RELATIVISTIC EFFECTS IN ELECTRON CYCLOTRON RESONANCE HEATING AND CURRENT DRIVE

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SYNOPSIS

• A brief description of the evaluation of the relativistic conductivity tensor.

• The effect of relativity on the propagation and damping of waves in the electron cyclotron range of frequencies.

• The relevance, to ITER, of studies on electron cyclotron heating in spherical tori.
There are essentially two steps in determining the homogeneous plasma conductivity tensor.

Linearizing the Vlasov equation

\[ \frac{\partial f}{\partial t} + \frac{\vec{p}}{m\gamma} \cdot \nabla f + q \left( \vec{E} + \frac{\vec{p}}{m\gamma} \times \vec{B} \right) \cdot \nabla_p f = 0 \]

where \( \gamma = (1 + p^2/m^2c^2)^{1/2} \).

\[ f \equiv f (\vec{r}, \vec{p}, t) = f_0 (\vec{p}) + f_1 (\vec{r}, \vec{p}, t), \]

\[ \vec{B} \equiv \vec{B} (\vec{r}, t) = \vec{B}_0 + \vec{B}_1 (\vec{r}, t), \]

\[ \vec{E} \equiv \vec{E} (\vec{r}, t) = \vec{E}_1 (\vec{r}, t) \]
RELATIVISTIC CONDUCTIVITY TENSOR

- \[ \frac{df_1}{dt} = -q \left( \vec{E}_1 + \frac{\vec{p}}{m\gamma} \times \vec{B}_1 \right) \cdot \nabla_p f_0 \]

- on the left hand side is the convective derivative.

- Formally:

\[ f_1 (\vec{r}, \vec{p}, t) = -q \int_{-\infty}^{t} dt' \left( \vec{E}_1' + \frac{\vec{p}'}{m\gamma'} \times \vec{B}_1' \right) \cdot \nabla_{p'} f_0 \]

where the integration is along the unperturbed orbits.
The wave fields are given by the Faraday-Ampere system of equations

\[ \nabla \times (\nabla \times \vec{E}) + \frac{\partial^2}{\partial t^2} \vec{E} = \vec{J} \]

where \( \vec{J} \) is the current density

\[ \vec{J}(\vec{r}, t) = qn_0 \int d^3p \left( \frac{\vec{p}}{m\gamma} \right) f_1(\vec{r}, \vec{p}, t) \]

The plasma conductivity is obtained from

\[ \vec{J} = \vec{\sigma} \cdot \vec{E} \]

\[ \Rightarrow \vec{\sigma} \propto \int_{-\infty}^{t} dt' \int d^3p \ldots \]
For a Maxwellian $f_0(\vec{p})$ only one of the integrals can be done analytically.

For a non-Maxwellian $f_0(\vec{p})$, in general, only the time history integral can be done analytically.

A numerical code $R2D2$ has been developed to evaluate fully relativistic $\vec{\sigma}$ (Ram et al. - 2002, 2003)

- do the time history integrals numerically (highly oscillatory integrands);
- do the momentum space integrals numerically (singular integrands)
- do the momentum space integrals numerically (significant analytical massaging) – one to two orders of magnitude reduction in time.
COMPARISON BETWEEN RELATIVISTIC AND NON-RELATIVISTIC WAVE CHARACTERISTICS

\[ \frac{\omega}{2\pi} = 170 \text{ GHz}, \quad \frac{\omega_{pe}}{\omega} = 0.75, \quad n_\parallel = 0.1, \quad T_e = 5 \text{ keV} \]
COMPARISON BETWEEN RELATIVISTIC AND NON-RELATIVISTIC WAVE CHARACTERISTICS

\[
\frac{\omega}{2\pi} = 170 \text{ GHz}, \quad \frac{\omega_{pe}}{\omega} = 0.75, \quad n_{||} = 0.1, \quad T_e = 10 \text{ keV}
\]
DAMPING PROPERTIES OF THE O MODE

\[ \text{Im}(n_\perp) \]

\[ \omega / \omega_{ce} \]

\( n_\parallel = 0.1 \)

Dashed line: \( \omega_{pe}/\omega = 0.75 \); solid line: \( \omega_{pe}/\omega = 0.92 \)
The electron cyclotron plasma waves usually used in experiments are the X and O modes.

In the same frequency range there also exist electron Bernstein waves

- heating and current drive demonstrated, e.g., in Wendelstein 7-AS and Compass-D;
- emission and coupling studies in spherical tori (NSTX, MAST), reversed field pinch (MST), and tokamak (TCV).
The differences between X & O modes and electron Bernstein waves are

- vacuum waves, plasma waves;
- directly coupled to in a plasma, coupling to a plasma through mode conversion;
- can have density cutoffs in a plasma, no density cutoffs in a plasma;
- strong cyclotron resonance damping at low ($\leq 3f_{ce}$) harmonics, strong cyclotron resonance damping at all harmonics;
- not useful in plasma regimes where $f_{pe}/f_{ce} \gg 1$, useful waves in overdense plasmas (NSTX, MAST).
ADVANTAGES OF STUDYING
EBW PHYSICS IN SPHERICAL TORI

- Electron Bernstein wave physics in a spherical torus can be of help in understanding electron cyclotron wave physics in general
  - high power sources at low frequencies are more readily available;
  - the physics of wave damping and of the wave-particle interaction in the vicinity of the electron cyclotron resonances has similar features to those of X and O modes;
  - the current drive physics also has similar physical characteristics.
Comparation between relativistic and non-relativistic EBWs

$\omega_{pe}/\omega_{ce} = 6, \ \omega/\omega_{ce} = 1.9, \ n_\parallel = 0.2$

Relativistic effects become important for $T_e > 500 \text{ eV}$. 
IMPLICATIONS OF RELATIVISTIC EFFECTS

- Approaching the electron cyclotron resonance
  - from the low field side \((\omega > n\omega_{ce})\): relativity narrows the absorption profile;
  - from the high field side \((\omega < n\omega_{ce})\): relativity broadens the absorption profile.
COMPARISON BETWEEN RELATIVISTIC AND NON-RELATIVISTIC WAVE CHARACTERISTICS

\[ \frac{\omega}{2\pi} = 170 \text{ GHz}, \quad \frac{\omega_{pe}}{\omega} = 0.75, \quad n_\parallel = 0.1, \quad T_e = 10 \text{ keV} \]
For EBWs: \( 200 \lesssim \tau_n \lesssim 1000 \).

EBWs interact with electrons in the range \( 3 \lesssim p_\parallel/p_{te} \lesssim 4 \).
• Various physical time scales
  – Landau damping time: \( \tau_{LD} \)
  – Decorrelation time: \( \tau_d \sim 1/(v_{te}\Delta k) \)
  – Transit time: \( \tau_{TR} \sim \Delta_b/v_{te} \)
• Quasilinear time scale: \( \tau_{QL} \sim \tau_{LD} \sim \tau_d \sim \tau_{TR} \)
• Nonlinear time scale: \( \tau_{NL} \sim \sqrt{me\over e|E||k|} \)
• Validity of quasilinear theory: \( \tau_{QL} \ll \tau_{NL} \)
  – this could be easily violated by electron Bernstein waves in spherical tori.
R2D2 has been coupled to LUKE - a drift kinetic Fokker-Planck solver (J. Decker & Y. Peysson)

\[
\frac{\partial f}{\partial t} + v_{GC} = C(f) + Q(f)
\]

- \( f \) is the guiding center distribution function

- \( v_{GC} = v_\parallel \hat{b} + v_D \).

This set of codes is used to calculate current drive by electron Bernstein waves.
FISCH-BOOZER CURRENT DRIVE

- $P = 1$ MW, $I = 100$ kA.
- $\eta = \frac{m_e \nu_e \nu_{te}}{e} \frac{J}{P}$, $\xi_{CD} = 32.7 \frac{IRn_e[20]}{T_e[keV]P}$.
- $\eta_{peak} = 3.7$, $\xi_{CD} = 0.67$ (for ECCD: $\xi_{CD} \sim 0.3$).
CONCLUSIONS

- Relativistic effects play a significant role in the propagation and damping of electron cyclotron waves modification to the deposition profile and interaction with electrons.

- A set of codes (beam and ray tracing with relativistic conductivity, Fokker-Planck for the evolution of the distribution function) is being developed.

- Heating and current drive experiments with electron Bernstein waves on spherical tori can provide insight into the interaction of high power electron cyclotron waves with electrons

  - these experiments will also be a good test of the analytical and computational models.