Electron Cyclotron Heating modeling in large tokamaks and ITER with 3D full wave code

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Abstract

We present modelling results of basic Electron Cyclotron Heating scenarios in several tokamaks and ITER performed with updated 3D full wave STELEC (stellarator_ECH, tokamaks included as particular case) code [1]. Code includes all basic wave physics as interference, diffraction, wave tunnelling, mode conversion at Upper Hybrid (UH) resonance to electron Bernstein waves and appropriate boundary conditions. Code operates in real 3D magnetic geometry and use massive parallel terabyte computers and firstly permitted solution of above problem. The basic non waited from ray tracing technique results are not only influence of diffraction effects but discovering UH resonance importance both at X-mode antenna excitation and at O-mode antenna excitation for fundamental harmonic (last one is contrary to ray tracing predictions). Thus so called “O and X” modes are coupled ones in exact solution. These effects, partly experimentally supported by DIII-D tokamak observed heating efficiencies, lead to another power deposition profiles and their space location, in compare with ray tracing technique. Coupling to X-mode reveals strong role of Electron Bernstein Waves previously neglected in ITER ECH modeling. These results can influence ECH/CD NTM suppression predictions for ITER, in parallel significantly decreasing requirements (respectively price) on ECH hardware (converters, polarization, etc) .Code permitted to investigate recent urgent issue: O-X-B ECH scenario for over dense tokamak/stellarator plasma “is myth or reality”?

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Introduction

Electron Cyclotron Resonance plasma heating (ECH) and current drive (CD) in Fusion plasma research in tokamaks and stellarators plays a key role in investigation of basic wave - plasma interaction physics like electrons heating, local transport coefficients behaviour, CD and current profile tailoring, Internal Transport Barrier (ITB) creation, Neoclassical Tearing Modes (NTM) stabilization, etc. The method, initially supposed to be very local in space, is an essential and very expensive tool in present fusion machines and ITER. Contrary, theoretical and modelling support during last 30 years was performed by so called very simple geometric optic approach, named as “ray tracing”, dropping important physics like reflection, interference, diffraction, wave tunnelling, mode conversion to electron Bernstein waves and appropriate boundary conditions. Reason for that simple ray tracing approach over all world use was due to non sufficient power of world computers in the past for ECH method modelling to solve exact problem in 3D plasma with small scale waves. Respectively planning, interpretation and prediction of ECH/CD were unsatisfactory ones as shows below full wave 3D code modelling.

1. STELEC Code

Code [1] solves 3D full wave equation
In flux coordinates with appropriate boundary conditions, in 3D stellarator geometry, on real VMEC code equilibrium, using poloidal and toroidal harmonic expansions over respective angles and center difference scheme over radial (psi) coordinate ( \( k_0 = \omega/c, c - \text{speed of light} \)).

In eq.(1.1) \( j^{\text{ext}} \) is an imposed antenna current density and \( j_p \) is the RF plasma current being evaluated using the small Larmor radius approximation by procedure starting from the results of [2], in which the Vlasov equation is solved in plane-stratified geometry, and assuming that the vector form of \( j_p \) in this limit holds also in 3D toroidal geometry. Zero plasma response is given by dielectric components:

\[
\varepsilon_{ii}^{(0)} = 1 - \frac{\omega_{pe}^2}{2\omega^2} \left[ \frac{\mu}{F_{5/2}(l)} + \frac{3}{2\mu} F_{7/2}(l) \right] \left[ \frac{\omega}{\omega + \omega_{ce}} \right]
\]

\[
\varepsilon_{12}^{(0)} = -i \frac{\omega_{pe}^2}{2\omega^2} \left[ \frac{\mu}{F_{5/2}(l)} + \frac{3}{2\mu} F_{7/2}(l) \right] \left[ \frac{\omega}{\omega + \omega_{ce}} \right]
\]

\[
\varepsilon_{33}^{(0)} = 1 - \frac{\omega_{pe}^2}{\omega^2} \left[ \frac{\mu}{F_{5/2}(0)} + 2\Psi^2 F_{7/2}(0) \right]
\]

The finite electron Larmor radius wave induced current is given as

\[
\vec{J} = \frac{c^2}{4\pi i\omega} \vec{R} \{ \nabla_\perp (\sigma \nabla_\perp (\vec{R}\vec{E}_\perp) - i\delta \nabla_\perp (\hat{\vec{e}}_b \times \vec{R}\vec{E}_\perp)) - \\
(\hat{\vec{e}}_b \times \nabla_\perp) (\sigma \nabla_\perp (\hat{\vec{e}}_b \times \vec{R}\vec{E}_\perp) + i\delta \nabla_\perp (\vec{R}\vec{E}_\perp)) \}
\]

in 3D magnetic geometry, with metric tensor, both given by VMEC) and expressed through Shkarovsky functions \( F_q(n) \) [3]:

\[
\sigma = \frac{1}{4} \frac{\omega_{pe}^2}{\omega_{ce}^2} \frac{v_{Te}^2}{c^2} \left[ \frac{\mu}{F_{7/2}(2)} + \frac{\omega}{\omega + 2\omega_{ce}} \right]
\]

\[
\delta = \frac{1}{4} \frac{\omega_{pe}^2}{\omega_{ce}^2} \frac{v_{Te}^2}{c^2} \left[ -\mu F_{7/2}(2) + \frac{\omega}{\omega + 2\omega_{ce}} \right]
\]

\[
F_q(n) = F_q(\Phi^2, \Psi), \quad \mu = \frac{m_e c^2}{T_e}, \quad \Psi^2 = \frac{\mu}{2} N^2, \quad \Phi^2 = \Psi^2 - \mu \frac{\omega - n\omega_{ce}}{\omega}
\]

\[
N_\parallel = \frac{k_{\parallel}}{k_0}, \quad \text{where parallel wave number is expressed through contravariant}
\]
components of equilibrium 3D magnetic fields and toroidal and poloidal modes numbers n, m [4]:

\[ k_1 = \frac{n B_e}{|B|} + \frac{m B_e}{|B|} \]

Out-off diagonal wave induced currents, neglected in initial modeling [1] to stress role of UH resonance at reduced wave damping, are given similar [2] with relativistic updates:

\[
\begin{align*}
\mathcal{J}^{(1,1)} &= \frac{c^2}{8\pi \omega c} \left\{ (\tilde{e}_b \cdot \nabla) \left[ (\xi_{-1} + \xi_{+1}) \nabla \times E_b \tilde{e}_b - i(\xi_{-1} - \xi_{+1}) \left( \nabla \times E_b \tilde{e}_b \right) \right] \right. \\
&\quad + \tilde{e}_b \left[ \tilde{e}_b \cdot \nabla \times \left[ (\xi_{-1} + \xi_{+1}) \left( \tilde{e}_b \cdot \nabla \right) \tilde{E}_\perp - i(\xi_{-1} - \xi_{+1}) \left( \tilde{e}_b \cdot \nabla \right) \left( \tilde{E}_\perp \times \tilde{e}_b \right) \right] \right\}, \quad (1.5)
\end{align*}
\]

Huge matrix resulting after discretization is solved by “Progonka” Algorithm (Gaussian inversion specialized for three diagonal block system) for parallel processors computer option, distributing matrices across processors, and uses the ScaLapack library and message passing interface (MPI) for parallel matrix operations. The individual blocks are distributed across all processors. The compact storage methods with out of core solver can also be used for serial code option.

Antenna is located in vacuum layer between plasma and conducting wall and modeled by current straps. Divergent free currents of generally elliptical polarization are imposed according to ECHO-mode or X-mode polarization at plasma separatrix, calculated through the local dispersion relation. Antenna currents generally are corrected by additional multipliers, requested by Maxwell equations solutions in vacuum layer surrounding plasma. Faraday toroidal and poloidal screens options are also provided, when needed, for linear polarization of an antenna.

2. Similarity Laws

Analysis of above equations shows that only combinations of parameters are used

\[
\left( \frac{\omega_{pe}}{\omega^2}, \frac{\omega_{ce}}{\omega}, T_e, N_\parallel \right) \quad (\text{magnetic equilibrium})
\]

Thus keeping these fixed (similar) ones and providing multi mode regime for excited ECH waves ( \( N_\perp = \frac{k_1}{k_0}, \lambda_0 \) - vacuum wave length, a – plasma minor radius)

\[ a \gg \frac{1}{2\pi N_\perp \lambda_0} , \]
it is possible to model ECH scenarios in large scale fusion machines at reduced frequencies thus diminishing requirements on computer memory capabilities (important for ITER like machines).

3. NSTX Tokamak Fundamental Harmonic Modeling

NSTX tokamak with major radius $R = 85$ cm, $a = 62$ cm uses quasi perpendicular O-mode outside launch at 7.65 GHz. Contour plots of $|E_{\text{minus}}|$ wave electrical field are displayed in Fig. 1a. The axis magnetic field $B_0 = 0.2856$ T, plasma density $N_e(0) = 6.7 \times 10^{17}$ m$^{-3}$ (parabolic density profile, $\alpha_n = 1$; central density is $\sim N_{cr}$), $T_e(0) = 4.95$ kV ($\alpha_T = 1$), $I_p = 0$, $2$ MA, $N_r(0) = 0.037$, $q(0) = 1.5$, $q(\psi_{95}) = 15.5$. Upper Hybrid resonance $\omega^2 = \omega_{ce}^2 + \omega_{pe}^2$ layer manifests itself, contrary to ray tracing, by bright “mirror” broadly radiating Electron Bernstein Waves (EBW) and slow X-waves to cold ECR side. Averaged over flux surface power deposition, in Fig. 1b, shows peaked power deposition mainly responding to large electrical fields of EBW near cyclotron layer ($X = 3$ cm). This is confirmed by 2D power deposition in Fig. 1c. EBW wave activity is crucial one. Main power absorption is at right resonance zone wing. Parallel electrical fields are small ones in bulk plasma (central density is near critical one for O-mode) as shown in Fig. 2.
The Fig. 3 demonstrates $|\text{real}(E_{\psi})|$ contours at O-X-B oblique fundamental harmonic outside launch to over dense NSTX L-mode plasma with parameters $N_e(0) = 0.49$, $F=7.65$ GHz, $N_e(0)=1.4\times10^{18}$ m$^{-3}$, $T_e(0)=4.95$ kV, $B_o=0.2856$ T, $I_p=0.2$ MA, $q(0) = 1.5$, $q(\psi_{95}) = 15.5$. The EB waves freely propagate to over dense plasma core.

3.1 Physics of nonlinear EBW-driven flows in 2D toroidal geometry

The EB wave group velocity goes to zero near UH and cyclotron resonances thus creating very large fields amplitudes as was demonstrated above. Thus appears a possibility of large force acting on electrons in localized space. Quasi linear analysis with Vlasov equation, analogous to [5] for ICRF, shows:

- Flows are driven by dissipative terms corresponding to
  - direct absorption of photon momentum $k/\omega$
  - nonlinear dissipative stress tensor
- For EB Waves in plasma, with account to relativistic effects, driving force part due to momentum absorption is proportional to absorbed power (to be averaged over magnetic flux surface, with important Up/Down equatorial plane non symmetry of $K_{par}$ spectrum)

$$F_d \approx \frac{k_{\perp}}{\omega} P - \frac{k_{\perp}}{\omega} k_{\parallel}^2 \rho_{ec} \mu \text{Im}(F_{5/2}(1)) \frac{E_{\perp}^2}{4} \omega \epsilon_0$$

Thus in the presence of a poloidal magnetic field, mode conversion near the Upper Hybrid resonance is dominated by a transition from the O-mode to the X-mode wave. The poloidal field generates strong variations in the parallel wave spectrum that cause EB wave damping in a narrow layer near the electron cyclotron resonance surface. The resulting poloidal forces in this layer drive sheared poloidal flows comparable to those in direct launch electron Bernstein wave experiments. This EBW sheared flow generation possibility at fundamental harmonic is interesting one for ITER due to ITB creation and turbulence control.

Outside quasi perpendicular O-mode ECH launch STELEC full wave modeling shows for NSTX tokamak: 1) strong coupling to X-mode with respective mode conversion to small scale EB waves; 2) large amplitude EB waves and strong modification of $K_{par}$ spectrum provide power absorption on right side of resonant zone (contrary to usual analytic and ray tracing approach); 3) This effect must be accounted in analysis of ECH in large fusion machines and predictive ITER ECH modeling; 4) similarity laws permit to perform predictive modeling at reduced computer resources.

4. ITER predictive ECH modeling

We did initial ITER ECH scenarios STELEC modeling at reduced frequencies, making use similarity laws, for designed O-mode outside launch. Representative ITER scenario #4 machine and plasma parameters are shown in Table:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Major radius, $R$</td>
<td>6.20 m</td>
</tr>
<tr>
<td>Minor radius, $a$</td>
<td>1.90 m</td>
</tr>
<tr>
<td>Separatrix radius</td>
<td>1.75 m</td>
</tr>
</tbody>
</table>
Plasma elongation, $\kappa$ 1.85
Plasma triangularity, $\delta$ 0.44
Nominal plasma current, $I_p$ 0.107 MA
Toroidal field, $B_0$ 0.077 T at $R = 6.2$ m
MHD safety factors $q_0$, $q_{\psi95}$ 1.25/6.0
Central electron temperature 25.8 keV
Temperatures exponent, $\alpha_T$ 1.0
Density exponent, $\alpha_N$ 1.0
Central electron density $3.7 \times 10^{16}$ m$^{-3}$
RF frequency 2.55 GHz
RF power 1 MW

The Fig.4 shows fundamental harmonic O-mode launch to ITER plasma with far inside NTM suppression scenario displaying contours of fields $|\text{real}(E-)|$, electron accelerating component. Visible are High Field Side EC resonance position at $X = -92$ cm and UH resonances at $X = -40$ cm.

![FIGURE 4a](image1) | ![FIGURE 4b](image2) |
---|---
**FIGURE 4a** $|\text{real}(E_{-})|$ in ITER  | **FIGURE 4b** $|E_{-}|$ in ITER

The fine EB waves structure in equatorial ITER plane is shown in Fig.5a,b

![FIGURE 5a](image3) | ![FIGURE 5b](image4) |
---|---
**FIGURE 5a** $E_{\psi}$ in ITER  | **FIGURE 5b** $E_{\zeta}$ in ITER
The 2D power deposition to electrons is given in Fig.6. Main power absorption is at right resonance zone wing, contrary to usual O-mode waitings at quasi perpendicular launch. Flux surface averaged power deposition is shown in Fig.7.

**FIGURE 6** $P_e(\rho, \theta)$ in ITER

Fundamental harmonic O-mode power deposition at O-mode quasi perpendicular outside (LFS) launch for increased magnetic field $B_0=0.095$ T (EC resonance is located at +26.5 cm) scenario in under dense ITER L-mode plasma is demonstrated with oblique antenna by contours of $|\text{real}(E_\psi)|$, $N_e(0) = 0.037$, on Fig.7a. The EBW structure in equatorial ITER plane is at Fig.7b.

**FIGURE 7a** $|\text{real}(E_{\text{minus}})|$ in ITER at $B_0=0.095$ T

2D power deposition to electrons. Main power absorption is again located at right resonance zone wing, Fig.8 ($P_{rf} = 1$ MW).

For comparison we give X-mode outside quasi perpendicular launch in ITER dense plasma ($B_0=0.095$ T, $N_e(0) = 7.4 \times 10^{16}$ m$^{-3}$ $\sim N_{cr}$ for O-mode). Outside antenna poloidal angle is 62°. Contour plots of $|\text{real}(E_\psi)|$ are given in Fig.9.

**FIGURE 5a** $E_\zeta$ in ITER at $B_0=0.095$ T

**FIGURE 6** $P_e(\rho)$ in ITER

**FIGURE 5** $E_z$ in ITER at $B_0=0.095$ T
The EBW and e.m. modes activity in ITER at outside X-mode launch to dense plasma is presented by Fig.10a,b for $E_{\psi}$ in equatorial plain and 2D $|\text{Im}(E_{\zeta})|$ contour plots.

5. Previous Fundamental Harmonic Scenario modeling in ITER-like Machine

Previously [1] was modelled ITER regime for NTM stabilization, relying on above similarity laws, with fundamental harmonic O-mode equatorial 1 MW launch into plasma with $R_0=6.2$ m, elongation $\kappa=1.85$, triangularity $\delta=0.45$, $N=380$ ($N_t(0)=0.13$), $F=22.6$ GHz, $T_e=25.8$ kV, $N_e(0)=1.2\times10^{18}$ m$^{-3}$, $(\alpha_n=0.2, \alpha_T=2)$, $B_0=0.646$ T, $I_p=1.27$ MA, $q(0)=0.88$, $q(a)=4.2$. Classical Doppler broadening was assumed and wave induced Far out-off diagonal currents $J^{(1,1)}$, (1.5), were dropped ones. Results are displayed in
Figs. 11 by $P_{\phi}(r)$. One can see HFS EC resonance at X=-117cm and UH resonances at X=-62cm. Power deposition is in Fig.11b again indicates crucial role of UH resonance in broadening of EC power localization, leading to large deposited power outside of supposed NTM localization.

**FIGURE 11** Power deposition $P_{\phi}(\rho)$ in ITER at reduced wave damping

**Conclusions**

Thus 3D Full wave ECH STELEC code numerically well resolved modelling for NSTX and ITER tokamaks supported our previous finding [1]: 1) O-mode and X-mode coupling in toroidal plasmas at fundamental EC harmonic; 2) Electron Bernstein waves play crucial role at O-mode antenna polarization (contrary to ray tracing) and lead to broader EC power deposition profiles. Last ones are located in another space positions in compare with usual ray tracing predictions; 3) Fundamental harmonic O-mode scenario modelling for ITER at reduced frequencies, making use similarity laws, confirmed remarkable difference in power depositions in compare with ray tracing; 4) This new role of huge amplitudes mode converted EB waves provides a possibility of sheared flow generation ($\sim E^2$), important for ITB creation and plasma turbulence control. The poloidal magnetic field plays an essential role in allowing strong damping of the EB wave on electrons, which is optimal for flow drive.

Natural looks our (similar to [6]) idea of wave polarization transformation is due to POLOIDAL non uniformity of toroidal plasmas in tokamaks and stellarators to understand ECH full wave code’s news.

**REFERENCES**

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