Conservative Global Gyrokinetic Toroidal Full-\(f\) 5D Vlasov Simulation

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Outline

- Introduction
- Conservative Gyrokinetic Toroidal 5D full-\(f\) Vlasov code GT5D
- Source driven ITG turbulence simulation
- Summary
Background and motivation

- New gyrokinetic full-$f$ Vlasov code GT5D is developed to study interaction between turbulent transport and profile formations
  - full-$f$: $f_0$ and $\delta f$ are solved consistently ($\delta f$: solve only $\delta f$ by assuming fixed $f_0$)
    → profile stiffness, transport barriers

- Key features of ion transport is examined
  - Stiffness of $T_i$ profile
    → Performance of H-mode plasma
  - Intermittent heat flux
    → Avalanche like phenomena
  - Momentum transport
    → Intrinsic toroidal rotation
    → Qualitative validation of GT5D

JT60U experiment

Stiff $T_i$ profile in H-mode plasma (Urano, NF08)

Intrinsic rotation with steep $\nabla P$
(Sakamoto, NF01, Yoshida, PRL08)
Present status of GT5D code (Idomura, CPC08)

- Target problem
  - Full-$f$ gyrokinetic simulations of ion heat and momentum transport in electrostatic Ion Temperature Gradient turbulence

- Key features of GT5D
  - Robust and accurate full-$f$ simulation using non-dissipative FD scheme (Idomura, JCP07)
  - Shaped MHD configuration including axis
  - Linear Fokker-Planck operator (Xu, PFB91)
  - Interface of MEUDAS and TOPICS codes
  - Highly scalable on MPP Altix3700Bx2

- Numerical verification
  - Linear/nonlinear benchmarks against GT3D
  - Neoclassical benchmarks
First principle models used in GT5D

- Hamiltonian in the gyro-centre coordinates $Z_{GY}=(t;R,v_{\parallel},\mu,\alpha)$

$$H = \frac{1}{2}mv_{\parallel}^2 + \mu B + e\langle \phi \rangle_{\alpha}$$

- Gyrokinetic equations based on modern GK theory (Brizard, RMP07)

$$\frac{df}{dt} + \{f, H\} = C(f) + S$$

$$-\nabla_{\perp} \cdot \rho_{ii}^{2} \nabla_{\perp} \phi + \frac{1}{\lambda_{Di}^{2}} \left[ \phi - \langle \phi \rangle_{f} \right] = 4\pi e \int f \delta \left( [R + \rho] - x \right) d^6Z - n_{0e} \$$

- Linear Fokker-Planck collision operator (Xu, PFB91, Wang, PPCF99)

$$C(f) = \frac{\partial}{\partial v_{\perp}} \left( v_{\perp} v^2 f \right) + \frac{\partial}{\partial \hat{v}_{\parallel}} \left( v_{\parallel} \hat{v}_{\parallel} f \right)$$

$$+ \frac{1}{2} \frac{\partial^2}{\partial v_{\perp}^2} \left( v_{\perp} v^4 f \right) + \frac{1}{2} \frac{\partial^2}{\partial \hat{v}_{\parallel}^2} \left( v_{\parallel} v^2 f \right) + \frac{\partial^2}{\partial v_{\perp}^2 \partial \hat{v}_{\parallel}} \left( v_{\parallel} v^3 f \right) + C_F$$
Non-dissipative conservative finite difference

- Main features of NDCFD for gyrokinetics (Idomura, JCP07)
  - Keep numerical stability by conserving phase space volume, $f$ and $f^2$, which are inherent to Poisson bracket operator
    → Concept similar to (Arakawa, JCP66, Morinishi, JCP98)
  - Conservation properties essential for full-$f$ GK simulations
    → Exact conservation of particle number
    → Good energy and momentum conservation

(a) Vlasov code (~255 CPU hours)  
(b) $\delta f$ PIC code (~211 CPU hours)
Neoclassical benchmark tests against local theories

- Axisymmetric $n=0$ simulation with ion-ion FP collision operator
- Circular tokamak configuration ($R_0/a=5$, $a/\rho_{ti}=150$, $L_n=L_{ti}=R_0$, $v^* \sim 0.1$)

(a) Equilibrium $E_r$ (Hinton, RMP76)

(b) Ion heat transport (Chang, PF82)

✓ Kinetic equilibrium with $E_r$ is determined by $U_{li}$, $L_n$, $L_{ti}$, and $v^*$
✓ Neoclassical heat flux agrees with Chang-Hinton’s formula
✓ Physical dissipation of fine scale structures in velocity space
Source driven ITG turbulence simulations

- Calculation parameters (Cyclone case)
  \[ R_0/a = 2.8, \ a/\rho_{ti} = 150, \ q = 0.85 + 2.18(r/a)^2 \]
  \[ n_0 e \sim 5 \times 10^{19} \text{m}^{-3}, \ T_i \sim 2 \text{keV}, \ \nu^* = 0.02 \sim 0.1 \]
  \[ R_0/L_n = 2.2, \ R_0/L_{ti} = 10, \ U_{\parallel} \sim 0.1 v_{ti} \text{ at } t=0 \]
  1/3 wedge torus \( n = 0, 3, 6, \ldots 48 \)
  \( (N_R, N_\zeta, N_Z, N_{v_{\parallel}}, N_\mu) = (160, 32, 160, 80, 20) \)

- Source: \( S_{src} = A_{src}(\psi)[f_{M1} - f_{M2}] \)
  Fixed \( P_{in} \) without momentum/particle input
  \[ P_{in} = \int (m_i v^2/2) S_{src} d^6Z = 2 \text{MW, 4MW} \]
  \[ \int S_{src} d^6Z = 0, \int v_{\parallel} S_{src} d^6Z = 0 \]

- Sink: \( S_{snk} = A_{snk}(\psi) \tau_s^{-1}[f_0 - f] \)
  \( \tau_s^{-1} = 0.1 c_s / a, \ T_{\text{edge}} \sim 0.8 \text{keV}, \ U_{\parallel} \sim 0 \)
  Fixed \( T_{\text{edge}} \) limited by edge localized modes
  No slip boundary imposed by neutrals
Long time gyrokinetic simulation over collision time

- Time history of energy $\delta E_{fld}$, $\delta E_{kin}$, $E_{src}$, and $E_{snk}$ ($P_{in}=2 MW$)

![Graph showing energy changes over time](image)

- Kinetic equilibrium $\{f, H\} \sim C(f)$ formed through GAM damping
- ITG mode grows from kinetic equilibrium state with $E_r$
- Initial transient bursts leading to quick adjustment of $T_i$ profile
- Power balance $dE_{src}/dt \sim dE_{snk}/dt$ is established after $t_{vi}/R_0 > 300$
- Energy conservation $(\delta E_{fld} + \delta E_{kin}) - (E_{src} + E_{snk}) \sim 0$ is sustained
Spatio-temporal plots of $\chi_i$, $L_{ii}$, $dE_r/dr$, $U_{||}$ ($P_{in}=2$MW)

Outward propagation
Avalanches
Inward propagation
Globally constant $L_{ii}$
Positive $E_r$ shear
Negative $E_r$ shear
Formation of notch structure of $V_{ctr}$
Steady $V_{co}$ without momentum input
Stiffness of $T_i$ profile observed in power scan

- $T_i$ and $L_{ti} = \nabla T_i / T_i$ in power scan with $P_{in} = 2$MW and $P_{in} = 4$MW

(a) $T_i$ profile (average $t/\tau_{ii} = 0.7$~1.2)

(b) $L_{ti}$ profile (average $t/\tau_{ii} = 0.7$~1.2)

- Stiff $T_i$ profiles are observed with $P_{in} = 2$MW and $P_{in} = 4$MW
- Source free region ($r/a = 0.5$~0.9) is characterized by globally constant $L_{ti}$ at slightly above nonlinear critical value $R_0/L_{ti} \sim 6$
  → Typical feature of JT60U H-mode plasmas (Urano, NF08)
- Avalanches are produced in supercritical turbulence
Properties of avalanches observed in power scan

- Time history and power spectrum of avalanches

- Significant heat flux is produced by intermittent avalanches
- Amplitudes of avalanches are doubled with the same $L_{ti}$

Turbulence: $l_c \sim 5\rho_{ti}$, $\tau_c \sim 0.7R_0/v_{ti} \sim 2a/c_s$

Avalanches: $l_A \sim 20\rho_{ti}$, $\tau_A \sim 9R_0/v_{ti}$ (c.f. $2\pi/\omega_{GM} \sim 3R_0/v_{ti}$)

- $1/f$ spectrum in self-organized critical system (Politzer, PRL00)
Equilibrium $E_r$ in source driven ITG turbulence

- Parallel momentum balance relation (Hinton-Hazeltine, RMP76)

\[
\langle U_\parallel \rangle_f = \frac{T_i I}{m_i \Omega_i} \left( \frac{\partial \psi}{\partial r} \right)^{-1} \left[ (k-1) \frac{\partial \ln T_i}{\partial r} - \frac{\partial \ln n_i}{\partial r} - \frac{e}{T_i} E_r \right]
\]

- $E_r$ in core region ($r/a < 0.5$) is close to neoclassical level
- $E_r$ in avalanche region ($r/a > 0.5$) exceeds neoclassical level
- Avalanche component of $E_r$ shows clear correlation with $L_{ti}$
Avalanche propagation depending on $E_r$ shear

- Outward propagation of single avalanche front in $dE_r/dr > 0$ region

- $T_i$ flattening leads to local maxima of $L_{ti}$, $E_r$ at avalanche fronts
- Avalanche fronts are bounded by positive/negative $E_r$ shear
- Shift of $E_r$ shear by mean $E_r$ leads to asymmetric mode coupling
  → leading to one-sided avalanches depending on $E_r$ shear
Non-diffusive momentum flux depending on \( E_r \) shear

- Toroidal rotation (parallel flow) \( U_\parallel \) and momentum flux \( \Pi \)

\[ \text{Co- (Ctr-) toroidal rotation is sustained at } r/a < 0.5 \ (r/a = 0.6) \]
\[ \rightarrow \text{Notch structure formed in high } \nabla P_i \text{ region (Yoshida, PRL08)} \]

\[ \text{Outward (inward) momentum flux } \Pi \text{ with } dE_r/dr > 0 \ (dE_r/dr < 0) \]
\[ \rightarrow E_r \text{ shear stress due to asymmetric shift of } k_\parallel \text{ spectrum} \]
\[ \text{(Dominguez, PFB93, Grucan, POP07)} \]
Summary

- Conservative Gyrokinetic Toroidal 5D full-$f$ Vlasov code GT5D
  - Robust and accurate long time full-$f$ simulation by NDCFD
  - Neoclassical physics implemented with linear FP operator
  - Source models with flexible particle/momentum/power input
    → Long time source driven ITG turbulence simulation

- Key features of ion turbulent transport are recovered in GT5D
  - Stiff $T_i$ profile with globally constant $L_{ti}$ tied to critical value
  - Significant heat flux is produced by avalanches with $1/f$ spectra
  - Equilibrium $E_r$ shear determines directions of avalanche propagation and non-diffusive momentum flux
  - Intrinsic toroidal rotation in Co- (Ctr-) direction is sustained in core (outer) region without momentum input near axis
    → Physics ingredients essential for quantitative validation