Optimum Corrugations for low-loss square and cylindrical Waveguides

B. Plaum, E. Holzhauer, M. Grüner, H. Kumrić

Institut für Plasmaforschung, Universität Stuttgart, Pfaffenwaldring 31, D-70569 Stuttgart, Germany
plaum@ipf.uni-stuttgart.de

A method for calculating the microwave reflection losses of corrugated metallic surfaces is presented. These losses are of interest for the design of corrugated waveguides and polarizers. The calculation is done by modelling the electromagnetic fields with a 2D FDTD algorithm. The metal is treated as a plasma with an electron-plasma frequency far above the microwave frequency. This leads to similar properties as a metal with a skin-depth far below the wavelength. The plasma currents are squared and integrated over the whole area resulting in an expression proportional to the dissipated power. The final calibration is done with the well known losses of a smooth metal surface under perpendicular incidence. Periodic boundary conditions limit the calculation grid to one corrugation period. An additional phase detector is employed to characterize the corrugation with respect to perfectly balanced hybrid modes. The results show, that the losses in E-Plane polarization, where the fields penetrate into the grooves, are most critical. By optimizing the groove profile, a significant improvement can be achieved.

1. Introduction

Circular corrugated waveguides are widely used for transmission due to low loss propagation of the HE11 mode. Rectangular corrugated waveguides can be used for compact remote-steering antennas and power splitters. For high-power applications, the ohmic loss becomes important. The experience however has shown, that analytical formulas for the losses often are too optimistic. This is mostly due to the fact, that they calculate the fields and losses by modelling the corrugated wall as a surface with anisotropic impedance [1]. Thus, the fields and wall-currents inside the grooves, which contribute to the losses, are neglected. Other numerical and semi-analytical methods ([2],[3]) calculate the fields using the boundary conditions of a perfectly conducting wall. While this approach leads to very good results in many cases, the error caused by the assumption of a perfectly conducting wall becomes unacceptably large, if the frequency is near a resonance.

2. Calculation method

The new approach is based on the fact, that the propagation in a waveguide is similar to the periodic reflection of a plane wave at a planar wall, where the specular angle is the Brillouin-angle of the considered waveguide mode. The calculation is done by modelling the electromagnetic fields with a FDTD algorithm [4]. Fig. 1 illustrates the calculation scheme.

![Calculation grid](image)

Fig. 1: Calculation grid. The angle \( \phi \) is the specular angle (90° - angle of incidence).

The calculation grid is excited by a plane wave. Both polarizations can be investigated, where the antenna always excites the component perpendicular to the calculation plane (\( E_z \) for H-plane, \( B_z \) for E-plane). The absorber prevents reflections from the upper boundary. At the left and right borders, periodical boundary conditions (sine-cosine method [4]) are employed, which
reduce the calculation grid to a single corrugation period. The metal is treated as an unmagnetized plasma with an electron plasma frequency \( \omega_{pe} \) far above the microwave frequency. This leads to a skin depth, which is small compared to the wavelength and the typical dimensions of the corrugation.

For calculating the losses, the absolute values of the plasma current \( J \) are squared and summed for all grid points. Since the effective power injected into the system is proportional to \( \sin^{-1}(\phi) \), the expression characterizing the losses becomes:

\[
\frac{P_i}{P_i} \propto \sin(\phi) \sum |J|^2
\]

where \( P_i \) and \( P_i \) are the dissipated and incident powers, respectively. For the following calculations, these losses are normalized to the losses of a smooth wall with perpendicular incidence.

3. Examples

Figures 2a and 2b show the losses of a rectangular corrugation profile as a function of the corrugation depth and the angle, respectively.

![Figure 2: Calculated reflection losses as a function of the corrugation depth (left) and the angle (right). The corrugation profile is rectangular, the polarization is E-Plane at 158 GHz.](image)

One can see, that the losses become very low for small angles and corrugation depths around \( \lambda/4 \). This is the expected behaviour found in corrugated waveguides. At \( \lambda/2 \), the losses become very large, and the angular dependency is reversed. This result cannot be obtained by impedance model calculations, because the impedance model treats a corrugation depth of \( \lambda/2 \) like a smooth wall. For the smooth wall, the theoretical angular dependence of the losses in E-plane polarization is proportional to \( \sin^{-1}(\phi) \); this behaviour is confirmed in Fig. 2b. For corrugations depths around \( \lambda/4 \), the losses are proportional to \( \sin(\phi) \). This dependence is in agreement with the analytical formula for the HE11 loss in corrugated cylindrical waveguides as discussed below.

Fig. 3 shows the losses as a function of the aspect ratio (groove width / corrugation period) at a depth of \( \lambda/4 \). The E-plane losses become very large for narrow grooves. The reason is that the overall field structure is more similar to that of a smooth wall, especially the field maximum moves closer to the wall. Due to the boundary conditions, this increased field near the wall causes an extremely large gradient in the area of the groove entrance and hence a very high
field inside the groove. For low-loss corrugations, it becomes therefore important to make the aspect ratio as large as possible (limited by mechanical stability).

![Graph showing normalized reflection losses as a function of the aspect ratio.]

**Fig. 3:** Normalized reflection losses as a function of the aspect ratio.

### 4. Comparison with measurements

In earlier works [5], reflection losses have been measured with the 3-mirror resonator technique. Fig 4 shows a comparison of the calculated and measured data. In this case, the absolute losses (in %) are shown. The calibration factor needed for the calculation (reflection loss of a smooth wall at perpendicular incidence) was taken as 0.212% from measurements on Aluminum alloy extrapolated to 158 GHz, $\phi = 90^\circ$. One can see, that a good agreement between measurement and calculation could be achieved.

![Graph comparing calculated and measured reflection losses.]

**Fig. 4:** Comparison with measurements at 158 GHz. Shown are the reflection losses of smooth and corrugated aluminium walls in both E- and H-Plane. The lines are calculated, the symbols are measured values.

### 5. Comparison of corrugation profiles for 170 GHz

In the framework of the ITER remote steering design, several corrugation profiles have been investigated. The first one is the rectangular profile according to e.g. Fig. 4 scaled to 170 GHz. The second is the corrugated wall forseen in the current design for the ITER remote steering.
antenna. The third is a compromise of large aspect ratio, rounded edges and mechanical stability. Fig. 5 summarizes the losses and the phase difference for E- and H-plane reflection at an angle of 10°. For perfectly balanced hybrid modes, the phase difference for E- and H-plane reflection must be 180°. The most critical polarization is always the E-Plane, since for H-Plane, the fields cannot penetrate into the grooves and the dependence on the groove profile is weak. For E-Plane polarization, losses for the rectangular and ITER-RS corrugations are similar, while the optimized profile is much better. The phase differences are close to 180° in all cases, they could be further improved by a slight variation of the corrugation depth.

<table>
<thead>
<tr>
<th>Normalized losses</th>
<th>Rectangular</th>
<th>ITER RS</th>
<th>Optimized</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>H-plane</td>
<td>E-plane</td>
<td>H-plane</td>
</tr>
<tr>
<td>Losses</td>
<td>0.246</td>
<td>0.676</td>
<td>0.231</td>
</tr>
<tr>
<td>E/H Phase difference</td>
<td>175.23°</td>
<td>177.28°</td>
<td>182.15°</td>
</tr>
</tbody>
</table>

Fig. 5: Summary of losses and E/H phase differences for some corrugation profiles relevant for operation in ITER at 170 GHz.

6. Implications for the losses in rectangular and cylindrical waveguides

From the results presented above, one can draw conclusions for the losses in corrugated waveguides. In the Remote Steering antenna, which was proposed for ITER, it is a good approximation to treat the mode propagation like a single beam, which is reflected at the walls and propagates along a zig-zag path. Summing up the losses for each reflection leads to results, which agree very well with the measured values even though the actual field pattern looks quite different due to different phase constants of the propagating modes [6].

In cylindrical waveguides, the propagation of the HE_{11} mode is of major interest. The generic formula for ohmic attenuation given in [1] can be simplified for the ideally balanced case (infinite longitudinal wall impedance) and small Brillouin angles $\phi$:

$$\alpha_{th} = \frac{R_s}{2Z_a} \sin^2(\phi)$$

Here, $R_s$ is the surface resistance, $Z$ is the free space wave impedance and $a$ is the waveguide radius. This formula neglects the fields inside the grooves, but the dependence on the Brillouin-angle is correct, since it denotes the number of wall reflections multiplied by the loss per reflection. To obtain exact numbers for the ohmic attenuation, it would be possible to find a calibration factor, which depends on the corrugation profile. With the given results (e.g. Fig. 5), it is already possible to to a quantitative comparison of corrugation profiles, since the waveguide losses scale identically to the reflection losses of a corrugated plate.
7. Summary

A method was developed to calculate microwave reflection losses at corrugated metallic walls. It works for arbitrary corrugation profiles and both polarizations. The results agree very well with measurements. The method allows the design and optimization of corrugation profiles for oversized waveguides as well as polarizers. In principle, it is also possible to find corrugations with extraordinary high losses (e.g. E-plane with a corrugation depth around $\lambda/2$), which would allow the design of pure metallic absorber loads (i.e. without coating). The calculation of the phase difference between E- and H-plane is also important, since it affects the shape of the hybrid modes and therefore the coupling to free space modes in antennas.

8. References


