and clarifying the relevance of available 3-D theoretical models.

2. EXPERIMENTS AND DISCUSSIONS

The target plasmas have been produced by ECH (28 and 53 GHz) or ICRF (7.5 and 13 MHz) and heated by two balance injected neutral beams with a total power of 1.7 MW. The magnetic field has been operated in the range between 0.5 T and 1.6 T.

The procedure reconstructing a finite-β equilibrium from the experimental kinetic data of quasi-steady plasmas using the 3-D equilibrium code VMEC[5] has been established for the CHS experiments[6]. The description of finite-β equilibria is a prerequisite for MHD stability analysis as well as various transport analyses. The reconstructed equilibrium with anisotropic pressure due to a tangentially injected beam[7] is in good agreement with the experimental observations. Figure 1(a) shows a comparison of the Shafranov shift estimated from the reconstructed equilibrium with that from the ion temperature profile measured by 34 ch charge exchange recombination spectroscopy with 7.5 mm spatial resolution[8]. The shift of the plasma outboard boundary derived from the VUV emission profile (350 Å−1700 Å) also indicates good correlation with the 3-D computation of the last closed flux surface (see Fig.1(b)). It should be noted that the quality of the discharge significantly affects the effective minor
FIG. 1. Equilibrium characteristics: (a) Comparison of Shafranov shift of the reconstructed equilibrium from experimental data, $\Delta_{Sh}^{exp}$, with that from the peak position of $T_i$, $\Delta_{Sh}^{exp}$, at the vertically elongated cross-section ($\phi = 0$). The data are normalized by the radius of the shorter axis, $a_s$. The top abscissa is the toroidally averaged value of the Shafranov shift. The database covers $R_{st} = 0.89-1.02$ m, $B_r = 0.5-1.6$ T, $n_e = (0.8-5.5) \times 10^{13}$ m$^{-3}$, $P_{abs} = 190-760$ kW and $<\beta> = 0.15-1.2\%$. (b) Shift of the outboard boundary derived from the emission profile of VUV compared with that of the last closed flux surface given by the 3-D computation.
radius of the plasma; the discharges with lower temperature have smaller minor radius.

The maximum value of $<\beta>$ including the beam pressure in the available profile database is 1.2% in the case of the vacuum magnetic axis position $R_{ax} = 0.92$ m, $B_t = 0.5$ T, $n_e = 5.0 \times 10^{19}$ m$^{-3}$, $P_{abs} = 720$ kW. On a single shot basis, the highest measured diamagnetic $<\beta_{dia}>$ in quasi-steady state is 1.5% when $R_{ax} = 0.92$ m, $B_t = 0.5$ T, $n_e = 5.4 \times 10^{19}$ m$^{-3}$, and $P_{abs} = 770$ kW. In the transient phase realized by a reheated mode [9], $<\beta_{dia}>$ has reached 1.7% when $R_{ax} = 0.95$ m, $B_t = 0.65$ T, $n_e = 5.7 \times 10^{19}$ m$^{-3}$, and $P_{abs} = 820$ kW. If we assume a reasonable parabolic profile, these shots have $<\beta>$ of 1.8% and 1.9%, and $\beta_0$ of 5.4% and 5.8%, respectively; the last value corresponds to 80% of the conventional equilibrium $\beta$ limit defined by the Shafranov shift reaching a value of half the minor radius.

Poloidal field flexibility in CHS enables external control of the magnetic well and shear by changing the magnetic axis position. In a significantly inward shifted configuration, i.e. $R_{ax} = 0.89$ m, where the magnetic hill is increased, sawtooth phenomena in the core region ($p \sim 0.25$) accompanying magnetic fluctuation with resonant $m/n = 3/1$ mode have sometimes been observed by soft X-ray diagnostics. In other experimental magnetic configurations, self-stabilization of the ideal interchange mode due to spontaneously generated magnetic well enables a Mercier stable discharge towards the equilibrium $\beta$ limit. Strong instabilities invoking major disruption and sawtooth oscillation have not been observed experimentally except for the case of $R_{ax} = 0.89$ m. However, since the magnetic hill cannot be excluded in the peripheral region, the resistive interchange mode is destabilized, even for low $\beta$[10]. A magnetic fluctuation of a level of up to 0.1% of the poloidal equilibrium magnetic field at the surface has been observed experimentally. The dominant component has a globally coherent structure with $m \leq 4$ and $n \leq 3$, with its resonance in the magnetic hill region. These modes usually rotate in the ion diamagnetic direction and are not synchronized with the bulk plasma rotation estimated from the Doppler shift of impurity line emission. The characteristics of magnetic fluctuation depend on the pressure profile. Figure 2(a) shows the pressure profiles in the intermediate region for different discharges with $R_{ax} = 0.89$ m and $B_t = 0.95$ T. Periodic bursts of the coherent $m/n = 2/1$ mode have been detected for the lowest pressure case (see Fig.2(b)). No events correlating with these bursts have been observed in soft X-ray and fast ion loss. In spite of the appearance of the instability, the pressure gradient around $\tau = 1/2$ can be increased by increasing density and/or power. This suggests that the coherent instability with low mode number does not limit local confinement. With increasing pressure, the ensemble averaged fluctuation continued increasing but the coherent component was suppressed. Also the burst of $m/n = 2/1$ mode became less distinct and the power spectrum of the magnetic fluctuation
FIG. 2. Instability characteristics: (a) Electron kinetic pressure profile measured by Thomson scattering. $n_e$, $P_{kin}$ and $\langle \beta \rangle$ for Case 1 are given by: $1.0 \times 10^{19} \text{ m}^{-3}$, 180 kW and 0.16%, for Case 2 by: $2.5 \times 10^{19} \text{ m}^{-3}$, 480 kW and 0.40%, for Case 3 by: $4.3 \times 10^{19} \text{ m}^{-3}$, 740 kW and 0.57% and for Case 4 by: $5.3 \times 10^{19} \text{ m}^{-3}$, 970 kW and 0.74%. Arrows show the location of $\iota = 1/2$.
(b) Observed magnetic fluctuations with the frequency band between 3 kHz and 100 kHz.
FIG. 3. Confinement characteristics: (a) Comparison of experimental \(<\beta_{\text{exp}}\) based on quasi-steady plasma with the scaling derived from LHD scaling. Closed symbols refer to \(B_i < 0.6\) T. The database covers \(R = 0.89-1.02\) m, \(B_i = 0.5-1.6\) T, \(\bar{n}_e = (0.6-7.1) \times 10^{19} \text{ m}^{-3}\), \(P_{\text{abs}} = 160-950\) kW. Symbols are plotted for different density regions with reference to the empirical critical density limit on helical devices [11]: \(n_c = 0.25P_{\text{abs}}^{0.5}B_i^{0.5}a^{-1}R^{-0.5}\). (b) Thermal diffusivity as a function of the local scale length of \(\beta\). The database is the same as described in Fig. 1(a).
changed into a 1/f like shape. In the highest pressure case, an \( m/n = 1 \) mode emerged instead of the \( m/n = 2/1 \) mode.

Confinement scaling can easily be rewritten for \( \beta \) scaling. When we apply the LHD scaling[11], we get
\[
\beta(\%) = 1.44 P_{\text{abs}}^{0.42} \tilde{n}_e^{0.69} B_t^{-1.16} R^{-0.25},
\]
where \( P_{\text{abs}}, \tilde{n}_e, B_t \) and \( R \) are in MW, \( 10^{20} \text{m}^{-3} \), T and m units, respectively. The experimental data based on diamagnetic measurements follow this scaling well in the realized range with \( <\beta_{\text{dia}}><1.5\% \) (see Fig.3(a)). A small decline in the higher \( \beta \) region corresponds to high density discharges. Since the dependence of \( \beta \) on \( B_t \) is not degraded down to 0.5 T operation, the trend of degradation in the high-\( \beta \) regime is supposed to be due to confinement saturation with increasing density or power, and not with an MHD effect related to the increase of \( \beta \). Improvement of confinement by inward shifted configuration[12], which is unfavourable to the interchange mode, could be realized in high-\( \beta \) oriented discharges down to 0.5 T.

Although the global mode destabilized in the magnetic hill region does not seem to degrade the confinement in CHS, it is probable that the resistive interchange mode with microscopic scale dominates transport [13]. If anomalous transport in CHS plasmas is related to resistive interchange mode, the thermal diffusivity \( \chi \) should show a dependence on the pressure gradient and change its characteristics in the magnetic well region. To distinguish these features, \( \chi_e \) and \( \chi_i \) derived from the power balance for discharges in the profile database are normalized by \( T_e^{3/2} B_t^{-2} \) and shown as a function of the local scale length of \( \beta \), \( L_\beta \), in Fig.2(b). Strong coincidence of \( \chi_e \) and \( \chi_i \) has been observed, which is a common feature of the microinstability. The normalized \( \chi \) is a decreasing function of \( L_\beta \), and the dependence appears to be enhanced in the magnetic hill region (\( \propto L_\beta^{-0.8} \) in the hill region and \( \propto L_\beta^{-0.3} \) in the well region).

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DISCUSSION

M. WAKATANI: Is a maximum central beta of 5.8% consistent with central density and central temperatures?
H. YAMADA: Yes. Note that the fast ion pressure contribution is about 40%.
M. WAKATANI: What mode triggers sawtooth oscillation in the CHS?
H. YAMADA: Sawtooth activity occurs at r/a = 0.25 where the \( \ell = 1/3 \) surface exists. We have observed the magnetic fluctuation simultaneously with the \( m = 3/n = 1 \) mode.
BOUNDARY TOPOLOGY, EDGE TRANSPORT
AND IMPURITY CONTROL IN
THE WENDELSTEIN 7-AS STELLARATOR

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Abstract

BOUNDARY TOPOLOGY, EDGE TRANSPORT AND IMPURITY CONTROL IN THE WENDELSTEIN 7-AS STELLARATOR.

In W7-AS, both limiter dominated (rotational transform $i < 1/2$) and separatrix dominated ($i > 1/2$) operations are possible. In the first case the edge topology is characterized by inhomogeneous connection lengths $L_c$ introduced by two asymmetric rail limiters. For this case, a 1-D radial plasma

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transport model for the scrape-off layer (SOL) is applied to the coherent flux bundles intersected by a Langmuir probe. Local transport coefficients in the scrape-off layer (SOL) are obtained by adjusting calculated to measured plasma parameter profiles for low density ECRH discharges. Particle diffusion coefficients reasonably match $D$ values inside the last closed magnetic surface (LCMS). $D$ was found to scale uniformly with the inverse local density within an $n_e$ range from $5 \times 10^{11}$ to $2 \times 10^{13}$ cm$^{-3}$.

Impurity control is strongly affected by the edge topology. In limiter dominated discharges, boronization has been proven to efficiently reduce impurities. For separatrix dominated regimes, enhanced plasma-wall interaction quickly erodes protective coatings. Further improvement has to utilize the divertor potential of the boundary configuration.

1. INTRODUCTION

The flexibility of the magnetic field configuration of the W7-AS stellarator (modular design with $n = 5$ field periods, low shear, $B \leq 2.5$ T, $R = 2$ m, $a \leq 0.18$ m) along with two movable rail limiters gives access to various plasma boundary regimes [1–3]. The objective of this study is, after briefly reviewing some topological characteristics, to investigate scrape-off layer (SOL) transport for selected conditions and to give a summary of the impact of boundary conditions on impurity control.

2. BOUNDARY TOPOLOGY

The plasma edge is limiter dominated for small rotational transforms ($\iota < 0.4$). With increasing $\iota$, 'natural' boundary islands associated with the rational values $\iota = 5/m$ reduce the range of undisturbed flux surfaces such that for $\iota > 0.5$ the respective separatrix is inside the limiter radius (separatrix dominated configuration). The existence of the 5/m perturbations as well as the transition to separatrix dominated regimes is, beside field line tracing, evident from experimental flux surface mapping [3], from 2-D resolving Langmuir probe results and limiter calorimetry [2]. As a consequence of these divertor-like structures, plasma outflow is — increasing with $\iota$ — directed away from the limiters to the torus wall and/or other installations. These losses seem to concentrate mainly towards a limited region at the radial outside ('helical edge' [4]), which is an important prerequisite for utilizing a future divertor concept. At the present state without adequate target plates, this wall loading at higher rotational transforms introduces severe limitations with respect to impurity control, especially to the effectiveness of wall coatings, as will be shown below.

3. PLASMA EDGE TRANSPORT

In general, the two asymmetrically placed local rail limiters introduce additional inhomogeneities in the SOL via different connection lengths $L_c$. Nevertheless, in
limiter dominated cases at small $t$, where smooth flux surfaces extend well into the limiter shadow, poloidally extended, coherent flux bundles with homogeneous connection lengths (one, two and three toroidal turns are dominating) exist [5]. Restricting ourselves to this case and neglecting net transport across the poloidal boundaries we applied a 1-D radial transport model to special flux bundles which are intersected by a fast reciprocating Langmuir triple probe.

The model [5] includes the usual diffusive particle fluxes, diffusive and convective electron energy fluxes. The losses at the limiters are parametrized by parallel residence times with an appropriate transmission coefficient for the electron energy [6]. Ionization particle sources and energy losses, averaged over the flux bundles in question, are taken from 3-D DEGAS neutral transport calculations [7]. $T_i = T_e$ is assumed throughout the SOL. This assumption is not crucial because of the weak sensitivity of the ion saturation current to the ion temperature and the vanishing electron–ion collisional coupling in the SOL for sufficiently low densities (see below) [6]. Owing to the non-axisymmetry of the magnetic topology in W7-AS, the particle and heat fluxes had to be averaged along the flux tubes between the limiters. A parallel density drop to the limiter by a factor of two is assumed. $T_e$ is taken to be constant along the flux tubes which restricts the analysis to sufficiently low densities and high temperatures at the last closed magnetic surface (LCMS).

Local transport coefficients $D$ and $\chi_e$ are obtained from least squares fitting of the calculated to the measured ion saturation current and $T_e$ profiles. The variation parameters are four coefficients for the density power dependence of $D$ and $\chi_e$ (additional power scaling with the electron temperature was found to be negligible).

The analysis has been applied to three ECRH discharge series at $B = 1.25$ T, $t = 0.35$ with $n_e(0) = 2.2$, 1.4 and $0.5 \times 10^{13}$ cm$^{-3}$. Heating powers were 270 kW (highest density case) and 210 kW (other cases). The transport coefficients $D$ and $\chi_e$ in the plasma core are obtained from the particle and electron power balance. Ionization sources from limiter recycling are derived from DEGAS code calculations and calibrated with H$_a$ data [8]. Radiative losses from the SOL are taken into account by assuming an impurity level of 5% carbon and by using the maximum value of the non-equilibrium radiation power function, $P/n_e n_{imp} = 10^{-31}$ W·m$^{-3}$ [9]. Neglecting radiation would decrease the $\chi_e$ values by a factor of about two for the highest density case and has no influence at the lowest density. Both $D$ and $\chi_e$ in the SOL are found to increase with radius (see Fig. 1(a), (b)). While $D$ matches the values inside the LCMS without any additional assumptions in a satisfactory way, the total electronic energy losses in the SOL calculated by the model had to be doubled in order to adapt the $\chi_e$ values outside the LCMS to the core values within reasonable limits (done in Fig. 1(b)). This could be due to losses by secondary electron emission, which were neglected, and/or to larger volume losses. By relating $D$ to the local $n_e$ for the discharges analysed, a uniform local density scaling is indicated, extending from the plasma periphery inside the LCMS ($r_{eff} \approx 8$ cm) deep into the SOL ($r_{eff} \approx 20$ cm, Fig. 1(c)). Values at the LCMS are close to the INTOR/ALCATOR scaling [6]. A similar scaling has already
FIG. 1. Plasma edge transport inside and outside the LCMS for ECRH discharges at $B = 1.25\ T$, $\iota = 0.35$. Limiters are at the position $r_{ee} = 12.3\ cm$.

(a) $D$ versus $r_{ee}$. Values in the core close to the LCMS are derived from DEGAS code calculations combined with $H_\alpha$ data, values in the limiter shadow from SOL model and Langmuir probe data. Errors from statistics: less than a factor of $\pm 1.5$.

(b) $\chi_e$ ($r_{ee}$) from electron power balance and from SOL model, respectively.

(c) Results as in (a) versus the local plasma density. $D$ values at the LCMS derived from INTOR/ALCATOR scaling for the three discharge series investigated and values in the plasma core from Al ablation (ECRH discharges at $B = 2.5\ T$) are shown for comparison.

been found for the averaged $D$ as a function of the LCMS density for several ECRH and NBI discharges [2]. Furthermore, diffusion coefficients estimated from an analysis of the temporal evolution of central aluminium lines (Al ablation into 280–350 kW ECRH discharges at $B = 2.5\ T$ and $\iota = 0.35$) [1] well extrapolate the $D$ scaling to higher density values (Fig. 1(c)). However, more data are required in order to enable
an assessment of the validity of this scaling for an extended range of densities and 
ECF heating powers.

Test calculations without neutral gas show that neglecting the ionization terms 
has almost no effect on the results for the low density case, whereas for the higher 
density case it leads to a decrease of D and an increase of $\chi_e$ by about a factor of 
two.

4. IMPURITY CONTROL

Impurity production is closely related to the boundary topology. With increasing $i$ the limiter efficiency degrades and fluxes are directed towards the wall. This 
tendency is clearly seen from the anticorrelated $i$ dependence of $H_a$ emission at the 
limiters and the wall. In parallel, impurity line radiation and total radiation power 
increase with $i$. Superimposed to this general trend is strongly enhanced impurity 
radiation at 5/m-resonant $i$ vaues. This is supposed to arise from locally concentrated 
plasma flow to unprotected wall areas along the divertor-like structures formed by 
boundary islands.

For impurity control, protective low Z coatings (carbon and boron layers) were 
applied to the inner vacuum vessel, and the limiter material was exchanged from 
titanium carbide coated graphite (TiC) to bulk boronized graphite (BC) [10]. Oxygen 
and carbon impurity levels as well as the recycling were comparable for both limiter 
materials. Advantages in plasma performance with bulk boronized graphite were due 
to generally lower radiation losses because titanium was eliminated as a critical impu­
ritiy. For example, in standard electron cyclotron frequency (ECF) discharges 
($n_e(0) = 3 \times 10^{19} \text{ m}^{-3}$, $i = 0.34$, $P_{ECF} = 350 \text{ kW}$, limiter regime with maximum 
aperture), $T_e$ increased from 1.5 keV to 2.3 keV. No significant impact of sputter 
boronization on the oxygen level was observed. A significant reduction of oxygen by 
a factor of about ten was achieved only after conventional gas boronization.

The successive progress in impurity control is evident from the total radiation 
losses (in per cent of heating power) and $Z_{eff}$, which were for standard discharges 
20% and 4 with TiC limiters, irrespective of the wall conditions, 10% and 3 with 
BC limiters and stainless steel wall, and finally 6% and 2 with BC limiters and 
boronized wall.

Boronization effectively reduced the impurities in the limiter regime. However, 
coatings are short lived in erosion dominated areas, leading to a reincrease of impuri­
ties in the separatrix regime because of locally enhanced plasma-wall interaction at 
resonant $i$ values.

Further improvement of the impurity situation requires, therefore, an application 
of additional limiters at the edge, or — if the divertor potential of the configuration is used — properly adapted target plates outside the separatrix (similar to the 
W7-X concept [11]). In view of the localized plasma outflow at the helical edge and 
of the stellarator specific high edge densities, a divertor-like solution should be preferred.
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DISCUSSION

M. MURAKAMI: Could you please comment on the power accountability under normal operational conditions?

R. BRAKEL: For moderate densities of $n_e < 5 \times 10^{19} \text{ m}^{-3}$ and a small rotational transform of $\iota < 0.4$, typically 50% of the absorbed heating power (ECRH or NBI) is deposited onto the limiters. This is more or less independent of the limiter aperture. If we include radiative losses of $\leq 20\%$ the power accountability is typically $\leq 70\%$ for these operational regimes. As $\iota$ and density increase, the power deposition onto the limiters decreases. The radiated power, on the other hand, increases. The power accountability drops to typically 50%, indicating that a larger power fraction is directed to the wall in these cases.
HEATING EXPERIMENTS USING NEUTRAL BEAMS WITH VARIABLE INJECTION ANGLES AND ICRF WAVES IN A COMPACT HELICAL SYSTEM


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Abstract

HEATING EXPERIMENTS USING NEUTRAL BEAMS WITH VARIABLE INJECTION ANGLES AND ICRF WAVES IN A COMPACT HELICAL SYSTEM.

Confinement of ripple trapped energetic ions is studied in a low aspect ratio helical system using a neutral beam with variable injection angle. The heating efficiency was found to be very low for perpendicular injection compared with the tangential one, in the magnetic field configuration which is optimized for the global confinement of tangential NBI plasmas. The confinement of beam ions was improved by shifting the magnetic axis inward, which was confirmed by the measurement of their energy spectrum. The results are consistent with a calculation by a Monte Carlo beam ion thermalization code. The poloidal rotation near the plasma edge is measured for different injection angles; larger values appear for perpendicular injection. Fast wave ICRF heating is applied to a low aspect ratio helical system. Substantial energy increase is obtained for the NBI target plasma. A high energy ion tail is observed in the limited pitch angle range.

1. Introduction

In helical systems, transport related to magnetic field ripple is one of the most important problems. This transport appears as direct orbit loss of high energy particles trapped in the magnetic field ripple as well as additional non-axisymmetric terms in the neo-classical transport calculations. In low aspect ratio heliotron/torsatrons, because the drift orbits of trapped particles largely

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deviate from the magnetic surfaces, the problem of the orbit losses is more important [1]. It is, however, possible to improve the confinement of trapped particles by optimizing the magnetic field configuration [2]. This paper reports experimental studies of these problems comparing the heating efficiencies of tangential and perpendicular neutral beam injection for different magnetic field configurations.

The radial electric field in the plasma has been receiving attention especially for helical systems in relation to confinement improvement [3,4]. The electric field is determined primarily by the bulk plasma density and temperature profiles but additional particle loss channels are also important in determining the electric field. In particular, orbit losses of trapped ions possibly contribute to electric field generation in plasmas.

ICRF heating has been successful in tokamak research. In helical systems, it was successful so far in Heliotron-E, which has a high aspect ratio [5]. The applicability of ICRF heating to low aspect ratio machines has been under discussion because of the uncertainty of sufficient confinement of high energy ions produced by ICRF waves. Actual experiments are hence necessary.

2. NBI experiments with variable injection angle

CHS (Compact Helical System) [6] is a low aspect ratio heliotron/torsatron which has an aspect ratio of 5 (major radius: \( R = 1 \) m, minor radius: \( a = 0.2 \) m) and a toroidal period number of \( m = 8 \). The maximum magnetic field on axis is \( B_t = 2 \) T. Two neutral beam injectors are installed. NBI-1 \((E_0 = 36 \) keV, port-through power \( P_{\text{NBI-1}} = 0.9 \) MW) is movable and its injection angle is varied from tangential to perpendicular. NBI-2 \((E_0 = 32 \) keV, \( P_{\text{NBI-2}} = 0.6 \) MW) is fixed for tangential injection, making a balanced injection possible with NBI-1. The NBI-1 beam was injected onto a target plasma produced by NBI-2. The magnetic axis position \( R_{ax} \) was 92.1 cm (in vacuum field), which provides the best global energy confinement for tangential NBI plasmas in CHS. It is possible to keep the plasma density constant by shaping a gas puffing throughout the discharge. The parameters of the target plasma were: diamagnetic energy \( W_{\text{dia}} = 2.5 \) kJ, \( n_e(0) = 5 \times 10^{13} \) cm\(^{-3} \), \( T_e(0) = 250 \) eV, \( B_t = 1.5 \) T.

Figure 1(a) shows the incremental energies produced by the additional heating of NBI-1 as a function of the injection angle (\( \alpha_{\text{inj}} \) is the angle formed by the beam ray and the magnetic axis). The incremental energy for the tangential injection of NBI-1 is almost the same as the value expected from the power
FIG. 1. (a) Dependence of incremental energy, \( \Delta W_{\text{dia}} \), on NBI-1 injection angle, \( \alpha_{\text{inj}} \). Monte Carlo calculation results for the fractions of injected beam power are plotted with thinner lines as a function of \( \alpha_{\text{inj}} \). (b) Energy spectra of beam ions for perpendicular injection measured by a neutral particle energy analyser (FNA) for three different positions of the magnetic axis, \( R_{\text{ax}} \). The observation angle is the same as the beam injection angle, \( \alpha_{\text{inj}} = 74^\circ \). The injection energy is \( E_0 = 36 \text{ keV} \).

scaling law: \( W \propto P_{\text{net}}^{0.4} \) (\( P_{\text{net}} \) is the net input power) [7]. When the injection angles of NBI-1 were \( 19^\circ < \alpha_{\text{inj}} < 44^\circ \), almost the same heating efficiency was obtained. It is because most beam ions are not trapped in the magnetic field ripple for these angles. The heating efficiency decreases for \( \alpha_{\text{inj}} > 44^\circ \) and almost no global heating effect is found for the case \( \alpha_{\text{inj}} = 74^\circ \).
In this paper, the injection with $\alpha_{inj} = 74^\circ$ is referred to as a perpendicular injection because the pitch angles of beam ions are almost $90^\circ$ in the plasma edge region, owing to the bumpy structure of the helical field. Calculations of collisionless orbits show that the injected ions are completely lost when they are born in the region $r/a > 0.4$ ($a$ is the plasma radius) on the outboard side of the torus.

In order to estimate beam ion dynamics more quantitatively, a calculation is made with the Monte Carlo code HELIOS [8], which traces the particles in a realistic magnetic field configuration. Fast ions are traced in the entire vacuum vessel including the region outside the plasma. Figure 1(a) shows the calculated fractions of injected beam power: shinethrough, orbit loss and the power thermalized in the bulk plasma. The calculated value for the charge exchange power loss is less than 15% for $\alpha_{inj} = 29^\circ$ and still lower for the perpendicular injection. The simulation results are consistent with the experiments.

The beam ion energy spectra are obtained by a fast neutral particle energy analyser (FNA), which is capable of scanning the observation angle. For perpendicular injection, the fast neutral flux was reduced significantly and the energy spectrum of the beam ions shows a large drop in the range $E_0/2 < E < E_0$, indicating the orbit losses during the thermalization process ($R_{ax} = 92.1$ cm case in Fig. 1(b)). Since the radial electric field was negative in this experiment, the helical resonance increases the orbit loss of the trapped ions. The Monte Carlo simulation shows some effect of the electric field for $E_0/2$ and $E_0/3$ ions, but very little effect for $E_0$ ions.

It is shown by the orbit analysis that the confinement of trapped ions can be improved by controlling the magnetic field configuration, especially by shifting the magnetic axis position. With the inward shift of the magnetic axis, the deviation of the orbits of trapped particles from the magnetic surfaces becomes smaller. Figure 1(b) shows the energy spectra of beam ions measured by FNA for three different positions of the magnetic axis. The observed fast ion flux increased with the inward shift of the magnetic axis, especially in the energy range of $E_0/2 < E < E_0$. The incremental energy increased for the inward shifted configuration even though the plasma volume decreased and hence the shinethrough power increased. $\Delta W = 0.4$ kJ is obtained with perpendicular injection for the case of $R_{ax} = 87.8$ cm.

To investigate the correlation between the beam ion loss and the radial electric field, the poloidal rotation of the plasma was measured by the charge
exchange recombination spectroscopy [9] for the experiments shown in Fig.
1(a). The measurement takes the CVI impurity line emission (λ = 5292 Å),
which is induced by the charge exchange process with the residual neutrals in
the plasma instead of beam neutrals, because the NBI-1 beam path is varied.
Figure 2(a) shows the product of the poloidal rotation speed and the local
toroidal magnetic field in the plasma edge region (r/a = 0.7 - 0.8) for the dif­
ferent injection angles of NBI-1. The ion pressure gradient term is about 40
V/cm for these plasmas. The electron temperature and density profiles are
shown in Fig. 2(b) for α_{inj} = 19° (tangential injection) and 74° (perpendicular)
cases. The density profiles are almost the same, and the difference in tempera­
ture profiles corresponds to the total energy difference. For the α_{inj} = 74° case,
the electron temperature did not increase by additional NBI-1 injection. In these
experiments, an increase of the electric field in the outer region (r/a ~ 0.8) was
observed for perpendicular injection of NBI-1, but no substantial change in
global confinement was found.

3. ICRF heating experiments

Two-ion hybrid (H or $^3$He minorities in D) fast wave heating experiments
were successful in CHS using two poloidal half-turn antennas. The antenna
position was determined to fit the plasma with the $R_{Ax} = 92.1$ cm configura­tion.
With ECH initial target plasmas, plasmas are heated solely by ICRF (P = 300
kW), and the resultant stored energy was 0.65 kJ for a plasma density of $2 \times
10^{13} \text{ cm}^{-3}$ ($B_t = 1 \text{ T, } f_{ICRF} = 14 \text{ MHz}$). However, the radiation loss increased
during the discharge (20 ms heating pulse length). The ion temperature was
200 - 350 eV, which was comparable to the electron temperature. Figure 3(a)
shows time traces of plasma parameters for the combined heating experiments
using NBI target plasmas ($B_t = 1.4 \text{ T, } f_{ICRF} = 22 \text{ MHz}$). The increase in stored
energy was 0.85 kJ with 500 kW ICRF power for an initial plasma energy of
1.2 kJ.

The ion energy spectrum was measured by FNA for different observation
angles. The spectrum is of the two-component Maxwellian type, and the tail
component extends to 10 keV energy (shown in Fig. 3(b)). Tail formation was
observed at the same time as the bulk ion heating occurred for various experi­
mental conditions. The energy ratio of tail and bulk components has a pitch
angle dependence as shown in Fig. 3(c), though the tail temperature is almost
constant. With the assumption of adiabatic motion of fast ions, this dependence
FIG. 2. (a) Dependence of poloidal rotation on injection angle of NBI-1 at two different radial positions for the $R_{inj} = 92.1$ cm configuration. The outboard boundary of the plasma is $R = 108$ cm. (b) Profiles of electron temperature and density for $\alpha_{inj} = 19^\circ$ (tangential injection) and $74^\circ$ (perpendicular injection). The density of the target plasma produced by NBI-2 shows a hollow profile which is different from both profiles in this figure.
FIG. 3. (a) Time traces of heating power and gas puff (upper), diamagnetic energy, $W_{dia}$, and radiation power (central) and average density (lower). Traces for two shots with and without additional ICRF heating for NBI plasma are shown for comparison. (b) Spectrum of fast neutral particles measured by FNA in ICRF heating experiment. (c) Dependence of energy ratio of tail to bulk component and tail component temperature on the observation angle.
reflects the difference in magnetic field strengths between the observation points and the ion cyclotron resonance points where the ions are accelerated perpendicularly. It is expected that such an anisotropic distribution of high energy ions is generated by strong ion heating and energy transfer to electrons.

Acknowledgement

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DISCUSSION

V. ERCKMANN: Can you please comment on the density rise during the application of ICRH?

S. OKAMURA: In the Compact Helical System (CHS) we tend to observe an increase in density with the application of ICRF under various experimental conditions. Our understanding is that the ICRF wave field causes some desorption processes of gas from the wall.

M. MURAKAMI: Monte Carlo calculations show that the heating efficiency of the counter-injection is sensitive to the exact geometry of the boundary. What determines the plasma radius for your NBI experiments? Did you observe any change in the experimental heating rate when you varied the boundary condition?

S. OKAMURA: The CHS is normally operated without a limiter. For a magnetic axis position of $R_{ax} \leq 97$ cm, the plasma boundary is determined by the inboard side of the vacuum chamber wall. The heating rate varies to some extent as the magnetic axis position is shifted; however, this is due not only to the change in the plasma boundary condition but also to the change in the magnetic surface configuration.
THE ROLE OF NEUTRAL HYDROGEN IN COMPACT HELICAL SYSTEM PLASMAS WITH REHEAT AND COLLAPSE, AND COMPARISON WITH JIPP T-IIU TOKAMAK PLASMAS


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Abstract

THE ROLE OF NEUTRAL HYDROGEN IN COMPACT HELICAL SYSTEM PLASMAS WITH REHEAT AND COLLAPSE, AND COMPARISON WITH JIPP T-IIU TOKAMAK PLASMAS.

Results on NBI plasmas of the Compact Helical System (CHS) are described. An increase in the stored energy, which is called plasma 'reheat', is observed with density peaking when the gas puffing is turned off in the high density region. A plasma collapse with large increase in radiation loss occurs even in discharges whose Z_{eff} values (typically, less than 2-3) do not show any increase when the gas puffing is continued. Both phenomena are basically explained by the edge electron temperature due to the difference in the amount of edge hydrogen neutrals. After turning off the gas puffing, the central electron density n_e shows an increase of 80% and the density peaking factor (n_e/n_i) changes from 1.0 to 20, in typical cases; a high inward velocity of the impurities appears (v = 20 m/s). The accumulation is studied in relation to poloidal rotation and edge temperature. These results are compared with results from plasmas with improved Ohmic confinement and H-modes in the JIPP T-IIU tokamak.

1. INTRODUCTION

The Compact Helical System (CHS) (m/I = 8/2, R = 10, a = 0.2 m, B_t = 2.0 T) is a low aspect ratio heliotron/torsatron; it has favourable MHD characteristics [1]. In contrast, the plasma-wall distance is not long enough (typically 0–2 cm) and no clear divertor configuration is formed. On the other hand, in the JIPP T-IIU tokamak (R = 0.93 m, a = 0.24 m, B_t = 3.0 T, I_p = 300 kA) an

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advanced confinement regime is obtained for H and improved Ohmic confinement (IOC) modes as a result of optimized wall conditioning and gas puff control [2]. Then, it is especially important for CHS to study the relation between core plasma characteristics and edge particle behaviour and to compare the influence of edge neutrals in helical and tokamak devices.

An NBI heating experiment in CHS has been carried out with the vacuum wall conditioned by Ti gettering which covers 80% of the vacuum wall area. Plasmas following large helical device (LHD) scaling [3] are successfully obtained in the low density region. However, the increase in plasma stored energy $W_p$ saturates in the high density region at $n_e \geq (4-5) \times 10^{13} \text{cm}^{-3}$ for 1 T operation when continuous gas puffing is carried out.

Recently, it has been observed that $W_p$ increases in time with density peaking and impurity accumulation in the high density region when the gas puffing is switched off. In this paper, results are reported on the increase in energy and particle confinement obtained after switching off the gas puffing, and a comparison is made with the JIPP T-IIU tokamak. In a series of studies, results are obtained on experiments using a single beam line of tangentially injected NBI ($P_{\text{NBI}} \leq 1.1 \text{MW}$, $E_b (H^0) \leq 40 \text{keV}$).

2. PLASMA REHEAT AND COLLAPSE

Plasma 'reheat' phenomena, accompanied by a temporal increase in the stored energy which is followed by impurity accumulation and density peaking, are observed in high density operations in CHS when the continuous fuelling by gas puffing is switched off (Fig. 1). After switching off the gas puffing at $t = 100 \text{ms}$, the $H_\alpha$ signal viewing the horizontal chord decreases gradually, and $H_\alpha$ decreases rapidly in the toroidal section where the gas puffing is carried out. Together with the decrease, the edge ion and electron temperatures increase significantly. Both temperatures become twice as high as those in the gas puff phase at $t = 100 \text{ms}$, whereas the central ion and electron temperatures change only slightly. The temporal increase in $W_p$ can be explained by the increase in the temperatures in the outer region ($\rho > 0.7$) of the plasma.

A similar operation is carried out in Heliotron-E, and it is reported that $W_p$ does not show any increase although density peaking is observed with the appearance of MHD activities [4]. In CHS the MHD activity is suppressed by the formation of the magnetic well due to the large Shafranov shift, except for cases of an inward shifted axis ($R_{\text{ax}} \leq 88.8 \text{cm}$) [5].

The stored energy obtained in the reheat mode is plotted against the line averaged electron density $\bar{n}_e$ in Fig. 2(a). The solid line designates LHD scaling between $W_p$ and $\bar{n}_e$. The reheat mode appears in the saturated region above $4.5 \times 10^{13} \text{cm}^{-3}$. In this case, the reheat operation makes an improvement of up to 30% of the saturation level of the stored energy. Thus, $W_p$ recovers up to the LHD.
FIG. 1. Reheat mode of NBI plasmas in CHS ($B_0 = 1$ T, $R_w = 92.1$ cm). The neutral beam is injected with a port-through power of 800 kW.

scaling level in the conditions of the reheat mode. This behaviour is very similar to that of the IOC mode [6] in the JIPP T-IIU tokamak, as shown in Fig. 2(b).

When the gas puffing is switched off, the deposited power from the fast ions ($E_b = 40$ keV) of NBI may be raised, because of the decrease in the charge exchange loss, and may contribute to the increase in $W_p$. In the case of Fig. 2(a), however, the charge exchange loss of the fast ions is estimated from computational analyses to be only 8% at $n_e = 5 \times 10^{13}$ cm$^{-3}$ for a port-through power of 1 MW. Then, the reheat mode gives an increase in energy confinement time from $\tau_E = 3.0$ ms to 3.5 ms for Fig. 2(a). The origin of the plasma reheat is mainly
related to the recovery of the edge temperature, a drop of the radiation loss and the appearance of density peaking.

Radiation collapse is also a severe problem in helical systems [7, 8]. To suppress it, Ti gettering and discharge cleaning are carried out to reduce mainly hydrogen and oxygen in CHS. After Ti gettering, the radiation loss is kept at a rather low level (normally, less than 300 kW), but radiation collapse is not avoided.

When the radiation collapse occurs, the radiation loss is usually large. There are, however, some cases where \( Z_{\text{eff}} \) does not show any increase, as shown in Fig. 2(c). Figure 2(d) shows a comparison of the \( H_{\alpha} \) intensities between CHS and
JIPP T-IIU, measured along the midplane from the horizontal outboard ports. The inverse of the vertical axis unit ($\tilde{n}_v/H_n$) generally gives the particle confinement time $\tau_p$ at the plasma edge. This proves that discharges with higher $\tau_E$ involve better $\tau_p$ at the plasma edge. Low $\tau_p$ in CHS leads to a drop of temperature in the edge region. The influence of the edge neutrals, of course, becomes larger in the high density region; it is accompanied by a growth in the electron density in the scrape-off layer ($\rho > 1.0$), as is shown in Fig. 1. The flat density profiles in the high density region enhance the radiation loss and accelerate the radiation collapse.

3. DENSITY PEAKING AND IMPURITY ACCUMULATION

Switching off the gas puffing also triggers density peaking and impurity accumulation. In typical cases, the density peaking factor, $n_{io}/\tilde{n}_e$, changes from 1.0 to 2.0 during 50 ms after switching off the gas puffing for high density NBI discharges.

The electron density and temperature profiles are shown in Fig. 3 for discharges in the low density region. It is seen that the hollow density profile changes to a peaked profile after turning off the gas puffing. There is a tendency for the density peaking to become remarkable at higher magnetic field and electron density. According to the density rise at the plasma centre, the ion temperature increases. The same phenomenon is also obtained with the IOC mode in JIPP T-IIU. This fact indicates that there is a common role of density peaking.

Impurity accumulation is also investigated for a study of particle transport. The measurement is carried out with a scanning mirror VUV spectrometer system providing the radial profiles of the impurity line emissions. The analysis from Ti XII (460.7 Å) profiles in a typical case yields impurity peaking parameters $C_v (= a_v v/2D)$ of 0.1–0.3, when $n_{io}/\tilde{n}_e = 1.0$ (before switching off the gas puffing) and of 3.5–4.5, when $n_{io}/\tilde{n}_e = 1.5$ (afterwards). This indicates the appearance of a large inward velocity and yields inward velocities of $v = 1.0$ m/s (before) and 20.0 m/s (after) for a diffusion coefficient of $D = 0.5$ m$^2$/s.

As a possible reason for this, the poloidal rotation is measured, as is shown in Fig. 3(d). The poloidal rotation goes back to the initial value at $t = 108$ ms, although the rotation changes rapidly after turning off the gas puffing. Some discharges in other cases do not show any change of the rotation after turning off the gas puffing. There is, however, a good correlation between the density peaking factor, $n_{io}/\tilde{n}_e$, and the absolute value of the poloidal rotation. In addition, the impurity accumulation is enhanced for counter-NBI injected plasma. In the counter case a larger negative electric field is observed. This behaviour is also the same as in JIPP T-IIU NBI plasmas. Then, at least, it is clear that the density peaking and the impurity accumulation are closely connected with the negative electric field, the edge temperature and the amount of edge neutrals.
In conclusion, from a series of studies, it is understood that also in helical systems particle control at the plasma edge is important in obtaining confinement improvement and avoiding radiation collapse. To realize particle control, high temperature divertor operation in the next LHD [9] may be a possible candidate.

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DISCUSSION

H. ZUSHI: In the CHS the incremental stored energy can be well explained by the beam deposition power, as concluded in paper C-2-4 by S. Okamura et al. When the gas puff is switched off, deposited power may increase because of the reduction in charge exchange loss. If you take this into account, is the energy confinement time actually improved?

S. MORITA: Computational results give an estimated charge exchange loss for NBI fast ions of less than 10% of input power. If we take this effect into account, the analytical result shows an increment of 20–30% in the energy confinement time in the high density region.
COMPARISON OF DENSITY LIMIT PHYSICS ON THE ASDEX TOKAMAK AND THE WENDELSTEIN 7-AS STELLARATOR

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Abstract

COMPARISON OF DENSITY LIMIT PHYSICS ON THE ASDEX TOKAMAK AND THE WENDELSTEIN 7-AS STELLARATOR.

The attempt to continuously raise the plasma density in the ASDEX tokamak leads to fast MHD disruption and in the W7-AS stellarator to a slower reduction of the plasma energy content. The stellarator allows significantly higher densities to be achieved. In both devices, processes close to the plasma edge which are described in some detail are responsible for the observed limitation to the density.

1. INTRODUCTION

High densities are required in reactor size fusion devices to keep the edge temperature and, as a consequence, the divertor plate erosion sufficiently low. For tokamaks this implies operation near the possibly disruptive density limit. It is, therefore, worth while to study the density limit and the plasma behaviour at high densities in other confinement devices with reactor potential. In this paper, the density limit physics as having evolved from systematic studies on the ASDEX divertor tokamak (R = 1.65 m, a = 0.4 m) is summarized and compared with more recent and yet preliminary results from the W7-AS stellarator (limiter, R = 2 m, aeff = 0.18 m) in order to identify common physics as well as device specific differences.

2. SUMMARY OF DENSITY LIMIT STUDIES ON ASDEX

The density limit in ASDEX [1] is, in all cases, a disruptive limit. In gas fuelled Ohmic plasmas (broad \( n_e \) profiles) the density limit is well represented in a Hugill diagram. For \( q_a > 2.4 \), a linear relation is found: \( \tilde{n}_{e,DL}Rq_a/B_t = 1.7 \times 10^{20} \, \text{m}^{-2} \cdot \text{T}^{-1} \)

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for boronized walls. Less clean plasmas have a lower density limit. With neutral injection (NI) heating, $n_{e,DL}$ increases; for $q_a > 2.4$, $n_{e,DL}$ scales as $P_{tot}^{0.25}$ (Fig. 1).

In ASDEX, the density limit is a limit to the edge density. Significantly higher $n_{e,DL}$ are found in peaked density discharges (counter-NI, pellet fuelling, improved Ohmic confinement regime, high $q_a$ He plasmas); however, the local density close to the plasma edge just before the disruption is essentially the same as that obtained in corresponding discharges with rather broad density profiles. An 'edge density limit' is confirmed by JET [2] and JT-60 [3].

The total plasma radiation at the density limit is always well below the heating power (30–40% of $P_{tot}$), if no strong marfes [4] are present. For $q_a > 2.4$, marfes arising at the inboard edge are detected as precursors to the disruption in $D_e$ (not in He) plasmas. At high $q_a$, an increase of $\ell$ is observed after the marfe onset, indicating a peaking of the current profile. With continued gas puffing, the MHD disruption is initiated by a fast growth (10–20 ms) of an $m = 2$ mode. The time interval between marfe formation and disruption increases with $q_a$.

The disruption coincides with the attainment of $T_e \approx 5$ eV in the outer divertor ($q_a > 2.4$). The power flow into this region remains, however, high enough to sustain a cold, high divertor recycling. Significantly less power flows to the high field side (as measured by target plate calorimetry) which should, therefore, first be sensitive to any thermal instability. A density limit model based on the (high field side) scrape-off layer (SOL)/divertor physics is in rather good quantitative agreement with the ASDEX results [5]. However, no divertor instability could be detected with the diagnostics available.
FIG. 2. Contours of \( n_e \) and \( T_e \) close to the plasma edge as obtained in a simulation of a high density discharge of ASDEX using the B2-EIRENE code [6]. The region of the high field side marfe is enlarged.

At the high field side edge a marfe instability occurs when the temperature approaches the low Z radiation maximum (\( T \leq 15 \) eV). The marfe, being a strong local heat sink, leads to further cooling and thereby to its deeper penetration into the plasma. Consequently, the flux surface averaged plasma resistance near the edge increases, resulting in the observed shrinking of the current channel. The destabilization of the \( m = 2 \) mode occurs when the steepening of \( j(r) \) reaches the \( q = 2 \) surface. This qualitative picture of edge cooling and marfe formation has been successfully simulated for a realistic ASDEX DN geometry by using the multifluid tokamak edge code B2 coupled to the EIRENE neutral gas code [6]. Figure 2 shows contours of \( n_e \) and \( T_e \) close to the edge. A high field side marfe and, simultaneously, a cold dense outer divertor is obtained, as was observed experimentally.

3. HIGH DENSITY DISCHARGES IN THE WENDELSTEIN 7-AS STELLARATOR

Systematic attempts to maximize the density of net current free, NI heated discharges in W7-AS are so far restricted to half of the maximum field (i.e. \( B_t = 1.28 \) T), \( \iota(a) = 1/q_a \approx 0.33 \) and \( a_{\text{eff}} \approx 0.16 \) m, with and without a boronized ves-
FIG. 3. Signals from a W7-AS ($B_t = 1.28 \, T$, $\psi(a) = 0.33$) discharge where a maximum density is aimed at by gas puffing. From top to bottom: energy content ($W_{\text{dia}}$); line integrated density ($\int n \, d\Omega$); external gas flux ($T$); injected and total radiated power.

Owing to the density scaling of the stellarator confinement time, high $\bar{n}_e$ values are correlated with high energy content ($W_{\text{dia}}$).

Figure 3 shows the attempt to raise the density above a level where steady state operation is still possible. Moderate gas puffing leads, after a transient reduction, to a slow further rise of $\bar{n}_e$ which quickly saturates and then decreases, following an earlier decrease of $W_{\text{dia}}$. Stronger or longer gas pulses result in a pronounced collapse. The highest values of $\bar{n}_e$ and $W_{\text{dia}}$ are often found after closing the gas valve. If at the highest densities the heating power is reduced (e.g. by switching off one of the beams) the plasma collapses completely, even without gas puffing. These observations indicate that the density in stellarators at a given heating power is limited; any attempt to raise $\bar{n}_e$ further leads to a reduction of the energy content or even to a collapse.

The maximum density strongly depends on the wall conditions. After boronization, a density (and $W_{\text{dia}}$) of up to twice as high as before was achieved. Figure 1 shows the maximum $\bar{n}_e$ obtained at different heating powers: $\bar{n}_{e,\text{max}}$ roughly scales as
Remarkably high densities are found; $n_e$ exceeds $10^{20}$ m$^{-3}$ even at low heating power. A density limit scaling for stellarators has been proposed mainly on the basis of Heliotron E results [7]: $n_e, DL = 0.25 \times (P_{tot}B_t/a^2R)^{0.5}$ [$10^{20}$ m$^{-3}$, MW, T, m]. The power scaling is confirmed, but the proportionality factor is slightly higher on W7-AS. Higher $n_e$ values were found earlier in W7-AS at $B_t = 2.5$ T; further experiments are, however, needed to establish a $B_t$ scaling and, possibly, an additional $\iota(a)$ dependence.

The total plasma radiation increases roughly as $n_e^2$ with $P_{Rad} \approx 0.3 \times P_{tot}$ at $n_e,max$. A value of $P_{Rad}$ well below $P_{tot}$ is also reported from ATF when, as a result of efforts to increase $n_e$, a similar thermal collapse occurs [8]. This collapse, therefore, seems to result from a local rather than the global power balance.

The sensitivity of high $n_e$ plasmas to gas puffing suggests that the processes limiting the density are localized near the edge. Profiles of $n_e$ and $T_e$ measured at unperturbed high density and shortly after a collapse differ mainly in the peripheral region: at $(r/a) \approx 0.75$, $n_e$ is lower by a factor of two and an even stronger reduction is found for $T_e$, despite similar central values. This shrinking of the plasma is also seen in soft X ray profiles. Calorimetric measurements show that the limiter loading during the phase of reduced energy content is lower by, typically, a factor of three than during the phase of high energy content. A lower mean energy of the low energy charge exchange neutrals from the plasma edge after the collapse confirms the occurrence of substantial edge cooling.

Whenever a rather strong collapse is observed, pictures of tangentially viewing video cameras show, near the inner plasma edge, the sudden appearance of a bright area which changes with time. Correlated with this observation are discontinuous reductions of the $H_a$ signals from the limiters and a sudden rise in the C III radiation, indicating a transition to very low temperatures. This resembles marfe observations on ASDEX and, therefore, indicates the presence of marfes in W7-AS plasmas, as well; their onset coincides with a significant drop of the energy content.

### 4. COMPARISON OF ASDEX AND W7-AS RESULTS

The Hugill plot presentation of the (tokamak) density limit expresses it in terms of $\bar{n}_eR_q/B_t$. Peaked density, pellet fuelled discharges in ASDEX have reached $\bar{n}_eR_q/B_t = 3 \times 10^{20}$ m$^{-2} \cdot$T$^{-1}$ ($P_{NI} = 2.5$ MW, $1/qL = 0.35$). These highest values ever achieved in ASDEX are by about a factor of three lower than the W7-AS results of Fig. 1: $\bar{n}_eR_q/B_t = 8.5 \times 10^{20}$ m$^{-2} \cdot$T$^{-1}$, which underlines the potential of high density operation in a stellarator. In both devices, higher densities can be sustained in cleaner plasmas (boronization) and with increasing heating power.

The density limit in ASDEX is always manifested by a fast MHD disruption. On the other hand, a density saturation or a thermal collapse is observed in W7-AS, depending on the rate of gas puffing. The collapse, being less violent than a disruption, occurs typically on the time-scale of $\tau_E$. A non-disruptive ‘fuelling limit’ was also found for certain conditions on JET [9].
A further common observation is that the density limit is not characterized by
\[ P_{\text{rad}} \approx P_{\text{tot}} \], indicating that it is determined by a local thermal instability. Evidence
has been given that processes at the plasma edge are responsible for the limitation
to the density in both devices. On ASDEX the thermal instability leading to a marfe
as a precursor to the density limit could be simulated on the basis of an edge model
which describes many features of the edge physics. Plasma edge measurements on
W7-AS can be rather well described in terms of SOL models similar to the ones for
tokamaks [10]. It is, therefore, not surprising that observations at high densities in
W7-AS such as edge cooling or marfe formation should not be very different from
what is seen on ASDEX. So far, however, the detailed mechanisms leading to the
density limitation in W7-AS are less well understood, and more experiments are
required for further clarification.

REFERENCES


DISCUSSION

J.A. WESSON: Is it not surprising that when the outer edge of the plasma cools
and contracts there is still sufficient temperature gradient to drive out 65% of the
power?

A. STÄBLER: The change in the temperature profile due to the marfe is asym­
metric. On the low field side, there may still be a sufficiently strong gradient to drive
out an appreciable amount of power, which is measured in the outer divertor. In addi­
tion, the marfe, being a region of strong radiation, is a source of significant power
loss, particularly at high \( q_a \). In the latter case the amount of radiated power exceeds
35% of the input power once the marfe is fully developed.

K. TOI: You have shown that the operational region of electron density in
W7-AS is wider than in ASDEX. My question is whether the global energy confinement
time deviates from the W7-AS scaling dependence if \( n_e \) is increased up to the
density limit.
A. STÄBLER: As long as the density is rising to rather high values, there is no deviation from the W7-AS energy confinement scaling. In a thermal collapse, however, the energy content decreases first, followed by a density decrease after about 20 ms. This transient phase may not be in complete conformity with the confinement scaling.

R. YOSHINO: In JT-60U, marfes and detached plasmas are always observed just before an energy quench. Can you comment on the ASDEX case?

A. STÄBLER: In ASDEX deuterium (hydrogen) plasmas, marfes are detected before the density limit, but no detachment is detected, at least not from the outer divertor. In helium discharges, however, a strong, poloidally symmetric radiating shell is seen, but no marfes, and for high $q_a$ the plasma is more or less detached from the divertor.

R. YOSHINO: In W7-AS the marfe seems to be a detached plasma with contraction of the temperature profile. Can you explain how this differs from the detached plasma state?

A. STÄBLER: The thermal collapse of a W7-AS plasma leads to a state which is very similar to a detached plasma. Measurements indicate that during this state marfes occur close to the plasma edge.

V. DHYANI: You concluded that the collapse in W7-AS is less violent than the disruption in ASDEX. Would you please clarify how the two cases differ?

A. STÄBLER: The major disruption in ASDEX is governed by processes on the fast time-scale of MHD events. In W7-AS the rate of loss of energy is typically the energy confinement time, and in addition the energy loss does not lead to a termination of the plasma pulse.
CURRENT DENSITY FLUCTUATIONS, NONLINEAR COUPLING AND TRANSPORT IN MST


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Abstract

CURRENT DENSITY FLUCTUATIONS, NONLINEAR COUPLING AND TRANSPORT IN MST.

New information on magnetic fluctuations and transport in toroidal devices has been obtained in the MST reversed field pinch through measurement of nonlinear coupling of three waves in k-space, and measurement of current density fluctuations. Measurements of nonlinear coupling of magnetic fluctuations reveal that (1) two poloidal mode number m = 1 modes couple strongly to an m = 2 mode; (2) toroidal mode coupling is broad, extending up to n = 20; (3) these features agree with predictions for tearing fluctuations from a nonlinear MHD code; (4) during a sawtooth crash the number of modes involved in nonlinear interactions increases dramatically and the k-spectrum broadens simultaneously. Measurements of current density fluctuations over the outer 20% of the minor radius reveal that (1) low frequency fluctuations are consistent with tearing modes; (2) high frequency fluctuations are localized turbulence which maintains resonance with the equilibrium field as q changes with radius; (3) particle transport from magnetic fluctuations is ambipolar (i.e. $\langle j_1 \delta B_2 \rangle = 0$).

1. INTRODUCTION

Two new measurements in the MST reversed field pinch yield information on the cause of magnetic fluctuations and its influence on transport. First, nonlinear three wave coupling of spontaneously occurring tearing fluctuations has been measured. Nonlinear interaction of tearing modes is critical to dynamics in tokamaks and reversed field pinches. Wave-wave coupling is expected to in part determine the fluctuation spectrum and possibly underlie transport, sawteeth, and dynamo phenomena. We have applied bispectral analysis to extensive spatial measurements of magnetic field fluctuations ($\delta B$) to determine the coupling between
three spatial Fourier modes which satisfy the wave vector sum rule $k_1 = k_2 + k_3$. The experimental data is compared with predictions of resistive MHD computation of tearing modes, yielding reasonably good agreement. In addition it is observed that during a sawtooth crash the nonlinear coupling is strongly enhanced concomitant with a broadening of the wave number spectra. Second, we have measured the current density fluctuations ($\delta j$) in the outer portion of the plasma. The current density fluctuations support the view that low frequency fluctuations in the RFP are tearing modes. It is also observed that at high frequency (> 50 kHz) the $k$ vector (of $\delta B$) changes direction with radius, so that the poloidal-to-toroidal mode number ratio, $m/n$, follows the radial variation in safety factor. Thus, we conclude that high frequency turbulence is locally resonant. Finally, by correlating current fluctuations parallel to $B$ with radial magnetic field fluctuations we demonstrate that particle transport from parallel motion along a fluctuating magnetic field is ambipolar (i.e., $\Gamma_i - \Gamma_e \approx \langle \delta j|| \delta B_r \rangle = 0$, where $\Gamma_i$ and $\Gamma_e$ are the ion and electron particle fluxes). This ambipolarity is in agreement with the expectation for tearing modes (at low frequency) and with localized turbulence at high frequency.

2. MEASUREMENT OF NONLINEAR INTERACTIONS

With magnetic pickup coils attached to the vacuum shell at 64 toroidal locations and 16 poloidal locations we resolve spatial Fourier modes up to $n=32$ and $m=8$, and frequency up to 250 kHz. Three wave coupling between modes is directly obtained [1] from the "bicoherence" $b(k_1,k_2,k_3)$ defined as [2]

$$b(k_1,k_2,k_3) = A \langle B(k_1)B(k_2)B(k_3) \rangle$$

where $A = \langle |B(k_1)|^2 |B(k_2)|^2 \rangle^{-1/2}$, $k_3 = k_1 + k_2$, $B(k)$ is the Fourier transform of $\delta B$, and $\langle \rangle$ denotes an ensemble average (experimentally consisting of 256 time records of 256 $\mu$s duration, sampled at 1 MHz). The proportionality constant, $A$, is chosen so that if $B(k_3)$ is wholly composed from coupling with $B(k_1)$ and $B(k_2)$, then $b = 1$. The bicoherence is nonzero if there is a statistically significant phase relation between the three modes. To compare experiment with tearing mode theory, we employ an initial value, 3D, resistive MHD code. The zero pressure code has been used
extensively to describe the nonlinear, self-consistent evolution of fluctuations and mean (equilibrium) quantities in the RFP [3]. The bicoherence is evaluated from the edge magnetic field values generated by the code, as is done for experimental data. An ensemble is generated from 256 code runs with random initial phase relations between spectral components.

Most of the fluctuation energy resides in internally resonant modes, as seen elsewhere [4], with \( m = 1, n = 5-7 \) in MST. Typically, these modes phase-lock in the experiment [5,6]; i.e., they constructively interfere, presumably as a result of the nonlinear coupling revealed by the bicoherence. Experimental coupling of poloidal modes (obtained from coils distributed poloidally) is shown in Fig. 1a. The two horizontal axes represent \( m_1 \) and \( m_2 \), and the vertical axis represents the bicoherence corresponding to \( m_3 = m_1 + m_2 \). There are two dominant peaks at \( m_1=m_2=1, m_3=2 \) and \( m_1=-1, m_2=2, m_3=1 \). The sign of the mode number indicates
sense of propagation. The bicoherence calculated from the MHD code predicts these particular nonlinear interactions. These results are consistent with theoretical expectation that unstable \( m=1 \) modes drive stable \( m=2 \) modes [7]. The toroidal bicoherence, displayed in Fig. 1b, indicates that a larger number of toroidal modes are involved in nonlinear coupling in the experiment. In both experiment and MHD the dominant mode coupling occurs at intermediate \( n \) values; i.e., modes with \( n \) about unity and \( n > 20 \) are only weakly coupled to other modes. Thus, the observed phase-locking occurs through intermediate mode numbers; for example, the \( n=5 \) and \( n=6 \) modes couple through \( n=11 \) rather than through \( n=1 \). Experimental mode coupling involves more modes than predicted by the MHD computation, perhaps due to the lower Lundquist number of the code.

During a sawtooth crash [8], nonlinear interactions become suddenly more extensive. Poloidal mode coupling between two \( m=1 \) modes and \( m=4 \) becomes dominant, and toroidal mode coupling is strong for all resolved toroidal modes, from \( n=1 \) to \( n=32 \). Simultaneously, the \( n \)-spectrum broadens (as seen in TPE-1RM15 [9]), consistent with the notion that the broadening is from nonlinear energy transfer.

3. CURRENT DENSITY FLUCTUATIONS AND AMBIPOLARITY OF TRANSPORT

Current density fluctuations are measured over the outer 20\% of MST (42 cm < \( r < 52 \) cm) with insertable pickup coils configured to obtain curl \( B \) [10]. For discussion, we separate the frequency spectrum into low frequency (\( f < 50 \) kHz) and high frequency (50 kHz < \( f < 250 \) kHz). At low frequency, the phase and amplitude of \( \delta j \) (\( \delta j/j \approx 0.1 \)) are consistent with tearing fluctuations. The \( k \) spectra of \( \delta B \) at low frequency are unchanging with radius, also consistent with global tearing modes. In contrast the high frequency fluctuations (Fig. 2a) display the interesting feature that the \( n \) values reverse sign across the reversal surface at \( r=44 \) cm. The \( m \) values do not change across the reversal surface (Fig. 2a). Thus, \( m/n \) reverses, thereby maintaining resonance (\( q = m/n \)) with the local magnetic field. Hence, high frequency fluctuations are localized, resonant turbulence.

Magnetic fluctuations drive particle transport in the radial direction by parallel motion of particles along the fluctuating
magnetic field. Since parallel motion is mass dependent, it is not obvious whether such transport is ambipolar. Electron and ion particle fluxes are given by \( \Gamma_i = \langle \delta j_{\|i} \delta B_r \rangle / n e B \) and \( \Gamma_e = \langle \delta j_{\|e} \delta B_r \rangle / n e B \), where \( \delta j_i \) and \( \delta j_e \) are the ion and electron current density fluctuations parallel to the magnetic field. The nonambipolar part of the particle transport is given by \( \Gamma_i - \Gamma_e = \langle \delta j_{\||} \delta B_r \rangle / n e B \), where \( \delta j_{\||} = \delta j_{\|i} + \delta j_{\|e} \). We have measured \( \langle \delta j_{\||} \delta B_r \rangle \) to be zero, to within experimental uncertainty. The individual magnitudes of \( \delta j_{\||} \) and \( \delta B_r \) are large. The resulting nonambipolar flux is small because the phase difference is approximately \( \pi/2 \), as seen in Fig. 2b. The 5% inaccuracy in the phase measurements yields an upper bound on the nonambipolar flux of \( 5 \times 10^{19} \text{ cm}^{-2} \text{ sec}^{-1} \) which is about a factor of 20 smaller than the actual particle flux.

FIG. 2. (a) Toroidal and poloidal mode number spectra (at high frequency, \( f > 50 \text{ kHz} \)) inside the reversal surface at \( r = 42 \text{ cm} \) (solid line) and outside the reversal surface at \( r = 46 \text{ cm} \) (dotted line). Spectra approximated from two-point correlations. (b) Phase between parallel current density (\( \delta J_i \)) and radial magnetic field (\( \delta B_r \)) fluctuations.
At low frequency, ambipolarity is required by the expected phase relations of tearing modes. At high frequency, the results are consistent with theoretical expectation for modes in which the parallel component of $\delta B$ is zero, which is approximately true at high frequency. For this case, Ampere's law implies that $\langle \delta j_{||} \delta B_r \rangle$ vanishes upon integration over the narrow radial width of a mode [11]. The ambipolarity in the experiment is somewhat more stringent, since $\langle \delta j_{||} \delta B_r \rangle$ is pointwise zero, subject to the radial averaging effect of the finite radial extent of the diagnostic, which is about 1 cm, less than the radial correlation length of the turbulence.

4. SUMMARY

Three wave nonlinear interactions of tearing modes have been measured, and are qualitatively similar to predictions of MHD. During a sawtooth crash (not modeled by the MHD computation employed here) nonlinear coupling is strongly enhanced, simultaneous with spectral broadening. These measurements add credibility to the nonlinear MHD theory used to describe both RFPs and tokamaks and demonstrate the importance of nonlinear interactions in experimental plasmas.

Measurement of current density fluctuations support the identification of tearing modes at low frequency. The k-spectrum at high frequency changes dramatically with radius so as to maintain resonance with the local magnetic field. The differential loss rate of ions and electrons, generated by magnetic fluctuations, is obtained by correlating $\delta j_{||}$ with $\delta B_r$. It is shown that the particle loss is ambipolar, consistent with theoretical expectation. The loss rate of the individual species is not directly obtained; however with the reasonable assumption that the ion current density is small ($\delta j_i \ll \delta j_e$), then ambipolarity leads to the conclusion that magnetic fluctuation induced particle transport is small. Measurement of the current from individual species is underway to confirm this expectation.

In addition, measurement of the magnetic fluctuation induced energy flux is underway through measurement of $\langle \delta q_{||} \delta B_r \rangle$ where $\delta q_{||}$ is the fluctuating heat flux (measured on the edge with a fast bolometer [12]). The understanding of fluctuation dynamics and transport in the RFP has evolved to the point that research has
been initiated in active control. Experiments are beginning in MST in which current is injected from electrodes in order to flatten the current density profile in an attempt to suppress the tearing fluctuations believed to contribute to transport [13].

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ANOMALOUS ION HEATING AND
SUPERHEMPHAL ELECTROMS IN THE
MST REVERSED-FIELD PINCH

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Abstract

ANOMALOUS ION HEATING AND SUPERHEMPHAL ELECTROMS IN THE MST REVERSED-
FIELD PINCH.

Anomalous ion heating and superthermal electron populations have been studied in the MST
reversed-field pinch. The ion heating is much stronger than that given by classical electron-ion friction,
and is particularly strong during dynamo bursts. The heating displays a marked density dependence:
in a 350 kA discharge with a maximum \( \bar{n} = 0.9 \times 10^{13} \) cm\(^{-3} \), \( T_i \) rises sharply as \( \bar{n} \) drops below
0.4 \( \times 10^{13} \) cm\(^{-3} \) late in the discharge. Superthermal electrons are produced in the core, with tempera-
tures of \( T_{eh} = 350-700 \) eV, while the bulk core temperature is \( T_{ch} = 130-230 \) eV. The fraction of
superthermal electrons decreases with increasing density, from 40% at \( \bar{n} = 0.5 \times 10^{13} \) cm\(^{-3} \) to 8%
at \( \bar{n} = 1.9 \times 10^{13} \) cm\(^{-3} \) at \( I = 350 \) kA. However, data with similar plasma parameters but higher
oxygen impurity content had a lower \( T_{eh} \) and a higher hot fraction. The edge superthermal electron
distribution is well fit by a drifted bi-Maxwellian distribution with \( T_i \sim T_{eh} \) and relative drift speed
\( v_d/v_{th} = 0.4 \). With the assumption that the parallel heat flux measured with a pyroelectric probe is
carried by superthermal electrons, the measured electron current is consistent with \( T_i \sim T_{eh} \sim T_{eh}/3 \)
and accounts for over half of the total edge parallel current measured with magnetic probes.

1. INTRODUCTION

Anomalous ion heating [1,2,3,4] and the presence of superthermal electrons both in the core [5] and edge [6,7] have been observed on several RFP experiments. It has been proposed that the anomalous ion heating process involves viscous damping of fluid fluctuations produced by MHD ‘dynamo’ instabilities [8,9] or ion cyclotron resonance as the dynamo modes cascade to shorter wavelengths [10]. The core superthermal electrons resemble a classical slideaway population in the presence of the large value of \( E/E_{cr} \), where \( E \) is the electric field and \( E_{cr} \) is the critical field for
FIG. 1. A rise in ion temperature during decay of density is seen via: (a) time history of three CXA energy channels; (b) CXA spectra at the times indicated with dotted lines in (a); (c) density for the two discharges used; and (d) C⁴⁺ ion temperature (28May92 data).
runaway. The edge superthermal electrons, which have temperatures comparable to the core bulk temperature and carry a substantial portion of the poloidal plasma current, may result from enhanced transport due to magnetic fluctuations [11,12] or may be generated locally by the effective dynamo electric field \((\mathbf{V} \times \mathbf{B})\).

These phenomena have been investigated on the MST reversed-field pinch using a five-channel charge-exchange analyzer (CXA) for central proton \(T_p\), measurement of the Doppler-broadened CV 2271Å line for the \(\mathrm{C}^{+4}\) ion temperature, central Thomson scattering for \(T_e0\) and \(n_e0\), a Si(Li) x-ray spectrometer for the core superthermal electron temperature \(T_{eh}\) and fraction \(n_{eh}/n_e\), an electrostatic electron analyzer (EEA) for measurement of the edge electron distribution, a pyroelectric for measurement of the parallel heat flux at the edge, as well as the standard diagnostics for plasma current, line-averaged density, etc.

2. **ION HEATING**

Both the CXA and CV measurements indicate that ion heating far exceeds that expected from classical electron-ion friction. Particularly strong ion heating is observed [3,4] during the periods of discrete dynamo events [13]. A recent theoretical model [10] proposes that this heating arises from ion cyclotron damping of dynamo fluctuations at wavelengths large compared to those required for efficient viscous damping of perpendicular flow fluctuations; edge fluctuating magnetic field spectra may indeed have a feature at \(\omega \sim \omega_i\) [3,4].

A strong increase of \(T_i\) has been observed to occur in the late portion of discharges in which the density is allowed to decrease. Figure 1 displays CXA spectra in different energy regions gathered from two similar discharges with peak \(\bar{n} = 0.9 \times 10^{13} \text{ cm}^{-3}\) and \(I = 350 \text{ kA}\). As \(\bar{n}\) decreases, the high energy channels increase, resulting in an increase of \(T_i\) from 190 to 270 eV; this is also seen in the CV temperature, which rises from 100 to 130 eV. (The CXA spectrum shows a low energy component with \(T = 40\) eV, due to charge exchange in the edge.) The rise in \(T_i\) is not enough to compensate for the decrease in \(\bar{n}\); \(\bar{n}T_i\) continues to decrease. Although the \(T_i\) rise may simply be the result of power balance with decreasing density and profile changes, there appears to be a critical density involved. Future radially resolved measurements will allow us to unfold profile changes that may be taking place.
Table I: Results of analysis of Si(Li) spectra for a series of 350 kA discharges with different density, along with a similar set from a different date. $\beta_{eo}$ is calculated using $\bar{n}$ and $T_{e0}$ alone while $\beta_{e}$ is calculated using $T_{e0}$ and $T_{eh}$ with the respective density fractions.

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<th>$T_{e0}$</th>
<th>$T_{eh}$</th>
<th>$n_{eh}/n_e$</th>
<th>$E/E_{cr}$</th>
<th>$\beta_{eo}$ (%)</th>
<th>$\beta_{e}$ (%)</th>
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</tbody>
</table>

3. SUPERThERMAL CORE ELECTRONS

The effect of electric field strength, or $E/E_{cr}$, on superthermal electron generation was investigated in a series of discharges at four density values $\bar{n} = 0.5-2.0 \times 10^{13}$ cm$^{-3}$, summarized in Table I. While $T_{eh}/T_{e0}$ remained in the range 3-4, $n_{eh}/n_e$ decreased from 40% to 7% and $E/E_{cr}$, calculated using $T_{e0}$ from Thomson scattering, decreased from 0.14 to 0.04. In contrast, a 1% fraction of superthermal electrons was observed in much hotter HBTX plasmas ($T_{e0} = 600$ eV) with $T_{eh}/T_{e0} \sim 10$ and $E/E_{cr} = 0.9$ [5]. As Table I indicates, the inclusion of the hot component substantially increases the calculated core electron beta; however, the total beta still increases with increasing density.

It appears, however, that the superthermal component does not have a simple dependence on $E/E_{cr}$, as indicated by data taken under similar conditions on a different day (04July92). In this case, $T_{eh}$ was lower and $n_{eh}/n_e$ was higher than the comparable case quoted above. Three spectra, with two-temperature Maxwellian fits, are displayed in Fig. 2. The presence of 1.5 keV Al K radiation is evident, as well as an increase above the continuum below 0.8 keV thought to be due to oxygen K lines. The only known difference in the 04July92 set is a higher level of oxygen radiation, both visible and x-ray.

4. SUPERThERMAL EDGE ELECTRONS

The distribution of electrons flowing along the magnetic field at the edge is measured with the EEA to fit a drifted bi-Maxwellian distribution with $T_{||} \sim T_{e0} \sim 115$ eV and relative drift speed $v_d/v_{th} = 0.4$ in low-density 220
FIG. 2. Si(Li) spectra, summed over the entire discharge, with two-temperature fits. Most counts are registered during the peak current portion. $T_e0$ was not measured for the 04July92 set but is expected to be similar to that of the corresponding 28May92 set.

kA discharges, accounting for roughly 10% of the local density. With the assumption that the parallel heat flux measured with the pyroelectric probe is carried by superthermal electrons, the measured current is consistent with $T_L \sim T_e0 \sim 40$ eV and accounts for over half of the total edge current measured with magnetic probes. The superthermal current increases by as much as a factor of 5 during discrete dynamo events, at which time the toroidal magnetic flux increases sharply, implying that these electrons play a major role in the fast time-scale magnetic behavior. The ratio of electron currents parallel and anti-parallel to the total magnetic field remains constant during a dynamo burst, implying that the shape of the distribution is constant.

As superthermal electrons appear to play an important role in the energy confinement and magnetics, it is important to determine their production mechanism. We plan to measure the radial profile of superthermal electrons to distinguish between production in the edge region and enhanced transport from the core.
5. CONCLUSIONS

It appears that both anomalous ion heating and superthermal electron production are a consequence of strong MHD activity characteristic of the RFP. This results in large flow and magnetic fluctuations as well as enhanced resistivity leading to larger \( E/E_{cr} \) values than found in tokamaks with similar temperature and density. Since these fluctuations drive the poloidal current which sustains the RFP state, one might expect both effects to be a fundamental aspect of the RFP. However, as the results reported here show, both effects depend on plasma conditions, particularly on density: high density discharges have lower ion temperatures and a small core superthermal fraction, but a larger overall beta. Work will continue on MST to elucidate the heating processes via profile measurements and a wider range of parameter variation. The RFX experiment, now operating, will be able to investigate these effects at current and density values over four times higher than achieved on MST to help determine how these processes scale to a reactor.

This work was supported by the US Department of Energy.

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DISCUSSION

T. TAMANO: If ion cyclotron damping is the ion heating mechanism, we would expect anisotropy in $T_i$. Do you detect any sign of anisotropy?

S.A. HOKIN: Our charge exchange analyser views charge exchange neutrals escaping perpendicular to the magnetic field. We are unable to measure ion anisotropy at this time.

T. HELLSTEN: With reference to the explanation of ion heating by ion cyclotron absorption, doesn't the fact that the observed fluctuation spectrum decreases towards the ion cyclotron frequency and then stays constant suggest emission rather than absorption of cyclotron waves?

S.A. HOKIN: In fact, one might also expect enhanced cyclotron emission in the case of a cyclotron damping heating mechanism. So far we have not been able to determine the significance of the $\omega_{ci}$ spectral feature.

T.R. JARBOE: Do you plan to run MST in the much stabler and much higher confinement regime of low $\theta$ ($\theta \leq 1.5$)?

S.A. HOKIN: We've tried some changes in the toroidal field system without much success. MST has high $\theta$ and sawteeth even in non-reversed cases, and we still don't understand why it is so strongly driven.
FLUCTUATION INDUCED TRANSPORT AND ANOMALOUS HEATING IN RFP, ULQ AND VLQ PLASMAS


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Abstract

FLUCTUATION INDUCED TRANSPORT AND ANOMALOUS HEATING IN RFP, ULQ AND VLQ PLASMAS.

The electron heat flux resulting from electrostatic fluctuations is too small in a reversed field pinch to account for the heat flux estimated from the power balance, suggesting that magnetic perturbations dominate the electron heat flux. The particle transport inferred from triple Langmuir probe measurements can account for the particle confinement time $\tau_{pa}$ derived from $H_a$ measurements. The value of $\tau_{pa}$ increases from 20 to 180 $\mu$s with the electron density. The MHD dynamo sustains the poloidal current inside the reversal surface and the transport of fast electrons due to magnetic fluctuations is more important in the edge plasma region. Observations of anomalous ion heating show that a part of the dissipated magnetic energy in the relaxation process is directly absorbed by ions. Theoretically the parallel ion viscosities, both collisional and collisionless, play an essential role in MHD relaxations.

1. INTRODUCTION

In the REPUTE-1 experiment we have provided the first direct experimental demonstration that the electron heat flux resulting from electrostatic fluctuations is too small to account for the electron heat flux estimated from the power balance [1].

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The measurement of the electrostatic fluctuations using triple Langmuir probes [2] in the outer region \((a/2 \leq r \leq a)\) of the REPUTE-1 \((R = 82 \text{ cm}, \ a = 22 \text{ cm})\) plasma shows that the density fluctuations have a negative correlation with the electron temperature fluctuations, so that the electron heat flux due to electrostatic fluctuations is not significant. Recently similar results have been reported in the edge region of MST [3] and ZT-40M [4]. Thus the RFP plasma is distinguished from the tokamak plasma [5] by this phenomenon. The temperature of \(C^{4+}\) ions measured by Doppler broadening of \(C(V)\) \((227.1 \text{ nm})\) is anomalously high in the lower density region \((n_e \leq 5 \times 10^{19} \text{ m}^{-3})\) [6]. Anomalous ion heating is a common phenomenon of RFP and ULQ plasmas [7]. A higher toroidal field regime of VLQ is studied on REPUTE-1Q \((R = 55 \text{ cm}, \ a = 15 \text{ cm})\).

2. FLUCTUATION INDUCED TRANSPORT

The fluctuation power is concentrated mostly in the poloidal mode numbers \(m = 0\) and 1. The radial profile of the toroidal mode number \(n\) is measured by two sets of soft X ray tomography systems at different toroidal locations. The observed \(n\) value of the \(m = 1\) poloidal mode increases from 6 to 8 with the radius; it agrees qualitatively with the \(q\) profile estimated from \(F\) and \(\theta\) values. The relative level of the radial magnetic fluctuation \(B_r (r = a/2)\) is 3\% at \(I_p = 110 \text{ kA}\). The electron heat flux \(q_E\) due to stochastic magnetic fields is estimated using Harvey’s expression [8] and experimental data. The estimated value of \(q_E = 2.0 \text{ MW} \cdot \text{m}^{-2}\) is comparable to the outward electron heat flux \(q_{\text{total}}\) calculated from the power balance. Another possible mechanism of the electron heat transport is the frictional heat flux resulting from the non-linear parallel current, which is directly correlated with the helicity flux [9].

The particle confinement time \(\tau_{\text{pa}}\) derived from \(H_\alpha\) measurements increases from 20 to 180 \(\mu\text{s}\) with the electron density. The particle flux \(\Gamma (r = a/2) = 2.2 \times 10^{22} \text{ m}^{-2} \cdot \text{s}^{-1}\) from triple Langmuir probe measurements can account for the observed particle confinement time \(\tau_{\text{pa}}\).

3. FLUCTUATIONS AND SUSTAINMENT OF POLOIDAL CURRENT

The values of \(\eta j_l - E_l\) and the fluctuation induced electric field \(\langle \nabla \times \vec{B} \rangle\) are plotted in Fig. 1, where \(\eta j_l\) and \(E_l\) are parallel components of the resistive and the applied electric field, respectively. The electric resistivity \(\eta\) and the parallel current density \(j_l\) are obtained from the measured profile of the electron temperature and the magnetic fields. The fluctuation induced electric field measured by Langmuir probes and magnetic probes is not sufficient to account for \(\eta j_l - E_l\) in the outer region \((r \geq 16 \text{ cm})\) [10]. Fast electrons are detected at the edge by the electrostatic energy analyser. The velocity fluctuations \(\nabla\) of the impurity are observed from O(V).
(278.1 nm) Doppler shift measurements, as shown in Fig. 2. The levels of the observed velocity fluctuations and the magnetic fluctuations \( \tilde{B}_r \) (\( r = 0.68a \), the radius where the correlation of the fluctuations of the toroidal magnetic field changes its sign) can explain the sustainment of the poloidal current, assuming the maximum correlation between \( \tilde{v}_{\text{toroidal}} \) and \( \tilde{B}_r \). These observations give us the following picture of the flux generation in the RFP: the MHD dynamo sustains the poloidal current inside the reversal surface and the transport of fast electrons due to magnetic fluctuations is more important in the edge plasma region.

4. ANOMALOUS ION HEATING IN RFPs

The temperature of the \( \text{C}^{4+} \) ions is higher than the bulk ion (hydrogen) temperature measured by a neutral particle analyser. The ratio of the \( \text{C}^{4+} \) temperature to the hydrogen temperature increases as the plasma resistance increases and it ranges from 1 to 3. It indicates the existence of anomalous heating of the impurity, since the ion temperature measured by the neutral particle analyser represents essentially the central temperature (within 20%) and the \( \text{C}^{4+} \) temperature likewise represents the central carbon temperature. Both the bulk ion and the impurity temperature increase as the line averaged electron density \( \bar{n}_e \) decreases and as the loop voltage \( V_{\text{loop}} \) increases. The bulk ion temperature \( T_i \) scales as \( T_i (eV) = 0.36\bar{n}_e (10^{20} \text{ m}^{-3})^{0.31} V_{\text{loop}} (V)^{1.24} \), as shown in Fig. 3. \( T_i \) increases as the high frequency (\( > 50 \text{ kHz} \)) fluctuation increases. The reconnection of the magnetic field related to the generation of the toroidal flux is considered as a mechanism of the direct heating
of ions. An estimation of this power due to reconnection shows that it can account for about 20% of the total input power, which is of the same order as the estimated power input to ions.

5. COMPARISON WITH ULQ AND VLQ DISCHARGES

Experiments on the ULQ (0 < q< 1) regime of toroidal discharges found an appreciable level of resistance anomaly, viz., the toroidal loop voltage of a discharge is much larger than can be accounted for by the kinematic resistivity and the mean current. The anomalous resistance enhances the inductive power influx even in a high electron temperature regime, and a part of the excess energy input goes directly into ions, resulting in anomalous ion heating. The strong heating of ions is correlated with magnetic fluctuations with frequencies smaller than the ion cyclotron frequency. As a consequence, there is a strong dependence of the heating power on the mass of ions [7]. A scaling study of the anomalous resistance shows that the loop voltage has almost no relation to the electron temperature, while it is a rapid function of the applied toroidal magnetic field, suggesting that helicity conservation plays an essential role in the energy dissipation process [11, 12].
VLQ \((1 < q < 2)\) discharges were also compared with ULQ plasmas on REPUTE-1Q. In a VLQ discharge, electrons are selectively heated to around 1 keV, while ions are cold (around 0.2 keV) on a time-scale shorter than the classical thermal equilibration time. This discrimination of the heating power partition for different species can be attributed to the difference in dominant MHD activities. As discussed in the next section, the finite magnetic compression induced by a fast reconnection yields a selective heating of ions.

6. ROLE OF THE ION VISCOSITY IN THE RELAXATION PROCESS

Experimental observations of the anomalous resistance and the anomalous ion heating suggest that the energy dissipation mechanism in the MHD relaxation process of RFP and ULQ plasmas is based on a fluctuation dissipation that is dominated by the ion viscosity. The viscosity dominated fluctuation dissipation is possible when fast reconnections occur [13] to yield a finite magnetic compression. The gyro-relaxation and the transit time damping yield a large parallel ion viscosity for a collisional and a collisionless plasma, respectively. This specific mechanism of dissipation is characterized by fluctuating electric fields perpendicular to the ambient magnetic field, and consequently it does not change the helicity, since the helicity dissipation results only from the parallel electric field. The parallel viscosity dissipates the divergent component of the rate-of-strain tensor, while the curl derivative
terms are unaffected. Therefore the vorticity of the fluctuations, which plays an essential role in the non-linear dynamics, is retained through the relaxation process. The kinetic effect of the transit time magnetic pumping becomes dominant when the ion temperature is much larger than 0.1 keV if the ion density is of the order of $10^{19} \text{ m}^{-3}$ and the magnetic field is around 0.3 T [14].

7. CONCLUSION

Experimental results of the REPUTE-1 RFP suggest that magnetic fluctuations are more responsible for the electron heat transport than are electrostatic fluctuations. The transport of fast electrons due to magnetic fluctuations is more important in the edge region. Observations of anomalous ion heating in the REPUTE-1 RFP and REPUTE-1Q ULQ plasmas show that a part of the dissipated magnetic energy in the relaxation process is directly absorbed by ions. Theoretically the parallel ion viscosities, both collisional and collisionless, play an essential role in the MHD relaxation.

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ADVANCES IN REVERSED FIELD PINCH THEORY AND COMPUTATION

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Abstract

ADVANCES IN REVERSED FIELD PINCH THEORY AND COMPUTATION.

Advances in theory and computations related to the reversed field pinch (RFP) are presented. These are: (1) the effect of the dynamo on thermal transport; (2) a theory of ion heating due to dynamo fluctuations; (3) studies of active and passive feedback schemes for controlling dynamo fluctuations; and (4) an analytic model for coupled g-mode and rippling turbulence in the RFP edge.

1. FINITE PRESSURE DYNAMICS AND TRANSPORT

Self-reversal, sustained reversal (the dynamo) and characteristic long wavelength magnetic fluctuations in the RFP can be self-consistently explained with the non-linear, force-free, resistive MHD model [1–3]. This 3-D simulation model
has been extended to include pressure forces, Ohmic heating and anisotropic thermal conduction.

Pressure driven modes have been found to cause spectral broadening, but otherwise have little effect on field reversal and sustainment, while the current driven modes that are responsible for the RFP dynamo have been found to have a significant effect on energy confinement.

The evolution of a cylindrical RFP discharge from a cold, zero current, uniform flux state through start up and self-reversal and into the sustainment phase has been simulated. The spatially uniform initial conditions were $J_e = J_z = 0$, $B_z = 1$ kG, $n_0 = 10^{14}$ cm$^{-3}$, $T_0 = 13.6$ eV, $a = 20$ cm and $R/a = 1$. This state yields values of $S = 2.6 \times 10^3$, and $\beta_0 = 0.055$. The toroidal flux was held constant throughout the calculation; the toroidal current was linearly ramped from zero to a value corresponding to a pinch parameter $\Theta = B_\psi(a)/\langle B_z \rangle = 1.8$ and then held fixed in time. The final sustained state has repeated sawtooth oscillations, with a poloidal $\beta$ value of $\beta_\rho = 8\pi \langle \rho \rangle / B_\psi(a)^2 \approx 0.1$, $S = 1.7 \times 10^5$, $I = 180$ kA, and $\tau_E = \langle \rho \rangle / \langle \eta j^2 \rangle \approx 1.8$ ms.

Stochastic magnetic field lines are generated by the dynamo fluctuations that sustain the discharge during this phase. While the plasma core remains stochastic as a result of the $m = 1$ dynamo modes throughout the sawtooth period, the edge confinement clearly improves during the sawtooth heating phase; the sawtooth crash is accompanied by a bursting of the $m = 0$ magnetic island generated by the non-linear interaction of the dynamo modes in the edge and a corresponding rapid transport of heat along the open field lines to the wall.

The anomalous radial transport arises from classical parallel thermal conduction along the stochastic magnetic field lines that are caused by the dynamo fluctuations sustaining the discharge. Convection is found to be a relatively unimportant transport mechanism.

2. THEORY OF ANOMALOUS ION HEATING

A theory has been developed of anomalous, inviscid ion heating from the MHD dynamo (Fig. 1). The inductive drive supplies free energy, some of which drives $m = 1$ tearing modes, which non-linearly couple to drive the toroidal minimum energy component. The remaining free energy relaxes to supply the poloidal minimum energy component. A portion of the $m = 1$ energy also cascades turbulently to shorter wavelength and dissipates to electron or ion heat. Overall, the energy branches at three points, A, B and C, for which the ratios are next calculated.

The branching ratio A is calculated from the requirement that the dynamo drive balance Ohmic dissipation of the Taylor state energy, yielding

$$\frac{IV_L - \epsilon}{\epsilon} = \frac{\eta \langle j \dot{\phi} \rangle}{\eta \langle j \phi \rangle - \epsilon f_{\text{tor}}}$$ (1)
where $\varepsilon^{\text{for}}$ is the forward cascaded power from the $m = 1$ tearing mode. The branching ratio $B$ is calculated from energy and helicity conservation in a steady circuit, and from the requirement that the energy/helicity flow ratios into both the $m = 1$ tearing modes and the relaxed toroidal field should equal the energy/helicity ratios in these components. This yields the forward/inverse power ratio cascaded from the $m = 1$ tearing mode:

$$\frac{\varepsilon^{\text{for}}}{\varepsilon^{\text{inv}}} = \frac{E_{1,n}K_{0,0}}{K_{1,n}E_{0,0}} - 1 \quad (2)$$

The energies, $E$, and helicities, $K$, are calculated for nearly relaxed profiles and from linear tearing mode theory [4]. Current gradient effects cancel, giving energy/helicity ratios that depend only on $\Theta$ and the dynamo mode number, $n$. The branching ratio $C$ is calculated by comparing the cascade rate from 3-D MHD turbulence theory [5] with various damping rates. Ion cyclotron and electron Landau dampings dominate, and contribute in roughly equal portions when the cascade approaches the ion cyclotron frequency. Ratios $A$ and $B$ give an anomalous to Spitzer loop voltage ratio of

$$\frac{V_{L,\text{anom}}}{V_{L,\text{Sp}}} = \frac{-\pi a r_2^2 \Delta^{(0)}[\psi^{(0)}(r_1)]^2}{(1 + n^2 r_2^2/a^2) \Theta K_0^0} \left[ \frac{2\Theta I_0^0 + J_1^0}{2\Theta I_0^2 + J_1^2} - J_0 J_1 \right] \quad (3)$$
where $A'(0)$ and $K^{(0)}_0$ are the flux jump and helicity of the $m = 1$ tearing mode for a Taylor profile, $r_s$ is the rational surface of the dynamo mode and $J_n = J_n(2\Theta)$. This is a function of only $\Theta$ and $na/R$, and typically near 1. The ion to electron heating ratio is thus about 1:2. Measurements on MST [6] made after this theory had been proposed support several aspects.

3. FEEDBACK CONTROL OF MAGNETIC FLUCTUATIONS

The $m = 1$ MHD modes that generate the dynamo also can cause enhanced energy transport and highly anomalous loop voltage if the boundary is not perfectly conducting [7]. Three-dimensional MHD computation is employed to investigate the possibility of suppression of these modes by feedback. Previous simulations [8] suggest the possibility of using feedback to sustain a resistive shell RFP at conducting shell fluctuation levels.

Two feedback schemes have been studied in which the boundary radial magnetic field for specific helical modes is set to a non-zero value. In the first scheme, the time derivative of the boundary field is

$$\frac{\partial b_r}{\partial t} = -\frac{a}{\tau_{rb}} \left\{ \left[ \frac{\partial b_r}{\partial r} \right]_a + \min \left( \frac{c_1^4}{(c_1^2 - b_r^2)^2}, 10^{10} \right) \frac{b_r}{c_2} \right\} \quad (4)$$

where [ ] indicates the jump across a resistive shell, and $c_1$ and $c_2$ are adjustable coefficients. Single mode simulations including quasi-linear modification of the mean fields demonstrate saturated fluctuation levels roughly half of that obtained with a conducting shell. The parameters $\tau_{rb}$, $c_1$ and $c_2$ must be carefully chosen so as not to induce phase flip instabilities [9]. Full non-linear runs with this feedback method do not yield correspondingly positive results and require further optimization.

In the second feedback scheme, the edge field is varied in response to the field at the resonant surface so as to minimize the jump in the logarithmic derivative of the radial field ($A'$) at the resonant surface. This approach reduces fluctuation amplitudes in linear runs, but is prone to the phase flip instability in non-linear tests.

4. THEORY OF EDGE TURBULENCE

The RFP edge region resembles a confinement zone where transport may be governed by resistive interchange modes [10]. Observations [11] indicate that high frequency, small scale electrostatic fluctuations can account for the large particle losses in MST. A study of the resistive interchange model, generalized to include the free energy in the resistivity gradient, has been undertaken [12] and applied to the RFP edge and MST data.
FIG. 2. Diffusivity ($m^2/s$) and electrostatic fluctuation level versus $r/a$ for MST parameters: $T = 350$ eV, $n = 10^{13}$ cm$^{-3}$, $B_{\text{eir}} = 1.7$ kG, $a = 52$ cm, $Z_{\text{eff}} = 2.5$, $\theta = 1.58$, reversal radius = 0.865 and $f = -0.235$.

The model couples equations for vorticity, density and temperature evolution, and Ohm’s law. A renormalized eigenmode equation determines the amplitude and structure of the fluctuations in steady state. Saturation occurs through a balance of line bending with the resistivity gradient and pressure gradient (curvature) source terms as mediated by a turbulently broadened mode width. The result consists of two equations for the diffusivity $D$ and the turbulent mode width $\Delta$:

$$D = \kappa \left( \frac{r}{R} \right)^2 \left( \frac{d q}{d r} \right)^{-2} k_x^2 \left( \frac{\eta_0 P_0}{L_T B_0^2} \right) \left( 1 + \frac{\Delta^3}{\Delta^3} \right)^{-1}$$ (5)

and

$$\left[ D + B_0^2 k_x^2 \left( \frac{d q}{d r} \right)^2 \Delta^6 \frac{(R/r)^2}{\eta_0 \rho_0} \right] \left[ D + \chi_1 \Delta^4 k_x^2 \left( \frac{d q}{d r} \right)^2 \left( \frac{R}{r} \right)^2 \right]$$

$$= - \frac{3}{2} \Delta^7 \frac{B_0 J_0 k_x}{L_T}$$ (6)

where $\Delta^3 = k_0 J_0 |\eta_0| (r/R)^3 (dq/dr)^3 B_0^{-1}$. Here, $\kappa$ is the curvature, $q$ is the safety factor, $L_T (\sim L_r)$ is the temperature gradient scale length, and $\chi_1$, $\eta_0$, $J_0$, $B_0$, $P_0$ and $\rho_0$ have their usual meanings.
Only a modest enhancement of turbulence levels above the level of the g-mode is found (Fig. 2). The spectrum integrated electrostatic potential amplitude only exceeds \((\langle e\phi/T_e \rangle^2)^{1/2} \sim 0.01\) at the extreme edge \((r/a > 0.975)\) with a maximum value of five percent. The spectrum of electrostatic fluctuations observed in MST can be integrated over all frequencies above a low frequency cut-off of 20 kHz, chosen in order to eliminate the contribution of low frequency global tearing modes. The result from MST (Fig. 2 of Ref. [11]) is approximately 0.2 for \(0.9 < r/a \leq 1.0\). The disparity between measured and predicted fluctuation levels indicates that it is unlikely for g-modes (or rippling modes) to govern edge transport or account for high frequency fluctuations activity in RFP devices.

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DISCUSSION

B. COPPI: You mentioned the excitation of \(m = 1\) resistive modes. There are two characteristic regimes of these modes: one where the transition layer growth rate is proportional to the power 2/5 of the resistivity, and another where the growth rate is proportional to the power 1/3 of the resistivity. Which one did you consider?

D.D. SCHNACK: The reversed field pinch is a non-linear driven system characterized by many finite amplitude, non-linearly interacting \(m = 1\) modes. The behaviour of such a system is quite different from the isolated behaviour of its individual components. We have found little correlation between the non-linear evolution of the complete system and the linear or non-linear theories of various
individual modes. Linearly, the individual modes vary considerably, ranging from resonant, non-constant $\psi$, resistive modes to non-resonant ideal modes. Non-linearly, their evolution as a system has little to do with their linear behaviour. Therefore, I probably cannot answer your question to your satisfaction, since we have not studied their linear behaviour in detail.
3D SIMULATIONS OF THE DYNAMO EFFECT IN THE REVERSED-FIELD PINCH

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\textit{Presented by D.D. Schnack}

Abstract

3D SIMULATIONS OF THE DYNAMO EFFECT IN THE REVERSED-FIELD PINCH.

3D computer simulations of a reversed-field pinch have been carried out in order to investigate the validity of the quasi-linear approximation in describing such a device. The simulation solves the non-linear equations of incompressible resistive MHD in a periodic cylinder. It has been found that three terms ignored in the quasi-linear induction equation have an active role in determining the magnitude and phase of the perturbed magnetic field. Specifically these terms are those due to the presence of a cylindrically averaged part of the velocity field and the mode-mode interactions. It is known that the dynamo effect involves a phase shift between the fluctuations in velocity and magnetic field, and the presence of a mean flow in the simulations was found to adjust this phase shift. This allows dynamo action with a lower level of fluctuations in the magnetic field. The role of this averaged velocity was examined with a simple linear code which showed that a change in the phase of a linear mode is a natural consequence of assuming the existence of a mean flow. It was found that the ion heating rate could exceed the Ohmic heating rate depending on the Prandtl number.

1. Introduction

In this paper the results from a series of computer simulations of a reversed-field pinch (RFP) are presented. The simulations were carried out in order to investigate whether the quasi-linear approximation of the resistive MHD equations is an adequate description of an RFP. This problem was investigated using the plasma simulation code MERCURY \cite{1}, which solves, non-linearly, the equations of incompressible resistive magnetohydrodynamics. MERCURY uses periodic cylindrical geometry and treats the ignorable coordinates, $\theta$ and $z$, spectrally. A working model of an RFP was set up and simulations carried out at various values of the Prandtl number, as this represented the ratio of the dissipative forces to which the dynamo term $\nabla \wedge (\langle \mathbf{v} \wedge \mathbf{b} \rangle)$ is subject (where $\langle \rangle$ is the cylindrical average over the ignorable coordinates).

\textsuperscript{1} AEA Fusion, Culham Laboratory, Abingdon, Oxfordshire, UK.
2. Quasi-Linear Theory and the Dynamo Effect

In linear MHD theory, the equations of magnetohydrodynamics are linearised about some equilibrium state and the non-linear terms are discarded. This leads to the equations being split up into a set of equations for the equilibrium quantities and a set of equations for the first-order quantities. In quasi-linear theory the space, or time, average of the non-linear terms appears in the zeroth order equation and in this way non-linear effects can begin to be treated. In the context of an RFP the quasi-linear approximation predicts the following.

If the linearisation is performed about the cylindrically averaged quantities then the quasi-linearised induction equation becomes

$$\frac{\partial \mathbf{B}_0}{\partial t} = \nabla \times (\langle \mathbf{v} \times \mathbf{b} \rangle) - \nabla \times (\eta \nabla \mathbf{B}_0)$$

(1)

$$\frac{\partial \mathbf{b}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}_0) - \nabla \times (\eta \nabla \mathbf{b})$$

(2)

where the angled brackets represent the cylindrical average with \( \mathbf{B} = \mathbf{B}_0 + \mathbf{b} \) and \( \langle \mathbf{B} \rangle = \mathbf{B}_0 \).

The first term in equation one is equal to zero in a steady state and in equilibrium there is necessarily a balance between the terms \( \nabla \times (\langle \mathbf{v} \times \mathbf{b} \rangle) \) and \( \nabla \times (\eta \nabla \mathbf{B}_0) \). This is the essence of the dynamo effect, the \( \mathbf{v} \times \mathbf{b} \) interaction between the fluctuations in \( \mathbf{v} \) and \( \mathbf{b} \) opposes the resistive diffusion of the equilibrium magnetic field.

Equation two may be temporally averaged and this eliminates the first term as it is a perfect differential in time. This leaves a balance between the terms \( \nabla \times (\mathbf{v} \times \mathbf{B}_0) \) and \( \nabla \times (\eta \nabla \mathbf{b}) \).

With the knowledge of the balance between the terms detailed above it is possible to make an order of magnitude estimate for the fluctuation level \( \mathbf{b}/\mathbf{B}_0 \). This estimate can be made by considering the following,

$$\nabla \times (\langle \mathbf{v} \times \mathbf{b} \rangle) \sim \nabla \times (\eta \nabla \mathbf{B}_0),$$

$$\nabla \times (\hat{\mathbf{v}} \times \mathbf{B}_0) \sim \nabla \times (\eta \nabla \mathbf{b}).$$

It can be argued that \( (\mathbf{v} \times \mathbf{b})/L \sim (\eta \mathbf{B}_0)/L^2 \) and \( (\hat{\mathbf{v}} \mathbf{B}_0)/L \sim (\eta \mathbf{b})/L^2 \). Examination of these relations leads to the conclusion that \( (\mathbf{b}/\mathbf{B}_0)^2 \sim 1 \) if it is assumed that the instabilities are perfectly matched in phase. This was an assumption which could be verified by the simulation.

The conclusion that \( \mathbf{b}/\mathbf{B}_0 \) is of order one is in contradiction to the fluctuation level found in this MHD code which is of order one tenth (in experiment the fluctuation level is smaller and of order one hundredth). This is the problem with the quasi-linear approximation. By setting up
a computer simulation of an RFP the importance of each term in the induction equation could be examined and in this way the problem could be investigated.

3. Results from the Simulations

The simulation code assumed an isotropic resistivity, which was a function of radius only, and uniform isotropic viscosity. The code was initialised with a flat $B_z$ and an axial current consistent with a constant electric field at the conducting wall and the resistivity profile. Noise was applied to the magnetic field and this system was allowed to evolve in time. The spectral grid chosen allowed the existence of 60 Fourier modes, poloidal modes 0 to 3 and axial modes -7 to 7. The radial grid typically had 32 points, though runs at 64 and 128 were also tried. The Lundquist number ($S$) was varied between 100 and 1000 while the Prandtl number ($P$) was varied between 0.01 and 10. These simulations were examined to see if they reproduced the phenomena of relaxation and sustained reversal. Once a reversed quasi-steady state was reached, the terms in the induction equation were calculated to see which were important and in this way the approximations of quasi-linear theory could be examined.

Results like figure 1 are typical and similar to those of Schnack et al [2]. This figure shows the temporal evolution of the cylindrically averaged poloidal and axial magnetic fields and it can be seen that the axial magnetic field starts off with a flat radial profile and evolves into a sustained reversed profile. With the simulation giving this characteristic result of dynamic equilibrium the terms in the induction equation could be examined.

Figure 1: This figure shows the temporal evolution of the cylindrically averaged poloidal and axial magnetic fields. The time scale is irregular, to allow for extra temporal resolution in the initial stages, where the figures in brackets indicate the time in Alfvén transit times. $S=100$, $P=1$. 
| $S=100, \ P=0.01$ | 4.1 | 14.6 | 32.8 | 32.9 | 15.5 |
| $S=1000, \ P=0.1$ | 3.5 | 13.3 | 33.9 | 35.8 | 13.5 |
| $S=100, \ P=1$  | 3.1 | 11.9 | 33.4 | 32.1 | 19.5 |

\[ \varepsilon_{NL} = \tilde{\mathbf{v}} \wedge \tilde{\mathbf{b}} - \langle \tilde{\mathbf{v}} \wedge \tilde{\mathbf{b}} \rangle \]

This table shows the relative magnitudes of each term in the axial component of the induction equation for various parameters. The numbers represent the percentage that each term makes in the equation. This percentage was evaluated for all grid points and then averaged over all $r$, $\theta$, $z$. These results are taken at one specific time during the quasi-steady state. Time averaged values during the quasi-steady state give similar results.

It was found that the resistive diffusion of the equilibrium magnetic fields was indeed balanced by the term $\nabla \wedge ((\tilde{\mathbf{v}} \wedge \tilde{\mathbf{b}}))$. The mode spectrum of the fluctuations $\tilde{\mathbf{v}}$ and $\tilde{\mathbf{b}}$ was dominated by $m=1$ modes and $n$ numbers in the range -2 to -5 which corresponds to the characteristic dynamo modes [2,3]. In table I the relative percentage made by each term in the first order induction equation is shown for the toroidal component and three different cases. It can be seen that three terms ignored by quasi-linear theory are present and involved in determining the dynamo effect via an influence on the perturbed magnetic field. Physically these terms are those due to the presence of a cylindrically averaged part of the velocity field ($\mathbf{V}_0$), and the non-linear interaction of the fluctuations which did not directly contribute to the dynamo effect. The presence of these terms obviates the arguments of the previous section.

The dynamo effect directly depends on the phase between the modes $\tilde{\mathbf{v}}$ and $\tilde{\mathbf{b}}$ and the dynamo effect could comprise ill-matched fluctuations with a high amplitude or well matched modes with a low amplitude. In the simulations it was found that the phase matching of the fluctuations was high with $|\tilde{\mathbf{v}} \wedge \tilde{\mathbf{b}}| \sim 0.8 |\tilde{\mathbf{v}}||\tilde{\mathbf{b}}|$. This validated the perfect phase matching assumption of section two.

The presence of the mean flow $\mathbf{V}_0$ is interesting, as it is consistent with the observed differences between compressible and incompressible simulation codes [4]. This difference may be ascribed to the absence of the mean flow $\mathbf{V}_0$ in the incompressible simulation codes. The temporal
evolution of the cylindrically averaged poloidal and toroidal flows is shown in figure 2. These flows conserve axial and angular momentum and the source of these flows was examined and found to be the terms $(\mathbf{j} \times \mathbf{b})$ and $\langle \rho \mathbf{v} \cdot \nabla \mathbf{v} \rangle$. Using similar arguments to section 2 it was found that the fluctuation level should be proportional to the quantity $(SV_0)^{-\frac{1}{2}}$, where $V_0$ is measured in Alfvén transit times, and this was verified. This would seem to imply that the dynamo effect is in some way affected by the rotation of the plasma. The effect of this flow is not essential, and simulations where the mean velocity is systematically eliminated at every time step still produce a dynamo effect, though one with fifty percent higher fluctuation levels and a smaller field reversal. So it appears that the role of the mean flow is one which assists the efficiency of the dynamo effect. The presence of a differential plasma rotation must therefore affect the phase between the fluctuations in an RFP. This was investigated by using a linear code to examine the effect of $V_0$ on the phase between the linear modes $\mathbf{v}$ and $\mathbf{b}$. It was found that the mean flow $V_0$ adjusted the phase between $\mathbf{v}$ and $\mathbf{b}$ by interacting mainly with the perturbed velocities.

It was also found that for high values of $P$ the viscous damping rate could be larger than the Ohmic heating rate. If it is assumed that viscous damping preferentially heats the ions and Ohmic heating heats the electrons, then anomalous ion heating could be explained by the viscous damping of the velocity fluctuations in an RFP.

4. Conclusions

- The quasi-linear approximation is inadequate to describe the physics in a reversed-field pinch.
• The cylindrically averaged velocity $V_0$ exists and modifies the growth and phase of linear instabilities.
• The dynamo effect is made more efficient by the presence of $V_0$ because of the induced phase shift between $\mathbf{v}$ and $\mathbf{b}$.
• The viscous heating rate can be greater than the Ohmic heating rate, and this could lead to anomalous ion heating via the viscous damping of the velocity perturbations.

Acknowledgements

One of the Authors (JPW) would like to thank Peter Kirby for the use of the simulation code MERCURY in this analysis and the SERC for the maintenance grant. It is also a pleasure to acknowledge discussions with Mike Bevir at the Culham Laboratory.

References

PLASMA IMPROVEMENT WITH ERGODIC MAGNETIC FIELDS IN JFT-2M


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Abstract

PLASMA IMPROVEMENT WITH ERGODIC MAGNETIC FIELDS IN JFT-2M.

Two significant effects of DC helical magnetic perturbations on MHD instabilities and on confinement properties have been found experimentally in JFT-2M. The m/n = 2/1 or m/n = 3/1 helical field has effectively suppressed the m/n = 2/1 locked mode growing during the plasma current ramp-up phase; as a result, disruptions caused by the locked mode have been avoided. The high n (n ~ 4 in our cases) and high m (m = 12-20) helical fields with resonant magnetic surfaces close to the separatrix have induced frequent edge localized modes (ELMs) and, therefore, reduced a density increase in the H-mode without large degradation of energy confinement.

1. INTRODUCTION

The effects of small amount of DC helical magnetic perturbations on MHD instabilities and on confinement properties are investigated systematically in the JFT-2M tokamak. To

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FIG. 1. Ergodic magnetic field coil systems in JFT-2M. Three types of ergodic magnetic coil systems are installed in JFT-2M. The main Fourier components for each coil system can be changed depending on the connections of the coil elements:

- **EML coil system**: $m = 3-7 \text{ or } 10-16$, $n = \text{even or odd}$.
- **Ladder coil system**: $m = 5 \text{ or } 10$, $n = 1, 2 \text{ or } 3$.
- **Saddle coil system**: $m = 2$, $n = 1 \text{ or } 2$.

Toroidal and poloidal mode numbers are denoted by $n$ and $m$, respectively.

compare the effects of different helical perturbations, three types of ergodic magnetic field coil systems are installed in JFT-2M as shown in Fig. 1. Two significant effects have been found and studied experimentally, related to (1) the avoidance of disruptions due to locked modes and (2) the realization of steady H-modes.

2. LOCKED MODE SUPPRESSION

Plasma disruptions can be caused by the $m/n = 2/1$ locked modes during the current ramp-up phase in tokamak devices. It has been found in JFT-2M that the application of an ergodic magnetic field can suppress this type of MHD activity. The time behavior of plasma current, $q_S$-value at the plasma surface and pick-up of magnetic fluctuations is shown in Fig. 2(a) for a disrupted discharge. Here the plasma disruption occurs at 400 ms because of an MHD instability that begins to grow when the $q$-value reaches a value of about 4. It has been found from the analysis of the magnetic pick-up signals that the main component of the MHD activities has an $m/n = 4/1$ helical structure at the
beginning of this instability but changes to $m/n = 2/1$ at an early stage. Following this structural change, the frequency decreases gradually from about 5 kHz with increasing amplitude. When the frequency reaches about zero and the fluctuating magnetic field reaches about 30 G, the plasma disrupts.

This type of instability, usually called locked mode, is suppressed by the helical magnetic perturbation of the Saddle ($m=2$, $n=1$) coil system; as a result, the disruption is avoided, as shown in Fig. 2(b). Here a Saddle coil DC current of 0.9 kA has been applied from 200 ms to 250 ms, after the rise of the instability. The $m/n = 2/1$ helical magnetic field is the main component in the Saddle ($m=2$, $n=1$) coil system; the radial field is about 10 G near the $q = 2$ surface, about 1% of the poloidal field. The amplitude of the magnetic fluctuations decreases by an order of magnitude with a 20% reduction of the frequency.

![Figure 2](image-url)

FIG. 2. Suppression of locked mode by helical magnetic perturbations in a limiter discharge. (a) Time behavior of plasma current, plasma surface $q$-value and magnetic fluctuation for the case of no Saddle ($m = 2$, $n = 1$) coil current. (b) Time behavior of above mentioned plasma parameters with Saddle ($m = 2$, $n = 1$) coil current. The MHD instability growing during the plasma current ramp-up phase is suppressed by the Saddle ($m = 2$, $n = 1$) coil DC current of 0.9 kA. The plasma parameters at plasma current flat-top are: toroidal field: 1 T, plasma current: 220 kA, horizontal minor radius: 0.33 m, elongation factor: 1.3. (c) Amplitudes of magnetic pick-up signals as functions of Saddle ($m = 2$, $n = 1$) coil current (closed circles) and Ladder ($m = 5$, $n = 1$) coil current (open circles). Since the suppression of the magnetic fluctuation does not depend on the current direction, our stabilization of the locked mode is not due to the error field compensation by the external helical field.
In DIII-D the locked mode could be suppressed by an error field compensation with an external coil, where the suppression depends on the direction of the applied field[1]. However, in our experiments the suppression of the \( m/n = 2/1 \) mode is not affected by the field direction, as shown in Fig 2(c), in analogy to stabilizations obtained in small tokamaks[2][3]. A significant reduction of the magnetic fluctuation is obtained when the absolute value of the coil currents exceeds 0.5 kA in the Saddle \(( m=2, n=1)\) coil system and 1.3 kA in the Ladder \(( m=5, n=1)\) coil system. The threshold radial field of the \( m/n = 2/1 \) component is estimated to be 5.3 G in the Saddle \(( m=2, n=1)\) coil system and 2.6 G in the Ladder \(( m=5, n=1)\) coil system; indeed they are in the same range. Our experiments confirm that other coil systems with the same range of \( m/n = 2/1 \) fields, e.g. EML \(( m=10-16, n=\text{odd})\), also have the same stabilizing effect. The \( m/n = 3/1 \) field also remains another possible candidate for the effective field.

3. CONTROL OF EDGE LOCALIZED MODE IN H-MODE

Plasma density and radiation loss increase continuously in an edge localized mode (ELM) -free H-mode, even without gas input. This plasma density rise has to be suppressed to realize steady H-mode operations. We have succeeded in an active density control during the H-mode with ergodic magnetic fields \((EML\ (m=10-16, n=\text{even})\) coils + Ladder \(( m=10, n=2)\) coils\), as shown in Fig. 3(a). The plasma is heated in a single null divertor configuration with tangential neutral beam injections of 1 MW from 600 ms to 800 ms. The electron temperature at \( r/a = 0.57 \) decreases gradually after the jump-up at the ELM-free H-transition. The ergodic magnetic field is applied from 700 ms, with a rise time of 50 ms. Many ELMs, indicated by \( \text{H}\alpha \) bursts, are induced in the flat-top of the ergodic magnetic field. The increases in density and the radiation loss are suppressed by this edge phenomenon. The temperature also stops decreasing and remains constant during the ELM phase. The steady H-mode is achieved with the help of these ELMs induced by the ergodic magnetic field.

In order to find out the helical components which effectively induce the ELMs, their repetition rates in different ergodic field coil operations have been compared. The repetition rate is high
Fig. 3. Steady H-mode with ergodic magnetic fields. (a) Time behavior of (1) line averaged electron density ($n_e$); (2) radiation loss power ($P_{\text{rad}}$); (3) electron temperature at $r/a = 0.57$ obtained by electron cyclotron emission (ECE); (4) emission of $H_\alpha$ line ($H_\alpha$); (5) H-factor; (6) EML ($m = 10-16$, $n = \text{even}$) coil current ($I_{\text{EML}}$); (7) Ladder ($m = 10$, $n = 2$) coil current ($I_{\text{Ladder}}$). The hydrogen plasma in the single null divertor configuration was heated by $H^0$ NBI of 1 MW, and an ELM-free H-mode was obtained. Frequent ELMs were induced by a combination of EML coil and Ladder coil fields; as a result, a steady H-mode was achieved. (b) Repetition rates of ELMs as functions of calculated $m/n = 16/4$ island width produced by a combination of EML ($m = 10-16$, $n = \text{even}$) coil field and Ladder ($m = 10$, $n = 2$) coil field. The Ladder coil currents are $+1.3$ kA in cases denoted by "+ Ladder (10/2)" and $-1.3$ kA in cases denoted by "-Ladder (10/2)". The EML coil currents are always $+5$ kA in this figure. The horizontal axis corresponds to the normalized magnetic island width due to the external $m/n = 16/4$ helical field. Here, a neutral beam of 1 MW was injected into the hydrogen divertor plasma. The $q$-value at 95\%$\Psi$ is $\sim 3.5.$
when the field of the positive current in the Ladder \((m=10, n=2)\) coil is added to the field of the EML \((m=10-16, n=\text{even})\) coil system and is almost zero when a field of the negative current in the Ladder coil is added. The repetition rate with only the EML coil system seems to be a little higher than that with the addition of the negative current Ladder coil. Thus, a density rise in the H-mode is effectively suppressed with an addition of a positive current in the Ladder coil. The repetition rate of ELMs is an increasing function of the \(m/n = 16/4\) magnetic island width estimated from a Fourier analysis of the ergodic magnetic field, as shown in Fig. 3(b). Same dependence is obtained for the \(m = 12-20\) islands with the \(n\)-value of 4. It means that the \(m/n = 12-20/4\) helical field is one of the possible candidates for the effective field inducing ELMs. These islands are produced on the \(q = 3-5\) surfaces close to the separatrix. We can find no obvious correlation between the repetition rate and the island width at a low \(n\)-value \((n = 1, 2 \text{ or } 3)\). Thus, the ELMs seem to be induced by the \(n \sim 4\) and high \(m \ (m = 12-20)\) helical fields whose resonant surfaces are close to the separatrix.

4. SUMMARY

Our experimental observations show that a small amount of helical field can affect instabilities. The \(m/n = 2/1\) or \(m/n = 3/1\) helical field effectively suppresses the \(m/n = 2/1\) locked mode growing during the plasma current ramp-up phase. The high \(n\) and high \(m\) helical fields with resonant magnetic surfaces close to the separatrix effectively induce frequent ELMs and, therefore, reduce the density increase in the H-mode.

References

T.N. TODD: Do you have integrated $\tilde{B}_p$ (perturbed poloidal field) data, or are your deductions based only upon $\tilde{B}_p$ measurements?

M. MORI: We used integrated $\tilde{B}_p$ data in our analysis.

T.N. TODD: Are you claiming suppression of the natural plasma $m/n = 2/1$ mode with the production of a locked mode by the external resonant magnetic perturbation, or without?

M. MORI: Our analysis shows that both the $m/n = 2/1$ field and the $m/n = 3/1$ field can suppress the $2/1$ mode. Here the $m/n = 3/1$ field is not a resonant perturbation. It is not clear whether there is production of a magnetic island (locked mode) by the external helical perturbation.

P.E. STOTT: What is the mechanism by which the externally applied $m = 2$, $n = 1$ field suppresses the growing unstable $m = 2$, $n = 1$ mode in the plasma?

M. MORI: We do not yet know.

P.E. STOTT: What is the phase of the applied field relative to the plasma instability?

M. MORI: We applied a DC field; therefore the magnetic island due to the instability rotates relative to the applied field.
OBSERVATION OF 'SPHERICAL TOKAMAK' PLASMAS IN THE START EXPERIMENT


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Abstract

OBSERVATION OF 'SPHERICAL TOKAMAK' PLASMAS IN THE START EXPERIMENT. The START (Small Tight Aspect Ratio Tokamak) experiment became operational in January 1991 and is the first device to provide information on the behaviour of hot tokamak plasmas at aspect ratios down to \( A = R/a \approx 1.3 \). This paper describes the START device and discusses the energy confinement time in this regime, the MHD properties of these naturally D-shaped, highly toroidal plasmas, and the general operational features of low aspect ratio devices.

1. The START Experiment
Description, Main Parameters and Diagnostics

The main parameters of the device are reported in [1]. A cut-away schematic of the START apparatus is shown in Fig. 1. The stainless steel jacketed poloidal field coils are situated inside a large aluminium vacuum tank. The central vacuum barrier is a stainless steel tube, clad on its vacuum side by a graphite limiter. The central copper rod of the eight-limb single-turn toroidal field magnet is situated inside the barrier. A small solenoid, wound round the copper central rod, was installed to produce external poloidal flux (up to 10 mV sec) to support the plasma current in the low aspect ratio plasma configuration. The pre-programmed pulse gas puffing is produced by four high-speed valves

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FIG. 1. Schematic of START.

TABLE I. MAIN PARAMETERS OF START DISCHARGES

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Major radius, R (cm)</td>
<td>18-22</td>
</tr>
<tr>
<td>Minor radius, a (cm)</td>
<td>13-17</td>
</tr>
<tr>
<td>Aspect ratio, A</td>
<td>1.3-1.5</td>
</tr>
<tr>
<td>Elongation, $\kappa$</td>
<td>1.3-2.0</td>
</tr>
<tr>
<td>Triangularity, $\delta$</td>
<td>0.2-0.4</td>
</tr>
<tr>
<td>Plasma current, $I_{pd}$ (kA)</td>
<td>80-200</td>
</tr>
<tr>
<td>Toroidal field, $B_{tot}$ (T)</td>
<td>0.4-0.6</td>
</tr>
<tr>
<td>Safety factor, $q_s$</td>
<td>6-15</td>
</tr>
<tr>
<td>Pulse duration, $\tau_{pol}$ (ms)</td>
<td>$\leq 18$</td>
</tr>
<tr>
<td>Electron density, $n_e$ (m$^{-3}$)</td>
<td>$(2-8) \times 10^{19}$</td>
</tr>
<tr>
<td>Electron temperature, $T_{e0}$ (keV)</td>
<td>0.3-1</td>
</tr>
<tr>
<td>Ion temperature, $T_{i0}$ (eV)</td>
<td>100-150</td>
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<tr>
<td>Energy confinement, $\tau_R$ (ms)</td>
<td>1.8 (for 100 kA)</td>
</tr>
<tr>
<td>$Z_{eff}$</td>
<td>2-3 (for 100 kA)</td>
</tr>
</tbody>
</table>
Equilibrium reconstruction of the poloidal flux structure
(a) breakdown  (b) before compression  (c) after compression

Evolution of electron temperature after compression (Thomson scattering)

FIG. 2. Low aspect ratio plasma formation in START.
positioned close to the plasma region, see Fig. 1. Stable shots with repeatable plasma parameters in a wide range of plasma currents, plasma densities and aspect ratios from $A = 1.3$ can be produced. Main parameters of START discharges are presented in Table I.

2. Experimental Results

Low-Aspect Ratio Plasma Formation

The first results from the START experiment were presented in [2]. Low aspect ratio plasmas are obtained in START using major radius compression of the initial plasma formed at higher aspect ratio, Fig. 2. The breakdown can be obtained with a loop voltage of about 30V, at a low toroidal field of 0.25T at the radius of the breakdown. It has been observed from light and soft x-ray emission that the breakdown phase is accompanied by the appearance of plasma around the induction coils and the quadrupole-null point Fig. 2(a). During the initial phase the plasma current rises to 50-80 kA. A small additional vertical field supports the position of plasma at this stage, Fig. 2(b). The plasma is then compressed to a low aspect ratio configuration, Fig. 2(c). The increase in C-III emission arises from the interaction of the plasma with the central graphite limiter during compression. The increase in current during compression is in good agreement with flux conservation taking into account an additional current rise produced by the external poloidal flux.

After compression, in the absence of an additional external flux supply, the plasma current decreases due to resistive decay and change in the current density profile. An additional flux produced by the solenoid can compensate this decay and support the current flat-top. A flux of 5mV sec was enough to produce a 5.5ms flat-top at 110kA, see Fig. 2. Figure 2 also shows the evolution of electron temperature profile after compression, as measured by Thomson scattering. In this shot, the temperature reaches 500eV at a plasma current of 80kA.

3. Energy Confinement in START

As START is the world's first hot tokamak operating at aspect ratios in the region from 1.3 to 2, it provides a unique opportunity to extend heating and confinement studies to a regime where toroidal effects are greatly enhanced. At low aspect ratio existing scaling laws diverge, with predicted energy confinement times varying by up to an order of magnitude. To test confinement predictions is one of the primary aims of the START experiment.

Estimates of the energy confinement time obtained from kinetic and diamagnetic energies and by simulations with the Tokamak Simulation
FIG. 3. Evolution of parameters in a typical START discharge.

Code TSC, suggest that this exceeds the predictions of scaling laws such as neo-Alcator and Merezhkin-Mukhovatov. For the typical 110kA discharge shown in Fig. 3, it is estimated that $\tau_E = 1.8 \pm 0.7$ msec during the flat-top, compared to the neo-Alcator value of 0.5msec.

Even in this low-current high $q (>10)$ regime the value of beta is quite high; estimates of the central beta give $(8 \pm 2)\%$. The average beta value ($\approx 2\%$) is about half the Troyon value. For the measured poloidal beta, $\beta_p = (0.54 \pm 0.17)$, a self-consistent neo-classical equilibrium code predicts a high bootstrap current fraction of $62\%$.

4. MHD Properties of Low-Aspect Ratio Tokamak Plasmas

Low aspect ratio tokamak plasmas are naturally elongated, the elongation being strongly dependent upon the current density profile. START plasmas with elongation up to 2 have been observed to be vertically stable in the absence of active feedback stabilisation.
However, attempts to increase the elongation by using a positive gradient of vertical field lead to vertical instability [4]. The speed of vertical movement is less than 100 m/sec due to passive stabilisation from the machine structure.

The improvement in both ideal and resistive stability in low aspect ratio tokamaks has been theoretically predicted in [5]. In the low aspect ratio phase, with \( A = (1.3 - 1.5) \), no major disruptions due to MHD resonance activity or strong gas puffing have been observed in 10 000 discharges over a considerable range of plasma current (60 - 220kA) and density \( (\bar{n}_e \approx 1 - 8 \times 10^{19} \ m^{-3}) \) [4].

A typical START discharge is stable with low level MHD activity, see Fig. 3. An increase in filling pressure may cause the appearance of sawtooth-like activity. After a series of sawteeth, a large reconnection can appear. This is accompanied by a positive spike on the plasma current which can be up to 50%. This can be explained by a flattening of the current density profile assuming flux conservation. An accompanying increase in the plasma elongation (to 2.0) supports this explanation, since the vertical field decay index remains constant.

A sufficient increase in filling pressure causes the appearance of a strong perturbation of the plasma shape with a boundary perturbation up to 40%. Free boundary external kink mode simulations show a similar form of surface perturbation. The observed non-linear saturation of this large amplitude kink mode may be related to the absence of the major disruption at high density in START.

5. Plasma Exhaust in START

The low aspect ratio configuration produces a "natural divertor" effect, as shown in Fig. 2c. From the confinement zone outwards, the scrape-off layer is divided into three regions by the last closed magnetic surface touching the central column, the magnetic separatrix and the open flux surface tangent to the centre column. Outside the latter is a region of very long connection length limited in the vertical direction only by the horizontal diagnostic frames. The scrape-off thickness at the outside mid-plane is such that plasma escapes into all three regions, the last of which forms the exhaust plume seen on photographs in visible light and which probe measurements suggest may carry about half of the particle and power exhaust. Internal reconnections generate large increases in the particle exhaust in the scrape-off layer lasting for \(~100\mu s\).
6. Conclusion

Results from the START device have shown that hot plasmas with good confinement can be obtained at low aspect ratio. These plasmas are resilient to major disruptions, and much of their power and particle exhaust is carried by the observed natural exhaust plume.

Acknowledgement

This work was funded jointly by the United Kingdom Department of Trade and Industry, and Euratom.

REFERENCES


DISCUSSION

M.E. MAUEL: In your experiments and in your stability calculations, what was the value of the central safety factor q(0)?

M. GRYAZNEVICH: In some shots we had ‘snake type’ activity, so a c = 1 surface was present, and q(0) was ≤ 1. This ‘snake’ disappeared after the internal reconnection event. In our calculations we took q(0) < 1 and q(0) > 1, as q(0) could vary in the experiment.
FIRST RESULTS ON THE RFX REVERSED FIELD PINCH EXPERIMENT

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Abstract

FIRST RESULTS ON THE RFX REVERSED FIELD PINCH EXPERIMENT.

Initial results obtained on the RFX Reversed Field Pinch experiment are presented. Plasma currents beyond 0.5 MA with pulse lengths up to ~ 80 ms have been obtained by using 4 of the 15 available volt-seconds. The plasma temperature on axis is ~ 0.5 keV with an average density of $2 \times 10^{19} \text{ m}^{-3}$, corresponding to $\beta_0 \leq 0.1$ and global energy confinement times of up to ~ 1 ms.

1. INTRODUCTION

The first plasma was obtained 21 November 1991 on RFX [1, 2], the large European Reversed Field Pinch (RFP) experiment designed to operate up to 2 MA plasma current. The main parameters of the experiment and the target plasma parameters expected in RFX at full performance are listed in Table I.

The initial cleaning and the conditioning techniques of the graphite first wall included hydrogen glow and pulsed discharge cleaning as well as baking up to 250°C. On 28 January 1992, during baking, a diagnostic window broke, imploded, and its fragments scattered throughout the vacuum vessel. Because of the small dimensions of the access ports which characterize this high quality magnetic configuration experiment, the removal of the glass fragments had to be carried out by means of a remote handling system [3].

The experiments were resumed in June 1992. Both baking at 350°C and the use of glow discharge cleaning [4] in helium were effective in controlling the graphite hydrogen retention and recycling. The first RFP plasma was obtained on 14 August 1992.

This paper presents the initial results obtained by using up to 4 V·s and without using the power supplies to sustain the plasma current and the reversed toroidal field at the wall.

Preliminary results on plasma startup and RFP formation, density behaviour, equilibrium and field error control, impurities and radiation losses, magnetic fluctuations, edge plasma properties and electron and ion temperature measurements are discussed in the following sections.
TABLE I. RFX PARAMETERS

<table>
<thead>
<tr>
<th>Experiment</th>
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<tbody>
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<td>Major radius</td>
</tr>
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</tr>
<tr>
<td>Flux swing</td>
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<tr>
<td>Maximum applied toroidal field</td>
</tr>
<tr>
<td>Inductive stored energy</td>
</tr>
<tr>
<td>Capacitive stored energy</td>
</tr>
</tbody>
</table>

Target plasma parameters

<table>
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</tr>
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<tbody>
<tr>
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<td>$2 \times 10^{-14}$ A·m</td>
</tr>
<tr>
<td>I</td>
<td>2 MA</td>
</tr>
<tr>
<td>$\tau_E$</td>
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<tr>
<td>$T(0)$</td>
<td>1 keV</td>
</tr>
<tr>
<td>$\langle n \rangle$</td>
<td>$10^{20}$ m$^{-3}$</td>
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</tbody>
</table>

2. STARTUP AND RFP FORMATION

The formation phase of a reversed field pinch configuration is in general quite turbulent because the plasma is forced to pass through MHD unstable states characterized by values of the safety factor at the edge, $q_a$, in the range $0 < q_a < 1$. The initial operation of RFX has shown the importance of rapidly passing through this phase.

Rather flexible toroidal field programming can be achieved by the 4.8 MJ, 7 kV capacitor bank [5] which can be separated into six independent sections and is used both to establish the bias toroidal field and to control the field reversal at the plasma edge.

In Fig. 1 typical waveforms of applied loop voltage, plasma current, toroidal field at the wall and toroidal flux are shown. The RFP is obtained by driving the toroidal field negative at the wall during the current rise phase. Once field reversal is achieved, the toroidal flux is amplified and sustained by the plasma. Initially the current increases rapidly, then pauses, decreasing somewhat during the reversal phase and increases again up to its maximum. The first peak in the current corresponds to $0.2 \leq q_a \leq 0.3$ as is shown in Fig. 2, where $q_a$ is represented as a function of the first current peak.
FIG. 1. Example of loop voltage, toroidal plasma current, toroidal field at the graphite surface and toroidal flux waveforms.

FIG. 2. Safety factor at the graphite surface, $q_a$, at the time of the first current peak plotted as a function of the corresponding values of the plasma current.
Poloidal mode analysis and preliminary results with a reduced set of pick-up coils in the toroidal direction show the onset and the decay of resonant low $n$ instabilities with dominant poloidal periodicities $m = 0, 1, 2$. In particular, when $q_a = 0.2-0.3$, several modes of significant amplitude are simultaneously present which could be responsible for enhanced plasma resistance and wall interaction.

Observations of the inner wall with a CCD camera confirm the occurrence of strong interactions in the area of the shell vertical insulating gaps.

A fairly resistive startup is a characteristic of the RFP formation. However, initial optimization of the toroidal field programming has led to rapid progress; plasma current waveforms obtained during this initial operational phase are shown in Fig. 3.

3. WALL CONDITIONING AND DENSITY CONTROL

The main part of the initial parameter optimization work has been dedicated to wall conditioning and plasma fuelling as dictated by the full graphite first wall of the machine. Indeed, not surprisingly, we have observed greatly different density behaviour depending on the inventory of H loaded into the first wall. In particular, the wall can be ‘emptied’ by baking at 350°C or by glow discharge in helium, and heavily ‘loaded’ by glow or pulsed discharge cleaning in hydrogen.

We have recovered from the air vented by the window accident with two periods of baking at 350°C of approximately 24 and 36 h, respectively. No windows were installed on the vessel for this operation. The diagnostic windows were then mounted, and we presently operate with the vessel at ambient temperature until flanges with

![FIG. 3. Plasma current waveforms for different toroidal field programms.](image-url)
modified glass sealing are mounted. Therefore, wall conditioning has been carried out only by H or He glow discharge cleaning.

Depending on the H inventory in the graphite we operate either with only an initial H filling pressure of \((0.2-1.2) \times 10^{-3}\) mbar or also with gas puffing by a preprogrammed electromagnetic valve with equivalent rates of \(\leq 5 \times 10^{18}\) m\(^{-3}\) particles per millisecond. The best performances have been obtained when the current to line density ratio, \(I/N\), is relatively low, i.e. in the \((2-4) \times 10^{-14}\) A\(\cdot\)m range. When plasma position and field errors are well controlled, the density profiles measured by a two colour CO\(_2\) interferometer [6] are usually fairly flat. The density versus time behaviour measured along four vertical chords is shown in Fig. 4.
4. EQUILIBRIUM AND FIELD ERROR CONTROL

In RFX the plasma equilibrium is in part passively controlled by the thick shell (6.5 cm aluminium, with a time constant of ~400 ms for the penetration of the vertical magnetic field) which surrounds the vacuum vessel and in part can be controlled by means of 16 external coils (hereinafter named field shaping winding (FSW)) which can be used to produce a bias vertical field and to provide an active control of the field error at the two poloidal shell gaps during the pulse.

Initial experiments on equilibrium control have been performed by applying a small bias vertical field (~5 mT) in order to centre the plasma within the shell. A significant improvement was obtained in plasma current and pulse length as is shown in Fig. 5. A comparison of the displacement of the circular magnetic surface tangential to the first wall for a discharge without applied vertical field (No. 1075) and one with a bias vertical field of ~5 mT (No. 1076) is shown in Fig. 6.

![Fig. 5](image1.png)

**FIG. 5.** Comparison of plasma current waveforms for two consecutive shots: No. 1075 without bias vertical field; No. 1076 with a 5 mT bias vertical field.

![Fig. 6](image2.png)

**FIG. 6.** Plasma displacement for shots with (Nos 1076 and 1208) and without (No. 1075) vertical field control.
In order to reduce the field errors at the poloidal gaps during the discharge various waveforms for the FSW amplifiers have been tested. Figure 7 shows the field error at the gap, $\delta B_g$, normalized to the average poloidal field at the shell, $\langle B_p \rangle$, for a typical discharge (No. 1099) without active FSW control and for three different waveforms of the currents in the FSW. The quantity $\delta B_g$ is obtained by subtracting the imposed bias vertical field from the average of the measurements provided by the saddle coils mounted on the inside surface of the shell (top and bottom) and spanning the two poloidal gaps. The positive values of the curve represent an excess of external vertical field with respect to the value inside the shell associated with the plasma. On the other hand, the negative values are related to a lack of external field. A large negative field error is found in shot No. 1099, where the FSW currents decay without any active control. For shot No. 1105 the power supplies were programmed to reduce the positive field error during plasma current rise, while the programming for shot No. 1120 aimed at influencing the subsequent field error time evolution. Shot No. 1208 is a trade-off between the previous cases in terms of field error during the RFP phase and represents the best results achieved up to now. The plasma displacement for shot No. 1208 is also shown in Fig. 6.

5. IMPURITY MEASUREMENTS AND RADIATION LOSSES

Observations of the impurity behaviour on RFX have been carried out by means of four spectrometers, two of which are absolutely calibrated in the visible and UV ranges, covering the wavelength spectrum from 10 to 8000 Å. Carbon, oxygen and nitrogen are the main impurities in the RFX plasma, with carbon clearly predominant, while no evidence for the presence of metals has been found.
Figure 8 shows two typical emission spectra for a 0.5 MA discharge taken with a Schwob–Fraenkel grazing incidence [7] and a survey (SPRED) [8] spectrometer, respectively.

Typical values for the impurity content in a shot with an average electron density of $2 \times 10^{19}$ m$^{-3}$ and a current of 0.4 MA are $3 \times 10^{18}$ m$^{-3}$ of carbon, $2 \times 10^{17}$ m$^{-3}$ of oxygen and $2 \times 10^{16}$ m$^{-3}$ of nitrogen, corresponding to
FIG. 9. Plasma current and ratio between total radiated power and Ohmic input power.

FIG. 10. Amplitude of magnetic fluctuations, normalized to the magnetic field at the wall, versus time. The toroidal current waveform is also shown.

FIG. 11. Poloidal mode spectrum for the poloidal field component normalized to the total magnetic field at the wall.
The carbon content appears to increase with the peak value of plasma current and to be determined by processes occurring during the initial phase of the discharge, while, within the precision of the measurements, it remains constant during the RFP phase.

The total radiated power, $P_{rad}$, has been measured by a miniaturized multichannel metal film bolometer [9] with 1 kHz bandwidth that looks at the plasma along a central chord. A typical waveform of the fraction $P_{rad}/P_{ohm}$, where $P_{ohm}$ is the Ohmic input power, is shown in Fig. 9, from which it can be seen that $P_{rad}/P_{ohm}$, after an initial peak during the set-up phase of the configuration, decreases once the plasma has reached the RFP state and remains $\leq 20\%$ until the current terminates.

6. MAGNETIC FLUCTUATIONS

A series of pick-up coils [10] are positioned on the inner surface of the shell. All signals are analogically filtered at 5 kHz and sampled at 10 kHz. The measured fluctuation power spectrum decays as the inverse of the square of the frequency in the range $100 \text{ Hz} < f < 2-3 \text{ kHz}$.

In Fig. 10 the fluctuation amplitude is reported as a function of time for a 0.5 MA discharge. Data shown are root mean square values averaged over a 5 ms time interval and normalized to the total magnetic field at the wall. In general, the fluctuation amplitude decreases during the current sustainment phase. On the assumption of a dominant mode $m = 1$, $n = 10$, the attenuation due to the vacuum vessel is estimated to be of the order of 2. Thus the corresponding $b/B$ just inside the vacuum vessel for the frequency range considered is estimated to be as low as $\sim 0.1\%$ for the poloidal component and $\sim 0.4\%$ for the toroidal component.

In a vertical section 16 probes are used to measure the poloidal mode power spectrum for the poloidal field component. Figure 11 shows a typical spectrum measured during the current sustainment phase for a series of discharges at 0.5 MA. Most of the power is concentrated in the $m = 0, 1, 2$ modes. Preliminary results of the toroidal mode analysis indicate that the toroidal spectrum is peaked approximately around $n = 10$ during the sustainment phase.

7. EDGE PLASMA PROPERTIES

A movable mushroom shaped graphite limiter [11] has been inserted into the plasma edge through a 150 mm diameter horizontal port. It is instrumented with Langmuir and heat flux probes and can rotate to search for directional effects. The measured angular dependence of the power deposition is in agreement with the presence of a fast electron stream aligned with the local magnetic field, with a factor of up to six in the asymmetry between the temperature increment measured on the electron and on the ion drift sides [12]. There is a steep onset of the asymmetry when
the limiter tip is withdrawn 3 mm from the surface of the graphite tiles. At the same position the floating potential of the limiter changes sign. The energy flux density \( q_1 \) and the particle flux density \( \Gamma_1 \) have been calculated by using the temperature increment of thermal sensors and the ion saturation current of the Langmuir probes, respectively. The characteristic decay lengths in the shadow of the graphite and on the ion drift side are \( \lambda_q \approx 4 \) mm and \( \lambda_T \approx 6 \) mm, while the electron temperature profile is flatter, with \( T_e \approx 15 \) eV close to the wall. The measured values of \( q_1 \) and \( \Gamma_1 \) are in reasonable agreement when the expression \( q_1 = \gamma \Gamma_1 k T_e \) is used with \( \gamma \approx 10 \). The total radial particle flux density (including impurities) is calculated from the particle balance equation \( \Gamma_\perp \approx \Gamma_T \lambda_T / L \), with \( L \) equal to half the post diameter, and gives \( \Gamma_\perp \approx (0.5) \times 10^{22} \) m\(^{-2}\) s\(^{-1}\) at the plasma surface (the uncertainty is mainly due to the actual collection surface).

8. ELECTRON AND ION TEMPERATURE MEASUREMENTS

The electron temperature has been measured with a four diode Si(Li) detector, while the ion temperature has been monitored by means of a 4 m long time of flight (TOF) neutral particle analyser and the Doppler broadening evaluation of impurity lines, i.e. C V 2272 Å and 2278 Å, C III 2296 Å and O VII 1623 Å. The latter data are yielded by the 1.3 m vacuum Czerny–Turner equipped with a FOMA detector capable of recording a whole reticon diode array in 0.25 ms [13]. Figure 12 reports the time evolution of \( T_e \) and \( T_i \) — from TOF and the Doppler broadened O VII line — in a 0.5 MA discharge. The electron temperature has also been measured from the intensity of the intercombination to the resonant line ratios of the helium like

![FIG. 12. Time evolution of \( T_e \) from Si(Li) detector, and \( T_i \) — from TOF and the Doppler broadened O VII line — in a 0.5 MA discharge.](image-url)
sequences of carbon and oxygen. Since the measurements are chord integrated, the plasma radius to be associated with the temperature measurements has been deduced by means of a collisional–radiative, one-dimensional impurity transport code computing the emission radial profile of the monitored lines [14]. Analogously, the TOF data have been analysed by means of a Monte Carlo transport code [15], which evaluates the opacity effects of the plasma over the escaping neutral particles. In the transport codes a fairly peaked temperature profile had to be used to order the experimental data along the plasma radius consistently. A summary of the above results is shown in Fig. 13, where the measured data are plotted versus the plasma radius and the point given by the Si(Li) detector is considered pertaining to the centre of the plasma.
In Fig. 14 the values of poloidal beta, $\beta_p$, computed on the assumption of the above temperature profiles and fairly flat density profiles as indicated by the interferometer data, are shown as a function of I/N. Though the I/N parameter ranges over a relatively small interval, from $2 \text{ to } 6 \times 10^{-14} \text{ A} \cdot \text{m}$, the obtained $\beta_p$ values confirm (within the uncertainties due to the profile information available) the well known dependence according to which the best confined RFP plasmas are those associated with low I/N values. The corresponding estimate of the global energy confinement time gives values of up to $\sim 1.0 \text{ ms}$.

Hydrogen influx during the discharge has been estimated from the Balmer-$\alpha$ line filter monitors. Influxes of the order of $3 \times 10^{21} \text{ m}^{-2} \cdot \text{s}^{-1}$ have been found, from which, on the assumption of a steady state balance with the outflux, particle confinement times of 0.5–1 ms may be inferred.

REFERENCES


DISCUSSION

H.A.B. BODIN: The Padova Group should be congratulated on reaching such interesting and encouraging parameters in their preliminary experiments at relatively low currents. My question is — can you say more about your planned schedule of operation up to 2 MA (1 MA in 1993, you say) and can the RFX machine reach even higher currents (>2 MA)?

S. ORTOLANI: The plan is to study reversed field pinch plasmas at I ~ 1 MA in 1993. Subsequently, we will increase the plasma current up to 2 MA. It seems to me premature to comment on whether RFX will operate at even higher current, but it is certainly a possibility.
RESONANT LOSS AND MHD EFFECTS ON PERPENDICULARLY INJECTED FAST IONS IN HELIOTRON E

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Abstract

RESONANT LOSS AND MHD EFFECTS ON PERPENDICULARLY INJECTED FAST IONS IN HELIOTRON E.

Neutral particle energy analysers were used to study the behaviour of trapped ions injected perpendicularly to the magnetic axis in Heliotron E. It is found that the horizontal distribution of the charge exchange flux with $E \sim E_{inj}$ is shifted inward by 10-15% of the minor radius, which is consistent with the toroidal effect on orbits of deeply trapped ions. A significant depletion of fast ions was observed in the energy range of $E_{inj}/3 \leq E \leq E_{inj}$. The effects of limiter insertion, pellet injection and magnetic configuration on the depletion are examined. An $E_r$ induced loss mechanism for the depletion is discussed. The effects of the pressure driven instability on the slowing-down process of trapped ions were studied. Depletion, enhancement and sawtooth-like oscillations of the charge exchange flux below $E_{inj}$ were seen in relatively low beta limiter plasmas. Some mechanisms for this sudden change in the spectrum are discussed.

1. INTRODUCTION

In order to study the confinement properties of energetic trapped ions, two charge exchange (CX) analysers (NPA) were used in Heliotron E. The energetic particles ($E_{inj} \leq 26$ keV) were injected perpendicularly to the magnetic axis by neutral beam injectors. One NPA was installed on the top of the device and scanned poloidally to study the toroidal effect on orbits of deeply trapped particles. The other NPA was installed at a different toroidal location and used to investigate the slowing-down spectrum along the major radius in the equatorial plane. A substantial depletion of the perpendicular slowing-down spectrum between $E_{inj}/3$ and $E_{inj}$ was first observed in a beam heated limiter plasma. The effects of magnetic configuration and pellet injection on the depletion were also studied. Depending on the limiter position, the

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effects of the MHD activity on the slowing-down spectrum were clearly observed. During a sawtooth crash phase, sudden enhancement and drop of CX signals were measured at two adjacent energies. Even in a more MHD stable sawtooth free plasma or during two consecutive sawtooth events, periodic oscillations of the CX flux were observed.

2. HORIZONTAL SHIFT OF TRAPPED ION DISTRIBUTION ALONG THE MAJOR RADIUS

A low density plasma ($n_e \sim 0.4 \times 10^{13}$ cm$^{-3}$) is initiated by second harmonic ECH at $B_0 = 0.94$ T, and test particles (23 keV) are injected for about 15 ms during the ECH pulse ($\sim 60$ ms). An NPA is located in the toroidal direction at about 90° from the beam injectors. By scanning this NPA poloidally in the meridian plane the horizontal distribution of the ions with $E \sim E_{\text{inj}}$ is measured. Since the sight lines of the NPA are almost perpendicular to the magnetic field line, only deeply trapped particles can be detected. Figure 1 shows that the observed distribution of $\Psi_{\text{CX}}(\theta)$ is shifted inward by about 10–15% of the minor radius ($a$), which agrees well with the orbit theory stating that the drift surface is shifted by $\Delta \psi / (a) = \epsilon_{a} / 2 \epsilon_{h} \sim 15\%$ for confining $v_t = 0$ particles [1]. Here $\epsilon_{a}$ and $\epsilon_{h}$ are the toroidal and helical ripples at $r = a$, respectively.

The effects of the magnetic axis shift, $\Delta_v = R_{\text{axis}} - R_0$, on $\Psi_{\text{CX}}(\theta)$ are also examined by applying an additional vertical field. It is found that $\Psi_{\text{CX}}(\theta)$ is almost independent of $\Delta_v (-8 \text{ cm} < \Delta_v < +4 \text{ cm})$. This result can be understood from the fact that the orbits of deeply trapped particles are determined by the minimum-B contour and are insensitive to the magnetic surface. For the extremely outward shift
case of $\Delta_n = +6.5$ cm, fast ions were also detected along the inside sight line ($R < R_o$) far from the inside edge of the plasma. The decay time of $\psi_{\text{CX}}$ after the beam had been turned off was fairly short (less than 1 ms). This suggests that deeply trapped ions travel on orbits beyond the plasma boundary, but confinement is dominated by CX loss in the region of high neutral density outside the plasma.

3. RESONANT LOSS CONE STRUCTURE ON THE HIGH ENERGY SPECTRUM

The second NPA was located toroidally at 90° and 180° from the perpendicular beam injectors and used to study the slowing-down spectrum. A significant depletion of fast ions was observed for several cases. The clearest case was in a limiter plasma ($nf < 6 \times 10^{15}$ cm$^{-2}$, $T_e \lesssim 500$ eV and $B_0 \approx 1.9$ T). As a carbon rail limiter was inserted at $Z_{\text{lim}}$ below the torus midplane, the depletion of the fast ions was clearly seen in the energy range of $E_{\text{inj}}/3 < E < E_{\text{inj}}$ ($\sim 26$ keV), as is shown in Fig. 2(a).

During the temperature recovering phase after pellet injection, a similar depletion of the slowing-down spectrum was observed (Fig. 2(b)). In the former case, $H_\alpha$ decreased as the limiter was inserted further, suggesting that the CX loss is not responsible for this depletion [2]. Since the line density was kept almost constant and the volume averaged temperature ($T_e$) was slightly reduced, flattening of the slowing-down spectrum by stronger electron drag effects could be expected. Actually, even for the case of $Z_{\text{lim}} = 20$ cm, no depletion was observed in a 'cold' high density plasma. In the latter case, the limiter was not inserted ($Z_{\text{lim}} > 30$ cm).
FIG. 3. (a) Expanded view of one of the sawtooth oscillations in a biased plasma. Burst and delayed sawtooth oscillations are observed at $E = 16.4$ keV, 13.7 keV and 11.4 keV, respectively. (b) Expanded view of two successive sawtooth events. Phase inversion is seen between fluxes at $E = 16.4$ keV and 13.7 keV.
Before pellet injection, there was no depletion in the energy range of interest. The depletion occurs at $E = 10$ keV at injection and varies in time, and then the spectrum recovers the pre-injection spectrum. By comparison with the former, the limiter position is not essential for the depletion. During this phase ($T_e$) increases gradually, resulting in weaker electron drag effects. Thus, from these two facts it is concluded that this depletion of fast ions is not caused by CX loss, geometrical intersection between the limiter and specified orbits or change in the electron drag. We interpret this depletion to be consistent with the $E_r$ induced loss cone structure [3]. By simple calculations assuming parabolic profiles of fast ion density and potential and an exponential neutral density profile, the observed depletion can be explained if a negative potential ($-2$ to $-1$ keV) exists [4]. The $\Delta_r$ effect on the depletion is also studied. By shifting the axis by $\Delta_r = -2$ cm, at which the trapped particles are well confined, the depletion energy decreased from 10 keV to 7 keV ($\Delta_r = 0$ cm), and the degree of depletion was reduced by a factor of 10. This tendency is consistent with the orbit calculations including both $E_r$ and $\Delta_r$ effects [5].

4. BURSTS OF ENERGETIC ION FLUX CORRELATED WITH MHD INSTABILITIES

In the standard configuration without limiter, a plasma with a peaked density profile is stable for a central beta value $\beta(0)$ below $\sim 1\%$ [6]. For $16$ cm $< Z_{\text{lim}} < 22$ cm ($0.87 < \iota_{\text{lim}} < 1.33$), however, sawtooth oscillations appeared around $\beta(0)$ of 0.4%, and significant MHD effects on the CX flux were only observed in the high energy range. When the limiter was biased with respect to the chamber [7], both the sawtooth behaviour (the phase inversion radius, $\langle r \rangle_{\text{inv}}$, of the soft X ray (SX) and the relative amplitude change, $\Delta A / A$) and the high energy slowing-down spectrum were varied. For $Z_{\text{lim}} = 20$ cm, $\langle r \rangle_{\text{inv}}$ was changed from 11 cm ($V_B = 0$) to 13 cm ($V_B = -180$ V) and roughly corresponded to $\iota_s = 4/5$ and $\iota_s = 1$, respectively. Figure 3(a) shows a typical example. The bias voltage was $-180$ V. A burst of the CX flux at $E = 16.4$ keV was observed at $t \sim 361$ ms during the Mirnov signal having a 10 kHz oscillation, and the flux was kept at a lower level after the burst. In some cases, a depletion of flux was seen at a different energy. The sampling time was 200 $\mu$s for this CX flux measurement. Then the CX flux at $E = 11.4$ keV (13.7 keV) exhibited ‘normal’ (‘inverted’) sawtooth oscillations, that is, a phase inversion occurred in the velocity space. If both ions with two different energies are transported radially towards the region of higher neutral density near the edge, the observed signals should show an ‘inverted’ sawtooth. For $V_B = 0$, $\psi_{\text{CX}}$ at $E = 25$ keV showed a rapid spike at the sawtooth crash, but $\psi_{\text{CX}}$ from 5.8 keV to 22 keV did not show any sawtooth oscillations. Thus, bursts, depletions and sawtooth oscillations of the CX flux are observed, presumably depending on $\iota_s$ in MHD unstable plasmas.
If the \( \mu \) conservation law is valid at the crash, the energy spectrum may be downshifted because fast ions are transported radially from the high B region. The sudden change in the \( E_r \) profile associated with MHD instabilities may also shift the spectrum up/down, depending on the radial profile. When both mechanisms act on a spectrum with a loss cone structure, enhancement and depletion of fluxes at different energies may be observed. The gradually varying sawtooth oscillations of fluxes at two adjacent energies are difficult to explain. Radial variation in the energy dependent loss cone structure and radial transport of fast ions may cause phase inversion of fluxes at different energies. Figure 3(b) shows sawtooth-like oscillations of CX fluxes between two successive sawtooth events. These oscillations do not seem to be correlated with \( B_e \), SX electron density or any other edge parameters. Similar oscillations are also observed in an MHD stable plasma \( (Z_{\text{lim}} = 24 \text{ cm}) \) and in a plasma with sinusoidal density fluctuations, \( n_e/n_e \sim 6\% \) \( (Z_{\text{lim}} = 14 \text{ cm}) \). A further investigation of sawtooth-like oscillations is needed.

5. CONCLUSIONS

The study of fast ion trajectories, significant depletion of the slowing-down spectrum, and bursts of the CX flux associated with MHD activity was performed by two NPAs in perpendicularly beam injected plasmas. The inward shift of the observed horizontal flux distribution is in good agreement with the toroidal effect on the orbits of deeply trapped particles. A significant depletion of the spectrum was observed. An \( E_r \) induced loss mechanism for this depletion is the most plausible explanation. Effects of MHD activity on the slowing-down spectrum were observed in relatively low beta limiter plasmas. The effects of radial transport of fast ions and \( E_r \) profile change at the sawtooth crash were discussed in connection with depletion and enhancement of the CX flux. Sawtooth-like oscillations of the flux were observed even in an MHD stable plasma or between two successive sawtooth events. These do not seem to be directly correlated with MHD activity, including enhancement or drop of neutral density.

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The authors wish to express their appreciation to the operating staff of Heliotron E.

REFERENCES


DESIGN STUDY OF LHD HELICAL DIVERTOR 
AND HIGH TEMPERATURE 
DIVERTOR PLASMA OPERATION

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Abstract

DESIGN STUDY OF LHD HELICAL DIVERTOR AND HIGH TEMPERATURE DIVERTOR PLASMA OPERATION.

The Large Helical Device (LHD) now under construction is a heliotron/torsatron device with a closed divertor system. The LHD divertor magnetic structure has been studied in detail. A peculiar feature of the configuration is the existence of edge surface layers, a complicated three-dimensional magnetic structure, which does not, however, seem to hamper the expected divertor functions. As a confinement improvement scheme in LHD, high temperature divertor plasma operation has been proposed in which a divertor plasma with a temperature of a few keV, generated by efficient pumping, leads to confinement improvement.

1. INTRODUCTION

With the inherent advantage of the stellarator as an attractive steady state reactor, there has been growing interest in the stellarator. We are constructing a large superconducting heliotron/torsatron device (B = 4T, R = 3.9 m), called the Large Helical Device (LHD), aiming at demonstrating the attractiveness of the helical device at more reactor relevant plasma parameters [1]. A built-in divertor configuration exists for heliotron/torsatron devices. This advantage has not been explored in any existing helical device, and the LHD device will be the first helical device to demonstrate various divertor functions such as impurity control and enhancement in the energy confinement. The LHD divertor configuration should be as flexible as possible so as to be able to accommodate a wide range of operational scenarios of the divertor. To this end, we have designed a large vacuum vessel for the installation of closed divertor chambers of a reasonable size.

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2. FEATURES OF THE EDGE MAGNETIC CONFIGURATION

The magnetic topology and the associated divertor plasma behaviour needs to be understood theoretically before the LHD divertor system is designed. A helical divertor magnetic configuration in LHD is depicted in Fig. 1(a),(b). A closed surface region is surrounded by a stochastic region generated by overlapping of the islands (n = 10) which somewhat naturally exist in the outer region. The field lines escaping from this region pass through thin, curved surface layers before reaching the X-point and then the divertor plate. The existence of edge surface layers is a peculiar feature of this type of helical divertor [2]. A surface layer consists of several layers, each of which again consists of multiple layers. Such a structure is generated by successive folding and stretching processes as the field lines rotate poloidally. The former process occurs because the radial position of the X-point changes with poloidal angle as much as ~1/3 of the plasma radius. The latter is caused by the high local rotational transform and shear at the edge on the larger major radius side of the torus.

**FIG. 1.** (a) Poloidal cross-section (θ = 0°) of LHD helical divertor. (b) LHD edge magnetic configuration (θ = 18°). (c) Puncture plots of field lines in a helical plane (θ = -(5θ + θ₀ + 0.1 \times \sin (5θ + θ₀))) in the divertor chamber (shown in (a)) for three different random walk parameters.
To study the structure of the divertor channel, the field lines are traced from the stochastic region until reaching a helical plane ( \( \theta = -[5\phi + \theta_0 + 0.1 \sin(5\phi + \theta_0)] \) ) rotating with helical coils. In the real device, the divertor plates are located at \( r = 1.55 \) m near this plane. The effect of the perpendicular plasma transport is taken into account by field line tracing with a random walk process (at every 0.2 m step, positions of the field lines deviate by \( \delta \) in the plane perpendicular to the field lines with random azimuthal angles). Puncture plots of the field lines in this plane are shown in Fig. 1(c). A strong poloidal asymmetry is seen in the plots. The fine structure of the edge surface layer is clearly seen when it is traced exactly, i.e. \( \delta = 0 \). For a random walk process with \( \delta = 1.2 \) mm which corresponds to an effective perpendicular diffusion coefficient of 0.5 m\(^2\)/s for a plasma with a temperature of 100 eV, i.e. a value that is smaller than a typically observed edge value, the fine structure is smoothed out completely. We expect that the distributions of the heat and particle fluxes in the plane for the LHD experiments are close to those with \( \delta = 1.2 \) mm. On the basis of this estimate, the maximum heat flux on the divertor plate with a field line incident angle of 3 CP is expected to be \( \sim 5 \text{ MWm}^{-2} \) for a standard 20 MW LHD discharge.

Divertor operation with a high density, cold divertor plasma [3,4] can reduce impurity sputtering and enhance edge radiation, a promising boundary control which we plan to pursue in the LHD experiments. The vague boundary discussed above has poor confinement properties and thus may provide a cold, radiative edge volume reducing the heat flux on the divertor plate [4], but may, in turn, prevent the formation of an H-mode edge temperature pedestal, which leads to an improvement of core energy confinement [5]. This has led us to propose a high temperature divertor plasma operation, as will be discussed in the following section.

3. HIGH TEMPERATURE DIVERTOR PLASMA OPERATION

High temperature divertor plasma operation has been proposed to improve the energy confinement in helical devices as well as in tokamaks [6]. In this operational mode, the divertor plasma temperature is raised by efficient pumping in the divertor chamber. An elevated divertor temperature will lead to an improvement of the core plasma, as observed in H-mode discharges.

The divertor temperature \( (T_{\text{div}}) \) is estimated from power balance in the divertor channel. We consider a steady state discharge heated \( (Q_{\text{in}} \text{ (input power)}) \) and fuelled \( (\Gamma_{\text{in}} \text{ (particle flux)}) \) by neutral beam injection alone, illustrated in Fig. 2(a). We assume that the pumping efficiency of the divertor is \( \xi \), i.e. a fraction \( \xi \) of the particles reaching the divertor plates \( (\Gamma_{\text{div}}) \) are pumped and the same amount of the particles need to be fuelled by neutral beam injection, i.e. \( \xi \Gamma_{\text{div}} = \Gamma_{\text{in}} \). The power injected into the main plasma...
FIG. 2. (a) Simplified heat and particle balances in the high temperature divertor plasma operation ($\xi$ is the pumping efficiency). (b) One-dimensional model with collisionless divertor plasma. (c) Electron velocity distributions in model divertor (b) at two different collisionalities: $\lambda_e/L = 36$ (3) for the left hand side case and $\lambda_e/L = 360$ (240) for the right hand side case. Here $\lambda_e$ is the electron mean free path at a temperature of $T_0$ (the numbers in the parentheses are those estimated from the temperature (average kinetic energy) of the trapped electrons) and $L$ is half the distance between the divertor plates ($V_0 = (kT_0/m)^{1/2}$).

region ($Q_{in}$) flows into the divertor channel, and the power balance ($Q_{in} = \gamma T_{div} \Gamma_{div}$) at the sheath of the divertor plate, is satisfied, where $\gamma$ is the heat transmission coefficient. From these relations, the divertor temperature is given by $T_{div} = (Q_{in} / \Gamma_{in})^{1/2} / \gamma$. For a parameter set (beam energy ($Q_{in} / \Gamma_{in}$) ~ 200 keV, $\gamma$ ~ 10, $\xi$ ~ 0.2), $T_{div}$ becomes as high as 4 keV, which is significantly higher than the values observed at the pedestal of the H-mode discharges.

We have studied a high temperature divertor plasma, i.e. a collisionless divertor plasma in a one-dimensional model as illustrated in Fig.2(b). An equal number of ions and electrons with temperature $T_0$ are supplied between the divertor plates. Ions simply flow to the divertor plates. A negative electric potential is set up for the ambipolar flow condition and electrons with parallel kinetic energy less than the potential amplitude are trapped by the potential. The electron distribution functions in the divertor channel, calculated by a Fokker-Planck code [7], are shown for two different collisionalities. When the mean free path becomes much longer than $(M/m)^{1/2} L$ (where $M/m$ is the ion-electron mass ratio and $L$ is the distance between the divertor plates), the
trapped electron density becomes nearly equal to the ion density, even with a potential amplitude of \(< eT_0\) and the average perpendicular energy of the trapped electrons is much greater than the parallel energy as is shown in the case on the right hand side of Fig.2c. On the other hand, for a marginally collisionless case (on the left hand side of Fig. 2c), the potential amplitude is less than \(eT_0\), but the temperature distribution is isotropic.

Such a collisionless effect becomes important for the energy balance with secondary electron emission. At temperatures above 100 eV, secondary electrons emitted from the divertor plate become a source of cold particles, which lower the divertor electron temperature. This effect can be included in \(\gamma\) [8]; \(\gamma = 7.8\) without secondary electron emission and \(\gamma \approx 10\) when the secondary emission rate is 0.7. But when it exceeds \(-0.7\), \(\gamma\) increases rapidly and then saturates at \(-23\), because of the space charge limit. When the divertor plasma electrons are collisionless, the secondary electrons emitted from the divertor are first accelerated by the sheath potential and barely trapped by the potentials. They eventually hit the divertor plate during the thermalization process. The parallel energy with which they hit the plates is nearly zero and the perpendicular average energy is a fraction of the sheath potential, in contrast to the conventional collisional sheath model where both average energies at which they strike the plates are equal to \(T_e\), the electron temperature. Thus the collisionless effect substantially reduces the cooling effect by secondary electrons and, hence, \(\gamma\).

In this operation, the density profile is maintained by a combination of deep fuelling such as pellet or neutral beam injection and particle pumping. Thus the diffusion coefficient (\(D\)) and hence the particle confinement become important in determining the energy confinement. This is desirable for the energy confinement in LHD where the high neoclassical ripple induced electron heat loss (\(1/v\) regime) tends to suppress the temperature gradient. However, the effective \(D\) may not be high because the ions are confined by \(E \times B\) drift (\(v\)-regime). The radial electric field in such a plasma regime is positive and, hence, a neoclassical outward impurity pinch [9] may prevent impurity contamination in the core plasma.

For this operation in LHD, we plan to install a pump system with an overall pumping efficiency of \(-20\%\) in the divertor chamber. For reactor design, we are trying to find divertor configurations which guide the heat and particle fluxes towards a remote region, away from the main coil system, thereby making particle pumping and heat removal achievable.

REFERENCES

REDUCTION OF LOOP VOLTAGE AND IMPROVEMENT OF CONFINEMENT IN A REVERSED FIELD PINCH PLASMA

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Abstract

REDUCTION OF LOOP VOLTAGE AND IMPROVEMENT OF CONFINEMENT IN A REVERSED FIELD PINCH PLASMA.

In order to decrease the anomalous part of the loop voltage which exists in reversed field pinch plasmas, the front-end system of the reversed field pinch TPE-1RM15 was modified to TPE-1RM20. The new machine has an improved shell proximity with a triple shell structure. The $b/a$ value has decreased to 1.08 from 1.18, where $b$ and $a$ are the minor radii of the shell and the plasma. Results from the new machine show that the loop voltage anomaly which was 14-21 V in TPE-1RM15 has been reduced to 3-8 V in TPE-1RM20. The square of the normalized radial magnetic field fluctuation amplitude at the plasma surface has decreased by a factor of 3-4 in the frequency range of $1 < f < 250$ kHz. The total loop voltage has decreased from 30 V to 15 V and the energy confinement time has doubled to $0.40 \pm 0.15$ ms with respect to the previous machine. The MHD mode analysis at $\Theta = 1.55$ showed tearing instabilities of $(m, n) = (1, 7-9)$, continuously rotating as a rigid body. Discrete events in the high $\Theta$ region are also observed and classified into two different types.

1. INTRODUCTION

A loop voltage anomaly has been observed in many reversed field pinch (RFP) machines. This is one of the issues that should be understood and, if possible, also overcome in the near future. In the TPE-1RM15 RFP device, we observed that the loop voltage deduced from the helicity balance equation using the Spitzer's formula could only explain about 14-21 out of 30 V of the total loop voltage, $V_{\text{loop}}$, for $I_p = 130$ kA discharges [1,2]. Here, a flat $Z_{\text{eff}}$ profile ($= 3-5$) and $T_e(r) = (T_e(0) - T_e(a)) (1-(r/a)^3) + T_e(a)$ were assumed. The same assumptions about $Z_{\text{eff}}$ and $T_e(r)$ are used throughout this paper. The remainder of $V_{\text{loop}}$, about 9-16 V, was thus attributed to the anomalous part.

This anomalous part of the loop voltage is attributable to the helicity loss at the plasma surface [3] and/or to the enhanced fast electron radial diffusion because of the magnetic field line stochasticity [4]. The helicity loss at the plasma surface will decrease if $\delta B_r^2$ ($B_r$ is the radial magnetic field component) decreases at the plasma surface. The fast electron loss will decrease if $\delta B^2$ decreases in the core plasma region. Experimentally, it was observed in

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TPE-1RM15 that the loop voltage linearly increased with the square of the magnetic field fluctuation level [5].

Motivated by these models and observations, we have modified TPE-1RM15, mainly in order to suppress the magnetic field fluctuation level by means of a close fitting shell system. To improve the shell proximity, we have increased the plasma minor radius from 0.137 to 0.192 m and adapted a triple shell structure consisting of an inner double thin copper shell and an outer single thick copper shell. The $b/a$ value has been improved from 1.18 to 1.08, where $b$ is the inner minor radius of the first shell and $a$ is the minor radius of the plasma. The major radius was also increased from 0.70 to 0.75 m.

2. GLOBAL CONFINEMENT PROPERTIES OF TPE-1RM20

The experiments in TPE-1RM20 have started recently. The confinement properties measured at $t = 4.5$ ms of 160-165 kA discharges are $T_{e0} = 650 \pm 190$ eV, $\langle n_e \rangle = (1.2 \pm 0.1) \times 10^{19}$ m$^{-3}$, $\beta_p = 0.11 \pm 0.04$, $\tau_E = 0.40 \pm 0.15$ ms, where $n_e T_e = n_i T_i$ and a parabolic electron temperature profile are assumed. The energy confinement time of TPE-1RM15 at $I_p = 160$ kA was $0.17 \pm 0.06$ ms. Thus, $\tau_E$ of TPE-1RM20 has increased by a factor of 2.5 from that of TPE-1RM15. $\beta_p$ is the same as the previous value, within a shot to shot variation. The electron density seems to have changed such that the $I/N$ value becomes the same, where $N = n a^2 \langle n_e \rangle$. Consequently, $T_{e0}$ did not change very much with respect to the previous machine. $T_{e0}$, $\langle n_e \rangle$ and $T_I$ all scale linearly with $I_p$ for $I_p < 250$ kA. The plasma current scaling is summarized as follows: $T_{e0} \text{(eV)} = (3.8 \pm 0.1) \times I_p \text{(kA)}$, $\langle n_e \rangle \text{(}10^{17} \text{m}^{-3}) = (0.7 \pm 0.1) \times I_p \text{(kA)}$ and $T_I \text{(eV)} = (2.6 \pm 0.1) \times I_p \text{(kA)} = (0.68 \pm 0.03) \times T_{e0} \text{(eV)}$. The ion temperature is measured by a neutral particle energy analyser. In Fig. 1, the $\tau_E$ values of both machines are compared as a function of $I_p$. The scaling of $\tau_E$ is strongly affected by the scaling of $V_{loop}$.

FIG. 1. Comparison of plasma current scaling of energy confinement time between TPE-1RM15 and TPE-1RM20.
3. COMPARISON OF LOOP VOLTAGES

The values of $V_{\text{loop}}$ of the old and the new machines are compared. For $I_p = 130$ kA discharges in TPE-1RM15, 14-21 out of 30 V were attributable to the anomalous part, where $T_{e0} = 660 \pm 190$ eV, $\Theta = 1.45$ and $F = -0.07$. It is possible to carry out a rough estimate of the anomalous part of $V_{\text{loop}}$ from the ratio of the magnetic shielding properties between the old and the new front-end systems. The magnetic shielding property of conducting walls can be described in terms of $(\delta B_t/\delta B)$, where $\delta B_t$ is the magnetic field fluctuation component tangential to the plasma surface. The ratio of these values of the new and old machines was numerically estimated to be 60-65% in the frequency range of $10 \text{ Hz} < f < 100 \text{ kHz}$. If the ratio of the loop voltage anomalous parts between two machines scale with the square of this reduction ratio, on the assumption that $\delta B_t$ is the same in both machines, then the loop voltage anomaly in TPE-1RM20 was expected to be 6-7 V.

In TPE-1RM20, we have attained $V_{\text{loop}}$ of $15 \pm 1$ V for $I_p = 130$ kA, where $T_{e0} = 580 \pm 100$ eV, $\Theta = 1.54$ and $F = -0.11$. Figure 2 shows a comparison of $V_{\text{loop}}$ and its dependence on $I_p$ between the old and the new machines. The loop voltage anomalous part is estimated to be about 3-8 V, which is comparable to the expected value. This reduction is larger than the uncertainties in the loop voltage estimate. It is seen in Fig. 2 that the qualitative tendency is almost the same in both machines in that $V_{\text{loop}}$ is almost constant up to a certain plasma current level and then increases gradually with $I_p$. This increase in $V_{\text{loop}}$ is attributed to the increase of the impurity content or the charge state, from the observation of the high Z impurity radiation. We note that the vacuum vessels in both machines have molybdenum fixed limiters with stainless steel liners. All data shown in Fig. 2 were taken by applying an optimum value of the DC vertical magnetic field to control plasma equilibrium for each $I_p$. Otherwise, $V_{\text{loop}}$ would increase up to twice as high as the optimum case [6].

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FIG. 3. Comparison of power spectra normalized by the square of the poloidal magnetic field at the plasma boundary between TPE-1RM15 and TPE-1RM20. (a) Radial component; (b) toroidal component.

4. INTERPRETATION OF $V_{\text{loop}}$

In order to confirm the phenomenological scaling of the loop voltage anomaly with the magnetic field fluctuation amplitude [5], a comparison of the typical magnetic field fluctuation level between the two machines was made by using identical, insertable magnetic probes placed at the same radial distance, $X = 20$ mm, from the plasma surface. The result shows (Fig. 3 (a)) that the square of the radial fluctuation amplitude, $\delta B_r^2$, normalized by the poloidal field at the plasma surface, $B_{pa}$, decreases by a factor of about ten, especially in the frequency region of $f < 10$ kHz and $f > 100$ kHz. The frequency range around 20-60 kHz, which has a broad peak in the power spectrum (Figs. 3 (a) and 3 (b)), corresponds to rotation frequency times mode numbers of the dominant modes $(m, n) = (1, 7-9)$ in TPE-1RM20. If $(\delta B_r/B_{pa})^2$ is averaged over the entire frequency region, $1$ kHz $< f < 250$ kHz, it has decreased to 25-33% of the previous value. On the other hand, the square of the toroidal magnetic
fluctuation amplitude, $\delta B_t$ normalized by $B_{pa}$, has not changed very much in $f < 100$ kHz (Fig. 3 (b)). It has only shown some decrease in $f > 100$ kHz. The averaged value of $(\delta B_t/B_{pa})^2$ was 80-100% of that of TPE-1RM15. The poloidal magnetic field fluctuation level has shown a similar tendency as $\delta B_t$. Thus, the reduction of the loop voltage anomaly is closely connected with the decrease in the radial magnetic field fluctuation amplitude near the plasma surface.

The experimentally obtained $(\delta B_t'/\delta B_t)^2$, averaged over $1$ kHz < $f$ < $250$ kHz, has decreased to $32 \pm 1\%$ of the previous value. This averaged ratio of the magnetic shielding properties agrees well with the numerically estimated value of $39 \pm 3\%$.

The loop voltage anomaly originating from the edge helicity loss model [3], $\Delta V_{H-a}$, scales as $\Delta V_{H-a} \propto (R/a) T_e(a) (\delta B_t/B_{pa})^2$, where $R$ is the major radius and $T_e(a)$ is the electron temperature at the plasma surface. Here, we assume that the fluctuation amplitude of the surface potential difference, $\delta \chi$, is proportional to $T_e(a) (\delta B_t/B_{pa})$. This scaling formula gives an estimate for $\Delta V_{H-a}$ of TPE-1RM20 as 20-30% of the value of TPE-1RM15, where $T_e(a)$ is assumed to be the same. The experimentally obtained ratio of the loop voltage anomaly for $I_p = 130$ kA is 21-38%, comparable to the estimated reduction ratio for $\Delta V_{H-a}$.

In TPE-1RM15, we measured the surface potential difference by the floating potential on the electron drift side and the ion drift side in the edge region. After smoothing out the pulses on the signals, a finite potential difference of about 15 V remained, which was comparable to the local cold edge electron temperature. The polarity was such as to increase the local helicity loss. A correlation analysis between the fluctuation of the surface potential difference and the magnetic field fluctuation is currently under way.

Next, the contribution of the fast electron loss to the loop voltage anomaly is examined [4]. Fast electrons have been observed in many RFP plasmas, as well as in TPE-1RM15 [7] and TPE-1RM20. The loop voltage anomaly originating from this fast electron loss, $\Delta V_{FE}$, scales as $\Delta V_{FE} \propto V_K (P/\Theta) (E_c/E_C) \propto (R/a^2) (P/\Theta) (V_{loop}/R) (Z_{eff} l_p/\sqrt{T_e a_e})$, where $V_K$ is the helicity balance loop voltage from Spitzer's resistivity, $P$ is the power deposition asymmetry and $E_C$ is the critical electric field for thermal electrons to run away. The power deposition asymmetry has decreased from 0.65 to 0.55, indicating that the power loss by fast electrons has decreased in TPE-1RM20. The estimated value of $\Delta V_{FE}$ in TPE-1RM20 deduced from the above scaling is 40% of $\Delta V_{FE}$ in TPE-1RM15. This percentage is slightly higher than the experimentally obtained reduction rate of the loop voltage anomaly, 21-38%.

From these estimates, it is seen that the edge helicity loss model gives a closer estimate for the observed change in the loop voltage anomaly after the modification of the machine than the fast electron loss model. Further estimates of the contribution from each model need more accurate information about the plasma resistivity profile and $Z_{eff}$.

5. MHD MODE ANALYSIS

Magnetic field fluctuation mode spectra have been analysed in TPE-1RM20. At normal operating conditions ($\Theta = 1.55$), the dominant fluctuations are coherent $(m, n) = (1, 7-9)$ modes, resonant near the axis, and continuously rotating as a rigid body with poloidal and toroidal rotation frequencies of 130-170 kHz and 11-14 kHz, respectively. At $\Theta > 1.6$, discrete events characterized by large drops in the soft X-ray radiation (SXR) are observed. The events are of two different types. The first type is preceded by a wall locked single $(m, n) =$

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3 Brunsell, P. R., et al., in preparation.
FIG. 4. Time evolution of sine amplitude of (m, n) = (1, 7-9) at an event where the saturated wall locked mode appears.

(1, 7) mode (Fig. 4). After the wall locking, the mode spectra cascade into higher \( n \) numbers, in agreement with numerical MHD simulations. The second type of discrete event is characterized by phase locking and appearance of large \((m, n) = (0, 1-5)\) modes during the subsequent drop of SXR. This type of events is accompanied by a continuous rotation of \((m, n) = (1, 7-9)\), similar to the normal operating conditions. These rotating modes resonant near the axis are likely to be resistive tearing mode instabilities.

6. CONCLUSIONS

In order to reduce the loop voltage anomaly, the front-end system of TPE-1RM15 was replaced by a new one, TPE-1RM20, which has a larger volume and improved shell proximity. After the modification, the total loop voltage decreased from 30 to 15 V. The anomalous part of the loop voltage has decreased from 14-21 to 3-8 V. This reduction of the loop voltage is closely connected with the observed reduction of the radial magnetic field fluctuation level at the edge, which supports the edge helicity loss model. The fast electron loss has also decreased, and its contribution to the change of the loop voltage anomaly may not be neglected. The global confinement properties of both machines are compared. The energy confinement time in TPE-1RM20 has shown a factor of 2.5 increase from \(0.17 \pm 0.06\) ms to \(0.40 \pm 0.15\) ms at \(I_p = 160-165\) kA. An MHD mode analysis in the normal operating conditions of TPE-1RM20 shows coherent modes of \((m, n) = (1, 7-9)\), rotating as a rigid body. These modes are resistive tearing modes resonant near the axis. Two types of discrete event are found in high \(\theta\) conditions, coincident with soft X-ray radiation crashes. One has a similar mode spectrum as the normal \(\theta\) case and the other one shows a distinct, single \((m, n) = (1, 7)\) mode which locks to the wall just before the sawtooth crashes.
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MERGING OF TWO SPHEROMAKS
AND ITS APPLICATION TO THE SLOW FORMATION
OF A FIELD REVERSED CONFIGURATION

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Abstract

MERGING OF TWO SPHEROMAKS AND ITS APPLICATION TO THE SLOW FORMATION OF
A FIELD REVERSED CONFIGURATION.

A new slow formation method of the field reversed configuration (FRC) has been developed by
using two axially colliding spheromaks with oppositely directed toroidal magnetic fluxes, $\Phi_1 (>0)$ and
$-\Phi_2 (<0)$. The merging spheromaks are found to relax either to a high beta FRC with toroidal field
$B_t = 0$ or to a new low beta spheromak with $B_t = B_p$ (poloidal field), depending on whether the initial
flux ratio $\Phi_2/\Phi_1$ is larger or smaller than a certain threshold value ($0 < \Phi_2/\Phi_1 < 1$). Unlike the
Taylor relaxation to the spheromak, the relaxation to the FRC annihilates the initial magnetic helicity,
suggesting that the stability of the FRC is as robust as that of the Taylor state spheromak. The new for­
mation method also allows the FRC to have a centre current transformer (OH coil), allowing toroidal
current drive of the FRC.

1. INTRODUCTION

A spheromak and a field reversed configuration (FRC) are both compact toroids
whose simply connected topologies permit their axial translation and merging [1, 2].
The spheromak, with a balanced amount of toroidal and poloidal magnetic fields $B_t$
and $B_p$, is in the Taylor minimum energy state, ensuring its global magneto-
hydrodynamic (MHD) stability [3]. On the other hand, the FRC with a single field
component $B_p$ and high $\beta$ (plasma pressure/magnetic field pressure) is far away
from the Taylor state. It needs some kinetic effect such as the finite Larmor radius
effect to keep its global stability [2, 4]. Especially, the lack of MHD stability limits
the FRC formation to the uneconomical fast $\theta$-pinch method [2]. This is the reason
why research into the slow formation of FRCs has received increased attention during
the past few years [5].

This paper deals with three important issues: (1) Is slow FRC formation possi­
ble if we use two merging spheromaks with opposite toroidal fluxes? (2) Is there any
new relaxation leading to FRC equilibrium?, and (3) Is FRC sustainment possible if
we drive its toroidal current by using a current transformer?

We have for the first time used two merging spheromaks with opposite toroidal
fluxes to produce the FRC at a slow time-scale. As is shown in Fig. 1(a), two sphero­
maks with toroidal fluxes, $\Phi_1 (>0)$ and $-\Phi_2 (<0)$, are produced along the central
FIG. 1. Two merging spheromaks with opposite toroidal fluxes, $\Phi_1 (> 0)$ and $-\Phi_2 (< 0)$, and the following relaxations to an FRC (a) and to a spheromak (b), experimental set-ups in the TS-3 device for the gun type (spheromak) formation (c) and for the $z$-$\theta$ type formation (d). In (d), a current transformer (OH coil) is located along the central axis for the toroidal current drive experiment.
geometrical axis \((0 \leq \Phi_2 \leq \Phi_1)\) and then are merged in the axial direction. The plasma produced is expected to have the sum of the toroidal fluxes, \(\Phi = \Phi_1 - \Phi_2\), \((\geq 0)\), indicating that an FRC is formed when \(\Phi_1 = \Phi_2\). The advantages of the proposed formation method are as follows: (1) the formation time of the FRC is as long as that of the spheromak (quasi-static), eliminating the need for the fast speed capacitor bank system; (2) since the stability of the spheromaks is ensured by Taylor's principle, it is easy to produce a large size FRC; and (3) the \(s\) number \((= \text{plasma separatrix radius/ion gyroradius})\) of the produced FRC is expected to be as large as that of the initial spheromaks.

An important finding is that the merging spheromaks relax either to an FRC with \(\Phi \approx 0\) (Fig. 1(a)) or to a new spheromak with finite \(\Phi\) (Fig. 1(b)), depending on the initial flux ratio \(\Phi_2/\Phi_1\). When \(\Phi_1 \neq \Phi_2\), the relaxation to the FRC annihilates the initial magnetic helicity \(K (= K_1 - K_2)\), revealing another robust relaxation different from the Taylor relaxation to the spheromak that conserves \(K\). Since this formation method allows the FRC to have a central current transformer, we have successfully performed toroidal current drive of the FRC over 200 \(\mu s\) for the first time.

2. EXPERIMENTAL SET-UPS

Figures 1(c) and (d) show the two experimental set-ups in the TS-3 device characterized by the ‘gun type’ [6] and ‘z-\(\theta\) type’ [6, 7] formation methods of the initial two spheromaks, respectively. In Fig. 1(c), a coaxial plasma gun is located on either end of the cylindrical vacuum vessel to produce the spheromak. In Fig. 1(d), eight pairs of electrodes and a poloidal field coil are located on either side of the vessel to produce the spheromak. The polarities of \(\Phi_i\) and \(K_i\) for the two spheromaks are determined independently by the directions of the gun discharge currents in Fig. 1(c) or by those of the Z-discharge currents in Fig. 1(d). In Fig. 1(d), the current transformer (OH coil) is located along the central axis to drive the toroidal current of the produced FRC. Its thin shell also maintains the plasma stability against the \(n = 1\) (tilt and/or shift) modes. A two-dimensional magnetic probe array is placed in the \(r-z\) plane of the vessel to measure the 2-D profiles of \(B_z\) and \(B_t\) on a single shot. This \(5 \times 7\) array is composed of 35 pick-up coils covered with thin glass tubes of 5 mm diameter. An electrostatic probe is inserted along the axial line at \(r = 14\) cm to measure the profiles of the electron temperature \(T_e\) and the electron density \(n_e\). The value of \(n_e\) is calibrated by using the \(\text{CO}_2\) laser interferometer.

3. EXPERIMENTAL RESULTS

Figures 2(a) and (b) show the time evolution of the poloidal flux contours in the \(r-z\) plane and those of the \(B_z\) profiles along the axial line at \(r = 14\) cm for the typical two cases of the merging. In Fig. 2(a), the two spheromaks produced have toroi-
FIG. 2. Time evolution of poloidal flux contours in the $r$–$z$ plane and toroidal field profiles along the axial line at $r = 14$ cm for two typical cases of merging: 
(a) $\Phi_2/\Phi_1 = 0.9$ ($K/W = 0.1$) ($K/W_{Taylor}$); (b) $\Phi_2/\Phi_1 = 0.3$ ($K/W = 0.8$) ($K/W_{Taylor}$). The two merging spheromaks have toroidal fluxes $\Phi_1$ and $-\Phi_2$ initially at $t = 80 \mu$s.
Dal fluxes of $\Phi_1$ and $-\Phi_2 \approx -0.9 \Phi_1$ at $t = 80 \mu s$, indicating that the total toroidal flux $\Phi = \Phi_1 - \Phi_2 \approx 0.1 \Phi_1$ (and the total magnetic helicity) is small but finite. The merging proceeds from $t = 80 \mu s$ to $t = 100 \mu s$, increasing the reconnected flux acceleratively. Finally, at $t = 100 \mu s$, the completion of the merging forms a spherical plasma with a single magnetic axis. During this process, the $B_t$ profile, initially with both polarities, is transformed into a nearly uniform profile with $B_t \approx 0$. The eigenvalue $\lambda (= \mu P/B_p)$ is no longer spatially uniform, unlike those of the initial Taylor state spheromaks. It is noted that the plasma $\beta$ increases from 0.2–0.3 to 0.7–0.9 during the merging. The line averaged electron density $n_e$ increases from $1.5 \times 10^{14} \text{ cm}^{-3}$ to $2.5 \times 10^{14} \text{ cm}^{-3}$, the electron temperature $T_e$ stays uniform around $10 \pm 3 \text{ eV}$, and the average magnetic field decreases from 0.65 kG to 0.5 kG. This value, $\tilde{\beta} \approx 0.8$, is high enough for the final state to be called an FRC.

In Fig. 2(b), $\Phi_2/\Phi_1$ is set to about 0.3, indicating that the total toroidal flux, $\Phi \approx 0.7\Phi_1$, is closer to the value for the Taylor state. After the merging has been completed ($t \approx 95 \mu s$), the $B_t$ profile does not relax to zero but to a peaked profile characteristic of typical spheromaks. Its $\lambda$ profile is observed to be spatially uniform, indicating relaxation to the Taylor state spheromak. Figures 2 and further merging experiments with various $\Phi_2/\Phi_1$ values ($0 \leq \Phi_2/\Phi_1 \leq 1$) reveal that the two merging spheromaks relax either to a high $\beta$ FRC with $B_t \approx 0$ or to a low $\beta$ spheromak with $B_t \approx B_p$, depending on whether $\Phi_2/\Phi_1 (\leq 1)$ is larger or smaller than a certain threshold value. The most probable interpretation of these phenomena is that the relaxation processes in the low $K/W$ (magnetic helicity/magnetic energy) region are classified into two types: the new relaxation to an FRC when $0 \leq (K/W)/(K/W)_{\text{Taylor}} < R$ (threshold value), and the well known Taylor relaxation to a spheromak when $R < (K/W)/(K/W)_{\text{Taylor}} < 1$, where $(K/W)_{\text{Taylor}}$ is the $K/W$ value for the Taylor state. As $\Phi_2/\Phi_1 (\equiv \alpha)$ increases from 0 to 1, $K/W$ of the merging toroid deviates from $(K/W)_{\text{Taylor}}$ as follows:

$$\frac{K/W}{(K/W)_{\text{Taylor}}} = \frac{(K_1 - K_2)/(W_1 + W_2)}{(K/W)_{\text{Taylor}}} \sim \frac{\Psi_1 \Phi_1 - \Psi_2 \Phi_2}{\Psi_1 \Phi_1 + \Psi_2 \Phi_2} \sim \frac{1 - \alpha^2}{1 + \alpha^2} \quad (1)$$

where the two initial (Taylor state) spheromaks have a fixed flux ratio of $\Psi_1/\Phi_1 = \Psi_2/\Phi_2$. It is noted that the relaxation to the FRC is robust enough to annihilate the initial $K = K_1 - K_2 \propto \Psi_1 \Phi_1 - \Psi_2 \Phi_2 \sim \Psi_1 \Phi_1 (1 - \alpha^2)$ when $\Phi_1 \neq \Phi_2$. This is in sharp contrast with the Taylor relaxation to the spheromak that conserves $K$. The existence of the two equally robust relaxations suggests that the stability of the FRC is as robust as that of the Taylor state spheromak. Experimentally, the relaxation to the FRC is consistent with the disappearance of small $B_t$ ($I_t$ current) observed in the TOR FRC [8]. Theoretically, it provides experimental evidence in favour of the theoretical analysis on relaxed FRCs by Hameiri and Hammer [9].
FIG. 3. Time evolution of peak poloidal fluxes and poloidal flux contours when $d\Psi/dt$ ($= V_{\text{loop}}$) = 0 V, 28 V and 40 V (maximum values) are applied to the produced FRCs by using the current transformer.

Another advantage of our slow formation method is to allow the FRC to have the central current transformer realizing the toroidal current drive experiment, as shown in Fig. 1(d). Figures 3 show the time evolution of the poloidal fluxes and the poloidal flux contours when $d\Psi/dt$ ($= V_{\text{loop}}$) = 0, 28 and 40 V (maximum values) are applied to the produced FRCs by using the current transformer. From $t = 85 \mu s$ to $t = 100 \mu s$, $V_{\text{loop}}$ increases and then gradually decreases to zero until $t = 350 \mu s$. Just as in Figs 2, the FRC with $B_t = 0$ is produced at about $t = 95$-$100 \mu s$. Without the current drive ($V_{\text{loop}} = 0$), the FRC is observed to decay resistively by $t = 120 \mu s$. However, when $V_{\text{loop}} = 40 V$, the FRC is successfully sustained longer than $200 \mu s$. During the current drive, the poloidal flux decreases gradually from 2 mWb to 0.6 mWb, because $V_{\text{loop}}$ is limited in the present set-up. Our preliminary experiment with the limited $V_{\text{loop}}$ clearly reveals that the FRC plasma can be sustained by driving its toroidal current, $d\Psi/dt$. 
In summary, we have proposed a new slow formation method of the FRC and have demonstrated its validity. We have made clear that there exist two relaxation processes of the two merging spheromaks in the small $\Phi$, $K$ region: they relax either to the high $\beta$ FRC with $B_t = 0$ or to the low $\beta$ spheromak with $B_t < B_p$, depending on whether $\Phi_2/\Phi_1 (\leq 1)$ is larger or smaller than the threshold value. The relaxation to the FRC annihilates the initial finite $\Phi$ and $K$, suggesting the stability of the FRC to be as robust as that of the Taylor state spheromak. Our successful toroidal current drive experiment for the FRC indicates the possibility of its additional heating and flux amplification making use of the current transformer and its future steady state operation by radio frequency current drive.

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FRC CONFINEMENT AND STABILITY MEASUREMENTS IN LSX

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Abstract

FRC CONFINEMENT AND STABILITY MEASUREMENTS IN LSX.

A new 90-cm diameter, 4.5-m long FRC facility, the Large s Experiment (LSX), was built to explore FRC confinement as the kinetic s parameter was increased. Formation techniques were developed to produce quiescent equilibria with s up to 5, and confinement was noted to scale in agreement with previous empirical measurements at s < 2. Although formation became increasingly difficult as s was increased, no evidence was seen of previously reported tilt instabilities.

1. INTRODUCTION

The Large s Experiment (LSX) was constructed during the time period from 1986 to 1990, and was operated for one year. It was built to study the stability and confinement properties of Field Reversed Configurations (FRC) as the ratio of radial scale length to ion gyroradius (characterized by the parameter s [1]) was increased. The LSX formation coils have a diameter of 90-cm and a total length of 4.5-m, with the last 0.75-m at each end devoted to three independent coils used to control and help symmeterize the formation process [2]. The LSX facility was designed with the capability of forming FRCs with s as large as 8.

Production of FRCs in a large field reversed theta pinch (FRTP) such as LSX is made difficult by the demand that the plasma be heated from a few eV to several hundred eV, and achieve a quiescent equilibrium, on an Alfvén timescale. In order for this to occur, special formation techniques were employed to minimize initial (preionization) plasma turbulence, and to control the dynamic formation process. Using optimum formation sequencing, quiescent equilibria with s as large as 5 could be attained. These equilibria were characterized by axial excluded flux distributions which were symmetric and parabolic, and by end-on emission profiles which were peaked at the field null. Complete stability to all but the controllable n=2 rotational distortion, and continued good confinement scaling, were observed. Higher s FRCs
could be formed, but they tended to be both axially and azimuthally asymmetrical, with flat end-on emission profiles, and with consequent degraded confinement.

2. FORMATION AND STABILITY

The ability to form FRCs in an FRTP, and their ultimate stability, particularly at higher s, are questions that are not easily separated. Because of the dynamic nature of the formation process, strong radial and axial accelerations are produced which tend to drive radial flutes. These flutes, and other azimuthal asymmetries, are observed to decay with time, but if they are too severe, they can either prevent formation of an FRC, or result in the poor equilibrium profiles described above. The severity of the fluting depends on the initial preionization uniformity and the axial implosion dynamics. Lack of formation, or formation of asymmetric equilibria can occur at any s value if the formation symmetry is poor, but the formation requirements become more constrained as s is increased. s = 5 is the upper limit at which symmetric, well confined FRCs could be formed at the end of the one year of LSX operation.

It has previously been reported that FRC formation was limited to about s = 2 by the development of a tilt instability during, or immediately following the formation process in the FRX-C/LSM device [3]. No such tilt instability has been seen in LSX, even at much higher s values. Using the same type of Bφ probe arrays employed in Ref. 3, many dynamic modes were measured during formation on LSX, corresponding roughly with the fluting mentioned above, but there was no clear pattern to these modes, and their amplitude decreased after an equilibrium was established. An example is shown on Fig. 1 of measurements obtained at the highest s value for which symmetric FRCs could be formed. There are strong, oscillating n=1 components during the 25 μsec formation process, but Bφ arrays at the ends do not show the phase coherence symptomatic of an n=1 tilt. Rather, all mode components seen on LSX during formation are generally incoherent, and the n=1 component is usually not the dominant mode when the distortions become so severe as to prevent FRC formation.

The data on Fig. 1 shows the establishment of an equilibrium FRC at s = 5, in spite of the formation asymmetries (Bφ field amplitudes). The FRC persists, with low flux and energy loss rates, until the development of the n=2 rotational instability at about 0.3 msec. Multipole fields, which have been demonstrated to stabilize the n=2 distortion [4], were not employed for these experiments. The evolution of the rotating n=2 distortion is seen on the cross tube emission detector (∫Edł), and the Bφ array clearly shows a dominant, growing n=2 mode. (By 0.3 msec the FRC has drifted somewhat from the tube center, so that even an axially uniform distortion produces a Bφ signal at the center array location.) At such high s values, other mode components are also evident during the development of the n=2 mode, which reflects the MHD nature of the plasma, and is consistent with numerical MHD simulations.
FIG. 1. Azimuthal field mode amplitudes measured during formation and equilibrium of high s FRC. (Plasma conditions correspond to the 5–6 mTorr entry on Table I.)
<table>
<thead>
<tr>
<th>FILL PRESSURE (mTorr)</th>
<th>LIFT-OFF FLUX (mWb)</th>
<th>EXT FIELD (kG)</th>
<th>SEPARATRIX RADIUS (cm)</th>
<th>POLOIDAL FLUX (mWb)</th>
<th>ELECTRON DENSITY ((10^{13} \text{ cm}^{-3}))</th>
<th>TOTAL TEMP (eV)</th>
<th>ELECTRON TEMP (eV)</th>
<th>(\tau_{\phi}/\tau_{p1}) (cm/1(^2))</th>
<th>s</th>
<th>(\tau_{\phi}) ((\mu\text{sec}))</th>
<th>(\tau_{E}) ((\mu\text{sec}))</th>
<th>(\tau_{N}) ((\mu\text{sec}))</th>
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<td><strong>Z-DISCHARGE PI</strong></td>
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<td></td>
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<tr>
<td>0.7</td>
<td>11-16</td>
<td>7.6</td>
<td>14.3</td>
<td>135</td>
<td>4.8</td>
<td>0.9</td>
<td>1500</td>
<td>500</td>
<td>9.1</td>
<td>1.17</td>
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<td>63 +11 -11</td>
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<td>11-12</td>
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<td>12.4</td>
<td>120</td>
<td>3.5</td>
<td>0.95</td>
<td>1800</td>
<td>(575)</td>
<td>8.0</td>
<td>0.91</td>
<td>100</td>
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<td>1.3</td>
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<td>7.7</td>
<td>14.8</td>
<td>140</td>
<td>5.4</td>
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<td>280</td>
<td>(130)</td>
<td>14.8</td>
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<td>25-35</td>
<td>3.9</td>
<td>21.8</td>
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<tr>
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<td>280</td>
<td>(130)</td>
<td>18.3</td>
<td>4.69</td>
<td>535</td>
<td>345</td>
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</table>

TABLE I
LSX FRC LIFETIMES FOR VARIOUS OPERATING CONDITIONS
3. LIFETIMES

The flux, energy, and particle lifetimes measured on LSX for various operating conditions, are shown on Table I. Two types of preionization were used. In order to achieve the initial uniformity required for successful formation with Z-discharge preionization, the Z-discharge was initiated prior to the application of the reverse bias field. Subsequent end flow reduced the effective plasma density by the time of field reversal, which resulted in the high temperatures shown for that type of preionization. The total temperatures \( (T_e + T_i) \) were deduced from pressure balance, with the electron temperatures measured through Thomson scattering. Where Thomson scattering was not performed, the electron temperatures were calculated from a model balancing electron-ion equilibration with radiation loss rates. The low fill pressure, high temperature results represent a new operating regime for FRTPs, with measured D-D neutron emission rates of about \( 10^{14} \text{ sec}^{-1} \).

The flux, energy, and particle lifetimes shown respectively on Table I are the averages over several shots. When enough data was taken, the plus and minus ranges of the lifetimes are also indicated. Particle lifetime data for individual shots is plotted on Fig. 2 versus the s parameter. This data was taken from a set of shots that were judged to have the good equilibrium qualities described above, and for which detailed time histories of the plasma density were available. The data points which are shown by open squares are part of a more restrictive database where very high equilibrium symmetry was achieved. That database was used to calculate the average plasma conditions and all the FRC lifetimes shown on Table I.
The listed s value is based on a rigid-rotor flux distribution such that $s = 0.16 s_r s / \rho_i$. $r_s$ is the separatrix radius, $r_c$ is the formation coil radius, $x_s = r_s / r_c$, and $\rho_i$ is the ion gyroradius in the external field. The particle lifetime is seen to scale approximately with $s^{1.6}$, or $(r_s / \rho_i)^{1.6}$, in agreement with the scaling observed in the smaller TRX device [5].

When added to previous results from smaller machines [5], the new LSX measurements make lifetime data available over about a factor of 5 in the dominant $r_s / \sqrt{\rho_i}$ parameter, and a factor of 50 in lifetime. A regression analysis including the important $x_s$ parameter yields a particle lifetime $\tau_N (\text{msec}) = 11 x_s^{0.64} r_s^{-2.32} (\text{m}) / \rho_i^{1.62} (\text{cm})$. Similar regression analysis for the flux and particle lifetimes yield $\tau_\phi (\text{msec}) = 8.4 x_s^{0.52} r_s^{-2.19} (\text{m}) / \rho_i^{1.23} (\text{cm})$ and $\tau_E (\text{msec}) = 3.3 x_s^{0.43} r_s^{-2.16} (\text{m}) / \rho_i^{1.48} (\text{cm})$. The $r_s$ scaling should be very accurate since a wide range of machine sizes is included, but in almost all experiments $x_s$ and $\rho_i$ are generally not independent parameters since low fill pressure operation, which results in high temperatures and large ion gyroradii, also produces low $x_s$ FRCs. Thus, the exact exponents for $x_s$ and $\rho_i$ may be off by as much as 50% (too low for $x_s$ and too high for $\rho_i$).

4. CONCLUSIONS

All the data show an approximate $r_s^2$ scaling, indicating that the governing transport processes for FRCs continue to be diffusive, even to $s$ values as high as 5. Poor confinement at higher $s$ appears to be related to formation problems in creating a symmetric, quiescent equilibrium. Coupling in the generation process between $x_s$ and the plasma temperature does not allow extremely detailed conclusions to be drawn about scaling dependencies on other parameters, but any regression analysis (including ion or electron temperatures as independent variables) shows a strong inverse scaling with ion gyroradius. The nearly equal flux and particle confinement times implies that the resistivity profile is approximately uniform. Using a rigid-rotor profile to relate the plasma resistivity to the measured flux lifetime, $(\eta / \mu_i) = r_s^{-2/16}$, $\tau_\phi = 7.5 \rho_i^{1.23} (\text{cm}) / x_s^{-5} r_s^{0.19} (\text{m})$ m$^2$/sec. Considering the high beta nature of the FRC, this can also be written as $(\eta / \mu_i) = 3 / x_s^{-5} n^{0.6} (10^{15} \text{ cm}^{-3}) r_s^{0.19} (\text{m})$ m$^2$/sec. The average FRC beta value is $<\beta> = 1 - x_s^{-2/3}$, and the decrease in effective resistivity with increasing $x_s$ implies that the basic transport coefficients are sensitive to profile changes, decreasing as the average beta is decreased from near unity at low $x_s$.

Assuming that quiescent FRCs can be formed, and kept stable at higher $s$, the presently observed empirical scaling would lead to several meter radius FRC reactors if the density were about $10^{15}$ cm$^{-3}$. There is a great advantage toward reducing the physical size by increasing the density, since the ratio of empirical to required confinement time scales as $n^{3/2}$. Higher densities can easily be reached through adiabatic compression at sub-10T magnetic fields, but the enormous fusion power densities would probably necessitate a pulsed reactor. If small ($r_s$
≈ 1 m), sub-GW steady-state thermal reactors are desired, about an order of magnitude improvement is required over the present empirical lifetime scaling.

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DENSE Z-PINCH RESEARCH — EXPERIMENT AND THEORY

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Abstract

DENSE Z-PINCH RESEARCH — EXPERIMENT AND THEORY.

Dense Z-pinch research at Imperial College is currently dominated by the design and construction of the MAGPIE generator which can deliver up to 2 MA at 2.4 MV for 200 ns into a Z-pinch load. It will be operational in 1993 and allow an investigation into radiative collapse at currents above the Pease–Braginskii current, as well as the attainment of a Z-pinch under thermonuclear conditions. In a joint experiment at the Ecole polytechnique, enhanced stability has recently been found by collapsing a current carrying plasma jet on to a fibre. In theoretical work a general class of non-stationary Z-pinch equilibria has been studied, distinguishing between cases of rising and falling current. A stability analysis of the m = 0 mode using the Chew, Goldberger and Low (CGL) model (valid in the small Larmor radius limit) with an anisotropic equilibrium shows that somewhat enhanced stability can be achieved if $P_{x0} > P_{E0}$. However, CGL absolute stability to the m = 0 mode can only be achieved in the gas embedded pinch. For stability with finite or large ion Larmor radius effects, two methods based on the Vlasov fluid model are being studied; the first is a linear initial value code, FIGARO, and the second uses a variational formulation of the eigenvalue equation. For a parabolic pressure profile the growth rate falls sharply to a minimum value as the ion Larmor radius is increased. Non-linear MHD evolution of m = 1 kink is studied with a simplified model. Finally, Ettinghausen and Nernst effects are shown to remove a surface thermal instability and cause a slight central peaking of the temperature.

1. INTRODUCTION

Two years ago we reported to the 13th IAEA Conference [1] that we had made great progress in understanding the stability properties of Z-pinches. A universal $I^a$ versus N diagram had been found [2] which showed that ideal MHD stability theory

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only applied to a small region of parameter space (I is the pinch current, a the radius
and N the line density). In particular, in the parameter space relevant to controlled
fusion, or to significant radiative collapse, large ion Larmor radius (a¡) effects will
be important. Indeed, we also reported then on the greatly enhanced stability of an
experimental compression Z-pinch for a¡/a = 0.18, though the value of £¡71 has
been corrected to 0.7, showing that collisions reduced the stabilizing potential (£¡ is
the ion cyclotron frequency and 71 the ion-ion collision time). In Section 6 of this
paper we report on significant progress in analysing the difficult theoretical problem
of large ion Larmor radius stability, especially to the m = 0 (sausage) mode. In the
small Larmor radius limit the Chew, Goldberger and Low (CGL) [3] model has
been applied to anisotropic equilibria (Section 5). The pinch is often expanding or
contracting because of a mismatch of the current waveform with the Ohmic heating
of the plasma. It is of interest therefore to explore the self-similar equilibria that are
possible under these circumstances (Section 4). Profiles are also affected by the in­
culsion of cross-phenomena in the transport arising from the velocity dependence of
the collision frequency. In Section 8 the combined effect of the Ettinghausen and Nernst
terms is explored. A simplified model of the non-linear development of the
m = 1 kink instability is presented in Section 7.

The possibility of obtaining a spectacular radiative collapse at currents signifi­
cantly above the Pease-Braginskii current was also shown theoretically two years ago
[1] and in more detail elsewhere [4, 5]. Densities as high as 10^4 to 10^5 times solid
densities were predicted in both 0-D [4] and 1-D [5] simulations, the latter limited
only by degenerate electron pressure and opacity effects. As a result we have been
funded by SERC (the UK Science and Engineering Research Council) to build a
pulsed power generator that could be employed not only to investigate this phenome­
on but also to explore lower density conditions (at about one tenth of solid density)
that could be relevant to thermonuclear fusion. This generator, called MAGPIE, is
now nearing completion, and is described in Section 2. Section 3 reports on an
experiment that shows how a fibre pinch can be stabilized by surrounding it with a
low density plasma jet that initially carries the current.

2. PROGRESS ON THE MAGPIE GENERATOR

The MAGPIE generator consists of four 86 kJ Marx generators at 2.4 MV that
feed into four water dielectric pulse forming lines. The 5 Ω lines are switched simul­
taneously into a single matched 1.25 Ω water transfer line. The final part carries a
load such as a magnetically insulated transfer section leading to a highly inductive
fibre pinch or some other Z-pinch configuration. By May 1992, one Marx generator
and pulse forming line had been tested successfully for 1000 shots with an acceptable
jitter. Currently, all four generators are being assembled together with the central
single water transfer line that will be able to carry currents up to 2 MA. The genera­
tor will be operational early in 1993.
3. ALUMINIUM PLASMA EMBEDDED FIBRE Z-PINCH

When a high current is passed through a thin metallic wire it usually explodes radially, accompanied by the development of large scale $m = 0$ instabilities. This in part is because of the failure to raise the current in time at a fast enough rate to satisfy the Haines–Hammel curve [6]; but it is also because of the MHD instability, enhanced by the inward acceleration of the outer coronal plasma as the current rises more sharply. Most experiments have too high a line density for large ion Larmor orbits to occur, but even if the line density were low enough [7] the Rayleigh–Taylor growth rate in the low density corona can be comparable to, or larger than, the ion cyclotron frequency so that there is insufficient time for the finite size of the ion Larmor orbits to have an effect. Experiments on carbon fibres [7] and cryogenic deuterium fibres [8, 9] reveal severe $m = 0$ instabilities, which are supported by 2-D MHD numerical simulations [10].

In a recent collaborative experiment at the École polytechnique a new Z-pinch configuration was employed in which an aluminium plasma jet was first formed around an aluminium wire. This jet carried the initial current and was pinched onto

![Image](image_url)

FIG. 1. Time integrated X ray pinhole photographs of discharges from a 10 μm diameter W fibre, obtained with (A) 50 μm diameter pinhole with 25 μm Be filter, (B) 50 μm diameter pinhole with 1.5 μm Ta filter, (C) 200 μm diameter pinhole with 10 μm Ti filter.
the wire which was radiatively photo-ionized. The current diffused into the wire, while the tamper and pinch effects of the outer plasma jet prevented significant expansion.

The experiments were carried out on GAEL, a 0.1 TW pulsed power generator providing a 200 kA, 50 ns current pulse from a 2 Ω pulse line. An auxiliary 8.48 μF capacitor is discharged into an aluminium foil which evaporates and expands through a collimating nozzle into the region between the anode and the cathode. This jet then surrounds the wire that connects the anode and the cathode. Then the current pulse is applied from GAEL and the jet forms a submillimetre diameter plasma that remains stable for 20 to 40 ns.

Results from three channels of a time integrated X ray pinhole camera with various filters are shown in Figs 1 and 2, without and with the presence of the aluminium plasma jet on a 10 μm diameter tungsten wire. It can be seen that the tungsten wire alone has the well known m = 0 instabilities, observed on all channels, i.e. from 1 keV to 10 keV X ray emission. In contrast with the aluminium jet surrounding the tungsten wire (Fig. 2) there is no apparent instability. Preliminary results have been reported [11].

FIG. 2. Time integrated X ray pinhole photographs of discharges from a 10 μm diameter W fibre immersed in an Al jet plasma, obtained with (A) 50 μm diameter pinhole with 25 μm Be filter, (B) 50 μm diameter pinhole with 10 μm Be filter, (C) 400 μm diameter pinhole with 10.5 μm Ti filter.
4. SELF-SIMILAR EQUILIBRIA

We are studying a general class of self-similar non-stationary Z-pinch equilibria. These are solutions of the coupled non-linear partial differential equations describing time dependent states in which pressure balance is maintained. By 'self-similar' we mean that the profile shape as a function of some similarity variable is preserved, but the scale changes with time. Previous work has shown that self-similar equilibria arise spontaneously [12, 13]. Thus they provide a good basis for studies of realistic Z-pinch configurations.

We use a modified version of Glasser's [14] formulation of the problem. This assumes temperature equipartition, and the magnetized plasma form of cross-field resistivity and thermal conductivity, but neglects radiation. The solutions impose a current time dependence of the form

\[ I \propto \left( 1 + \gamma \frac{t}{t_0} \right)^\alpha \]  

where \( \alpha \) and \( \gamma \) are free parameters, and \( t_0 \) is a characteristic time, the value of which is determined as part of the solutions. In our formulation we set \( \gamma = \pm 1 \). There are two more free parameters in the equations. The first sets the position of the edge. In practice this is done on the basis of the density profile. We define the edge as the position of either the first minimum in the particle number density, \( n_0(r) \), (in which case the equilibrium has finite density at the edge, corresponding to the gas embedded pinch) or the first zero in \( n_0(r) \) (in which case the equilibrium corresponds to a thermally isolated pinch in vacuo). The second free parameter, \( \Omega \), determines the form of the heat flow at the edge. By varying the free parameters a wide range of pinch profiles can be generated.

Figure 3 shows the \((\alpha, \Omega)\) parameter space (for \( \gamma = +1 \)). It can be partitioned into zones corresponding to various types of solution. In region I the solutions are thermally isolated, and heat flow at the edge \( (q_h) \) is inwards, arising from a skin current. In region II the solutions are gas embedded, with an outward heat flow to the gas. The solutions in region III are not physically accessible since they correspond to a gas embedded pinch with an inward heat flow from the gas.

The line \( \Omega = \Omega_{\text{crit}} \) corresponds to the physically important solutions in which the pinch is thermally isolated with no skin current. This probably provides the best representation of fibre pinches. For \( \gamma = +1 \) we find that such solutions do not exist for \( \alpha < -1/2 \). Thus, in the \( \alpha = +1 \) case, corresponding to a linearly rising current, there is a well behaved solution of this type. However, for \( \gamma = -1 \) and \( \alpha = +1 \), corresponding to a linearly falling current, such a solution does not exist. It is interesting to note that in the NRL fibre pinch experiment [9], in which the current waveform was approximately triangular, the plasma remained quiescent during the current rise, but the start of the current fall was strongly correlated with a high level of MHD activity, which may have been triggered by the absence of a suitable physical equilibrium.
Turning to stability, we have studied the instantaneous linear MHD stability to both \( m = 0 \) and \( m = 1 \) modes of the self-similar equilibria. By neglecting the effect of equilibrium dynamics, we isolate the effect of profile changes. This also provides a good approximation to the full solution at the late times for a large subset of the equilibria. Varying \( \alpha \) (the parameter determining the current time history) we find almost no change in the linear growth rate. Thus the MHD stability properties cannot be significantly improved solely by tailoring the current waveform to achieve favourable profile shapes.

5. CGL THEORY

Previous work using the Chew–Goldberger–Low (CGL) [3] model has shown that, in the collisionless, small Larmor radius regime, anisotropy of the perturbed pressure can provide a significant enhancement of linear stability over ideal MHD even for isotropic equilibria [15, 16]. This work has been extended to treat anisotropic equilibria [17].

The CGL equations neglect parallel heat flow, an assumption which usually invalidates the model. However, there is no problem in the case of the Z-pinch if we use the equations to study either equilibria or the linear \( m = 0 \) instability.
For an anisotropic Z-pinch equilibrium, the pressure balance condition is

$$\frac{dP_{\perp 0}}{dr} = -j_0 B_0 + \frac{(P_{10} - P_{\perp 0})}{r}$$

(2)

For a general 1-D equilibrium, the 'r' in the denominator of the second term on the right hand side is replaced by the radius of curvature of the field lines. Pressure anisotropy therefore has a stronger effect on equilibria in the case of the Z-pinch than for any other 1-D configuration.

We quantify the anisotropy of an equilibrium in the following way. By analogy with the conventional Bennett temperature (which is a mean value weighted by density) we define perpendicular and parallel 'Bennett temperatures':

$$\bar{T}_\perp = \frac{2\pi}{N} \int_0^\infty n_0 T_{\perp 0} r \, dr, \quad \bar{T}_1 = \frac{2\pi}{N} \int_0^\infty n_0 T_{10} r \, dr$$

(3)

We then characterize the degree of anisotropy by the single dimensionless parameter $\Theta = T_{\perp 0}/T_{10}$. We note that the Bennett relation can be written as

$$\frac{\mu_0 l^2}{16\pi N k_B} = \frac{1}{2} \left( T_\perp + T_1 \right)$$

(4)

where $k_B$ is Boltzmann's constant.

The strong effect which anisotropy has on pressure balance, in the case of the Z-pinch, severely restricts the form of allowed equilibria. In particular, the anisotropy must satisfy two conditions: (i) the pressure must be locally isotropic at $r = 0$, (ii) the pressures must everywhere satisfy the following inequality:

$$\int_0^r [P_{\perp 0}(r') + P_{10}(r')] \, r' \, dr' \geq r^2 P_{\perp 0}(r)$$

(5)

The latter condition makes it very hard to find equilibria in which $P_{\perp 0}/P_{10}$ has a very large value anywhere.

As an example of an anisotropic equilibrium, we consider the generalization of the familiar parabolic pressure equilibrium, in which the current density is uniform ($j_0 = I/\pi a^2$) everywhere. In this case

$$P_{\perp 0} = P^* \left[ 1 - \left( \frac{r}{a} \right)^\nu \right]$$

(6a)

$$P_{10} = P^* \left[ 1 + \frac{2\nu}{2 + \nu} \left( 1 + \frac{1}{\Theta} \right) \left( \frac{r}{a} \right)^2 - (1 + \nu) \left( \frac{r}{a} \right)^\nu \right]$$

(6b)

where

$$P^* = \frac{\mu_0 j_0^2 a^2 \Theta}{4(1 + \Theta)} \left( 1 + \frac{2}{\nu} \right)$$

(7)
For any \( \nu \) the maximum value of \( \Theta \) for which \( P_{\nu 0} \geq 0 \) everywhere is \( 2/\nu \). Since we require \( \nu \geq 1 \) to avoid singular pressure gradients on axis, we are limited to \( 0 < \Theta \leq 2 \).

The linear \( m = 0 \) mode is amenable to CGL treatment because it has \( \mathbf{B}_0 \cdot \nabla = 0 \) everywhere. In this case we can explicitly solve the CGL eigenvalue equation. Figure 4 shows the growth rate (normalized to the radial ion thermal transit time) as a function of \( \Theta \) for three values of \( ka \) (\( k \) is the axial wavenumber) for the uniform current density equilibrium described above. Two values of \( \nu \) are illustrated. We find that increasing \( \Theta \) reduces the growth rates at all wavelengths. Short wavelength modes show the strongest dependence on \( \Theta \) (a 70% reduction in growth rate can be achieved at \( \Theta_{\text{max}} \) for \( ka = 10 \)). This is probably due to the localization of the instability eigenfunction near the plasma edge, where the anisotropy is strongest.

Other anisotropic equilibria show broadly similar behaviour. However, changing the degree of anisotropy can be associated with changes in the current density and
pressure profile shapes. These variations (particularly the latter) can be destabilizing and more than offset the beneficial effect of increasing $\Theta$.

As in the ideal MHD case, we can formulate the CGL linear stability problem as an energy principle [18]. We find that the perturbed potential energy, $\delta W$, can be expressed as the sum of three terms: $\delta W_i$, an integral over the plasma volume; $\delta W_s$, an integral over the plasma surface; $\delta W_e$, an integral over the external region. Again we consider only $m = 0$. In this case, $\delta W_i$ is positive if the equilibrium variables satisfy the following inequality:

$$4 \frac{B_0^2}{\mu_0} (3P_{\perp 0} + 4P_{\parallel 0}) + 20P_{\perp 0}P_{\parallel 0} - (P_{\perp 0} - P_{\parallel 0})^2$$

$$+ \left[ \frac{B_0^2}{\mu_0} + 2P_{\perp 0} \right] r \frac{d}{dr} (P_{\perp 0} + P_{\parallel 0}) > 0$$

If $P_{\perp 0}(a) = 0$, the condition is violated. Thus a pinch in vacuum (implying $\delta W_e = 0$) with no skin current is CGL unstable to $m = 0$.

$\delta W_s$ is non-zero only if $\nabla (P_{\perp 0} + B_0^2/2\mu_0)$ is discontinuous at $r = a$. This can be achieved either with $P_{\perp 0}(a) \neq 0$ or by allowing a skin current in equilibrium. In either of these cases it can be shown that a trial displacement exists which makes $\delta W_s$ both negative and larger in magnitude than the other two terms [19], i.e. the pinch is unstable.

Setting aside these situations (i.e. assuming $\delta W_s = 0$) we turn to the third contribution to $\delta W$. $\delta W_e$ is zero if the pinch is in vacuum, but if it is surrounded by a gas then

$$\delta W_e = \frac{5}{6} P_0^g \int_{\text{gas}} (\nabla \cdot \vec{E})^2 d^3 r$$

where $P_0^g$ is the equilibrium gas pressure (which is uniform). If there is no skin current then $P_0^g = P_{\perp 0}(a)$. Thus, if a given equilibrium has $\delta W_i > 0$, i.e. satisfies inequality (8), then $\delta W > 0$, and, conversely, if a displacement of plasma exists which makes $\delta W_i < 0$, then the same displacement plus an incompressible displacement of the gas gives $\delta W < 0$. In other words, Eq. (8) is a necessary and sufficient condition for stability. This result has been confirmed by direct solution of the eigenvalue equation for the gas embedded case and has been generalized to include a conducting external gas [19]. We conclude that, in the small Larmor radius limit, the only type of Z-pinch which can be absolutely stable to the $m = 0$ mode is the gas embedded device, and this result is probably crucial to understanding why the $m = 0$ mode is never seen experimentally in pinches of this type.

6. LARGE LARMOR RADIUS STABILITY THEORY

We have developed two alternative approaches to the linear stability problem in the collisionless, finite and large ion Larmor radius regime. This represents an
extremely formidable problem, and the availability of two different formulations is valuable as a mutual check. Both methods are based on the Vlasov fluid model [20] which treats the ions fully kinetically, using the exact Vlasov equation, and the electrons as a cold background fluid, maintaining quasi-neutrality. However, they differ in their techniques for solving the linear stability problem.

The first approach is based on a linear initial value code, FIGARO. A perturbation is applied and its time evolution is followed. After a few growth times the fastest growing mode will dominate the behaviour. We use the term 'mode' even though we are not solving an eigenvalue equation since the radial structure of the perturbed variables which eventually emerges should correspond to the eigenfunction with the highest growth rate. The Vlasov fluid eigenvalue, \( \omega \), is complex; \( \text{Im } \omega \) is the growth rate, \( \text{Re } \omega \) is the oscillation frequency of the instability. The method used in the code is based on integrating the first order Vlasov equation along the unperturbed ion trajectories of an ensemble of particles to obtain the time dependent values of the first order ion distribution function, at the time dependent positions of the particles in phase space. Moments are taken from these values of \( f_1 \) to find the perturbed fluid variables and fields. At present the code is confined to the \( m = 0 \) mode and assumes a fixed boundary.

The second approach is a variational formulation, based on the dispersion functional derived from the Vlasov fluid eigenvalue equation [21]. Symmetry properties of the Z-pinch allow the autocorrelation integrals which appear in more complex geometry [22] to be replaced with a Fourier expansion of the trajectory integrals. Thus trajectory integrals need only be performed over half of an ion gyroperiod [23]. CGL eigenfunctions and Bessel functions have been used as basis elements in constructing the dispersion matrix for \( m = 0 \) internal modes. While both sets of function expansions converge to the same eigenvalue, it has been found that fewer CGL functions are required to achieve a given accuracy.

Two equilibria are being studied: (a) the Bennett equilibrium, corresponding to a uniform velocity of the current carriers, and (b) the parabolic equilibrium, corresponding to a uniform current density. Both codes have been benchmarked against the CGL result in the limit of very small Larmor radius.

Preliminary results indicate that the behaviour is strikingly different for the two equilibria. In the Bennett case, increasing the average Larmor radius actually increases the linear growth rate. The eigenfunction structure displays a curious variation as \( a_1/a \) is increased. In the zero Larmor radius limit we have the CGL solution, which has a well defined unperturbed inner region. As the average Larmor radius increases, the structure changes and the unperturbed region vanishes, but beyond a certain point the CGL eigenvalue is approximately recovered.

For the physically more realistic parabolic equilibrium we find that, as the average Larmor radius is increased, the growth rate falls sharply to a minimum value beyond which it again increases with \( a_1 \). Although the magnitude of the growth rate reduction at the minimum depends on \( ka \), the value of \( a_1/a \) at which the minimum occurs is almost independent of wavelength. Since \( a_1/a \) depends only on the line den-
sity, it would appear that, for the uniform current density pinch, there is a favoured region of parameter space. In the case of a hydrogen fibre Z-pinch, for example, the results suggest that a significant reduction in growth rate can be achieved by using a fibre radius in the range of 5 to 10 \( \mu \text{m} \).

7. NON-LINEAR EVOLUTION OF THE \( m = 1 \) MODE

Linear ideal MHD stability theory normally uses boundary conditions equivalent to the assumption that the plasma and the surrounding vacuum (or gas) are completely isolated from the outside world. With these boundary conditions the ideal MHD force operator is self-adjoint, and this property is highly convenient both because it facilitates the direct solution of the eigenvalue equation and allows the problem to be formulated as an energy principle [18].

For instabilities that have grown to large amplitude, however, we must include the interaction between the plasma and the external circuit, allowing both the total energy of the plasma/vacuum system (i.e. plasma kinetic energy + plasma internal energy + total field energy) and the plasma current to vary.

We have studied the \( m = 1 \) (helical) instability in the Z-pinch, using a simple model which has these required features. The \( m = 1 \) mode is routinely seen in gas embedded Z-pinches (considerations relevant to the absence of the \( m = 0 \) mode in experiments of this type are discussed above). Detailed observations of the Imperial College gas embedded Z-pinch [24] showed that the instability initially has maximum amplitude on axis, but that the pinch evolves into a well defined helical structure. During this process the wavelength increases (to a maximum corresponding to \( ka \approx 1.7 \)), and the pinch undergoes a very rapid radial expansion with a peak velocity larger than the ion thermal velocity by a factor of between three and four. Subsequently, the expansion is damped, probably through mixing of the plasma and the surrounding gas.

Ideal MHD linear theory can explain the initial eigenfunction structure [25], but is unable to account for other aspects of the behaviour, in particular the choice of wavelength (it predicts that the fastest growing mode corresponds to \( k = \infty \)) and the origin of the rapid expansion.

We use a simplified circuit representation of a pinch driven by a transmission line. The pinch is treated as a variable inductance \( L_p \), in series with an external inductance \( L_e \) and a line impedance \( R \), with a constant voltage source, \( V \), representing the voltage on the line. It is assumed that the pinch remains cylindrical, but that the current follows a helical path on the surface. This crude model gives a good approximation to the inductance of a gross helical structure for \( ka_0 \geq 1 \) and \( r_{\text{wall}} \gg a_0 \) (\( a_0 \) is the initial pinch radius and \( r_{\text{wall}} \) is the radius of the return conductor surrounding the plasma). The plasma is assumed to behave adiabatically in order to allow the evaluation of the internal energy.
The dynamics are determined by the circuit equation and power balance, which can be written in normalized form as follows:

\[
\frac{\text{d}I}{\text{d}t} = \frac{1 - I - I\frac{\text{d}l_p}{\text{d}a}}{L_x + L_p} \tag{9}
\]

\[
\frac{\text{d}v}{\text{d}t} = \Phi \left[ I^2 \frac{\text{d}l_p}{\text{d}a} + I_0^2 \left( \frac{a_0^4}{a^2} \right)^{1/3} \right] \tag{10}
\]

where \( I \) is the pinch current, \( v = \frac{\text{d}a}{\text{d}t} \), \( I_0 \equiv I(t = 0) \) (where \( t = 0 \) is defined as the time at which the instability first manifests itself),

\[
\Phi = 4.782 \times 10^6 \frac{V^2}{R^4} \frac{\alpha^2}{N} \tag{11}
\]

\( \alpha \) is the aspect ratio of the pinch chamber (i.e. length to radius) and \( N \) is the pinch line density. The normalization has been carried out with respect to the following

**FIG. 5.** Simple model of large amplitude \( m = 1 \) mode. Kinetic energy released as a function of time for various values of \( ka \): (a) 0.801, (b) 0.802, (c) 1, (d) 2, (e) 5.
variables: $\hat{\lambda} = r_{\text{wall}}$, $\hat{L} = \mu_0 \theta / 2\pi$ ($\ell$ is the pinch length), $\hat{t} = \hat{L} / R$, $\varphi = \hat{\lambda} / \hat{t}$, $\hat{I} = V / R$.

We choose suitable values for $L_x$, $\Phi$, $a_0$, and $I_0$ and then integrate the equations for various values of $k_\alpha$. For small values of $k_\alpha$ the plasma contracts under the influence of the pinch effect. The model neglects Ohmic heating, which in real situations can increase the outward force and maintain pressure balance [6]. However, at a well defined threshold value of $k_\alpha$ there is an abrupt transition from the pinching solutions, corresponding to a well confined plasma, to rapidly expanding solutions, corresponding to an unconfined (i.e. highly unstable) plasma.

Figure 5 shows the time evolution of the pinch kinetic energy for various values of $k_\alpha$. Parameters relevant to the Imperial College gas embedded Z-pinch [24] have been used. The sudden change of solution type at $k_\alpha = 0.802$ is illustrated, and we see that the energy release is higher for the unstable solutions, as we would expect. At early times the released energy is maximized by choosing a very large value of $k_\alpha$ (in agreement with conventional linear theory), but, as time advances, the ‘most unstable’ value of $k_\alpha$ falls until the threshold is reached. These features are broadly in agreement with experiment. However, the model neglects the external gas and cannot describe the damping of the expansion. During the damping phase, electrical breakdown between the turns of the helix can occur, and the current will tend to flow in the axial direction again.

8. INCLUSION OF ETTINGHAUSEN AND NERNST EFFECTS

The two cross-phenomena that we have included in the 1-D code represent the additional transport effects caused by the velocity dependence of the collision frequency. The radial component of the electron heat flux $q_{er}$ is given by

$$q_{er} = -k_{\perp e} \frac{\partial T_e}{\partial r} - \frac{T_e}{e} \beta_\perp I_x$$  \hspace{1cm} (12)

while the axial component of Ohm’s law becomes

$$E_x + \frac{v_r B_\theta}{c} = \frac{\alpha_{\perp e}}{e} I_x + \frac{\beta_{\perp e}}{e} \frac{\partial T_e}{\partial r}$$  \hspace{1cm} (13)

where Braginskii’s notation is employed. The Ettinghausen term, describing heat flux in the direction of $\vec{J} \times \vec{B}$, and the Nernst term, describing the convection of the magnetic field by the hotter electrons associated with the heat flux, have a common coefficient $\beta_{\perp e}$ through Onsager’s relations.

Figure 6 shows the electron and ion temperature radial profiles for a line density of $5 \times 10^{18} \text{ m}^{-1}$ without the $\beta_{\perp e}$ terms, while Fig. 7 is the corresponding case with the $\beta_{\perp e}$ terms included. Both cases are for a quasi-equilibrium where there is pressure balance, a parabolic density profile, and no radial motion. We note that the high
FIG. 6. Temperature profiles without the Nernst and Ettinghausen terms (hydrogen, \( N = 5 \times 10^{18} \text{ m}^{-1} \)).

FIG. 7. Temperature profiles with the Nernst and Ettinghausen terms (hydrogen, \( N = 5 \times 10^{18} \text{ m}^{-1} \)).
electron temperature near the pinch surface in Fig. 6 (which is associated with a low
number density and therefore low equipartition and high electron drift velocity) is
completely removed by the addition of the cross-phenomena. Indeed, a slight central
peaking of the electron temperature occurs instead.

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CONFINEMENT SCALING, HEATING AND STABILITY IN THE GAMMA 10 AND HIEI TANDEM MIRRORS

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Abstract

CONFINEMENT SCALING, HEATING AND STABILITY IN THE GAMMA 10 AND HIEI TANDEM MIRRORS.

Recent results of tandem mirror experiments on GAMMA 10 and HIEI are reported. The ion confining potential of GAMMA 10 has been extended to 2 keV and the axial ion confinement time has improved according to the Pastukhov scaling law. The scaling laws for the confining potential $\phi_c$ and the ion confinement time $\tau_e$ have been examined for larger sets of plasma parameters (central plasma density $n_c \leq 5 \times 10^{18} \text{ m}^{-3}$ and ion temperature $T_i \leq 5 \text{ keV}$). Their dependence on plug ECRH power and plasma density is clarified. The importance of mode conversion of ICRF waves for heating ions and electrons in a non-uniform and/or non-axisymmetric magnetic field configuration is experimentally demonstrated. Fluctuations associated with the flute interchange mode, Alfvén ion cyclotron mode, drift wave mode and rotational mode are identified. The radial electric field is controlled by end plate biasing in GAMMA 10 and by limiter biasing in HIEI. The effects of the instability induced fluctuations on the plasma confinement have been investigated.

1. CONFINEMENT SCALING IN THE GAMMA 10 TANDEM MIRROR

The GAMMA 10 consists of an axisymmetric central mirror cell for confining the main plasma, anchor cells with minimum B coils for suppressing MHD instabilities, and axisymmetric mirror cells for forming both plug and barrier potentials. The total length of GAMMA 10 is 27 m, with the central cell vessel 6 m in length and 1 m in diameter. The ion confining potential $\phi_c$ and the thermal barrier potential $\phi_b$
FIG. 1. (a) Energy confinement time $\tau_E$ versus plug ECRH power for a given density $n_c$ of the GAMMA 10 central cell. Dashed lines show $\tau_E$ predicted from the Pastukhov formula by using the measured ion confining potential $\phi_i$. (b) Summation of barrier potential depth $\phi_b$ and ion confining potential $\phi_i$ versus central cell density $n_c$ for a given plug ECRH power in GAMMA 10. Dashed curves are predicted from the strong ECRH theory combined with the experimental scaling for plug ECRH power and central cell electron heating.
at each end are formed by using two 28 GHz, 200 kW gyrotrons for the experiments described in this paper. The details of GAMMA 10 and the heating systems are described in Refs [1, 2].

Plasma confinement in GAMMA 10 has been studied with sets of plasma parameters larger than those previously reported [1, 2]. The measured ion confining potential $\phi_c$ for a given $n_c$ increases nearly linearly with the plug ECRH power $P$. The energy confinement time $\tau_E$ is estimated from measured ion losses at the ends (Fig. 1(a)). It is also estimated from the Pastukhov formula by using the measured $\phi_c$, as shown by the dashed lines in Fig. 1(a). The two agree, and $\tau_E$ increases exponentially with $P$ for a given $n_c$.

The barrier potential depth $\phi_b$ and $\phi_c$ are studied as a function of $n_c$. Applying the Boltzmann relation for $\phi_b$ and the theoretical formula for strong ECRH [3] to the GAMMA 10 geometry we obtain:

$$\phi_b + \phi_c \sim T_{ec} \{0.665(n_p/n_c)^{2/3}(n_c/n_b)^{50/63}\}$$

(1)

where $T_{ec}$, $n_p$ and $n_b$ are the central cell electron temperature and the plug and barrier densities, respectively. Since the observed electron pressure $n_e T_{ec}$ is nearly proportional to $P$ for the given $n_p/n_c$, the above formula becomes:

$$\phi_b + \phi_c \propto n_c^{0.21} n_b^{0.79} P$$

(2)

The obtained values of $\phi_b + \phi_c$ after minor corrections due to a small variation in $n_b$ are plotted versus $n_c$ in Fig. 1(b). The above relation, Eq. (2), explains well the experimental result, which is shown by the dashed lines. This supports the potential formation mechanism employed in the strong ECRH theory.

Pitch angle and energy distributions of ions escaping over the plug potential have been determined with a particle analyser that is placed near the end wall. This provides independent verification of the formation of a plug potential, which is used in the scaling studies.

Neutral density in the central cell plasma has been estimated from the measured $H_\alpha$ profiles by the use of an improved neutral particle code (the DEGAS code), which includes the effects of excited hydrogen molecules. The result confirms that the charge exchange process can be neglected in the evaluation of ion confinement scaling near the axis under a good confinement condition.

2. HEATING AND STABILITY IN THE GAMMA 10 TANDEM MIRROR

Two types of ICRF antennas driven at different frequencies — 6.3 and 9.9 MHz — are installed in the central cell to heat the central cell ions and the anchor cell ions, respectively. An $m = \pm 1$ fast Alfvén wave, which is excited in the central cell, is efficiently converted to an $m = -1$ slow Alfvén wave in the anchor cell through resonant mode coupling that is caused by an elliptical, azimuthal modulation ($m = \pm 2$) of the flux tube in the quadrupole transition region [4, 5]. Accordingly,
FIG. 2. (a) Flute stability boundary of GAMMA 10 plasma. The central cell on-axis beta value $\beta_c(0)$ versus the anchor cell beta $\beta_a(0)$, which is an arithmetical average of east and west anchor betas. The symbol \( \perp \) represents a component perpendicular to the magnetic field. The dashed line is the predicted boundary for an isotropic plasma in the central cell ($\beta_{\perp c} = 0.17\beta_{\perp a}$), while the solid line is for an anisotropic plasma with an axially non-uniform pressure profile ($\beta_{\perp c} = 4.0\beta_{\perp a}$). (b) Squared ion temperature anisotropy $(T_i/T_e)^2$ versus average beta $\beta_{\perp c}$ in the GAMMA 10 central cell. Solid and open circles indicate cases with and without magnetic fluctuations due to the AIC instability. Solid curves represent theoretically predicted unstable modes with a fixed growth rate $\gamma$. 
the anchor cell ions are heated not by a slow wave but by a fast wave that is excited in the central cell.

A stability boundary for the flute interchange mode has been experimentally obtained on GAMMA 10, as shown in Fig. 2(a), for widely varied beta values $\beta_{\perp A}$ and $\beta_{\perp C}$ in the anchor cell and the central cell. No plasma can be sustained above the boundary owing to the appearance of violent density fluctuations. The central cell beta value greatly exceeds the stability limit predicted by a simple flute interchange theory for an isotropic plasma. This discrepancy is explained by taking into account the pressure anisotropy due to an intense ICRF wave heating in the central cell.

Microscopic, Alfvén ion cyclotron (AIC) mode instability has been identified in ICRF heated plasmas of the GAMMA 10 central cell. The magnetic field fluctuations strongly depend on temperature anisotropy $T_1/T_2$ and average beta value $\beta_{\perp C}$. The boundary between the regions with and without the magnetic fluctuations is found in $(T_1/T_2)^{2-}\beta_{\perp C}$ space (Fig. 2(b)) and agrees reasonably well with the theoretically predicted stability boundary of a convectively unstable AIC mode [6]. Relaxation of the temperature anisotropy due to AIC mode fluctuations has been observed. However, the plasma confinement has not been affected at the present level of the fluctuations.

Fluctuation studies are performed using the Fraunhofer diffraction method [7] in the GAMMA 10 central cell. The radial electric field $E_r$ is varied by controlling the bias voltage on the radially separated end plate segments, which are installed near the end wall. The unstable modes are identified as electron drift waves. The fluctuation level is at a maximum when $E_r = 0$ and decreases with an increase in $|E_r|$, regardless of its sign. The radial confinement time estimated from the particle balance increases as the fluctuation level decreases. This dependence agrees well with that evaluated from the quasi-linear theory of drift wave turbulence. In plasmas with a relatively peaked density profile, low frequency fluctuations due to the $E_r \times B_z$ driven rotational mode are found in the core region from the reflectometer measurement. The fluctuation level is minimal at weak $E_r$, which is contrary to the case of the drift wave.

Hot electrons with the maximum beta value, which is approximately 20%, are produced in the anchor cell by means of second harmonic ECRH. The spatial distribution of the soft X ray emission, which is measured with pinhole cameras, varies in response to the location of the resonance layer.

3. HEATING AND CONFINEMENT IN THE HIEI TANDEM MIRROR

The HIEI device is a three cell axisymmetric tandem mirror [8]. A helicon/fast wave is excited in the central cell plasma with two ion species (He and H). It has been observed that the fast wave propagating towards the mirror throat mode converts into a slow wave and heats the central cell ions only when an ion–ion hybrid resonance ($\Omega_{i\perp}$) layer exists in the central cell [8]. Minority H ion heating [9] is
FIG. 3. (a) Plasma potential at the ion-ion (He-H) hybrid layer in the HIEI plug cell versus plug electron temperature $T_{ep}$ for cases with (open circles) and without (solid circles) ion-ion hybrid resonance. (b) Limiter biasing experiments on HIEI. Effects of bias voltage on the on-axis density $n(0)$, diamagnetism and density fluctuations.
identified as a heating mechanism by the use of a mass resolved charge exchange analyser and magnetic probes. The fast wave partly propagates into the plug cells and encounters the $\Omega_{ii}$ layer from the high field sides. Strong electron heating and potential formation localized at the $\Omega_{ii}$ layer have been observed (Fig. 3(a)). The cold plasma dispersion relation including the electron inertia predicts that the fast wave incident from the high field side mode converts into a short wavelength electrostatic wave that is resonant at the $\Omega_{ii}$ layer. The Landau damping of the latter wave may lead to the electron heating and the associated potential formation.

The effect of the radial electric field on drift waves has also been studied in HIEI. Application of a bias voltage on the central cell limiter changes the potential at the periphery of the plasma. Beyond the critical bias voltage an abrupt decrease of the fluctuation level (Fig. 3(b)), an abrupt change of the azimuthal rotation velocity and a steepening of the peripheral density gradient have been observed. The fluctuation level exhibits a bifurcation with respect to the bias voltage. The plasma density and energy increase with a decrease of the fluctuation level, which indicates improved radial confinement.

4. SUMMARY

In GAMMA 10 the scaling laws for the ion confining potential and the ion confinement time have been examined for large data sets of plasma parameters. They are consistent with the Pastukhov scaling law. The approximately proportional dependence of the confining potential on the plug ECRH power and its weak dependence on the central cell density are encouraging for the feasibility of a tandem mirror reactor. An MHD stability boundary has been observed. This boundary is consistent with the flute mode theory that takes into account the anisotropic pressure of the ICRF heated plasma. Density fluctuations caused by the drift wave and the rotational mode have been identified. The fluctuation due to the drift wave is suppressed by a strong radial electric field regardless of its sign, and thus the effects on radial confinement become small when the confining potential is formed. The rotational mode that is observed in plasmas with a peaked density profile could be minimized by controlling the end plate biasing.

In HIEI, a helicon/fast wave excited in a two ion species (He–H) central cell plasma is found to mode convert into an electrostatic wave in the plug mirror as well as into a slow wave in the central cell. The former conversion is effective for heating plug electrons and for enhancing the plug potential, the latter for heating central cell ions. Limiter biasing is found to be effective for suppressing the drift wave induced fluctuations and for improving the radial confinement.

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TWO STAGE DENSE PLASMA HEATING IN THE GOL-3 DEVICE

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Abstract

TWO STAGE DENSE PLASMA HEATING IN THE GOL-3 DEVICE.

A review of the activities of the GOL-3 programme is presented. The first group of results concern the experiments on uniform plasma heating by a 100 kJ electron beam. Heating of the bulk plasma electrons up to 1 keV at $10^{15} \text{cm}^{-3}$ density is achieved. The second group of experiments deal with ‘two stage’ dense plasma heating. Hydrogen clouds of $10^{16}-10^{17} \text{cm}^{-3}$ density and 0.5-3 m length are produced by gas puffing. The energy delivered by the electron beam to the $10^{15} \text{cm}^{-3}$ background plasma occupying the rest of the device is then efficiently transferred to these clouds, giving rise to the formation of a hot dense plasma.

1. INTRODUCTION

Previous experiments on collective plasma heating by high power microsecond relativistic electron beams have shown that under certain conditions the electron beam can transfer to the plasma up to 25% of its initial energy (see, for example, Ref. [1]). High efficiency of beam relaxation is reached with a plasma density of less than $2 \times 10^{15} \text{cm}^{-3}$. The energy lost by the electron beam is transferred mainly to plasma electrons.

To raise the plasma density accessible for this heating technique, a ‘two stage’ heating scheme was proposed [2] in which hot plasma electrons were supposed to serve as a secondary source of heating of the dense plasma clouds, adjacent to the region of lower density (where effective electron beam–plasma interaction is observed).

The present experiments were performed on the GOL-3 facility (Fig. 1), which is a 7 m long mirror trap with a 5.5 T homogeneous magnetic field and an 11 T field in the end mirrors. After the discharge chamber was filled with hydrogen of $10^{14}-10^{15} \text{cm}^{-3}$ density, a gas cloud with a density of up to $2 \times 10^{17} \text{cm}^{-3}$ and a length of 0.5–3 m was created by gas puffing. Then preliminary gas ionization was produced by a linear discharge. The electron beam (0.8 MeV, 40 kA, 3–5 $\mu$s) was injected into this plasma with a controlled delay time with respect to the gas puff that allowed the gas cloud length to be varied.
FIG. 1. Layout of the GOL-3 experiment.
FIG. 2. (a) Axial hydrogen atomic density distribution; (b) axial plasma pressure distribution; (c) plasma pressure at 40 cm distance from the entrance foil. Thin lines refer to the case of injection into a homogeneous plasma; thick lines refer to the short gas cloud. Large dot: Thomson scattering data; other symbols: diamagnetic loop data.
The plasma that can in principle be obtained in this device after raising the beam energy content up to the design value of 0.5 MJ is of interest for a broad spectrum of applications, such as controlled fusion, pulsed neutron sources, X ray flash lamps and UV lasers. At present the experiments are carried out at the first stage of the device, with a beam energy content of up to 100 kJ.

2. HEATING OF HOMOGENEOUS PLASMA

Uniform plasma heating was studied using new diagnostics, including two Thomson scattering systems. Figure 2(c) shows the typical time evolution of the plasma pressure, obtained from the diamagnetic measurements. The temperature obtained from laser measurements \( z = 270 \text{ cm} \) is shown to be \( 0.6 \pm 0.2 \text{ keV} \) at the heating maximum at \( (1 \pm 0.2) \times 10^{15} \text{ cm}^{-3} \) density, and the values of the 'laser' and the 'diamagnetic' temperatures coincide within the measurement accuracy. If at other distances from the entrance foil the measured value of the diamagnetic temperature is also basically determined by the Maxwellian electrons, then the plasma electron temperature in the vicinity of \( z \sim 0.5 \text{ m} \) should be \( 0.8-1 \text{ keV} \) at the density given above at the heating maximum.

The calculations of heat transport in the plasma show that the heat conductivity should be heavily suppressed during the beam injection to allow high temperature gradients in the relatively long lived sub-keV plasma. After the end of the heating pulse the level of plasma turbulence decreases rapidly and the plasma heat conductivity becomes classical with \( Z_{\text{eff}} \sim 1.2-1.6 \).

Soft X ray measurements show that the high energy tail of the electron distribution has a mean energy of at least 10 keV and a density of several per cent of the plasma density up to the end of the heating pulse. The role of these electrons in the energy balance is relatively greater owing to their short transit time and non-classical scattering. Measurement of the energy spectrum of escaping electrons at the exit mirror gives \( \sim 10 \text{ MW/cm}^2 \) flux of 10-100 keV electrons from the plasma to the ends of the device. This energy flux can be utilized in the two stage scheme of dense plasma heating.

3. TWO STAGE DENSE PLASMA HEATING

Here we present results on the two stage heating of the plasma using short, dense \( (10^{16}-10^{17} \text{ cm}^{-3}) \) gas clouds in a long, low density \( (10^{14}-10^{15} \text{ cm}^{-3}) \) background plasma column. Detailed discussion of these experiments can be found in Ref. [3].

The major part of the experiments were performed with a background homogeneous plasma density of \( (3-5) \times 10^{14} \text{ cm}^{-3} \), at which the electron beam interacts effectively with the plasma. The measurements by the exit calorimeter and
FIG. 3. (a) Axial density and temperature distribution in the case of a long cloud; (b) plasma pressure over the plasma column length. Dots: Thomson scattering data.
two beam energy analysers show that the total beam energy losses are up to 20–25% in these experiments, which for different shots corresponds to absolute losses of 10–15 kJ.

The effect of two stage heating of the gas cloud is illustrated in Fig. 2. The plasma pressure $nT$ at the point $z = 40$ cm (near the maximum of cloud density) becomes 3–4 times higher than that of the homogeneous plasma. With a cloud length of $\sim 50$ cm the pressure reaches its maximum of $nT = 2.7 \times 10^{18}$ eV/cm$^3$ at a local plasma density of $\sim 2 \times 10^{16}$ cm$^{-3}$.

With the change of the initial cloud length the region of maximum pressure follows the cloud boundary with the background rare plasma. The plasma pressure inside the cloud is lower but always exceeds substantially the pressure obtained through the direct beam–plasma interaction at this cloud density.

For the case of a long cloud (Fig. 3) the peak electron temperature reaches $\sim 0.2$ keV at $6 \times 10^{15}$ cm$^{-3}$ density at the point of the laser measurements ($z = 270$ cm). By changing the initial density distribution an electron temperature of 0.1–0.3 keV can be obtained with $10^{16}$ to $3.5 \times 10^{15}$ cm$^{-3}$ density at this point.

The expansion rate of the gas cloud substantially changes with the start of heating. A complex picture of the plasma flow and pressure wave generation with secondary maxima on the diamagnetic signals is observed after termination of the beam injection.

4. CONCLUSIONS

(a) An electron temperature of 1 keV is achieved on the GOL-3 device during the heating of a 7 m long, $10^{15}$ cm$^{-3}$ plasma by a 100 kJ electron beam.

(b) The feasibility of the two stage heating of a $10^{16}$–$10^{17}$ cm$^{-3}$ plasma has been demonstrated in experiments with dense plasma clouds. A threefold increase in the plasma energy density with respect to the case of a uniform plasma is obtained.

(c) The experiments performed show the possibility of obtaining dense plasma clouds with $\beta \sim 1$ on the GOL-3 facility. For this purpose further upgrading of the solenoid length and initial beam energy content is planned.

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PHYSICS STUDIES FOR THE TORSATRON TJ-IU

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Abstract

PHYSICS STUDIES FOR THE TORSATRON TJ-IU.

TJ-I Upgrade (TJ-IU) is a six period, \( t = 1 \) torsatron in the final stages of construction at CIEMAT, Madrid. Its major radius is 0.6 m and its average plasma radius 0.1 m. The device was designed to bridge the gap between the TJ-I tokamak actually working at CIEMAT and the Heliac TJ-II that will start operation in 1995. TJ-IU will share with TJ-I its power supplies and peripherals — hence its name. The main theoretical predictions as to equilibrium, stability, transport and kinetic theory are described.

1. DESCRIPTION OF THE DEVICE

The required magnetic field of TJ-I Upgrade (TJ-IU) is created by the five coil system shown in Fig. 1. The main helical coil winding follows the law:

\[ \phi = \left( \frac{1}{6} \right) (\theta + 0.4 \sin \theta) \]

where \( \phi \) is the toroidal and \( \theta \) the poloidal angle. This modulation is essential to achieving a deep magnetic well with moderate ripple at axis. The helical coil has 20 turns, with a total current of 280 kA turn. In addition to the helical coil, two pairs of vertical field coils are needed. The external pair has a radius of 1 m and is placed at 0.35 m of height, symmetrically up and down the equatorial plane; the internal pair has a radius of 0.30 m and a height of 0.40 m. For the standard case the external pair current is 122 kA turn, opposite to the helical current, while the internal pair current is 49 kA turn. Two pairs of additional coils are also used in order to compensate small currents in the plasma.

The device has been designed to have a wide operational flexibility, i.e. different values of rotational transform and magnetic well depth can be achieved by changing the ratio between the currents of the vertical field and the helical coils. This change shifts the magnetic axis of the position, thereby producing the basic configuration variation. With due account for the current limits imposed by the available power supplies, TJ-IU can shift its magnetic axis between 6 cm inside and 3 cm outside the nominal \( R_0 \) (\( R_0 = 0.58 \) m). This shift produces a range for the rotational transform at axis of \( 0.14 \leq t(0) \leq 0.40 \) while the shear may be changed between \(-5\%\) and \(37\%\) and the magnetic well depth between \(0\%\) and \(7\%\), always maintaining
an average plasma radius greater than 7 cm and the magnetic field intensity of 0.52 T at the magnetic axis with a magnetic ripple of 7.6%. Startup and heating will be done initially with a 200 kW, 28 GHz gyrotron working at the second harmonic; an ion cyclotron heating (ICH) system is under design. Magnetic surface mapping experiments are expected to begin in November 1992 and first plasmas by spring 1993.

2. EQUILIBRIUM AND STABILITY

The strong modulation of the helical coil produces a deep vacuum magnetic well (7%). This is the main stability mechanism in the machine since its shear is almost negligible for the configuration with the magnetic axis at its 'standard' position (0.58 m). The average radius of the last closed magnetic surface inside the vacuum chamber is $\langle a \rangle \approx 10$ cm. The average toroidal field at axis is 0.62 T, with a magnetic ripple of 7.6%.

With the fixed boundary version of the 3-D code VMEC [1], equilibria for two different configurations of the machine have been obtained, and a sequence of zero net current equilibria has been generated. Figure 2 shows the equilibrium magnetic axis shift for the standard configuration, and Fig. 3 shows the flux surfaces at four toroidal positions for this case $\langle \beta \rangle \approx 2\%$. The Shafranov shift of the surfaces and the helicity of the magnetic axis are to be seen clearly. The position at $\xi = 30^\circ$, corresponding to the most triangular cross-section, is the one chosen for microwave injection. The pressure profile used is the usual $p \propto (1 - \Phi)^2$, where $\Phi$ is the toroidal flux normalized to 1 at the edge. The two configurations considered are the 'standard' one for which parameters such as plasma volume, magnetic well depth, etc.
FIG. 2. Toroidal and helical shifts for increasing pressure.

FIG. 3. Flux surfaces of the standard configuration at $\langle \beta \rangle = 2\%$ for four toroidal positions ($\xi = 0^\circ$, $15^\circ$, $30^\circ$ and $45^\circ$).

FIG. 4. Mercier criterion ($D_M$) in arbitrary units for several values of the pressure versus radius.
have been optimized and the 'inward' configuration, characterized for a position of the magnetic axis in vacuum shifted 6 cm towards the axis of the machine. The equilibrium quantities obtained have then been used to evaluate the 3-D Mercier stability criterion for local instabilities [2]. Figure 4 summarizes the results obtained for the standard configuration. A positive $D_M$ satisfies the stability criterion for Mercier modes. The configuration is stable to Mercier modes for the whole radius and pressures considered, even at $\langle \beta \rangle = 3\%$. The valley in the criterion at average radius 0.4 is due to the passing through a lower order resonance of the iota profile that should flatten the pressure profile. We have chosen Mercier modes for our $\beta$ studies since they have been shown to give a limit to the pressure in other torsatrons [3]. Nevertheless, a study is under way to check the TJ-IU stability to ballooning modes.

From the Mercier studies we can conclude that the $\beta$ limit in TJ-IU is well above the achievable $\beta$ value for the 200 kW of ECH heating power available initially although the possibility to inject 600 kW of ICH in a second phase will permit us to reach $\beta$ values closer to its theoretical limit in some configurations.

This stability picture changes dramatically when we push the magnetic axis inwards, towards the centre of the machine. The stabilizing vacuum magnetic well is destroyed and the configuration becomes unstable to Mercier modes, even at very low values of the pressure. This strong stability antagonism between both configurations will permit us to experimentally check the influence, if any, of the fraction of trapped particles on the stability of the machine. This fraction changes considerably from 11\% for the standard configuration to 68\% for the inward one.

3. TRANSPORT

Calculations of neoclassical transport coefficients for the TJ-IU have been made by means of a Monte Carlo code [4, 5]. Figure 5 shows the neoclassical electron heat conductivity versus collisionality for several values of the radial electric field. The
axisymmetric coefficient for equal rotational transform is also shown for comparison as a dashed line. Evidently, the neoclassical transport in TJ-IU is higher than in the equivalent tokamak for all regimes including the plateau and collisional regime. Displacing the magnetic axis inwards we can change the magnetic field configuration and obtain a lower effective field ripple, improving the transport properties by a factor of two. It is worth while to note that transport and stability seem to improve in opposite direction with respect to magnetic axis shift.

By assuming parabolic profiles for density, temperature and radial electric potential, a simple estimate of the neoclassical confinement times on the half-radius surface was found. The magnitude of the electric field was taken from the solution of the ambipolarity condition, \( \Gamma_e(\Phi') = \Gamma_i(\Phi') \). Taking as central temperatures for an electron cyclotron heating (ECH) scenario, \( T_e(0) = 400 \text{ eV}, T_i(0) = 50 \text{ eV} \) and a density of \( n_c(0) = 0.5 \times 10^{13} \text{ cm}^{-3} \), the ambipolar electric potential was found to be \( e\Phi = 350 \text{ eV} \), the particle confinement time \( \tau_p = 0.37 \text{ ms} \) and the energy confinement time \( \tau_E = 0.17 \text{ ms} \) in the reference case. In the case of a displaced magnetic axis (6 cm) the results were \( e\Phi = 100 \text{ eV}, \tau_p = 0.50 \text{ ms} \) and \( \tau_E = 0.25 \text{ ms} \).

Calculations of bootstrap currents in TJ-IU have been also made by means of the analytical expressions for non-axisymmetric devices of Ref. [6, 7]. Figure 6 shows the expected current profiles for the reference case and for the case with inwards displacement of the magnetic axis. The total currents are 3.7 kA and 1.6 kA, respectively, with their direction opposite to the magnetic field. As in radial transport, the magnitude is lower for the displaced axis case.

4. ELECTRON CYCLOTRON CURRENT DRIVE

One of the main topics of research on this machine will be the study of ECH plasmas and, in particular, of the possibility to compensate induced currents such as
bootstrap and rotational transform effects by using electron cyclotron (EC) waves. The induced current, parallel to the magnetic field, in the TJ-IU torsatron was computed in terms of the relativistic response function and the absorbed power density [8]:

$$J_i(r) = -\frac{mc^2}{8\pi n A e^3} \int d\vec{v} \eta(\vec{v}) \sum_s w_s(\vec{v})$$

where \(n, m\) and \(e\) are the electron density, mass and charge, respectively, \(c\) is the speed of light, \(A\) is the Coulomb logarithm and \(\vec{v} = \vec{p}/mc\).

For arbitrary diffusion in momentum space, i.e. for any kind of wave, the microscopic efficiency can be written as [9]:

$$\eta(\vec{v}) = \frac{\delta I_i}{\delta P_d} = G(v) \left[ N_i - \frac{v_i}{v^2} (\gamma + 1 + Z) \right] + \frac{2vv_i}{\gamma^2}$$

where we have introduced the function

$$G(v) = \frac{2}{v} \left[ \frac{\gamma(v) + 1}{\gamma(v) - 1} \right]^{1+Z/2} \int_0^v dx \left[ \frac{x}{\gamma(x)} \right]^3 \left[ \frac{\gamma(x) - 1}{\gamma(x) + 1} \right]^{1+Z/2}$$

and \(\gamma(x) = (1 + x^2)^{1/2}\).

The absorbed power density in phase space is calculated for EC waves at a harmonic of order \(s\), for a Maxwellian distribution function and at first order in the Larmor radius. We introduce a macroscopic current drive efficiency in the plasma:

$$\gamma(\vec{r}) = -\frac{mc^2}{8\pi n A e^3} \int d\vec{v} \eta(\vec{v}) \sum_s w_s(\vec{v})$$

FIG. 7. Absorbed power and induced current density versus radius obtained with one 200 kW, 28 GHz gyrotron launched in the X-mode \((n_e = 0.5 \times 10^{19} m^{-3} \text{ and } T_e = 0.5 \text{ keV})\).
and the induced current density will be

\[ J_1(\mathbf{r}) = \gamma(\mathbf{r}) W_1(\mathbf{r}) \]

The mean absorbed power density at a magnetic surface is calculated by using the ray tracing code RAYS [10] and the volume of the magnetic surface considered is computed in terms of magnetic co-ordinates. The plasma parameters needed to calculate the efficiency are also given by the code.

The microwave beam is considered to be 5 cm wide which is not negligible because the plasma minor radius is about 10 cm. To take this fact into account we simulate the microwave beam by a set of 64 rays weighted by a Gaussian. So the actual current density will be given by

\[ \int J_{\text{actual}}(\mathbf{r}) = \sum_{i=\text{rays}} \sum_{j=\text{rays}} J_i(\langle r \rangle) \exp\left(-\frac{x^2}{(d/2)^2}\right) \]

The results for a typical case are shown in Fig. 7, where the absorbed power and the induced current densities are plotted against the mean minor radius. The launched power is 200 kW at the second harmonic (f = 28 GHz) of the X-mode. Even though the plasma density and temperature are not very high (n = 0.5 × 10^{19} m^{-3} and T = 0.5 keV) we obtain a rather high current of 2.5 kA because the volume of the device is small and therefore we usually have a high power density. The total current is obtained by integrating the current density, which is a function of the particular magnetic surface:

\[ I = \int \mathbf{J} \cdot d\mathbf{S} = 2\pi \int_0^a \langle r \rangle J(\langle r \rangle) d\langle r \rangle \]

5. CONCLUSIONS

The standard configuration of TJ-IU is stable to ideal interchange modes for values of \( \beta \) much higher than those achievable with the available power. Trapped particle effects could be visible experimentally.

Neoclassical transport and bootstrap current are very sensitive to the shift of the magnetic axis in TJ-IU. This feature will allow experimental testing of theoretical models.

TJ-IU high power density value enables the attaining of relatively high EC driven currents that will make possible the compensation of plasma currents induced by bootstrap or iota effects.
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EDGE TURBULENCE AND TRANSPORT STUDIES*

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Abstract

EDGE TURBULENCE AND TRANSPORT STUDIES.

Using data from Langmuir probe measurements in tokamak, stellarator and reversed field pinch devices the authors compare and contrast fluctuation and transport related phenomena; namely the electrostatic and electromagnetic nature of the fluctuations, the scaling of various transport related parameters, and comparison of turbulence driven fluxes with estimates of the total radial fluxes.

Introduction

Study of edge turbulence and transport in different toroidal confinement systems [1-3] allows comparisons of experimental results with theoretical predictions over a wide range of parameters. To this end, a self-contained Langmuir probe diagnostic facility equipped with its own data acquisition electronics and data analysis package has been deployed in collaborative measurements in various devices [4-6] including the stellarator ATF, the reversed field pinch ZT40M and the tokamaks Phaedrus-T, TEXT, TFTR, and Versator. The diagnostic provides simultaneous local measurements of equilibrium and fluctuating (typically 1 - 500 kHz) electron temperature ($T_e$), density ($n$), and plasma potential ($\varphi_p$), as well as turbulence induced particle and electron energy fluxes. Table 1 summarizes some of the discharge and edge plasma parameters.

Turbulence characteristics

Significant levels of fluctuations exist in all the devices studied (Table 1). The ordering of the normalized rms amplitude ($e_1 \varphi_p l / T_e > l p l / p > l n l / n > l T_e V_e$ where $p = nT_e$) and the anti-phase correlation between density and temperature fluctuations [7] imply that thermal drive [8] is not likely to be the principal driving mechanism for these non-Boltzmann fluctuations though the thermal condensation drive might still be relevant. In tokamaks, a velocity shear layer having thickness $\sim 1$cm and poloidal velocity shear $dV_\theta / dr \sim 5 - 10 \times 10^5$ s$^{-1}$ is found close to the last closed flux surface. In the pinch ZT40M, no corresponding toroidal velocity shear layer was observed. Near the shear layer, local changes in the turbulence characteristics (e.g. reduction in the average frequency and increase in decorrelation) support the idea of shear suppression of turbulence [9,10].
Table 1. Edge plasma parameters close to $r = a$ or the last closed flux surface. The average phase velocity $V_{ph}$ is obtained from a two-point cross-correlation and $V_{ExB}$ is the poloidal rotation derived from the radial electric field.

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<td>0.2-0.4</td>
<td>0.6-0.8</td>
<td>0.3-0.4</td>
<td>0.4-0.5</td>
<td>0.5-0.6</td>
</tr>
<tr>
<td>$</td>
<td>V_{ph}/V_{ExB}</td>
<td>$</td>
<td>$\geq 1$</td>
<td>$&gt; 1$</td>
<td>$\geq 1$</td>
</tr>
<tr>
<td>$k_{\perp}$ (cm$^{-1}$)</td>
<td>2</td>
<td>0.1</td>
<td>2.5</td>
<td>1.5</td>
<td>0.8</td>
</tr>
<tr>
<td>$k_{\perp}\rho_s$</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.02</td>
</tr>
<tr>
<td>$\Gamma_{s\perp}$ ($10^{21}$ m$^{-2}$s$^{-1}$)</td>
<td>0.02</td>
<td>4</td>
<td>0.7</td>
<td>0.7</td>
<td>0.05</td>
</tr>
<tr>
<td>$\Gamma_{\infty}$ ($10^{21}$ m$^{-2}$s$^{-1}$)</td>
<td>0.02</td>
<td>7</td>
<td>0.5</td>
<td>1</td>
<td>0.06</td>
</tr>
</tbody>
</table>

It is found that $\bar{n}/n$ can be approximately described by a simple mixing length estimate using an average power-weighted wavevector $\vec{k}_{\perp}$ to represent the radial structure i.e. $\bar{n}/n = \vec{k}_{\perp} \cdot L_n$ with $L_n$ the density scale length. Generally $\vec{k}_{\perp}$ varies with $B$, or in normalized quantity $\vec{k}_{\perp}\rho_s = 0.05$ to 0.1, with $\rho_s$ an ion gyroradius using $T_e$. The TFTR data has smaller $\vec{k}_{\perp}\rho_s \sim 0.01$ to 0.02. Drift wave theories (e.g. 11) expect $\vec{k}_{\perp}\rho_s \sim 1$.

To model the turbulence, it is necessary to know if magnetic contributions must be considered i.e. if the 'electrostatic' approximation is valid. According to
FIG. 1. Estimates of the parameter $g$, the ratio of parallel electric field induced from magnetic activity to the total. The values of $g_1$ and $g_2$ may be considered as the upper and lower bound estimates of $g$. Radius $a_s$ corresponds to the last closed flux surface or the leading edge of the limiter. The two straight lines are exponential fits to the TEXT data.

The Ampere's law and the parallel Ohm's law, $$(\eta/\mu_0)\nabla \cdot \mathbf{A}_\perp = \eta J_{\parallel} = -\nabla \phi_\parallel - i\omega \mathbf{A}_\parallel,$$
the 'electrostatic' limit corresponds to $g = \mu_0 \omega / \eta \nabla \mathbf{A}_\perp \approx 1$. The parameter $g$, a ratio of a 'skin time' to the mode period, is evaluated by approximating $\nabla \mathbf{A}_\perp$ by the measured $k_\perp^2$ to give $g_1$ (fine scale electrostatic turbulence limit) and by taking an estimate of the magnetic mode width (i.e. $\nabla \mathbf{A}_\perp \approx (3\rho_s)^{-2}$) to give $g_2$. From these values (fig. 1), we conclude that it is generally accurate to describe the fluctuations as electrostatic in the scrape off layer (SOL) except for ZT40M data.

**Transport**

We found that generally the density and temperature scale lengths (or e-folding lengths $L_n$ and $L_T e$), often the only data available for scaling study of transport related parameters, are themselves related to transport properties (particle and energy fluxes) so that other published data is available for scaling purposes. Over a range of nearly 3 orders of magnitude, the measured electrostatic turbulence accounts for a large part of the density dependence in the SOL. This means that
FIG. 2. Scale lengths of temperature $L_T$ and density $L_n$ in the scrape off layer. The two lines correspond to $\zeta = 0$ and $0.4$ with an electron parallel energy transmission coefficient of $\gamma_t = 5$.

locally the fluctuation driven particle flux $\Gamma_{\text{E}}$ can be approximated by $\Gamma_{\text{SOL}} = 0.5n c s L_n/L_c$. We note that correction for temperature fluctuations [7] is essential in obtaining these measurements of $\Gamma_{\text{E}} = <\bar{n}E_\perp>/B$ where $<...>$ denotes an ensemble average.

The electrostatic fluctuation driven electron energy flux consists of a convective part $Q_{T e \bar{E}} = 5T_e <\bar{n}E_\perp>/2 = 5T_e <\bar{n}E_\perp>/(2B)$ and a conductive part $Q_{nT e \bar{E}} = 5n <T_e E_\perp>/2B$. We found that in tokamaks electrostatic turbulence explains edge energy transport, with a ratio of electron energy conduction to convection $\zeta = 0.2$ to $0.4$, and in pinches magnetic turbulence explains energy transport, with electrostatic turbulence explaining particle transport. These conclusions can be tested by investigating the relationship between $L_n$ and $L_T$ in the SOL. If conduction and convection are the only means of perpendicular energy loss, and if there are no plasma sources, then balancing perpendicular and parallel particle and energy fluxes leads to $\zeta = 2\gamma_t (L_T/L_n + \frac{1}{2})/(L_T/L_n + \frac{3}{2}) - 1$ where $\gamma_t$ denotes the electron parallel energy transmission coefficient [12]. Only two machines exhibit values of $L_T/L_n > 2$, which imply $\zeta > 0.4$ (fig. 2). The ZT40M result was expected, because of the high levels of magnetic fluctuations measured in this low field device. The ATF results might be explained by the presence of a stochastic magnetic field associated with the destruction of the separatrix.
Conclusions

There are notable similarities in the properties of the edge turbulence in different toroidal confinement systems. Generally these fluctuations have $k_p \sim 0.1$ and can be approximately described by a simple mixing length estimate. Among others, thermal condensation might be a relevant driving mechanism. It is generally accurate to describe the fluctuations as electrostatic in the scrape off layer, except for ZT40M. This is consistent with the results that electrostatic turbulence can explain edge density scale lengths in all three device types and determine edge thermal transport in tokamaks and stellarators, while magnetic fluctuations explain edge thermal transport in reversed field pinches.

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CONFINEMENT OF RF HEATED PLASMA IN THE URAGAN-3M TORSATRON

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Abstract

CONFINEMENT OF RF HEATED PLASMA IN THE URAGAN-3M TORSATRON.

The latest results of RF heated plasma confinement experiments with a helical divertor in the Uragan-3M torsatron are presented together with their theoretical analysis. A new, compact antenna for Alfvén plasma heating has been used in these experiments together with the traditional frame type antenna. The results of operation with both types of antenna are presented.

1. INTRODUCTION

The Uragan-3M (U-3M) magnetic system is located in the large vacuum chamber to ensure spatial divertor operation [1]. The system includes the helical winding (ℓ = 3, m = 9, R = 1 m) and four vertical magnetic field coils. Results have been obtained for two values of B⊥: B⊥/B0 = 0% and B⊥/B0 = 1.2%. In the latter case the magnetic axis is shifted with respect to the geometrical axis by 5 cm to the outer side (aρ ≤ 12.5 cm, c(aρ) = 0.29 [2]).

Antennas of two types have been used in plasma production and heating experiments in the ion cyclotron range of frequencies ω < ωci. Both types of antenna use Alfvén resonance excitation in a multimode regime. An electrostatic mode which appears at the Alfvén resonance layer is strongly absorbed by electrons. The first antenna, which is a frame type antenna (FTA) [3] (Fig. 1(a)), has been used for
FIG. 1. Schematic view of the two types of antenna used in the experiments: (a) the FTA for initial plasma production and heating; (b) the new THTA for plasma heating.

plasma production and heating with a fixed input power $P_{RF} \leq 200$ kW in the range of values $1 \times 10^{12} \leq n_e \leq 9 \times 10^{12}$ cm$^{-3}$ and $0.45 \leq B_0 \leq 1.3$ T. The regimes of stationary gas puffing, pulse gas puffing and hydrogen pellet injection have been studied. The second antenna, a three-half-turn antenna (THTA) [4] (Fig. 1(b)), has been used for the heating of plasma initially produced by the FTA.

2. CONFINEMENT OF PLASMA PRODUCED AND HEATED BY THE FTA

2.1. Low density plasma regime

For low density plasma ($n_e < 5 \times 10^{12}$ cm$^{-3}$), radial distributions have been measured of plasma density $n_e(r)$ (by multichord interferometry and reflectometry) and temperature $T_e(r)$ (by laser scattering and EC emission diagnostics) (Fig. 2). A hollow temperature profile and a relatively peaked density profile have been observed. The energy spectra of charge exchange neutrals measured in tangential and perpendicular directions to the toroidal plane show a two temperature ion distribution (Fig. 3(a)). Doppler broadening of the C(V) line (227.1 nm) indicates that these impurity ions are in equilibrium with the lower temperature part of the hydrogen ion distribution $T_i(C(V)) \sim T_{\|}$ during almost the whole RF pulse duration. By changing the entrance slit width of the longitudinal charge exchange analyser, a qualitative picture of the spatial localization of ions with $T_{\|}$ and $T_{\perp}$ has been obtained (Fig. 3(b)). After the RF pulse was switched off the high energy ion temperature decay time was three times less than that of the low energy ions. This result indicates high energy ion generation at the plasma edge.

For the low density regime ($n_e < 5 \times 10^{12}$ cm$^{-3}$), the positive electric field value in the plasma periphery, $E_r \sim 100$ V/cm [5], has been measured.

The plasma energy global confinement time for the low density plasma regime ($n_e = (2-3) \times 10^{12}$ cm$^{-3}$ and $B_0 = 0.46$ T), $\tau_{\|}^E = W/P_{RF} = 0.3$ ms, has been found to be approximately two times less than the decay time of $T_e$ after the RF pulse switch-off. This indicates enhanced plasma particle and heat fluxes during the RF pulse. Plasma flux from the confinement volume to the divertor layer increased with
the RF power input, evidently confirming particle loss dependence on the RF power level [6].

In the U-3M torsatron the hydrogen atom density in the central part of the plasma column \((r = 4 \text{ cm})\) has been measured by means of laser fluorescent spectroscopy. The resonance excitation of the \(H_a\) line has been produced by a pulse dye laser with lamp pumping. The spectral width was 1 nm. The spectral flux of the probing beam was \(\sim 20 \text{ kW cm}^{-2} \text{ nm}^{-1}\), which enabled measurements in the saturation regime (saturation parameter \(s > 4\)) to be performed. A two channel scheme of registration was used. The detection solid angle was \(2 \times 10^{-3} \text{ sr}\). In the low plasma density regime \((n_a = (2-3) \times 10^{12} \text{ cm}^{-3}\), the population of excited states due to the dissociation channel affected the results of the hydrogen neutral density \((n_a)\) measurements. In the central part of the plasma the hydrogen atom density value \(n_a = 8 \times 10^9 \text{ cm}^{-3}\) has been obtained. The particle confinement time \(\tau_n = 4 \text{ ms}\) has been calculated from this value of neutral atom density. The Langmuir probe measurements of particle flux through the LCMS give an estimate of the particle confinement time at the plasma periphery of \(\tau_n = 0.5 \text{ ms}\) (Fig. 4).
All the experimental results have been obtained in the regime with stationary hydrogen pressure into the vacuum chamber.

The experimental data obtained have been analysed by 1-D numerical simulation to study particle and heat transport in the low density RF plasma discharge in U-3M. The modelling performed is distinguished by the simultaneous application of the transport code and the code calculating power deposition profiles [7]. The power deposition to electrons and ions has been calculated by numerical solution of the Maxwell equations with realistic antenna RF currents. The model of a non-uniform cylinder with identical ends has been used.

The excitation of both the fast (electromagnetic) and slow (kinetic Alfvén) waves and the effects of their mutual conversion have been studied. The linear mechanisms of the electron Cherenkov and ion cyclotron absorption have been taken into account. Ion cyclotron absorption of RF power was negligibly small.

The calculations have shown the amplitudes of excited waves to be high enough that the relative velocity of electrons and ions $u_E$ becomes comparable with the ion thermal velocity $v_T$. In this case the short wavelength ion Bernstein waves can be excited owing to instability [8], resulting in turbulent heating of ions and electrons with temperature growth rate $\tau_{\text{heat}} = \omega_{ci}(u_E/v_T)^5$ [9]. The ion temperature growth rate becomes very high (of the order of $10^5$ s$^{-1}$) at the plasma periphery on the low field side where the ion cyclotron frequency is very close to the wave frequency and peripheral long wavelength modes are effectively excited by the FTA, whereby the existence of the high temperature ion fraction observed in the CX measurements can be explained.

In the balance code the particle and energy fluxes take into account all regimes of neoclassical transport [10] and anomalous transport. The phenomenological elec-
tron and ion anomalous transport coefficients were fitted in the region of destroyed magnetic surfaces to obtain a satisfactory agreement between measured and calculated density and temperature profiles. The anomalous transport coefficients in particle and heat fluxes appeared to be

\[ \chi_{e}^{an} = 2 \times 10^{5}(1.2r/a)_{p}^{6}(T_{e}/100)^{1/2} \text{ cm}^{2}/\text{s} \]

for electrons, whereas for ions

\[ \chi_{i}^{an} = 2 \times 10^{5}(1.2r/a)_{p}^{6}(T_{i}/100)^{1/2}(m_{e}/m_{i})^{1/2} \text{ cm}^{2}/\text{s} \]

where \( T_{e,i} \) is in eV. The radial electric field value was obtained from the condition of electron and ion fluxes being equal. As \( \chi_{e}^{an} \gg \chi_{i}^{an} \) in the region of destroyed magnetic surfaces the ambipolar electric field is positive at the plasma edge, in agreement with the measurements [5]. The comparison of experimental data and results of numerical modelling is presented in Figs 2 and 4. It was assumed that the main radiation losses \( (W_{rad} \sim 7 \text{ kW}) \) occur from the central region of the plasma.

### 2.2. High density plasma regime

The high density plasma confinement regime has been studied with both stationary gas puffing and hydrogen pellet injection into the plasma.
Figure 5 shows the temporal evolution of some plasma parameters during the RF pulse and hydrogen pellet injection. The pellet, with a total number of atoms $\sim 2 \times 10^{19}$, was injected tangentially into the target plasma with density $n_e = 4 \times 10^{12} \text{ cm}^{-3}$ ($B_0 = 1.2 \text{ T}$). The fast density rise corresponds to the input of approximately $2 \times 10^{19}$ new particles, i.e. only 10% of the pellet particles were evaporated, because of a rather fast electron temperature reduction. Similar electron temperature behaviour has been observed during the plasma density rise because of the increase of hydrogen pressure in the vacuum vessel.

The plasma energy global confinement time $\tau_E^G$ at different plasma density and confining magnetic field values in U-3M is in agreement with Heliotron-E $\tau_E^Q$ scaling [11] (Fig. 6).

3. NEW, COMPACT THREE-HALF-TURN ANTENNA FOR PLASMA HEATING

One dimensional numerical simulation has shown that the efficiency of plasma heating by the FTA is reduced with the plasma density rise. The FTA's current spectrum maximum corresponds to waves with $k_1 = 0.1 \text{ cm}^{-1}$. For $B_0 = 0.45 \text{ T}$ and

![Graph showing temporal evolution of plasma parameters]
\( n_e = 10^{12} \text{ cm}^{-3} \) the position of the conversion layer for these waves corresponds to the inner part of the plasma column, but the waves with long wavelength excited by the antenna are absorbed at the plasma periphery. With the density rise the region of local Alfvén resonances for the whole antenna spectrum is shifted to the plasma periphery, reducing the plasma heating efficiency (Fig. 7, curves 1).

To avoid the deleterious effect of the power deposition profile shift to the plasma periphery with the plasma density rise a new, compact THTA system was proposed and has been studied numerically and adapted for the U-3M torsatron. The 1-D calculations showed that the power deposition profile of the THTA was always better than that of the FTA (Fig. 7, curves 2). For the low density case the FTA has a much higher value of coupled power owing to more effective excitation of long wavelength modes. Long wavelength mode excitation by the THTA has been deliberately reduced in order to avoid plasma periphery heating in the high density regime.

In the first experiments with the new THTA (pulse gas puffing, \( B_0 = 0.47 \text{ T} \), \( P_{RF} \) from the FTA \( \leq 200 \text{ kW} \), \( P_{RF} \) from the THTA \( \leq 100 \text{ kW} \)), RF breakdown and plasma production were produced by the FTA (\( n_e = 4 \times 10^{12} \text{ cm}^{-3} \)). Plasma production by the THTA alone was not observed, as had been predicted by calculation for this case. Efficient THTA operation was observed when the plasma density value was higher than \( n_e \geq 10^{12} \text{ cm}^{-3} \). Turning on RF power to the THTA resulted in plasma density rise (up to \( 1.6 \times 10^{13} \text{ cm}^{-3} \), Fig. 8(a)). In these experiments RF power input from the THTA was approximately two times less than that from the FTA (the calculated FTA plasma loading resistance was two times greater than for

\[ \tau_{E}^{H-E}, \text{ms} \]

\[ \tau_{E}^{G}(\text{exp}), \text{ms} \]

**FIG. 6.** Experimental U-3M energy confinement time compared with empirical heliotron/stellarator energy confinement scaling.
the THTA). However, the electron temperature was approximately the same \((T_e \sim 70-80 \text{ eV})\) for both antennas, confirming the more favourable heating capabilities of the THTA in the high density regime. During THTA operation the high energy neutral signal was not observed in the CX measurements. The value of the ion temperature corresponded to the lower value of temperature measured by CX during FTA operation (see Section 2.1). Moreover, the shift of the THTA power deposition profile to the inner part of the plasma has made it possible to achieve a new plasma confinement regime. This regime is characterized by a reduction of the \(H_n\) line intensity (Fig. 8(b)) and Langmuir probe ion saturation current \(J_i\) (Fig. 8(e)) at higher plasma density. The steepness of the plasma density gradient at the boundary also was greater in this regime (Fig. 9).
The value of plasma density up to $2.7 \times 10^{13}$ cm$^{-3}$ obtained at a confining magnetic field value of $B_0 = 0.47$ T owing to improved electron heating in the central part of the plasma column by means of the THTA was not attainable earlier using the FTA.

4. CONCLUSIONS

The main physical mechanisms of the U-3M torsatron operation have been studied experimentally and theoretically.
A new, compact THTA for Alfvén plasma heating was designed for the U-3M torsatron.

As a result of the new THTA operation a new regime with improved global energy confinement time and steeper plasma density profile at the plasma edge was obtained.

This regime of plasma heating and confinement is now being studied.

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