Physics of Non-Diffusive Turbulent Transport of Momentum and the Origins of Spontaneous Rotation in Tokamaks


1) University of California at San Diego, La Jolla, CA 92093-0424 USA
2) Princeton Plasma Physics Laboratory, Princeton, NJ 08543-0451 USA
3) University of California at Irvine, Irvine, CA 92697-4575 USA
4) Association EURATOM-Risø DTU, Roskilde, DK-4000 Denmark
5) Institute for Plasma Research, Bhat, Gandhinagar, 382428 India
6) CEA Cadarache, 13108 St Paul Lez Durance France

e-mail contact of main author: pdiamond@ucsd.edu

Abstract. Recent results in the theory of turbulent momentum transport and intrinsic rotation are summarized. Special attention is devoted to the physics of the residual stress. The critical external torque required to balance intrinsic rotation is calculated. A simple model of profile evolution and velocity scaling for intrinsic rotation is summarized.

1. Introduction

The needs for understanding of, and predictive capacity for, both the off-diagonal flux of toroidal angular momentum and the origins of spontaneous or intrinsic rotation are now well established and accepted. In general, the mean field momentum flux driven by electrostatic turbulence is given by

\[ \Pi_r = n \langle \vec{v}_r \vec{v}_\phi \rangle + \langle \vec{v}_r \vec{n} \rangle \langle \vec{v}_\phi \rangle. \]  

(1)

Here the first term is the toroidal Reynolds stress and the second is the familiar convective flux, hereafter neglected. The Reynolds stress may be further decomposed as

\[ \langle \vec{v}_r \vec{v}_\phi \rangle = -\chi_\phi \frac{\partial \langle \vec{v}_\phi \rangle}{\partial r} + V \langle \vec{v}_\phi \rangle + \Pi_r^{R}, \]  

(2)

where \( \chi_\phi \) is the turbulent viscosity, \( V \) is the convective velocity (i.e. the momentum pinch) and \( \Pi_r^{R} \) is the residual stress. Note that \( \chi_\phi \) and \( V \) have well-known analogues in the theory of the particle flux, while \( \Pi_r^{R} \) does not. In this paper, we discuss the status of our current understanding of \( \chi_\phi, V \) and \( \Pi_r^{R} \), and the physics of turbulent transport of toroidal momentum and intrinsic rotation. In Section 2, we survey the constituents of the turbulent momentum flux and their underlying physics. In Section 3, we discuss the physics of the residual stress, which is the most unusual and counter-intuitive element of the turbulent momentum flux, but also the piece most important to intrinsic rotation. In Section 4, we outline a simple model which captures many of the basic scaling trends for intrinsic rotation. Section 5 consists of a brief discussion of future work.
2. Survey of Turbulent Momentum Flux Physics

The turbulent viscosity $\chi_\phi$ is now relatively well understood. As was realized 20 years ago[1], $\chi_\phi$ is closely related to the ion thermal diffusivity $\chi_i$ for drift wave turbulence. Recent simulation[2] and theory[3] works have discovered that near ITG marginality, when transport is dominated by the resonant scattering of slightly suprathermal ions (with $s = \omega/k||V_{T_i} \sim 2$), then

$$\frac{\chi_\phi}{\chi_i} = \frac{\langle s^2 \rangle}{1 + \langle s^2 \rangle + \langle s^4 \rangle/2},$$

(3)

where the average is to be taken over the mean distribution function. This reveals that in stiff profile regimes, the intrinsic Prandtl number $Pr \neq 1$, but rather $Pr \sim 0.2 \rightarrow 0.5$, due to the inherent difference between wave-particle auto-correlation times for $\tilde{v}_\phi$ and $\tilde{T}$. Here, it is important to note that the intrinsic Prandtl number is defined by the ratio of the purely diffusive fluxes, and differs from the conventionally quoted 'raw' Prandtl number $Pr = \left(\frac{\langle v \rangle}{\langle T \rangle}/\frac{\langle v \rangle}{\langle T \rangle}\right)$, defined without regard to the presence of non-diffusive fluxes.

The past two years have witnessed intensive interest in and study of the convective momentum velocity $V$. Recent detailed theoretical work on the momentum pinch is reported in Refs [4, 5, 6]. In general, the toroidal pinch may be decomposed into a TEP and thermoelectric (TH) piece

$$V = V_{\text{TEP}} + V_{\text{TH}}.$$

(4)

The turbulent equipartition convection velocity is purely inward (corresponding to a pinch) and is robust and mode independent.

Like the TEP pinch for density, the origin of the TEP pinch is in the compressibility of the $E \times B$ velocity in toroidal geometry ($\nabla \cdot V_{E \times B} \neq 0$), so that magnetically weighted angular momentum $V_{||}R/B^2$ (rather than simply $V_{||}R$) is locally conserved. Thus, it is no surprise that the TEP momentum and particle pinches are strongly correlated. The TEP pinch has been derived from detailed gyrokinetic analysis[4,5] and general considerations of angular momentum homogenization [6]. For a stationary profile in the absence of a residual stress $V_{\text{TEP}}/\chi_\phi = -B^2/R d/dr(R^2) \approx -3/R$. The thermoelectric velocity $V_{\text{TH}}$ is related to the ion temperature fluctuation $\tilde{T}_i$ and is given by

$$V_{\text{TH}} = 4 \left(\frac{e}{B} \sum_k \omega_d \phi^*_{-k} \tilde{\phi}_{\perp} / T_0 \right).$$

(5)

In distinct contrast to the TEP pinch, the thermoelectric convection velocity is very sensitive to mode characteristics which determine the phase angle between $\tilde{\phi}$ and $\tilde{T}_i$. In general, $V_{\text{TH}} < 0$ (i.e inward) for electron direction ($V_{e}$) modes such as CTEM, while typically $V_{\text{TH}} > 0$ (i.e. outward) for ion direction ($V_{i}$) modes such as ITG.
The third element in the momentum flux is the residual stress, \( \Pi^R_{r,\phi} \). The residual stress is defined as that part of the Reynolds stress which is not directly proportional to either \( \partial \langle v_\phi \rangle / \partial r \) or \( \langle v_\phi \rangle \), i.e. the portion other than the diffusive and convective flux. The residual stress has no counterpart in the theory of the turbulent particle flux, since momentum can obviously be exchanged between waves and particles, while density cannot. The residual stress defines an effective local internal toroidal momentum source

\[
\frac{\partial \langle p_\phi \rangle}{\partial t} = S_{\phi,internal} = -\frac{\partial}{\partial r} \langle n \Pi^R_{r,\phi} \rangle,
\]  \tag{6}

and so is crucial to the formation of intrinsic rotation profiles. However, the residual stress only affects net rotation by its value at the separatrix or plasma boundary. The physics of the residual stress is discussed at length in the next section.

3. Physics of the Turbulent Residual (Radiation) Stress

The residual stress \( \Pi^R_{r,\phi} \) is that part of the Reynolds stress \( \langle \tilde{v}_r, \tilde{v}_\phi \rangle \) which remains after turbulent diffusion and convection are subtracted. Its existence is a necessary consequence of wave-particle momentum exchange, which is enforced by outgoing wave boundary conditions even in a purely fluid theory. Physically, the residual stress \( \Pi^R_{r,\phi} = \Pi(\nabla T, \nabla T_e, \nabla P, \nabla P_e, \nabla n...) \) converts part of the driving heat flux \( Q_i \) or \( Q_e \) to a net toroidal flow. Observe that the residual stress is the only way to spin-up the plasma from rest, i.e. for \( \langle v_\phi \rangle = 0 \), \( \langle v_\phi \rangle' = 0 \),

\[
\partial_t \int_0^a \langle p_\phi \rangle dr = -(n) \Pi^R_{r,\phi} \bigg|_0^a = -(n) \Pi^R_{r,\phi} \bigg|_a,
\]  \tag{7}

so the radially integrated momentum drive is set by the gradients (particularly pressure) at the plasma edge, acting through the residual stress. Note that a pinch alone cannot spin-up the plasma from rest, but instead requires some toroidal flow at the separatrix to initiate rotation, i.e. it needs \( \langle V_\phi(a) \rangle \neq 0 \). Of course, the critical pinch is that acting at \( r = a \). For \( r < a \), \( V \) simply re-distributes the plasma momentum in radius, but cannot change the net plasma momentum. The separatrix boundary condition is also critical. The total stress corresponds to a net momentum flux, which along with the boundary condition on \( \langle V_\phi \rangle \), determines the profile. In particular, for the relevant prototypical case where \( V_\phi(a) = 0 \) (corresponding to a no-slip boundary, enforced by strong neutral drag), we have for zero net momentum flux (corresponding to an intrinsic rotation solution).

\[
\langle V_\phi(r) \rangle = -\int_r^a \langle v_\phi \rangle' \Pi^R_{r,\phi}(r) / \chi_\phi(r),
\]  \tag{8}

so that \( \Pi^R < 0 \) corresponds to co-rotation while \( \Pi^R > 0 \) corresponds to counter-rotation. Note that either sign of \( \Pi^R \) can generate a flow. Of course, \( \Pi^R_{r,\phi}(r) \) can change sign in radius, and so produce internal flow reversals. The sign dependence of \( \Pi^R \) should be contrasted to that for convection, where \( V > 0 \) is unfavorable while \( V < 0 \) is favorable. Thus, we see that \( \Pi^R_{r,\phi} \) is conceptually distinct from a pinch or other convective effect.
The micro-physics of the residual stress is governed by resonant and non-resonant turbulent transport acting in the presence of broken parallel reflection symmetry (i.e. $k_\parallel$ symmetry breaking). The calculation of $\Pi_{r,\phi}^k$ in the resonant limit is discussed in the literature[8]. Here we focus on the non-resonant or "wave" contribution. Intuitively, this is the most appealing way to envision the origin of intrinsic rotation, namely as a consequence of the modulation of an anisotropic quasi-particle pressure. Taking the turbulent $\chi_\phi$ momentum diffusivity as already determined, the mean flow $\langle V_\phi \rangle$ then satisfies

$$\partial_t \langle V_\phi \rangle - \partial_r \chi_\phi \partial_r \langle V_\phi \rangle = -\partial_r \Pi_{r,\phi}^{\text{wave}},$$

(9a)

where

$$\Pi_{r,\phi}^{\text{wave}} = \int d\vec{k} v_g \cdot k_\parallel N.$$

(9b)

Here, $\Pi_{r,\phi}^{\text{wave}}$ is the net radial flux of parallel wave momentum $k_\parallel N$. The quasi-particle population density is just $N(x,k,t)$, which obeys a wave-kinetic equation[9]. Defining $S_\parallel = \delta \langle V_\phi \rangle$, the modulation in toroidal velocity shear, we have

$$\partial_t S_\parallel - \partial_r \chi_\phi \partial_r S_\parallel = -\partial_r^2 \int d\vec{k} v_g \cdot k_\parallel N.$$

(10)

The RHS effectively accounts for the quasi-particle induced residual stress. Formulation of the problem as one of modulational instability is useful for clarifying the dynamics of flow shear amplification. Note that the edge boundary condition discussed above guarantees that flow shear amplification leads to net flow amplification. Linearizing the wave kinetic equation then gives

$$\delta N = \tau_{c,\text{mod}} \left[ k_\theta \nabla_{\vec{k}} \cdot \frac{\partial \langle N \rangle}{\partial k_r} - v_{sr} \frac{\partial \langle N \rangle}{\partial r} \right].$$

(11)

Here $\nabla_{\vec{k}}$ is the electric field shear modulation and $\tau_{c,\text{mod}}$ is the $\delta N$ response correlation time. Thus

$$\Pi_{r,\phi}^{\text{wave}} = \int d\vec{k} k_\parallel v_g \tau_c \left\{ k_\theta \frac{\partial \langle N \rangle}{\partial k_r} (\nabla_{\vec{k}} \cdot V_E) - v_{sr} \frac{\partial \langle N \rangle}{\partial r} \right\}.$$  

(12a)

If a net external torque $T^{\text{ext}}$ modulation was retained, the condition for a stationary state in the presence of the wave stress given by Eqn. (12a) can easily be shown to be

$$T^{\text{ext}} = \int d\vec{k} k_\parallel v_g \tau_c \left\{ k_\theta \frac{\partial \langle N \rangle}{\partial k_r} (\nabla_{\vec{k}} \cdot V_E) - v_{sr} \frac{\partial \langle N \rangle}{\partial r} \right\} \bigg|_{a} - nm \chi_\phi \frac{\partial \langle V_\phi \rangle}{\partial r} \bigg|_{a}.$$  

(12b)
Several observations are in order here. First note that the net residual stress is driven by the quasi-particle population gradients in both $k_r$ and $r$. The $k_r$ gradient $\partial \langle N \rangle / \partial k_r$ induces a stress via shearing when $k_\theta \partial v_{gr}/\partial k_r \neq 0$, so that the net $k_r$-space flow is compressible. Note that for drift waves, $k_\theta \partial v_{gr}/\partial k_r \equiv -2 k_\theta \rho_s^2 v_s / \left( 1 + k_\perp \rho_s \right)^2$, so the integrated contribution to the stress is even in $k_\theta$ and $k_r$, and exhibits some mode dependence via $v_{se}$. We expect this trend to be generic. The $r$-gradient $\partial \langle N \rangle / \partial k_r$ induces a radiative diffusive inward flux of wave momentum, which may be either co or counter direction, depending on the sign of $k_\parallel$. The radiative diffusion flux $\sim -D_r \partial \langle P_\parallel \rangle / \partial r$, where $\langle P_\parallel \rangle_w$ is the wave parallel momentum density and $D_r \sim v_{gr}^2 \tau_c$ is the quanta diffusivity. Note $D_r \sim D_{GB}$.

Second, before proceeding to calculate $\partial \langle v_{\phi} \rangle / \partial r \big|_a$, the edge rotation gradient, we note that

$$\langle V_E \rangle' = \frac{\partial \langle v_{\phi} \rangle}{\partial r} + \langle V_E \rangle'_0,$$  \hspace{1cm} (13)

i.e. the net electric field shear is the sum of the contributions due to toroidal rotation and the other pieces, denoted by $\langle V_E \rangle'_0$. The latter includes both diamagnetic (i.e. $\nabla P_\parallel$-driven) velocity shear and poloidal velocity shear. Of course this means that absent $\langle V_E \rangle'_0$, $\partial \langle v_{\phi} \rangle / \partial r$ can feed back on itself, as in a modulational instability. Indeed, one can easily extend the analysis presented here to demonstrate that toroidal zonal flows are modulationally unstable. Such toroidal zonal flows have been observed in gyrokinetic particle simulation[10].

More generally, this result suggests that any intrinsic rotation feeds back on itself via electric field shearing, and so renormalizes the effective $\chi_{\phi}$. To see this, observe that plugging Eqn. (13) into Eqn. (12b) and re-writing gives

$$\frac{\partial \langle v_{\phi} \rangle}{\partial r} \bigg|_a \left[ T_{ex} - \left\{ \int d^2 k k_{\parallel} v_{gr} \tau_c k_\theta \frac{\partial \langle N \rangle}{\partial k_r} \right\} \langle V_E \rangle'_0 ight] + D_{rad} \frac{\partial \langle P_\parallel \rangle}{\partial r} \bigg|_a \right]/nm \chi_{\phi,eff} \big|_a,$$  \hspace{1cm} (14a)

where

$$nm \chi_{\phi,eff} = nm \chi_{\phi} \bigg|_a \left\{ \int d^2 k k_{\parallel} v_{gr} \tau_c k_\theta \frac{\partial \langle N \rangle}{\partial k_r} \right\} \bigg|_a$$  \hspace{1cm} (14b)
is the ‘renormalized’ $\chi_\phi$ which includes self-induced rotation feedback via $\langle V_E \rangle'$. Note that the sign of the $\chi_\phi$ renormalization is determined by the group velocity $v_{gr}$, the spectral population gradient $\partial(N)/\partial k_r$ (which is usually negative) and the spectrally weighted $k_\parallel$. Observe that the correction to $\chi_\phi$ can be positive and so it is at least conceivable that the observed $\chi_\phi$-deduced, say, from momentum perturbation experiments—may exceed the observed $\chi_i$. $\chi_\phi > \chi_i$ has been observed in JT60U perturbation experiments[11].

Third, observe that Eqn. (14a) defines an effective critical torque which zeroes the edge velocity gradient, i.e. $T_{crit}^{ext}$ for $\partial\langle v_\phi \rangle/\partial r \big|_a \to 0$. This may be thought of as defining a critical torque which exactly cancels the residual stress-driven intrinsic rotation[12]. Here, the critical torque is

$$T_{crit}^{ext} = \left\{ d_E^r \left(k_\parallel v_{gr} \tau_c k_\parallel \frac{\partial(N)}{\partial k_r} \langle N_E \rangle_0 + D_{rad} \frac{\partial\langle p_{\parallel} \rangle_w}{\partial r} \right) \right\}_e .$$

Note that the critical torque is determined by $\langle V_E \rangle_0$, i.e. the electric field shear due to diamagnetic and poloidal rotation, the mode propagation velocity (in $v_{gr}$), the turbulence spectrum (in $\partial(N)/\partial k_r$), the wave momentum density profile $\langle p_{\parallel} \rangle_w$ and $D_{rad}$, $\tau_c$, etc. Of course, the critical torque defines the off-set in the linear plot of $\partial\langle v_\phi \rangle/\partial r \big|_a$ vs. $T_{crit}^{ext}$. Interestingly, it is renormalized $\chi_\phi$ - i.e. $\chi_{\phi,\text{eff}}$ - which sets the slope of this linear relation. Thus, the feedback loop physics of intrinsic rotation enters more than just the off-set! Finally, we should recall that if the edge rotation velocity is finite,

$$\frac{\partial\langle v_\phi \rangle}{\partial r} \big|_a = \frac{-1}{nm \chi_{\phi,\text{eff}}} \left( T_{crit}^{ext} - \Pi_{\phi,\text{eff}} \big|_a - V\langle v_\phi \rangle \big|_a \right) .$$

In this case, the edge pinch velocity also enters the determination of $\partial\langle v_\phi \rangle/\partial r \big|_a$.

Interestingly, only the edge momentum pinch is relevant to intrinsic rotation. We speculate here that SOL physics in general, and SOL flow effects in particular[13], couple to core intrinsic rotation via the edge momentum pinch. The TEP momentum pinch, discussed in reference [4], is surely operative at the edge. Analysis of other possible contributions requires a study of the regime with collisionless fluid ion and dissipative/collisional electron dynamics. In particular, it would be interesting to see if a momentum analogue of the familiar in mixing mode density pinch[14] exists. In closing this section, we remark that coupling of intrinsic rotation to $v_\phi(a)$ should also manifest itself as a sensitivity of the critical torque to SOL asymmetry - i.e. $T_{crit}^{ext}$ should differ between single null and double null operation.

Virtually all of the results in this discussion are sensitive to spectrally averaged $k_\parallel$, i.e. $\langle k_\parallel \rangle$. Following Eqns. (36a,b) of Ref. [8], we can balance nonlinear decay with shearing to obtain
\[ \langle k_0 \rangle = -\int d_k^2 \frac{\partial k_\parallel}{\partial k_r} k_\theta \langle V_E \rangle' \langle N \rangle / \gamma_{NL}, \tag{17a} \]

where

\[ \langle k_0 \rangle = \int d_k k_\parallel \langle N \rangle. \tag{17b} \]

Here \( \gamma_{NL} \) is the nonlinear decorrelation rate (i.e. inverse mode lifetime) for wave-vector \( \vec{k} \). \( \partial k_\parallel / \partial k_r \neq 0 \) requires magnetic shear. This description is equivalent to that developed in real space, in which the shift of the spectrum off the resonant surface induced by the electric field shear sets the mean \( k_\parallel \)[7,15,16].

5. Simple Model for Intrinsic Rotation Scalings

It's interesting to note that Equation (14a) effectively states that \( \partial \langle v_\phi \rangle / \partial r \big|_a \) - and thus the net intrinsic rotation - will increase with \( \langle V_E \rangle' \big|_0 \). Since \( \langle V_E \rangle' \big|_0 = \partial r (\partial \langle P \rangle / \partial r / ne B_\theta) - \partial r (\langle v_\phi \rangle B_0) \) increases with edge pressure gradient, one direct prediction of this theory is a correlation between edge pressure gradient and intrinsic rotation velocity. This is qualitatively suggestive of the \( \Delta \langle v_\phi \rangle \sim \Delta W_p / I_p \) scaling proposed by J. Rice[17], but now expressed in terms of more physical, local gradient quantities. One can go further and develop a transport model which evolves the

i.) toroidal momentum profile, in terms of \( \chi_\phi, V \) and \( \Pi_{r,\phi} \) acting along with the external torques,

ii.) density profile, in terms of \( D, V_n \) and fueling,

iii.) ion temperature profile, in terms of \( \chi \) and heating,

iv.) fluctuation intensity, evolved by simple \( E \times B \) shear-induced quenching[18,19].

This model represents a generalized Hinton model[20]. The model may be solved numerically, and also analytically, assuming a piecewise linear profile structure. Results indicate that the central rotation velocity is determined primarily by the pedestal velocity, and that the latter scales as

\[ \Delta \langle v_\phi \rangle / \nu_{\nu_1} \sim (\Delta r_c / a)(\Delta_{ped}/a) \sim \rho_\nu^\alpha (\Delta_{ped}/a). \tag{18c} \]

Here \( \Delta_{ped} \) is the pedestal width and \( \Delta r_c \) is the turbulence correlation length. Thus, \( \alpha \sim 1 \) corresponds to Gyro-Bohm edge turbulence while \( \alpha \sim 0 \) corresponds to Bohm. The pedestal width is proportional to the pedestal pressure, i.e. \( \Delta_{ped} \sim P_{ped} \), so \( \Delta v_\phi \sim P_{ped} \sim \Delta W_p \), the increment in the stored energy, as in the Rice scaling. More interestingly, we note that if:

i.) the edge turbulence exhibits Bohm scaling, so \( \Delta r_c / a \sim \left( \rho_\nu \right)^\alpha \sim 1 \),

ii.) we assume the Snyder, et al.[21] empirical pedestal width scaling \( \Delta_{ped}/a \sim \beta_p^{1/2} \),
we then recover \( \Delta \nu_\theta/\nu_{Ti} \sim \beta_p^{1/2} \) which is effectively equivalent to the Rice scaling \( \Delta \nu_\theta \sim \Delta W_p/I_p \). Interestingly, the unfavorable current scaling of intrinsic rotation appears as a consequence of the unfavorable current scaling of the pedestal width. This seems plausible, since otherwise transport scalings with current are nearly universally favorable. Note that in this scenario, intrinsic rotation is strongly tied to pedestal physics, which is also suggested by the experimental results. The absence of \( \rho_\psi \) scaling of intrinsic rotation velocity appears as a consequence of Bohm scaling of the pedestal turbulence. The persistence of this favorable trend into the regime of ITER parameters is far from certain.

5. Future Work

Ongoing and future work will focus on studies of electron heat transport driven regimes[22], electromagnetic coupling and saturation, alternative symmetry breaking mechanisms (especially GAM shearing), poloidal rotation effects, SOL-core coupling and detailed modelling work. Understanding the edge pinch of momentum and its interaction with the edge rotation velocity driven by SOL flows is a particularly important near-term goal. Finally we also plan to apply the theory to the interesting TCV internal momentum transport bifurcations[23].

Acknowledgements

This research was supported by Department of Energy Grant Nos. DE-FG02-04ER54738, DE-FC02-08ER54959 and DE-FC02-08ER54983. We thank J. Rice, M. Yoshida, Y. Kamada, W. Solomon, S. Kaye, K. Ida, X. Garbet, L. Eriksson, J. deGrassie, C.-S. Chang, F. Hinton, K. Burrell, C. Hidalgo, K. Itoh, S.-I. Itoh, J. Myra, B. LaBombard, P. Snyder, R. Groebner, B. Duval, and G. Tynan for useful conversations.

References: