Stability analysis of runaway-driven waves in a tokamak

Pavel Aleynikov\textsuperscript{1,2} and Boris N. Breizman\textsuperscript{3}

\textsuperscript{1} Max Planck Institute for Plasma Physics, Greifswald, Germany
\textsuperscript{2} ITER Organization, Route de Vinon sur Verdon, 13115 St Paul Lez Durance, France
\textsuperscript{3} Institute for Fusion Studies, UT Austin, USA

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Outline

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• Ray-tracing code COIN
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  1. Week collisional dumping for whistlers
  2. Internal reflection and mode conversions
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Motivation

• High RE current (several MA) can be generated during disruptions in ITER.
• Large fraction of plasma magnetic energy can be converted into RE kinetic energy. Average electron energy could be > 10 MeV.
• Uncontrolled loss of such REs can cause localized damage of the Plasma Facing Component (PFC).

Kinetic instabilities could be potentially beneficial for RE suppression.

A carbon dust particles produced from a plasma-facing component under the impact of runaway electrons caused by a disruption in Tore Supra. (Photo courtesy: CEA).
Experimental observations of RE driven instabilities


2) B.C Schokker, P.C. de Vries et al, 21st EPS, 1994 (Montpellier, France) “Characterization of the RTP slidaway regime and its fast electron population”


Earlier assessments related to ITER

Post-disruption plasma is different. Higher density, lower temperature.


   Considered only the upper-hybrid mode. The robust stability is predicted for ITER.


   1) Collisional damping ($\Gamma_d = 1.5 \nu_e$) is overestimated;

   2) Convective damping is considered. $\Gamma_d = \frac{\partial \omega / \partial k_{\perp}}{4L_r}$ ($L_r$-runaway beam radius)

   It is a rough estimate due to the possibility of the reflection of the wave from the plasma boundary and the evolution of the wave vector during propagation which may break the resonant condition.
Candidate modes

Dispersion relation:  \[ \varepsilon N_\perp^4 + N_\perp^2 \left[ g^2 - (\varepsilon + \eta)(\varepsilon - N_\parallel^2) \right] - g^2 \eta + \eta(\varepsilon - N_\parallel^2)^2 = 0 \]

Dielectric tensor components:  
\[ \varepsilon = 1 - \sum_i \frac{\omega_{pi}^2}{\omega^2 - \omega_{ci}^2}; \quad g = \sum_i \frac{\omega_{ci}}{\omega} \left( \frac{\omega_{pi}^2}{\omega^2 - \omega_{ci}^2} \right); \quad \eta = 1 - \sum_i \frac{\omega_{pi}^2}{\omega^2} \]

High frequency electron waves:

Upper-hybrid wave:  \[ \omega = \sqrt{\omega_c^2 + \omega_p^2} \]

Magnetized plasma wave:  \[ \omega = \omega_p \frac{|k||c|}{\sqrt{k^2 c^2 + \omega_p^2}} \]

Whistler:  \[ \omega = |\omega_c| \frac{|k||kc^2| \sqrt{1 + k^2 c^2 \omega_c^2 / \omega_p^4}}{\omega_p^2 \sqrt{1 + k^2 c^2 \omega_c^2 / \omega_p^4}} \]

Waves can be excited via:
- anomalous Doppler resonance:  \[ \omega + |\omega_c| / \gamma = k_\parallel V_\parallel \]
- Cherenkov resonance:  \[ \omega = k_\parallel V_\parallel \]
The substitution $\omega \Rightarrow \omega + i\nu_e$ in the conductivity tensor is valid. Plasma dielectric tensor becomes:

$$
\varepsilon_{\alpha\beta} = \begin{pmatrix}
1 - \frac{\omega_p^2}{(\omega + i\nu_e)^2 - \omega_c^2} & \frac{\omega + i\nu_e}{\omega} & i \frac{\omega_p^2}{(\omega + i\nu_e)^2 - \omega_c^2} |\omega_c| & 0 \\
-\frac{\omega_p^2}{(\omega + i\nu_e)^2 - \omega_c^2} |\omega_c| & 1 - \frac{\omega_p^2}{(\omega + i\nu_e)^2 - \omega_c^2} & \frac{\omega + i\nu_e}{\omega} & 0 \\
0 & 0 & 1 - \frac{\omega_p^2}{(\omega + i\nu_e)^2 - \omega_c^2} & \omega
\end{pmatrix}
$$

And the damping rate: $\Gamma_v = i e^*_\alpha e_\beta \omega^2 \varepsilon^{A}_{\alpha\beta}$ is a function of:

$$
e^*_\alpha e_\beta \frac{\partial}{\partial \omega} \omega^2 \varepsilon^{H}_{\alpha\beta} \omega; N_I; \omega_p (n_e); \omega_c (B); \nu_e (T)
$$

The damping exhibits strong frequency dependence.
Collisional damping

The collisional damping rate is much smaller than $\nu_e$ for the low frequency branch of the resonant waves, which makes it easier for the runaway electrons to excite such waves.

Collisional damping rates for waves driven via anomalous Doppler resonance ($\gamma = 20; \omega_p / \omega_c = 0.5$).
Kinetic drive

A function of: \( \omega; N_{\|}; \omega_p(n_e); \omega_c(B); \frac{\partial F_b}{\partial p}(p,\theta); \frac{\partial F_b}{\partial \theta}(p,\theta) \)

\[
\Gamma_b = \frac{2\pi^2 e^2 \omega \int d^3 p \sum_{n=-\infty}^{\infty} Q_n \left[ V \frac{\partial F_b}{\partial p} + \frac{V n\omega_{cb} - \omega \sin^2 \theta}{p} \omega \cos \theta \sin \theta \frac{\partial F_b}{\partial \theta} \right] \delta(\omega - k_\| V \cos \theta - n\omega_{cb})}{(1 + e_2^2) \frac{1}{\omega} \frac{\partial}{\partial \omega} \omega^2 \varepsilon + 2i e_2 \frac{1}{\omega} \frac{\partial}{\partial \omega} \omega^2 g + e_3^2 \frac{1}{\omega} \frac{\partial}{\partial \omega} \omega^2 \eta}
\]

\[
Q_n \equiv \left\{ \frac{n\omega_{cb}}{k_\perp V} J_n + e_3 \cos \theta J_n + i e_2 \sin \theta J'_n \right\}^2
\]

Bessel function argument: \( k_\perp V \sin \theta / \omega_{cb} \);
\( \omega_{cb} \) - beam electron cyclotron frequency;
\( n_{re} = \int F_b d^3 p \)

Polarization vector components:

\[
e_2 = i \frac{|g|}{\varepsilon - N_{\|}^2 - N_{\perp}^2}
\]

\[
e_3 = - \frac{N_{\|} N_{\perp}}{\eta - N_{\perp}^2}
\]

Comparison with earlier experiments

Reported [*] plasma parameters:

\[ B = 3T \]

\[ T_e \approx 10 - 15 \text{eV} \]

Runaway parameters:

\[ E \approx 0.5 - 2.0 \text{MeV} \]

\[ E_\perp \approx 7 - 60 \text{keV} \]

\[ \theta_0 \approx 0.1 - 0.3 \]

Implying:

\[ F_b \sim A \exp\left( -\frac{p}{p_0} - \frac{\theta^2}{\theta_0^2} \right) \]

Calculated and experimental instability threshold.

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The lines represent calculations for different average energies of the runaway electrons (in MeV) indicated next to the lines.

Sample ITER parameters

\[ B = 5.2T \]
\[ j_{re} = 1.2 \, \text{MA} / m^2 \]
\[ E \approx 15 \, \text{MeV} \]
\[ \theta_0 \approx 0.1 \]

Implying exponential spectrum:

\[ F_b \sim A \exp \left( -\frac{p}{p_0} - \frac{\theta^2}{\theta_0^2} \right) \]

The instability window is close to the disruption parameters in ITER. Further steps are required.
Ray tracing equations:

\[
\frac{d\mathbf{k}}{dt} = -\frac{\partial}{\partial \mathbf{r}} \omega(\mathbf{k}; \mathbf{r})
\]

\[
\frac{d\mathbf{r}}{dt} = \frac{\partial}{\partial \mathbf{k}} \omega(\mathbf{k}; \mathbf{r})
\]

Integrating growth rate along the trajectory:

\[
\int \Gamma_b - \Gamma_v \, dt
\]
Internal reflection

The dispersion relations for whistlers and magnetized plasma waves preclude their escape from the plasma in the WKB limit.

\[
\omega = \left| \omega_c \right| \frac{|k_\parallel| k c^2}{\omega_p^2 \sqrt{1 + k^2 c^2 \omega_c^2 / \omega_p^4}} \quad \rightarrow \quad k c / \omega_p^2 = \text{const} \quad \rightarrow \quad n_{\text{reflect}} \geq n_0 |k_\parallel| / k_0
\]
Conversion of whistlers into magnetized waves

\[ \varepsilon N^4 + \varepsilon N^2 \left[ g^2 - (\varepsilon + \eta)(\varepsilon - N^2) \right] - g^2 \eta + \eta \left( \varepsilon - N^2 \right)^2 = 0 \]

The situation, when the discriminant of the dispersion relation vanishes and the two roots coincide does not produce any singularity on the trajectory, but the two branches become indistinguishable at this point.

Whistler wave coming to the zero discriminant location can continue its path as a magnetized plasma wave and vice versa.

The collisional damping for magnetized waves is significantly higher than for whistlers. This conversion may terminate the wave.
The COIN code

- Fortran90
- Modularized
- MPI statistics gathering
- GUI + post analysis (Python+QT4)

The input data:
- EQDSK equilibrium
- Plasma profiles \( (n_e, T_e, j_{re}) \) on \( \psi \) or \( R \)
- Runaway distribution function
  - Arbitrary analytical
  - \( f(p, \lambda) \) data file

The initial coordinates: \( (r, z, \omega, N_{||}, \theta) \)

```fortran
program main
  use MagneticField
  use PlasmaProfiles
  !<...>
  call MF_init_('ITER.EQDSK') !initializing equilibrium
  call PP_init_('ITER.PROF',1.0,1.0,1.0) !initializing plasma profiles
  call MF_Metric_('metric2.dat','eqplot.dat')!metric calculation
  call rgrt(dt0,r0,z0,W_W_c,N_pa,theta,wm,nmax,n_total1,solution1)!launch
  !<...write/analyze solution...>
end program main
```

Physics highlights

What we have learned from the following preliminary analysis:

1. The collisional damping for whistlers is weak $\Gamma \ll \nu_e$;

2. The conversion of whistler wave into magnetized wave can terminate the wave;

3. The runaways electron beam may create the ‘reflection layer’ due to the effects on the plasma profiles;

4. The plasma density in modern machines (DIII-D, JET, ITER, etc) is too high to excite the “fan”-like instability seen in [T-6, T-3, TFR];

5. The results are sensitive to RE distribution function;

6. Ray-tracing (plasma inhomogeneity) effects may shift the instability threshold significantly in ITER.
1) Experimental energy spectrum is different from the avalanche theory [RP].

\[
f(E, \lambda) = A e^{\left( \frac{-E}{4} - \frac{\theta^2}{0.02} \right)}
\]

High energy RE \( \lambda \sim 0.02 \) is not possible for the whole beam.

Most of the beam is isotropic.

2) Avalanche distribution function:

\[
f(p, \lambda) = A e^{\left( -\frac{p}{22.0} - \frac{\lambda}{0.02} \right)}
\]

\[
j_{re} = ceA \int_{0.001}^{100} \beta(E, \lambda) e^{\left( \frac{-E}{4} - \frac{\lambda}{0.02} \right)} E^{0.7} dE d\lambda = 37 \text{ MA} / m^2
\]

[E. Hollmann, Nucl. Fusion 53 (2013)]

Is there an experimentally achievable parameters for instability?

No. With avalanche model spectrum DIII-D is well below the threshold.
High density of resonant electrons suggests lower instability threshold.

However, no magnetized wave at any reasonable plasma parameters.
No magnetized wave at any reasonable plasma parameters due to absorption on Cherenkov resonance.

Experimentally measured distribution function prohibits excitation of the magnetized waves in DIII-D
Statistical analysis

Trajectories diverges after several reflections with small level of fluctuations.

This uncertainty calls for statistical analysis:
Launch many waves at reference temperature >> Calculate drive and damping separately >> locate trajectories with maximum gain $\forall t_1, t_2 \in (t_1 > t_2) \rightarrow E_{\text{max}} = \max \left\{ \int_{t_1}^{t_2} \Gamma dt \right\}$
>> scale damping with temperature >> Find minimal temperature for instability to appear.
ITER Results

Kinetic Drive and Dumping

Growth rate for 22 eV

\[ n_e \sim 1.3 \cdot 10^{20} \text{ m}^{-3} \quad T_e \sim 22\text{ eV} \quad j_{re} \sim 1.0 \text{ MA} / \text{ m}^2 \quad E_{re} \sim 15\text{ MeV} \]

The first wave with positive growth rate appears at plasma core at \( T_e \sim 12\text{ eV} \).

The ray-tracing analysis indicates significantly higher temperature for the instability onset.
Example trajectories of long-living waves
Numerically calculated RE distribution function

A possibility of peaked RE distribution function was reported recently [*]

Moderately peaked distribution function


Strongly peaked distribution function

Increased maximum growth rate.

However, the instability requires a higher plasma temperature.
Summary

- A ray-tracing code have been developed and benchmarked for linear stability analysis of runaway electrons in realistic tokamak geometry.

- Whistlers and magnetized plasma waves are primary candidates for instabilities.

- Radial non-uniformity of the tokamak plasma creates a cavity for these waves.

- DIII-D RE beam is robustly stable to high-frequency kinetically driven modes.

- The results are sensitive to RE distribution function.

- The ray-tracing (plasma inhomogeneity) effects may shift the instability threshold significantly in ITER.