Measurement of the neutral hydrogen density in LHD core plasmas based on the spectral inversion

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Hydrogen transport and recycling

Core

Edge

Wall

Ionization

Recombination

\[ H_2 \rightarrow H + H^+ \]

\[ H^+ + e^- \rightarrow H + e^- \]

\[ H + H^+ \rightarrow H_2 + e^- \]

\[ n_H \text{ (m}^{-3}\text{)} \]

\[ r_{\text{eff}} \text{ (m)} \]

\[ T_p \text{ (keV)} \]

\[ n_H \text{ (m}^{-3}\text{)} \]

\[ r_{\text{eff}} \text{ (m)} \]
Hydrogen transport and recycling
Hydrogen transport in core regions

- Charge exchange
- Ionization
- Recombination

Graph showing $n_H$ (m$^{-3}$) vs. $r_{eff}$ (m)
Hydrogen transport and plasma confinement

Before wall conditioning (Large neutral flux)

After wall conditioning (Small neutral flux)

$n_e (x 10^{19} \text{ m}^{-3})$

$T_i (\text{keV})$

Core

Edge

Wall

Ionization

Recombination

Hydrogen transport in fusion plasmas

Intense edge emission (10^3 times more intense) always overwraps to the core emission
Modelling the Hydrogen transport

Atom transport can be modeled by

- Monte-Carlo method
- Diffusion equation

\[ 0 = \nabla \cdot \left\{ \mu_H \nabla \left( n_H kT_H \right) \right\} - R_{\text{ion}} n_e n_H \]

Diffusion equation for number density of atoms

\[ 0 = \frac{5}{2} \nabla \cdot \left( \mu_H \nabla \left\{ n_H (kT_H)^2 \right\} \right) - \frac{3}{2} R_{\text{ion}} n_e n_H kT_H + \frac{3}{2} R_{\text{CX}} n_{H^+} n_H k(T_{H^+} - T_H) \]

Diffusion equation for energy density of atoms
Boundary conditions make a significant impact on $n_H$ in the core.

Atom transport can be modeled ...

Atom density (& distribution)  
Atom temperature (& distribution)

Boundary conditions make a significant impact on $n_H$ in the core.

It has been difficult to reduce the uncertainty by experiments.
Modelling the Hydrogen transport

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Atom density (& distribution)
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Boundary conditions make a significant impact on $n_H$ in the core.

It has been difficult to reduce the uncertainty by experiments.
Hydrogen transport and plasma confinement

Hydrogen recycling strongly relates to the plasma confinement.
1. Hydrogen atom dynamics in fusion plasmas

2. Spectroscopic approach to detect the atom emission in the core plasmas

3. Forward modeling: \( n_H \) distribution and observed spectrum.

4. Spectral inversion

5. Some applications
1. Hydrogen atom dynamics in fusion plasmas

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5. Some applications
Balmer-$$\alpha$$ spectrum

also observed in tokamaks
Balmer-$\alpha$ spectrum

Atoms generated in core region

Charge exchange
Our first finding:

Atom emission from core region can be separately detected in far wings!
Problem: Lack of the dynamic range

The problem of the conventional spectrometer

Lack of the dynamic range ($< 10^4$).
Development of HDR spectroscopic system

Increase the throughput.

**Suppression of the photon noise**

- Large diameter camera lens
- High efficiency grating (grism)

**Increase in the resolution of the AD conversion by software binning**

- Low noise CMOS camera with high density image sensor
Development of HDR spectroscopic system

\[10^6\] dynamic range is achieved!

It became possible to observe the core emission separately.

**conventional**  
(dynamic range < \(10^4\))

**HDR spectroscopy**  
(dynamic range \(\sim 10^6\))
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High Dynamic Range Balmer-$\alpha$ spectrum

$T_e$

$T_i$

$n_e$ (x10$^{19}$ m$^{-3}$)

$r_{\text{eff}}$ (m)

Intensity (W m$^{-1}$ nm$^{-1}$ sr$^{-1}$)

Wavelength (nm)
Charge exchange location and Spectral line shape

The broadening has information of $T_i$ at the charge exchange location.

The velocity distribution of atoms that experience C.X. at $r$ is Maxwellian at $T_i(r)$. 
Forward modeling

\[ I(\lambda) \cong \sum_{i=1}^{i=30} r_{\text{CX}} n_{H^+}(r_i) n_H(r_i) \frac{\Delta V(r_i)}{2\pi R_{ax}} P_\alpha \sqrt{\frac{m_H}{\pi kT_i(r_i)}} \exp \left[ - \frac{m_H (\lambda - \lambda_0)^2}{2kT_i(r_i)} \right] \]

Number of atoms generated at \( r_i \) by C.X. per unit time

Velocity distribution of the generated atoms is a Maxwellian at \( T_i \)
Forward modeling

\[ I(\lambda) \approx \sum_{i=1}^{i=30} r_{\text{CX}} n_{H^+}(r_i) n_H(r_i) \]

\[ \frac{\Delta V(r_i)}{2\pi R_{\text{ax}}} P_\alpha \sqrt{\frac{m_H}{\pi k T_i(r_i)}} \exp \left[ -\frac{m_H (\lambda - \lambda_0)^2}{2k T_i(r_i)} \right] \]

The spectrum can be calculated by \( T_i \) and \( n_e \).
Inverse problem

Cannot be solved without prior knowledge.

Forward modeling

\[ I_j \approx \sum_{i=1}^{i=30} n_{Hi} K_{ij} \]
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Prior knowledge

Prior knowledge from the diffusion model of atoms

Uniform prior for the boundary condition is approximated from the two extreme cases.

Laplace approximation is made. The results of the two extreme cases are considered as a confidential range.
Evaluation of $n_H$

**Posterior**

$n_H$ (m$^{-3}$)

$r_{\text{eff}}$ (m)

**Prior**

$n_H$ (m$^{-3}$)

$r_{\text{eff}}$ (m)

**Likelihood**

$$
\left( I_j - \sum_{i=1}^{i=30} n_{Hi} K_{ij} \right)^2 \frac{1}{\sigma_j} + \left( \frac{d \log n_{Hi}^{\text{model}} / dr}{D_i} - \frac{d \log n_{Hi} / dr}{D_i} \right)^2
$$
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$n_e$ dependence of $n_H$

In higher $n_e$ plasma, less penetration of H in the core plasma.
Evaluation of $\Gamma_i$ & $\Gamma_e$

$S_{\text{rcycl}} = R_{\text{ion}} \ n_e \ n_H$

$$\Gamma_i = \Gamma_e = \frac{1}{r} \int r S_{\text{rcycl}} \, dr + \frac{1}{r} \int r S_{\text{NBI}} \, dr + \frac{1}{r} \int r \frac{dN_e}{dt} \, dr$$
We have developed an evaluation method of the atom density in the core plasmas.
Future work
The inversion result is consistent with the simulated one.
Evaluation of $n_H$

$$\Phi = \sum_{j} I_{ij}^{\text{obs}} - \sum_{i} n_{Hi} K_{ij} + \lambda^2 \sum_{i} \frac{dn_{Hi}}{dr^2}$$

Observed spectrum

Synthetic spectrum
Hydrogen transport simulation

Conventional spectroscopic system has $\sim 10^4$ dynamic range.

More dynamic range is necessary to observe the core atom emission.
Hydrogen transport simulation

1. Atom trajectories are traced.
Hydrogen transport simulation

1. Atom trajectories are traced.

Charge exchange

\[ T_H \text{ (keV)} \]

\[ n_H \text{ (m}^{-3}\text{)} \]
Hydrogen transport simulation

2. Reconstruction of the emission spectrum

Atom emission from edge region

$T_H$ (keV)

$n_H$ (m$^{-3}$)

$r_{eff}$ (m)

intensity (W sr$^{-1}$ m$^{-2}$ nm$^{-1}$)

wavelength (nm)
Hydrogen transport simulation

2. Reconstruction of the emission spectrum

Atoms generated in core region
Our first finding:

Atom emission from core region can be separately detected in far wings!