Turbulence and intermittent transport in edge/SOL of a toroidal plasmas

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Motivation

Cross field transport of particles and heat in the edge/scrape-off-layer (SOL) region of a magnetically confined plasma is strongly intermittent and characterized by:

- large-amplitude, radially propagating blob-like structures of particles and heat,
- generated close to the last closed flux surface (LCFS),
- resulting in asymmetric conditional wave forms, and skewed and flattened PDFs,
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Observed under a variety of conditions:
Density blob observations

Observations of density blobs at the outboard midplane of ALCATOR C-mod ($D_\alpha$ - light)

O. Grulke et al. PSI-2004.

Structure along B.  
Radial propagation, $V \approx 0.05c_s$. 
Density blobs theory

Several recent works:
Krasheninnikov PLA 283, 268 (2001)
Curvature drift: charging of a density blob → radial propagation, velocity fraction of $c_s$, linear model, no self-consistent blob formation
D’Ippolito et al, PoP 9, 222 (2002)
D’Ippolito et al, CPP 44, 205 (2004)

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Similar mechanism for uprising density bubbles in ionospheric E-F layer – inverse stratification in a gravitational field.
e.g. Kelley and Ott, JGR 83, 4369 (1978).
Overview

A self-consistent description of fluctuations and intermittent transport in the edge/SOL by employing the RISØ ESEL (Edge SOL Electrostatic) model for interchange dynamics that:

- include separate plasma production “edge” and loss region “SOL”,
- allow self-consistent flows and profile relaxations,
- conserve particles and energy in collective dynamics.

Results are in good agreement with experimental observations

Garcia, Naulin, Nielsen, Rasmussen, PRL 92 165003 (2004); PoP 2005 submitted.
Geometry and Coordinates

We consider the outboard midplane of a toroidal plasma.

The non-uniform magnetic field is \( \mathbf{B} = -(B_0R_0/R)\hat{\Theta} \), described in elementary cylindrical coordinates \((R, \Theta, Z)\).

Applying a local slab approximation: \( x = R - R_a, \ y = Z, \ z = -\Theta \).
Model Equations

Fluid model cold ions and quasi-neutrality

\[
\frac{dn}{dt} + nC(\phi) - C(nT) = \nu_n \nabla_{\perp}^2 n - \sigma_n (n - 1) + S_n,
\]

\[
\frac{dT}{dt} + \frac{2T}{3} C(\phi) - \frac{7T}{3} C(T) - \frac{2T^2}{3n} C(n) = \nu_T \nabla_{\perp}^2 T - \sigma_T (T - 1) + S_T,
\]

\[
\frac{d\Omega}{dt} - C(nT) = \nu_\Omega \nabla_{\perp}^2 \Omega - \sigma_\Omega \Omega, \quad \Omega = \nabla_{\perp}^2 \phi.
\]

Adveective derivative and curvature operators defined by

\[
\frac{d}{dt} = \frac{\partial}{\partial t} + \frac{1}{B} \hat{z} \times \nabla \phi \cdot \nabla, \quad C = \nabla \left( \frac{1}{B} \right) \cdot \hat{z} \times \nabla, \quad B(x) = \frac{1}{1 + \varepsilon + \zeta x}.
\]

Conservation of particles and global energy (lowest order in $\zeta$)

\[
E(t) = \int dx \left[ \frac{1}{2} (\nabla_{\perp} \phi)^2 + \frac{3}{2} nT \right].
\]
Interchange instability: \( N = -B'(p'_0 - \frac{5}{3}B') \leq 0 \) instability at low field side.

Naulin et al. PRL 81, 4148 (1998); PoP 10, 1075 (2003)

Define the kinetic energy of the fluctuating and poloidal mean motions,

\[
v_0(x, t) = \frac{1}{L_y} \int_0^{L_y} v_y(x, t) dy = \partial \phi_0 / \partial x:
\]

\[
K(t) = \int \frac{1}{2} \left( \nabla \phi \right)^2 dx, \quad U(t) = \int \frac{1}{2} v_0^2 dx.
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\[ K(t) = \int \frac{1}{2} \left( \nabla_\perp \phi \right)^2 dx, \quad U(t) = \int \frac{1}{2} v_0^2 dx. \]

Energy transfer rates from thermal energy to the fluctuating motions, and from the fluctuating to the poloidal mean flow:

\[ F_p(t) = \int pC(\phi) dx, \quad F_v(t) = \int \tilde{v}_x \tilde{v}_y \frac{\partial v_0}{\partial x} dx. \]

\( F_p \) is also a measure of the turbulent energy transport.
Domain $L_x = 2L_y = 200$, resolution $512 \times 256$, $x_{\text{LCFS}} = 50$. SOL damping rates $\sigma_n = \sigma_\Omega = \sigma_T/5 = 3\zeta/2\pi q$ with $q = 3$; magnetic curvature $\varepsilon = 0.25$, $\zeta = 5 \times 10^{-4}$; collisional diffusion $\nu = 10^{-2}$; timespan $4 \times 10^6$.
Density blob propagation: The blob is generated inside LCFS ($x_{LCFS} = 50$). Radial propagation velocity of the structures is estimated to $\approx 0.05c_s$; but with large variance. Envisage the density blob as a filament elongated along the magnetic field with a ballooning structure.
Bursting: Kinetic energy contained by the mean $U$ and fluctuating $K$ motions and the collective energy transfer terms $F_p$ and $F_v$. 


Bursting: Expanded time scale, \((\nu = 10^{-2})\).

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Garcia et al  
2nd IAEA meeting: Theory of Plasma Instabilities, March 2 - 4, 2005, Trieste, Italy

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Energy Transfer

Bursting: Half viscosity \( \nu = 5 \times 10^{-3} \rightarrow \text{double time span} \)

Robust behavior
Time averaged profile of density, $\bar{n}_0$ and temperature, $\bar{T}_0$: Strong gradients in the edge region ($x < x_{LCFS} = 50$) and flat profiles in the SOL.

Time averaged profile of the poloidal flow, $\bar{v}_0$, and vorticity, $\bar{\Omega}_0$ ($\nu = 10^{-2}$)
Shear flow stabilization?

Influence of a background shear flow \( V(x) \) on the classical interchange instability


Numerical solution of linear dispersion relation: \( V(x) = V_0 \tanh(x) \), \( V_0 = 0, 0.5, 1.0, 2.0 \)

NOTE: Stability for \( 2\pi/L_y > k_c \)
Energy flow

- Instability drive in the edge
- Turbulence propagating into the SOL
- Saturates and via particle and momentum fluxes
- Profile modification and flow generation; weak transport
- Profile steepening, flow damping (viscous timescale)
- Instability drive

Bursting period related to viscous timescale.
Cross-correlations of density fluctuations between probe $P_4$ and the other probes $P_i$ in the simulations:
Cross-correlations between probe at $\rho = 13.8\, mm$ and the other probes in ALCATOR C-Mod (Grulke et al)
Single-Point PDFs

Probability distribution functions (count number) of density at $P_i$. $i > 2$ exponential tails, indicating strong blob structures.

Coarse grained PDF at $P_3$. Time intervals $\tau$. Increasing $\tau$: skewness decreases: $2.6 \rightarrow 0.06$, flatness factor decreases: $12.0 \rightarrow 3.1$. Absence of self-similarity for all scales ($\tau > 10^4$): intermittency
Probability distribution functions (count number) of density at $P_i$.

$i > 2$ exponential tails, indicating strong blob structures.

PDF from experiment in ALCATOR C-Mod (Grulke et al)

Detailed comparisons with density fluctuation PDFs at TCV are in progress. Universal PDF: One parameter Gamma-distribution.

Conditional averaging of the density signal: asymmetric, large-amplitude wave forms significantly exceeding the back-ground levels; decaying outwards in the SOL. Condition $n(x_{P_i}) - \bar{n}(x_{P_i}) > 3n_{\text{rms}}(x_{P_i})$

Conditional averaging of radial velocity; maximum $> 0$ in blob center.
Conditional averaging of the density signal: compared with experimental results from the ALCATOR C-Mod (Grulke et al)
“Laminar” Burst

Temperature structure, faster decay (higher sheath transmissivity) than density structure; the potential is subtracted the poloidially averaged potential.
Probability density functions of poloidally averaged particle density flux, $\Gamma_0 = \langle n v_x \rangle$ at $P_1, P_3$. Exponential tails: flux dominated by strong bursts. Coarse grained PDF at $P_3$. Increasing $\tau = 2.5m$: skewness and flatness factor decreases $\rightarrow$ Gaussian for large $\tau >$ burst intervals. Absence of self-similarity for all scales: intermittency
Conditional particle flux $\Gamma_\alpha = \langle \Gamma_0 | \Gamma_0 - \bar{\Gamma}_0 > \alpha \Gamma_0 \text{rms} \rangle$ relative to total flux $\Gamma_\Sigma$ at $P_3$.

Relative count number $N_\alpha$ of sub-records.

Few events contain most of the flux. Burst rate $\propto$ viscosity.

Transport characterized by the Flux PDF; not diffusive: find the “unique PDF“

Prediction of loads to divertor plates and PFC.
Impurity dynamics are modeled by tracing passive particles convected by the turbulent field.
Assumptions: Impurity density low, fully ionized, cold.

Impurity convection: \( \frac{d\bar{x}}{dt} = \bar{v}_{part} = \frac{1}{B} \hat{z} \times \nabla \phi \quad \nabla \cdot \bar{v}_{part} \neq 0 \) due to curvature.

Neglecting inertia effects \( \propto M_{imp}/Z_{m_i} \): only lighter impurities.

Impurity density: \( D(n_{imp}/B)/Dt = 0 \): Total mixing \( \propto n_{imp} \propto B \)
\( \propto \) Curvature pinch (Naulin, PRE 71, 015402 (2005))
Impurity dynamics

Impurity density: \[ D\left(\frac{n_{\text{imp}}}{B}\right)/Dt = 0 \] : Total mixing :: \[ n_{\text{imp}} \propto B \]
:: Curvature pinch (Naulin, PRE 71, 015402 (2005))

Trajectory of a test particle released inside LCFS

Distribution of impurities, that are initially released at \( x = 160 \): Turbulent mixing. Inward (curvature) pinch
Generalizations: 3D effects

Connect regions of good curvature (HFS) with outboard midplane (LFS) in the edge region to mimic the dynamics along the field lines.

First approach \( \frac{dn_L}{dt} \cdot = \cdot \alpha(n_H - n_L), \quad \frac{dn_H}{dt} \cdot = \cdot \alpha(n_L - n_H) \)

Ballooning nature of fluctuations.
Conclusions and Outlook

The non-linear dynamics of interchange turbulence (2-D ESEL-code) yields very good agreement with experimental measurements:

- the formation of blobs due to profile relaxations,
- radial propagation velocities around 0.1 acoustic speed,
- asymmetric wave forms; skew and flat PDFs.
- intermittent transport.

More complete modelling of edge and SOL turbulence should be 3-D, non-local, and energy-conserving,

- with geometry effects and boundary conditions,
- address the relation to ELMs.