Study of the redistribution of energetic and impurity ions by sawteeth in ASDEX Upgrade

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Abstract.

In the non-linear phase of the sawtooth, the complete reconnection of field lines around the \( q = 1 \) flux surface often occurs resulting in the radial displacement of the plasma core. A complete time-dependent electromagnetic model of this type of reconnection has been developed (based on the analysis of [10]) and implemented in the EBdyna_go code [7]. This contribution aims at studying the behaviour of ions, both impurity and fast particles, in the pattern of reconnecting field lines during sawtoothing plasma experiments in the ASDEX Upgrade tokamak by using the newly developed numerical framework.

The results of detailed simulations of the sawtooth crashes are compared to measurements of various diagnostics such as Soft X-Ray (SXR) tomography, Collective Thomson Scattering (CTS) and Fast-Ion D-Alpha (FIDA) spectroscopy.

1. Introduction

A sawtooth crash is triggered when the \( m = 1 \) mode is destabilized [14], where \( m \) is a poloidal mode number. We have built a numerical model (named EBdyna_go or EBdyna for conciseness [7]) of the sawtooth collapse based on the original full reconnection pattern suggested by Kadomtsev [8] with the dynamical evolution of the electromagnetic fields during the collapse as introduced by Kolesnichenko et al. in [10]. The code evolves the ions according to their momentum equation. The first part of the sawtooth ’collapse’ is the ’crash’ phase where reconnection occurs. EBdyna is applied here in the specific experimental configuration of the ASDEX Upgrade tokamak to understand the effect of the collapse on ions with different orbit categories (passing or trapped) and with a large range of velocities (from thermal impurity ions up to NBI generated 60keV fast ions).

The model for the dynamical evolution of the electromagnetic field during a sawtooth collapse is discussed in Section 2. An analysis of the effect of the reconnection on tungsten impurity ions is given in section 3. The numerical modelling is then applied
in section 4 to a population representative of a 60keV Neutral Beam Injection (NBI) system, as obtained by the NUBEAM-TRANSP solver [12]. We compare CTS [11] and FIDA [3] measurements with the EBdyna simulations. Finally, Section 5 comments on the various dynamical behaviour of the NBI ions depending on their energy and pitch.

2. Dynamical modelling of the sawtooth collapse in ASDEX Upgrade

2.1. Experimental plasma conditions

The experiments that we consider in this paper include a significant population of fast particles from the co-current NBI system, representing up to one third of the bulk pressure. These particles affect the stability of the plasma [2], yielding a sawtooth cycle with large sawteeth where the radial size of the mixing region $r_{\text{mix}}$ extends up to half of the minor radius $a \approx 0.51 \text{m}$. The major radius of the torus is $R_0 \approx 1.63 \text{m}$ in FINESSE.

On a given flux surface, the safety factor $q = d\theta/d\varphi$ describes the helicity of the field line ($\theta$ being a poloidal angle and $\varphi$ being the toroidal angle). In table 1 we give values of the plasma parameters (or their average) at the pre-crash position of the $q = 1$ flux surface (subscript 1). Without diamagnetic corrections (damping), the $m = 1$ mode growth rate in the large Larmor radius collisionless limit is [13]:

$$\gamma_{de} = \left[ \frac{2}{\pi} \left( 1 + \frac{T_{e1}}{T_{i1}} \right) \right]^{1/3} \frac{d_{e1}}{r_1} \left( \frac{\rho_{Li1}}{d_{e1}} \right)^{2/3} \text{ with } \hat{\omega}_A = \frac{v_{Al1}s_1}{\sqrt{3}R_0},$$

where $\hat{\omega}_A$ is an effective Alfvén frequency ($s_1 = r_1(dq/dr)_{r=r_1}$) and $v_{Al1}$ is the Alfvén velocity.

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Table 1. Comparison of the experimental parameters just before the considered sawteeth. The global plasma current was $I_p \approx 1 \text{MA}$ and the position of the magnetic axis was approximately at $R_{\text{axis}} \approx 1.68 \text{m}$. Successively listed are: the total magnetic field, the poloidal beta, toroidal rotation, radial position, electron temperature, ion temperature, electron density, Larmor radius, electron skin depth $d_{e1}$, helical flux, theoretical growth rate and experimental reconnection duration.
2.2. Pre-collapse equilibrium and coordinates

The total pressure profile and the safety factor profile were obtained from TRANSP. $B_0$ is the experimental magnetic field on axis. The equilibrium code FINESSE [9] was then used to reproduce these data. The simulations were performed with a current in the opposite direction as that of the toroidal magnetic field, as in the experiments. The unit vectors obey $e_\varphi = e_R \times e_Z$: this results in a positive toroidal field component and a negative current (figure 1). The output of the equilibrium code gives us a mesh in the poloidal plane representative of the poloidal flux contours and of a straight-field-line poloidal angle. It provides the horizontal coordinate $X(\psi, \theta) = R(\psi, \theta) - R_0$ and the vertical one $Z(\psi, \theta)$. Here $\psi$ is the (divided by $2\pi$) poloidal magnetic flux. Regarding the description of the particles, the parallel velocity $v_\parallel$ of co-current circulating particles is thus negative. We define the pitch $\mathcal{P}$ of a passing ion as $\mathcal{P} = -\langle v_\parallel \rangle / v$ where the average is taken over one orbit and $v$ is the total velocity, assumed approximately conserved. Correspondingly we also define the trapping parameter $\lambda_0 = \mu B_0 / \mathcal{E}_{\text{kin}}$ where $\mu$ is the magnetic moment of an ion and $\mathcal{E}_{\text{kin}}$ its kinetic energy.

For convenience, we use a length index $r$ to label flux surfaces. It is defined as $r = \sqrt{S(\psi) / \pi}$ where $S(\psi)$ is the area of the surface enclosed by the poloidal contour of the flux surface $\psi$. We also use the normalized radial flux coordinates $\rho_{\text{tor}} = \sqrt{\psi_{\text{tor}} / \max(\psi_{\text{tor}})}$, where $\psi_{\text{tor}}$ is the toroidal magnetic flux.

2.3. Description of the sawtooth collapse

The global description considered for the reconnection occurring during a sawtooth collapse is designed according to Kadomtsev [8] . Surfaces that have identical helical flux $\psi_*$ on both sides of the separatrix reconnect during the crash and create a new helical flux surface contour. The helical flux corresponds to an auxiliary helical field.
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**Figure 2.** Left: Representation of the contours of the electric potential Φ in V for the sawtooth reconnection of AUG#308015@2.97s. The red line indicate the position of the \( m = 1 \) island separatrix. Middle: compared trajectories of NBI ions of about 33keV during this sawtooth. These particles were initially in the core and are redistributed to the mixing radius region. The possible change of orbit category and thus of pitch \( P \) for these particles is clearly seen in the bottom figure. Right: SXR measurements (AUG#308015@2.97s) compared with simulation of W (radial position of maximum of density and magnetic core).

\[ \mathbf{B}_* = (\mathbf{e}_\phi \times \nabla \psi_*) / R. \] The complete description of the evolution of the electromagnetic potentials was described in [10] and implemented in [7]. Figure 2 illustrates the contours of the electric potential Φ during the reconnection in the geometry of ASDEX Upgrade.

The crash time that we consider in this study is typically of about \( \tau_{cr} \sim 120\mu s \). In table 1, the values are based on the radial position of the maximum of emission as seen in the Soft-X-ray tomography (see for example figure 2 (right)). The total collapse simulation is assumed to last about \( \tau_{coll} \sim 250\mu s \). The dynamics is supposed to be close to the one given by the model described in [10] where we consider that the reconnection phase of duration \( \tau_{cr} \) represents roughly 48% of \( \tau_{coll} \).

Since in some crashes considered the impurity density is not peaked yet, a large uncertainty of about 50% remains on the real dynamics of the reconnection. The next section investigates the peculiar behaviour of the maximum of emission of W due to the high centrifugal force.

### 3. Comparison of Soft X-Ray tomography with W impurity simulations

#### 3.1. Modifications of the motions of ions to take into account a centrifugal force

An arbitrary steep density profile of tungsten (W) ions having charge \( Z_i = 35 \) [16] is specified. The velocity of the tungsten is distributed according to Maxwell-Boltzmann statistics with a temperature equal to that of the background ions. This population undergoes a significant effect of the plasma rotation that can be described as the centrifugal force in the reference frame of the plasma: \( \mathbf{f}_c = mR\Omega^2 \mathbf{e}_R \). The effect of the Coriolis force is assumed to be negligible. The momentum equation and the algorithm to evolve the position are found in [7]. The total electric field \( \mathbf{E} \) felt by the particles is
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implemented as the sum of the sawtooth field and a centrifugal contribution:

\[ E = E_{sth} + E_c \]

where \[ E_{sth} = -\nabla \Phi + \frac{\dot{\psi}_s}{R} e_\varphi \]

and \[ E_c = \frac{m}{Z_i e} R \Omega^2 e_R \]. (2)

This results in the following equation for the total energy evolution:

\[ \frac{dE}{dt} = (Z_i e) \left[ \frac{v_A}{R} \left( \frac{\partial \psi}{\partial t} \right) + \left( \frac{\partial \Phi}{\partial t} \right) \right] + f_c \cdot \mathbf{v} . \] (3)

Furthermore, to avoid the creation of a large population of W ions that have zero kinetic energy, we arbitrarily limit the minimum value of the kinetic energy to 1keV during the simulations, mimicking the effect of the collisions.

3.2. Dynamics of the maximum of emission

The Soft X-Ray (SXR) diagnostics at ASDEX Upgrade consist of several pinhole cameras gathering 208 lines of sights lying in a poloidal plane at a given toroidal position. The diode currents are originally recorded at sample rates of 500 kHz or 2MHz, allowing a time resolution of about 2\( \mu \)s and a good coverage of fast MHD phenomena. The cameras further allow tomographic reconstruction of the plasma SXR emission [1].

The plasma of ASDEX Upgrade contains a significant amount of tungsten impurity because W is the main material of most plasma facing components. The emission of SXR can thus be considered proportional to the density of W in the plasma. The approximate description of the tungsten distribution is used in EBdyna to give a qualitative interpretation of the Soft X-Ray measurements obtained during some of the sawteeth considered in terms of radial displacement of the core \( \xi \). An illustration is given in figure 1 (middle) and in figure 2 (right) where it is seen that the complete reconnection model can qualitatively account for the measured emission.

3.3. Complete reconnection estimate of the crash duration \( \tau_{cr} \)

It is important to note that the sawteeth considered are likely incomplete [6]: simulations do not account for the remaining \( m = 1 \) feature as seen by Mirnov coils (figure 1, right). However, in first approximation, the complete reconnection model can be further analyzed in terms of the dissipation of magnetic energy during the reconnection. This representation is also considered good enough since it accounts for the redistribution of the ions. A simple geometrical analysis yields, according to mass conservation:

\[ \xi \Delta = 2v_A \delta \] (4)

where \( v_A \) is the outflow velocity, \( \delta \) is the radial layer width and \( \Delta \) its approximate length in the poloidal plane. At one instant in time, the plasma volume \( \delta V_1 \) enters the layer with the helicity \( B_{s1} \) from the core and the volume \( \delta V_2 \) with \( B_{s2} \) enters from the outer region. The plasma of mass density \( m_i n_i \) then leaves the layer with the helicity \( B_{s3} \). The energy entering the reconnection region per unit time is mostly magnetic. The difference of magnetic pressure \( \Delta P_{mag} \) between the inflow and the outflow has to
be reflected by an increase of the kinetic energy of the plasma or be dissipated through Joule heating inside the layer through the term $\Delta E_\eta$. The overall energy density balance in the layer is thus approximately:

$$\Delta P_{\text{layer}} = \Delta P_{\text{mag}} - \frac{\Delta E_\eta}{\delta V_3} \simeq \frac{1}{2\mu_0} \left( \frac{B_{s1}^2 + B_{s2}^2}{2} - B_{s3}^2 \right) - \frac{\Delta E_\eta}{\delta V_3} = \frac{1}{2} m_i n_i v_{A*}^2 \delta V_3$$

where $\delta V_3$ corresponds to the global volume of plasma transiting through the layer during the time interval $dt$ and we have considered $\delta V_1 \sim \delta V_2 \sim \delta V_3/2$; the losses through Joule heating are expressed as: $\Delta E_\eta = \eta_{||} \langle \tilde{j} \rangle^2 \delta V_{\text{layer}}$ with $|\langle \tilde{j} \rangle| \simeq (|B_{s1} - B_{s3}|)/(2\mu_0\delta)$ and $V_{\text{layer}} = 2\pi R_0 \delta d_e$. It is possible to use an extended Ohm’s law to estimate the width of the layer $\delta$. Following the interpretation given in [19], we consider:

$$\delta \simeq d_e \left( \frac{1}{\tau_{A*}} + (1 - q_0) \frac{v_{\text{the}}}{R} \right)^{1/2} \tau_{A*}^{1/2}$$

with $v_{\text{the}} = \sqrt{\frac{2T_e}{m_e}}$ (6)

and we retain for the transit time of the ions in the layer the expression $\tau_{A*} = 2\Delta/v_{A*}$. A simple numerical solver using as an initial guess the dynamics for $\xi$ introduced in [10] converges to similar values of $\tau_{cr}$ as that observed experimentally, provided Joule dissipation or plasma toroidal rotation do not become dominant. However, the relatively large resistivity in the considered ASDEX Upgrade experiments, as well as the magnetic signal remaining after the reconnection, makes the overall model subjective.

4. Redistribution of Neutral Beam ions compared with the measurements

4.1. Collective Thomson Scattering (CTS) measurements

![Figure 3](image.png)

Figure 3. Left: Illustration of the probe and receiver lines of the CTS geometry (top view). The passive view is used to subtract spurious background emission [11]. Middle and Right: illustration of the spectrum obtained from the measurements (error bars). The intensity can be compared with a reconstructed emission based on the NBI simulation and measured thermal-ion plasma properties for the case of the crash AUG#30382 at 2.3s (middle) and AUG#30382 at 2.5s (right).

CTS offers the capability to measure the fast ion distribution locally. As it propagates through the plasma, the radiation from a high power probe beam combines
Study of the redistribution of energetic and impurity ions by sawteeth in ASDEX Upgrade with plasma fluctuations, causing the emission of ‘scattered’ radiation. The received signal bears a signature of the ion velocity distribution (due to Doppler shift of the signal) provided that collective effects dominate the triggered fluctuations. If we denote \( \lambda_D \) the Debye length, this requires that \( |\lambda_D k_\delta| < 1 \). Here the resolved fluctuation wave vector is defined as \( k_\delta = k_s - k_i \), where \( i \) and \( s \) refer to the incident and scattered wave vectors, respectively. A frequency shift in scattered radiation \( \nu_\delta \) can be approximately related to an ion velocity \( v_{\text{ion}} \) by \( \nu_\delta = \nu_s - \nu_i = v_{\text{ion}} \cdot k_\delta / 2\pi \).

In ASDEX Upgrade the incident diagnostic beam is created by a gyrotron with an output frequency of 105GHz and typical output power of 500kW [15]. The overall reconstructed emission (adding reconstructed NBI and thermal emission) closely match the measured ones (figure 3). Further interpretation of the spectra in terms of distribution of velocities is discussed by J. Rasmussen’s in his contribution, also in the proceedings of this meeting.

4.2. Fast Ions Deuterium-Alpha (FIDA) spectroscopic measurements

The FIDA diagnostic technique is Charge-eXchange Recombination Spectroscopy (CXRS) applied to fast D-ions: it uses the Balmer alpha emission line [3] with a time resolution of 2ms. We can compare reconstructed signals from two different views with the measured intensity as is done in figure 4. Good agreement is found for AUG#30382 at 2.5s where the remaining discrepancy can be explained by the absence of a source term (NBI fueling) in EBdyna. The FIDA diagnostic can further be used as a tomographic system ([17],[18]) aimed at resolving the velocity space of fast Deuterium in the experiment [4]. The tomographic inversion is shown in figure 5. Although qualitative agreement is found regarding the evolution of redistribution with kinetic energy for fully passing and barely passing ions, a significant discrepancy remains with the simulation, especially regarding passing particles with intermediate pitch (\( P \sim 0.6 \)).
5. Insights on the dynamics of the NBI fast ions during a sawtooth

5.1. Behaviour of trapped energetic ions

The trapped fast D particles represent in these experiments about 30% of the global NBI population. The main result of the previous work of Kolesnichenko et al. [10] is that, when equaling the crash time with the drift precession period, there is evidence of an energy threshold above which more than half of the energetic trapped ions is not redistributed by the sawtooth and is effectively detached from the reconnecting field line motion of the sawtooth crash. It scales as

$$E_{\text{crit}} \simeq r_1 R_0 (Z_i e) B_0 \omega_{\text{cr}} \quad \text{with} \quad \omega_{\text{cr}} \sim 1.5 \pi / \tau_{\text{cr}}.$$  (7)

We thus find the approximate scaling $E_{\text{crit}} \simeq 4 / \tau_{\text{cr}}$ in eV in the considered experiments. This roughly corresponds to the range $30 - 40$keV for Deuterium ions in our simulations.

Actually, the situation is made more complicated by the presence of a resonant [7] pitch (specific value of $\lambda_0$): a small fraction of the trapped population is not able to circulate in the helicity of the crash (precession $\omega_v D \sim 0$) and thus remains attached to it up to higher energies. This feature partly explains the lesser effect of the crash dynamics in figure 5 compared with what would be expected from expression (7).

![Figure 5.](image)

**Figure 5.** Left: comparison with tomography for the crash AUG#308015 at 2.29s. The considered poloidal positions were around ($R \simeq 1.74, Z \simeq 0$ or $\rho_{\text{tor}} \sim 0.1$). Middle: AUG30382, several dynamics yielding various values of $E_{\text{crit}}$ ($\Delta p_\phi$ refers to the change of $p_\phi$, and thus of radial position, during the sawtooth). Right: illustration of the influence of pitch and energy of co-current passing particles (in the core region $q < 0.97$) on their redistribution for the crash AUG#30815 at 2.29s.

5.2. Behaviour of passing energetic ions

The co-current circulating fast D particles represent about 58% of the global NBI population. The circulating particles have their behaviour mostly guided by the value of their pitch $\mathcal{P}$ or correspondingly their trapping parameter $\lambda_0$. As was studied in [7],
there is qualitatively an impact of the orbit width $\delta_r$ of the particles on their interaction with the perturbation.

Barely circulating ions have a specific dependence of their redistribution on kinetic energy. It is due to the increase of the ratio between drift precession $\omega_{vD}$ and longitudinal precession $\omega_\psi$ allowing partial detachment [7]. This is shown in figure 5 and is qualitatively recovered for extreme pitch values ($P \sim 0.2$ and $P \sim 1$) by the tomography.

6. Conclusions

We have implemented a numerical model, EBdyna, to simulate the effect of a sawtooth collapse on a population of high energy ions. We have successfully reconstructed various sawtooth crashes in realistic ASDEX Upgrade equilibria. The code assumes complete reconnection, whereas in the experiment incomplete reconnection is often observed. However, good agreement is found between simulations and FIDA or CTS measurements. On the other hand, despite qualitative agreement, we still find significant discrepancies in the pitch profiles as calculated by EBdyna and the ones reconstructed by FIDA tomography. Future work will address the refinement of the modelling.

Acknowledgments

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7. References

[16] Pütterich T. et al. 2010 Nucl. Fusion 50 025012